

# DRAFT – NOT FOR CIRCULATION

## REPORT ON NEW EQUIPMENT FOR COMMUNITY BROADBAND

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### 1. SUMMARY

The Demonstrating Digital Programme of Scottish Government provided a small grant to the University of Edinburgh to test wireless equipment that may be useful in the deployment of rural community broadband. Although most community networks are currently limited not by the speed of their wireless links, but by the available bandwidth, it is to be hoped that this will be improved with the current....

A manufacturer of low-cost wireless distribution hardware has recently released some low cost equipment operating in the 24GHz and 5GHz spectrum. If it lives up to its specifications, it offers an order of magnitude increase in the capacity of current point-to-point links over existing technology that operates in the spectra that are available to com munites. In addition we tested laser (free-space optics) equipment operating in an urban environment but also serving rural communities.

To summarise our findings.

- The 24 GHz equipment could prove useful over relatively short distances (up to 8 km) if licencing permitted use at the power for which it is designed.
- Given the current power limits and available bands in the 5GHz range, the 5GHz equipment probably provides only a minor improvement on what could be achieved by using two or three parallel links using lower cost equipment in different bands. There is, of course, a simplicity/redundancy trade-off. Moreover the 5GHz equipment was sensitive to tidal fading and occasionally failed completely (perhaps due to the presence of marine radar)



FIGURE 1. Scotch mist

- The free-space optics equipment performs, as expected, well over distances of a few hundred metres, but will not be useful for longer links in a rural (Scottish) environment<sup>1</sup>.

The main obstacle to using the 5 and 24 GHz equipment is licencing. Under current regulations the 24GHz is effectively useless, and the 5GHz equipment would be much more useful if (a) the power limits were to be raised and (b) Ofcom were to publish their database of licences in the C band.

## 2. INTRODUCTION

Many community broadband projects require a long-range wireless link for their connection to the Internet. The equipment of choice is currently based on wifi operating in the unlicensed 2.4 or 5GHz spectra. This is cheap: the equipment for a long-distance point-to-point link costs under £400 and can provide bi-directional throughput of about 50Mb/s. While this provides an improvement for the many rural communities that are served by long copper telephone lines, 50Mb/s is no longer adequate for a community of, say, 50 residences. Technically, there is no problem in getting more bandwidth in one of the licenced spectra, but the equipment is more expensive and the additional cost of the licence makes this option unaffordable for small communities.

Recently some new equipment operating at 24GHz has come on the market from Ubiquiti<sup>2</sup>. This is advertised as offering 1.4Gb/s at up to 13km. Although not as cheap (a point-to-point link costs about £3,000) it might present an opportunity for some rural communities to upgrade their service to be competitive with the current fibre based offerings in the UK, assuming they can find an internet connection with that bandwidth. A 5GHz variant is also produced by Ubiquiti claiming comparable speeds at upwards of 50km.

With this in mind, the Scottish Government's Demonstrating Digital programme provided the University of Edinburgh with funds to test this equipment "in the wild". Of course, there is ample evidence that the equipment works, but most of the evidence we have is from installations in urban areas over short distances. How will it perform over longer distances in West Highland weather? And what are the practical problems faced by communities who want to install it?

At the request of the Scottish Government we also evaluated an unrelated technology for use in urban areas – free-space optics. This means signalling by laser through the air instead

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<sup>1</sup>Figure 1 shows the "free-space" conditions on one of the shortest Tegola links. The camera is at one relay and the next relay is about 2km away on the hill in the background.

<sup>2</sup><http://www.ubnt.com/airfiber>

of through fibre-optic cabling. The main question here was, since these devices operate in the visible spectrum, how well do they operate in reduced visibility conditions? Do they operate *well enough* for use in this climate over short distances. Again it is a question of cost. The equipment is expensive (some £8-15k per link) and requires careful mounting and alignment, but compared to the civil engineering cost of running fibre in urban areas, or leasing it, it may be worth it.

