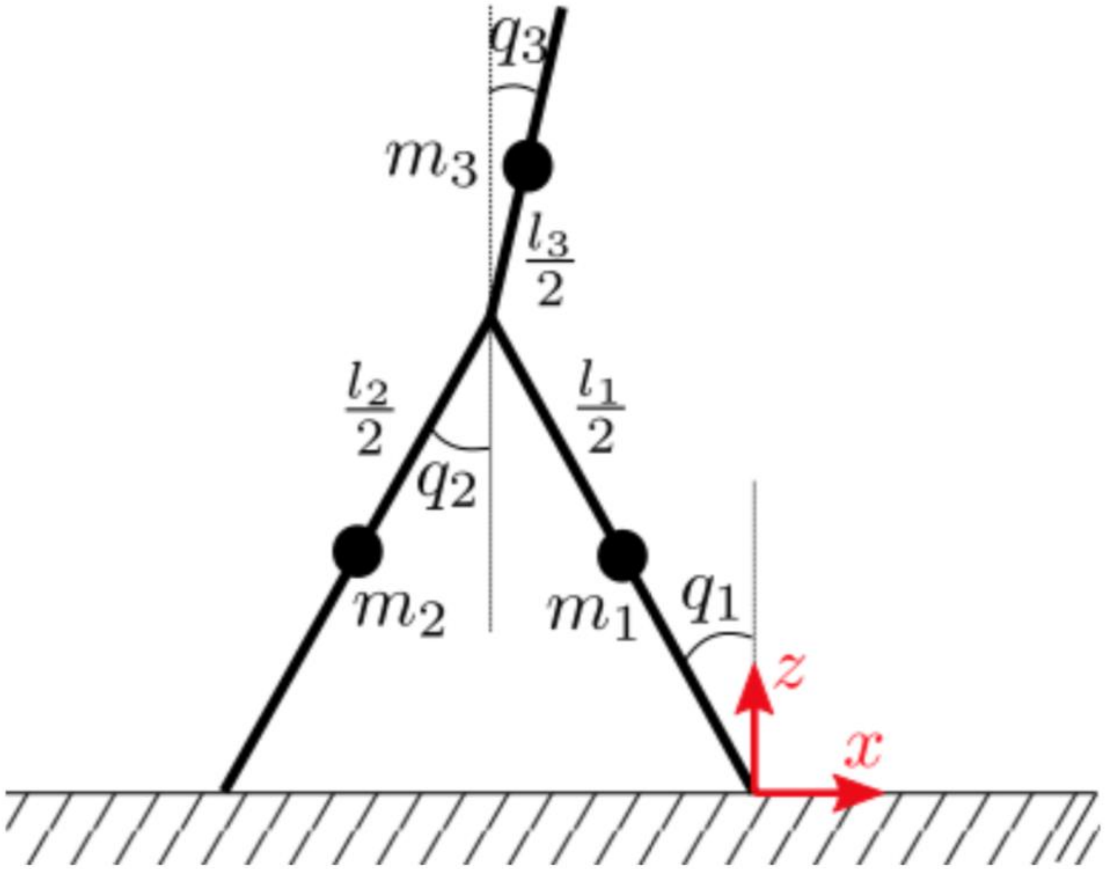
Control and optimization of a three-link 2D biped

Legged Robots

Overview

- Method of virtual constraints
- Gait quality measures
- Numerical optimization



Virtual constraints

- Model has two actuators to control three degrees of freedom

$$\begin{bmatrix} A_1\ddot{q}_1 + b_1 \\ A_2\ddot{q}_2 + b_2 \\ A_3\ddot{q}_3 + b_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

