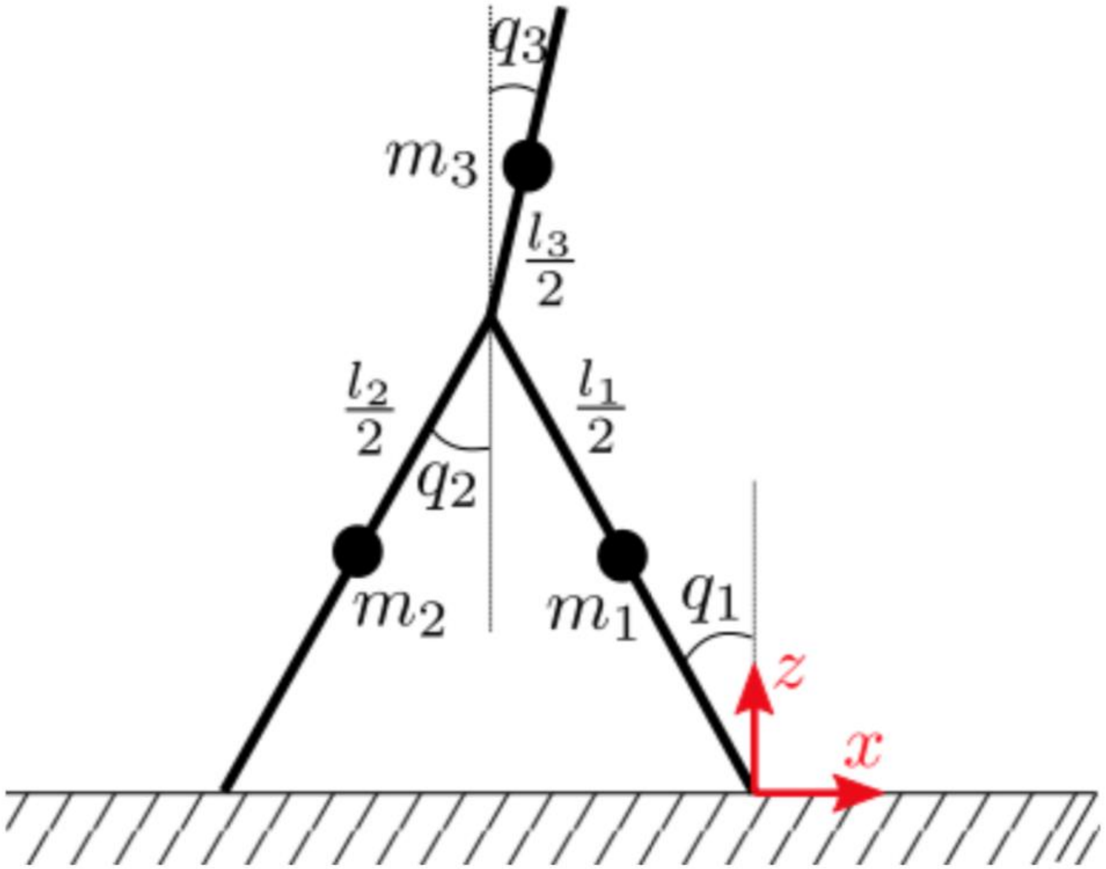
# Modeling of a three-link 2D biped

Legged Robots

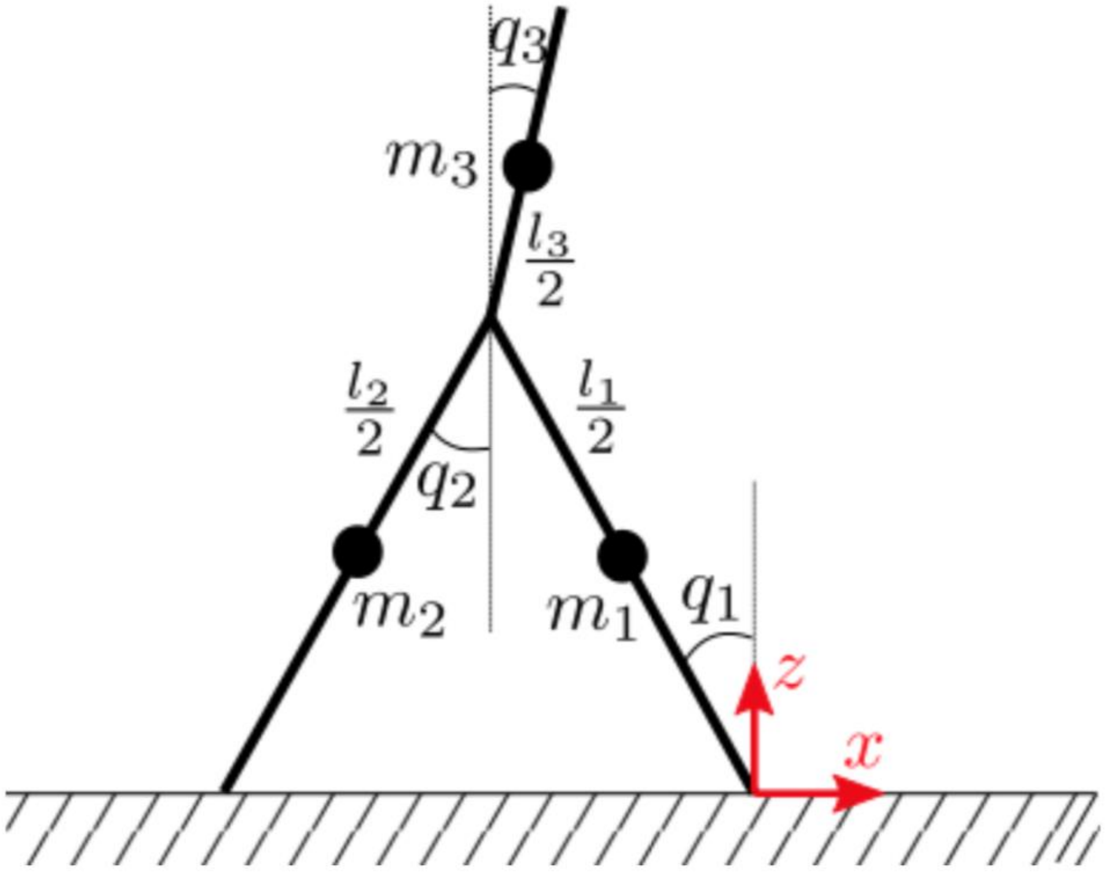
# Overview

- Model and visualize the three-link biped
- Solve the equation of motion of the three-link biped
- Design walking controllers, evaluate the resulting gaits and compare them



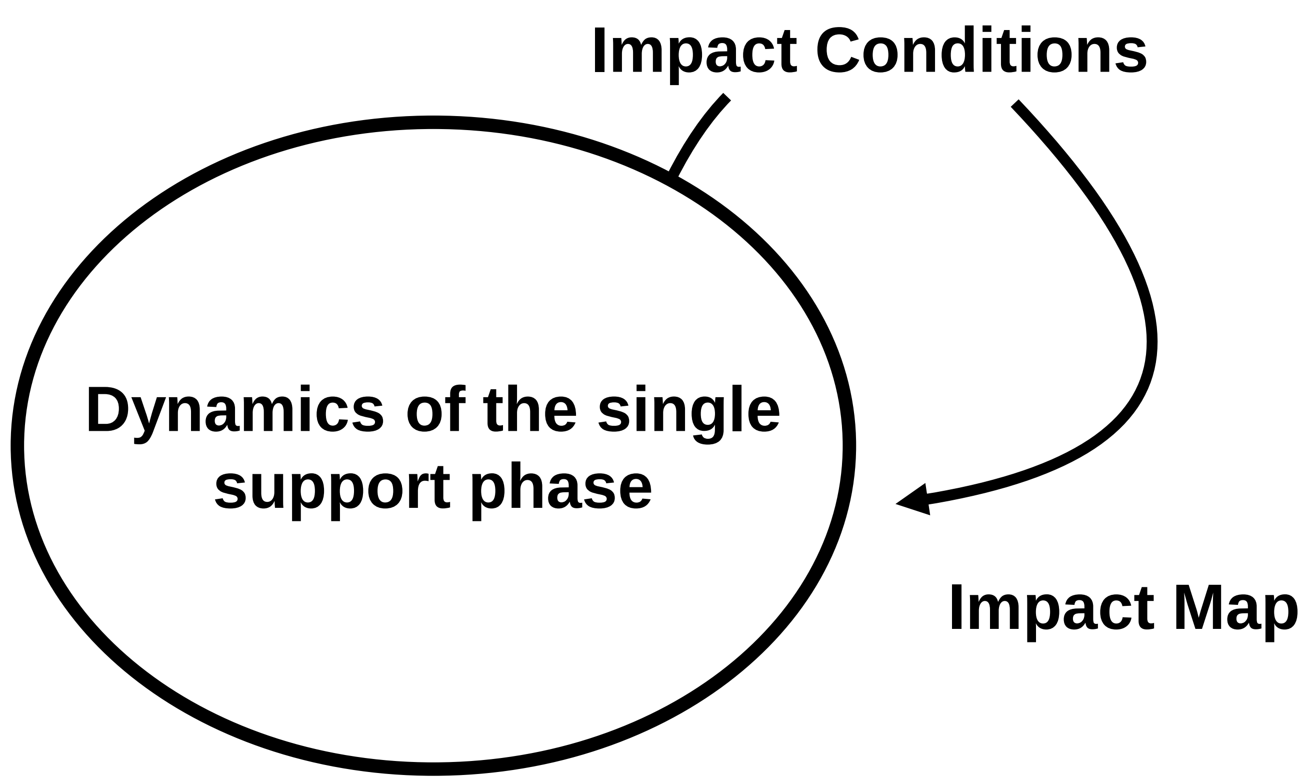
# Model and visualize the three-link biped

- Kinematics
- Dynamics
- Impact



# Hybrid model of walking

- Swing phase model
- Impact model




*Feedback Control of Dynamic Bipedal Robot Locomotion, [Eric R. Westervelt, Jessy W. Grizzle, et al.]*