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### 3. 24GHz AND 5.8GHz EXPERIMENTS

**3.1. Background.** Before going into an account of the the project, let us look at some of the pros and cons of using this equipment and what is already known about wireless transmission in the 24 GHz spectrum. We have already noted that the equipment is affordable. The advertised throughput of 1.4Gb/s presumably means 700Mb/s in each direction, but that would provide a satisfactory connection for a hundred or so residences. Moreover, transmission in this frequency is much less likely to be affected by tidal reflection (a significant problem in the Highlands and Islands)

There are some significant drawbacks, though.

- In the UK, the 24.050–24.350GHz band is partitioned into three sub-bands<sup>3</sup>, and these devices can use two of them. Of these two, one is reserved for government and amateur radio use and is not allowed for general use and the last is permitted at extremely low power densities ( $1.5 \text{ mW/m}^2$  as opposed to the although the  $13 \text{ W/m}^2$  supported by the equipment. We obtained a “non-operational” licence from Ofcom in order to test the equipment at the advertised power and in both sub-bands.
- Several of the links used by Tegola and related projects are longer than 13km
- Transmission in higher frequencies is adversely affected by high humidity and high temperatures. Scotland benefits from only one of these.
- The Ubiquiti equipment uses substantially more power than their 5GHz offerings – about 40W. This would make it unsuitable for solar and wind-powered relays.

Our initial plan was to test the equipment on existing Tegola relays one is a 6.5km link; the second 15.5km. Although the latter is over the advertised range, even a substantial fraction of the advertised throughput would be useful.

The following is a roughly chronological account of the project. The initial installation was done during a period of very high winds in early January 2014.

**3.2. Power output calculations.** Expanding on the first bullet point above, Ofcom has this to say about the part of the 24GHz radio spectrum used by this equipment in the UK Bandplan<sup>4</sup>:

*Non-government low power devices in the radiolocation services are limited to:*

- (a) Portable and fixed applications between 24.15-24.25 GHz; and
- (b) Mobile applications between 24.25-24.35 GHz on a NIB to the radi-onavigation service

*Power flux-density at 10 metres from the system antenna in the direction of maximum radiation is not to exceed  $1.5 \text{ mW/m}^2$  without approval*

*24.05-24.25 GHz is used by the Amateur service. The part of the allocation between 24.05 and 24.150 GHz may only be used with written consent of the Secretary of State.*

*Home Office/Office of The Scottish Executive for the Emergency Services between 24.05-24.15 GHz*

*ISM apparatus may use the band 24.0-24.25 GHz.*

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<sup>3</sup>Ofcom UK Bandplan

<sup>4</sup>[http://www.ofcom.org.uk/static/archive/ra/publication/ra\\_info/ra365.htm](http://www.ofcom.org.uk/static/archive/ra/publication/ra_info/ra365.htm)

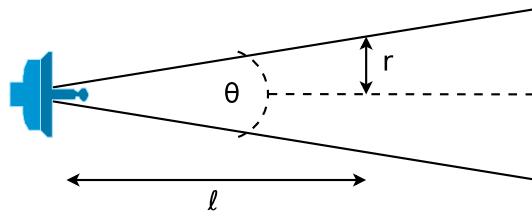


FIGURE 2. Directional radiation cone

Also, the UK Spectrum Strategy 2000 (appendix B)<sup>5</sup> allows this frequency range to be used for “Technology Development” for low-power fixed and portable devices. In this context, “low-power” means less than 1.5 mW/m<sup>2</sup>.

The regulations are confusingly laid out, it would appear from the arrangement of the text that the sentence “The part of the allocation between 24.05 and 24.15 GHz may only be used...” pertains only to the amateur service, but it seems to be a global restriction, particularly since another Ofcom document on Vehicle Mounted Radar Detectors<sup>6</sup> mentions that the police use this part of the band for radar speed meters.

So generally it is allowed to use this equipment in the UK, it is just necessary to check the output power and keep away from the bottom half of the band. The airFibre datasheet<sup>7</sup> gives dBm and the government, gives the permitted radiation energy per unit area at 10 meters from the system. So we need to do a bit of arithmetic.

The datasheet says that the beam-width is less than 3.5°. Let’s make a simplifying assumption that it is exactly that width and further that the cross-section of the main lobe of radiation is circular as depicted in figure 2. The radius of the cross-section will be,

$$(1) \quad r = \ell \sin\left(\frac{\theta}{2}\right) = 10 \text{ m} \times \sin(1.75^\circ) = 0.6 \text{ m}$$

The area of this circle is,  $\pi r^2 = 0.3 \text{ m}^2$ .

