Swing-Leg Retraction for Limit Cycle Walkers Improves Disturbance Rejection

Daan G. E. Hobbelen and Martijn Wisse, *Member, IEEE*

***Abstract*—Limit cycle walkers are bipeds that exhibit a stable cyclic gait without requiring local controllability at all times during gait. A well-known example of limit cycle walking is McGeer's "passive dynamic walking," but the concept expands to actuated bipeds as involved in this study. One of the stabilizing effects in limit cycle walkers is the dissipation of energy that occurs when the swing foot hits the ground. We hypothesize that this effect can be enhanced with a negative relation between the step length and step time. This relation is implemented through an open-loop strategy called swing-leg retraction; a predefined time trajectory for the swing leg makes the swing leg move backwards just prior to foot impact. In this paper, we study the effect of swing-leg retraction through three bipeds; a simple point mass simulation model, a realistic simulation model, and a physical prototype. Their stability is analyzed using Floquet multipliers, followed by an evaluation of how well disturbances are handled using the Gait Sensitivity Norm. We find that mild swing-leg retraction is optimal for the disturbance rejection of a limit cycle walker, as it results in a system response that is close to critically damped, rejecting the disturbance in the fewest steps. Slower retraction results in an overdamped response, characterized by a positive dominant Floquet multiplier. Likewise, faster retraction results in an underdamped response, characterized by a negative Floquet multiplier.**

***Index Terms*—Bipeds, disturbance rejection, dynamic walking, limit cycle walkers, swing-leg retraction.**

I. Introduction

BIPEDAL walking is a complex dynamic task, although humans can walk with apparent ease. We aim to understand the dynamic principles that underlie the elegant human combination of efficiency, robustness, and versatility. Such knowledge can be applied in various fields, for instance in improving the quality of prosthetics and orthotics or in the design of autonomous bipedal robots.

An important dynamic principle is described by the concept of "limit cycle walking" [3]. This concept says that it is possible to obtain stable periodic walking without locally stabilizing the walking motion at every instant during gait. The term is derived from the definition of a stable limit cycle, which is a closed trajectory in state–space (i.e., periodic motion) to which neighboring trajectories converge. A limit cycle walker utilizes the property that such a trajectory is stable when observing it from step to step (including step-to-step transitions), while it is not locally stabilized (or not even locally controllable) along the entire trajectory. Some examples of limit cycle walkers are shown in Fig. 1 and described by Collins *et al*. [4].

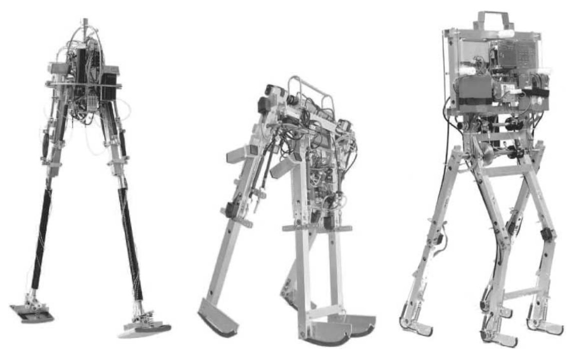

Fig. 1. Three limit cycle walkers that have been build at Delft University of Technology [1], [2]. The third walker is the subject of the current study.

A key stabilizing effect in limit cycle walking appears to be the dissipation of energy when the swing foot hits the ground. When the kinetic energy content (or speed) of the walker is larger than nominal, this generally results in a larger impact and increased energy dissipation at foot strike. Due to this self-regulating effect, the biped eventually (over multiple steps) returns to the nominal limit cycle.

Our hypothesis is that the stabilizing transition effect can be enhanced by adjusting the step length. We propose to make step length a function of step time, as step time is an indicator of the walker's kinetic energy content. The proposed function creates a negative relation between step length and step time, as longer step time indicates *smaller* kinetic energy content and longer step lengths induce *larger* energy loss at impact [5]. The same relation has previously been applied in a running model by Seyfarth *et al*. [6]. By applying it, a fast step (too much kinetic energy content) will result in a longer step length that increases the energy dissipation at foot strike. A slow step will result in a shorter step length and reduced energy dissipation.

The proposed negative relation between step length and step time for walking is based on observations in passive dynamic walkers pioneered by McGeer [7]. Passive dynamic walkers are examples of limit cycle walkers that are able to show stable cyclic gait without any control or actuation. The necessary external energy supply comes from gravity as they walk down a slope. Most passive dynamic walkers have two limit cycle gaits, one of which is stable and one of which is unstable [8]. The stable limit cycle gait, referred to as the "long period gait," shows

Manuscript received June 20, 2007; revised November 12, 2007. This paper was recommended for publication by Associate Editor B. J. Yu and Editor K. Lynch upon evaluation of the reviewers' comments. This work was supported in part by the Delft Centre for Mechatronics and Microsystems, in part by the Dutch Technology Foundation STW, in part by the Applied Science Division of NWO, and in part by the Technology Program of the Ministry of Economic Affairs.

The authors are with the Department of Mechanical Engineering, Delft University of Technology, Delft 2628CD, The Netherlands (e-mail: d.g.e.hobbelen@tudelft.nl; m.wisse@tudelft.nl).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TRO.2008.917002

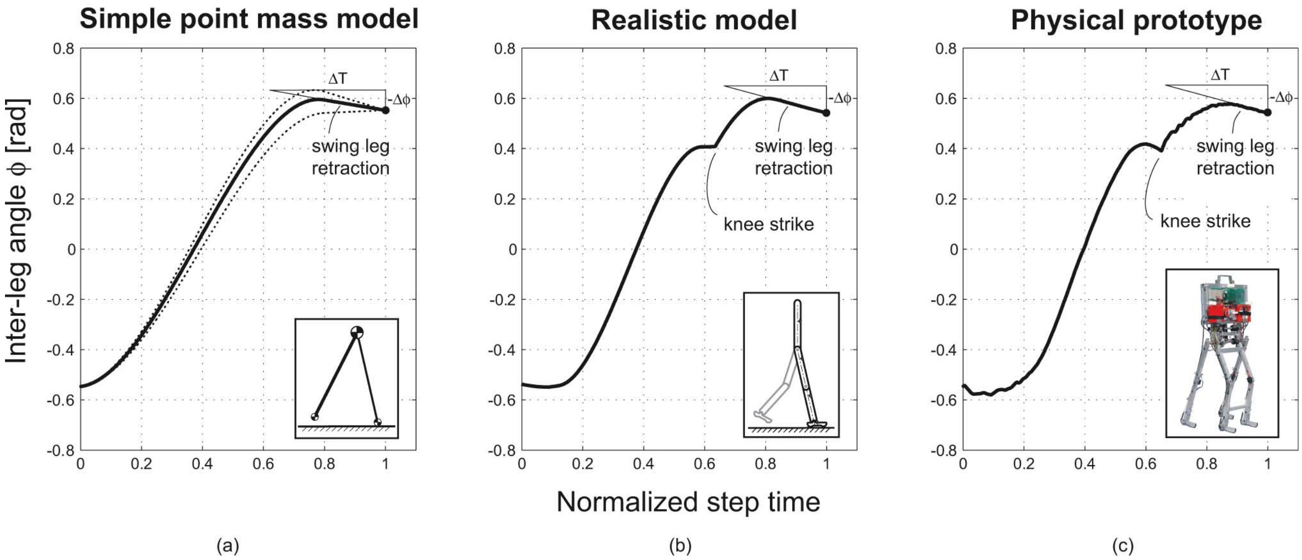

Fig. 2. (a) Implementation of swing-leg retraction on the simple point mass model. (b) Realistic model. (c) Physical prototype. Retracting the swing leg results in a negative relation between the walkers' step length and step time. The slope of the end of the swing leg trajectory ($\dot{\phi}^- = \Delta\phi/\Delta T$) is varied (exemplified by the dotted lines for the simple point mass model) to study the effect of swing-leg retraction speed on disturbance rejection. All trajectories pass through a specific point (indicated by the big dot) to ensure equal nominal step length and step time in all cases.

a gait in which a faster step is longer, i.e., a negative relation between step length and step time. The unstable limit cycle, the "short period gait," shows the opposite, a gait in which a faster step is shorter. This observation has motivated us to further study the stabilizing effect of the relation between step length and step time.

We implement a negative relation between a walker's step length and step time through "swing-leg retraction." Swing-leg retraction describes a motion of the swing leg in which it is moving backwards just prior to foot impact. This automatically results in longer step lengths for fast steps and shorter step lengths for slow steps. This retracting motion of the swing leg is the same for every step in the gait cycle and is thus not dependent on the walker's state; swing-leg retraction is an open-loop strategy.

Open-loop stabilizing behaviors similar to swing-leg retraction have been found in running and juggling, other examples of dynamical tasks with intermittent contact. The work on juggling [9] featured a robot that had to hit a ball that would then ballistically follow a vertical trajectory up and back down until it was hit again. The research showed that stable juggling occurs if the robot hand is following a well chosen trajectory, such that its upward motion is decelerating when hitting the ball. The stable juggling motion required no knowledge of the actual position of the ball. We feel that the motion of the hand and ball is analogous to that of the swing leg and stance leg, respectively. Also analogous is the work on the simple point mass running model that we mentioned earlier [6]. It was shown that the stability of the model was significantly improved by the implementation of a retraction phase in the swing leg motion.