# Swing phase model

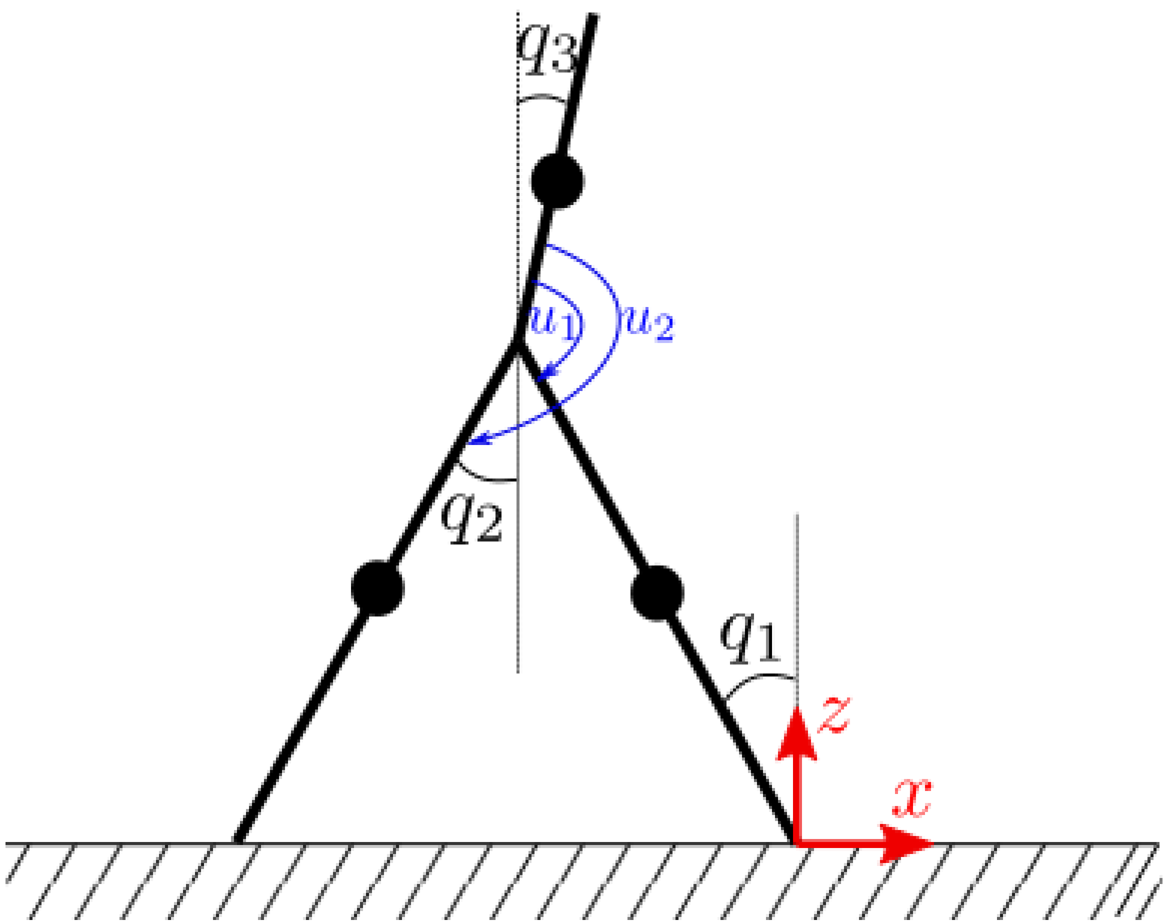
- Pinned open kinematic chain

- Lagrangian method

$$L(q, \dot{q}) = T(q, \dot{q}) - V(q)$$

$$M\ddot{q} + C\dot{q} + G(q) = Bu$$

- Model is under actuated (why?)




**Clockwise rotation is considered positive!**

# Selection matrix

$$\boldsymbol{\theta_1 = \pi - (-q_1) - q_3}$$

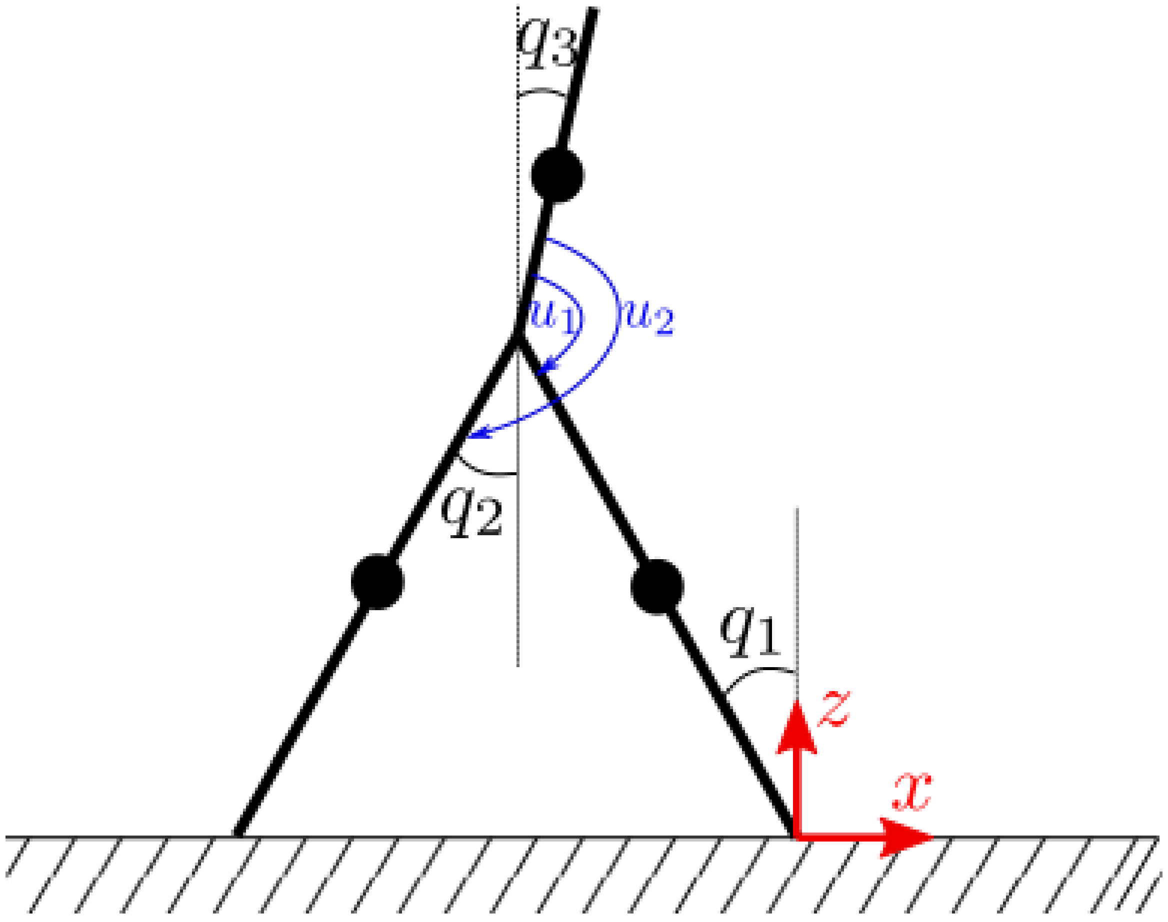
$$\boldsymbol{\theta_2 = \pi + q_2 - q_3}$$