So now, how much power goes through one square meter when transmitting at 33 dBm?

$$(2) \quad p = \frac{10^{\frac{1}{10}\beta}}{a} = \frac{10^{\frac{33}{10} \text{ dBm}}}{0.3 \text{ m}^2} = \frac{1995 \text{ mW}}{0.3 \text{ m}^2} = 6.7 \text{ W/m}^2$$

Clearly this is well in excess of the permitted output power.

**3.3. Link budgets.** This experiment is meant to compare the real world performance against the vendor’s claims, but first let us see how the claims stack up against the theoretical performance. To get an upper bound on possible throughput we use the Friis Transmission Equation,

$$(3) \quad P_r = P_t + G_t + G_r + 20 \log\left(\frac{\lambda}{4\pi d}\right)$$

where  $P_t$  and  $P_r$  are the transmitted and received power levels,  $G_t$  and  $G_r$  are the gains of the antennae at either end and the last term describes the path loss due to distance and wavelength. This is an ideal model and doesn’t account for other real-world sources of loss.

We will consider two cases, the usual legal limit and the 33 dBm transmit power. We know the legal limit is 1.5 mW/m<sup>2</sup> so again we need to do some arithmetic using equation 2

<sup>5</sup><http://www.ofcom.org.uk/static/archive/ra/topirum-strat/future/strat00/appendixb.pdf>

<sup>6</sup><http://www.ofcom.org.uk/static/archive/ra/topics/research/rctg/projects/project706.pdf>

<sup>7</sup>[http://www.ubnt.com/downloads/datasheets/airfiber/airFiber\\_DS.pdf](http://www.ubnt.com/downloads/datasheets/airfiber/airFiber_DS.pdf)

in the reverse direction,

$$(4) \quad \frac{10^{\frac{1}{10}\beta}}{0.3 \text{ m}^2} = 1.5 \text{ mW/m}^2$$

which we can re-arrange and solve for  $P_t + G_t = \beta = -3.5 \text{ dBm}$ .

To make use of equation 3 we need some more information. To find the wavelength, we use,

$$(5) \quad \lambda = \frac{c}{f} = \frac{299,792,458 \text{ m/s}}{24 \times 10^9 \text{ Hz}} = 0.0125 \text{ m}$$

where  $c$  is the speed of light. We also look up in the datasheet and see that the receive gain,  $G_r$  is 23 dB.

So as a first calculation, let us see what receive power can be expected at 13 km,

$$\begin{aligned} P_r &= -3.5 \text{ dBm} + 23 \text{ dB} + 20 \log\left(\frac{0.0125 \text{ m}}{4\pi \times 13,000 \text{ m}}\right) \\ &= -3.5 + 23 - 142 = -122 \text{ dBm} \end{aligned}$$

Referring again to the data sheet, we find that the weakest signal that can sustain a connection (at about 3 Mbps since we would need to use 10 MHz wide channels – the datasheet is misleading on this point since it only quotes throughput numbers for 50 MHz wide channels) is  $-95 \text{ dBm}$ . At the legal limit we are 27 dB or about 500 times weaker than we need to be to sustain a link at the advertised distance. Clearly transmitting at 33 dBm with our non-operational licence this is feasible, indeed we have 12 dB margin to spare which, again referring to the datasheet, should net us 128 Mbps over this distance.

So let us ask another question. Over what distance is it possible to sustain the maximum advertised throughput? For this, according to the datasheet, we need a receive power of at least  $P_r = -58 \text{ dBm}$ . Solving equation 3 for the distance,  $d$ , we get,

$$(6) \quad d = \frac{\lambda}{4\pi} 10^{\frac{1}{20}(-P_r + P_t + G_r + G_t)}$$

which works out to all of 7.5 m at the regulatory limit and 500 m at 33 dBm.

In our case we are planning to run this on links at most 6 km where the same calculations say that we can expect a receive signal level of around  $-79 \text{ dBm}$  which means that using 40 MHz wide channels we should be able to solidly sustain links at 256 Mbps or better for shorter links.