The goal of this paper is to investigate how swing-leg retraction affects a biped's ability to handle disturbances, i.e., its disturbance rejection. In Section II, the concept of swing-leg retraction and its implementation on two walker simulation models and one physical prototype will be explained. The disturbance rejection of all three walkers will be studied based on three measures, the Floquet multipliers in Section III, the disturbance response in Section IV, and the Gait Sensitivity Norm in Section V. The paper ends with a discussion and conclusion in Sections VI and VII.

II. Swing-Leg Retraction

The effect of swing-leg retraction will be studied in this paper using two walking models and one physical walking prototype. The first model is a simple point mass model (Section II-A), the second is a more realistic model (Section II-B) that closely resembles the physical properties of the real prototype (Section II-C). In all three cases, swing-leg retraction will be implemented as a fixed constant interleg angular speed at the end of each step of the gait cycle, just prior to foot impact. The swing leg will first be swung forward, which will take 80% of the total nominal swing time for all walkers involved in this study. This percentage is based on what can be obtained on the physical prototype given its dynamics and actuator properties. For the remainder of the swing phase, the swing leg follows a constant speed trajectory, as shown in Fig. 2. The magnitude of this speed will be varied to study the effect of the swing-leg retraction speed on the disturbance rejection of limit cycle walkers. The amount of swing-leg retraction is quantified by a dimensionless number $\bar{\dot{\phi}}^-$ given by the slope of the interleg angle at the end of a step ($\dot{\phi}^-$) divided by the nominal interleg angle at heel strike (ϕ^*) and multiplied by the nominal step time (T^*) of the walking model:

$$\bar{\dot{\phi}}^- = \frac{T^*}{\phi^*}\dot{\phi}^- = \frac{T^*}{\phi^*}\frac{\Delta\phi}{\Delta T}. \tag{1}$$

In this study, for all models, the nominal step length is kept constant at 54% of the models' leg length and step time is

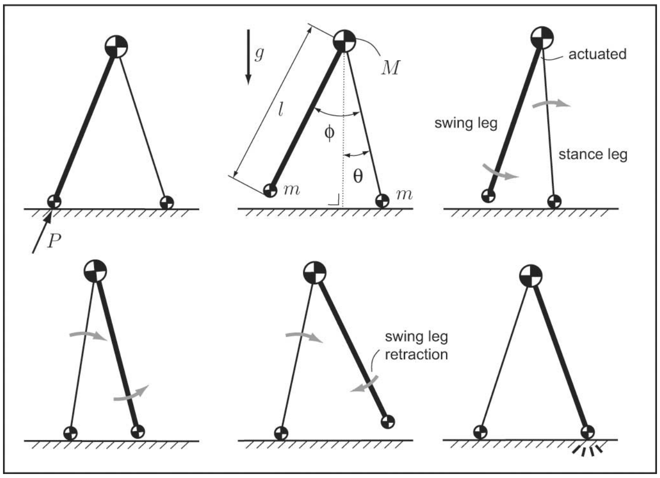

Fig. 3. Typical walking step of our simple point mass model. An impulsive push-off P is applied at the trailing leg just after foot strike has occurred, subsequently the actuated swing leg (heavy line) swings forward past the stance leg (thin line), retracts at the end of the step until the swing leg hits the ground, and a new step begins. θ is the angle between the stance leg and the slope normal, ϕ is the angle between the two legs, l is the leg length, M is the hip mass, m is the foot mass, g is the gravitational acceleration. Adapted from Garcia *et al.* [8].

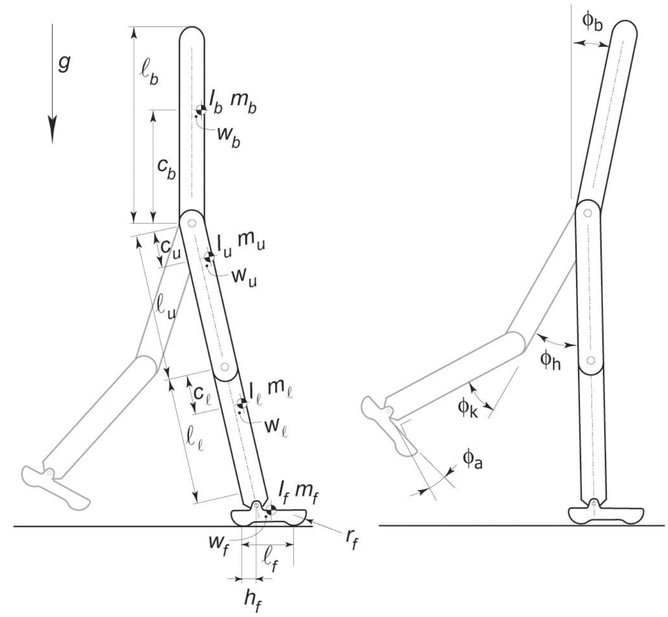

Fig. 4. 2-D seven-link realistic walking model. The actual parameter values are given in Table I.

kept constant at a Froude number of $Fr = v/\sqrt{gl} = 0.16$. This ensures that differences in the models' behaviors can only be contributed to the change in swing-leg retraction speed.

Fig. 2 shows this implementation of swing-leg retraction on the simple point mass model, the realistic model, and the physical prototype that will be used in this study.

A. Simple Point Mass Model

Simple straight legged point mass models of bipeds have been studied thoroughly [8], [10]–[12]. Due to the fact that the dynamics of these models are well understood and that their simulation time is relatively short, we use such a point mass model as the start of our research into swing-leg retraction.

Our simple point mass model closely resembles the simplest walking model by Garcia *et al.* [8]. Our model is a 2-D model consisting of two rigid links with unit length l, connected at the hip. There are three point masses in the model, one in the hip with unit mass M and two infinitesimally small masses m in the feet as shown in Fig. 3. The model walks in a gravity field with unit magnitude g.

During single stance, the stance leg acts as an inverted pendulum as the foot is a point and there is no torque acting between the foot and the floor. The equation of motion for the stance leg is

$$\ddot{\theta} = \sin\theta. \tag{2}$$

The main difference with Garcia's model is that our swing leg is not passive. It follows a predefined trajectory. This does not influence the stance leg dynamics, because the swing leg has negligible mass. The exact trajectory of the swing leg is defined by two knot points. The trajectory before the first knot point (located at 80% of the nominal swing time) is a third-order polynomial with four end constraints. It starts with the actual swing leg angle and velocity just after heel strike and ends at a specific swing leg angle enforcing velocity continuity to the second portion of the trajectory. This second portion of the trajectory is a straight line, the slope of which corresponds to the walker's swing-leg retraction speed.

The single stance phase ends with a discrete impact when the swing foot hits the ground, an event that occurs when $\phi = 2\theta$. The required propulsive energy for the walker is supplied by an impulsive push-off P at the trailing leg just after this foot strike. This is not the most energetically efficient way of pushing off [12], [13], but it best resembles the push-off that is currently applied in our physical prototype. The magnitude of the push-off P is regulated to add a constant amount of kinetic energy $E_{\text{push-off}}$ at every step. During the push-off, the leading leg is assumed to remain in contact with the ground. The combination of the foot impact and the impulsive push-off results in the following step-to-step transition equations:

$$\begin{aligned}
\theta^+ &= -\theta^- \\
\phi^+ &= -2\theta^- \\
\dot{\theta}^+ &= -\sqrt{E_{\text{push-off}} + (\cos(2\theta^-)\dot{\theta}^-)^2} \\
\dot{\phi}^+ &= -(1-\cos(2\theta^-))\sqrt{E_{\text{push-off}} + (\cos(2\theta^-)\dot{\theta}^-)^2}.
\end{aligned} \tag{3}$$

Note that the swing-leg retraction speed $\dot{\phi}^-$ is not explicitly present in these equations, but does in fact strongly affect the step-to-step transitions. With varying step duration, the swing-leg retraction speed changes the leg angle θ^- at which foot impact occurs.

B. Realistic Model

A more realistic model of a walker is the seven-link model depicted in Fig. 4. It is modeled after the prototype that is

TABLE I
PARAMETER VALUES FOR THE REALISTIC WALKING MODEL

	body	upper leg	lower leg	foot
mass m [kg]	8	0.7	0.7	0.1
mom. of Inertia I [kgm^2]	0.11	0.005	0.005	0.0001
length l [m]	0.4	0.3	0.3	0.085
vert. dist. CoM c [m]	0.2	0.15	0.15	0
hor. offset CoM w [m]	-0.02	0	0	0.0175
foot radius r_f [m]	-	-	-	0.02
foot heel-ankle h_f [m]	-	-	-	0.025

also used in this study and it consists of an upper body, two upper legs, two lower legs, and two feet. In the hip joint, a bisecting mechanism is present that makes the two upper legs move symmetrically relative to the upper body [14]. The knee joint holds a hyperextension stop and a latching mechanism. The latch is released at the start of the swing phase and locks the knee joint in the stretched position at the end of the swing phase and throughout stance. Due to these mechanisms, the maximum number of degrees of freedom of this model is 5 at any instant during gait.