$$\boldsymbol{\dot{\theta} = B^T \dot{q}}$$

$$\boldsymbol{\tau^T \dot{q} = u^T \dot{\theta}}$$

$$\boldsymbol{\tau^T \dot{q} = u^T B^T \dot{q}}$$

$$\boldsymbol{\tau = B u}$$

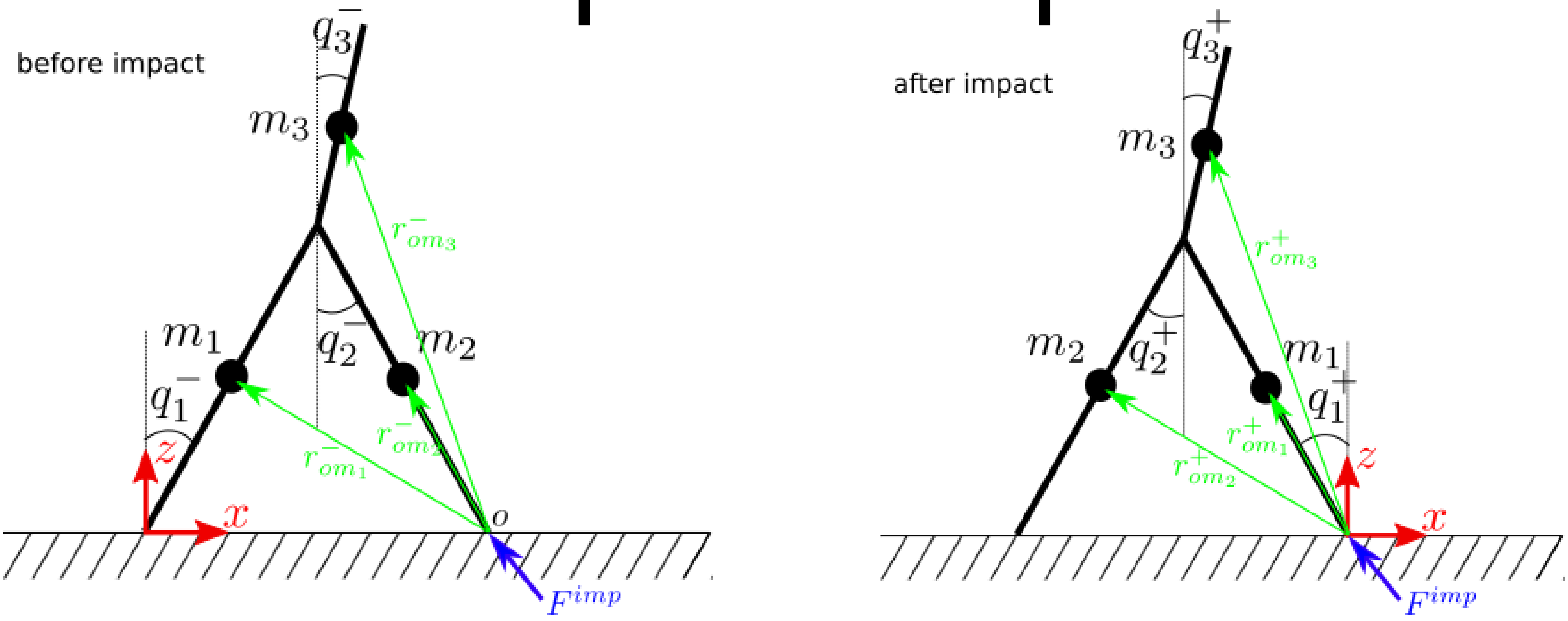



**Clockwise rotation is considered positive!**

# Impact map

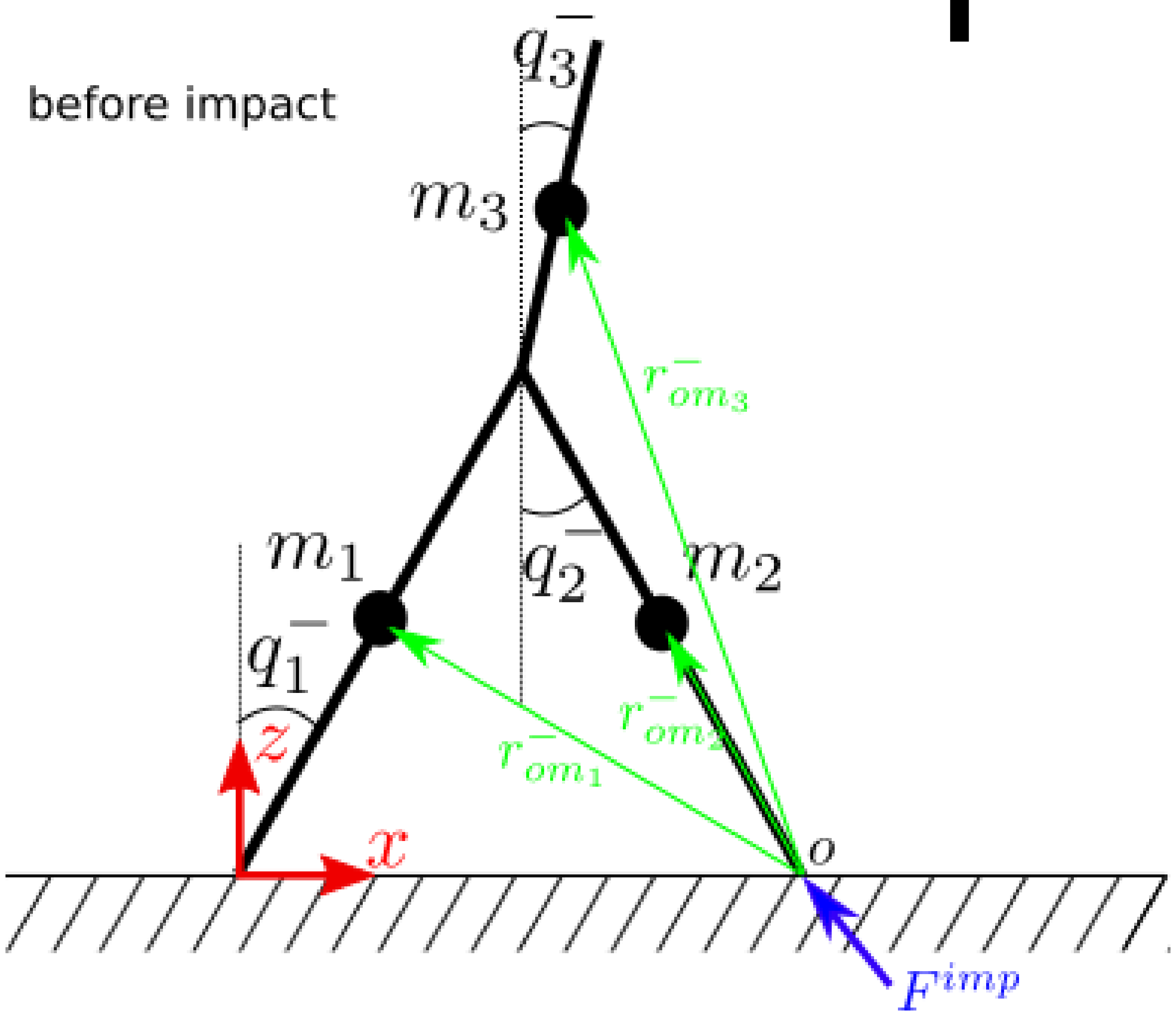
- The impact map, maps the state of the robot right before the impact to its state right after
- Assumption: the impact and switching of the leg roles are instantaneous and the stance leg does not slip

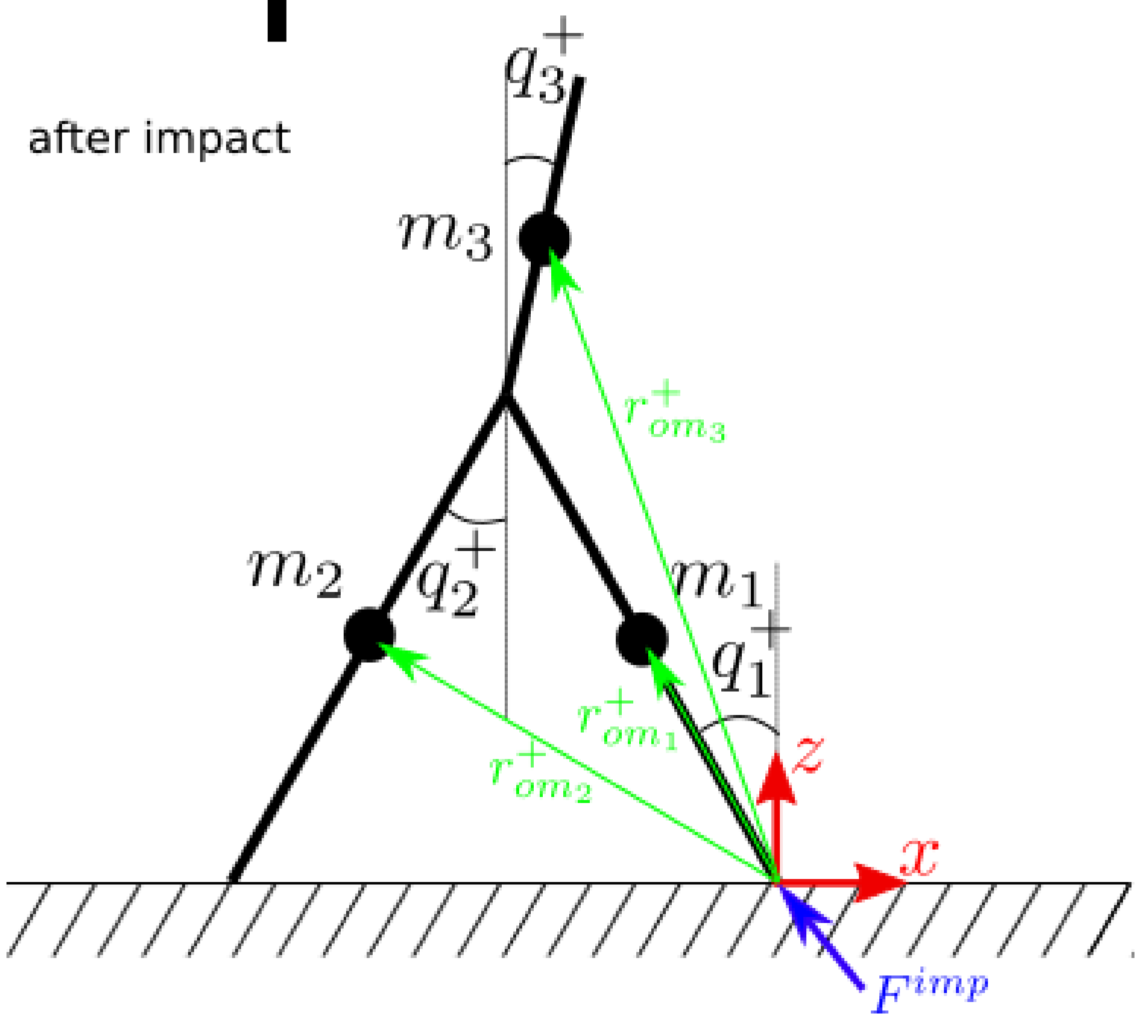
# Impact map



before impact
$q_3^-$
$m_3$
$r_{om_3}^-$
$m_1$
$q_2^-$
$m_2$
$q_1^-$
$r_{om_2}^-$
$r_{om_1}^-$
$z$
$x$
$o$
$F^{imp}$
after impact
$q_3^+$
$m_3$
$r_{om_3}^+$
$m_2$
$q_2^+$
$m_1$
$q_1^+$
$r_{om_1}^+$
$r_{om_2}^+$
$z$
$x$
$F^{imp}$