**3.4. Initial configuration and testing.** We ordered and received one pair of radios. Before deploying them we thought it would be a good idea to check that they were working and test them in ideal situation – our office corridor. One thing we immediately noticed was how critical alignment is. Even over a distance of 35m, the performance fell off dramatically if the antennae were slightly out of alignment. It's a very good idea to configure equipment before deploying it, but to do this we had to turn off synchronisation which relies on GPS and doesn't work indoors.



**3.5. Strengthening the masts.** Our basic relay construction (see the [relevant howto on the Tegola web site](#)) uses aluminium pegs to anchor the diagonal braces to the ground. Both sites were on terrain that consisted of bedrock covered by peat of varying depth. Although we have never had a problem with the pegs shifting, peat is a bit jelly-like, and the structures can wobble through a cm or two. The alignment of 24GHz is much more critical than for the lower bandwidths of 2.4 and 5.8GHz, so we replaced the pegs with epoxy bolts into the bedrock.



FIGURE 3. Pegged anchors (left) and epoxied anchors (right).

We also strengthened both relays. For example, at one of our relays we added an extra horizontal bar.



FIGURE 4. Corran mast before (left) and after (right)

**3.6. Installation and alignment.** The radios are reasonably light (under 10kg) but awkward to carry up hills. We used an old backpack frame that 30 years ago had been used for carrying batteries up to community TV relays. The radios come with a mounting frame that is first attached to the structure. The radio is then “slotted” into the mounting frame. This arrangement makes it quite easy to install the whole assembly when working from a ladder.

The antenna can be aligned through elevation and azimuth adjustment screws. Unfortunately there is a great deal of backlash in these screws, and they are almost useless if you are working in high winds. If the clamping bolts are loose, the antenna is blown around through the considerable travel allowed by the adjustment screws. The signal strength read-out is at the bottom of the antenna, and if the alignment is being done from a ladder, you almost certainly need someone below (with a hard hat) to squint up and call out the figures.

The installation instructions recommend an alternating process in which one end of the link is adjusted then the other, and so on. Unfortunately we were unable to complete this process before the weather closed in and our workforce departed. However, the alignment is good enough that we can start taking some measurements. The initial indications are that the link will work reasonably well over a distance of 6.5km.

### 3.7. Performance.

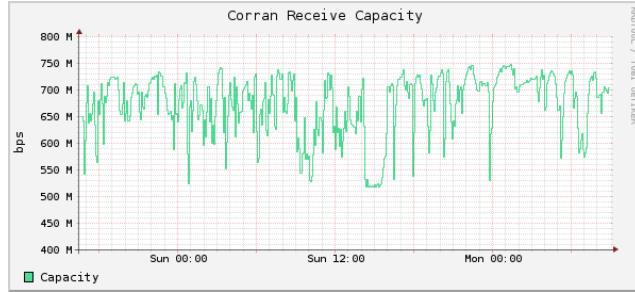


FIGURE 5. Corran receive capacity...

**3.8. Tidal Fading.** Tidal fading has been a serious problem in over water links that use to the 2.4 and 5.8 GHz ranges. They are less of a problem with higher frequencies (e.g. 24GHz) in which the Fresnel ellipsoids and beamwidth are narrower<sup>8</sup>. The 5GHz Airfiber link operates over a distance of approximately 20.8km with the two ends at heights of 300m and 20m above MSL. We have seen some tidal fading on normal 5GHz transmission over this link and disruptive fading over the same stretch of water when both ends are at heights of approximately 20m.

