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# The Role of Variability in Learning Generalization: A Computational Modeling Approach

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Researchers and educators have long been interested in how the variability of training examples experienced during learning influences subsequent generalization. However, previous studies have reported both beneficial and detrimental effects, leaving the nature of this relationship unclear. Furthermore, a formal computational modeling approach to account for the effects of training variability has been largely lacking within the domain of visuomotor skill learning. The current work presents two projects which shed light on this issue, each consisting of several experiments which compare varied training (i.e., multiple training items) in comparison to constant training (training with a single item). The empirical results, much like those of the broader literature, provide evidence both for and against a benefit of variable training in visuomotor skill learning, highlighting the complex interplay between training conditions and generalization performance. To provide a theoretical account of these findings, this dissertation employs both instance-based and connectionist computational modeling approaches. The instance-based modeling approach introduced in project 1 provides a theoretically justifiable method of quantifying/controlling for similarity between training and testing conditions, while also demonstrating that varied training may induce broader generalization in the similarity function relating training and test items. In project 2, the Extrapolation- Association Model (EXAM) provided the best account of the testing data across all experiments, capturing the constant groups’ ability to extrapolate linearly to novel regions despite limited training experience, while also revealing potential detriments of varied training for simple extrapolation tasks. By integrating empirical findings with computational modeling approaches, this dissertation sheds light on the complex relationship between training variability and generalization performance.

# Introduction

## Varied Training and Generalization

Varied training has been shown to influence learning in a wide array of different tasks and domains, including categorization (Hahn et al., 2005; Maddox & Filoteo, 2011; Morgenstern et al., 2019; Nosofsky et al., 2019; Plebanek & James, 2021; Posner & Keele, 1968), language learning (Brekelmans et al., 2022; Jones & Brandt, 2020; Perry et al., 2010; Twomey et al., 2018; Wonnacott et al., 2012), anagram completion (Goode et al., 2008), perceptual learning (Lovibond et al., 2020; Manenti et al., 2023; Robson et al., 2022; Zaman et al., 2021), trajectory extrapolation (Fulvio et al., 2014), cognitive control tasks (Moshon-Cohen et al., 2024; Sabah et al., 2019), associative learning (Fan et al., 2022; Lee et al., 2019; Livesey & McLaren, 2019; Prada & Garcia-Marques, 2020; Reichmann et al., 2023), visual search (George & Egner, 2021; Gonzalez & Madhavan, 2011; Kelley & Yantis, 2009), voice identity learning (Lavan et al., 2019), face recognition (Burton et al., 2016; Honig et al., 2022; Menon et al., 2015), the perception of social group heterogeneity (Gershman & Cikara, 2023; Konovalova & Le Mens, 2020; Linville & Fischer, 1993; Park & Hastie, 1987) , simple motor learning (Braun et al., 2009; Kerr & Booth, 1978; Roller et al., 2001; Willey & Liu, 2018), sports training (Breslin et al., 2012; Green et al., 1995; North et al., 2019), and complex skill learning (Hacques et al., 2022; Huet et al., 2011; Seow et al., 2019). See Czyż (2021) or Raviv et al. (2022) for more detailed reviews.

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