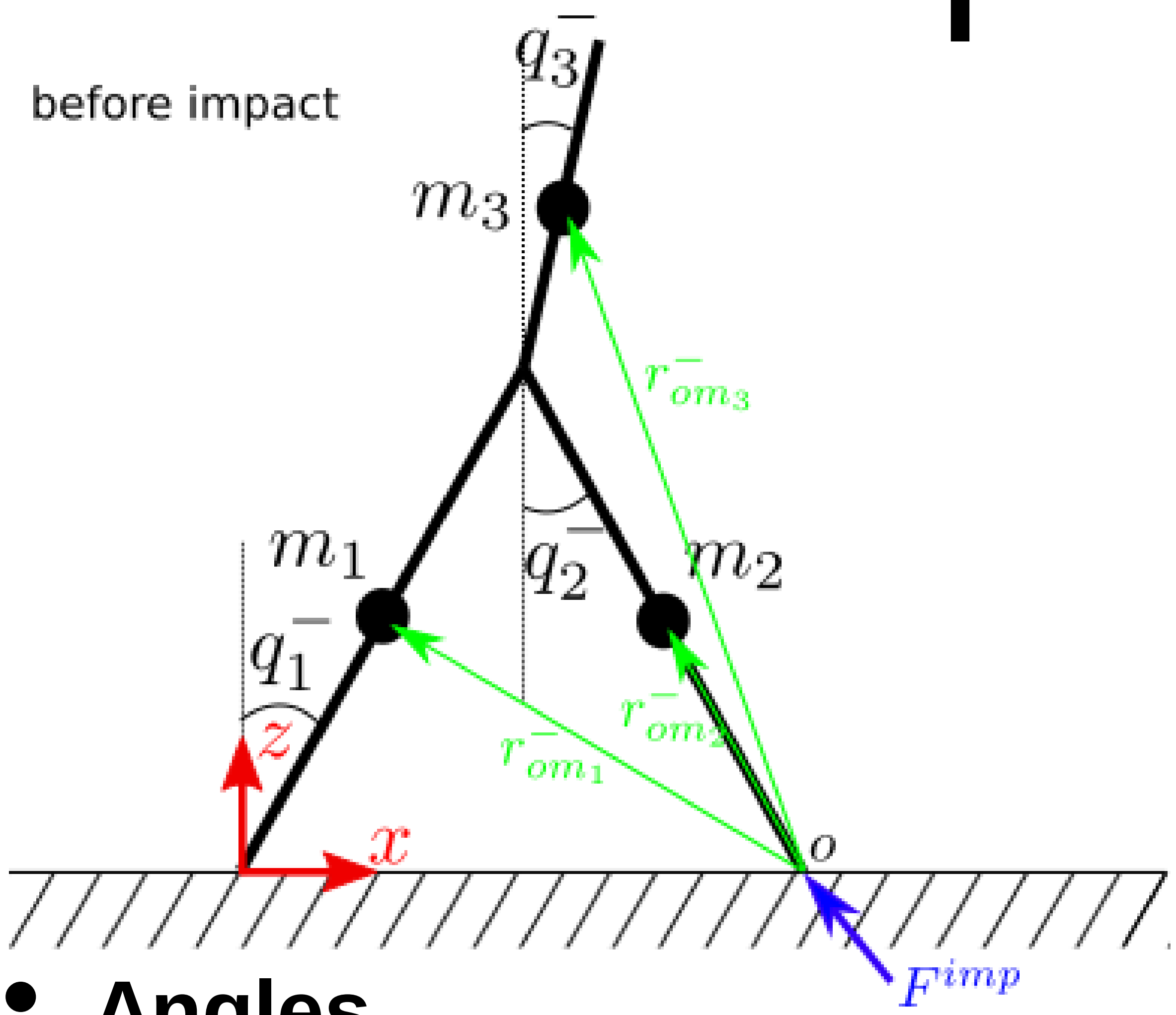
# Impact map

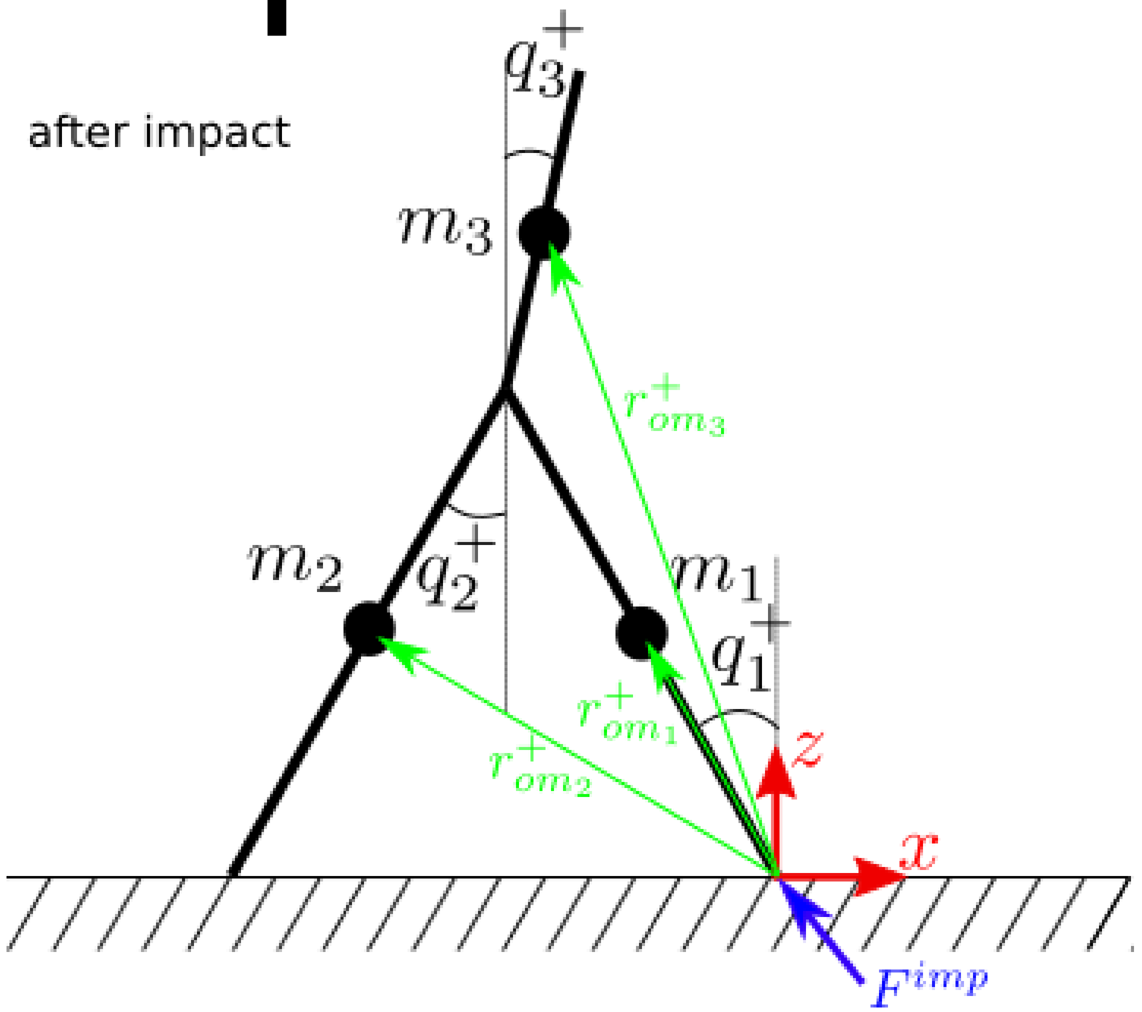



- **Differences**?
  - reference frame
  - masses (stance/swing)