The unilateral constraints between the foot and the floor as well as the hyperextension stop in the knee are modeled as rigid constraints. In the foot, there are two unilateral constraints (one horizontal and one vertical) when only the heel or only the toe of the foot is touching the ground and three constraints (one horizontal and two vertical) when the foot is fully flat on the floor. During simulation, these constraints can be created and released. To check whether a contact should be released, in every simulation step, we perform a separate calculation in which the constraint is released and observe in which direction the contact point would accelerate. In case this acceleration is away from the unilateral constraint (e.g., the heel comes up from the floor), we release the constraint in the actual simulation [15]. Constraints are created when a kinematic constraint violation is detected (e.g., heel passes the floor). After this, a discrete impact calculation is performed. In this impact calculation, initially existing unilateral constraints are assumed to remain. After the initial calculation, we check the impulsive forces of the existing unilateral constraints in a specific order. If impulsive forces exist that are in the opposite direction of the unilateral constraint, this constraint is released and the discrete impact calculation is repeated. This iterative process is continued until all impulsive forces are in the direction corresponding to the unilateral constraints. Continuous actuator forces are not included in the impact calculation as their effect is negligible in comparison with the discrete impact forces that are infinitely high forces occurring for an infinitely small duration.

The model has actuation in the hip joint and in the two ankle joints, the knee joints are fully passive. We assume ideal, unlimited force actuators that allows us to accurately obtain the desired swing leg trajectory for the purpose of establishing the effect of swing-leg retraction. The desired swing leg trajectory in this model is defined by three knot points and uses third-order splines. One knot point is added in comparison with the simple point mass model to get a more controlled motion of the knee joint and knee impact [the effect of this added knot point can be seen in Fig. 2(b)]. The end of the trajectory is a straight line again, whose slope corresponds to the model's swing-leg retraction speed. A proportional-derivative controller with high feedback gains ensures that the actual swing leg motion closely follows the desired trajectory.

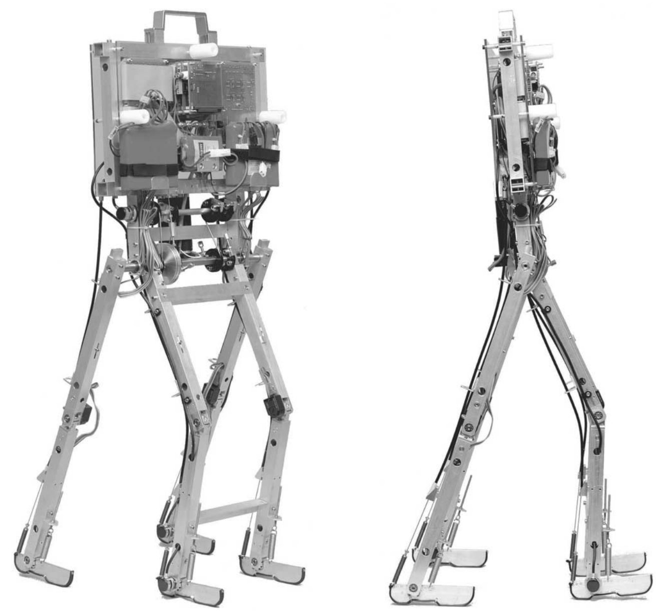

Fig. 5. Physical prototype "Meta." This 2-D biped consists of an upper body, two upper legs, two lower legs, and two feet. The hip joint and the two ankle joints are actuated by electric dc motors, the hip joint can be position controlled and the ankle joint can be position and force controlled through series elastic actuation [16]. All joints are equipped with digital encoders to measure the joint angles, the prototype is controlled at 1 kHz running MATLAB's xPC Target on a PC104 stack (400 MHz processor).

The actuation in the ankle joints is used to achieve push-off with the trailing leg during double stance. Push-off starts at the instant of heel strike with the leading leg. A constant amount of energy is added in every step by applying a constant torque in the ankle over a fixed angular range. During the swing phase, the angle of the ankle joint is controlled to a fixed value by a proportional-derivative position controller.

C. Physical Prototype "Meta"

Next to the two simulation models, this study involves a physical 2-D biped called "Meta," shown in Fig. 5. The prototype consists of an upper body, two upper legs, two lower legs, and two feet. Both legs are constructed in pairs (one outside and one inside) to achieve 2-D behavior.

The hip joint features a bisecting mechanism that is constructed using an auxiliary axle and a set of pulleys and cables [14]. The joint is actuated by an electric dc motor (Maxon RE35) through a 103:1 planetary gearbox and a cable drive. The knee joints are fully passive, but feature a hyperextension stop and latching mechanism that can be released through a solenoid. The ankle joints are actuated by two electric dc motors (Maxon RE35) through 23:1 planetary gearboxes and a cable drive that adds another reduction of 4:1. The ankle is series elastically actuated [16] by means of an elastic element in the cable. This actuation allows the joint to be force controlled as well as position controlled.

All joints are equipped with digital incremental encoders with a resolution of 2×10^{-4} rad for the actuated joints and 3×10^{-3} rad for the knees. The actuation force on the ankle joint is measured through the elongation of the series elastic element, which results in a torque resolution of 4×10^{-3} N $\cdot$ m. Microswitches located near the heels underneath the feet detect heel strike. The prototype is controlled from a PC104 stack that includes a 400 MHz processor, analog I/O, and counters for the digital encoders. This stack allows a controller to be run at 1 kHz using MATLAB's xPC Target.

In each step of the prototype, the controller detects heel strike through the microswitches that triggers a chain of control actions. First, the ankle push-off with the trailing leg is initiated; a constant ankle torque is commanded by controlling a constant elongation of the series elastic element. This action is maintained until the ankle joint has traveled a predefined angular range. After that, the swing phase starts and the ankle control switches to position control that ensures the toe is pointing upwards during swing to prevent toe stubbing. At the same time, the hip joint is controlled to follow a predefined time trajectory.

The desired swing leg trajectory is defined by three knot points, similar to the realistic simulation model. In the physical prototype, however, we have to deal with friction, noise, limited feedback gains to ensure stability, and limited actuator torque. These effects result in an actual swing leg motion that differs from step to step and from the desired trajectory. The relation between the prototype's step length (interleg angle at heel strike) and step time will not exactly be equal to the desired swing-leg retraction. Because of this, we need to measure the "effective" swing-leg retraction speed by establishing the actual relation between the prototype's interleg angle at heel strike and step time through measurements as shown in Fig. 6. The graph shows a collection of data points giving the step time and interleg angle at heel strike of 300 steps having the same desired swing leg trajectory (solid line). Indeed, the data points do not exactly coincide with the desired trajectory. The effective swing-leg retraction speed is defined as the slope of the least squares linear fit (dashed line) through the data points.

III. Floquet Multipliers

To understand the effect of swing-leg retraction on the disturbance rejection capability of limit cycle walkers, we will study how swing-leg retraction affects various properties of the walkers. The first property related to disturbance rejection is the Floquet multipliers.

A. Definition

The Floquet multipliers (λ_i) are obtained using the Poincaré mapping method, which observes how the biped states evolve on a step-to-step basis. It involves only small deviations from the nominal limit cycle, in which the initial conditions of each step are identical (the "fixed point" $\mathbf{v}^*$). The method perturbs these initial conditions of a step by a small amount $(\Delta \mathbf{v}_n)$ and measures how this affects the initial conditions to the subsequent step $(\mathbf{v}_{n+1})$. The step-to-step behavior of the walker is approximated by

$$\mathbf{v}^* + \Delta \mathbf{v}_{n+1} \approx \mathbf{v}^* + \mathbf{A}\,\Delta \mathbf{v}_n. \tag{4}$$

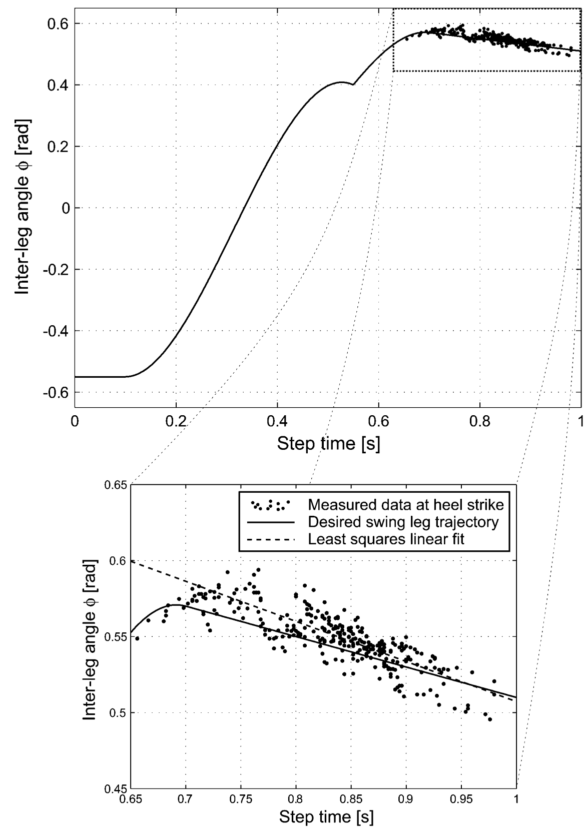

Fig. 6. Measurements of swing-leg retraction in the physical prototype; data are taken from 300 steps. The least squares linear fit through these data points (solid line) gives the best-fit estimate of the prototype's retraction speed.

The matrix $\mathbf{A}$ (Jacobian) is the partial derivative of the function that maps the initial conditions of a step to the initial conditions of the subsequent step and can be found through simulation. The Floquet multipliers are the eigenvalues of the matrix $\mathbf{A}$.

