

## **test\_types**

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```
pacman::p_load(dplyr, purrr, tidyr, tibble, ggplot2,  
  brms, tidybayes, rstanarm, emmeans, broom, bayestestR,  
  stringr, here, conflicted, patchwork, knitr, gt)  
  
out_type <- knitr::opts_knit$get("rmarkdown.pandoc.to")  
print(out_type)
```

```
[1] "latex"
```

```
primary=out_type == "html" || out_type == "pdf" || out_type == "latex"  
  
fmt_out <- knitr::pandoc_to()  
print(fmt_out)
```

```
[1] "latex"
```

```
out_type %in% c("html", "pdf")
```

```
[1] FALSE
```

```
# out_type = "html"

out_type == "html" || out_type == "pdf" || out_type == "docx"
```

[1] FALSE

Iris Flower Data  
A tibble: 150 × 5

| Sepal Characteristics |             |              |             |         |
|-----------------------|-------------|--------------|-------------|---------|
| Sepal.Length          | Sepal.Width | Petal.Length | Petal.Width | Species |
| 5.1                   | 3.5         | 1.4          | 0.2         | setosa  |
| 4.9                   | 3.0         | 1.4          | 0.2         | setosa  |
| 4.7                   | 3.2         | 1.3          | 0.2         | setosa  |
| 4.6                   | 3.1         | 1.5          | 0.2         | setosa  |
| 5.0                   | 3.6         | 1.4          | 0.2         | setosa  |
| 5.4                   | 3.9         | 1.7          | 0.4         | setosa  |
| 4.6                   | 3.4         | 1.4          | 0.3         | setosa  |
| 5.0                   | 3.4         | 1.5          | 0.2         | setosa  |
| 4.4                   | 2.9         | 1.4          | 0.2         | setosa  |
| 4.9                   | 3.1         | 1.5          | 0.1         | setosa  |
| 5.4                   | 3.7         | 1.5          | 0.2         | setosa  |
| 4.8                   | 3.4         | 1.6          | 0.2         | setosa  |
| 4.8                   | 3.0         | 1.4          | 0.1         | setosa  |
| 4.3                   | 3.0         | 1.1          | 0.1         | setosa  |

|     |     |     |     |        |
|-----|-----|-----|-----|--------|
| 5.8 | 4.0 | 1.2 | 0.2 | setosa |
| 5.7 | 4.4 | 1.5 | 0.4 | setosa |
| 5.4 | 3.9 | 1.3 | 0.4 | setosa |
| 5.1 | 3.5 | 1.4 | 0.3 | setosa |
| 5.7 | 3.8 | 1.7 | 0.3 | setosa |
| 5.1 | 3.8 | 1.5 | 0.3 | setosa |
| 5.4 | 3.4 | 1.7 | 0.2 | setosa |
| 5.1 | 3.7 | 1.5 | 0.4 | setosa |
| 4.6 | 3.6 | 1.0 | 0.2 | setosa |
| 5.1 | 3.3 | 1.7 | 0.5 | setosa |
| 4.8 | 3.4 | 1.9 | 0.2 | setosa |
| 5.0 | 3.0 | 1.6 | 0.2 | setosa |
| 5.0 | 3.4 | 1.6 | 0.4 | setosa |
| 5.2 | 3.5 | 1.5 | 0.2 | setosa |
| 5.2 | 3.4 | 1.4 | 0.2 | setosa |
| 4.7 | 3.2 | 1.6 | 0.2 | setosa |
| 4.8 | 3.1 | 1.6 | 0.2 | setosa |
| 5.4 | 3.4 | 1.5 | 0.4 | setosa |
| 5.2 | 4.1 | 1.5 | 0.1 | setosa |
| 5.5 | 4.2 | 1.4 | 0.2 | setosa |
| 4.9 | 3.1 | 1.5 | 0.2 | setosa |
| 5.0 | 3.2 | 1.2 | 0.2 | setosa |