In January 2015 we had a failure of our the operating link and, since the weather prohibited climbing up to the 300m high relay, we switched to the experimental 5GHz Airfiber equipment. Figure shows the received signal strengths on both polarizations. The classic quasi-cycloidal pattern is discernable in both plots, however it is in a range over with dips to -78 dBm, which might cause problems. However these extreme points are probably due to other causes. Examination of the RX capacity (see Figure ?? indicates that other effects may dominate tidal fading and that other things being equal, tidal fading is likely to be less of a problem on these links than it is on conventional 5GHz equipment. Readings were taken at 10 minute intervals from mid-January to mid-February

Tidal fading, or tidal multi-path interference is a known problem in radiofrequency engineering. It happens when the direct signal from a transmitting antenna and an indirect or reflected signal both arrive at the receiving antenna with opposite phase and cancel each other out. The degree to which this happens depends on how long the reflected path is, the direct path being fixed. Since in this case the reflections are caused by the water, as the tide rises and falls the reflected path gets longer and shorter. At certain points in the water's travel, the signals cancel out nearly completely – from the plots the loss from this effect is 12dB or a signal around 16 weaker than it would normally be.

The solution is to have multiple antennae spaced so that when the signal reaching one is being destructively interfered with, the other has a good signal and vice-versa. The picture to the right shows an inexpensive arrangement that implements this. It uses the standard 5GHz rocket radios from Ubiquiti and a non-standard arrangement with two aftermarket antennae. This has been found to work very well. Unfortunately such a solution is not possible with the AirFiber units because of their construction – the radios and the antennae are integrated into a single housing, making it impossible to



<sup>8</sup>our 24GHz link is not over water

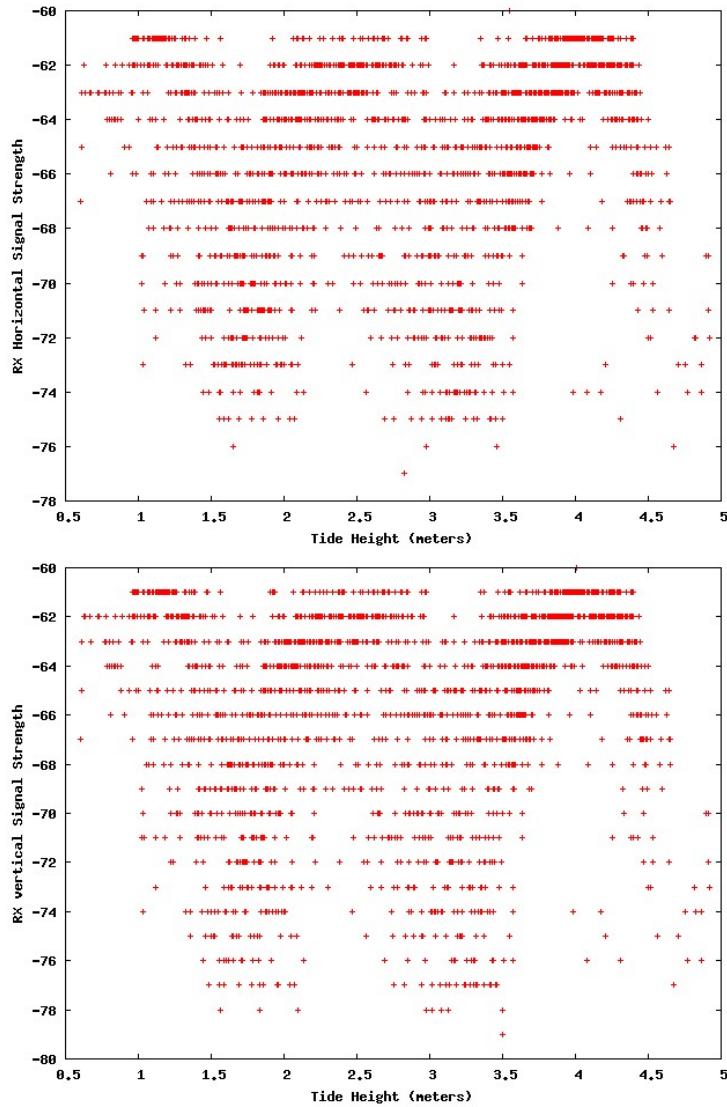


FIGURE 6. Scatter plot of signal strength against sea level for horizontal (top) and vertical polarizations

use different antennae. Nevertheless the AirFibre would be useful for links that do not traverse tidal bodies of water.

