Running on Four Legs As Though They Were One

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Abstract—**Simple locomotion algorithms provide balance for machines that run on one leg. The generalization of these one-leg algorithms for control of machines with several legs is explored. The generalization is quite simple when multilegged systems run with gaits that use the support legs one at a time. For these gaits the one-leg algorithms can be used to control multilegged running. The concept of a *virtual leg* is introduced to further extend the approach to gaits that use the legs in pairs, such as the trot, the pace, and the bound. These quadruped running gaits map into gaits that use one virtual leg for support at a time, for which the one-leg algorithms can provide control. This approach was used in laboratory experiments to control a quadruped machine that runs with a trotting gait.**

I. Introduction

RUNNING is a form of legged locomotion characterized by travel at high speed and by periods of ballistic flight during which all feet leave the ground. The basic control task is to establish a pattern of leg and body motions that stabilizes the attitude and altitude of the body while propelling the body in the desired direction at the desired speed [2]. The characteristics of high speed and ballistic flight suggest that dynamics and active stability are important to accomplishing this control task, and to getting a better general understanding of running.

In recent work the authors used dynamic techniques to control an experimental one-legged machine that balances itself as it runs by hopping [3, 4]. The goal of these experiments was to focus on the dynamic aspects of locomotion, while avoiding the difficult task of coordinating the behavior of several legs. This paper addresses the task of leg coordination by starting with the algorithms that were used to control the one-legged hopping machine, and extending them to control a quadruped running machine. The general goal is to develop a set of locomotion principles that applies to the dynamic behavior of diverse legged systems—machines and animals alike—independent of the number of legs.

Background

Workers first began to model and simulate dynamic legged locomotion in the late 1960's. Frank used a dynamic model to establish the feasibility of static control techniques for multilegged crawlers [5]. Vukobratović and Frank defined criteria for dynamic stability in legged locomotion and several others explored the control of walking in dynamic biped and quadruped models, assuming planar motion, and massless legs [5]–[9]. Extension to models having legs with mass [10]–[12] and motion in three dimensions [13, 14] followed. An important outcome of this period was the development of the *inverted pendulum model* of locomotion, which has been studied widely, e.g. [13], [15]–[18].

Physical legged machines that employed dynamic techniques for stability appeared in the late 1970's. Kato and his co-workers built a biped that walked with a quasi-dynamic gait, transferring support from one large foot to the other by temporarily destabilizing itself. A motion was used between transfers that preserved static stability [19]–[21]. Miura and Shimoyama built a three-dimensional biped that had legs like stilts with small feet. It balanced itself actively by rapidly respositioning a foot on each step [22], [23]. Unlike Kato's machine, which came to static equilibrium between each dynamic transfer, the stilt biped tipped all the time. Both Kato and Miura used the inverted pendulum model to characterize and predict the dynamic tipping behavior of their machines.

Matsuoka was the first to build a machine that ran, where running is defined by periods of ballistic flight with all feet leaving the ground [24], [25]. He considered an extreme form of running, for which nearly the entire cycle was spent in flight. This minimized the influence of tipping during support. Matsuoka derived a time-optimal state feedback controller that provided stability for hopping in place and for low speed translations. Experiments were done with a planar one-legged machine that operated by lying on a table inclined 10° from the horizontal. The machine rolled on ball bearings operating in an effective 18-percent gravity.

The present authors studied one-legged machines with springy legs. They found that a simple set of control algorithms provided balance when the control system was decomposed into separate parts for controlling hopping height, forward running velocity, and body attitude [3], [4], [26]. This paper extends the use of the three-part control decomposition to systems with more than one leg. The approach is to consider biped and quadruped running in terms of components that we understand from work with the one-legged systems. We argue that biped running has the same essential features as one-legged running, and that certain gaits of the quadruped are, in turn, like those of the biped. The first step is to show that a system with two legs can run as though it had just one. This is possible when the running gait uses just one leg for support at a time, with the other leg elevated and out of the way. The legs alternate like the barrels of a Gatling gun, with only one firing at a time. Humans run this way. The control algorithms for this mode of running are like those used to control one-legged machines.

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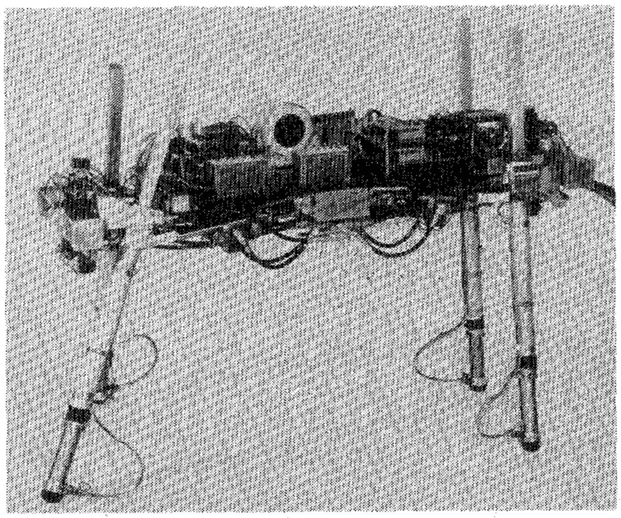

Fig. 1. Photograph of four-legged running machine used for experiments.

The second step in the approach considers the quadruped gaits that use the legs in pairs: the trot, the pace, and the bound. If the control system can make the two legs that form a pair work together as though they were one leg, then a quadruped can be controlled like an equivalent biped. Once this is done, the Gatling approach and the one-legged control algorithms can again be used. The result is a set of locomotion algorithms for controlling systems with several legs, that builds upon the algorithms originally used to control one leg. We have verified the feasibility of this approach with experiments on a four-legged machine that runs with a trotting gait (see Fig. 1).

Because this approach builds on the methods used for controlling one-legged systems, those methods are reviewed briefly in the next section. The subsequent sections extend the methods for control of several legs, first for a special class of running gaits that use only one leg for support at a time, and then for quadruped gaits that use pairs of legs in unison: the trot, the pace, and the bound. Section V reports experimental results from the physical four-legged running machine.

II. Review of One-Leg Algorithms

The essential features of the one-legged machines we studied are a body and a leg connected by a hinge-type hip. The body has its center of mass located at the hip, and its moment of inertia is several times larger than that of the leg. One actuator exerts a torque between the leg and the body at the hip, while another actuator powers axial leg motion. The leg has a spring in series with the axial actuator, so hopping is a resonant bouncing motion. The control system produces hopping by exciting the spring-mass system formed by the leg and the body. The locomotion cycle consists of an alteration between a stance phase, in which the foot touches the ground and the leg provides support to the body, and a flight phase, in which the leg lifts the foot from the ground in order to move it from one foothold to another.

The locomotion algorithms we explored decompose the control problem into three parts. One part of the control specifies the thrust to be delivered by the leg to the ground to regulate the amplitude of the machine's bouncing motion. A second part maintains the body in an upright posture. A third part of the control determines a forward position for the foot that stabilizes the forward running speed. Details of these three parts of the control system are as follows.

Hopping Height: The control system excites the cyclic hopping motion that underlies running while regulating how high the machine hops. The hopping motion is an oscillation governed by the mass of the body, the springiness of the leg, and gravity. During support, the body bounces on the springy leg and during flight, the system travels a ballistic trajectory. The control system delivers a vertical thrust with the leg during each support period to sustain the oscillation and to regulate its amplitude. Some of the energy needed for each hop is recovered by the leg spring from the previous hop.

Body Attitude: The control system maintains an erect body posture during running by generating a torque between the leg and the body about the hip during stance. Friction between the foot and the ground permits large torques to be applied to the body during stance without causing large accelerations of the leg. These torques are used to implement a linear servo that moves the body toward an erect posture once each step:

$$\tau = -k_p(\phi - \phi_d) - k_v(\dot{\phi}) \tag{1}$$

where

τ	hip torque
ϕ	inclination of the body
ϕ_d	desired inclination of the body
$\dot{\phi}$	inclination rate of the body
k_p, k_v	gains.