The magnitude of the Floquet multipliers indicates the local stability of a limit cycle walker. If all the Floquet multipliers are within the unit circle, the walker's limit cycle is locally stable, i.e., small deviations from the limit cycle will decay over time and the walking gait will eventually converge back to the nominal limit cycle. The closer the Floquet multipliers are to zero, the faster the convergence rate will be.

B. Results for the Simple Point Mass Model

The simple point mass model has two Floquet multipliers, because it has two independent initial conditions to a step. The Floquet multipliers of this model are shown in Fig. 7 for a range of swing-leg retraction speeds. A positive retraction speed indicates that the swing leg keeps moving forward prior to heel strike, zero retraction speed means the interleg angle is kept fixed, and a negative retraction speed indicates actual retraction of the swing leg. Fig. 7 shows that locally stable walking gaits exist for normalized retraction speeds between -0.80 and 0.12.

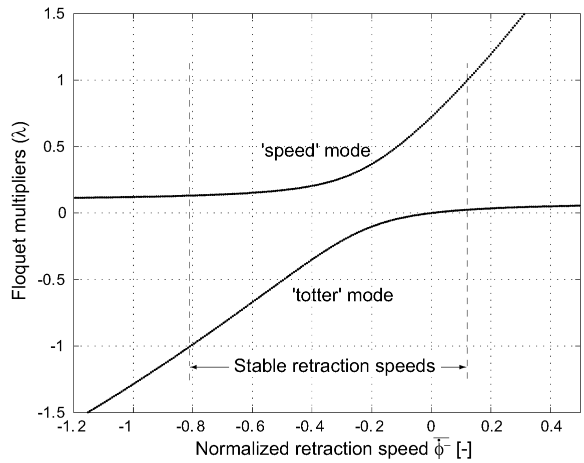

Fig. 7. Floquet multipliers of the simple point mass model for a range of swing-leg retraction speeds. The stable retraction speeds range from −0.80 to 0.12.

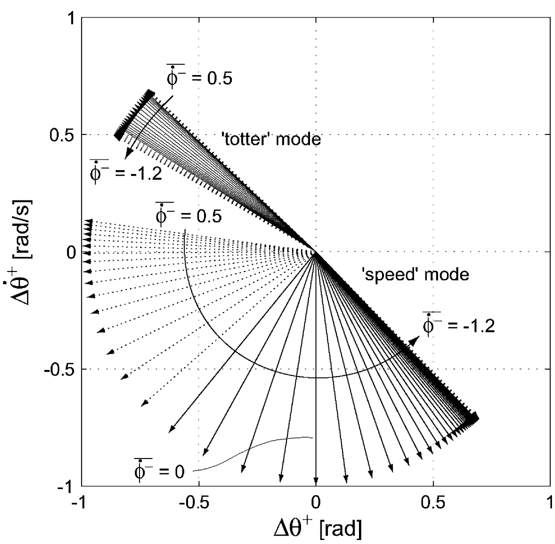

Fig. 8. Eigenvectors of the two eigenmodes of the simple point mass model for a range of swing-leg retraction speeds. The "speed" mode has a corresponding positive Floquet multiplier and the "totter" mode has a corresponding negative Floquet multiplier. The eigenvectors outside the stable range of retraction speeds are depicted as dotted arrows.

This means there is a larger range of negative speeds (retraction of the swing leg) that gives stable gaits than positive speeds.

Two separate eigenmodes with corresponding eigenvalues (Floquet multipliers) and eigenvectors (see Fig. 8) can be distinguished in the simple point mass model's behavior. One of the eigenmodes has a corresponding Floquet multiplier that is positive for all retraction speeds and is dominant (close to one) for positive retraction speeds and retraction speeds close to zero. This eigenmode has been referred to as the "speed" mode by McGeer [7]. When this eigenmode is excited, the system monotonically converges back to the nominal walking speed, i.e., in case the excitation makes the walker go faster than nominal, the consecutive steps will tend to get slower but will never get slower than nominal. Fig. 8 shows that steps in the "speed" mode that are faster than nominal ($\Delta\dot{\theta}^+ < 0$) will be shorter ($\Delta\theta^+ < 0$) for positive retraction speeds and longer ($\Delta\theta^+ < 0$) for negative retraction speeds. The second eigenmode shows a corresponding Floquet multiplier that is negative for most retraction speeds and is dominant at large negative retraction speeds. This mode has been named the "totter" mode, an oscillatory attempt to match step length with forward speed. The negative Floquet multiplier and the accompanying eigenvectors in Fig. 8 show that this mode distinguishes itself by an alternating succession of short, fast steps ($\Delta\theta^+ < 0$ and $\Delta\dot{\theta}^+ < 0$) and long, slow steps. The effect of both eigenmodes will become apparent when we look at the disturbance response of this model in Section IV.

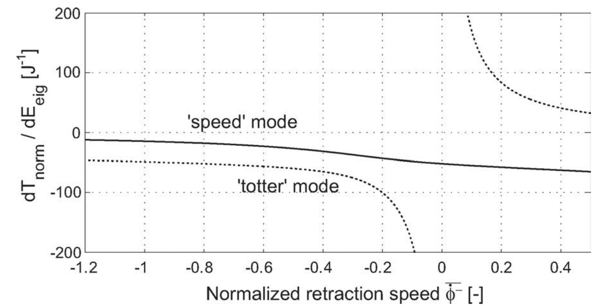

Fig. 9. Change in the duration of a step of the simple point mass model just after an excitation of either one of the eigenmodes of the model. A negative sensitivity of the step time to the perturbation ($dT_{\mathrm{norm}}/dE_{\mathrm{eig}}$) indicates a smaller step time when the perturbation adds energy to the walker. This negative sensitivity generally exists for all models with a negative retraction speed. $dT_{\mathrm{norm}}/dE_{\mathrm{eig}}$ does not exist for the "totter" mode at $\bar{\phi}^- = 0$ as a perturbation of this mode has zero energy content.

C. Discussion of the Simple Point Mass Model

The Floquet multipliers of the simple point mass model show that the majority of the stable limit cycles exist for retraction of the swing leg (a negative retraction speed). In this section, we will discuss the stabilizing feature that explains this result.

The stabilizing effect of swing-leg retraction as postulated in Section I was based on two assumptions. Firstly, it was assumed that steps with larger energy content would generally lead to shorter step times. Through swing-leg retraction, this leads to a longer step length and this would result in an increased energy loss at foot impact, the second assumption. This increased energy loss compensates for the larger energy content of the walker.

The relation between step time and energy content of the simple model is shown in Fig. 9. The quantity $dT_{\mathrm{norm}}/dE_{\mathrm{eig}}$ gives the sensitivity of the model's normalized step time to a perturbation with unit energy content in either one of the eigenmodes ("speed" and "totter"). A negative value of $dT_{\mathrm{norm}}/dE_{\mathrm{eig}}$ means that increased energy content leads to a shorter step time, confirming the first assumption. This negative value exists consistently for both eigenmodes for all negative retraction speeds (cases with swing-leg retraction). The first assumption that explains the stabilizing effect of swing-leg retraction holds.

The relation between impact energy loss and energy content of the simple model is shown in Fig. 10. The quantity $dE_{\mathrm{impact}}/dE_{\mathrm{eig}}$ gives the sensitivity of the model's

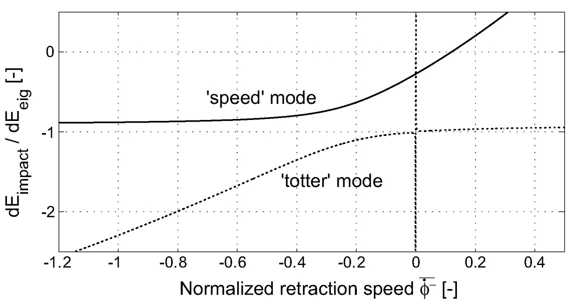

Fig. 10. Change in the amount of energy loss that occurs in the simple point mass model in the step after an excitation of either one of the eigenmodes of the model. A negative sensitivity $dE_{\mathrm{impact}}/dE_{\mathrm{eig}}$ indicates increased energy loss at foot impact when the energy content of the walker is increased. $dE_{\mathrm{impact}}/dE_{\mathrm{eig}}$ shows large resemblance with the Floquet multipliers in Fig. 7 indicating that the change in foot impact losses is the stabilizing factor in the model.

energy loss at the first foot impact after a perturbation with unit energy content in either one of the eigenmodes. A negative value of $dE_{\mathrm{impact}}/dE_{\mathrm{eig}}$ means that more energy is lost at impact for an increased energy content of the walker, the value $dE_{\mathrm{impact}}/dE_{\mathrm{eig}} = -1$ means that all of the added energy is lost immediately at the first foot impact. The impact energy loss gets larger (more negative) for more swing-leg retraction (more negative retraction speed), meaning that the aforementioned second assumption also holds.

The dependence of $dE_{\mathrm{impact}}/dE_{\mathrm{eig}}$ on retraction speed in Fig. 10 shows large resemblance to the dependence of the Floquet multipliers on retraction speed in Fig. 7, indicating that the variation of the foot impact losses is the stabilizing factor in the model. Roughly, the model shows stable gaits for $-2 < dE_{\mathrm{impact}}/dE_{\mathrm{eig}} < 0$. The value of $dE_{\mathrm{impact}}/dE_{\mathrm{eig}} = -1$ corresponds to a Floquet multiplier of 0 as an eigenmode perturbation is canceled completely by one foot impact.

