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Access the code, data, and analysis at https://github.com/tegorman13/Dissertation

Dissertation

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ABSTRACT In project 1, we applied model-based techniques to quantify and control for the similarity between training and testing experience, which in turn enabled us to account for the difference between varied and constant training via an extended version of a similarity based generalization model. In project 2, we will go a step further, implementing a full process model capable of both 1) producing novel responses and 2) modeling behavior in both the learning and testing stages of the experiment. Project 2 also places a greater emphasis on extrapolation performance following training - as varied training has often been purported to be particularly beneficial in such situations.

KEYWORDS Learning Generalization; Function Learning; Visuomotor learning; Training Variability

Project 1

Abstract

Exposing learners to variability during training has been demonstrated to improve performance in subsequent transfer testing. Such variability benefits are often accounted for by assuming that learners are developing some general task schema or structure. However much of this research has neglected to account for differences in similarity between varied and constant training conditions. In a between-groups manipulation, we trained participants on a simple projectile launching task, with either varied or constant conditions. We replicate previous findings showing a transfer advantage of varied over constant training. Furthermore, we show that a standard similarity model is insufficient to account for the benefits of variation, but, if the model is adjusted to assume that varied learners are tuned towards a broader generalization gradient, then a similarity-based model is sufficient to explain the observed benefits of variation. Our results therefore suggest that some variability benefits can be accommodated within instance-based models without positing the learning of some schemata or structure.

Introduction

The past century of research on human learning has produced ample evidence that although learners can improve at almost any task, such improvements are often specific to the trained task, with unreliable or even nonexistent transfer to novel tasks or conditions (Barnett & Ceci, 2002; Detterman, 1993). Such transfer challenges are of noteworthy practical relevance, given that educators, trainers, and rehabilitators typically intend for their students to be able to apply what they have learned to new situations. It is therefore important to better understand the factors that influence transfer, and to develop cognitive models that can predict when transfer is likely to occur. The factor of interest to the present investigation is variation during training. Our experiments add to the longstanding empirical investigation of the controversial relationship between training variation, and subsequent transfer. We also offer a novel explanation for such results in the form of an instance-based model that accounts for the benefits of variation in simple terms of psychological similarity. We first review the relevant concepts and literature.

Similarity and instance-based approaches to transfer of learning

Notions of similarity have long played a central role in many prominent models of generalization of learning, as well as in the longstanding theoretical issue of whether learners abstract an aggregate, summary representation, or if they simply store individual instances. Early models of learning often assumed that discrete experiences with some task or category were not stored individually in memory, but instead promoted the formation of a summary representation, often referred to as a prototype or schema, and that exposure to novel examples would then prompt the retrieval of whichever preexisting prototype was most similar (Posner & Keele, 1968). Prototype models were later challenged by the success of instance-based or exemplar models – which were shown to provide an account of generalization as good or better than prototype models, with the advantage of not assuming the explicit construction of an internal prototype (Estes, 1994; Hintzman, 1984; Medin & Schaffer, 1978; Nosofsky, 1986). Instance-based models assume that learners encode each experience with a task as a separate instance/exemplar/trace, and that each encoded trace is in turn compared against novel stimuli. As the number of stored instances increases, so does the likelihood that some previously stored instance will be retrieved to aid in the performance of a novel task. Stored instances are retrieved in the context of novel stimuli or tasks if they are sufficiently similar, thus suggesting that the process of computing similarity is of central importance to generalization.

Similarity, defined in this literature as a function of psychological distance between instances or categories, has provided a successful account of generalization across numerous tasks and domains. In an influential study demonstrating an ordinal similarity effect, experimenters employed a numerosity judgment task in which participants quickly report the number of dots flashed on a screen. Performance (in terms of response times to new patterns) on novel dot configurations varied as an inverse function of their similarity to previously trained dot configurations Palmeri (1997). That is, performance was better on novel configurations moderately similar to trained configurations than to configurations with low-similarity, and also better on low-similarity configurations than to even less similar, unrelated configurations. Instance-based approaches have had some success accounting for performance in certain sub-domains of motor learning (Cohen & Rosenbaum, 2004; Crump & Logan, 2010, 2010; Meigh et al., 2018; Poldrack et al., 1999; Wifall et al., 2017) trained participants to type words on an unfamiliar keyboard, while constraining the letters composing the training words to a pre-specified letter set. Following training, typing speed was tested on previously experienced words composed of previously experienced letters; novel words composed of letters from the trained letter set; and novel words composed of letters from an untrained letter set. Consistent with an instance-based account, transfer performance was graded such that participants were fastest at typing the words they had previously trained on, followed by novel words composed of letters they had trained on, and slowest performance for new words composed of untrained letters.

The effect of training variability on transfer

While similarity-based models account for transfer by the degree of similarity between previous and new experiences, a largely separate body of research has focused on improving transfer by manipulating characteristics of the initial training stage. Such characteristics have included training difficulty, spacing, temporal order, feedback schedules, and the primary focus of the current work – variability of training examples.

Research on the effects of varied training typically compares participants trained under constant, or minimal variability conditions to those trained from a variety of examples or conditions (Czyż, 2021; Soderstrom & Bjork, 2015). Varied training has been shown to influence learning in myriad domains including categorization of simple stimuli (Hahn et al., 2005; Maddox & Filoteo, 2011; Posner & Keele, 1968), complex categorization (Nosofsky et al., 2018), language learning (S. D. Jones & Brandt, 2020; Perry et al., 2010; Twomey et al., 2018; Wonnacott et al., 2012), anagram completion (Goode et al., 2008), trajectory extrapolation (Fulvio et al., 2014), task switching (Sabah et al., 2019), associative learning (Lee et al., 2019), visual search (George & Egner, 2021; Gonzalez & Madhavan, 2011; Kelley & Yantis, 2009), voice identity learning (Lavan et al., 2019), simple motor learning (Braun et al., 2009; Kerr & Booth, 1978; Roller et al., 2001; Willey & Liu, 2018a), sports training North et al. (2019), and training on a complex video game (Seow et al., 2019).

Training variation has received a particularly large amount of attention within the domain of visuomotor skill learning. Much of this research has been influenced by the work of Schmidt (1975), who proposed a schema-based account of motor learning as an attempt to address the longstanding problem of how novel movements are produced. According to Schema Theory, learners possess general motor programs for classes of movements (e.g. throwing a ball with an underhand movement), as well as schema rules that determine how a motor program is parameterized or scaled for a particular movement. Schema theory predicts that varied training results in the formation of a more general schema-rule, which can allow for transfer to novel movements within a given movement class. Experiments that test this hypothesis are often designed to compare the transfer performance of a constant-trained group against that of a varied-trained group. Both groups train on the same task, but the varied group practices from multiple levels of a task-relevant dimension that remains invariant for the constant group. For example, investigators might train two groups of participants to throw a projectile at a target, with a constant group that throws from a single location, and a varied group that throws from multiple locations. Both groups are then tested from novel locations. Empirically observed benefits of the varied-trained group are then attributed to the variation they received during training, a finding observed in numerous studies (Catalano & Kleiner, 1984; Chua et al., 2019; Goodwin et al., 1998; Kerr & Booth, 1978; Wulf, 1991), and the benefits of this variation are typically thought to be mediated by the development of a more general schema for the throwing motion.

Of course, the relationship between training variability and transfer is unlikely to be a simple function wherein increased variation is always beneficial. Numerous studies have found null, or in some cases negative effects of training variation (DeLosh et al., 1997; Sinkeviciute et al., 2019; Wrisberg et al., 1987), and many more have suggested that the benefits of variability may depend on additional factors such as prior task experience, the order of training trials, or the type of transfer being measured (Berniker et al., 2014; Braithwaite & Goldstone, 2015; Hahn et al., 2005; Lavan et al., 2019; North et al., 2019; Sadakata & McQueen, 2014; Zaman et al., 2021).

Issues with Previous Research

Although the benefits of training variation in visuomotor skill learning have been observed many times, null findings have also been repeatedly found, leading some researchers to question the veracity of the variability of practice hypothesis (Newell, 2003; Van Rossum, 1990). Critics have also pointed out that investigations of the effects of training variability, of the sort described above, often fail to control for the effect of similarity between training and testing conditions. For training tasks in which participants have numerous degrees of freedom (e.g. projectile throwing tasks where participants control the x and y velocity of the projectile), varied groups are likely to experience a wider range of the task space over the course of their training (e.g. more unique combinations of x and y velocities). Experimenters may attempt to account for this possibility by ensuring that the training location(s) of the varied and constant groups are an equal distance away from the eventual transfer locations, such that their training throws are, on average, equally similar to throws that would lead to good performance at the transfer locations. However, even this level of experimental control may still be insufficient to rule out the effect of similarity on transfer. Given that psychological similarity is typically best described as either a Gaussian or exponentially decaying function of psychological distance (Ennis et al., 1988; Ghahramani et al., 1996; Logan, 1988; Nosofsky, 1992; Shepard, 1987; Thoroughman & Taylor, 2005), it is plausible that a subset of the most similar training instances could have a disproportionate impact on generalization to transfer conditions, even if the average distance between training and transfer conditions is identical between groups. Figure 1 demonstrates the consequences of a generalization gradient that drops off as a Gaussian function of distance from training, as compared to a linear drop-off.

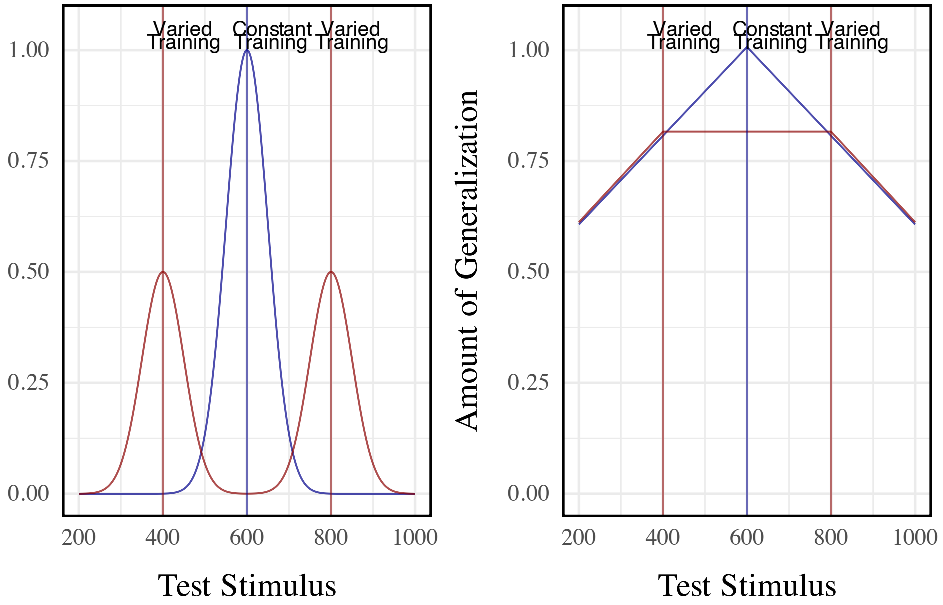


Figure 1: Left panel- Generalization predicted from a simple model that assumes a linear generalization function. A varied group (red vertical lines indicate the 2 training locations) trained from positions 400 and 800, and a constant group (blue vertical line), trained from position 600. Right panel- if a Gaussian generalization function is assumed, then varied training (400, 800) is predicted to result in better generalization to positions close to 400 and 800 than does constant training at 600. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In addition to largely overlooking the potential for non-linear generalization to confound interpretations of training manipulations, the visuomotor skill learning literature also rarely considers alternatives to schema representations (Chamberlin & Magill, 1992b). Although schema-theory remains influential within certain literatures, instance or exemplar-based models have accounted for human behavior across myriad domains (Jamieson et al., 2022; Logan, 2002). As mentioned above, instance based accounts have been shown to perform well on a variety of different tasks with motoric components (Crump & Logan, 2010; Gandolfo et al., 1996; Meigh et al., 2018; Rosenbaum et al., 1995; van Dam & Ernst, 2015). However, such accounts have received little attention within the subdomain of visuomotor skill learning focused on the benefits of varied training.

The present work examines whether the commonly observed benefits of varied training can be accounted for by between-group differences in similarity between training and testing throws. We first attempt to replicate previous work finding an advantage of varied training over constant training in a projectile launching task. We then examine the extent to which this advantage can be explained by an instance-based similarity model.

Experiment 1

Methods

Sample Size Estimation To obtain an independent estimate of effect size, we identified previous investigations which included between-subjects contrasts of varied and constant conditions following training on an accuracy based projectile launching task (Chua et al., 2019; Goodwin et al., 1998; Kerr & Booth, 1978; Wulf, 1991). We then averaged effects across these studies, yielding a Cohens f =.43. The GPower 3.1 software package (Faul et al., 2009), 2009) was then used to determine that a power of 80% requires a sample size of at least 23 participants per condition. All experiments reported in the present manuscript exceed this minimum number of participants per condition.

Participants Participants were recruited from an undergraduate population that is 63% female and consists almost entirely of individuals aged 18-22 years. A total of 110 Indiana University psychology students participated in Experiment 1. We subsequently excluded 34 participants poor performance at one of the dependent measures of the task (2.5-3 standard deviations worse than the median subject at the task) or for displaying a pattern of responses that was clearly indicative of a lack of engagement with the task (e.g. simply dropping the ball on each trial rather than throwing it at the target), or for reporting that they completed the experiment on a phone or tablet device, despite the instructions not to use one of these devices. A total of 74 participants were retained for the final analyses, 35 in the varied group and 39 in the constant group.