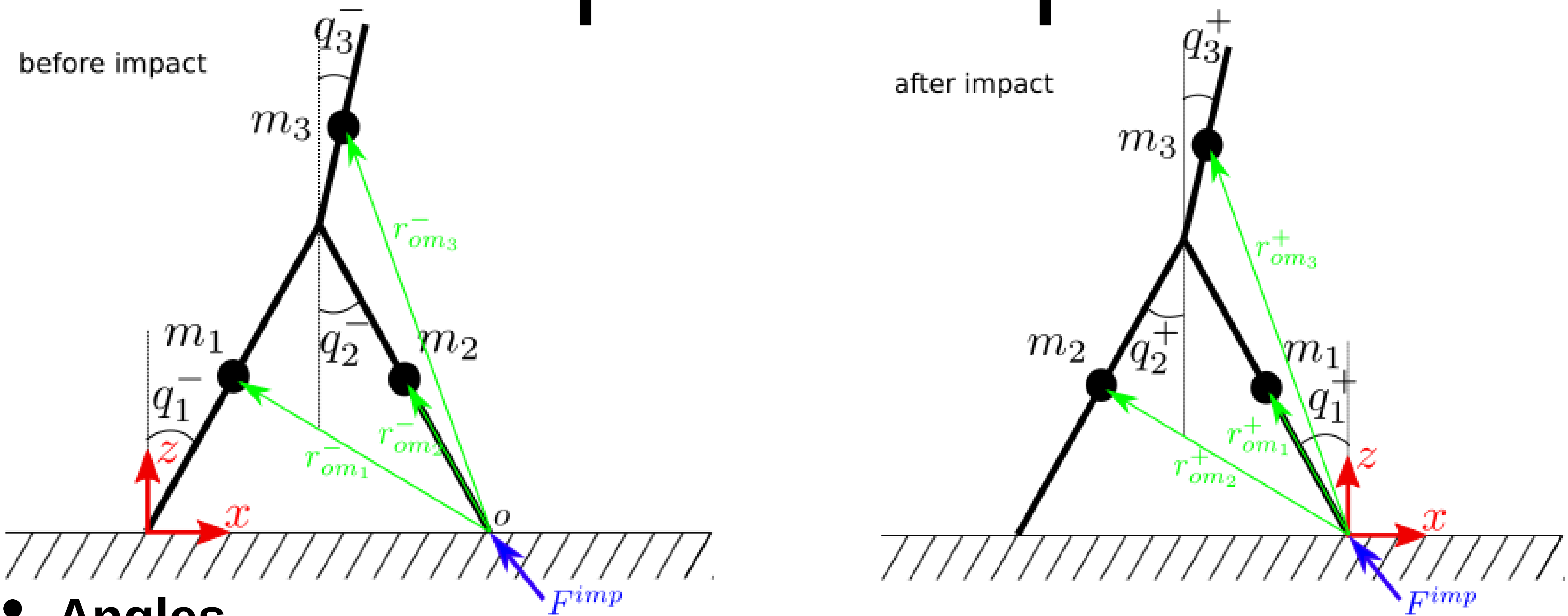
# Impact map




- **Angles**

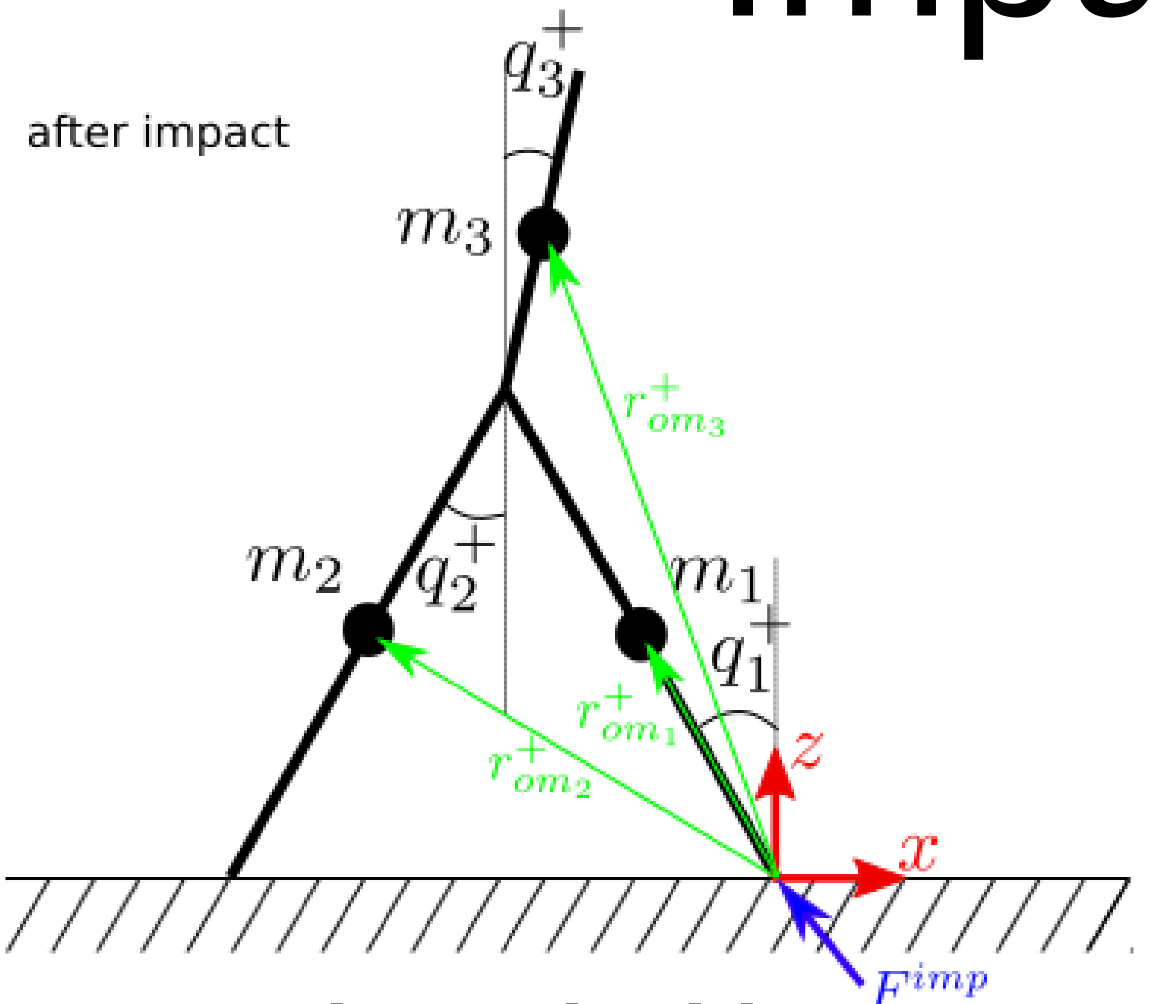
# Impact map

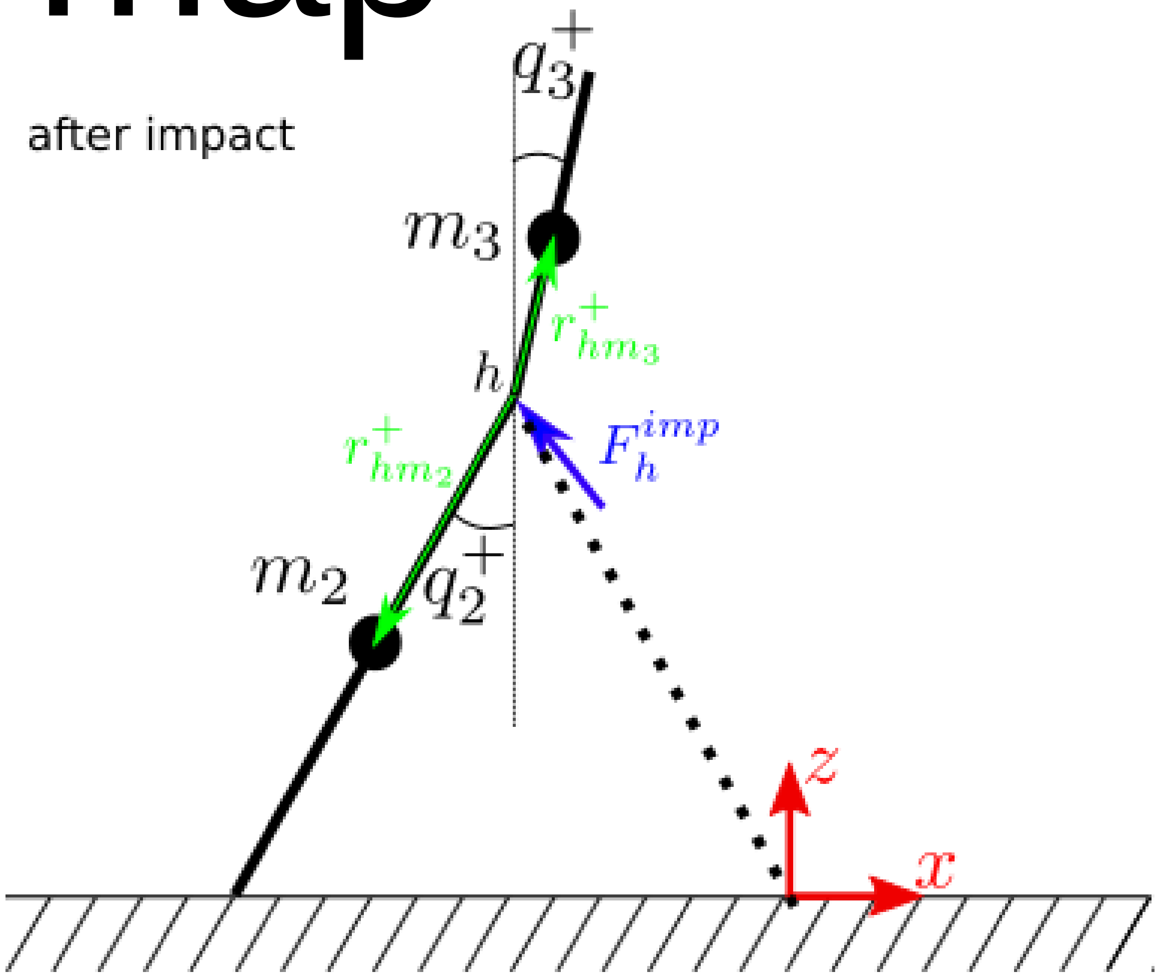



- **Angles**

$$\begin{bmatrix} q_1^+ \\ q_2^+ \\ q_3^+ \end{bmatrix} = \begin{bmatrix} q_2^- \\ q_1^- \\ q_3^- \end{bmatrix}$$