Forward Running Speed: The control system manipulates forward running speed by choosing a forward position for the foot during the ballistic flight portion of each stride that accelerates the body properly during the next support interval. The *CG-print*, shown in Fig. 2, is the locus of points on the ground over which the center of mass of the body will travel during stance. When the control system places the foot in the center of the CG-print, it generates no change in forward running speed. This point that produces no net acceleration is called the *neutral point*. When the control system displaces the foot from the neutral point, the body accelerates with magnitude and direction related to the magnitude and direction of the foot's displacement.

For instance, placing the foot behind the neutral point causes the system to run faster, and placing the foot forward of the neutral point causes it to run slower. Fig. 3 illustrates this effect. The control system estimates the length of the CG-print from the forward velocity and the duration of stance. The discrepancy between the measured velocity and the desired velocity determines how far to displace the foot from the neutral point. The overall calculation of foot placement is done with the following equation:

$$x_f = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d) \tag{2}$$

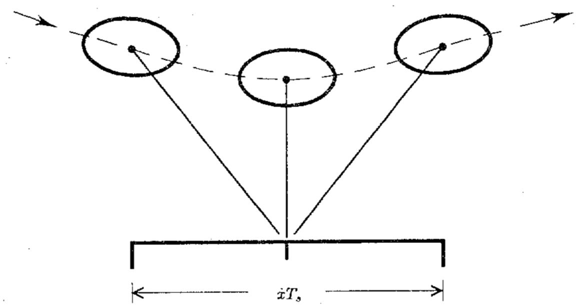

Fig. 2. Symmetry of motion in one-legged running. The diagrams depict running from left to right. When the foot is placed in the center of the CG-print, the neutral point, there is a symmetric motion of the body with respect to the foot. The figure shows the configuration just before the foot touches the ground (left), the configuration half way through stance when the leg is vertical and maximally compressed (center), the configuration just after the foot loses contact with the ground (right).

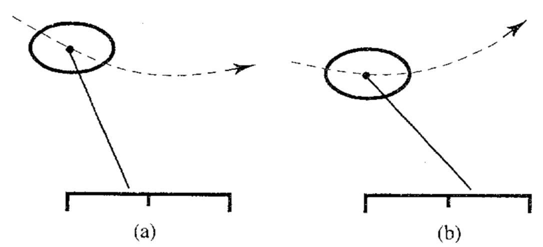

Fig. 3. Displacement of the foot from the neutral point accelerates the body by skewing the symmetry of the body's trajectory. (a) When the foot is placed behind the neutral point, the body accelerates forward during stance. (b) When the foot is placed in front of the neutral point, the body accelerates backward during stance. Horizontal lines under the each figure indicate the CG-print, and the dashed lines indicate the path of the body.

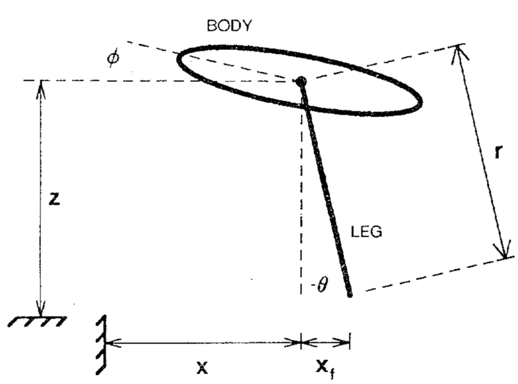

Fig. 4. Diagram of one-leg system showing variables used in calculating placement of the foot to control forward running speed.

where

- x_f forward placement of the foot with respect to the projection of the center of gravity (see Fig. 4)
- $\dot{x}$ forward velocity
- $\dot{x}_d$ desired forward velocity
- T_s duration of a support period
- $k_{\dot{x}}$ a gain.

Once the control system finds x_f, a kinematic transformation determines a hip angle that will position the foot as specified, and a servomechanism drives the hip actuator.

This three-part control emphasizes the separate actions of controlling the vertical bouncing motion, the body posture, and the rate of forward travel. The details of the individual control algorithms are not so important as the control framework provided by decomposing the problem into the three parts.

This approach was used to control the two-dimensional one-legged machine shown in Fig. 5, which was constrained mechanically to operate in a plane, and a three-dimensional one-legged machine that traveled freely about the laboratory. Detailed accounts of the control algorithms used for the one-legged machines and of the experimental results are given in [3]–[4]. In the sections that follow we describe how these algorithms form the basis for control of systems that run with several legs.

III. One-Foot Gaits

For the purpose of this discussion, let us assume that the three-part control system described in Section II is effective for controlling machines that have a leg, a body, and a suitable collection of actuators and sensors. It is not difficult to imagine a control system that uses these same three control algorithms for human-type biped running.

Despite a number of differences in detail,[1] running on two legs and running on one leg are remarkably similar. Rather than use one leg for support over and over again, as a system with only one leg must, a biped can alternate in the use of two separate legs. In human-type biped running, only one of the two legs provides support at a time, only one foot is held in place by friction during support at a time, and only one leg recovers to a forward position during flight. Therefore the thrust each leg delivers during support, the torque generated by each hip during support, and the forward placement of each foot during flight, can each be governed by control algorithms like those used for systems with one leg. Since there are two legs, more than one of the control algorithms may operate at the same time, but the overall behavior of the system during the locomotion cycle is not much different than before: alternation of support and ballistic flight.

We characterize the running gait used by one- and two-leg

[1] There are several important differences in the running behavior of systems with one and two legs.

1) Two legs permit running with substantially reduced pitching of the body. The recovery motion for a one-legged system can occur only during flight, when the angular momentum of the system is fixed, so every recovery motion of the leg must be compensated by a pitching motion of the body. A biped can overlap the backward motion of the supporting leg with the forward motion of the recovery leg, resulting in zero net angular momentum of the legs. The stance leg, whose foot is held in place by friction, can absorb any residual angular momentum by exerting a pitching torque on the body.

2) When the stance and recovery motions overlap in time, the duration of flight does not uniquely determine the time available for the recovery motion. Therefore, if the time it takes to recover a leg limits the running speed, then a biped can run faster than a comparable one-legged system.

3) If a biped is to recover a leg during stance, then it must be able to shorten that leg during recovery. The recovery leg must be substantially shorter than the support leg if it is to clear the ground without stubbing. Therefore a biped must have a mechanism that will permit the leg to shorten substantially during recovery, and to lengthen again in time for landing. The one-legged systems we explored did not have this requirement, because the recovery motion occurred when the body achieved peak height during flight.

4) A biped can generate yaw moments on the body when the legs counteroscillate, swinging fore and aft. This was not a problem with the one-legged systems because the leg swung in the plane containing the center of mass.