Task The experimental task was programmed in JavaScript, using packages from the Phaser physics engine (https://phaser.io) and the jsPsych library (de Leeuw, 2015). The stimuli, presented on a black background, consisted of a circular blue ball – controlled by the participant via the mouse or trackpad cursor; a rectangular green target; a red rectangular barrier located between the ball and the target; and an orange square within which the participant could control the ball before releasing it in a throw towards the target. Because the task was administered online, the absolute distance between stimuli could vary depending on the size of the computer monitor being used, but the relative distance between the stimuli was held constant. Likewise, the distance between the center of the target, and the training and testing locations was scaled such that relative distances were preserved regardless of screen size. For the sake of brevity, subsequent mentions of this relative distance between stimuli, or the position where the ball landed in relation to the center of the target, will be referred to simply as distance. Figure 2 displays the layout of the task, as it would appear to a participant at the start of a trial, with the ball appearing in the center of the orange square. Using a mouse or trackpad, participants click down on the ball to take control of the ball, connecting the movement of the ball to the movement of the cursor. Participants can then "wind up" the ball by dragging it (within the confines of the orange square) and then launch the ball by releasing the cursor. If the ball does not land on the target, participants are presented with feedback in red text at the top right of the screen, on how many units away they were from the center of the target. If the ball was thrown outside of the boundary of the screen participants are given feedback as to how far away from the target center the ball would have been if it had continued its trajectory. If the ball strikes the barrier (from the side or by landing on top), feedback is presented telling participants to avoid hitting the barrier. If participants drag the ball outside of the orange square before releasing it, the trial terminates, and they are reminded to release the ball within the orange square. If the ball lands on the target, feedback is presented in green text, confirming that the target was hit, and presenting additional feedback on how many units away the ball was from the exact center of the target.

Link to abbrevaited example of task.

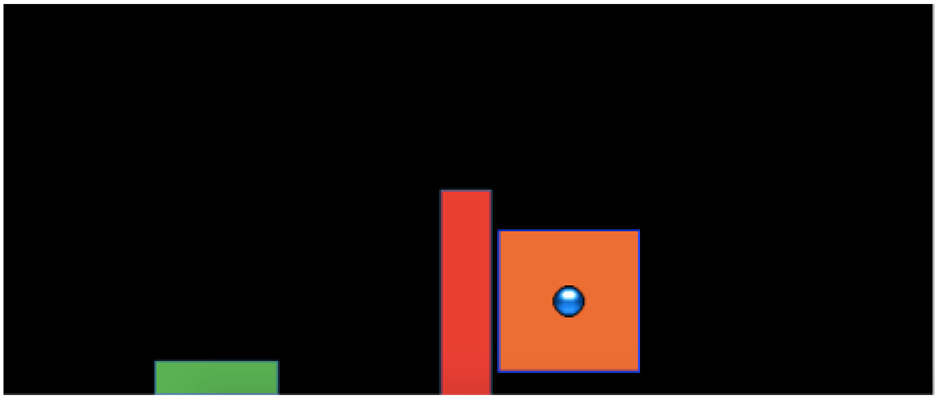


Figure 2: The stimuli of the task consisted of a blue ball, which the participants would launch at the green target, while avoiding the red barrier. On each trial, the ball would appear in the center of the orange square, with the position of the orange square varying between experimental conditions. Participants were constrained to release the ball within the square

Results

Data Processing and Statistical Packages To prepare the data, we first removed trials that were not easily interpretable as performance indicators in our task. Removed trials included: 1) those in which participants dragged the ball outside of the orange starting box without releasing it, 2) trials in which participants clicked on the ball, and then immediately released it, causing the ball to drop straight down, 3) outlier trials in which the ball was thrown more than 2.5 standard deviations further than the average throw (calculated separately for each throwing position), and 4) trials in which the ball struck the barrier. The primary measure of performance used in all analyses was the absolute distance away from the center of the target. The absolute distance was calculated on every trial, and then averaged within each subject to yield a single performance score, for each position. A consistent pattern across training and testing phases in both experiments was for participants to perform worse from throwing positions further away from the target – a pattern which we refer to as the difficulty of the positions. However, there were no interactions between throwing position and training conditions, allowing us to collapse across positions in cases where contrasts for specific positions were not of interest. All data processing and statistical analyses were performed in R version 4.03 (R Core Team, 2020). ANOVAs for group comparisons were performed using the rstatix package (Kassambara, 2021)****.

Training Phase Figure 3 below shows aggregate training performance binned into three stages representing the beginning, middle, and end of the training phase. Because the two conditions trained from target distances that were not equally difficult, it was not possible to directly compare performance between conditions in the training phase. Our focus for the training data analysis was instead to establish that participants did improve their performance over the course of training, and to examine whether there was any interaction between training stage and condition. Descriptive statistics for the intermittent testing phase are provided in the supplementary materials.

We performed an ANOVA comparison with stage as a within-group factor and condition as between-group factor. The analysis revealed a significant effect of training stage F(2,142)=62.4, p<.001, η^2_G = .17, such that performance improved over the course of training There was no significant effect of condition F(1,71)=1.42, p=.24, η^2_G = .02, and no significant interaction between condition and training stage, F(2,142)=.10, p=.91, η^2_G < .01.

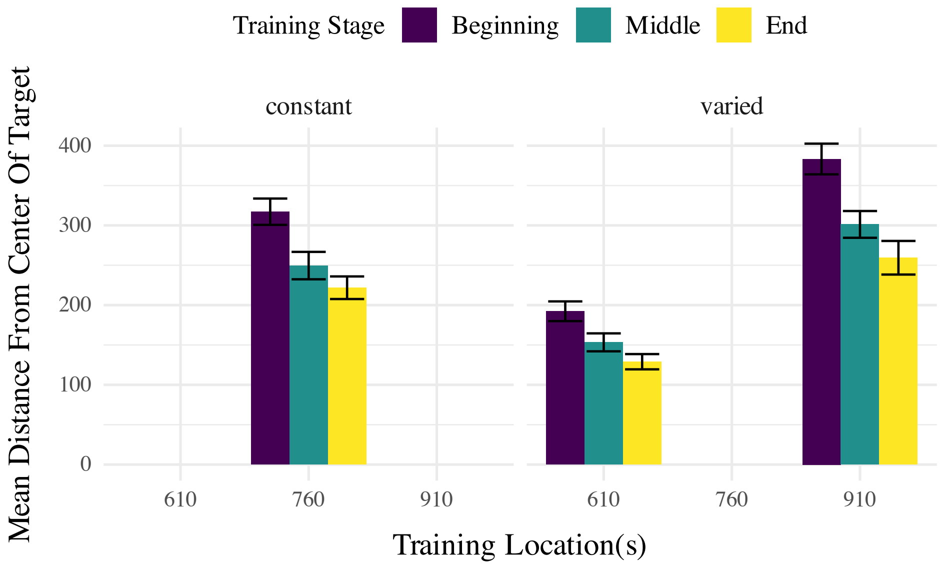


Figure 3: Training performance for varied and constant participants binned into three stages. Shorter bars indicate better performance (ball landing closer to the center of the target). Error bars indicate standard error of the mean.

Testing Phase

In Experiment 1, a single constant-trained group was compared against a single varied-trained group. At the transfer phase, all participants were tested from 3 positions: 1) the positions(s) from their own training, 2) the training position(s) of the other group, and 3) a position novel to both groups. Overall, group performance was compared with a mixed type III ANOVA, with condition (varied vs. constant) as a between-subject factor and throwing location as a within-subject variable. The effect of throwing position was strong, F(3,213) = 56.12, p<.001, η2G = .23. The effect of training condition was significant F(1,71)=8.19, p<.01, η2G = .07. There was no significant interaction between group and position, F(3,213)=1.81, p=.15, η2G = .01.

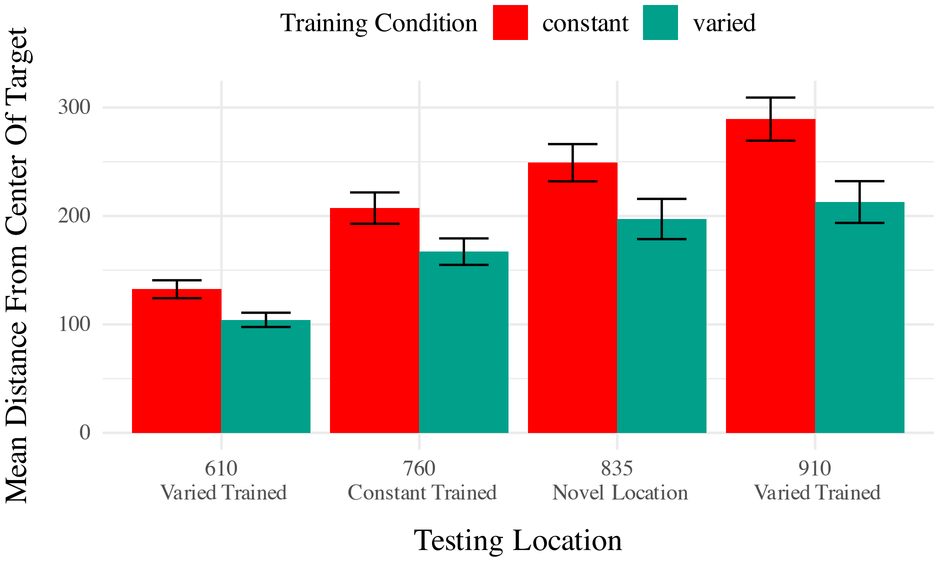


Figure 4: Testing performance for each of the 4 testing positions, compared between training conditions. Positions 610 and 910 were trained on by the varied group, and novel for the constant group. Position 760 was trained on by the constant group, and novel for the varied group. Position 835 was novel for both groups. Shorter bars are indicative of better performance (the ball landing closer to the center of the target). Error bars indicate standard error of the mean.

Table 1

Position	Constant	Varied
610	132.48(50.85)	104.2(38.92)
760	207.26(89.19)	167.12(72.29)
835	249.13(105.92)	197.22(109.71)
910	289.36(122.48)	212.86(113.93)

Discussion

In Experiment 1, we found that varied training resulted in superior testing performance than constant training, from both a position novel to both groups, and from the position at which the constant group was trained, which was novel to the varied condition.

The superiority of varied training over constant training even at the constant training position is of particular note, given that testing at this position should have been highly similar for participants in the constant condition. It should also be noted, though, that testing at the constant trained position is not exactly identical to training from that position, given that the context of testing is different in several ways from that of training, such as the testing trials from the different positions being intermixed, as well as a simple change in context as a function of time. Such contextual differences will be further considered in the General Discussion.

In addition to the variation of throwing position during training, the participants in the varied condition of Experiment 1 also received training practice from the closest/easiest position, as well as from the furthest/most difficult position that would later be encountered by all participants during testing. The varied condition also had the potential advantage of interpolating both of the novel positions from which they would later be tested. Experiment 2 thus sought to address these issues by comparing a varied condition to multiple constant conditions.

Experiment 2

In Experiment 2, we sought to replicate our findings from Experiment 1 with a new sample of participants, while also addressing the possibility of the pattern of results in Experiment 1 being explained by some idiosyncrasy of the particular training location of the constant group relative to the varied group. To this end, Experiment 2 employed the same basic procedure as Experiment 1, but was designed with six separate constant groups each trained from one of six different locations (400, 500, 625, 675, 800, or 900), and a varied group trained from two locations (500 and 800). Participants in all seven groups were then tested from each of the 6 unique positions.

Methods

Participants

A total of 306 Indiana University psychology students participated in Experiment 2, which was also conducted online. As was the case in experiment 1, the undergraduate population from which we recruited participants was 63% female and primarily composed of 18–22-year-old individuals. Using the same procedure as experiment 1, we excluded 98 participants for exceptionally poor performance at one of the dependent measures of the task, or for displaying a pattern of responses indicative of a lack of engagement with the task. A total of 208 participants were included in the final analyses with 31 in the varied group and 32, 28, 37, 25, 29, 26 participants in the constant groups training from location 400, 500, 625, 675, 800, and 900, respectively. All participants were compensated with course credit.

Task and Procedure

The task of Experiment 2 was identical to that of Experiment 1, in all but some minor adjustments to the height of the barrier, and the relative distance between the barrier and the target. Additionally, the intermittent testing trials featured in experiment 1 were not utilized in experiment 2, and all training and testing trials were presented with feedback. An abbreviated demo of the task used for Experiment 2 can be found at (https://pcl.sitehost.iu.edu/tg/demos/igas_expt2_demo.html).

The procedure for Experiment 2 was also quite similar to experiment 1. Participants completed 140 training trials, all of which were from the same position for the constant groups and split evenly (70 trials each - randomized) for the varied group. In the testing phase, participants completed 30 trials from each of the six locations that had been used separately across each of the constant groups during training. Each of the constant groups thus experience one trained location and five novel throwing locations in the testing phase, while the varied group experiences 2 previously trained, and 4 novel locations.

Results

Data Processing and Statistical Packages After confirming that condition and throwing position did not have any significant interactions, we standardized performance within each position, and then average across position to yield a single performance measure per participant. This standardization did not influence our pattern of results. As in experiment 1, we performed type III ANOVA's due to our unbalanced design, however the pattern of results presented below is not altered if type 1 or type III tests are used instead. The statistical software for the primary analyses was the same as for experiment 1. Individual learning rates in the testing phase, compared between groups in the supplementary analyses, were fit using the TEfit package in R (Cochrane, 2020).

Training Phase The different training conditions trained from positions that were not equivalently difficult and are thus not easily amenable to comparison. As previously stated, the primary interest of the training data is confirmation that some learning did occur. **?@fig-e2train** depicts the training performance of the varied group alongside that of the aggregate of the six constant groups (5a), and each of the 6 separate constant groups (5b). An ANOVA comparison with training stage (beginning, middle, end) as a within-group factor and group (the varied condition vs. the 6 constant conditions collapsed together) as a between-subject factor revealed no significant effect of group on training performance, F(1,206)=.55,p=.49, η^2_G <.01, a significant effect of training stage F(2,412)=77.91, p<.001, η^2_G =.05, and no significant interaction between group and training stage, F(2,412)=.489 p=.61, η^2_G <.01. We also tested for a difference in training performance between the varied group and the two constant groups that trained matching throwing positions (i.e., the constant groups training from position 500, and position 800). The results of our ANOVA on this limited dataset mirrors that of the full-group analysis, with no significant effect of group F(1,86)=.48, p=.49, η^2_G <.01, a significant effect of training stage F(2,172)=56.29, p<.001, η^2_G =.11, and no significant interaction between group and training stage, F(2,172)=.341 p=.71, η^2_G <.01.