# Impact map

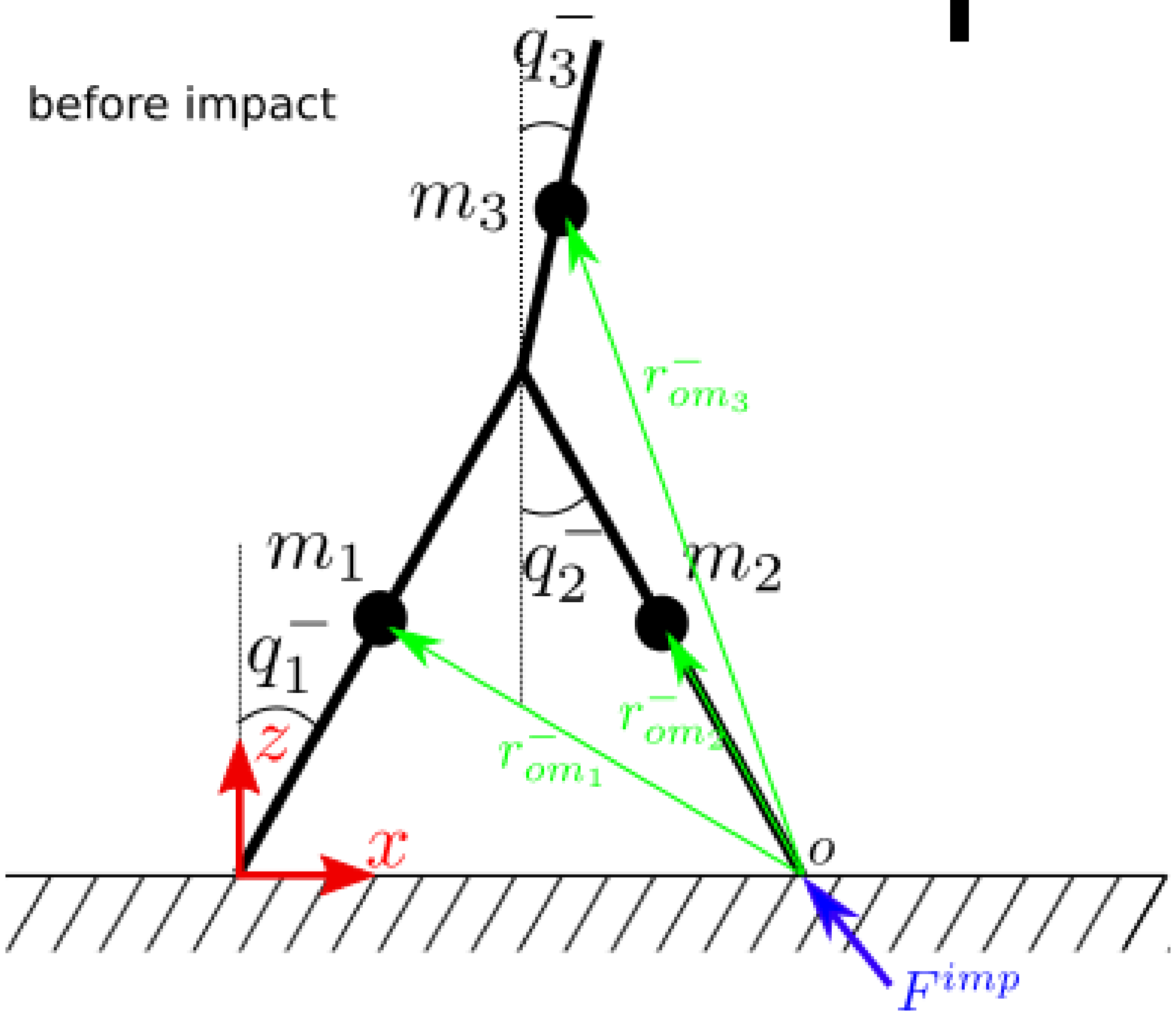


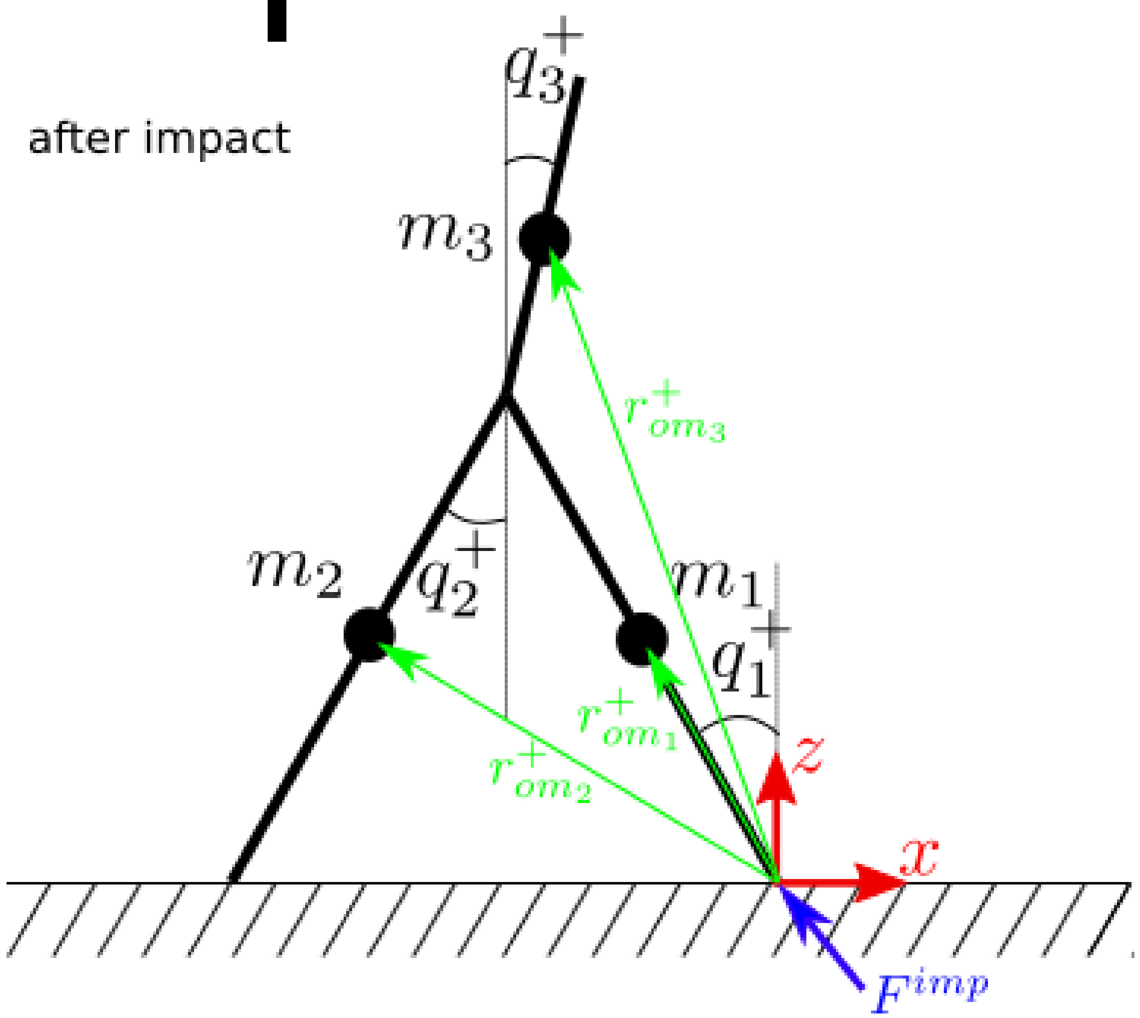

- **Angular Velocities**
  - Conservation of the angular momentum of **whole system about the post-transfer support point** and of the **trailing leg and torso about the hip**

*Stability and control of two dimensional bipedal walking, Tad McGeer, 1988*

# Impact map

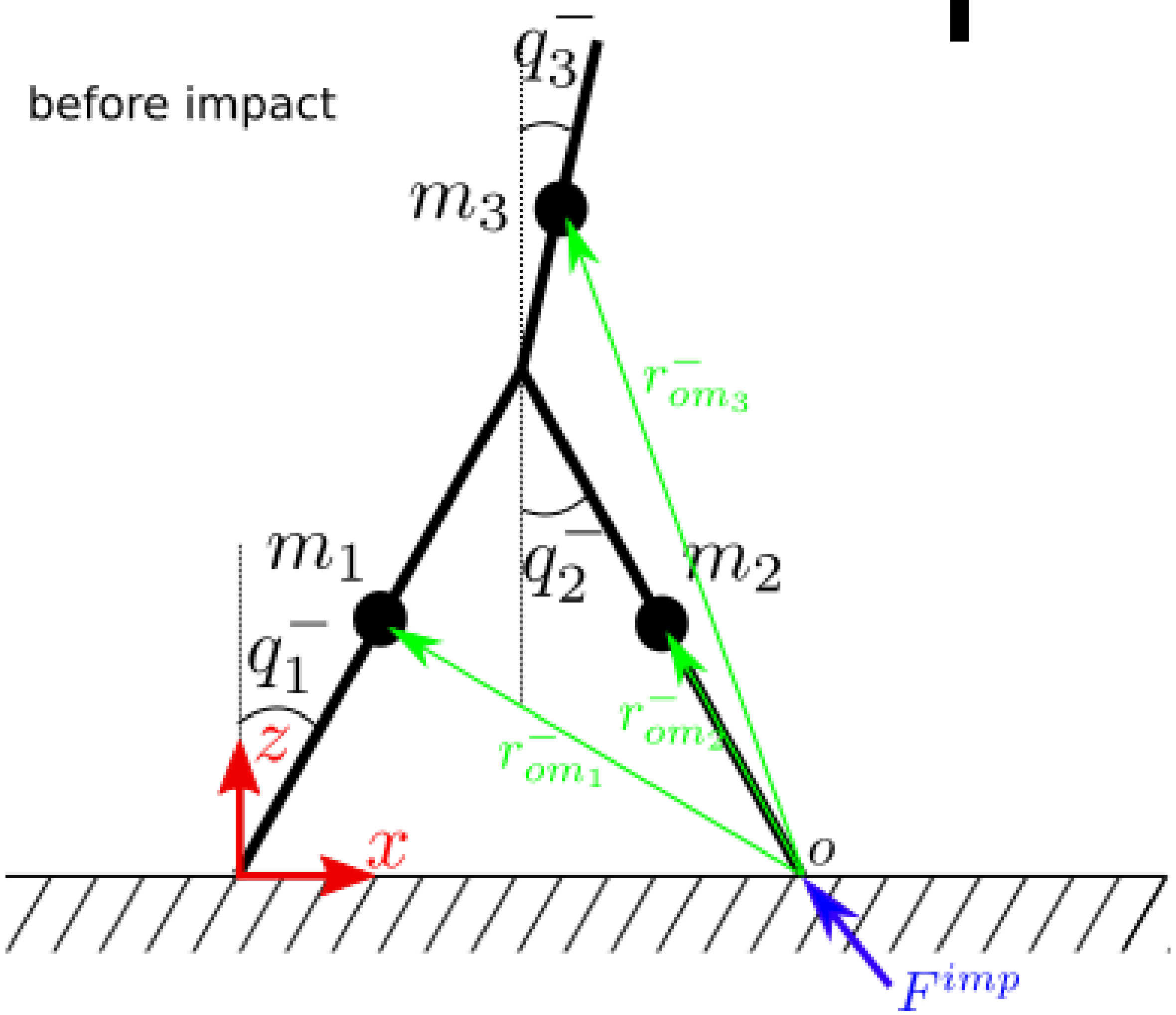


before impact
$q_3^-$
$m_3$
$m_1$
$m_2$
$q_2^-$
$q_1^-$
$r_{om_3}^-$
$r_{om_2}^-$
$r_{om_1}^-$
$z$
$x$
$o$
$F^{imp}$