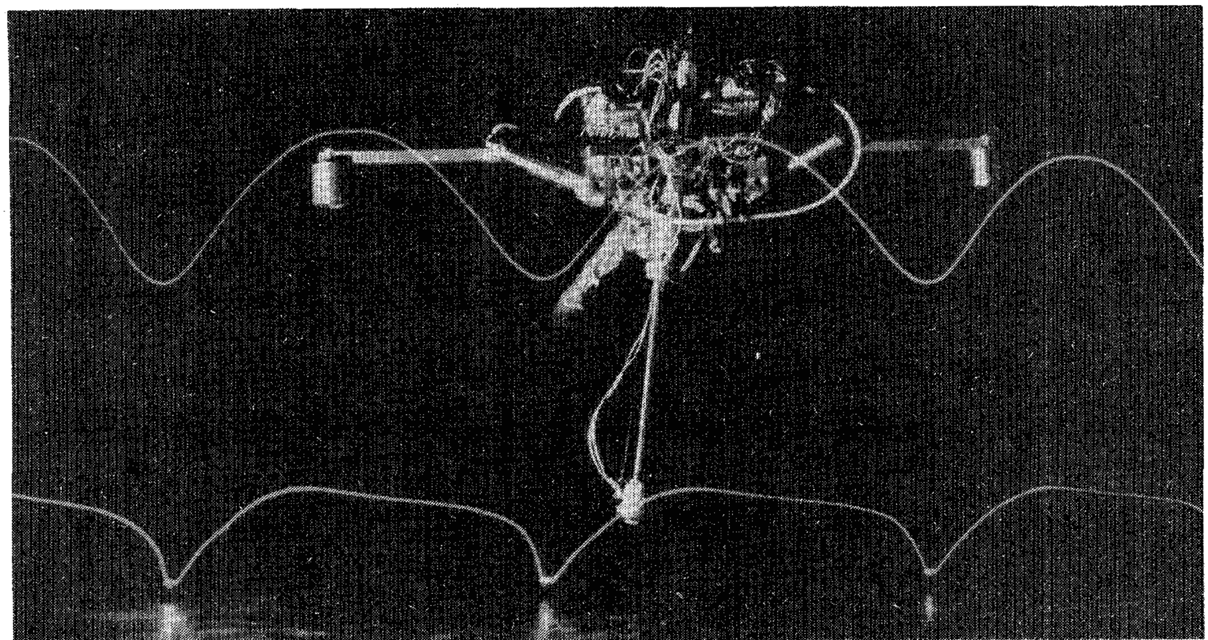

Fig. 5. Planar one-legged hopping machine. Action is from right to left, with light sources indicating the paths of the hip and the foot. The stride is about 0.5 m and running speed is 0.8 m/s [3].

systems in the following terms: 1) only one leg provides support at a time and 2) support phases and flight phases proceed in strict alternation. These characteristics define a class of running gaits that we call the one-foot gaits. All the feet remain elevated above the ground, except for the one whose turn it is to provide support. Humans normally run with a one-foot gait, as do the one-legged machines we have built. Hodgins *et al.* [27] have built a planar biped that runs with a one-foot gait using similar techniques.

The definition of a one-foot gait avoids limiting it to systems with one or two legs. One-foot gaits can, in principle, exist for systems with any number of legs. A quadruped performing a one-foot gait would cycle through the legs, perhaps in a regular order. During stance a leg would deliver a vertical thrust to maintain the vertical bouncing motion of the body, and its hip would provide a torque to correct the attitude of the body. The next leg would move to a forward position appropriate for landing, some time before it was due to provide support. This is the Gatling gait we mentioned earlier.

In addition to providing the three control functions used for locomotion on one leg, the control system for a quadruped one-foot gait would have to sequence the legs in their use. The sequencing mechanism would select the next leg to provide support so that it could move to a forward position for landing, and it would assign the hopping height and attitude control functions to the leg currently providing support. The sequencing mechanism would shorten the other legs to keep them out of the way of the ground until they were needed.

A practical problem with this sort of running is the difficulty of attaching the legs close enough to the center of the body to permit the feet to reach footholds that would provide balance. Each foot must be placed near the neutral point, so that the average point of support during the support interval is under the center of mass. It is not hard to attach one or two legs near the center of mass, but the design problem becomes severe with more legs. It is also hard to keep motions of many legs from interfering with one another when the legs are mounted close together. Nature has eased this problem by providing some animals with spines that are flexible enough to permit the feet to reach under the center of mass, even though the hips are located far from the center of mass. For example, see Muybridge's photograph of a running dog, reproduced in Fig. 6. Despite this adaptation, it is not clear that any natural quadruped employs a one-foot gate. Duikers and muntjaks would be the best candidates [29].

Even without putting the hips near the center of mass and without a flexible spine, it might be possible for a quadruped to use a one-foot gait if it were not required that each step by itself provide perfect balance. The feet might be positioned on either side of the center of mass on alternate steps. Then the system could tip in one direction during one stance phase, and in the opposite direction during the next stance phase. For such a system to balance, the step rate would have to be high compared to the tipping rate. Raibert [3] discusses this sort of balance, which relies on a sequence of steps that generate complementary tipping motions of the body.

To summarize this section, the one-foot is a class of running gaits for which only one leg provides support at a time and a flight phase occurs between each stance phase. In principle, one-foot gaits can be defined for systems with any number of legs, but leg interference becomes a practical problem when there are more than two legs. The locomotion algorithms for multi-legged one-foot gaits can be like those used for one-legged systems, provided that the control system also performs sequencing functions that assign the control functions to the legs.

The next step in generalizing from one leg to several legs is to consider pairs of legs that work together. The practical limitations of the quadruped one-foot gaits, namely the difficulty of placing the feet on the neutral point when the hips

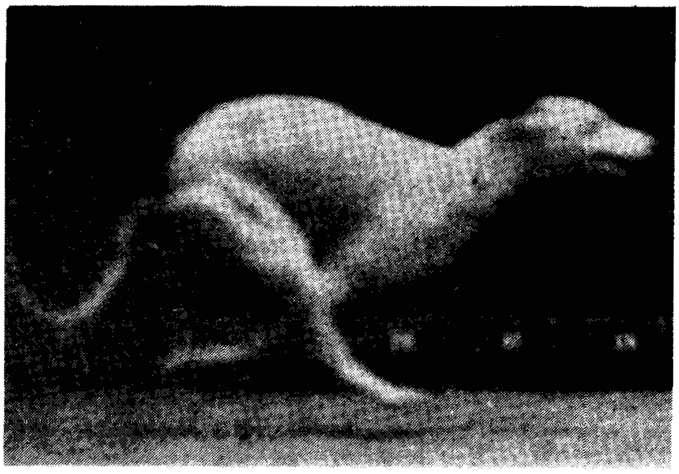

Fig. 6. Photograph of dog in crossed phase of bound. Flexibility in the back enables the feet to sweep under the center of mass, even with hips located far from the center of mass. (Photograph from [6, plate 121]. Reprinted with permission.)

are separated by feasible distances, can be reduced or eliminated when legs are used in pairs.

IV. Virtual Legs

Suppose a pair of legs were coordinated to work together, like one equivalent leg. Then a quadruped gait that used the four legs in two pairs could be viewed as an equivalent biped. Quadruped gaits that can be understood and produced in this way are the trot, the pace, and the bound. In each of these gaits the legs operate in pairs. The members of a pair strike the ground in unison and they leave the ground in unison. While one pair of legs provides support, the other pair of legs swings forward in preparation for the next step. Diagonal legs form pairs in the trot, lateral legs form pairs in the pace, and the front legs and rear legs each form a pair in the bound.

An important piece of background comes from Sutherland's work on the construction of a human-carrying walking machine [1], [30]. In order to design the hydraulic circuits that would coordinate the load-bearing operation of the machine, Sutherland introduced the idea of a virtual leg. He made two or more physical legs act like a single equivalent virtual leg. For instance, when two legs moved downward to lift the machine, each leg supported the same load because a parallel hydraulic circuit equalized the oil pressures. The two physical legs acted like one virtual leg located between them, as shown in Fig. 7. The forces and torques exerted on the body by the set of physical legs and by the virtual leg were equal, so the behavior of the body was the same in both cases. The virtual leg concept permitted Sutherland to express and analyze the complicated behavior of a machine with six legs in simpler terms.

In this paper we use the idea of the virtual leg to separate the problem of generating quadruped running gaits that use pairs of legs in unison, into two simpler problems. One problem is to control the physical legs of a pair, so that the virtual leg they form behaves as desired. A set of operations that coordinate the physical legs to produce desired virtual behavior are as follows.

Positioning: Choose positions for the physical feet that place the virtual feet in the desired location.

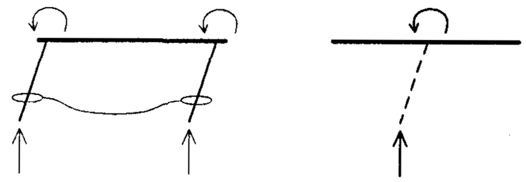

Fig. 7. Virtual legs. When two legs act in unison, they can be thought of as a functionally equivalent virtual leg. The original pair of legs and the virtual leg exert the same forces and moments of the body, so they both produce the same behavior of the body. A force-equalizing virtual leg is shown here. It locates the virtual leg half way between the two physical legs it represents.