|     |     |     |     |            |
|-----|-----|-----|-----|------------|
| 5.5 | 3.5 | 1.3 | 0.2 | setosa     |
| 4.9 | 3.6 | 1.4 | 0.1 | setosa     |
| 4.4 | 3.0 | 1.3 | 0.2 | setosa     |
| 5.1 | 3.4 | 1.5 | 0.2 | setosa     |
| 5.0 | 3.5 | 1.3 | 0.3 | setosa     |
| 4.5 | 2.3 | 1.3 | 0.3 | setosa     |
| 4.4 | 3.2 | 1.3 | 0.2 | setosa     |
| 5.0 | 3.5 | 1.6 | 0.6 | setosa     |
| 5.1 | 3.8 | 1.9 | 0.4 | setosa     |
| 4.8 | 3.0 | 1.4 | 0.3 | setosa     |
| 5.1 | 3.8 | 1.6 | 0.2 | setosa     |
| 4.6 | 3.2 | 1.4 | 0.2 | setosa     |
| 5.3 | 3.7 | 1.5 | 0.2 | setosa     |
| 5.0 | 3.3 | 1.4 | 0.2 | setosa     |
| 7.0 | 3.2 | 4.7 | 1.4 | versicolor |
| 6.4 | 3.2 | 4.5 | 1.5 | versicolor |
| 6.9 | 3.1 | 4.9 | 1.5 | versicolor |
| 5.5 | 2.3 | 4.0 | 1.3 | versicolor |
| 6.5 | 2.8 | 4.6 | 1.5 | versicolor |
| 5.7 | 2.8 | 4.5 | 1.3 | versicolor |
| 6.3 | 3.3 | 4.7 | 1.6 | versicolor |
| 4.9 | 2.4 | 3.3 | 1.0 | versicolor |

|     |     |     |     |            |
|-----|-----|-----|-----|------------|
| 6.6 | 2.9 | 4.6 | 1.3 | versicolor |
| 5.2 | 2.7 | 3.9 | 1.4 | versicolor |
| 5.0 | 2.0 | 3.5 | 1.0 | versicolor |
| 5.9 | 3.0 | 4.2 | 1.5 | versicolor |
| 6.0 | 2.2 | 4.0 | 1.0 | versicolor |
| 6.1 | 2.9 | 4.7 | 1.4 | versicolor |
| 5.6 | 2.9 | 3.6 | 1.3 | versicolor |
| 6.7 | 3.1 | 4.4 | 1.4 | versicolor |
| 5.6 | 3.0 | 4.5 | 1.5 | versicolor |
| 5.8 | 2.7 | 4.1 | 1.0 | versicolor |
| 6.2 | 2.2 | 4.5 | 1.5 | versicolor |
| 5.6 | 2.5 | 3.9 | 1.1 | versicolor |
| 5.9 | 3.2 | 4.8 | 1.8 | versicolor |
| 6.1 | 2.8 | 4.0 | 1.3 | versicolor |
| 6.3 | 2.5 | 4.9 | 1.5 | versicolor |
| 6.1 | 2.8 | 4.7 | 1.2 | versicolor |
| 6.4 | 2.9 | 4.3 | 1.3 | versicolor |
| 6.6 | 3.0 | 4.4 | 1.4 | versicolor |
| 6.8 | 2.8 | 4.8 | 1.4 | versicolor |
| 6.7 | 3.0 | 5.0 | 1.7 | versicolor |
| 6.0 | 2.9 | 4.5 | 1.5 | versicolor |
| 5.7 | 2.6 | 3.5 | 1.0 | versicolor |

|     |     |     |     |            |
|-----|-----|-----|-----|------------|
| 5.5 | 2.4 | 3.8 | 1.1 | versicolor |
| 5.5 | 2.4 | 3.7 | 1.0 | versicolor |
| 5.8 | 2.7 | 3.9 | 1.2 | versicolor |
| 6.0 | 2.7 | 5.1 | 1.6 | versicolor |
| 5.4 | 3.0 | 4.5 | 1.5 | versicolor |
| 6.0 | 3.4 | 4.5 | 1.6 | versicolor |
| 6.7 | 3.1 | 4.7 | 1.5 | versicolor |
| 6.3 | 2.3 | 4.4 | 1.3 | versicolor |
| 5.6 | 3.0 | 4.1 | 1.3 | versicolor |
| 5.5 | 2.5 | 4.0 | 1.3 | versicolor |
| 5.5 | 2.6 | 4.4 | 1.2 | versicolor |
| 6.1 | 3.0 | 4.6 | 1.4 | versicolor |
| 5.8 | 2.6 | 4.0 | 1.2 | versicolor |
| 5.0 | 2.3 | 3.3 | 1.0 | versicolor |
| 5.6 | 2.7 | 4.2 | 1.3 | versicolor |
| 5.7 | 3.0 | 4.2 | 1.2 | versicolor |
| 5.7 | 2.9 | 4.2 | 1.3 | versicolor |
| 6.2 | 2.9 | 4.3 | 1.3 | versicolor |
| 5.1 | 2.5 | 3.0 | 1.1 | versicolor |
| 5.7 | 2.8 | 4.1 | 1.3 | versicolor |
| 6.3 | 3.3 | 6.0 | 2.5 | virginica  |
| 5.8 | 2.7 | 5.1 | 1.9 | virginica  |

|     |     |     |     |           |
|-----|-----|-----|-----|-----------|
| 7.1 | 3.0 | 5.9 | 2.1 | virginica |