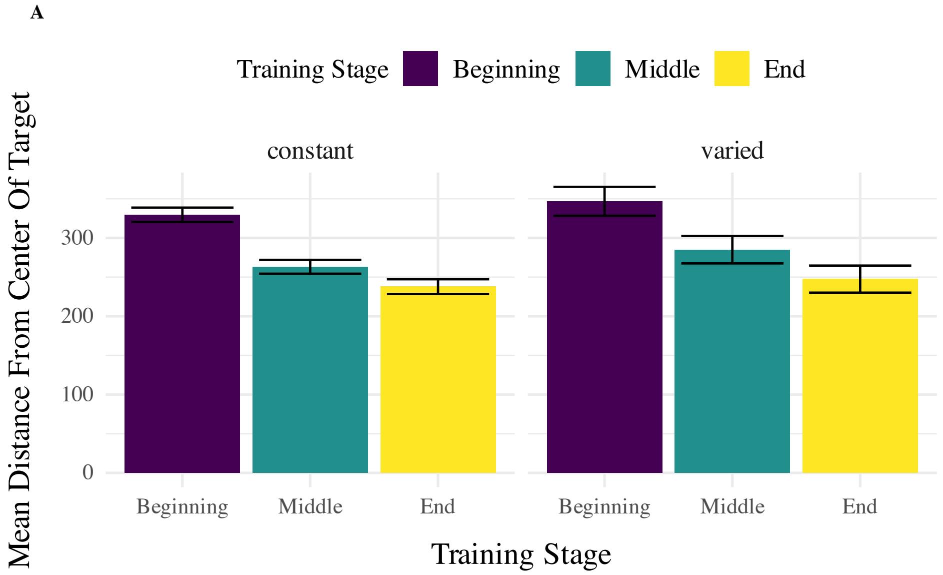


Figure 5: Training performance for the six constant conditions, and the varied condition, binned into three stages. On the left side, the six constant groups are averaged together, as are the two training positions for the varied group. On the right side, the six constant groups are shown separately, with each set of bars representing the beginning, middle, and end of training for a single constant group that trained from the position indicated on the x-axis. Figure 5b also shows training performance separately for both of the throwing locations trained by the varied group. Error bars indicate standard error of the mean.

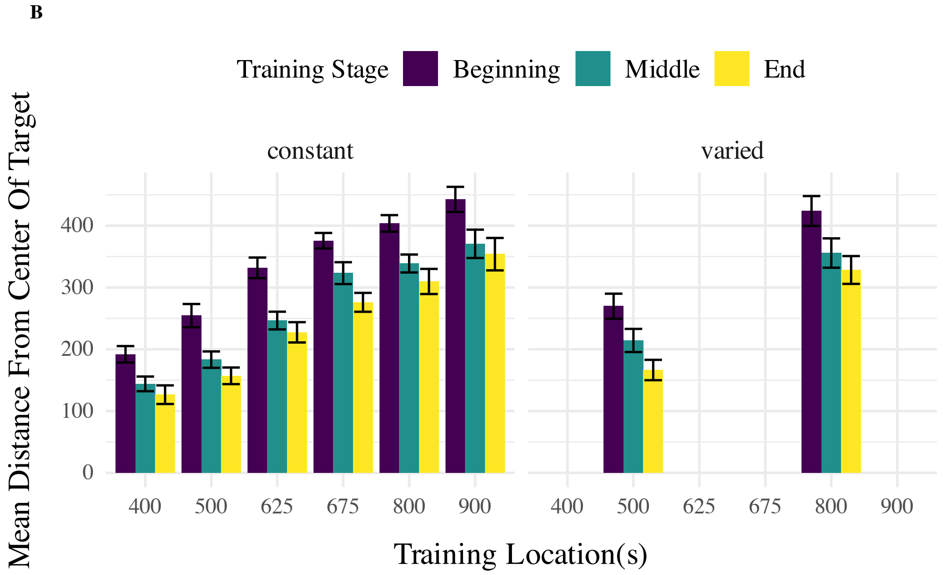


Figure 6: Training performance for the six constant conditions, and the varied condition, binned into three stages. On the left side, the six constant groups are averaged together, as are the two training positions for the varied group. On the right side, the six constant groups are shown separately, with each set of bars representing the beginning, middle, and end of training for a single constant group that trained from the position indicated on the x-axis. Figure 5b also shows training performance separately for both of the throwing locations trained by the varied group. Error bars indicate standard error of the mean.

Testing Phase In Experiment 2, a single varied condition (trained from two positions, 500 and 800), was compared against six separate constant groups (trained from a single position, 400, 500, 625, 675, 800 or 900). For the testing phase, all participants were tested from all six positions, four of which were novel for the varied condition, and five of which were novel for each of the constant groups. For a general comparison, we took the absolute deviations for each throwing position and computed standardized scores across all participants, and then averaged across throwing position. The six constant groups were then collapsed together allowing us to make a simple comparison between training conditions (constant vs. varied). A type III between-subjects ANOVA was performed, yielding a significant effect of condition F(1,206)=4.33, p=.039, η^2_G =.02. Descriptive statistics for each condition are shown in table 2. In Figure 7 visualizes the consistent advantage of the varied condition over the constant groups across the testing positions. Figure 7 shows performance between the varied condition and the individual constant groups.

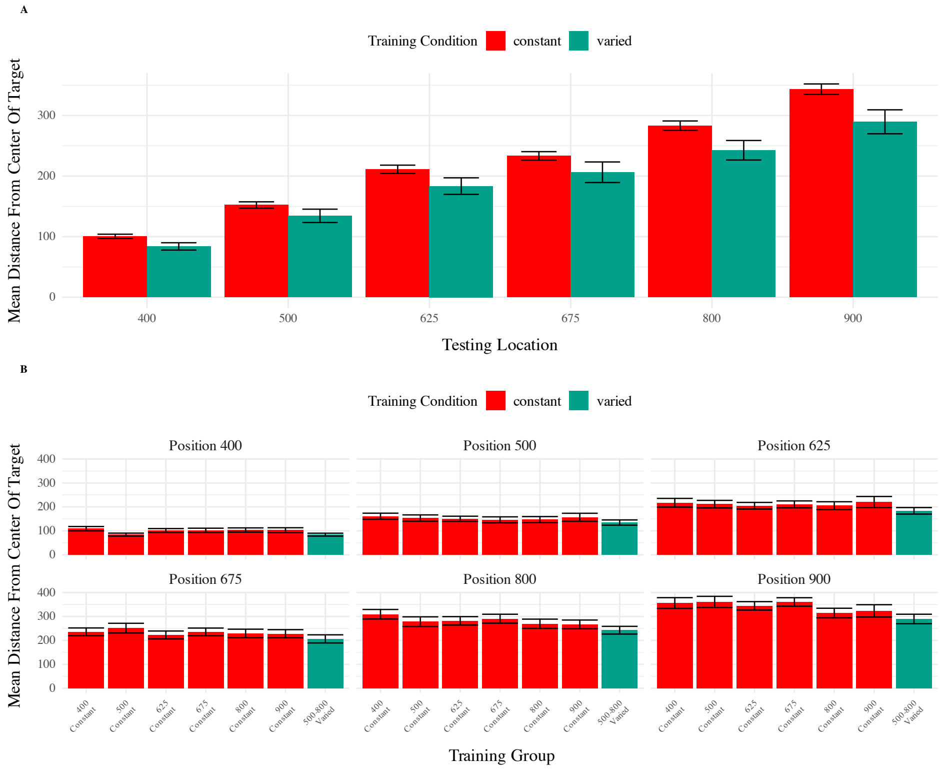


Figure 7: Testing phase performance from each of the six testing positions. The six constant conditions are averaged together into a single constant group, compared against the single varied-trained group.B) Transfer performance from each of the 6 throwing locations from which all participants were tested. Each bar represents performance from one of seven distinct training groups (six constant groups in red, one varied group in blue). The x axis labels indicate the location(s) from which each group trained. Lower values along the y axis reflect better performance at the task (closer distance to target center). Error bars indicate standard error of the mean.

Table 2: Transfer performance from each of the 6 throwing locations from which all participants were tested. Each bar represents performance from one of seven distinct training groups (six constant groups in red, one varied group in blue). The x axis labels indicate the location(s) from which each group trained. Lower values along the y axis reflect better performance at the task (closer distance to target center). Error bars indicate standard error of the mean.

Position	Constant	Varied
400	100.59(46.3)	83.92(33.76)
500	152.28(69.82)	134.38(61.38)
625	211.21(90.95)	183.51(75.92)
675	233.32(93.35)	206.32(94.64)
800	283.24(102.85)	242.65(89.73)
900	343.51(114.33)	289.62(110.07)

Next, we compared the testing performance of constant and varied groups from only positions that participants had not encountered during training. Constant participants each had 5 novel positions, whereas varied participants tested from 4 novel positions (400,625,675,900). We first standardized performance within in each position, and then averaged across positions. Here again, we found a significant effect of condition (constant vs. varied): F(1,206)=4.30, p=.039, η_G^2 = .02 .

Table 3: Testing performance from novel positions. Includes data only from positions that were not encountered during the training stage (e.g. excludes positions 500 and 800 for the varied group, and one of the six locations for each of the constant groups). Table presents Mean absolute deviations from the center of the target, and standard deviations in parenthesis.

Position	Constant	Varied
400	98.84(45.31)	83.92(33.76)
500	152.12(69.94)	
625	212.91(92.76)	183.51(75.92)
675	232.9(95.53)	206.32(94.64)
800	285.91(102.81)	
900	346.96(111.35)	289.62(110.07)

Finally, corresponding to the comparison of position 760 from experiment 1, we compared the test performance of the varied group against the constant group from only the positions that the constant groups trained. Such positions were novel to the varied group (thus this analysis omitted two constant groups that trained from positions 500 or 800 as those positions were not novel to the varied group). Figure 8 displays the particular subset of comparisons utilized for this analysis. Again, we standardized performance within each position before performing the analyses on the aggregated data. In this case, the effect of condition did not reach statistical significance F(1,149)=3.14, p=.079, η_G^2 = .02. Table 4 provides descriptive statistics.

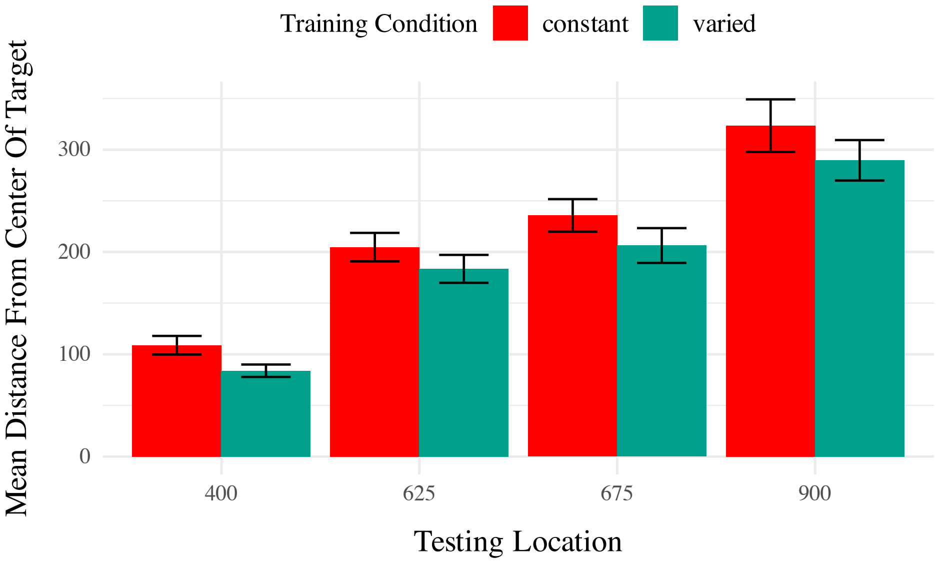


Figure 8: A comparison of throwing location that are identical to those trained by the constant participants (e.g. constant participants trained at position 900, tested from position 900), which are also novel to the varied-trained participants (thus excluding positions 500 and 800). Error bars indicate standard error of the mean.

Table 4: Testing performance from the locations trained by constant participants and novel to varied participants. Locations 500 and 800 are not included as these were trained by the varied participants. Table presents Mean absolute deviation from the center of the target, and standard deviations in parenthesis.

Position	Constant	Varied
400	108.85(50.63)	83.92(33.76)
625	204.75(84.66)	183.51(75.92)
675	235.75(81.15)	206.32(94.64)
900	323.5(130.9)	289.62(110.07)

Discussion

The results of experiment 2 largely conform to the findings of experiment 1. Participants in both varied and constant conditions improved at the task during the training phase. We did not observe the common finding of training under varied conditions producing worse performance during acquisition than training under constant conditions (Catalano & Kleiner, 1984; Wrisberg et al., 1987), which has been suggested to relate to the subsequent benefits of varied training in retention and generalization testing (Soderstrom & Bjork, 2015). However our finding of no difference in training performance between constant and varied groups has been observed in previous work (Chua et al., 2019; Moxley, 1979; Pigott & Shapiro, 1984).

In the testing phase, our varied group significantly outperformed the constant conditions in both a general comparison, and in an analysis limited to novel throwing positions. The observed benefit of varied over constant training echoes the findings of many previous visuomotor skill learning studies that have continued to emerge since the introduction of Schmidt's influential Schema Theory (Catalano & Kleiner, 1984; Chua et al., 2019; Goodwin et al., 1998; McCracken & Stelmach, 1977; Moxley, 1979; Newell & Shapiro, 1976; Pigott & Shapiro, 1984; Roller et al., 2001; Schmidt, 1975; Willey & Liu, 2018b; Wrisberg et al., 1987; Wulf, 1991). We also join a much smaller set of research to observe this pattern in a computerized task (Seow et al., 2019). One departure from the experiment 1 findings concerns the pattern wherein the varied group outperformed the constant group even from the training position of the constant group, which was significant in experiment 1, but did not reach significance in experiment 2. Although this pattern has been observed elsewhere in the literature (Goode et al., 2008; Kerr & Booth, 1978), the overall evidence for this effect appears to be far weaker than for the more general benefit of varied training in conditions novel to all training groups.