Synchronization: Ensure that both legs of the pair strike the ground in unison and leave the ground in unison.

Force Equalization: Ensure that both legs of the pair deliver the same axial thrust to the ground.

In order to implement each of these operations, the control system must coordinate the low-level behavior of the physical legs.

The other problem in generating quadruped running gaits that use legs in pairs is to provide locomotion algorithms that specify the desired behavior for the virtual system. Since the trot, the pace, and the bound reduce to virtual biped one-foot gaits, as shown in Fig. 8, the results of the last two sections apply. The locomotion algorithms described earlier for the one-legged systems can be used to specify the behavior of each virtual leg.

To make this approach workable, the control system needs a sequencing mechanism that keeps track of the legs and assigns each of the three control functions to the right leg at the right time. The sequencing mechanism would select the next virtual leg to provide support so that the pair of legs could move to a forward position for landing, and it would assign the hopping height and attitude control functions to the virtual leg currently providing support. The control system would also shorten the virtual leg in recovery to keep it from touching the ground while the other virtual leg provides support.

The *force-equalizing* virtual leg we have chosen is a special case of Sutherland's more general concept. When the control system equalizes the axial forces that the legs deliver to the ground, it moves the effective support point half-way between the physical feet. This choice makes the resulting behavior simple to analyze and similar to that of the one-legged systems we have already studied. It is possible to implement virtual legs that do not equalize axial forces, with correspondingly more complicated behavior. Equations that define the general virtual leg are given in the Appendix. We have considered only the force-equalizing type of virtual leg in this paper.

Relationships between the physical and virtual legs permit the control system to convert the desired behavior of the virtual leg into control signals for each physical leg. For instance, if the physical hips were separated a distance $2d$ in the fore-aft direction, then the control system could add d to the desired virtual foot position to find the desired position for the forward physical foot of the pair, and it could subtract d to find the desired position for the rear physical foot. A similar procedure could position the feet sideways.

Pairs of legs can act together to place the effective point of

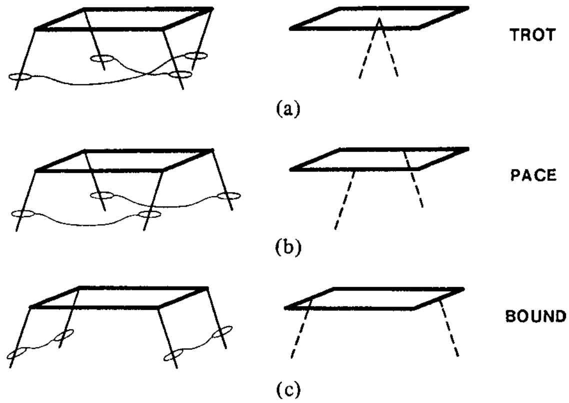

Fig. 8. Three quadruped gaits move the legs in pairs. Using the virtual leg idea, each of the gaits shown on the left reduces to the virtual biped one-foot gait shown on the right. (a) Trot, with diagonal pairs of legs acting in unison, as shown by the shackles. They strike the ground at the same time, they leave the ground at the same time, and they swing forward at the same time. (b) Pace, with lateral pairs of legs acting in unison. (c) Bound, with the front legs acting in unison, as do the rear legs.

support close to the center of mass, even though the physical legs are located a substantial distance from the center of the body. The effective point of support provided by a virtual leg lies halfway along the line connecting the two physical feet. The one-foot running gaits described in the previous section would be difficult for a quadruped to use, because it is difficult to locate four hips close enough to the center of mass to position the feet properly and avoid leg collisions at the same time. In the trot, the gait involving diagonal support pairs, these points are near the center of the body, which we assume is near the center of mass. In the pace, the gait involving lateral support pairs, the lines connecting the feet may pass under the center of mass if the legs are angled inward during stance, or they may pass quite close to the center of mass if the body is narrow.

In the bound, the virtual legs do not provide support under the center of mass. If the body is elongated in the fore-aft direction, then the virtual leg formed by the front legs may not be able to reach under the center of the body. This is the same problem described earlier for the one-foot gaits. For this reason it may be incorrect to include the bound in the same class with the trot and pace. Once again, a flexible spine may help to provide a solution, or the control system might be extended to tolerate the pitching motions that occur when the virtual feet do not reach far enough under the body.

To summarize, we use the concept of a virtual leg to separate the task of making a quadruped run into two simpler tasks. One task is to couple the behavior of two physical legs so their combined behavior conforms to the desired behavior of an equivalent virtual leg. The second task is to provide locomotion algorithms for virtual biped one-foot gaits. This approach applies to three quadruped gaits, the trot, the pace and perhaps the bound.

V. Quadruped Running Experiments

In order to explore the idea that locomotion algorithms for systems with one leg can be extended to systems with several legs, we built and experimented with a four-legged running machine. Figs. 9 and 10 describe the machine, Fig. 11 and Table I give the state diagram for sequencing the one-foot gait, and Fig. 12 shows photographs of the machine running with a trotting gait. Table II gives the physical parameters of the machine.

We designed the control system for these experiments along the lines discussed in the last two sections. The control system uses the one-leg algorithms to specify desired behavior for each virtual leg, it sequences the virtual legs according to the state in the running cycle, and it coordinates the behavior of pairs of physical legs to act like virtual legs.

One-Leg Algorithms

The locomotion algorithms previously used to control the one-legged hopping machines were reimplemented to control the virtual legs of the four-legged running machine. To control the forward running velocity, the control system positions the virtual legs with respect to the center of mass of the body during each flight phase:

$$x_{f,d}=\frac{\dot{x}T_s}{2}+k_{\dot{x}}(\dot{x}-\dot{x}_d) \tag{3}$$

$$y_{f,d}=\frac{\dot{y}T_s}{2}+k_{\dot{y}}(\dot{y}-\dot{y}_d) \tag{4}$$

where

$x_{f,d}$, $y_{f,d}$ desired displacement of the foot with respect to the projection of the center of mass,
$\dot{x}$, $\dot{y}$ forward running velocity,
$\dot{x}_d$, $\dot{y}_d$ desired forward running velocity,
T_s duration of a support period, and
$k_{\dot{x}}$, $k_{\dot{y}}$ gains.

In the experiments reported here an operator uses a two-axis joystick to specify the desired forward running velocity ($\dot{x}_d$, $\dot{y}_d$). The control system estimates the forward velocity of the body ($\dot{x}$, $\dot{y}$) with the assumption that the feet do not move with respect to the ground during stance. Under this assumption, the backward motion of a foot with respect to the body is equal to the forward motion of the body with respect to the ground. Gyroscope and hip angle measurements are used together with kinematics to make this estimate. The control system assumes that the forward running velocity does not change during flight. The control system measures the duration of stance T_s during each stance phase, and it uses the most recent value for control.

To control the pitch and roll attitude of the body during stance, the control system applies torques about the virtual hips during stance, using linear servos:

$$u_x=-k_{p,x}(\phi_P-\phi_{P,d})-k_{v,x}(\dot{\phi}_P)-k_{f,x}(f_x) \tag{5}$$

$$u_y=-k_{p,y}(\phi_R-\phi_{R,d})-k_{v,y}(\dot{\phi}_R)-k_{f,y}(f_y) \tag{6}$$

where

u_x, u_y servovalve output signals for the hip actuators,
ϕ_P, ϕ_R pitch and roll angles of the body,
f_x, f_y forces delivered by the hip actuators, and
k_p, k_v, k_f gains.