| 6.3 | 2.9 | 5.6 | 1.8 | virginica |
| 6.5 | 3.0 | 5.8 | 2.2 | virginica |
| 7.6 | 3.0 | 6.6 | 2.1 | virginica |
| 4.9 | 2.5 | 4.5 | 1.7 | virginica |
| 7.3 | 2.9 | 6.3 | 1.8 | virginica |
| 6.7 | 2.5 | 5.8 | 1.8 | virginica |
| 7.2 | 3.6 | 6.1 | 2.5 | virginica |
| 6.5 | 3.2 | 5.1 | 2.0 | virginica |
| 6.4 | 2.7 | 5.3 | 1.9 | virginica |
| 6.8 | 3.0 | 5.5 | 2.1 | virginica |
| 5.7 | 2.5 | 5.0 | 2.0 | virginica |
| 5.8 | 2.8 | 5.1 | 2.4 | virginica |
| 6.4 | 3.2 | 5.3 | 2.3 | virginica |
| 6.5 | 3.0 | 5.5 | 1.8 | virginica |
| 7.7 | 3.8 | 6.7 | 2.2 | virginica |
| 7.7 | 2.6 | 6.9 | 2.3 | virginica |
| 6.0 | 2.2 | 5.0 | 1.5 | virginica |
| 6.9 | 3.2 | 5.7 | 2.3 | virginica |
| 5.6 | 2.8 | 4.9 | 2.0 | virginica |
| 7.7 | 2.8 | 6.7 | 2.0 | virginica |
| 6.3 | 2.7 | 4.9 | 1.8 | virginica |



|     |     |     |     |           |
|-----|-----|-----|-----|-----------|
| 6.7 | 3.3 | 5.7 | 2.1 | virginica |
| 7.2 | 3.2 | 6.0 | 1.8 | virginica |
| 6.2 | 2.8 | 4.8 | 1.8 | virginica |
| 6.1 | 3.0 | 4.9 | 1.8 | virginica |
| 6.4 | 2.8 | 5.6 | 2.1 | virginica |
| 7.2 | 3.0 | 5.8 | 1.6 | virginica |
| 7.4 | 2.8 | 6.1 | 1.9 | virginica |
| 7.9 | 3.8 | 6.4 | 2.0 | virginica |
| 6.4 | 2.8 | 5.6 | 2.2 | virginica |
| 6.3 | 2.8 | 5.1 | 1.5 | virginica |
| 6.1 | 2.6 | 5.6 | 1.4 | virginica |
| 7.7 | 3.0 | 6.1 | 2.3 | virginica |
| 6.3 | 3.4 | 5.6 | 2.4 | virginica |
| 6.4 | 3.1 | 5.5 | 1.8 | virginica |
| 6.0 | 3.0 | 4.8 | 1.8 | virginica |
| 6.9 | 3.1 | 5.4 | 2.1 | virginica |
| 6.7 | 3.1 | 5.6 | 2.4 | virginica |
| 6.9 | 3.1 | 5.1 | 2.3 | virginica |
| 5.8 | 2.7 | 5.1 | 1.9 | virginica |
| 6.8 | 3.2 | 5.9 | 2.3 | virginica |
| 6.7 | 3.3 | 5.7 | 2.5 | virginica |
| 6.7 | 3.0 | 5.2 | 2.3 | virginica |

|     |     |     |     |           |
|-----|-----|-----|-----|-----------|
| 6.3 | 2.5 | 5.0 | 1.9 | virginica |
| 6.5 | 3.0 | 5.2 | 2.0 | virginica |
| 6.2 | 3.4 | 5.4 | 2.3 | virginica |
| 5.9 | 3.0 | 5.1 | 1.8 | virginica |

See Table 1 for a full specification of the equations that define ALM and EXAM, and ?@fig-alm-diagram for a visual representation of the ALM model.

### 💡 Approximate Bayesian Computation

To estimate the parameters of ALM and EXAM, we used approximate Bayesian computation (ABC), enabling us to obtain an estimate of the posterior distribution of the generalization and learning rate parameters for each individual. ABC belongs to the class of simulation-based inference methods (Cranmer et al., 2020), which have begun being used for parameter estimation in cognitive modeling relatively recently (Kangasrääsiö et al., 2019; Turner & Van Zandt, 2012; Turner et al., 2016). Although they can be applied to any model from which data can be simulated, ABC methods are most useful for complex models that lack an explicit likelihood function (e.g., many neural network models).