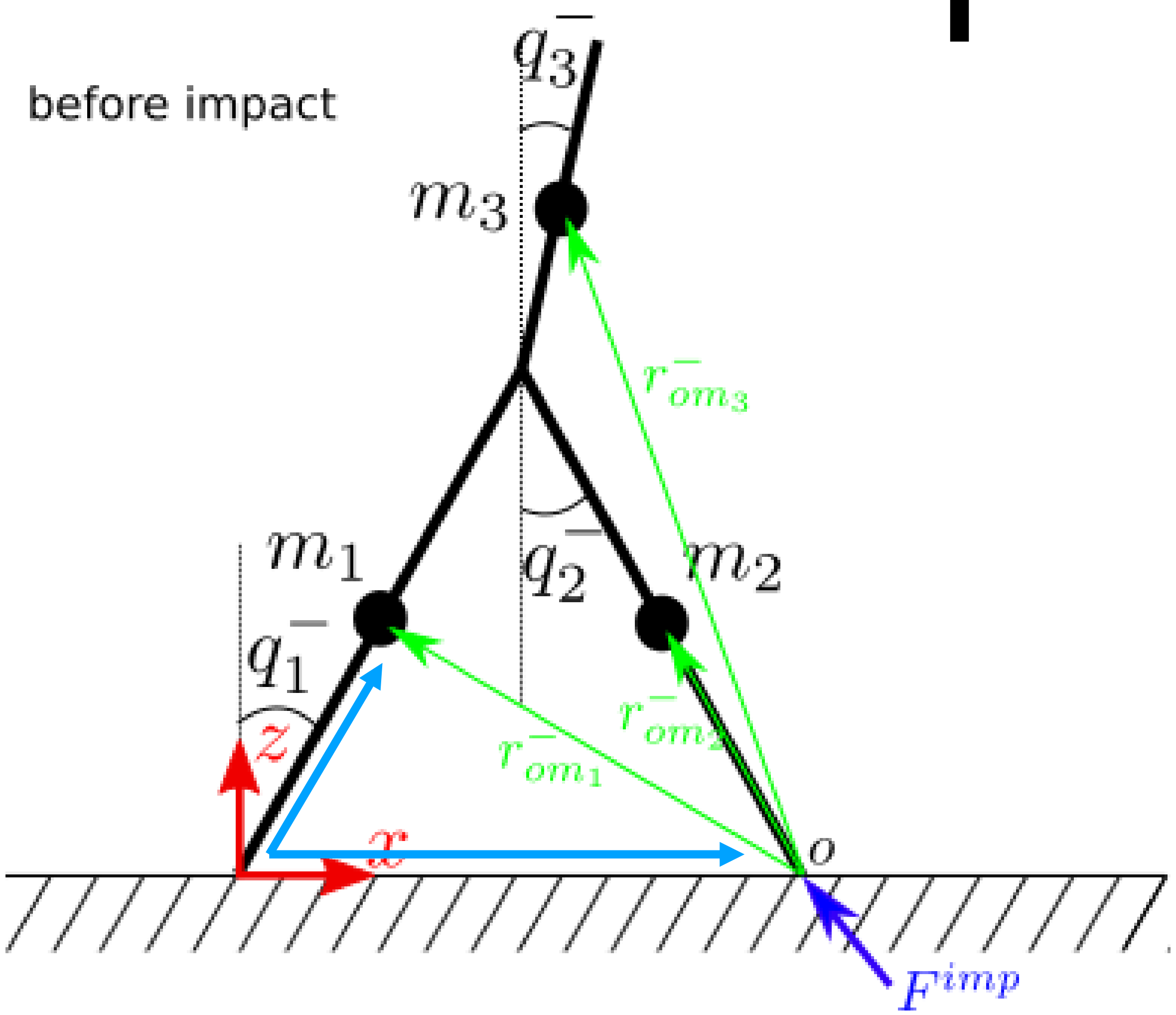
after impact
$q_3^+$
$m_3$
$m_2$
$m_1$
$q_2^+$
$q_1^+$
$r_{om_3}^+$
$r_{om_1}^+$
$r_{om_2}^+$
$z$
$x$
$F^{imp}$

# Impact map



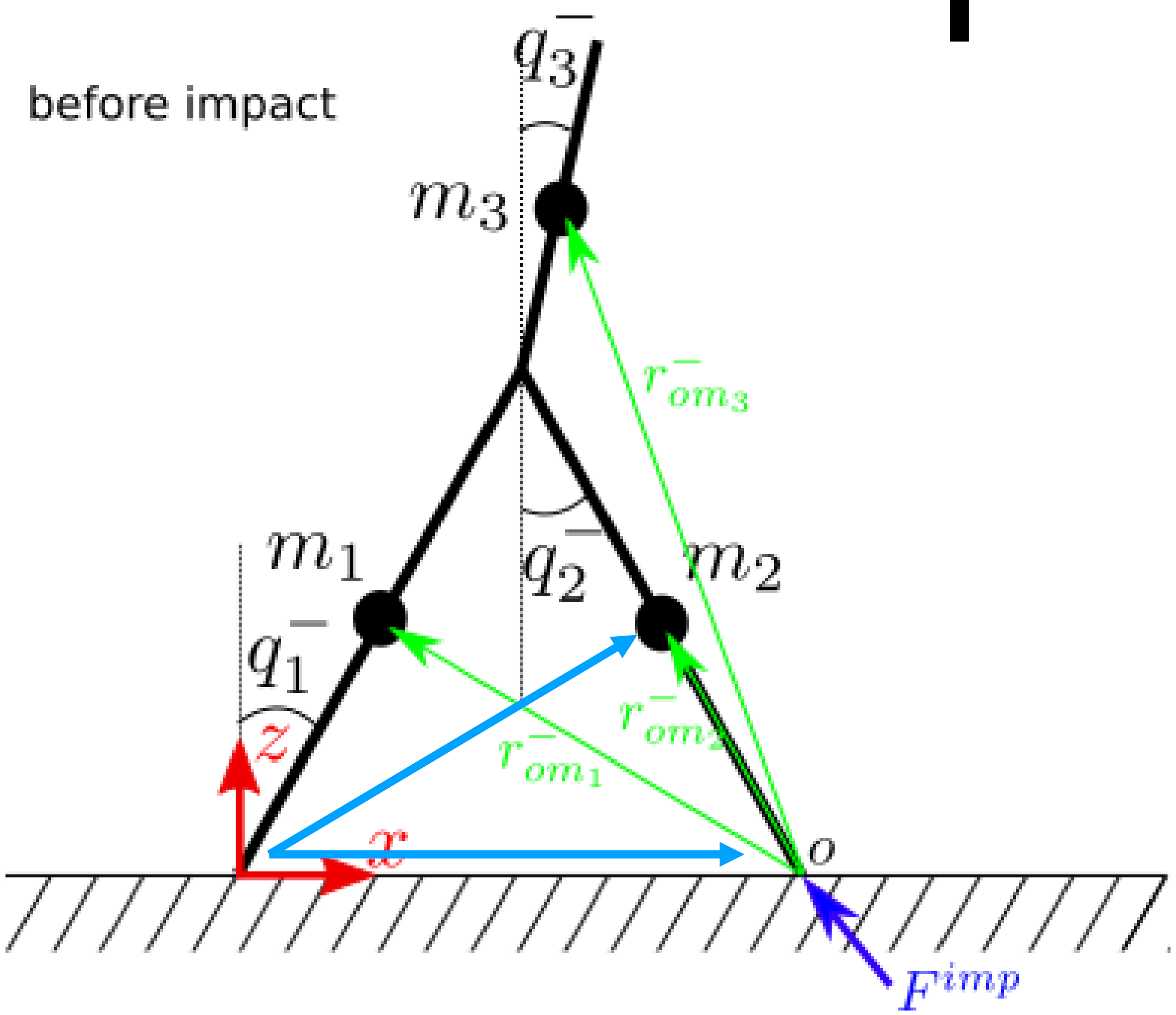

$$H_a^- = m\, r_{om_1}^- \times \dot{r}_1^- + m\, r_{om_2}^- \times \dot{r}_2^- + m_3\, r_{om_3}^- \times \dot{r}_3^-$$