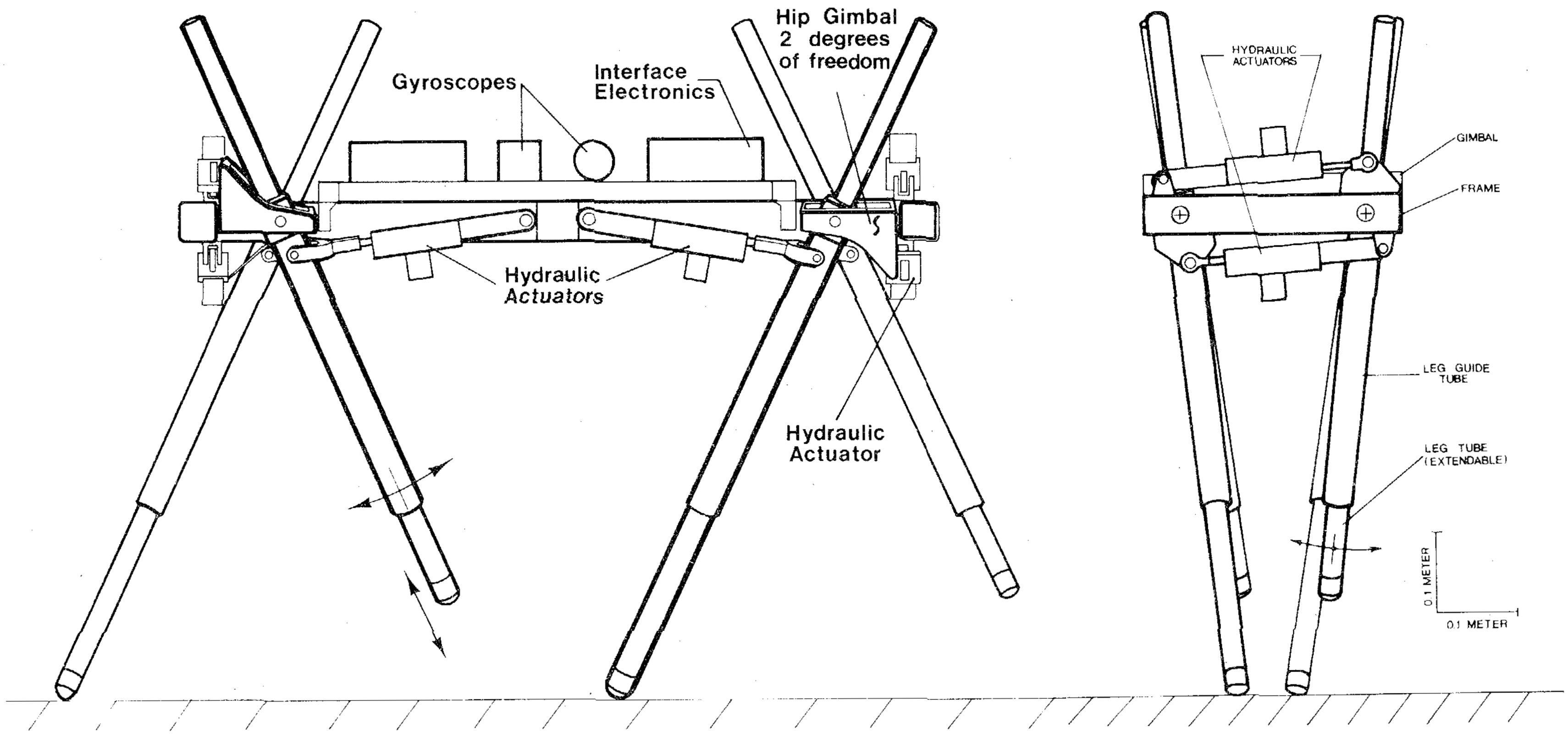

Fig. 9. Diagram of running machine used for experiments. The body is an aluminum frame, on which are mounted hip actuators, gyroscopes, and computer interface electronics. Each hip has two low friction hydraulic actuators that position a leg fore and aft, and sideways. An actuator within each leg changes its length, while an air spring makes the leg springy in the axial direction (see Fig. 10). Sensors measure the lengths of the legs, the positions and velocities of the hydraulic hip actuators, pressures in the leg air springs, contact between the feet and the floor, and the pitch and roll angles of the body. An umbilical cable connects the machine to hydraulic, pneumatic, and electrical power supplies, and to the control computer, all of which are located nearby in the laboratory. The arrangement of body, legs, hips, and actuators provides a means to control the position of the feet with respect to the body, to generate an axial thrust with each leg, and to provide hip torques during running. Fore/aft hip spacing is 0.78 m. Lateral hip spacing 0.24 m.

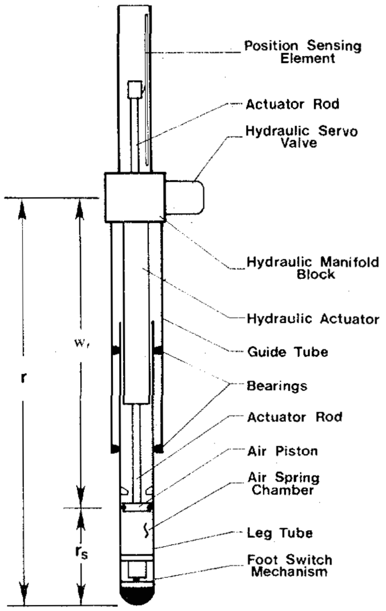

Fig. 10. Diagram of leg used in running machine. A hydraulic actuator acts in series with an air spring. The hydraulic actuator is used to drive resonant bouncing motion of the machine and to retract the leg during flight. It also acts in conjunction with the air spring to determine the axial force the leg exerts on the ground. Sensors measure the hydraulic length, the overall leg length, the air pressure in the spring, and loading on the foot.

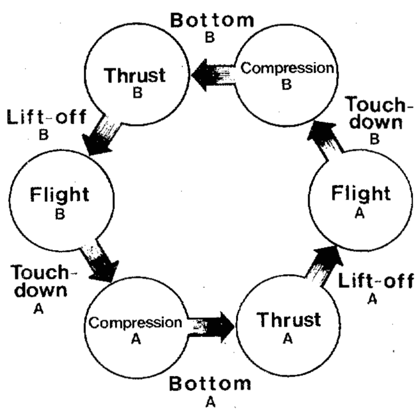

Fig. 11. Simplified state diagram used to sequence virtual biped one-foot gait. Virtual leg B_l recovers while leg A_l provides support, and vice versa. State transitions are determined by events related to the support leg. (See Table I for the details).

To control the hopping motion, the control system adjusts the hydraulic length of the virtual leg throughout the running cycle. When a virtual leg is in recovery, the desired hydraulic length is shortened, $w_{l,d} = L_1$, to keep the feet from touching the ground. This keeps the virtual leg out of the way. When a virtual leg is preparing for landing or compressing under load of the body, the desired hydraulic length is set to an intermediate value, $w_{l,d} = L_2$. During the second part of stance when the legs delivers a thrust to the body, the desired hydraulic length is increased, $w_{l,d} = L_3$. The operator specifies L_1, L_2, and L_3 from a control box with $L_1 < L_2 < L_3$.

Sequencing Virtual Legs

Throughout the running cycle, the control system uses a finite state machine to determine which control functions should be applied to which virtual legs. The cycle traverses ten states during each stride. Each state prescribes a set of sensor conditions that triggers transition into the state, and a set of control actions to be taken during the state. The state transitions synchronize the various control functions—vertical thrust, attitude control and foot placement—to the behavior of the running machine. Fig. 11 and Table I give the sequencing details as they are implemented for the trotting experiments.