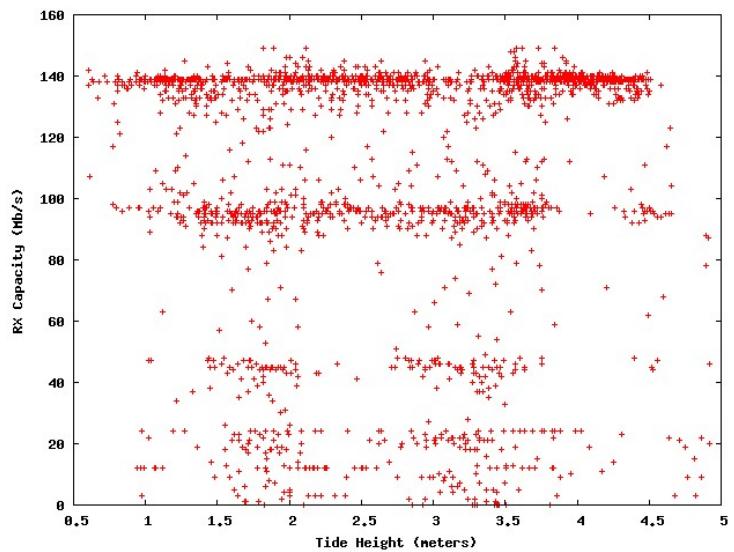


FIGURE 7. Scatter plot of RX channel capacity against sea level for horizontal (left) and vertical polarizations



#### 4. FREE-SPACE OPTICS

The purpose of the trials with the free-space optical (FSO) links was to ascertain the extent to which fog, a common feature of the Scottish environment especially in coastal areas, would affect their use. Obviously fog would attenuate the signal to some extent since the units operate in the near-infrared spectrum. We can quantify this effect theoretically as we do in §4.2 and §4.3, but we were primarily interested in finding out if their operational window was, on average, large enough to justify the considerable expense.

The Scottish Government had a preferred vendor, CableFree, for the equipment though we were formally free to select another. Our budget was only about £10,000 for the equipment. The FSO vendors that are well-regarded in industry such as F-Sona and Canon do not produce anything in this price range. Small wonder as their primary market is the military – optical links are much harder to eavesdrop upon compared to radio links. Two vendors were within our price range: Geodesy of Hungary and Cablefree.

We selected the suggested vendor partly because they were a local (i.e. UK-based) company on the theory that they might be more responsive and accessible. We were also given to believe – with no evidence – that their Hungarian competitor used inferior, older technology. It is difficult to tell the extent to which this was true without comparing the equipment from both manufacturers side by side. We were subjected to the hard-sell and given very little concrete information without signing non-disclosure agreements that would prevent our conducting any useful research. As it was, not executing the NDAs prevented us from properly instrumenting the equipment and measuring its behaviour at any but very coarse granularity.

The whole episode was without a doubt the most unpleasant interaction with an equipment vendor in our considerable experience.

**4.1. Link Design.** CableFree produced a link design as shown in Figure 8. In principle it is plausible, consisting of a laser link, and a back-up radio link between Appleton Tower and the Tech Cube, a distance of about 500m. At either end two switches would use the spanning-tree protocol (STP) configured with the radio link having a higher cost than the laser link. The principle being that traffic would flow over the lasers unless they were unavailable in which case the backup link would be used.

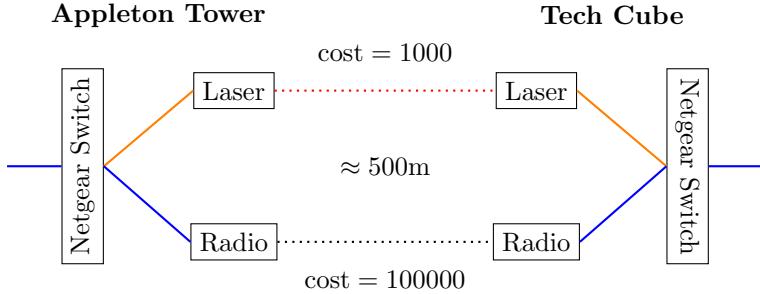


FIGURE 8. CableFree Link Design

There were, however, several problems with this design. The less serious was that this design was hardly appropriate for the local environment. We already had a significant amount of radio equipment at both sites – including a radio link between them – so an extra radio link was superfluous. The radios specified by the vendor were low-end Mikrotik radios with panel antennae in a waterproof enclosure. This is not to cast aspersions on Mikrotik equipment, indeed we make much use of it and it is generally quite good for the price. However Cablefree wished to sell this re-branded equipment at a significant markup.

