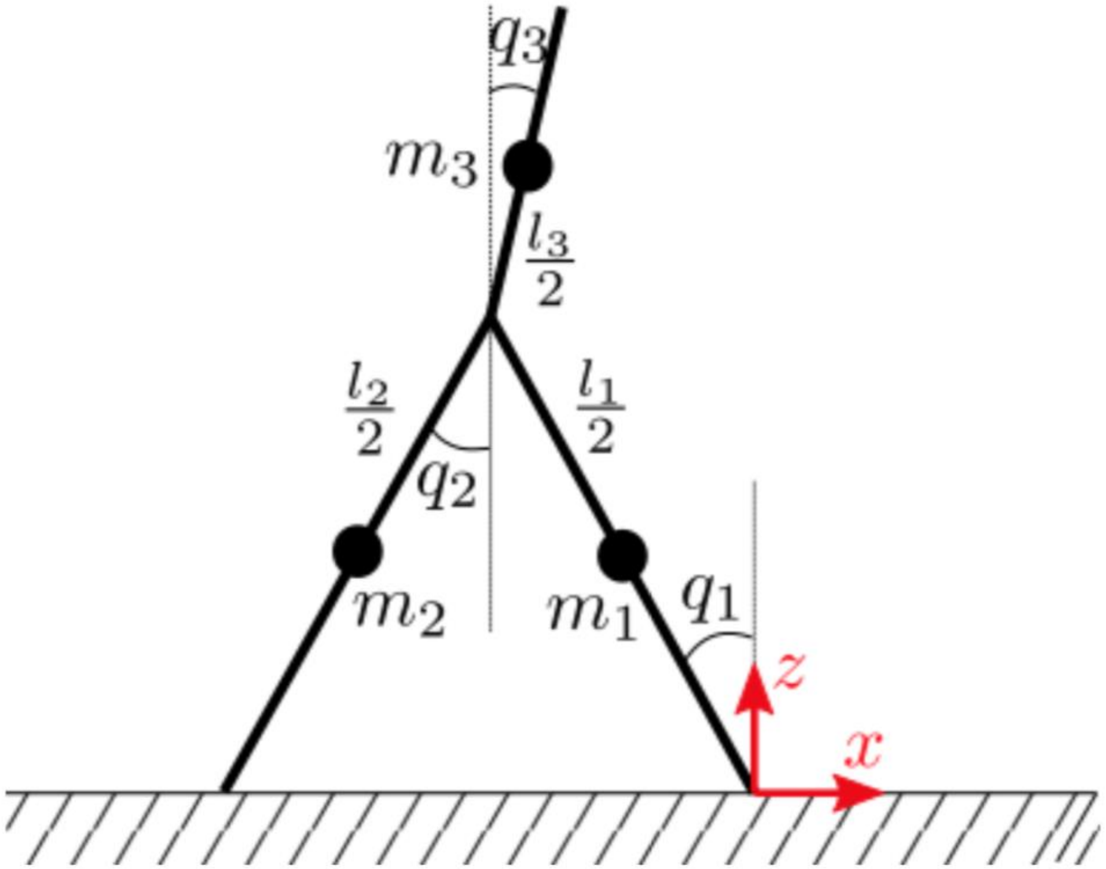
Exploration of control strategies for a three-link 2D biped