TABLE I
FINITE STATE SEQUENCE FOR TROTTING[1]

State	Trigger Event	Action
1) Loading A_l	A_l touches ground.	Equalize axial force A_l. Zero hip torque A_l. Shorten $\boldsymbol{B}_l$. Don't move hip $\boldsymbol{B}_l$.
2) Compression A_l	A_l air springs shortened.	Equalize axial force A_l. Erect body with hip A_l. Shorten $\boldsymbol{B}_l$. Position $\boldsymbol{B}_l$ for landing.
3) Thrust A_l	A_l air springs lengthening.	Extend A_l, equalizing force. Erect body with hip A_l. Keep $\boldsymbol{B}_l$ short. Position $\boldsymbol{B}_l$ for landing.
4) Unloading A_l	A_l air springs near full length.	Shorten A_l, equalizing force. Zero hip torques A_l. Keep $\boldsymbol{B}_l$ short. Position $\boldsymbol{B}_l$ for landing.
5) Flight A_l	A_l not touching ground.	Shorten A_l. Don't move hip A_l. Lengthen $\boldsymbol{B}_l$ for landing. Position $\boldsymbol{B}_l$ for landing.

[1] The state shown in the left column is entered when the event listed in the center column occurs. During normal running states advance sequentially. A_l refers to the virtual leg that uses physical legs 1 and 3 (left front and right rear). $\boldsymbol{B}_l$ refers to the virtual leg that uses physical legs 2 and 4 (left rear and right front). During states 1–5, A_l is the support leg and $\boldsymbol{B}_l$ is the recovery leg. States 6–10 repeat states 1–5, with A_l and B_l reversed.

Implementation of Virtual Legs

In order to make the legs work together in pairs, the control system coordinates positioning of the physical legs, synchronizes ground contact, and equalizes axial leg thrust. The geometry of the body simplifies the task of positioning the physical legs for trotting. Since the hips are located in symmetric positions about the center of mass, an x–y displacement of both physical feet from the projection of their hips results in a comparable x–y displacement of the virtual foot from the projection of the center of mass. Therefore, the desired position of the virtual foot with respect to the center of mass is used to specify the desired position of the physical legs with respect to their hips:

$$x_{h,i,d} = x_{h,j,d} = x_{f,d} \tag{7}$$

$$y_{h,i,d} = y_{h,j,d} = y_{f,d} \tag{8}$$

where

$x_{h,i,d}$, $y_{h,i,d}$ desired displacement of the ith foot with respect to the projection of the ith hip,

$x_{f,d}$, $y_{f,d}$ desired displacement of the virtual foot with

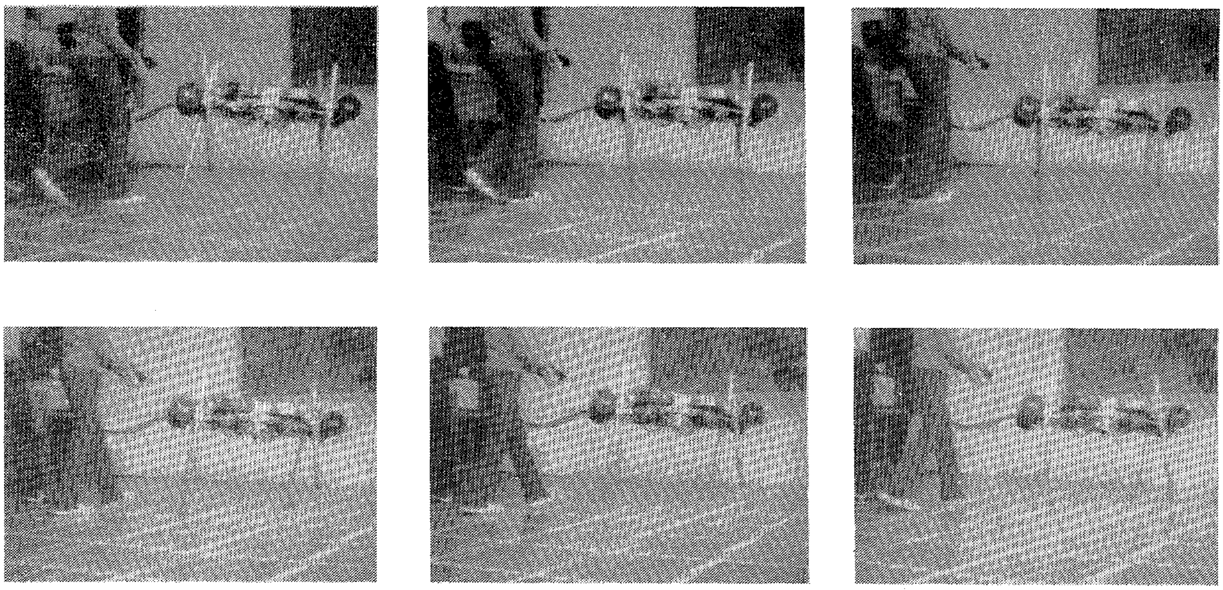

Fig. 12. Quadruped machine trotting. The trot is a gait that uses diagonal pairs of legs in synchrony. Running speed was about 1.0 m/s, with progress from left to right. Grid on floor indicates 0.5-m intervals. Adjacent frames separated in time by 84 ms. During these experiments wranglers held on to safety ropes attached to the machine.

TABLE II
PHYSICAL PARAMETERS OF FOUR-LEGGED RUNNING MACHINE[1]

Parameter	Metric Units	English Units
Overall length	1.05 m	41.2 in
Overall height	0.95 m	37.5 in
Overall width	0.35 m	13.8 in
Hip height (max)	0.668 m	26.31 in
Hip spacing (x)	0.776 m	30.56 in
Hip spacing (y)	0.239 m	9.40 in
Leg sweep angle (x)	±0.565 rad	±32.4°
Leg sweep angle (y)	±0.384 rad	±22.0°
Leg stroke (hydraulic)	0.229 m	9.0 in
Leg stroke (spring)	0.102 m	4.0 in
Body mass	25.2 kg	55.4 lb
Body moment of inertia (x)	0.257 kg-m^2	880 lb-in^2
Body moment of inertia (y)	1.60 kg-m^2	5470 lb-in^2
Body moment of inertia (z)	0.86 kg-m^2	6340 lb-in^2
Leg mass, total each	1.40 kg	3.08 lb
Leg mass, unsprung	0.286 kg	0.63 lbm
Leg moment of inertia (about hip)	0.14 kg-m^2	480 lb-in^2
Leg spring stiffness @20 psi (fully extended)	2100 N/m	12 lbf/in
Hip torque, @2000 psi (x)	111 N-m	983 in-lbf
Hip torque, @2000 psi (y)	77.6 N-m	687 in-lbf
Leg thrust, @2000 psi	765 N	172 lbf

[1] x—fore and aft, y—sideways, z—up and down.

respect to the projection of the center of mass, and

i, j indices of two physical legs that form one virtual leg.

Once these desired foot displacements are known, transformations based on the kinematics of the legs, hips, and actuators are used to find actuator lengths that will position the foot as desired. These transformations [3] take into account the pitch and roll orientations of the body and the lengths of the legs.

To synchronize the instant of ground contact for the two legs forming a virtual leg, the control system servos the leg lengths during flight, so that both feet have the same altitude. This adjustment affects only the difference in the lengths of the legs, while L_2 determines the average length. Pitch and roll measurements and a kinematic calculation are required to perform these adjustments.

This approach to synchronizing ground contact works correctly on a flat floor, but it would fail if used with a support surface that was not flat. An alternative approach that might better tolerate variations in the altitude of the terrain would be to servo the axial leg forces in anticipation of ground contact, perhaps using (9). When one foot struck the ground, its leg would retract and the other extend. When both feet touch the ground, the stance phase would proceed. This method would rely on fast response from the actuators that retract the legs.

To equalize the axial forces the legs deliver to the ground during stance, the control system servos the differential length of the leg hydraulic actuators:

$$w_{l,i,d} = w_{l,i} + \frac{r_{s,i} - r_{s,j}}{2} \tag{9}$$

where

$w_{l,i}$ hydraulic length of the ith leg,
$w_{l,i,d}$ desired hydraulic length of the ith leg, and
$r_{s,i}$ air spring length of the ith leg, and
r_i length of the ith leg, $r_i = w_{l,i} + r_{s,i}$.