- Express the control conditions and constraints

$$q_3 \approx \alpha \qquad y_1 = q_3 - \alpha \qquad \dot{y}_1 = \dot{q}_3$$

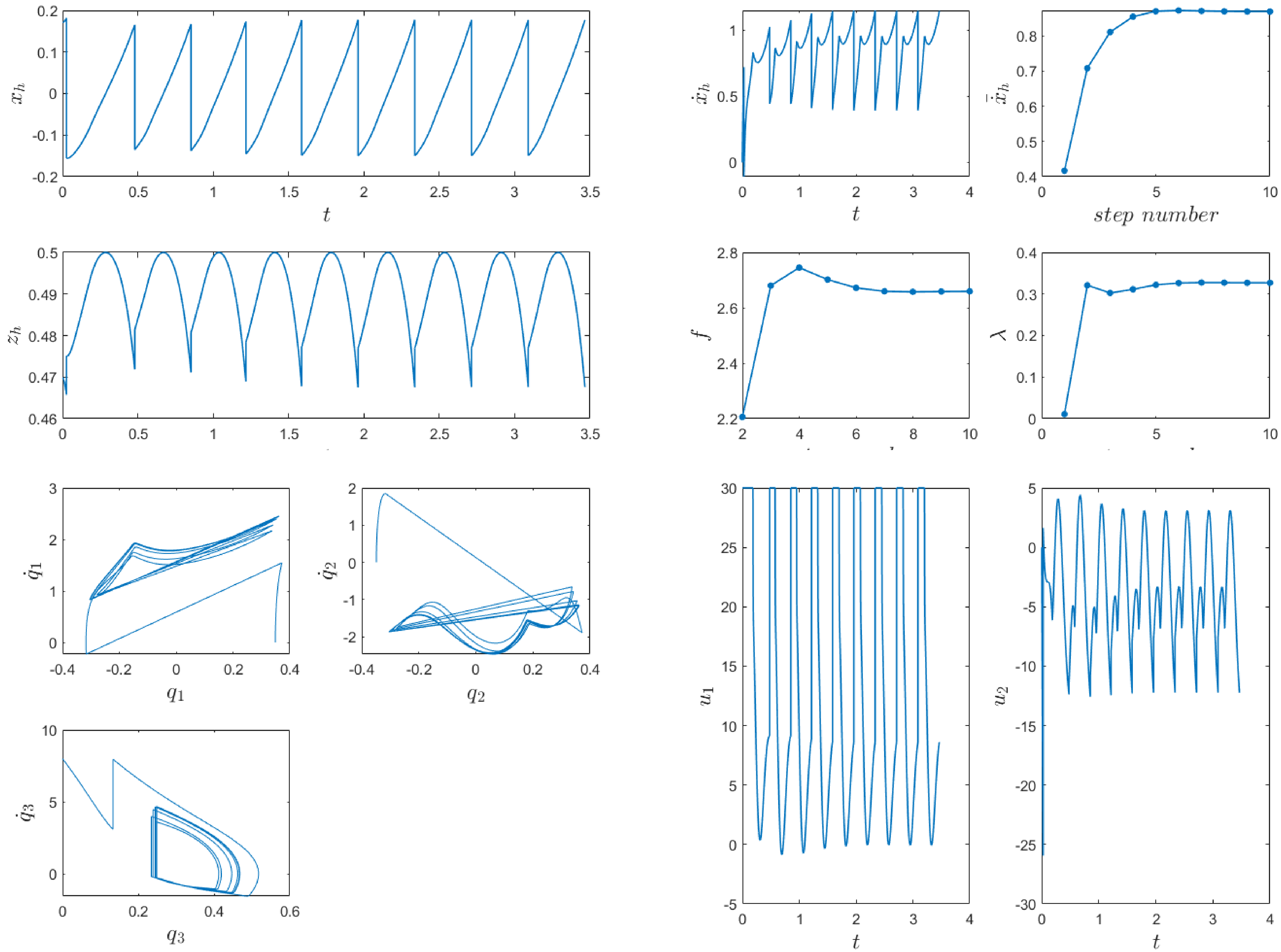
$$q_2 \approx -q_1 \qquad y_2 = -q_2 - q_1 \qquad \dot{y}_2 = -\dot{q}_2 - \dot{q}_1$$

- Close the loop on the virtual constraints

$$u_1 = k_1^P y_1 + k_1^D \dot{y}_1$$

$$u_2 = k_2^P y_2 + k_2^D \dot{y}_2$$

Gait quality measures



Numerical optimization

- Avoid hand tuning of parameters

$$\boldsymbol{p} = \left[\boldsymbol{q_0}, \dot{\boldsymbol{q}}_0, \boldsymbol{\alpha}, \boldsymbol{k_1^P}, \boldsymbol{k_1^D}, \boldsymbol{k_2^P}, \boldsymbol{k_2^D}\right]^{\boldsymbol{T}}$$

- Formulate an unconstrained optimization

$$\underset{\boldsymbol{p}}{\text{minimize}}\, \boldsymbol{f}(\boldsymbol{p})$$

- Define the objective function

$$\boldsymbol{effort} = \frac{\mathbf{1}}{\mathbf{2}\, \boldsymbol{T}\, \boldsymbol{U_{max}}} \sum_{\boldsymbol{i}}^{\boldsymbol{T}} \left(\boldsymbol{u_1^2}(\boldsymbol{i}) + \boldsymbol{u_2^2}(\boldsymbol{i})\right)$$

$$\boldsymbol{CoT} = \frac{\boldsymbol{effot}}{\boldsymbol{x_{hip}}(\boldsymbol{T}) - \boldsymbol{x_{hip}}(\mathbf{1})}$$

$$\boldsymbol{f}(\boldsymbol{p}) = \boldsymbol{w_1}\left|\dot{\boldsymbol{x}}_{\boldsymbol{hip}} - \dot{\bar{\boldsymbol{x}}}_{\boldsymbol{hip}}(\boldsymbol{p})\right| + \boldsymbol{w_2}\boldsymbol{CoT}$$