Legged Robots

Overview

- Final assignment
- Zero dynamics
- Task space control

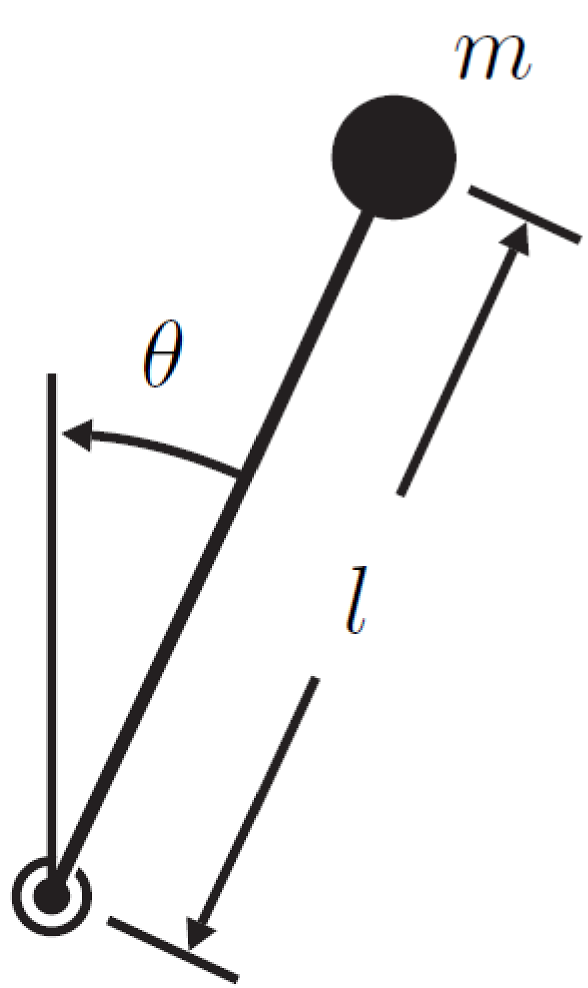


Final assignment

- Part 1: what we did so far
  - Section 1.1: kinematics
  - Section 1.2: dynamics (Lagrange, equations of motion, B)
  - Section 1.3: impact map (do not forget the questions in the code)
  - Section 1.4: model validation (tests)
  - Section 1.5: numerical integration and animation (do not forget the additional questions)
  - Section 1.6: control and optimization (controller, gait metrics, optimization criteria)
- Part 2: implement your own controller
  - the maximum number of steps that indicate stability
  - the minimum and maximum steady-state gait velocity
  - optimize for different step lengths (short, self-selected, and long)
  - optimize for different step frequencies
  - examine the robustness of the controller against external perturbations
  - examine the robustness of the controller against internal perturbations (sensory noise)
- Part 3: compete and reflect
  - novelty
  - compare performances

http://underactuated.csail.mit.edu/index.html

Zero dynamics



$$\underbrace{\begin{bmatrix} ml^2 & 0 \\ 0 & m \end{bmatrix}}_{D(q)} \underbrace{\begin{bmatrix} \ddot{\theta} \\ \ddot{l} \end{bmatrix}}_{\ddot{q}} + \underbrace{\begin{bmatrix} ml\dot{l} & ml\dot{\theta} \\ -ml\dot{\theta} & 0 \end{bmatrix}}_{C(q,\dot{q})} \underbrace{\begin{bmatrix} \dot{\theta} \\ \dot{l} \end{bmatrix}}_{\dot{q}} = \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$y = l - l_d(\theta) \qquad \dot{y} = \dot{l} - \frac{\partial l_d(\theta)}{\partial \theta}\dot{\theta}$$

$$\begin{aligned} \ddot{y} &= \ddot{l} - \frac{\partial^2 l_d(\theta)}{\partial \theta^2}\dot{\theta}^2 - \frac{\partial l_d(\theta)}{\partial \theta}\ddot{\theta} \\ &= l\dot{\theta}^2 - \frac{\partial^2 l_d(\theta)}{\partial \theta^2}\dot{\theta}^2 + \frac{2}{l}\frac{\partial l_d(\theta)}{\partial \theta}\dot{l}\dot{\theta} + \frac{1}{m}u \end{aligned}$$