This differential adjustment forces the air springs to assume equal lengths and therefore to generate equal axial force. Once again, values for L_2 and L_3 determine the average length of the hydraulic actuators contributing to the virtual leg.

12 linear servos act on the hydraulic actuators to position the hips and leg lengths once the desired actuator lengths are

known:

$$u_i = -k_p(w_i - w_{i,d}) - k_v(\dot{w}_i) \tag{10}$$

where

u_i	servovalve output signal for the ith actuator,
w_i, $w_{i,d}$, $\dot{w}_i$	position, desired position, and velocity of the ith actuator, and
k_p, k_v	position and velocity gains.

Data recorded during trotting experiments are shown in Figs. 13 and 14. They show that diagonal pairs of legs are used for support in alteration, as required for trotting. The synchronization of foot impacts and equalization of axial leg forces are controlled with good precision as shown by the small differences in axial forces between the legs of a pair. The vertical bouncing motion of the body is regular and quite smooth.

The control system's ability to regulate forward running velocity is rather poor, as shown in Fig. 14. We observe only a rough proportionality between the desired and actual running velocities. These errors are due to known limitations of the velocity control algorithm, as described in [8]. During forward running the inclination of the body about the pitch axis ϕ_P deviates from the desired value by as much as 8°. The magnitude and sign of this error are generally related to the forward running velocity. The control system keeps error about the roll axis to about 5° in these experiments, although control of this degree-of-freedom proves to be the most difficult task for the control system to perform. Despite these quantitative limitations in performance, the control system was adequate to balance the machine as it ran back and forth in the laboratory.

VI. Discussion

We have limited this paper to study of force-equalizing virtual legs. The force-equalizing virtual leg has the virtue of keeping the effective point of support, the virtual foot, located halfway between the physical points of support. This makes the task of positioning the point of support comparable to the one-legged system's task of positioning the foot. This similarity permits us to bring our experience with one-legged systems to bear on the quadruped control problem.

A consequence of using the force-equalizing virtual leg, however, is to limit the control system's performance. In restricting the control system to this special case, we have given up one degree-of-freedom in the control. Without this restriction the control system could adjust the differential axial leg force and the hip torque to manipulate the location of the virtual foot *during* the support interval. Such adjustments could correct the forward running velocity with finer temporal resolution than the once-per-step method described in this paper. However, this would be done at the expense of simplicity in the control. Such additional control might be particularly useful at low stepping rates, or for systems that tip over quickly because of very short legs.

Another consequence of using force-equalizing virtual legs is the loss of passive stability that a set of legs might otherwise

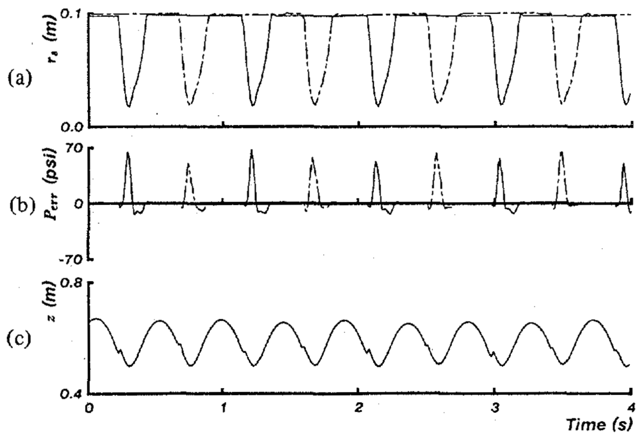

Fig. 13. Trotting data. These graphs describe the bouncing motion that underlies the machine's running. Curves (a) and (b) show compression of the air springs, and the equalization of axial forces during stance. Curve (c) shows the altitude of the body above the floor, as estimated by the control system. The discontinuities in z are from errors in estimating the vertical velocity of the body when the feet left the ground. (FL.295.3)

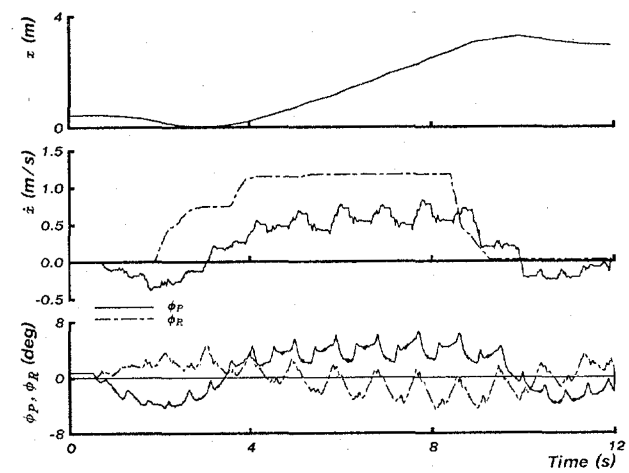

Fig. 14. Forward running. When the joystick specified a desired forward velocity, the machine accelerated forward. The forward running velocity was not controlled with high precision, as shown by the plots of desired and actual running velocity $\dot{x}$ and $\dot{x}_d$ (shown dashed). The body tipped in the direction of running, as shown by the pitch and roll angles. Positive pitch indicates nose down. (FL.295.3)

provide. A table resists tipping when unevenly loaded, because the legs near the load generate more force than the legs that are remote from the load. If a table had force-equalizing legs, then an uneven load would cause the legs near the load to shorten, the legs remote from the load to lengthen, and the surface to tip. This force-equalized behavior should be expected, since it is precisely the behavior of a table with just one leg located in the middle. This one-legged behavior is what we set out to accomplish in the first place.

Our experiments have shown that this approach which discards passive stability of the leg is workable, but it leaves us in a philosophical quandary. On the one hand, the force-equalizing virtual leg permits relatively sophisticated behavior with an extremely simple implementation, largely because it permits us to build on previous results. On the other hand, we believe that a well-engineered control system should take advantage of the intrinsic mechanical properties of the mechanism. If the machine is cleverly designed its intrinsic mechanical behavior will be the desired behavior. The control system need only fine tune this correct system behavior, not having to fight the mechanism to make it obey. Such an approach that splits responsibility for good behavior between the mechanical design and the control design leads to simpler, hardier, and more efficient machines. Because the control system that uses the force-equalizing virtual leg discards the passive stability available from the legs rather than somehow harnessing it, we expect that it will eventually be replaced by a better method.

Despite these limitations, it is entirely possible that four-legged animals use force equalization when they trot, pace, or bound. One might find out by measuring the axial forces that develop in the legs of running quadrupeds, perhaps using sets of force platforms. The experiment would disturb one of the feet during stance by shifting the support surface upward or downward. If force equalization were in effect, the difference in axial leg force would not be affected by the manipulation. Exact force equalization is unlikely to be found, because the distribution of mass in animals' bodies is skewed by the asymmetric placement of their heads and often by unequal lengths of the fore and hind legs. One might expect, therefore, to find that the forces delivered by the legs vary in proportion to the loading caused by distribution of mass in the body.

We have suggested that there are two quadruped running gaits, in addition to the trot, that should yield to the approach described in this paper. They are the pace and the bound. These gaits use pairs of legs in unison, as does the trot, but they also involve pitching and rolling motions of the body. We have not yet done experiments with these gaits, but we anticipate that they will require somewhat more sophisticated methods for controlling the attitude of the body and the forward-running velocity.

For instance, if the running motion has a rhythmic rolling or pitching motion of the body, it does not seem wise to servo the body to a fixed upright posture during stance. Likewise, running velocities in both the pace and the bound would normally vary on alternate steps, so the control system should accommodate those variations in a systematic way. While these additional elements needed for control of pacing and bounding are not provided by the existing control system, we believe that they represent minor additions that will fit into the existing locomotion framework without major renovation.