D. Results and Discussion of the Realistic Model

The Floquet multipliers of the realistic model are shown in Fig. 11. There are four Floquet multipliers for this model as it has four independent initial conditions. The imaginary parts of all the Floquet multipliers are close to zero. The range of normalized retraction speeds that result in stable gaits is between -1.1 and 0.4.

Despite great differences between the two simulation models, the Floquet multiplier results are surprisingly similar. In both cases, the largest part of the stable retraction speed range is negative, meaning the swing leg is moving backward before heel strike. Also, the shape in the graphs of Figs. 7 and 11 shows definite resemblance. Two similar modes ("speed" and "totter") can be distinguished and the extra modes in the realistic model (there are two extra eigenmodes) appear to be irrelevant as the corresponding eigenvalues are all close to zero. Even quantitatively, the stable ranges of retraction speeds are quite similar. These observations confirm the believe that the simple point mass model describes the essential dynamics of a more complex walker rather well and that the discussion in the previous section applies here as well.

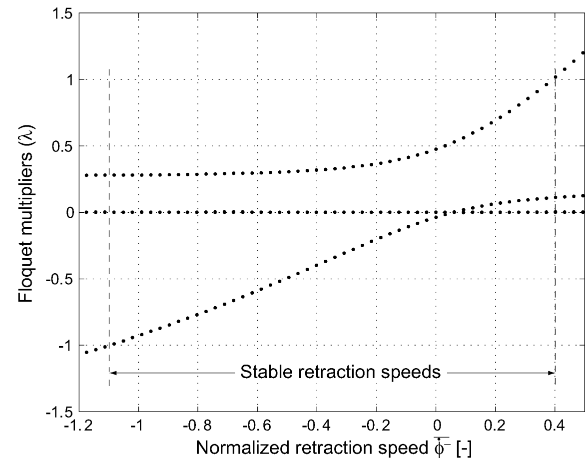

Fig. 11. Real part of the four Floquet multipliers of the realistic model for a range of swing-leg retraction speeds, the imaginary part of all Floquet mutlipliers is close to zero. The stable retraction speeds range from -1.1 to 0.4.

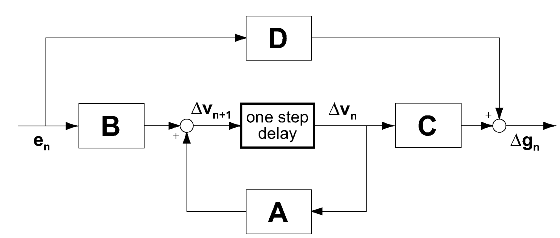

Fig. 12. Block diagram of the step-to-step discrete linear system description of a limit cycle walker.

IV. Disturbance Response

Previous research on various limit cycle walkers [17], [18] has shown that Floquet multipliers are not a good indicator for how well a walker is able to reject disturbances. The Floquet multipliers indicate how fast a walking gait converges after deviations from the nominal limit cycle (or fixed point) have occurred. However, they do not incorporate information on how heavily typical disturbances (e.g., floor irregularities, external pushes) induce such deviations. For this reason, we have expanded on the concept of the Floquet multipliers by looking at the step-to-step dynamic response of the walkers, adding the direct influence of physical disturbances, and assigning weights to the different eigenmodes. These weights are directly related to the possible failure modes of the walkers. This combination gives us a qualitative description of how the walker responds to disturbances as described in this section and a quantitative measure of disturbance rejection, the Gait Sensitivity Norm [17], which will be used in Section V.

A. Definition

When we assume small deviations from the nominal limit cycle motion as we did for the Floquet multipliers, we can describe the step-to-step behavior of a limit cycle walker as a discrete linear system, as depicted in Fig. 12.

In the core of this system description, one will find the mapping $\mathbf{A}$ of the initial conditions of a step $\mathbf{v}_n$ to the initial

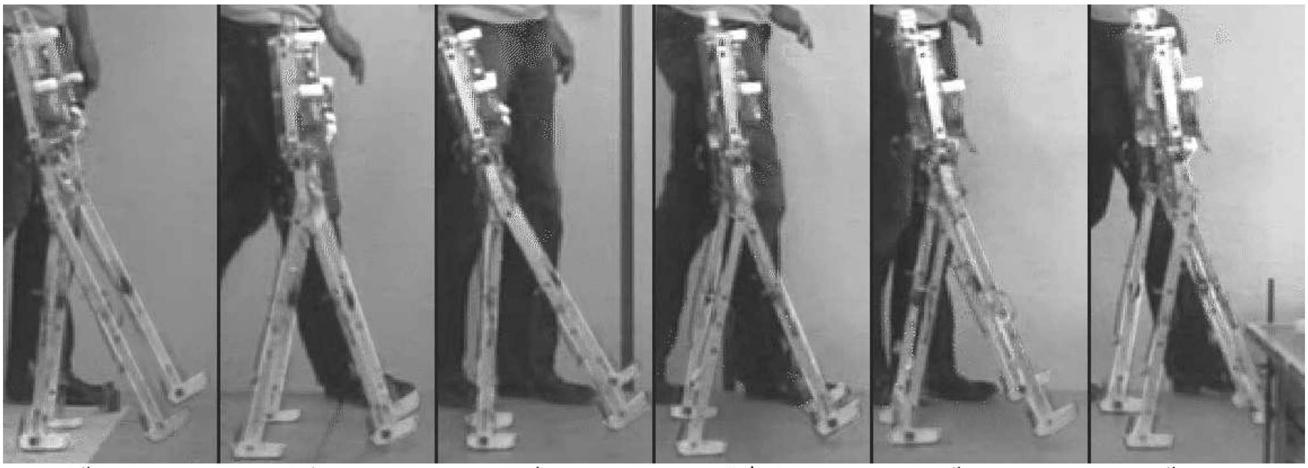

Fig. 13. Example of a trial performed with the physical prototype to get an estimate of its disturbance response. The prototype is disturbed by a step-down of 4 mm (left image). Due to this disturbance, in consecutive steps (five other images), the step times will deviate from the nominal step time of the prototype. These step time deviations are measured and give the prototype's impulse disturbance response as depicted in Fig. 15. The measurements are also used for the calculation of the walker's Gait Sensitivity Norm in Section V.

conditions of the subsequent step $\mathbf{v}_{n+1}$, which is identical to the definition of the Jacobian $\mathbf{A}$ used in the previous section. To incorporate the influence of actual physical disturbances on the walker, the system input $\mathbf{e}_n$ is defined to be (a set of) physical disturbances (e.g., floor irregularities, sensor noise). The system output $\mathbf{g}_n$ is (a set of) gait indicators, which are physical variables (e.g., step time, step length) that are directly related to the possible failure modes of the limit cycle walker. The use of these gait indicators is a way to assign meaningful weights to the various eigenmodes in the system, which is more insightful than observing the magnitudes of the eigenmodes themselves. The total step-to-step system description is given by two difference equations

$$\begin{aligned}\Delta \mathbf{v}_{n+1} &= \mathbf{A}\,\Delta \mathbf{v}_n + \mathbf{B}\,\Delta \mathbf{e}_n \\ \mathbf{g}_n &= \mathbf{C}\,\Delta \mathbf{v}_n + \mathbf{D}\,\Delta \mathbf{e}_n \end{aligned} \tag{5}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and $\mathbf{D}$ are all sensitivity matrices that can be found through simulation.

The choice of disturbances $\mathbf{e}$ and gait indicators $\mathbf{g}$ is vital to get a good description of the limit cycle walker's disturbance rejection. The set of disturbances should include all disturbances that the designer wants his biped to reject. The gait indicators should capture the presence of all vital failure mechanisms of the biped. It turns out that for 2-D walkers, as we are dealing with in this study, it is best to use floor height differences (h) as disturbance e and step time (T) as the gait indicator g. This choice gives a good prediction of the probability of the walker falling when it is faced with large random disturbances, i.e., its disturbance rejection [17]. The reason for this is that the most frequently occurring failure in these bipeds is a forward fall. Floor height differences induce this type of failure, and variations in step time indicate how well the biped is close to this failure mechanism. When a walker is close to a fall forward, it means that it hardly had sufficient time to swing its swing leg forward caused by a short step time.

An insightful qualitative description of a walker's disturbance rejection behavior is the system's impulse response, given by the variations in the walker's subsequent step times due to a single height difference in the floor. In case of the simulation models for which the sensitivity matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and $\mathbf{D}$ can be obtained, this disturbance response is defined as

$$\begin{aligned}\Delta T_k &= \mathbf{D}, \qquad \text{for} \quad k = 0 \\ \Delta T_k &= \mathbf{C}\mathbf{A}^{k-1}\mathbf{B}, \qquad \text{for} \quad k > 0\end{aligned} \tag{6}$$

where ΔT are the deviations of the walker's step time from the nominal step time and the subscript k indicates the amount of steps after the impulse disturbance has occurred.

In case of the prototype, we will use a direct input–output identification of the system's response by performing multiple measurement trials. In every trial, we let the prototype get in (or close to) its limit cycle after which we disturb the system by having a single $\Delta h = 4$ mm step-down in the floor (see Fig. 13). In the six consecutive steps, we record the deviations of prototype's step time T_k from its nominal limit cycle value T^*, which, in turn, is measured in the step just before the disturbance.