Computational Model

Controlling for the similarity between training and testing The primary goal of Experiment 2 was to examine whether the benefits of variability would persist after accounting for individual differences in the similarity between trained and tested throwing locations. To this end, we modelled each throw as a two-dimensional point in the space of x and y velocities applied to the projectile at the moment of release. For each participant, we took each individual training throw, and computed the similarity between that throw and the entire population of throws within the solution space for each of the 6 testing positions. We defined the solution space empirically as the set of all combinations of x and y throw velocities that resulted in hitting the target. We then summed each of the trial-level similarities to produce a single similarity for each testing position score relating how the participant threw the ball during training and the solutions that would result in target hits from each of the six testing positions – thus resulting in six separate similarity scores for each participant. **?@fig-taskSpace** visualizes the solution space for each location and illustrates how different combinations of x and y velocity result in successfully striking the target from different launching positions. As illustrated in **?@fig-taskSpace**, the solution throws represent just a small fraction of the entire space of velocity combinations used by participants throughout the experiment.

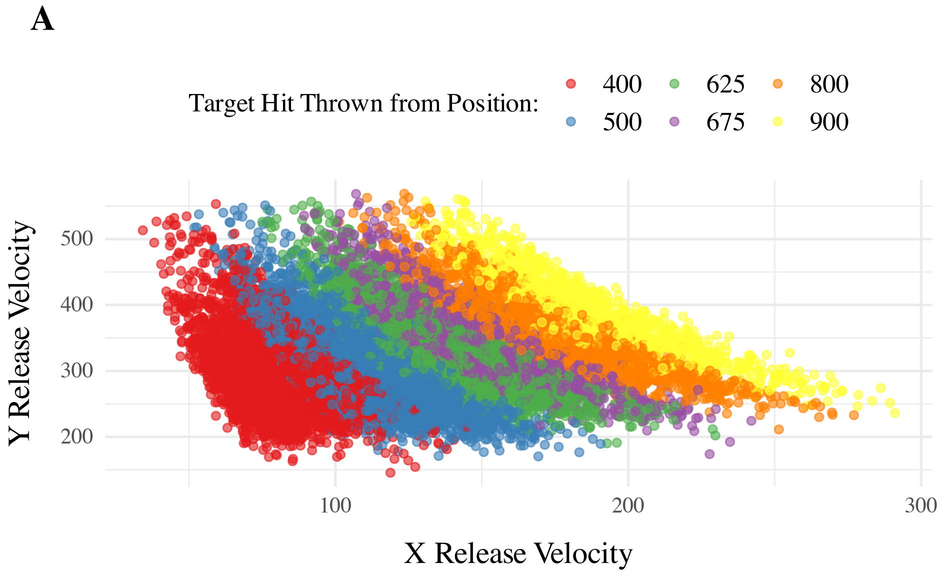


Figure 9: A visual representation of the combinations of throw parameters (x and y velocities applied to the ball at launch), which resulted in target hits during the testing phase. This empirical solution space was compiled from all of the participants in experiment 2. Figure 8B shows the solution space within the context of all of the throws made throughout the testing phase of the experiment.

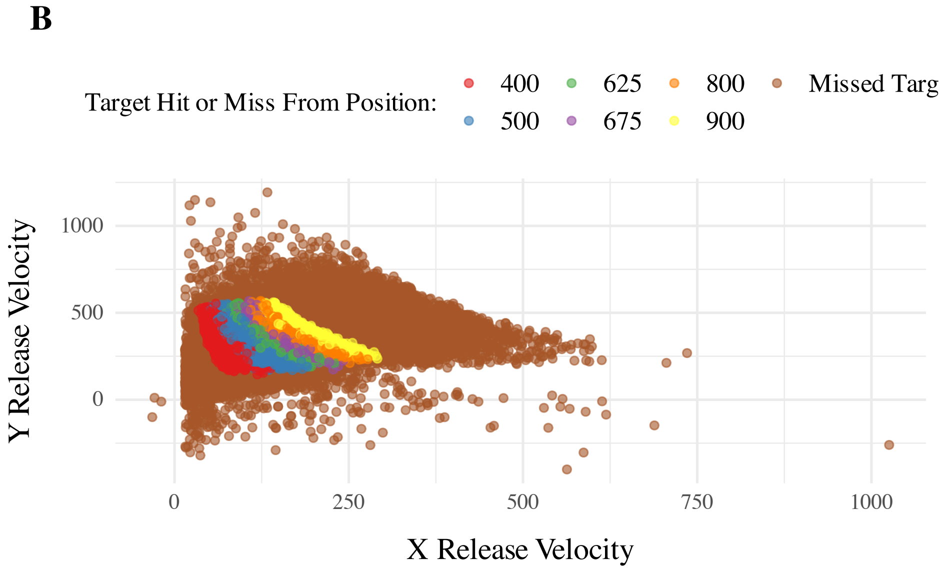


Figure 10: A visual representation of the combinations of throw parameters (x and y velocities applied to the ball at launch), which resulted in target hits during the testing phase. This empirical solution space was compiled from all of the participants in experiment 2. Figure 8B shows the solution space within the context of all of the throws made throughout the testing phase of the experiment.

For each individual trial, the Euclidean distance (Equation 1) was computed between the velocity components (x and y) of that trial and the velocity components of each individual solution throw for each of the 6 positions from which participants would be tested in the final phase of the study. The P parameter in Equation 1 is set equal to 2, reflecting a Gaussian similarity gradient. Then, as per an instance-based model of similarity (Logan, 2002; Nosofsky, 1992), these distances were multiplied by a sensitivity parameter, c, and then exponentiated to yield a similarity value. The parameter c controls the rate with which similarity-based generalization drops off as the Euclidean distance between two throws in x- and y-velocity space increases. If c has a large value, then even a small difference between two throws' velocities greatly decreases the extent of generalization from one to the other. A small value for c produces broad generalization from one throw to another despite relatively large differences in their velocities. The similarity values for each training individual throw made by a given participant were then summed to yield a final similarity score, with a separate score computed for each of the 6 testing positions. The final similarity score is construable as index of how accurate the throws a participant made during the training phase would be for each of the testing positions.

Equation 1: [Similarity_{I,J} = Σ*i = I*Σj=J e$^{\{-c}$·dp_{i,j}}]

Equation 2:

$$d_{i,j} = \sqrt{(x_{Train_i} - x_{Solution_j})^2 + (y_{Train_i} - y_{Solution_j})^2}$$

A simple linear regression revealed that these similarity scores were significantly predictive of performance in the transfer stage, t =-15.88, p<.01, r^2=.17, such that greater similarity between training throws and solution spaces for each of the test locations resulted in better performance. We then repeated the group comparisons above while including similarity as a covariate in the model. Comparing the varied and constant groups in testing performance from all testing positions yielded a significant effect of similarity, F(1, 205)=85.66, p<.001, η^2_G =.29, and also a significant effect of condition (varied vs. constant), F(1, 205)=6.03, p=.015, η^2_G =.03. The group comparison limited to only novel locations for the varied group pit against trained location for the constant group resulted in a significant effect of similarity, F(1,148)=31.12, p<.001, η^2_G =.18 as well as for condition F(1,148)=11.55, p<.001, η^2_G =.07. For all comparisons, the pattern of results was consistent with the initial findings from experiment 2, with the varied group still performing significantly better than the constant group.

Fitting model parameters separately by group To directly control for similarity in Experiment 2, we developed a model-based measure of the similarity between training throws and testing conditions. This similarity measure was a significant predictor of testing performance, e.g., participants whose training throws were more similar to throws that resulted in target hits from the testing positions, tended to perform better during the testing phase. Importantly, the similarity measure did not explain away the group-level benefits of varied training, which remained significant in our linear model predicting testing performance after similarity was added to the model. However, previous research has suggested that participants may differ in their level of generalization as a function of prior experience, and that such differences in generalization gradients can be captured by fitting the generalization parameter of an instance-based model separately to each group (Hahn et al., 2005; Lamberts, 1994). Relatedly, the influential Bayesian generalization model developed by Tenenbaum & Griffiths (2001) predicts that the breadth of generalization will increase when a rational agent encounters a wider variety of examples. Following these leads, we assume that in addition to learning the task itself, participants are also adjusting how generalizable their experience should be. Varied versus constant participants may be expected to learn to generalize their experience to different degrees. To accommodate this difference, the generalization parameter of the instance-based model (in the present case, the c parameter) can be allowed to vary between the two groups to reflect the tendency of learners to adaptively tune the extent of their generalization. One specific hypothesis is that people adaptively set a value of c to fit the variability of their training experience (Nosofsky & Johansen, 2000; Sakamoto et al., 2006). If one's training experience is relatively variable, as with the variable training condition, then one might infer that future test situations will also be variable, in which case a low value of c will allow better generalization because generalization will drop off slowly with training-to-testing distance. Conversely, if one's training experience has little variability, as found in the constant training conditions, then one might adopt a high value of c so that generalization falls off rapidly away from the trained positions.

To address this possibility, we compared the original instance-based model of similarity fit against a modified model which separately fits the generalization parameter, c, to varied and constant participants. To perform this parameter fitting, we used the optim function in R, and fit the model to find the c value(s) that maximized the correlation between similarity and testing performance.

Both models generate distinct similarity values between training and testing locations. Much like the analyses in Experiment 2, these similarity values are regressed against testing performance in models of the form shown below. As was the case previously, testing performance is defined as the mean absolute distance from the center of the target (with a separate score for each participant, from each position).

Linear models 1 and 3 both show that similarity is a significant predictor of testing performance (p<.01). Of greater interest is the difference between linear model 2, in which similarity is computed from a single c value fit from all participants (Similarity1c), with linear model 4, which fits the c parameter separately between groups (Similarity2c). In linear model 2, the effect of training group remains significant when controlling for Similarity1c (p<.01), with the varied group still performing significantly better. However, in linear model 4 the addition of the Similarity2c predictor results in the effect of training group becoming nonsignificant (p=.40), suggesting that the effect of varied vs. constant training is accounted for by the Similarity2c predictor. Next, to further establish a difference between the models, we performed nested model comparisons using ANOVA, to see if the addition of the training group parameter led to a significant improvement in model performance. In the first comparison, ANOVA(Linear Model 1, Linear Model 2), the addition of the training group predictor significantly improved the performance of the model (F=22.07, p<.01). However, in the second model comparison, ANOVA (Linear model 3, Linear Model 4) found no improvement in model performance with the addition of the training group predictor (F=1.61, p=.20).

Finally, we sought to confirm that similarity values generated from the adjusted Similarity2c model had more predictive power than those generated from the original Similarity1c model. Using the BIC function in R, we compared BIC values between linear model 1 (BIC=14604.00) and linear model 3 (BIC = 14587.64). The lower BIC value of model 3 suggests a modest advantage for predicting performance using a similarity measure computed with two c values over similarity computed with a single c value. When fit with separate c values, the best fitting c parameters for the model consistently optimized such that the c value for the varied group (c=.00008) was smaller in magnitude than the c value for the constant group(c= .00011). Recall that similarity decreases as a Gaussian function of distance (equation 1 above), and a smaller value of c will result in a more gradual drop-off in similarity as the distance between training throws and testing solutions increases.

In summary, our modeling suggests that an instance-based model which assumes equivalent generalization gradients between constant and varied trained participants is unable to account for the extent of benefits of varied over constant training observed at testing. The evidence for this in the comparative model fits is that when a varied/constant dummy-coded variable for condition is explicitly added to the model, the variable adds a significant contribution to the prediction of test performance, with the variable condition yielding better performance than the constant conditions. However, if the instance-based generalization model is modified to assume that the training groups can differ in the steepness of their generalization gradient, by incorporating a separate generalization parameter for each group, then the instance-based model can account for our experimental results without explicitly taking training group into account. Henceforth this model will be referred to as the Instance-based Generalization with Adaptive Similarity (IGAS) model.

General Discussion

Across two experiments, we found evidence in support of the benefits of variability hypothesis in a simple, computerized projectile throwing task. Generalization was observed in both constant and varied participants, in that both groups tended to perform better at novel positions in the testing phase than did participants who started with those positions in the training phase. However, varied trained participants consistently performed better than constant trained participants, in terms of both the testing phase in general, and in a comparison that only included untrained positions. We also found some evidence for the less commonly observed pattern wherein varied-trained participants outperform constant-trained participants even from conditions identical to the constant group training (Goode et al., 2008; Green et al., 1995; Kerr & Booth, 1978). In experiment 1 varied participants performed significantly better on this identity comparison. In Experiment 2, the comparison was not significant initially, but became significant after controlling for the similarity measure that incorporates only a single value for the steepness of similarity-based generalization (c). Furthermore, we showed that the general pattern of results from Experiment 2 could be parsimoniously accommodated by an instance-based similarity model, but only with the assumption that constant and varied participants generalize their training experience to different degrees. Our results thus suggest that the benefits of variation cannot be explained by the varied-trained participants simply covering a broader range of the task space. Rather, the modeling suggests that varied participants also learn to adaptively tune their generalization function such that throwing locations generalize more broadly to one another than they do in the constant condition. A learning system could end up adopting a higher c value in the constant than variable training conditions by monitoring the trial-by-trial variability of the training items. The c parameter would be adapted downwards when adjacent training items are dissimilar to each other and adapted upwards when adjacent training items are the same. In this fashion, contextually appropriate c values could be empirically learned. This learning procedure would capture the insight that if a situation has a high amount variability, then the learner should be predisposed toward thinking that subsequent test items will also show considerable variability, in which case generalization gradients should be broad, as is achieved by low values for c.