Most amusingly, the mounting brackets supplied with the re-branded Mikrotik radios were somewhat flimsy. It can get very windy at the top of tall buildings in Edinburgh. Wind speeds can reach several times more than at ground level. The installation is shown at right with the mounting bracket having worked loose in high winds leaving the radio pointing downwards.

We also had high end HP ProCurve J9050A switches at either end (loaned by the School of Informatics), far more capable network elements than the consumer-grade Netgear switches (see Figure 9) specified by the vendor. The deficiencies of the Netgear switch are that it is not manageable via telnet or ssh, which makes it very difficult to recover from outages or erroneous configurations in a network of any size or complexity, and more seriously, the cheap power adapter of the kind that are prone to failure or simply disconnection due to the use of barrel connectors. The power injector for the re-branded Mikrotik radio also suffers from this deficiency (the white apparatus connected into the switch in the photograph).



Unfortunately CableFree wished to sell a “complete solution” for us to evaluate rather than an “appropriate solution” for our circumstances and we succumbed to their hard-sell, “complete” with sub-standard parts. We ultimately ended up operating the link by connecting the vendor-supplied netgear switches to our HP switching core and removing the vendor-supplied radio equipment entirely.

The more serious deficiency with this design is the use of the spanning-tree protocol. We will have more to say about the detail of this below, but in brief, the mechanism is unstable in marginal conditions. If the weather is clear or if the weather is very foggy, the link operates as designed. However in the region between clear and very foggy, the path is liable to rapidly change between the (possibly unuseable) optical link and the radio link. Evidence for this is anecdotal due to the impossibility of properly instrumenting the optical equipment, of which also more below.



FIGURE 9. A “carrier-grade” ethernet switch.

**4.2. Absorption of light by water.** Unlike electromagnetic waves at radio frequencies which are not significantly attenuated by atmospheric gasses, in the visible and near-infrared part of the spectrum this effect plays a major role. We all know from our experience that this is the case, for example this effect explains why objects under water take on a blue-green tinge – because the red part of the light reflected from them is attenuated more by water than blue or green light. In fact we have data for this which is shown plotted in Figure 10 which ranges over wavelengths from the ultraviolet (circa 380nm) to the near-infrared (circa 800nm).

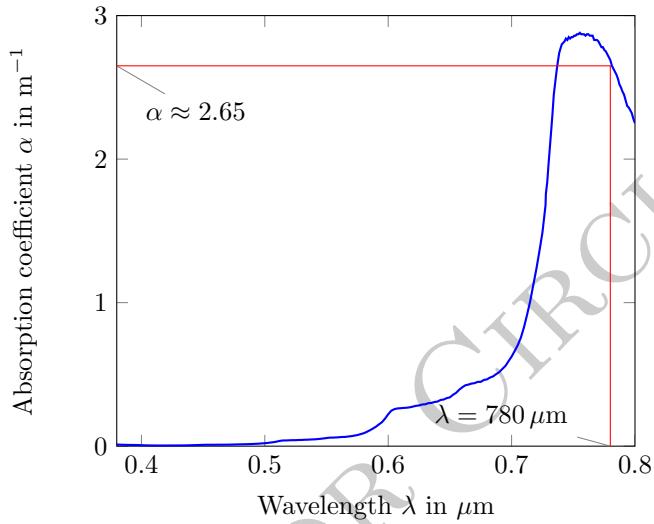


FIGURE 10. Absorption of light by frequency through liquid water (Jonasz 2007)

A line is marked on the diagram that shows the particular wavelength of interest for the free-space optical equipment, 780nm. This corresponds to an absorption coefficient of around  $2.65/\text{m}^{-1}$  which means that through liquid water, light at this wavelength decreases in intensity by 2.65 times for every meter of water that it travels through.