B. Results for the Simple Point Mass Model

Fig. 14 shows the disturbance response of the simple point mass model for three different normalized retraction speeds (−0.6, −0.3, and 0). These retraction speeds are picked to show the different system behaviors that exist within the stable range of retraction speeds. The effect that the single disturbance has on the first step after the disturbance is approximately equal and dominant for all three retraction speeds.

For a retraction speed of $\bar{\dot{\phi}}^- = -0.6$ (left), the step time deviations as a result of a step-down disturbance show an oscillating, poorly damped behavior. This response corresponds to the observation that, at this retraction speed, the dominant eigenmode of the model has a negative eigenvalue or Floquet multiplier

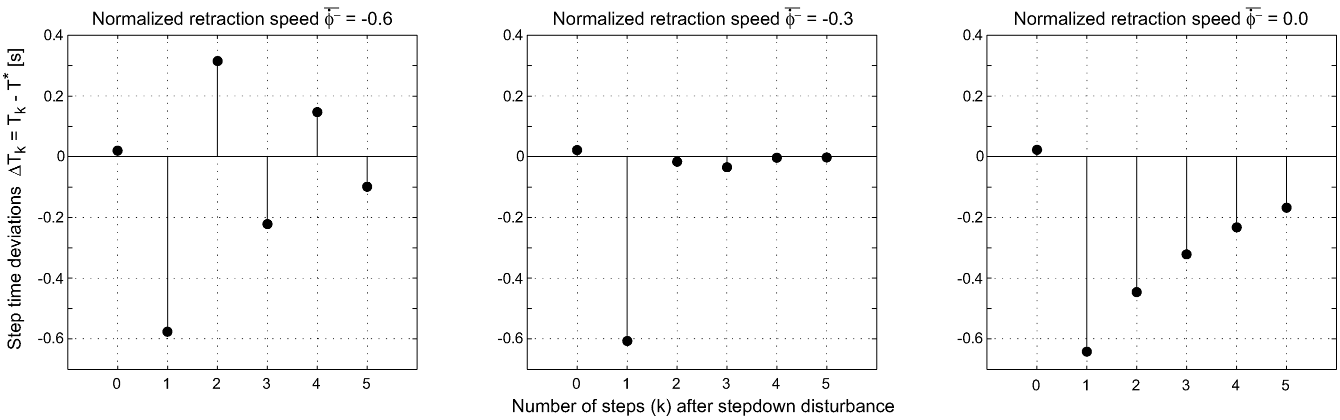

Fig. 14. Step time response to a 4 mm step-down disturbance of the simple point mass model for three different retraction speeds. On the left, the disturbance response for a highly negative retraction speed (-0.6) is shown, an oscillating, poorly damped response (totter mode). In the middle, the response for mild swing-leg retraction (-0.3) is shown, a response that resembles a critically damped system. The right graph shows the response at zero retraction speed, where the step time deviations are monotonically decreasing, resembling an overdamped system response(speed mode).

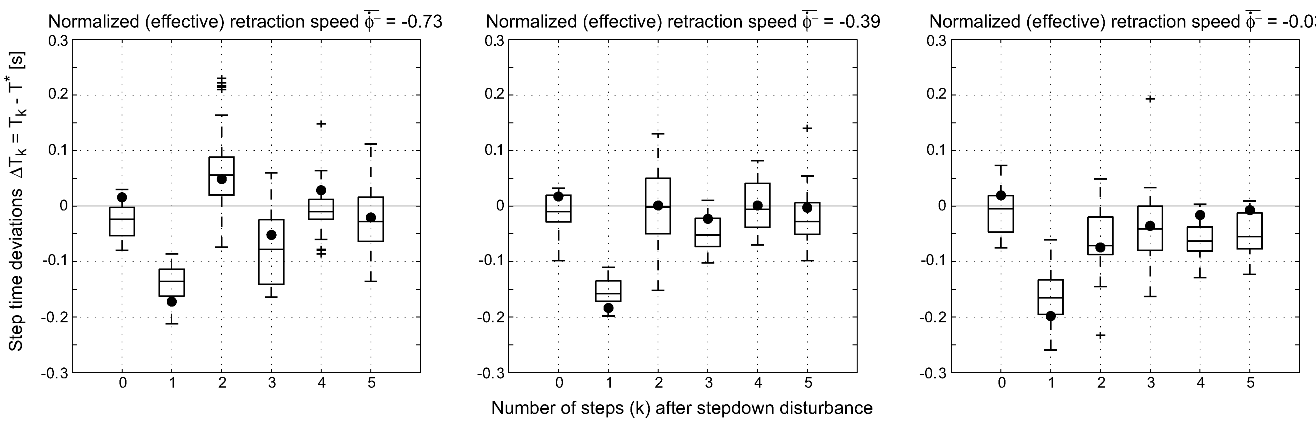

Fig. 15. Step time response to a 4 mm step-down disturbance of the realistic model (solid dots) and the physical prototype (boxplots). The results for the physical prototypes are taken from a measurement set of 50 trials per retraction speed. Trials for which the step time of the step preceding the disturbance deviated more than 5% from the nominal step time were rejected as the prototype was not close enough to its nominal limit cycle. Similar to the simple point mass model, a highly negative retraction speeds (left) results in an oscillating, poorly damped response and zero retraction speed (right) results in an overdamped response. For a mild retraction speed, the response is close to critically damped (middle).

(Section III and Fig. 7). At $\bar{\dot{\phi}}^- = 0$ (right), the step time deviations show a monotonic decrease over subsequent steps, analogous to an overdamped system. This response corresponds to the observed positive dominant eigenvalue at these speeds. For the intermediate case ($\bar{\dot{\phi}}^- = -0.3$) (middle), the response comes close to a critically damped system, which makes the step time deviations following the first step after the disturbance minimal.

C. *Discussion of the Simple Point Mass Model*

The disturbance response of the simple point mass model shows a convergence that is well explained by the "speed" and "totter" mode discussed in Section III-B. Besides that, the largest deviation in step time exists in the first step after the step-down disturbance and this deviation does not seem to be influenced much by swing-leg retraction. This fact can be explained by looking at the change in energy loss at impact of the step in which the disturbance takes place, similar to the discussion in Section III-C.

The energy loss at impact in the disturbed step changes little relative to the amount of energy that is added by the disturbance. Not more than 15% of the amount of potential energy that is added by a disturbance is compensated by an increased energy loss at impact. This means that the step after the disturbance starts with an energy content that has increased with at least 85% of the disturbance energy, having a large effect on this step's duration (see step 1 in Fig. 14). The specific amount of swing-leg retraction hardly changes this effect: over the whole range of stable retraction speeds, the amount of energy that remains in the first step after the disturbance is between 85% and 91% of the energy added by the disturbance. Altogether, swing-leg retraction does little to cancel the direct influence (i.e., within one step) of a floor height disturbance.

D. *Results and Discussion of the Realistic Model and Prototype*

Fig. 15 shows the disturbance response of the realistic model and the physical prototype. The disturbance response of the realistic model (solid dots) is deterministic, while the depicted response of the prototype is a statistical boxplot representation of the 50 trials that were performed to establish each of the three responses. The results show similar effects as with the simple point mass model. Fast swing-leg retraction results in an oscillatory, poorly damped response, mild retraction results in a response that resembles a critically damped situation, and no retraction results in an overdamped, monotonically converging response. The first step after the disturbance is hardly affected by swing-leg retraction.

V. Gait Sensitivity Norm

From the step-to-step dynamic system response described in the previous section, it is even possible to derive a quantitative measure for the walker's disturbance rejection, the Gait Sensitivity Norm [17]. This measure gives a good prediction of the disturbance rejection ability of a limit cycle walker, as will be shown in Section V-E.

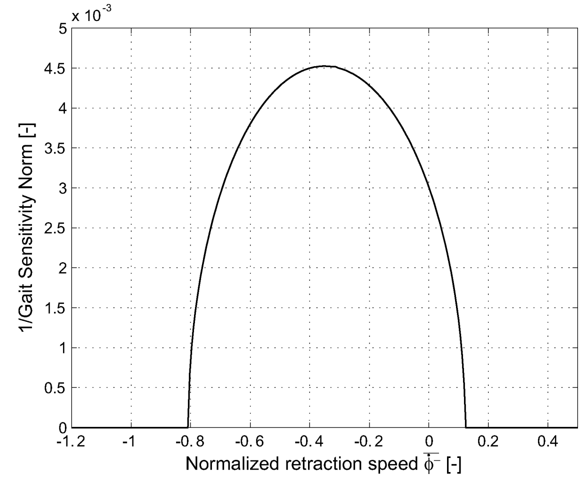

Fig. 16. Gait Sensitivity Norm of the simple point mass model. According to the Gait Sensitivity Norm, the model has the best disturbance rejection ability at a normalized retraction speed of -0.35.

A. Definition

The Gait Sensitivity Norm quantifies the disturbance response by taking the H_2-norm of the system given in Section IV and Fig. 12. The H_2-norm of a system gives the variance of the system output (gait indicators in our case) when the system input (physical disturbances) is unit white noise. Next to this, the H_2-norm is also equal to the root mean square of the system's impulse response.

