Derive the equations of motion of a double pendulum

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Introduction

I am sure by now you have seen many legged robots. The question is, if I give you a legged robot how would you make it walk? Where do you start? Of course, there is an interface where you can receive sensory data and send commands, but will you start by sitting and sending commands and see what happens? No. So, what is the starting point? The starting point is to model, then design control and finally simulate. The whole point of the mini-project is to realize the following **Control Design Pipeline**:

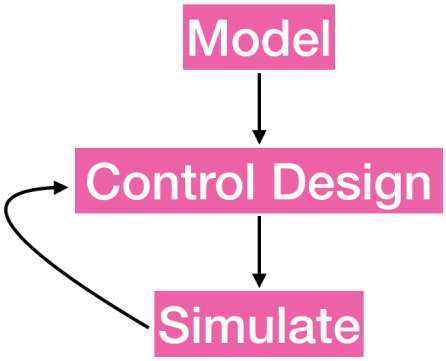

Figure 1: **Control Design Pipeline**

In order to understand the main principles behind the control design, we will work with a simple model: the **Three-link 2D Biped**, as represented in Figure 2. The project objectives can be divided into three main parts:

- Modelling and visualization of the 3-link;
- Solving the equations of motion of the 3-link biped and collision handling (simulation);
- Design of different walking controllers, evaluate the resultant gaits and compare the performances.

Each week, students should submit their answer to the tasks so that we can follow their progress and help them solve their issues during the practical sessions. At the end of the course, you will be asked to submit a final report, your code and demonstration videos related to the topics that will be covered during the course. It is advised to structure your final report according to the following sections:

- Introduction
- Methods

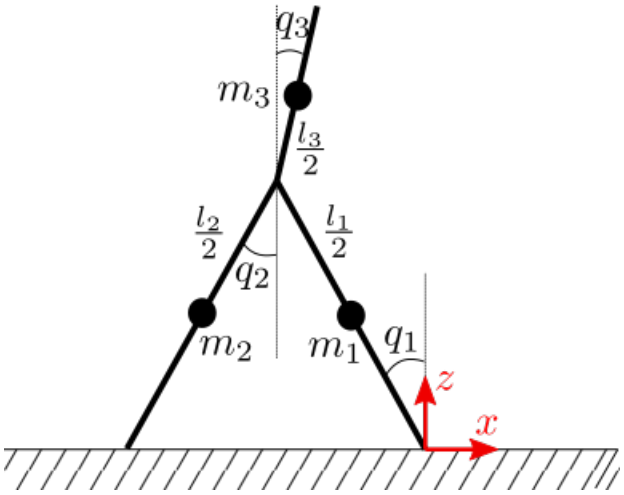

Figure 2: **Three Link Biped**

- Results
- Discussion
- Conclusion

In this lecture, we will get started with the first point, that is *Modelling and visualization of the 3-link*. This will be done by understanding what a "Model" is and how to develop the kinematic and dynamic equations of a double pendulum.

What is a "Model"?

What do we mean by model? We mean developing kinematics and dynamics from a simplified model (e.g., urdf: Universal Robot Description Format) of the real robot. Normally, for complex robots like the COMAN with 31 actuators, we do not do this from scratch and there are tools that convert (reduce) the mechanical design (CAD files) to simplified models (urdf or srdf). There are libraries that symbolically or numerically calculate the kinematics and dynamics (for instance, Robotran calculates the kinematics and dynamics symbolically, while RBDL does the same thing numerically). Software such as Gazebo take the urdf and internally calculate the kinematics and dynamics numerically, solve the dynamics and presents a graphical simulation of the real robot. However, it is a good practice to perform this process from scratch yourself. Indeed, the goal of the mini-project's modeling part is to implement the kinematics and the dynamics of a 3-link biped. In order to get familiar with such an implementation let us start with implementing the equations of motion of the double pendulum represented in Figure 3, that can be considered as a simplified version of the 3-link biped.

Exercise 1.1: Equations of motion of the double pendulum

In this exercise (**NOT GRADED**) you are asked to derive the equations of motion of the double pendulum and write them in the document *double_pendulum_start.mlx*.

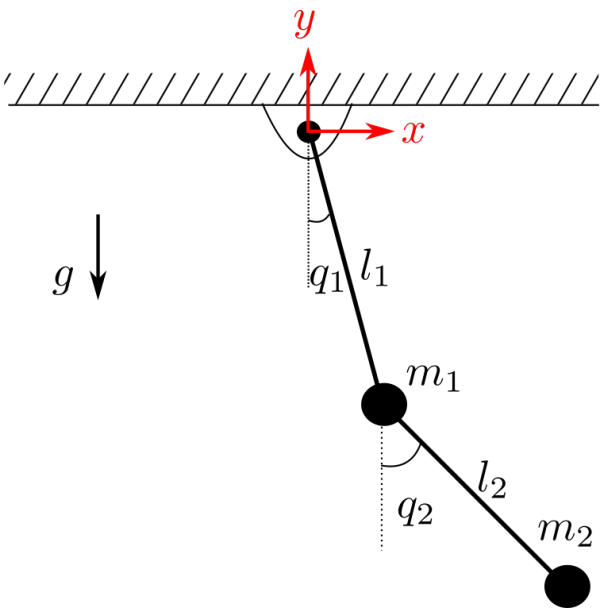

Figure 3: **Double Pendulum**