# Impact map



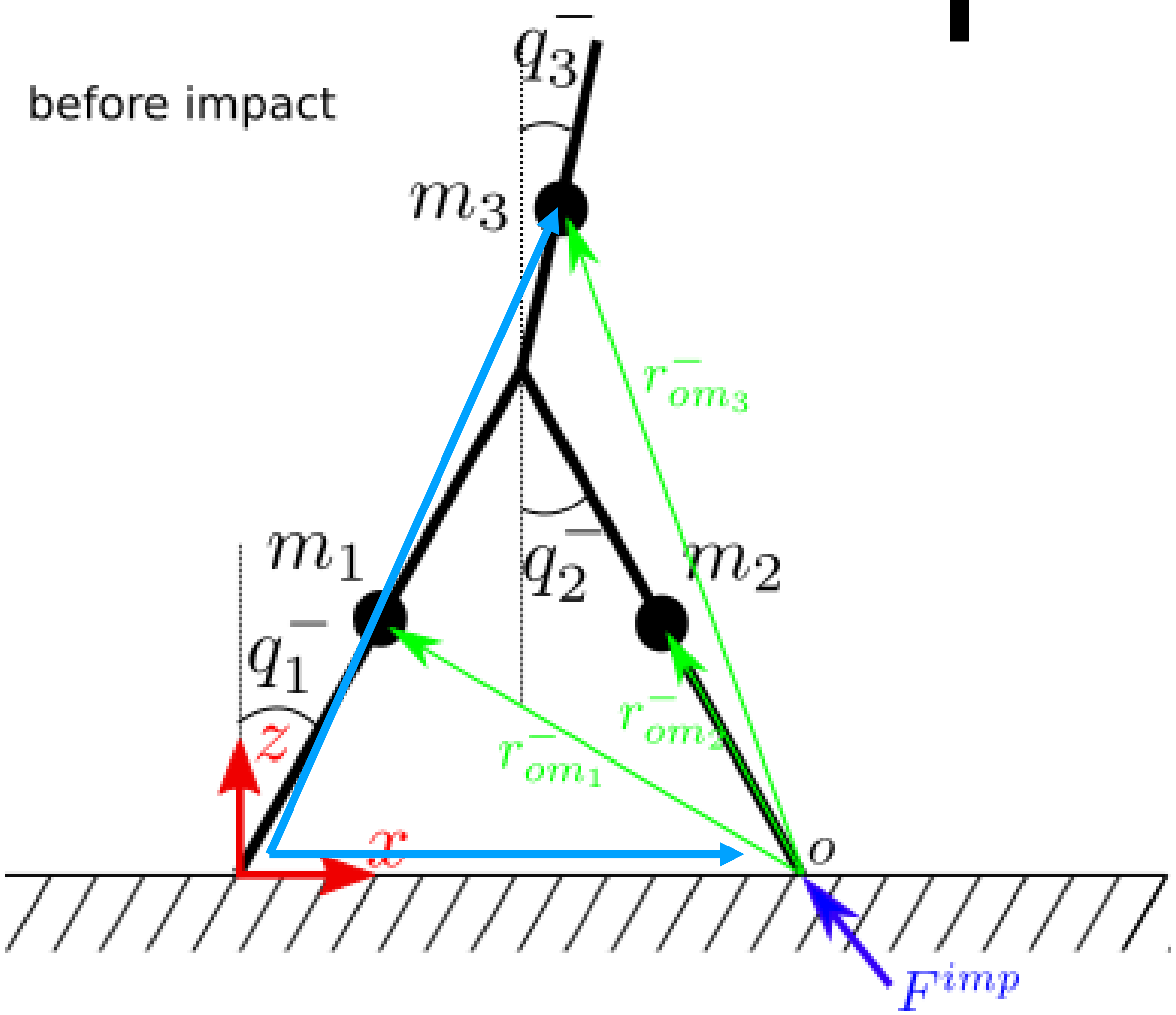

$$H_a^- = m\, r_{om_1}^- \times \dot{r}_1^- + m\, r_{om_2}^- \times \dot{r}_2^- + m_3\, r_{om_3}^- \times \dot{r}_3^-$$

# Impact map



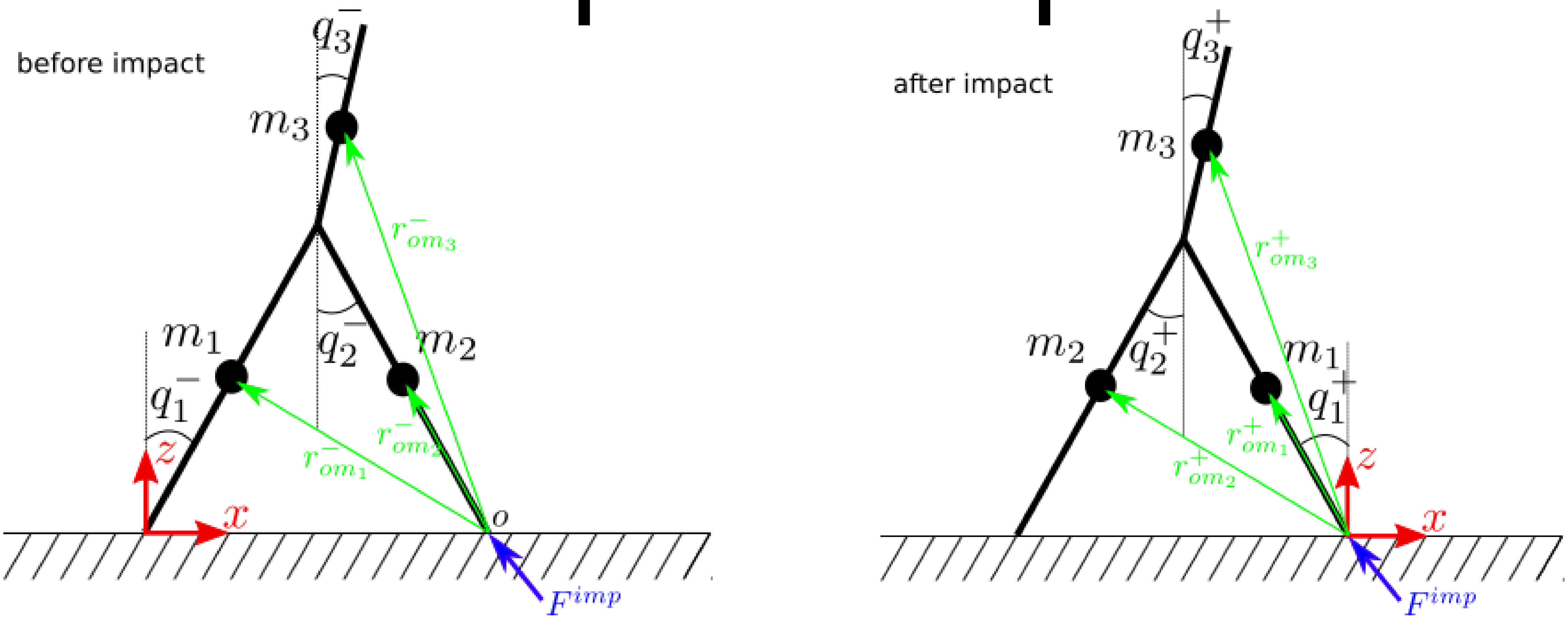

$$H_a^- = m\, r_{om_1}^- \times \dot{r}_1^- + m\, \boxed{r_{om_2}^-} \times \dot{r}_2^- + m_3\, r_{om_3}^- \times \dot{r}_3^-$$

# Impact map




$$H_a^- = m\, r_{om_1}^- \times \dot{r}_1^- + m\, r_{om_2}^- \times \dot{r}_2^- + m_3\, r_{om_3}^- \times \dot{r}_3^-$$

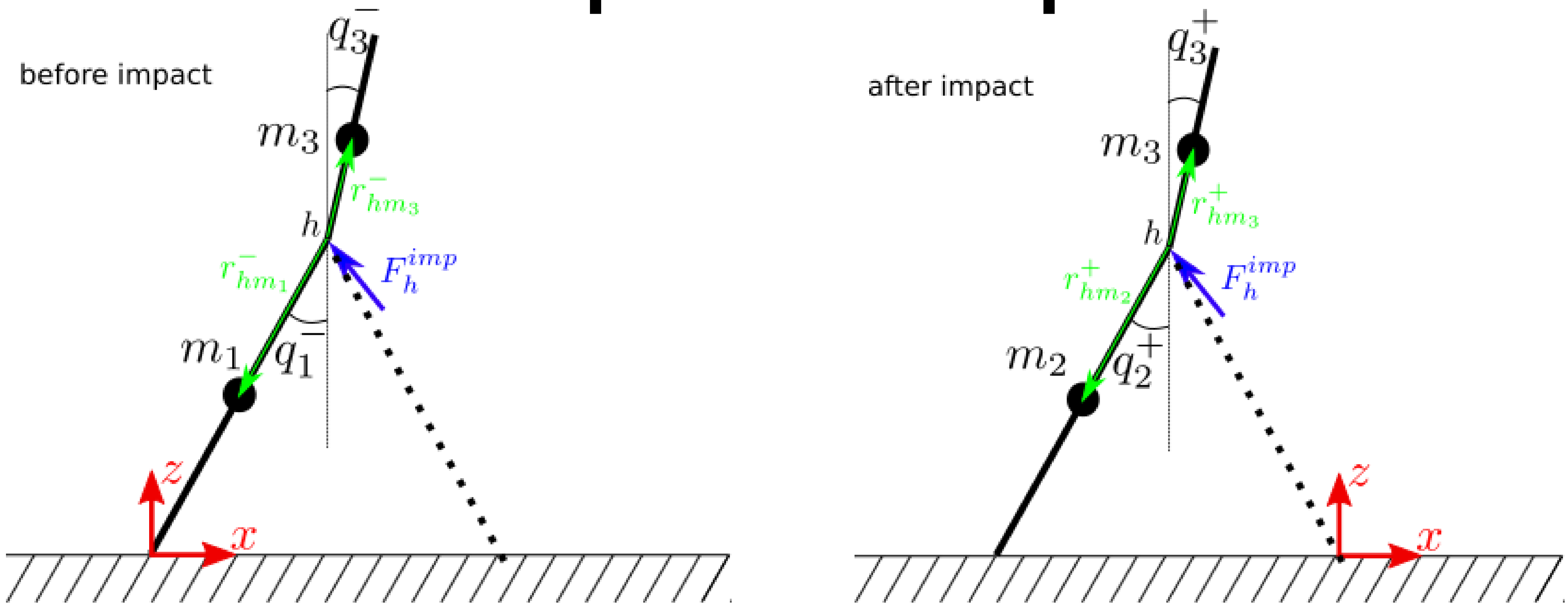
# Impact map




$$H_a^- = m\, r_{om_1}^- \times \dot{r}_1^- + m\, r_{om_2}^- \times \dot{r}_2^- + m_3\, r_{om_3}^- \times \dot{r}_3^-$$

$$H_a^+ = m\, r_{om_1}^+ \times \dot{r}_1^+ + m\, r_{om_2}^+ \times \dot{r}_2^+ + m_3\, r_{om_3}^+ \times \dot{r}_3^+$$

# Impact map




- Conservation of the angular momentum of **trailing leg and torso about the hip**

$$\boldsymbol{H}_{\boldsymbol{b}}^{-} = \cdots \qquad \boldsymbol{H}_{\boldsymbol{c}}^{-} = \cdots$$

$$\boldsymbol{H}_{\boldsymbol{b}}^{+} = \cdots \qquad \boldsymbol{H}_{\boldsymbol{c}}^{+} = \cdots$$

# Impact map

- Conservation of the angular momentum

$$H^- = [H_a^-; H_b^-; H_c^-] \qquad H^+ = [H_a^+; H_b^+; H_c^+]$$

$$H^+ = H^-$$

$$\dot{q}^+ = (A^+)^{-1} A^- \dot{q}^-$$