Section 5.1: http://web.eecs.umich.edu/~grizzle/biped_book_web/

Zero dynamics

$$\ddot{y} = l\dot{\theta}^2 - \frac{\partial^2 l_d(\theta)}{\partial \theta^2}\dot{\theta}^2 + \frac{2}{l}\frac{\partial l_d(\theta)}{\partial \theta}\dot{l}\dot{\theta} + \frac{1}{m}u$$

$$u = u^* + v$$

$$u^* = m\left(-l\dot{\theta}^2 + \frac{\partial^2 l_d(\theta)}{\partial \theta^2}\dot{\theta}^2 - \frac{2}{l}\frac{\partial l_d(\theta)}{\partial \theta}\dot{l}\dot{\theta}\right)$$

$$v = -m\left(K_D\dot{y} + K_P y\right)$$

$$\ddot{y} + K_D\dot{y} + K_P y = 0$$

$$\boxed{v = \ddot{y}_D + K_P\big(y_D - y(q)\big) + K_D\big(\dot{y}_D - \dot{y}(q)\big)}$$

Damped oscillators

$$\ddot{y} + K_D \dot{y} + K_P y = 0$$

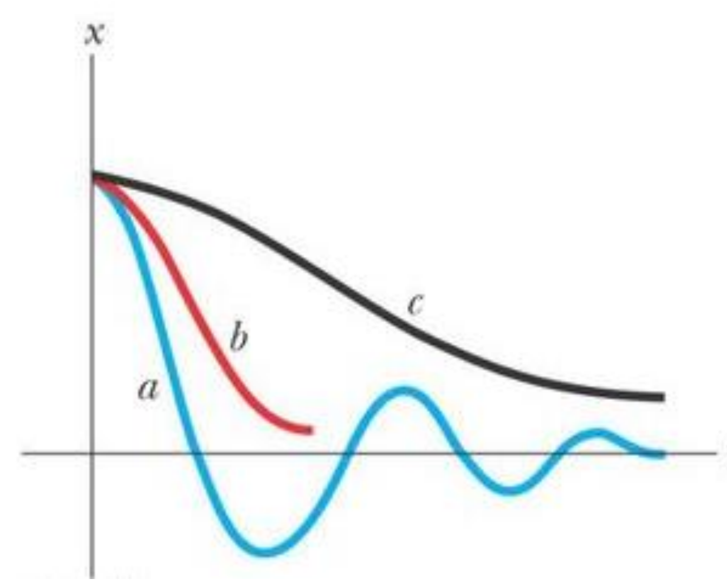
$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

Three important cases:

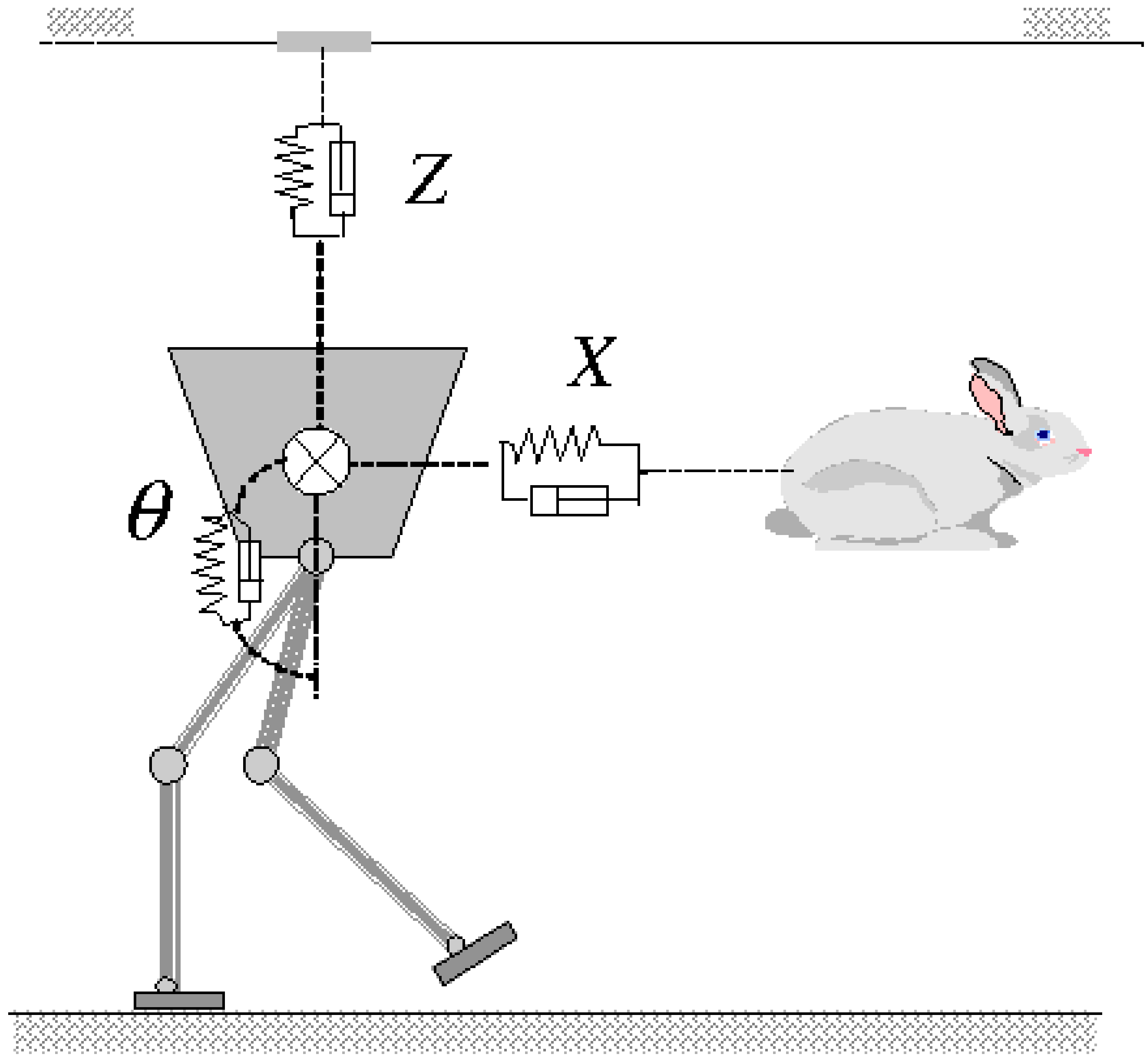
a = **underdamped** $(if\ b < 2\sqrt{mk})$