Also of interest is whether the IGAS model can predict the pattern of results wherein the varied condition outperforms the constant condition even from the position on which the constant condition trained. Although our models were fit using all of the Experiment 2 training and testing data, not just that of the identity comparisons, in Figure 11 we demonstrate how a simplified version of the IGAS model could in principle produce such a pattern. In addition to the assumption of differential generalization between varied and constant conditions, our simplified model makes explicit an assumption that is incorporated into the full IGAS model – namely that even when being tested from a position identical to that which was trained, there are always some psychological contextual differences between training and testing throws, resulting in a non-zero dissimilarity.

Figure 9.

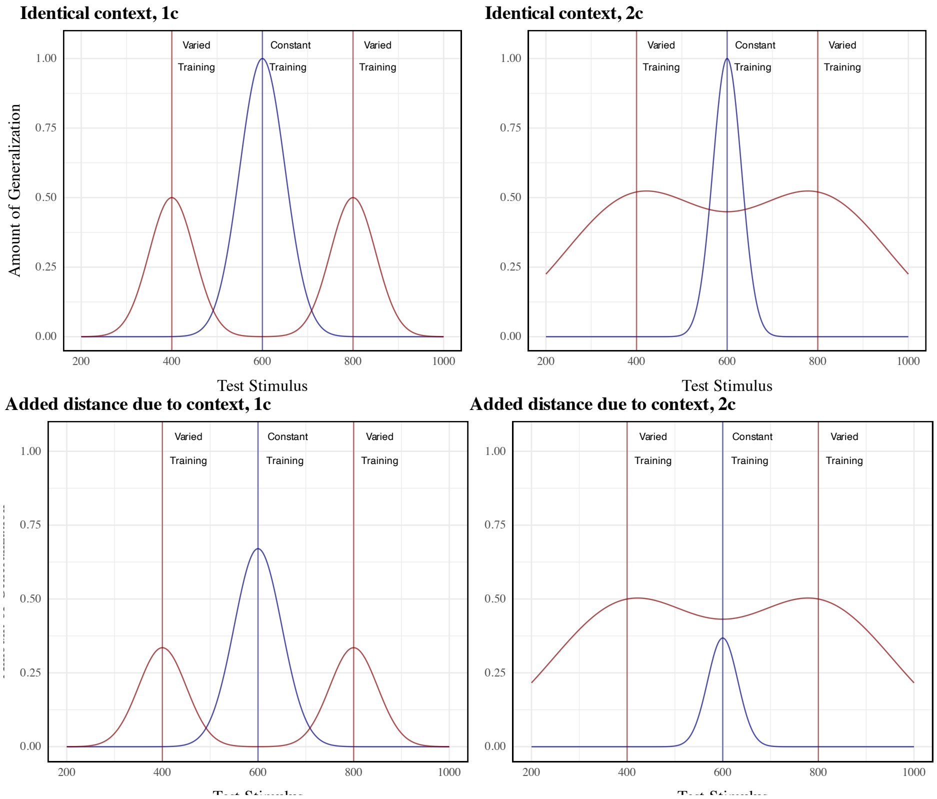


Figure 11: A simple model depicting the necessity of both of two separately fit generalization parameters, c, and a positive distance between training and testing contexts, in order for an instance model to predict a pattern of varied training from stimuli 400 and 800 outperforming constant training from position 600 at a test position of 600. For the top left panel, in which the generalization model assumes a single c value (-.008) for both varied and constant conditions, and identical contexts across training and testing, the equation which generates the varied condition is - Amount of Generalization = $e^{(ccdot|x-800|)} + e^{(ccdot|x-400|)}$, whereas the constant group generalization is generated from 2
$cdote^{(ccdot|x-600|)}$. For the top right panel, the c constants in the original equations are different for the 2 conditions, with $c = -.002$ for the varied condition, and $c = -.008$ for the constant condition. The bottom two panels are generated from identical equations to those immediately above, except for the addition of extra distance (100 units) to reflect the assumption of some change in context between training and testing conditions. Thus, the generalization model for the varied condition in the bottom-right panel is of the form - Amount of Generalization = $e^{(cvariedcdot|x-800|)} + e^{(cvariedcdot|x-400|)}$.

As mentioned above, the idea that learners flexibly adjust their generalization gradient based on prior experience does have precedent in the domains of category learning (Aha & Goldstone, 1992; Briscoe & Feldman, 2011; Hahn et al., 2005; Lamberts, 1994;

Op de Beeck et al., 2008), and sensorimotor adaptation (Marongelli & Thoroughman, 2013; Taylor & Ivry, 2013; Thoroughman & Taylor, 2005). Lamberts (1994) showed that a simple manipulation of background knowledge during a categorization test resulted in participants generalizing their training experience more or less broadly, and moreover that such a pattern could be captured by allowing the generalization parameter of an instance-based similarity model to be fit separately between conditions. The flexible generalization parameter has also successfully accounted for generalization behavior in cases where participants have been trained on categories that differ in their relative variability (Hahn et al., 2005; Sakamoto et al., 2006). However, to the best of our knowledge, IGAS is the first instance-based similarity model that has been put forward to account for the effect of varied training in a visuomotor skill task. Although IGAS was inspired by work in the domain of category learning, its success in a distinct domain may not be surprising in light of the numerous prior observations that at least certain aspects of learning and generalization may operate under common principles across different tasks and domains (Censor et al., 2012; Hills et al., 2010; Jamieson et al., 2022; Law & Gold, 2010; Roark et al., 2021; Vigo et al., 2018; Wall et al., 2021; Wu et al., 2020; Yang et al., 2020; ?).

Our modelling approach does differ from category learning implementations of instance-based models in several ways. One such difference is the nature of the training instances that are assumed to be stored. In category learning studies, instances are represented as points in a multidimensional space of all of the attributes that define a category item (e.g. size/color/shape). Rather than defining instances in terms of what stimuli learners experience, our approach assumes that stored, motor instances reflect how they act, in terms of the velocity applied to the ball on each throw. An advantage of many motor learning tasks is the relative ease with which task execution variables can be directly measured (e.g. movement force, velocity, angle, posture) in addition to the decision and response time measures that typically exhaust the data generated from more classical cognitive tasks. Of course, whether learners actually are storing each individual motor instance is a fundamental question beyond the scope of the current work – though as described in the introduction there is some evidence in support of this idea (Chamberlin & Magill, 1992a; Crump & Logan, 2010; Hommel, 1998; Meigh et al., 2018; Poldrack et al., 1999). A particularly noteworthy instance-based model of sensory-motor behavior is the Knowledge II model of Rosenbaum and colleagues (Cohen & Rosenbaum, 2004; Rosenbaum et al., 1995). Knowledge II explicitly defines instances as postures (joint combinations), and is thus far more detailed than IGAS in regards to the contents of stored instances. Knowledge II also differs from IGAS in that learning is accounted for by both the retrieval of stored postures, and the generation of novel postures via the modification of retrieved postures. A promising avenue for future research would be to combine the adaptive similarity mechanism of IGAS with the novel instance generation mechanisms of Knowledge II.

Our findings also have some conceptual overlap with an earlier study on the effects of varied training in a coincident timing task (Catalano & Kleiner, 1984). In this task, participants observe a series of lamps lighting up consecutively, and attempt to time a button press with the onset of the final lamp. The design consisted of four separate constant groups, each training from a single lighting velocity, and a single varied group training with all four of the lighting velocities used by the individual constant groups.

Participants were then split into four separate testing conditions, each of which were tested from a single novel lighting velocity of varying distance from the training conditions. The result of primary interest was that all participants performed worse as the distance between training and testing velocity increased – a typical generalization decrement. However, varied participants showed less of a decrement than did constant participants. The authors take this result as evidence that varied training results in a less-steep generalization gradient than does constant training. Although the experimental conclusions of Catalano and Kleiner are similar to our own, our work is novel in that we account for our results with a cognitive model, and without assuming the formation of a schema. Additionally, the way in which Catalano and Kleiner collapse their separate constant groups together may result in similarity confounds between varied and constant conditions that leaves their study open to methodological criticisms, especially in light of related work which demonstrated that the extent to which varied training may be beneficial can depend on whether the constant group they are compared against trained from similar conditions to those later tested (Wrisberg et al., 1987). Our study alleviates such concerns by explicitly controlling for similarity.

Limitations

A limitation of this study concerns the ordering of the testing/transfer trials at the conclusion of both experiments. Participants were tested from each separate position (4 in Experiment 1, 6 in Experiment 2) in a random, intermixed order. Because the varied group was trained from two positions that were also randomly ordered, they may have benefited from experience with this type of sequencing, whereas the constant groups had no experience with switching between positions trial to trial. This concern is somewhat ameliorated by the fact that the testing phase performance of the constant groups from their trained position was not significantly worse than their level of performance at the end of the training phase, suggesting that they were not harmed by random ordering of positions during testing. It should also be noted that the computerized task utilized in the present work is relatively simple compared to many of the real-world tasks utilized in prior research. It is thus conceivable that the effect of variability in more complex tasks is distinct from the process put forward in the present work. An important challenge for future work will be to assess the extent to which IGAS can account for generalization in relatively complex tasks with far more degrees of freedom.

It is common for psychological process models of categorization learning to use an approach such as multidimensional scaling so as to transform the stimuli from the physical dimensions used in the particular task into the psychological dimensions more reflective of the actual human representations (Nosofsky, 1992; Shepard, 1987). Such scaling typically entails having participants rate the similarity between individual items and using these similarity judgements to then compute the psychological distances between stimuli, which can then be fed into a subsequent model. In the present investigation, there was no such way to scale the x and y velocity components in terms of the psychological similarity, and thus our modelling does rely on the assumption that the psychological distances between the different throwing positions are proportional to absolute distances in the metric space of the task (e.g. the relative distance between positions 400 and 500 is equivalent to that between 800 and 900). However, an advantage of our approach is that we are measuring similarity in terms of how participants behave (applying a velocity to the ball), rather than the metric features of the task stimuli.

Conclusion

Our experiments demonstrate a reliable benefit of varied training in a simple projectile launching task. Such results were accounted for by an instance-based model that assumes that varied training results in the computation of a broader similarity-based generalization gradient. Instance-based models augmented with this assumption may be a valuable approach towards better understanding skill generalization and transfer.

Project 2

Introduction

In project 1, we applied model-based techniques to quantify and control for the similarity between training and testing experience, which in turn enabled us to account for the difference between varied and constant training via an extended version of a similarity based generalization model. In project 2, we will go a step further, implementing a full process model capable of both 1) producing novel responses and 2) modeling behavior in both the learning and testing stages of the experiment. Project 2 also places a greater emphasis on extrapolation performance following training - as varied training has often been purported to be particularly beneficial in such situations. Extrapolation has long been a focus of the literature on function learning (Brehmer, 1974; Carroll, 1963). Central questions of the function learning literature have included the relative difficulties of learning various functional forms (e.g. linear vs.bilinear vs. quadratic), and the relative effectiveness of rule-based vs. association-based exemplar models vs. various hybrid models (Bott & Heit, 2004; DeLosh et al., 1997; A. Jones et al., 2018; Kalish et al., 2004; M. Mcdaniel et al., 2009; M. A. Mcdaniel & Busemeyer, 2005). However the issue of training variation has received surprisingly little attention in this area.

Methods

Participants

Data was collected from 647 participants (after exclusions). The results shown below consider data from subjects in our initial experiment, which consisted of 196 participants (106 constant, 90 varied). The follow-up experiments entailed minor manipulations: 1) reversing the velocity bands that were trained on vs. novel during testing; 2) providing ordinal rather than numerical feedback during training (e.g. correct, too low, too high). The data from these subsequent experiments are largely consistently with our initial results shown below.

Task

We developed a novel visuomotor extrapolation task, termed the Hit The Wall task, wherein participants learned to launch a projectile such that it hit a rectangle at the far end of the screen with an appropriate amount of force. Although the projectile had both x and y velocity components, only the x-dimension was relevant for the task. Link to task demo

Procedure

Upon arrival at the laboratory, participants were provided with a description of the experiment and signed informed consent forms. They were then seated in front of a computer equipped with a mouse and were given instructions on how to perform the "Hit The Wall" (HTW) visuomotor extrapolation task.

The HTW task involved launching projectiles to hit a target displayed on the computer screen. Participants completed a total of 90 trials during the training stage. In the varied training condition, participants encountered three velocity bands (800-1000, 1000-1200, and 1200-1400). In contrast, participants in the constant training condition encountered only one velocity band (800-1000).

During the training stage, participants in both conditions also completed "no feedback" trials, where they received no information about their performance. These trials were randomly interleaved with the regular training trials.

Following the training stage, participants proceeded to the testing stage, which consisted of three phases. In the first phase, participants completed "no-feedback" testing from three novel extrapolation bands (100-300, 350-550, and 600-800), with each band consisting of 15 trials.

In the second phase of testing, participants completed "no-feedback" testing from the three velocity bands used during the training stage (800-1000, 1000-1200, and 1200-1400). In the constant training condition, two of these bands were novel, while in the varied training condition, all three bands were encountered during training.

The third and final phase of testing involved "feedback" testing for each of the three extrapolation bands (100-300, 350-550, and 600-800), with each band consisting of 10 trials. Participants received feedback on their performance during this phase.

Throughout the experiment, participants' performance was measured by calculating the distance between the produced x-velocity of the projectiles and the closest edge of the current velocity band. Lower distances indicated better performance.

After completing the experiment, participants were debriefed and provided with an opportunity to ask questions about the study.