We are not, however, intending to operate this equipment under water. Instead we are interested in water vapour in the form of clouds or fog, so we need to figure out how much water vapour is contained in a given amount of air when a cloud is present. We will take cloud formation to mean (greater than) 100% relative humidity. This means that the vapour pressure of water in the air is greater than the equilibrium vapour pressure at which it evaporates and condenses at equal rates. In fact it condenses faster than it evaporates and so the air becomes full of condensed water droplets, or in other words a cloud forms. The formula for this pressure commonly used in the literature is given by Buck 1981 as,

$$(7) \quad e_w = (1.0007 + 3.46 \times 10^{-6} P) \times 6.1121 e^{\frac{17.502T}{240.97+T}}$$

with the pressure  $P$  given in hPa and the temperature in  $^{\circ}\text{C}$ . This relation is plotted in figurefig:water-pressure for a pressure of one atmosphere.

Since this is the minimum water vapour pressure required to form a cloud, we may take this as a lower bound on the amount of water in the air. At  $10^{\circ}\text{C}$ , Buck's relation gives an equilibrium pressure of 1.23 kPa. Recalling our ideal gas law from high school,

$$(8) \quad PV = nRT$$

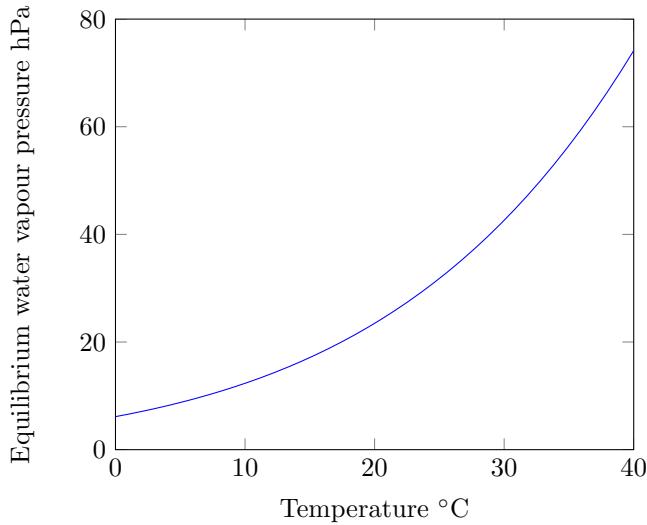


FIGURE 11. Water vapour density as a function of pressure

we can work out that,

$$n = \frac{PV}{RT} = \frac{1.23 \times 10^3 \times 1}{8.314 \times 283} = 0.528 \text{ mol}$$

we can look up the molar mass of water, which is 18.02g/mol, and so arrive at a density of water in the cloud as it is forming of 9.01g/m<sup>3</sup>.

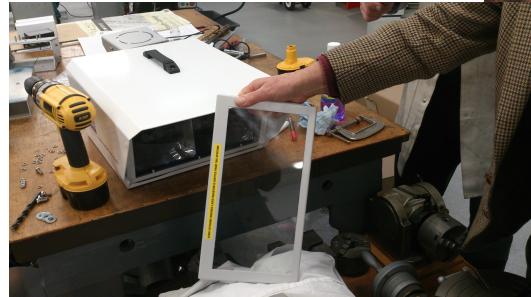
So what? Well, we know that, roughly, liquid water has a density of 1g/cm<sup>3</sup> and in terms of cubic meters this is  $1 \times 10^6 \text{ g/m}^3$ . So this tells us that we should scale the absorption factor for water by  $9.01 \times 10^{-6}$  at the point of cloud formation, which gives an adjusted absorption factor of  $2.39 \times 10^{-5}$ . Over the course of our 500m path, this means that loss due to absorption (heating of the cloud) is only about 1%.

In practice, this value is an overestimate, owing primarily to the fact that water vapour does not behave as an ideal gas near the phase transition from vapour to water (condensation). Experimental measurements (Whiteman and Melfi 1999) and more sophisticated models (Tampieri and Tomasi 1976; Hess, Koepke and Schult 1998) give values an order of magnitude lower for the density of water in fog.

**4.3. Scattering of light by water.** Another effect is scattering...

4.4. **Fittings.** Story about trials and tribulations of getting them mounted...

And also how they had to have the glass screwed down because it came off very easily...



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