b = **critically** damped $(if\ b = 2\sqrt{mk})$

c = **overdamped** $(if\ b > 2\sqrt{mk})$

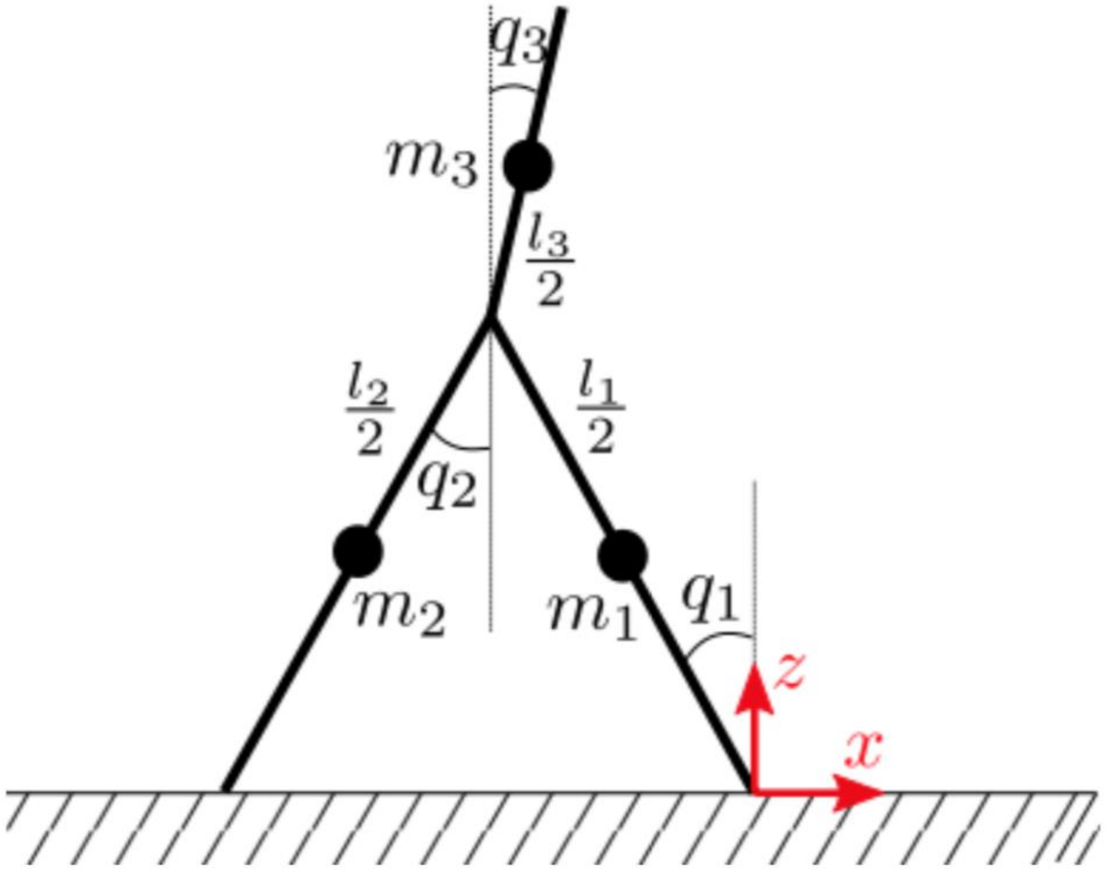


Virtual model control



Pratt, Jerry, et al. "Virtual model control: An intuitive approach for bipedal locomotion." *The International Journal of Robotics Research* 20.2 (2001): 129-143.

Task space projection



$$\boldsymbol{p_h} = \begin{bmatrix} x_h \\ z_h \end{bmatrix} = \begin{bmatrix} l_1 sin(q_1) \\ l_1 cos(q_1) \end{bmatrix} = \boldsymbol{f}(\boldsymbol{q})$$

$$\dot{\boldsymbol{p}}_{\boldsymbol{h}} = \begin{bmatrix} \dot{x}_h \\ \dot{z}_h \end{bmatrix} = \begin{bmatrix} l_1 cos(q_1) & 0 & 0 \\ -l_1 sin(q_1) & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \boldsymbol{J}_h(\boldsymbol{q})\dot{\boldsymbol{q}}$$

$$\boldsymbol{\tau}^T \dot{\boldsymbol{q}} = \boldsymbol{f}_{\boldsymbol{h}}^T \dot{\boldsymbol{p}}_{\boldsymbol{h}} \quad \boldsymbol{\tau} = \boldsymbol{J}_{\boldsymbol{h}}^T(\boldsymbol{q})\boldsymbol{f}_{\boldsymbol{h}}$$

$$\boxed{\boldsymbol{u} = \boldsymbol{B}^+ \boldsymbol{J}_{\boldsymbol{h}}^T(\boldsymbol{q})\boldsymbol{f}_{\boldsymbol{h}}}$$

$$\boldsymbol{f}_{\boldsymbol{h}} = \begin{bmatrix} \boldsymbol{k}_{\boldsymbol{p}}(\boldsymbol{x}_{\boldsymbol{d}} - \boldsymbol{x}_{\boldsymbol{h}}) + \boldsymbol{k}_{\boldsymbol{d}}(\dot{\boldsymbol{x}}_{\boldsymbol{d}} - \dot{\boldsymbol{x}}_{\boldsymbol{h}}) \\ \boldsymbol{k}_{\boldsymbol{p}}(\boldsymbol{z}_{\boldsymbol{d}} - \boldsymbol{z}_{\boldsymbol{h}}) + \boldsymbol{k}_{\boldsymbol{d}}(\dot{\boldsymbol{z}}_{\boldsymbol{d}} - \dot{\boldsymbol{z}}_{\boldsymbol{h}}) \end{bmatrix}$$