Analyses Strategy

All data processing and statistical analyses were performed in R version 4.31 Team (2020). To assess differences between groups, we used Bayesian Mixed Effects Regression. Model fitting was performed with the brms package in R Bürkner (2017), and descriptive stats and tables were extracted with the BayestestR package (?). Mixed effects regression enables us to take advantage of partial pooling, simultaneously estimating parameters at the individual and group level. Our use of Bayesian, rather than frequentist methods allows us to directly quantify the uncertainty in our parame-

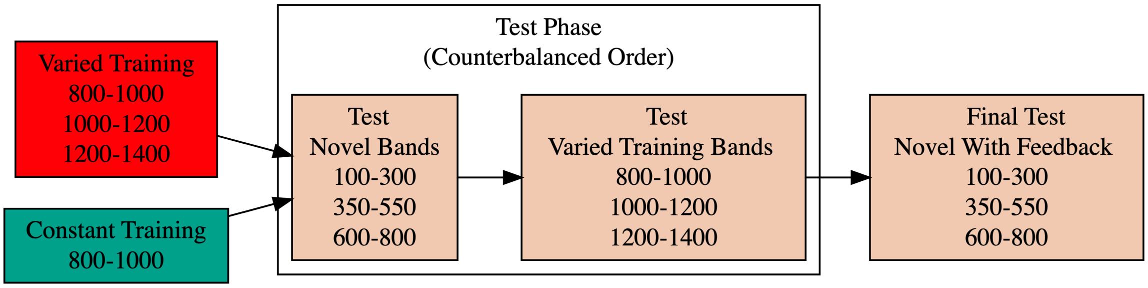


Figure 12: Experiment 1 Design. Constant and Varied participants complete different training conditions.

ter estimates, as well as circumventing convergence issues common to the frequentist analogues of our mixed models. For each model, we report the median values of the posterior distribution, and 95% credible intervals.

Each model was set to run with 4 chains, 5000 iterations per chain, with the first 2500 of which were discarded as warmup chains. Rhat values were generally within an acceptable range, with values <=1.02 (see appendix for diagnostic plots). We used uninformative priors for the fixed effects of the model (condition and velocity band), and weakly informative Student T distributions for for the random effects.

We compared varied and constant performance across two measures, deviation and discrimination. Deviation was quantified as the absolute deviation from the nearest boundary of the velocity band, or set to 0 if the throw velocity fell anywhere inside the target band. Thus, when the target band was 600-800, throws of 400, 650, and 1100 would result in deviation values of 200, 0, and 300, respectively. Discrimination was measured by fitting a linear model to the testing throws of each subjects, with the lower end of the target velocity band as the predicted variable, and the x velocity produced by the participants as the predictor variable. Participants who reliably discriminated between velocity bands tended to have positive slopes with values ~1, while participants who made throws irrespective of the current target band would have slopes ~0.

Table 5: Testing Deviation - Empirical Summary

Table 6: Summary of Deviation- Constant

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	254	148	298
350-550	Extrapolation	191	110	229
600-800	Extrapolation	150	84	184
800-1000	Trained	184	106	242
1000-1200	Extrapolation	233	157	282
1200-1400	Extrapolation	287	214	290

Table 7: Summary of Deviation- Varied

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	386	233	426
350-550	Extrapolation	285	149	340
600-800	Extrapolation	234	144	270
800-1000	Trained	221	149	248
1000-1200	Trained	208	142	226
1200-1400	Trained	242	182	235

Results

Testing Phase - No feedback.

In the first part of the testing phase, participants are tested from each of the velocity bands, and receive no feedback after each throw.

Deviation From Target Band Descriptive summaries testing deviation data are provided in Table 5 and Figure 13. To model differences in accuracy between groups, we used Bayesian mixed effects regression models to the trial level data from the testing phase. The primary model predicted the absolute deviation from the target velocity band (dist) as a function of training condition (condit), target velocity band (band), and their interaction, with random intercepts and slopes for each participant (id).

$$dist_{ij} = \beta_0 + \beta_1 \cdot condit_{ij} + \beta_2 \cdot band_{ij} + \beta_3 \cdot condit_{ij} \cdot band_{ij} + b_{0i} + b_{1i} \cdot band_{ij} + \epsilon_{ij} \qquad (1)$$

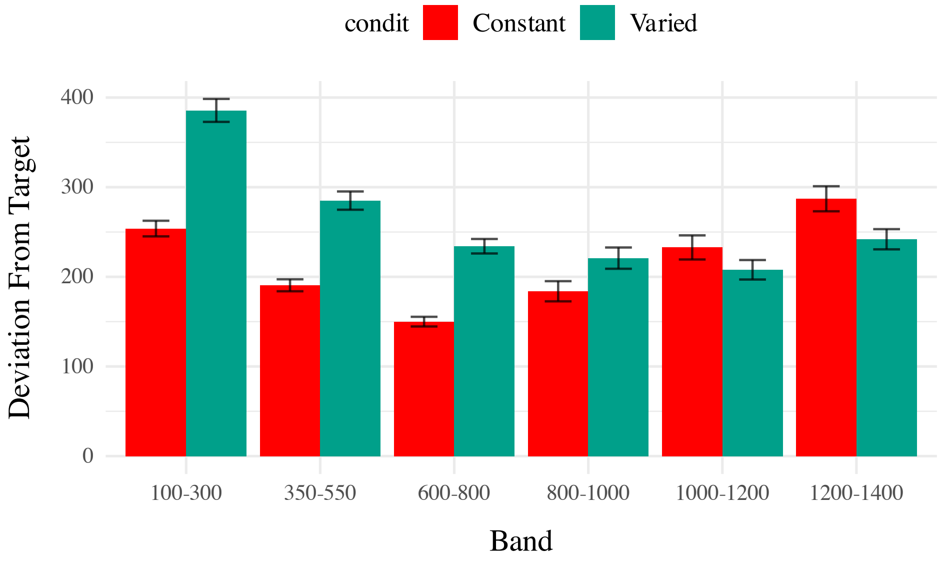


Figure 13: E1. Deviations from target band during testing without feedback stage.

Table 8: Experiment 1. Bayesian Mixed Model predicting absolute deviation as a function of condition (Constant vs. Varied) and Velocity Band

Table 9: Coefficients

Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	205.09	136.86	274.06	1.00
conditVaried	157.44	60.53	254.90	1.00
Band	0.01	-0.07	0.08	0.57
condit*Band	-0.16	-0.26	-0.06	1.00

contrast	Band	value	lower	upper	pd
Constant - Varied	100	-141.49	-229.2	-53.83	1.00
Constant - Varied	350	-101.79	-165.6	-36.32	1.00
Constant - Varied	600	-62.02	-106.2	-14.77	1.00
Constant - Varied	800	-30.11	-65.1	6.98	0.94
Constant - Varied	1000	2.05	-33.5	38.41	0.54
Constant - Varied	1200	33.96	-11.9	81.01	0.92

The model predicting absolute deviation (dist) showed clear effects of both training condition and target velocity band (Table X). Overall, the varied training group showed a larger deviation relative to the constant training group (β = 157.44, 95% CI [60.53, 254.9]). Deviation also depended on target velocity band, with lower bands showing less deviation. See Table 8 for full model output.

Discrimination between bands In addition to accuracy/deviation, we also assessed the ability of participants to reliably discriminate between the velocity bands (i.e. responding differently when prompted for band 600-800 than when prompted for band 150-350). Table 10 shows descriptive statistics of this measure, and Figure 1 visualizes the full distributions of throws for each combination of condition and velocity band. To quantify discrimination, we again fit Bayesian Mixed Models as above, but this time the dependent variable was the raw x velocity generated by participants on each testing trial.

$$vx_{ij} = \beta_0 + \beta_1 \cdot condit_{ij} + \beta_2 \cdot bandInt_{ij} + \beta_3 \cdot condit_{ij} \cdot bandInt_{ij} + b_{0i} + b_{1i} \cdot bandInt_{ij} + \epsilon_{ij} \qquad (2)$$

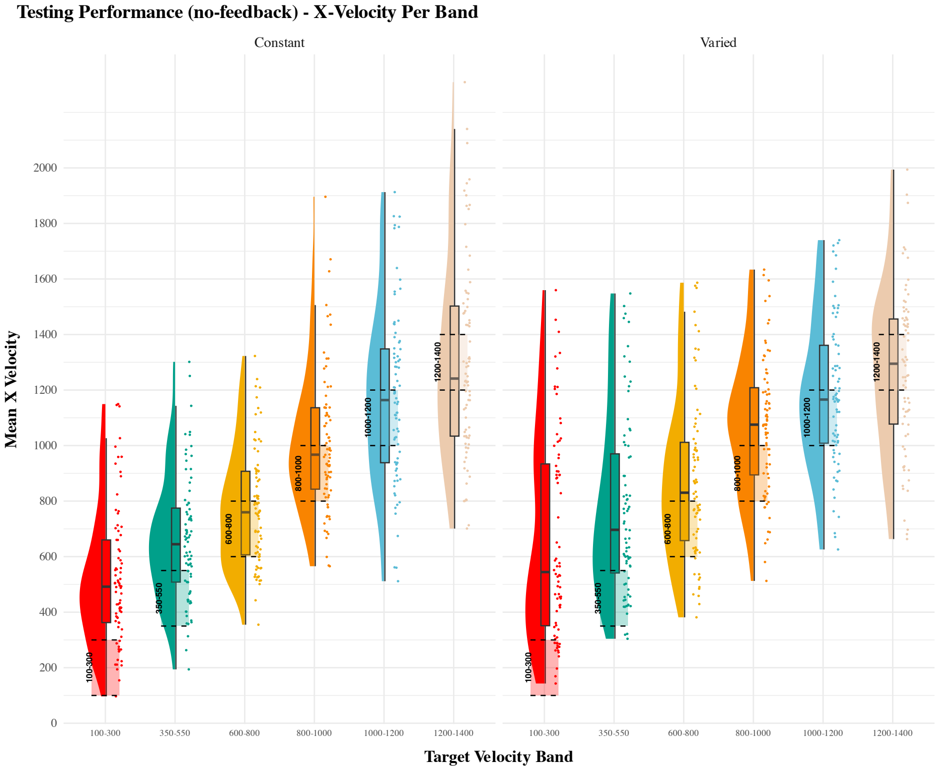


Figure 14: E1 testing x velocities. Translucent bands with dash lines indicate the correct range for each velocity band.

Table 10: Testing vx - Empirical Summary

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	524	448	327
350-550	Extrapolation	659	624	303
600-800	Extrapolation	770	724	300
800-1000	Trained	1001	940	357
1000-1200	Extrapolation	1167	1104	430
1200-1400	Extrapolation	1283	1225	483
Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	664	533	448
350-550	Extrapolation	768	677	402
600-800	Extrapolation	876	813	390
800-1000	Trained	1064	1029	370
1000-1200	Trained	1180	1179	372
1200-1400	Trained	1265	1249	412

Table 11: Experiment 1. Bayesian Mixed Model Predicting Vx as a function of condition (Constant vs. Varied) and Velocity Band

Table 12: Fit to all 6 bands

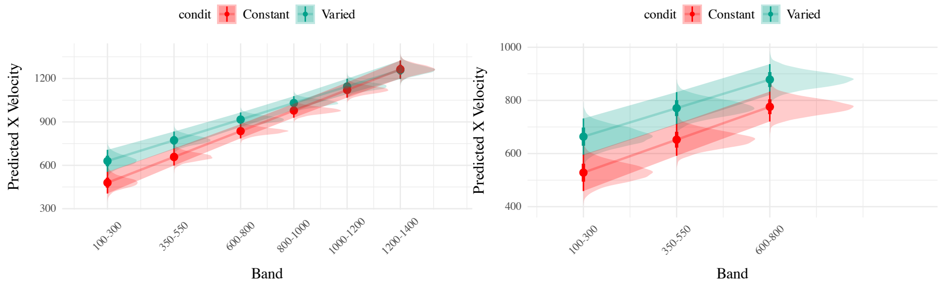
Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	408.55	327.00	490.61	1.00
conditVaried	164.05	45.50	278.85	1.00
Band	0.71	0.62	0.80	1.00
condit*Band	-0.14	-0.26	-0.01	0.98

Table 13: Fit to 3 extrapolation bands

Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	478.47	404.00	551.45	1.00
conditVaried	142.04	37.17	247.59	1.00
Band	0.50	0.42	0.57	1.00
condit*Band	-0.07	-0.17	0.04	0.89

See Table 11 for the full model results. The estimated coefficient for training condition (B = 164.05, 95% CrI [45.5, 278.85]) suggests that the varied group tends to produce harder throws than the constant group, but is not in and of itself useful for assessing discrimination. Most relevant to the issue of discrimination is the slope on Velocity Band (B = 0.71, 95% CrI [0.62, 0.8]). Although the median slope does fall underneath the ideal of value of 1, the fact that the 95% credible interval does not contain 0 provides strong evidence that participants exhibited some discrimination between bands. The estimate for the interaction between slope and condition (B = -0.14, 95% CrI [-0.26, -0.01]), suggests that the discrimination was somewhat modulated by training condition, with the varied participants showing less senitivity between vands than the constant condition. This difference is depicted visually in Figure 15.@tbl-e1-slope-quartile shows the average slope coefficients for varied and constant participants separately for each quartile. The constant participant participants appear to have larger slopes across quartiles, but the difference between conditions may be less pronounced for the top quartiles of subjects who show the strongest discrimination. Figure Figure 16 shows the distributions of slope values for each participant, and the compares the probability density of slope coefficients between training conditions. Figure 17

The second model, which focused solely on extrapolation bands, revealed similar patterns. The Velocity Band term (B = 0.5, 95% CrI [0.42, 0.57]) still demonstrates a high degree of discrimination ability. However, the posterior distribution for interaction term (B = -0.07, 95% CrI [-0.17, 0.04]) does across over 0, suggesting that the evidence for decreased discrimination ability for the varied participants is not as strong when considering only the three extrapolation bands.