In case of simulation models for which the sensitivity matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are known, the Gait Sensitivity Norm is defined as

$$\left\|\frac{\partial \mathbf{g}}{\partial \mathbf{e}}\right\|_2 = \sqrt{\mathrm{trace}(\mathbf{D}^T\mathbf{D}) + \sum_{k=0}^{\infty} \mathrm{trace}(\mathbf{B}^T(\mathbf{A}^T)^k\mathbf{C}^T\mathbf{C}\mathbf{A}^k\mathbf{B})}. \tag{7}$$

The Gait Sensitivity Norm is normalized by weighting the input disturbances and the output gait indicators. The unit input disturbance is a floor height difference of 1 m. The gait indicator step time is weighted by dividing the step time deviations by the value of a step time deviation for which failure (a fall) is expected. This way, the absolute value of the reciprocal of the Gait Sensitivity Norm is a prediction of the size of the input disturbance for which a fall is likely to occur (see Section V-E). In this case, the step time deviations are divided by the nominal duration of the swing-leg retraction (20% of the nominal step time) as a forward fall is not likely to occur when the swing leg is retracting. A fall forward occurs when the swing leg is not sufficiently brought forward, in time to catch the biped and start a new step [19]. When the swing leg is retracting, it has already been sufficiently brought forward, and thus, a fall forward will not occur. A fall forward will only occur before the swing leg reaches its most forward position and starts retracting.

An estimate of the Gait Sensitivity Norm can also be obtained through measurements on a physical prototype. This estimate is taken from the same measurement trials that were used to establish the prototype's disturbance response. As the system's H_2-norm is equal to the root mean square of the system's impulse response, the estimate of the Gait Sensitivity is calculated by

$$\left\|\frac{\partial \mathbf{g}}{\partial \mathbf{e}}\right\|_2 \approx \frac{1}{\Delta h}\sqrt{\sum_{k=0}^{6}\left(\frac{T_k - T^*}{0.2T^*}\right)^2}. \tag{8}$$

B. Results for the Simple Point Mass Model

For the simple point mass model, the reciprocal of the Gait Sensitivity Norm is shown in Fig. 16. It shows a nonzero value in the range of retraction speeds that result in stable gaits. The closer the reciprocal of the Gait Sensitivity Norm is to zero, the worse is the predicted disturbance rejection. For fast retraction speeds (e.g., $\bar{\dot{\phi}}^- = -0.8$), the oscillating, poorly damped response described in the previous section translates into bad disturbance rejection. On the other side, for zero or positive retraction speeds (e.g., $\bar{\dot{\phi}}^- = 0.1$), the predicted disturbance rejection is small too due to the slow monotonic convergence in the disturbance response.

The Gait Sensitivity Norm predicts an optimal disturbance rejection for mild retraction at a normalized retraction speed of -0.35. The critically damped system response that has been observed for mild retraction speeds results in the best disturbance rejection ability of the simple point mass model.

C. Parameter Sensitivity Analysis Simple Point Mass Model

Using the Gait Sensitivity Norm, we can establish how the effect of swing-leg retraction varies with changing walking parameters, such as step length and step time. The results are shown in Fig. 17. Although the exact value of the optimal retraction speed and the optimal disturbance rejection do change with step length and step time, the optimal retraction speed is always negative and in the middle of the range of speeds for which stable gaits are found. The optimal disturbance rejection that can be achieved increases with increasing step length and

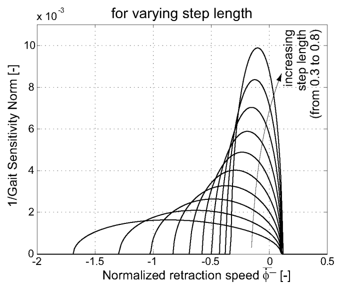

Fig. 17. Disturbance rejection of the simple point mass model as a function of swing-leg retraction speed for models with different step lengths and step times. The optimal retraction speed does change with these model parameters, but the optimal retraction speed is always negative. Generally, larger step lengths and shorter step times (both resulting in faster walking) increase the optimal disturbance rejection of the model.

decreasing step time, indicating that walking faster is beneficial for disturbance rejection.

D. Results for the Realistic Model and Prototype

For the realistic model, the reciprocal of the Gait Sensitivity Norm is shown in Fig. 18 by the solid line. The optimal disturbance rejection for this model is found at a normalized retraction speed of -0.50. Fig. 18 also shows the reciprocal of the Gait Sensitivity Norm as measured on the physical prototype by the boxplots. The measurements show a similar result as the realistic simulation model with an optimal normalized retraction speed of -0.54. For both the realistic model and the prototype, we find low predicted disturbance rejection at positive retraction speeds (swing leg going forward) and highly negative retraction speeds (swing leg going backward fast).

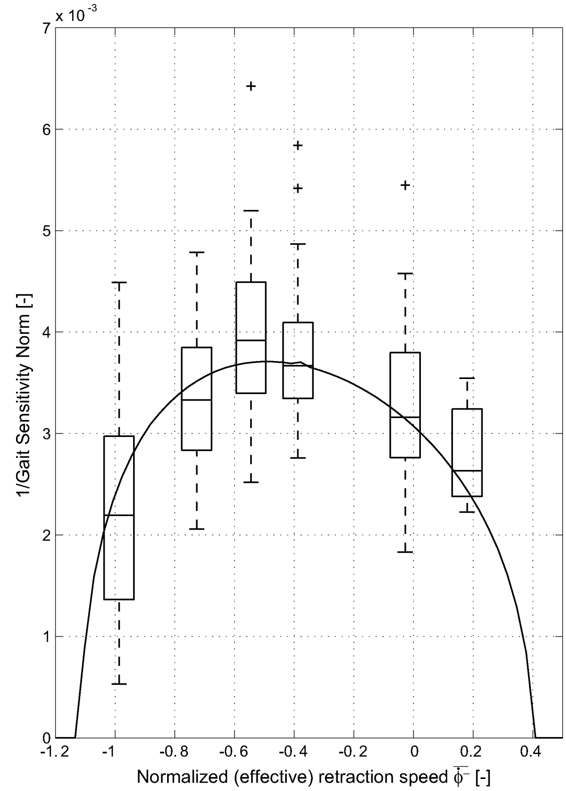

Fig. 18. Gait Sensitivity Norm for the realistic model and the physical prototype. The predicted optimal normalized retraction speed is around -0.50. The results for the physical prototypes are taken from a measurement set of 50 trials per retraction speed. Trials for which the step time of the step preceding the disturbance deviated more than 5% from the nominal step time were rejected as the prototype was not close enough to its nominal limit cycle. The boxplots are horizontally placed on the normalized "effective" retraction speed that is measured as explained in Section II-C and Fig. 6.

E. Correlation to Actual Disturbance Rejection

The Gait Sensitivity Norm provides a good prediction of a biped's ability to reject disturbances [17]. The hypothesis underlying this is that the variability or variance of the gait indicator is related to the chance the biped will fall. The way to validate the Gait Sensitivity Norm is to check how well it predicts the magnitude of a random disturbance that makes a walker fall, which we refer to as the actual disturbance rejection. Obtaining this actual disturbance rejection is very time-consuming as it involves the full nonlinear dynamics of the walker and stochastic input disturbances. For this reason, we only validate the Gait Sensitivity Norm on the fast computable simple point mass model and for three different retraction speeds only on the realistic model.

To establish the actual disturbance rejection, we perform 80 steps with the two models in which the floor has a Gaussian floor height distribution. We quantify the size of this disturbance as two times the standard deviation of the distribution, meaning that the floor height is within this $\pm 2\sigma$ value for 95% of the time. We perform a search for the size of this disturbance for which the model manages to successfully finish 97.5% of the 80 steps without falling. The 97.5% confidence interval is chosen instead of the 95% interval as the failure of falling forward is one sided, meaning that it only occurs when the step time is too short and not when it is too long.

Fig. 19 shows the actual disturbance rejection ability of the simple point mass model and the realistic model as well as the reciprocal of their Gait Sensitivity Norm. For the simple point mass model, the Gait Sensitivity Norm gives a good prediction of the relative effect of swing-leg retraction as the shapes of the two graphs are similar. Quantitatively, this is shown by the correlation between the reciprocal of the Gait Sensitivity Norm and the actual disturbance rejection that is $R^2 = 0.92$, meaning that 92% of the variation in the actual disturbance rejection is captured by the Gait Sensitivity Norm. The absolute value of the actual disturbance rejection is higher than the Gait Sensitivity Norm predicts. We think that this is due to the fact that the model can actually withstand step time variations larger than the conservatively assumed 20% of the nominal step time for the weighting of the Gait Sensitivity Norm.

In case of the realistic model (Fig. 19, right), the actual disturbance rejection seems to agree with the prediction by the Gait Sensitivity Norm both on the relative and the absolute effect of swing-leg retraction. The highest actual disturbance rejection we have found is 4.1 mm at the retraction speed that the Gait Sensitivity Norm predicts as optimal.