So far, we have considered virtual legs that represent the behavior of physical legs acting in unison. A plausible extension of the concept might represent the behavior of physical legs that act in sequence, possible overlapped in time. The simplest approach would separate each support interval into subintervals, during which a fixed set of legs would provide support. Then the entire support interval might be represented by a sequence of virtual support phases. For a rotary gallop the sequence of phases would be 1) right rear, 2) right rear and left rear, 3) left rear, 4) left rear and left front, 5) left front, 6) left front and right front, and 7) right front. A key challenge in this problem is to find a mechanism that can mediate the smooth exchange of support from one leg to another without disrupting the bouncing motion of the body. With such an approach one might understand and produce galloping with control techniques no more complicated that those described in this paper.

This paper discusses methods for generating several gaits, but it is silent on the issue of choosing which gait to use. In animals, the energetic cost of a gait seems to be an important factor in its selection. Animals change gait as they change speed in order to minimize the cost of transportation [31]. The geometry of the animal may also enter into gait selection. At low running speeds, for instance, long-legged animals use a pace rather than a trot, presumably to avoid interference between the front and rear legs on each side [32]. Other factors, such as the range of leg motion and stiffness, may also be important. Despite these potential factors, we do not yet have any clear criteria for selecting one gait over another.

VII. Summary

1) Previous work resulted in locomotion algorithms that were effective for controlling systems with a body and one springy leg. The control system was decomposed into a hopping part, a body attitude part, and a forward velocity part.

2) There is a class of gaits called the one-foot, for which only one foot touches the ground at a time. In principle, the locomotion algorithms that were effective for one-legged machines could be used to control a variety of systems executing one-foot gaits, independent of the number of legs. Control systems for N-legged one-foot gaits would also need to sequence the legs in their use.

3) The behavior of a pair of legs that act in unison can be represented by an equivalent virtual leg [1]. Virtual legs are used to map several quadruped running gaits—the trot, the pace, and the bound—into virtual biped one-foot gaits. This approach to running decomposes the control problem into a part that uses the behavior of the virtual legs to control the body, and a part that coordinates the actions of the legs.

4) Preliminary experiments with a four-legged running machine verify the general approach outlined in this paper. The control system uses the one-legged algorithms, a finite state machine, and virtual legs to make it run with a trotting gait.

Appendix

Equations for Virtual Leg

This appendix describes the relationships between a general planar virtual leg and the two physical legs it represents. The equations given below were developed to express the angular position θ, axial force f, and hip torque τ for the virtual leg in terms of variables that describe the behavior of the two physical legs. The analysis is for the static case. The configuration and variables are defined in Fig. 15.

We assume that the virtual foot is located on a line connecting the physical feet. This yields the geometric constraint

$$r_1 \cos \theta_1 + r_2 \cos \theta_2 = 2r \cos \theta = 2A. \quad (11)$$

Body angle ϕ is not independent of θ_1, θ_2, r_1, r_2, and d, so

$$r_1 \cos \theta_1 - r_2 \cos \theta_2 = 2d \sin \phi. \quad (12)$$

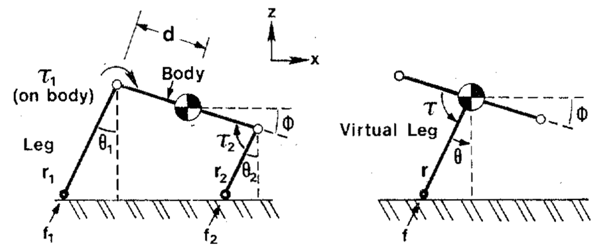

Fig. 15. Model used for analysis of virtual leg.

By summing forces and moments we obtain

$$\sum F_x = f_1 \sin\theta_1 + f_2 \sin\theta_2 - \frac{\tau_1}{r_1}\cos\theta_1 - \frac{\tau_2}{r_2}\cos\theta_2$$

$$= f\sin\theta - \frac{\tau}{r}\cos\theta$$

$$= B \tag{13}$$

$$\sum F_z = f_1\cos\theta_1 + f_2\cos\theta_2 + \frac{\tau_1}{r_1}\sin\theta_1 + \frac{\tau_2}{r_2}\sin\theta_2 - mg$$

$$= f\cos\theta - \frac{\tau}{r}\sin\theta - mg$$

$$= C \tag{14}$$

$$\sum M_{cg} = -f_1 d\cos(\theta_1 - \phi) + f_2 d\cos(\theta_2 - \phi)$$

$$-\frac{\tau_1}{r_1}(r_1 + d\sin(\theta_1 - \phi)) - \frac{\tau_2}{r_2}(r_2 - d\sin(\theta_2 - \phi))$$

$$= -\tau$$

$$= -D. \tag{15}$$

Using the lumped variables A, B, C, and D, we can solve (11)–(15) for θ, r, τ, and f:

$$r = \frac{A}{\cos\theta} \tag{16}$$

$$\theta = \arctan\left(\frac{B + D/A}{C}\right) \tag{17}$$

$$\tau = D, \tag{18}$$

$$f = \frac{B + (\tau/r)\cos\theta}{\sin\theta}. \tag{19}$$

How do these results square with the force-equalizing virtual leg described in the text? For $f_1 = f_2$, $r_1\sin\theta_1 = r_2\sin\theta_2$, and small ϕ

$$r \approx \frac{r_1 + r_2}{2} \tag{20}$$

$$\theta \approx \frac{\theta_1 + \theta_2}{2} \tag{21}$$

$$f \approx 2f_1 \tag{22}$$

$$\tau \approx \tau_1 + \tau_2. \tag{23}$$

This is the desired result.

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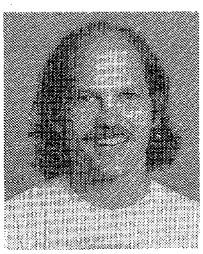
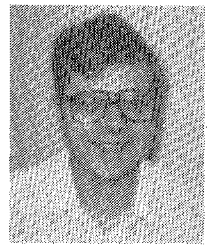


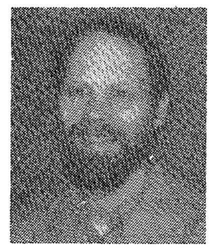
Marc Raibert (M'79) received the B.S.E.E. degree from Northeastern University, Boston, MA, in 1973 and the Ph.D. degree from the Massachusetts Institute of Technology, Cambridge, in 1977.

He was an engineer at the Jet Propulsion Laboratory, and a Visiting Associate in Computer Science at the California Institute of Technology from 1977 through 1980. He is currently an Associate Professor of Computer Science and a member of the Robotics Institute at Carnegie-Mellon University in Pittsburgh. His research addresses the control of movement in robots and biological motor systems. Recently he has concentrated on the principles of legged locomotion, both to understand how animals use their legs for transportation and to build legged robots.



Michael Chepponis (M'84) attended the Massachusetts Institute of Technology, Cambridge, from 1972 through 1976 and will receive the B.S. degree in 1986.

He was a programmer in Digital Equipment's Large Systems Group while a student at M.I.T. In 1977, he helped design and implement relational database systems for minicomputers. From 1979 to 1981, he developed real-time control systems for energy management. Since 1981, Mr. Chepponis has been a Senior Research Engineer at the Carnegie-Mellon University Robotics Institute. His research interests include computer systems for legged locomotion research, data capture and analysis, analog hardware design, and amateur radio packet communications networks.



H. Benjamin Brown, Jr., received the B.S.M.E. and M.S.M.E. degrees from Carnegie-Mellon University, Pittsburgh, PA, in 1967 and 1977, respectively.

From 1970 to 1976 he worked for Allegheny Ludlum Industries, McGraw-Edison Power Systems Division, and Jones & Laughlin Steel, doing design, research, and development on mechanical equipment. From 1976 to 1981 he was Senior Research Engineer at Verna Engineering, Inc. in Carnegie, Pa. Since 1981, Mr. Brown has been a Senior Research Engineer at the CMU Robotics Institute, and an Associate of Sutherland, Sproull, and Assoc. His research interests include legged locomotion, elastic energy storage systems, and human powered vehicles.