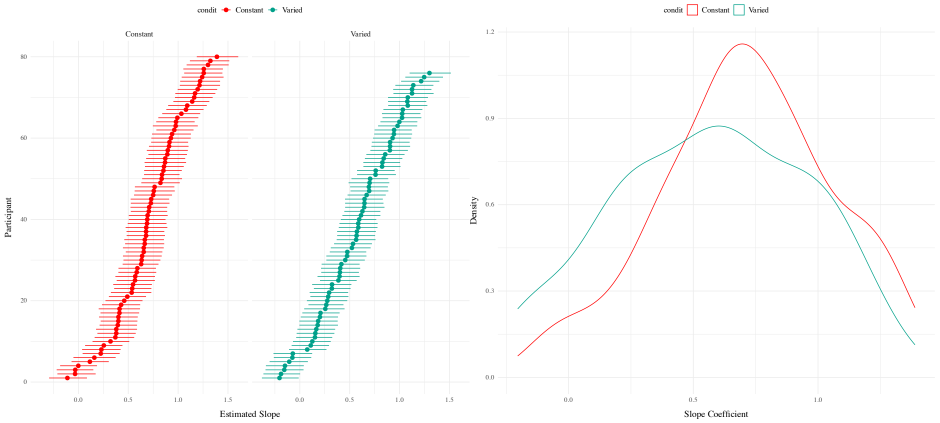
(a) Model fit to all 6 bands

(b) Model fit to only 3 extrapolation bands

Figure 15: Conditional effect of training condition and Band. Ribbons indicate 95% HDI. The steepness of the lines serves as an indicator of how well participants discriminated between velocity bands.

Table 14: Slope coefficients by quartile, per condition

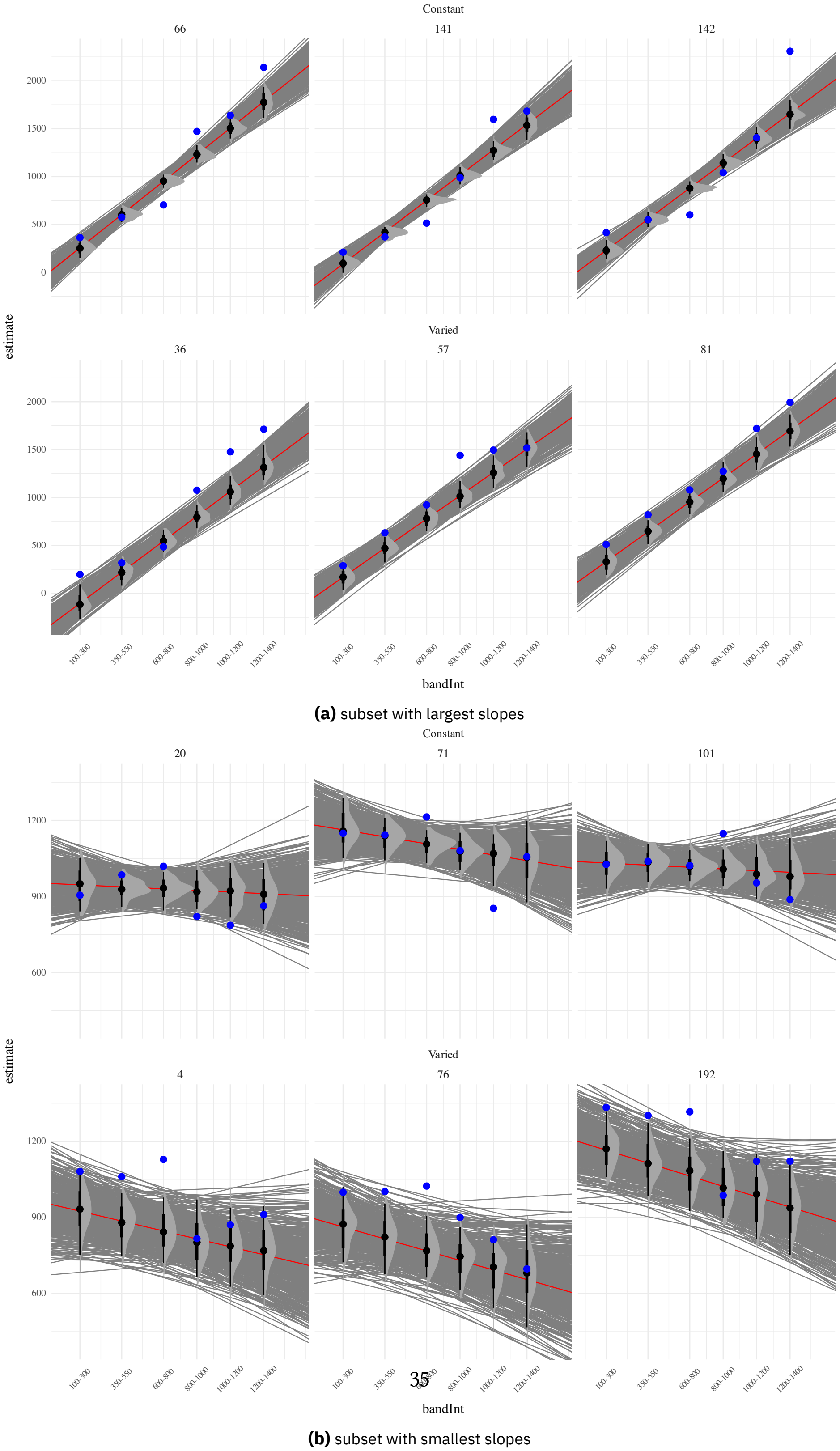
Condition	Q_0%_mean	Q_25%_mean	Q_50%_mean	Q_75%_mean	Q_100%_mean
Constant	-0.110	0.483	0.692	0.931	1.39
Varied	-0.204	0.273	0.586	0.903	1.30



(a) Slope estimates by participant - ordered from lowest to highest within each condition.

(b) Destiny of slope coefficients by training group

Figure 16: Slope distributions between condition



(a) subset with largest slopes

(b) subset with smallest slopes

Figure 17: Subset of Varied and Constant Participants with the smallest and largest estimated slope values. Red lines represent the best fitting line for each participant, gray lines are 200 random samples from the posterior distribution. Colored points and intervals at each band represent the empirical median and 95% HDI.

Experiment 2

Figure 18 illustrates the design of Experiment 2. The stages of the experiment (i.e. training, testing no-feedback, test with feedback), are identical to that of Experiment 1. The only change is that Experiment 2 participants train, and then test, on bands in the reverse order of Experiment 1 (i.e. training on the softer bands; and testing on the harder bands).

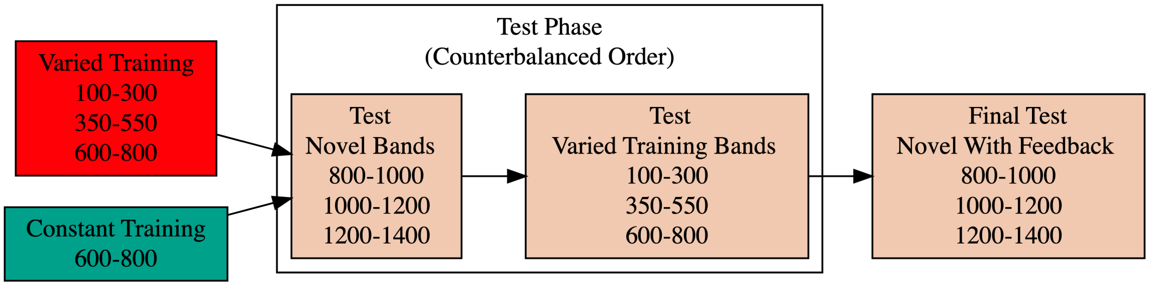


Figure 18: Experiment 2 Design. Constant and Varied participants complete different training conditions. The training and testing bands are the reverse of Experiment 1.

E2 Results

Testing Phase - No feedback.

In the first part of the testing phase, participants are tested from each of the velocity bands, and receive no feedback after each throw.

Deviation From Target Band Descriptive summaries testing deviation data are provided in Table 15 and Figure 19. To model differences in accuracy between groups, we used Bayesian mixed effects regression models to the trial level data from the testing phase. The primary model predicted the absolute deviation from the target velocity band (dist) as a function of training condition (condit), target velocity band (band), and their interaction, with random intercepts and slopes for each participant (id).

$$dist_{ij} = \beta_0 + \beta_1 \cdot condit_{ij} + \beta_2 \cdot band_{ij} + \beta_3 \cdot condit_{ij} \cdot band_{ij} + b_{0i} + b_{1i} \cdot band_{ij} + \epsilon_{ij} \tag{3}$$

Table 15: Testing Deviation - Empirical Summary

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	206	48	317
350-550	Extrapolation	194	86	268
600-800	Trained	182	112	240
800-1000	Extrapolation	200	129	233
1000-1200	Extrapolation	238	190	234
1200-1400	Extrapolation	311	254	288
Band	Band Type	Mean	Median	Sd
100-300	Trained	153	25	266
350-550	Trained	138	53	233
600-800	Trained	160	120	183
800-1000	Extrapolation	261	207	257
1000-1200	Extrapolation	305	258	273
1200-1400	Extrapolation	363	314	297

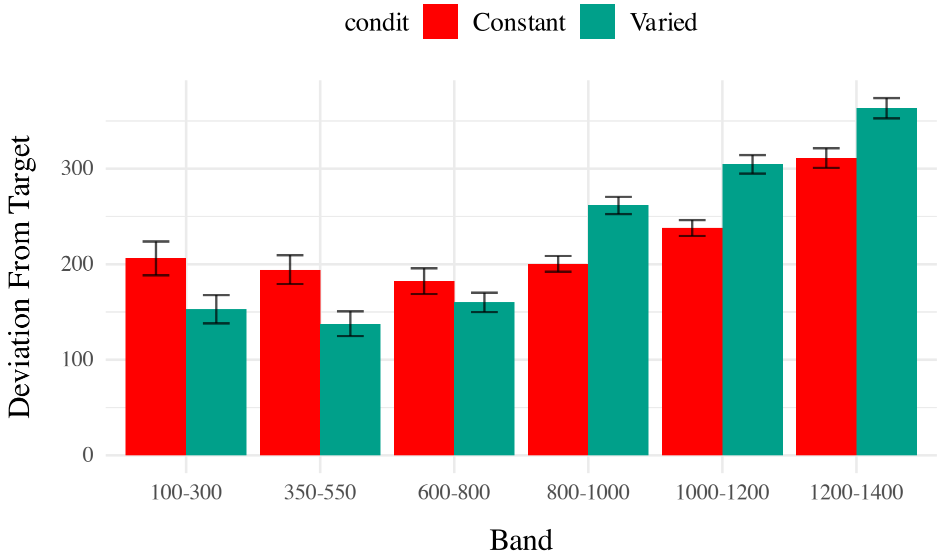


Figure 19: E2. Deviations from target band during testing without feedback stage.

Table 16: Experiment 2. Bayesian Mixed Model predicting absolute deviation as a function of condition (Constant vs. Varied) and Velocity Band

Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	151.71	90.51	215.86	1.00
conditVaried	-70.33	-156.87	16.66	0.94
Band	0.10	0.02	0.18	1.00
condit*Band	0.12	0.02	0.23	0.99

Table 17: Contrasts

contrast	Band	value	lower	upper	pd
Constant - Varied	100	57.6	-20.5	135.32	0.93
Constant - Varied	350	26.6	-30.9	83.84	0.83
Constant - Varied	600	-4.3	-46.7	38.52	0.58
Constant - Varied	800	-29.3	-69.4	11.29	0.92
Constant - Varied	1000	-54.6	-101.1	-5.32	0.98
Constant - Varied	1200	-79.6	-139.5	-15.45	0.99

The model predicting absolute deviation showed a modest tendency for the varied training group to have lower deviation compared to the constant training group (β = -70.33, 95% CI [-156.87, 16.66]),with 94% of the posterior distribution being less than 0. This suggests a potential benefit of training with variation, though the evidence is not definitive.

Experiment 3

The major manipulation adjustment of experiment 3 is for participants to receive ordinal feedback during training, in contrast to the continuous feedback of the earlier experiments. Ordinal feedback informs participants whether a throw was too soft, too hard, or fell within the target velocity range. Experiment 3 participants were randomly assigned to both a training condition (Constant vs. Varied) and a Band Order condition (original order used in Experiment 1, or the Reverse order of Experiment 2).

Results

Testing Phase - No feedback.

In the first part of the testing phase, participants are tested from each of the velocity bands, and receive no feedback after each throw. Note that these no-feedback testing trials are identical to those of Experiment 1 and 2, as the ordinal feedback only occurs during the training phase, and final testing phase, of Experiment 3.

Deviation From Target Band Descriptive summaries testing deviation data are provided in Table 18 and Figure 20. To model differences in accuracy between groups, we fit

Bayesian mixed effects regression models to the trial level data from the testing phase. The primary model predicted the absolute deviation from the target velocity band (dist) as a function of training condition (condit), target velocity band (band), and their interaction, with random intercepts and slopes for each participant (id).

