Numerical Methods 35006
Computer Lab 10: Numerical Solution to Differential Equations

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1. Create code that uses Euler's method to solve the initial value problem
$$y' = x(1+4y^2) \qquad , y(0) = 0$$
on the interval $[0, 5]$, and starting with a step size $h = 0.1$. Plot your solution.

2. Create python code that solves the problem in Question 1 using the mid-point (a.k.a 2nd-order Runge-Kutta) method, and compare your result with Euler's method for the same step-size.

3. Modify your code from Question 2 into a function `rk2solve` that solves a general initial value problem
$$y' = f(x)$$
using the call
```
x,y = rk2solve(f,x0,y0,xmax,h)
```
where `f` is a function of a single variable, `y0` gives the initial value at point `x0`, and the solution is given as a list of values `y` and `x`, where the value of `x` start at `x0` and end at `xmax`. Test your code with the example from Question 1, then save this function to a new module `myodes.py`.

4. Modify your function from the previous question into a 4th-order Runge-Kutta method called `rk4solve`, called
```
x,y = rk4solve(f,x0,y0,xmax,h)
```
and with the inputs and outputs from the previous question. Test this function using the example above and one additional example, then save it to the the `myodes.py` module.

5. a) Modify your `rk4solve` procedure so that it can solve a system of N equations. Test it on the following linear system:
$$\begin{aligned} y_1' &= y_1^2 + xy_2 - 1 \\ y_2' &= y_1 y_2 - xy_1 \end{aligned}$$
with initial values $y_1(0) = 1$, $y_2(0) = 2$.

   b) Check that your code still works with the single-equation linear system from Q1-4, then save the modified code to your `myodes` module.

6. Transform the 2nd-order linear initial value problem
$$y'' + 4y = 0, \quad y(0) = 1, \quad , y'(0) = 0$$
into a system of first-order initial value problems, and solve it using your `rk4solve` routine. Solve this system exactly and compare the numerical solution with the exact solution.