The general ABC procedure is to 1) define a prior distribution over model parameters. 2) sample candidate parameter values,  $\theta^*$ , from the prior. 3) Use  $\theta^*$  to generate a simulated dataset,  $Data_{sim}$ . 4) Compute a measure of discrepancy between the simulated and observed datasets,  $discrep(Data_{sim}, Data_{obs})$ . 5) Accept  $\theta^*$  if the discrepancy is less than the tolerance threshold,  $\epsilon$ , otherwise reject  $\theta^*$ . 6) Repeat until desired number of posterior samples are obtained.

Table 1: ALM &amp; EXAM Equations

| ALM Response Generation              |   |  |
|--------------------------------------|---|--|
| Input Activation                     | $a_i(X) = \frac{e^{-c(X-X_i)^2}}{\sum_{k=1}^M e^{-c(X-X_k)^2}}$                       | Input nodes activate as a function of Gaussian similarity to stimulus  |
| Output Activation                    | $O_j(X) = \sum_{k=1}^M w_{ji} \cdot a_i(X)$   | Output unit $O_j$ activation is the weighted sum of input activations and association weights                    |
| Output Probability                   | $P[Y_j X] = \frac{O_j(X)}{\sum_{k=1}^M O_k(X)}$                                       | The response, $Y_j$ probabilities computed via Luce's choice rule  |
| Mean Output                          | $m(X) = \sum_{j=1}^L Y_j \cdot \frac{O_j(X)}{\sum_{k=1}^M O_k(X)}$                    | Weighted average of probabilities determines response to X   |
| <b>ALM Learning</b>                  |   |  |
| Feedback                             | $f_j(Z) = e^{-c(Z-Y_j)^2}$  | feedback signal Z computed as similarity between ideal response and observed response                            |
| magnitude of error<br>Update Weights | $\Delta_{ji} = (f_j(Z) - o_j(X))a_i(X)$<br>$w_{ji}^{new} = w_{ji} + \eta \Delta_{ji}$ | Delta rule to update weights. Updates scaled by learning rate parameter $\eta$ .                                 |
| <b>EXAM Extrapolation</b>            |   |  |
| Instance Retrieval                   | $P[X_i X] = \frac{a_i(X)}{\sum_{k=1}^M a_k(X)}$                                       | Novel test stimulus X activates input nodes $X_i$  |
| Slope Computation                    | $S = \frac{m(X_1) - m(X_2)}{X_1 - X_2}$   | Slope value, S computed from nearest training instances  |
| Response                             | $E[Y X_i] = m(X_i) + S \cdot [X - X_i]$   | Final EXAM response is the ALM response for the nearest training stimulus, $m(X_i)$ , adjusted by local slope S. |

Although simple in the abstract, implementations of ABC require researchers to make a number of non-trivial decisions as to i) the discrepancy function between observed and simulated data, ii) whether to compute the discrepancy between trial level data, or a summary statistic of the datasets, iii) the value of the minimum tolerance  $\epsilon$  between simulated and observed data. For the present work, we follow the guidelines from previously published