Table 18: Testing Deviation - Empirical Summary

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	396	325	350
350-550	Extrapolation	278	176	299
600-800	Extrapolation	173	102	215
800-1000	Trained	225	126	284
1000-1200	Extrapolation	253	192	271
1200-1400	Extrapolation	277	210	262

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	383	254	385
350-550	Extrapolation	287	154	318
600-800	Extrapolation	213	140	244
800-1000	Trained	199	142	209
1000-1200	Trained	222	163	221
1200-1400	Trained	281	227	246

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	403	334	383
350-550	Extrapolation	246	149	287
600-800	Trained	155	82	209
800-1000	Extrapolation	207	151	241
1000-1200	Extrapolation	248	220	222
1200-1400	Extrapolation	322	281	264

Band	Band Type	Mean	Median	Sd
100-300	Trained	153	0	307
350-550	Trained	147	55	258
600-800	Trained	159	107	192
800-1000	Extrapolation	221	160	235
1000-1200	Extrapolation	244	185	235
1200-1400	Extrapolation	324	264	291

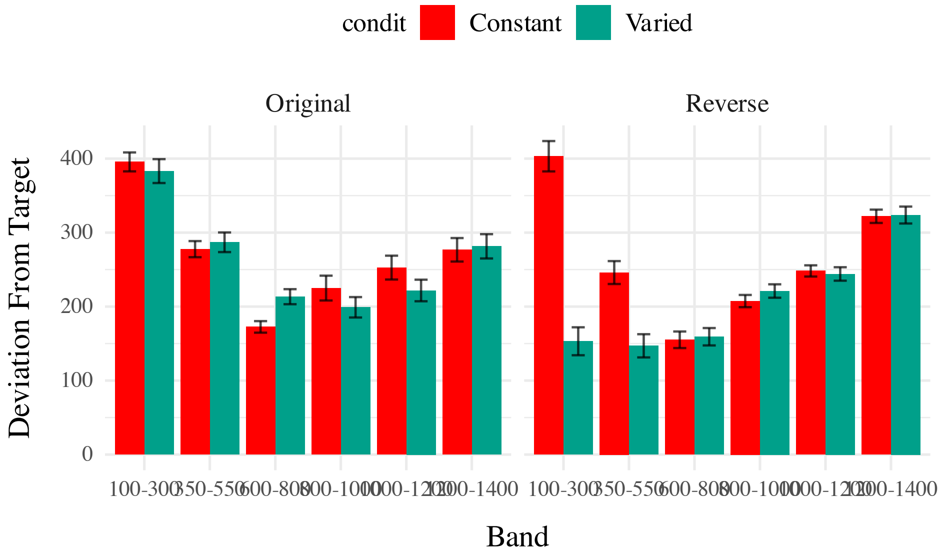


Figure 20: e3. Deviations from target band during testing without feedback stage.

Table 19: Experiment 3. Bayesian Mixed Model predicting absolute deviation as a function of condition (Constant vs. Varied) and Velocity Band

Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	306.47	243.89	368.75	1.00
conditVaried	-90.65	-182.79	3.75	0.97
Band	-0.07	-0.13	0.00	0.97
condit*Band	0.09	-0.01	0.19	0.96

The effect of training condition in Experiment 3 showed a similar pattern to Experiment 2, with the varied group tending to have lower deviation than the constant group (β = -90.65, 95% CrI [-182.79, 3.75]), with 97% of the posterior distribution falling under 0.

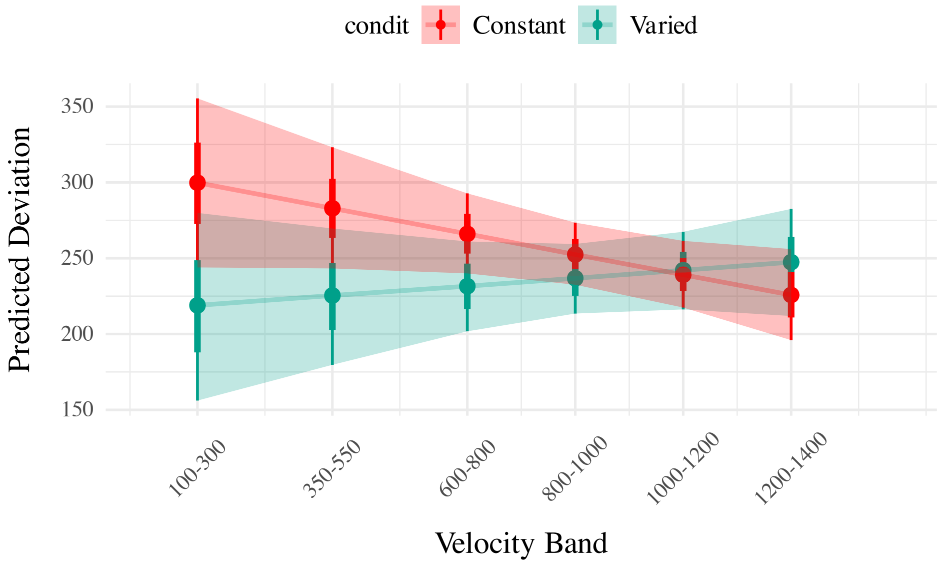


Figure 21: e3. Conditioinal Effect of Training Condition and Band. Ribbon indicated 95% Credible Intervals.

Discrimination between Velocity Bands In addition to accuracy/deviation. We also assessed the ability of participants to reliably discriminate between the velocity bands (i.e. responding differently when prompted for band 600-800 than when prompted for band 150-350). Table 20 shows descriptive statistics of this measure, and Figure 1 visualizes the full distributions of throws for each combination of condition and velocity band. To quantify discrimination, we again fit Bayesian Mixed Models as above, but this time the dependent variable was the raw x velocity generated by participants.

$$vx_{ij} = \beta_0 + \beta_1 \cdot condit_{ij} + \beta_2 \cdot bandInt_{ij} + \beta_3 \cdot condit_{ij} \cdot bandInt_{ij} + b_{0i} + b_{1i} \cdot bandInt_{ij} + \epsilon_{ij} \quad (4)$$

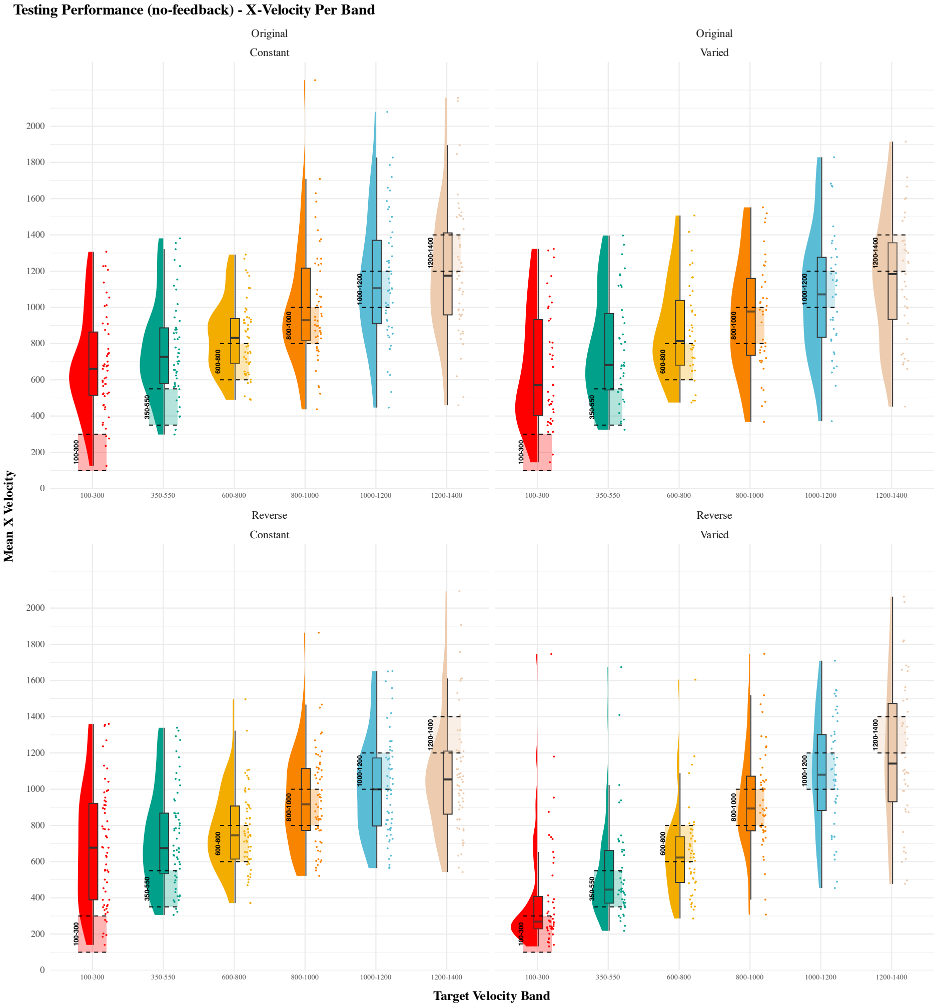


Figure 22: e3 testing x velocities. Translucent bands with dash lines indicate the correct range for each velocity band.

Table 20: Testing vx - Empirical Summary

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	680	625	370
350-550	Extrapolation	771	716	357
600-800	Extrapolation	832	786	318
800-1000	Trained	1006	916	417
1000-1200	Extrapolation	1149	1105	441
1200-1400	Extrapolation	1180	1112	443
Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	667	554	403
350-550	Extrapolation	770	688	383
600-800	Extrapolation	869	814	358
800-1000	Trained	953	928	359
1000-1200	Trained	1072	1066	388
1200-1400	Trained	1144	1093	426
Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	684	634	406
350-550	Extrapolation	729	679	350
600-800	Trained	776	721	318
800-1000	Extrapolation	941	883	387
1000-1200	Extrapolation	1014	956	403
1200-1400	Extrapolation	1072	1014	442
Band	Band Type	Mean	Median	Sd
100-300	Trained	392	270	343
350-550	Trained	540	442	343
600-800	Trained	642	588	315
800-1000	Extrapolation	943	899	394
1000-1200	Extrapolation	1081	1048	415
1200-1400	Extrapolation	1185	1129	500

Table 21: Experiment 3. Bayesian Mixed Model Predicting Vx as a function of condition (Constant vs. Varied) and Velocity Band

Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	607.67	536.02	679.87	1
conditVaried	-167.76	-277.14	-64.08	1
Band	0.44	0.35	0.52	1
condit*Band	0.18	0.06	0.31	1

See Table 21 for the full model results.

Slope estimates for experiment 3 suggest that participants were capable of distinguishing between velocity bands even when provided only ordinal feedback during training (β = 0.44, 95% CrI [0.35, 0.52]). Unlike the previous two experiments, the posterior distribution for the interaction between condition and band was consistently positive, suggestive of superior discrimination for the varied participants β = 0.18, 95% CrI [0.06, 0.31].

Modeling

In project 1, we applied model-based techniques to quantify and control for the similarity between training and testing experience, which in turn enabled us to account for the difference between varied and constant training via an extended version of a similarity based generalization model. In project 2, we will go a step further, implementing a full process model capable of both 1) producing novel responses and 2) modeling behavior in both the learning and testing stages of the experiment. For this purpose, we will apply the associative learning model (ALM) and the EXAM model of function learning (DeLosh 1997). ALM is a simple connectionist learning model which closely resembles Kruschke's ALCOVE model (Kruscke 1992), with modifications to allow for the generation of continuous responses.

ALM & Exam Description

DeLosh et al. (1997) introduced the associative learning model (ALM), a connectionist model within the popular class of radial-basis networks. ALM was inspired by, and closely resembles Kruschke's influential ALCOVE model of categorization (Kruschke, 1992).

ALM is a localist neural network model, with each input node corresponding to a particular stimulus, and each output node corresponding to a particular response value. The units in the input layer activate as a function of their Gaussian similarity to the input stimulus. So, for example, an input stimulus of value 55 would induce maximal activation of the input unit tuned to 55. Depending on thevalue of the generalization parameter, the nearby units (e.g. 54 and 56; 53 and 57) may also activate to some degree. ALM is structured with input and output nodes that correspond to regions of the stimulus space, and response space, respectively. The units in the input layer activate as a function of their similarity to a presented stimulus. As was the case with the exemplar-based models, similarity in ALM is exponentially decaying function of distance. The input layer is fully connected to the output layer, and the activation for any particular output node is simply the weighted sum of the connection weights between that node and the input activations. The network then produces a response by taking the weighted average of the output units (recall that each output unit has a value corresponding to a particular response). During training, the network receives feedback which activates each output unit as a function of its distance from the ideal level of activation necessary to produce the correct response. The connection weights between input and output units are then updated via the standard delta learning rule, where the magnitude of weight changes are controlled by a learning rate parameter.

See for a full specification of the equations that define ALM and EXAM.

Model Table

ALM Activation & Response

Step	Equation	Description
ALM Activation & Response		
Input Activation	$a_i(X) = \frac{e^{-c(X-X_i)^2}}{\sum_{k=1}^{M} e^{-c(X-X_k)^2}}$	Activation of each input node X_i, is a function of the Gaussian similarity between the node value and stimulus X.
Output Activation	$O_j(X) = \sum_{k=1}^{M} w_{ji} \cdot a_i(X)$	Activation of each Output unit O_j is the weighted sum of the input activations and association weights.
Output Probability	$P[Y_j \| X] = \frac{O_j(X)}{\sum_{k=1}^{M} O_k(X)}$	Each output node has associated response, Y_j. The probability of response Y_j is determined by the ratio of output activations.
Mean Output	$m(x) = \sum_{j=1}^{L} Y_j \cdot \frac{O_j(x)}{\sum_{k=1}^{M} O_k(X)}$	The response to stimulus x is the weighted average of the response probabilities.
ALM Learning		
Feedback Activation	$f_j(Z) = e^{-c(Z-Y_j)^2}$	After responding, feedback signal Z is presented, activating each output node via the Gaussian similarity to the ideal response.
Update Weights	$w_{ji}(t+1) = w_{ji}(t) + \alpha \cdot (f_j(Z(t)) - O_j(X(t)) \cdot a_i(X(t))$	Delta rule to update weights. Magnitude of weight changes controlled by learning rate parameter alpha.
EXAM		
Extrapolation	$P[X_i \| X] = \frac{a_i(X)}{\sum_{k=1}^{M} a_k(X)}$	Novel test stimulus X activates input nodes associated with trained stimuli.
	$E[Y \| X_i] = m(X_i) + \frac{m(X_{i+1}) - m(X_{i-1})}{X_{i+1} - X_{i-1}} \cdot [X - X_i]$	Slope value computed from nearest training instances and then added to the response associated with the nearest training instance,m(x)

Model Fitting and Comparison

Following the procedure used by M. Mcdaniel et al. (2009), we will assess the ability of both ALM and EXAM to account for the empirical data when fitting the models to 1) only the training data, and 2) both training and testing data. Models will be fit directly to the trial by trial data of each individual participants, both by minimizing the root-mean squared deviation (RMSE), and by maximizing log likelihood. Because ALM has been shown to do poorly at accounting for human patterns extrapolation (DeLosh et al., 1997), we will also fit the extended EXAM version of the model, which operates identically to ALM during training, but includes a linear extrapolation mechanism for generating novel responses during testing.

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