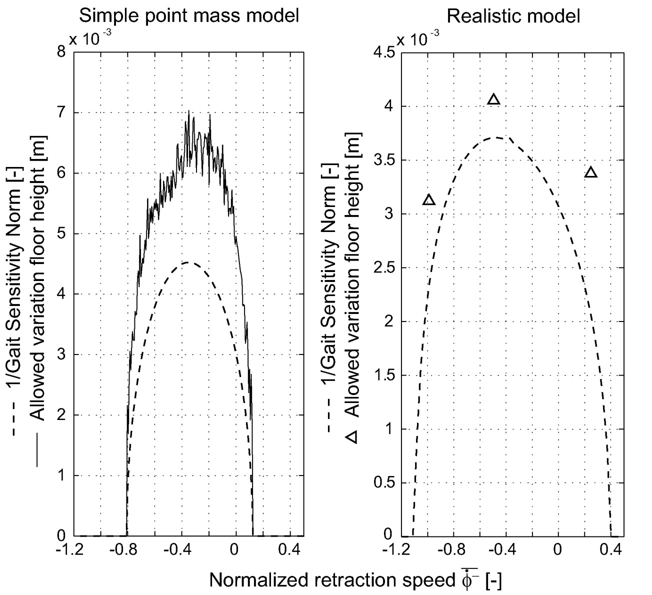

Fig. 19. Gait Sensitivity Norm and the actual disturbance rejection of the simple point mass model and the realistic model. The Gait Sensitivity Norm does a good job in estimating the actual disturbance rejection as it explains 93% of the variation in the actual disturbance rejection, according to the correlation between the two quantities.

VI. Discussion

In this paper, we research the effect that swing-leg retraction has on a biped's disturbance rejection. We find that there is an optimal swing-leg retraction speed for all three walkers involved in this study. Mildly, retracting the swing leg just prior to foot strike gives a system behavior that resembles a critically damped response, which means the effect of disturbances is rejected in the least amount of steps.

Quantitatively, there are some differences in the results from the different models, but qualitatively, all models agree. The behavior of the simple point mass walking model and the more realistic model is characterized by two similar dominant eigenmodes. The first eigenmode is dominant at zero or positive retraction speed and has a positive eigenvalue (Floquet multiplier) that results in a slow, monotonically converging system response. The second eigenmode is dominant at highly negative retraction speeds and has a negative eigenvalue that results in an oscillating, poorly damped system response. These typical system responses were also found through measurements on the physical prototype.

The walkers' disturbance rejection abilities are quantified by the Gait Sensitivity Norm. As expected from the qualitative disturbance response descriptions, the Gait Sensitivity Norm predicts an optimal retraction speed. Using the simple point mass model, the prediction made by the Gait Sensitivity Norm is validated by comparing the prediction to the "actual" disturbance rejection, defined as the maximal size of random floor height variations the model can handle without falling.

The Gait Sensitivity Norm predicts that our prototype Meta is able to walk over a normally distributed rough surface in which the floor height is within ± 3.7 mm for 95% of the time. Besides the measurements reported so far, we experimented with the prototype on a floor with random height variations in the order of 4 mm and it did manage to perform well under these conditions. Also, the prototype was able to handle a single step-down in the floor of 8 mm.

In this study, swing-leg retraction has been implemented through control to be able to distinguish its effect in isolation, keeping other factors as nominal speed and nominal step length equal. Swing-leg retraction can also be implemented by mechanical design, as a passive property of the system. Making the natural frequency of the swing leg slightly faster than the nominal walking frequency inherently results in swing-leg retraction and, given the results of this study, is beneficial for the performance of a limit cycle walker. We recommend taking this design aspect into account when building future limit cycle walkers.

An important shortcoming of swing-leg retraction is that it has a limited influence on the walking step directly following a disturbance. This creates a delay in the disturbance rejection limiting its capacity. We postulate that ankle push-off can drastically improve this part of the response to disturbances. To achieve this, ankle push-off will have to contribute a varying amount of energy to the motion per step instead of the constant amount used in this study. The amount of push-off energy will primarily have to be based on the initial conditions of that same step. This ankle push-off strategy will be topic of further research in the near future.

VII. Conclusion

In this paper, we study the effect of swing retraction on the ability of a biped to handle disturbances, i.e., its disturbance rejection. The effect of swing-leg retraction has been studied in three different cases, a simple point mass simulation model, a realistic simulation model, and a 2-D physical prototype. We conclude that:

1) mild swing-leg retraction is optimal for the disturbance rejection of a biped as it results in a critically damped disturbance response;
2) zero swing-leg retraction results in a slow, monotonically converging disturbance response corresponding to a positive dominant Floquet multiplier;
3) fast swing-leg retraction results in an oscillating, poorly damped disturbance response corresponding to a negative dominant Floquet multiplier;
4) the two walking models and the physical walking prototype give qualitatively similar results confirming the believe that simple point mass models can be a good representation of the dynamics of walking;
5) the optimal swing-leg retraction speed can be found by the Gait Sensitivity Norm, a disturbance rejection measure that gives a good prediction of a walker's ability to handle disturbances.

Acknowledgment

The authors thanks F. van der Helm for proofreading and J. van Frankenhuyzen for engineering the prototype.

REFERENCES

[1] R. Q. Linde, "Passive bipedal walking with phasic muscle contraction," *Biol. Cybern.*, vol. 81, pp. 227–237, 1999.

[2] M. Wisse and J. van Frankenhuyzen, "Design and construction of Mike: A 2D autonomous biped based on passive dynamic walking," presented at the 2nd Int. Symp. Adapt. Motion Animals Mach., Kyoto, Japan, 2003.

[3] D. G. E. Hobbelen and M. Wisse, "Limit cycle walking," in *Humanoid Robots, Human-Like Machines*, M. Hackel, Ed. Vienna, Austria: I-Tech Education and Publishing, 2007, ch. 14.

[4] S. H. Collins, A. Ruina, R. Tedrake, and M. Wisse, "Efficient bipedal robots based on passive-dynamic walkers," *Science*, vol. 307, no. 5712, pp. 1082–1085, 2005.

[5] M. Wisse, C. G. Atkeson, and D. K. Kloimwieder, "Swing leg retraction helps biped walking stability," in *Proc. IEEE Int. Conf. Humanoid Robots*, Dec. 2005, pp. 295–300.

[6] A. Seyfarth, H. Geyer, and H. Herr, "Swing-leg retraction: A simple control model for stable running," *J. Exp. Biol.*, vol. 206, pp. 2547–2555, 2003.

[7] T. McGeer, "Passive dynamic walking," *Int. J. Robot. Res.*, vol. 9, no. 2, pp. 62–82, 1990.

[8] M. S. Garcia, A. Chatterjee, A. Ruina, and M. J. Coleman, "The simplest walking model: Stability, complexity, and scaling," *ASME J. Biomech. Eng.*, vol. 120, no. 2, pp. 281–288, 1998.

[9] S. Schaal and C. G. Atkeson, "Open loop stable control strategies for robot juggling," in *Proc. IEEE Int. Conf. Robot. Autom.*, Atlanta, GA, May1993, vol. 3, pp. 913–918.

[10] S. Kajita and K. Tani, "Study of dynamic biped locomotion on rugged terrain—Derivation and application of the linear inverted pendulum mode," in *Proc. Int. Conf. Robot. Autom.*, Sacramento, CA, Apr. 1991, vol. 2, pp. 1405–1411.

[11] A. Goswami, B. Thuilot, and B. Espiau, "A study of the passive gait of a compass-like biped robot: Symmetry and chaos," *Int. J. Robot. Res.*, vol. 17, no. 12, pp. 1282–1301, 1998.

[12] A. D. Kuo, "Energetics of actively powered locomotion using the simplest walking model," *ASME J. Biomech. Eng.*, vol. 124, no. 2, pp. 113–120, 2002.

[13] A. Ruina, J. E. A. Bertram, and M. Srinivasan, "A collisional model of the energetic cost of support work qualitatively explains leg sequencing in walking and galloping, pseudo-elastic leg behavior in running and the walk-to-run transition," *J. Theor. Biol.*, vol. 237, no. 2, pp. 170–192, 2005.

[14] M. Wisse, D. G. E. Hobbelen, and A. L. Schwab, "Adding the upper body to passive dynamic walking robots by means of a bisecting hip mechanism," *IEEE Trans. Robot.*, vol. 23, no. 1, pp. 112–123, Feb. 2007.

[15] V. Duindam, "Port-based modeling and control for efficient bipedal walking robots" Ph.D. dissertation, Univ. Twente, Enschede, The Netherlands, 2006.

[16] G. A. Pratt and M. M. Williamson, "Series elastic actuators," in *Proc. IEEE Int. Conf. Intell. Robots Syst.*, Pittsburgh, PA, Aug. 1995, vol. 1, pp. 399–406.

[17] D. G. E. Hobbelen and M. Wisse, "A disturbance rejection measure for limit cycle walkers: The Gait Sensitivity Norm," *IEEE Trans. Robot.*, vol. 23, no. 6, pp. 1213–1224, Dec. 2007.

[18] A. L. Schwab and M. Wisse, "Basin of attraction of the simplest walking model," presented at the ASME Des. Eng. Tech. Conf., Pittsburgh, PA, 2001.

[19] M. Wisse, A. L. Schwab, R. Q. van der Linde, and F. C. T. van der Helm, "How to keep from falling forward: Elementary swing leg action for passive dynamic walkers," *IEEE Trans. Robot.*, vol. 21, no. 3, pp. 393–401, Jun. 2005.

Daan G. E. Hobbelen received the M.Sc. degree in mechanical engineering in 2003 from Delft University of Technology, Delft, The Netherlands, where he is currently working toward the Ph.D. degree.

His current research interests include mechatronics, bioinspired robotics, legged locomotion, (multibody) dynamics, and (compliant) control.

Martijn Wisse (M'02) received the M.S. and Ph.D. degrees in mechanical engineering from Delft University of Technology, Delft, The Netherlands, in 2000 and 2004, respectively.

He is currently an Assistant Professor at Delft University of Technology, where he is engaged in research in the field of humanoid robotics.