ABC tutorials (Farrell & Lewandowsky, 2018; Turner & Van Zandt, 2012). For the test stage, we summarized datasets with mean velocity of each band in the observed dataset as  $V_{obs}^{(k)}$  and in the simulated dataset as  $V_{sim}^{(k)}$ , where  $k$  represents each of the six velocity bands. For computing the discrepancy between datasets in the training stage, we aggregated training trials into three equally sized blocks (separately for each velocity band in the case of the varied group). After obtaining the summary statistics of the simulated and observed datasets, the discrepancy was computed as the mean of the absolute difference between simulated and observed datasets (Equation 1 and Equation 2). For the models fit to both training and testing data, discrepancies were computed for both stages, and then averaged together.

$$discrep_{Test}(Data_{sim}, Data_{obs}) = \frac{1}{6} \sum_{k=1}^6 |V_{obs}^{(k)} - V_{sim}^{(k)}| \quad (1)$$

$$discrep_{Train,constant}(Data_{sim}, Data_{obs}) = \frac{1}{N_{blocks}} \sum_{j=1}^{N_{blocks}} |V_{obs,constant}^{(j)} - V_{sim,constant}^{(j)}| \quad (2)$$

$$discrep_{Train,varied}(Data_{sim}, Data_{obs}) = \frac{1}{N_{blocks} \times 3} \sum_{j=1}^{N_{blocks}} \sum_{k=1}^3 |V_{obs,varied}^{(j,k)} - V_{sim,varied}^{(j,k)}|$$

The final component of our ABC implementation is the determination of an appropriate value of  $\epsilon$ . The setting of  $\epsilon$  exerts strong influence on the approximated posterior distribution. Smaller values of  $\epsilon$  increase the rejection rate, and improve the fidelity of the approximated posterior, while larger values result in an ABC sampler that simply reproduces the prior distribution. Because the individual participants in our dataset differed substantially in terms of the noisiness of their data, we employed an adaptive tolerance setting strategy to tailor  $\epsilon$  to each individual. The initial value of  $\epsilon$  was set to the overall standard deviation of each individuals velocity values. Thus, sampled parameter values that generated simulated data within a standard deviation of the observed data were accepted, while worse performing parameters were rejected. After every 300 samples the tolerance was allowed to increase only if the current acceptance rate of the algorithm was less than 1%. In such cases, the tolerance was shifted towards the average discrepancy of the 5 best samples obtained thus far. To ensure the acceptance rate did not become overly permissive,  $\epsilon$  was also allowed to decrease every time a sample was accepted into the posterior.

## Appendix

[Appendix](https://tegorman13.github.io/Dissertation/Sections/Appendix.html) available at [tegorman13.github.io/Dissertation/Sections/Appendix.html](https://tegorman13.github.io/Dissertation/Sections/Appendix.html)

### References

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