Chapter 1

Introduction to Operations Research

1. INTRODUCTION

Operations Research (OR) is a science which deals with problem, formulation, solutions and finally appropriate decision making. This subject is new and started after World War II, when the failures of missions were very high. Scientists and technocrates formed team to study the problem arising out of difficult situations and at the later stage solutions to these problems. It is research designed to determine most efficient way to do something new. OR is the use of mathematical models, statistics and algorithm to aid in decision-making. It is most often used to analyze complex real life problems typically with the goal of improving or optimizing performance. Decision making is the main activity of an engineer/manager. Some decisions can be taken by common sense, sound judgement and experience without using mathematics, and some cases this may not be possible and use of other techniques is inevitable.

With the growth of technology, the World has seen a remarkable changes in the size and complexity of organisations. An integral part of this had been the division of labour and segmentation of management responsibilities in these organisations. The results have been remarkable but with this, increasing specialisation has created a new problem to meet out organisational challenges. The allocation of limited resources to various activities has gained significant importance in the competitive market. These types of problems need immediate attention which is made possible by the application of OR techniques.

The tools of operations research are not from any one discipline, rather Mathematics, Statistics, Economics, Engineering, Psychology, etc. have contributed to this newer discipline of knowledge. Today, it has become a professional discipline that deals with the application of scientific methods for decision-making, and especially to the allocation of scare resources.

In India first unit of OR started in the year 1957 with its base at RRL Hyderabad. The other group was set up in Defence Science Laboratory which was followed by similar units at different parts of the country. The popular journal of OPSEARCH was established in 1963, to promote research in this field. Keeping in view the critical economic situation which required drastic increase in production efficiency, OR activities were directed, in all areas of business activities. In the late 50's OR was introduced at university level. With the development of PC's the use of OR techniques became prominent and effective tool as large amount of computation is required to handle complex problems. In

recent years application of OR techniques have achieved significance in all walk of life, may it be industry or office work for making strategical decisions more scientifically.

2. BACKGROUND OF OPERATIONS RESEARCH

The effectiveness of operations research in military spread interest in it to other governmental departments and industry. In the U.S.A. the National Research Council formed a committee on operations research in 1951, and the first book on the subject "Methods of Operations Research", by Morse and Kimball, was published. In 1952 the Operations Research Society of America came into being.

Today, almost all organisations make use of OR techniques for decision-making at all levels. This general acceptance to OR has come as managers have learned the advantage of the scientific approach to all industrial problems. Some of the Indian organisations using operations research techniques to solve their varied complex problems are: Railways, Defence, Indian Airlines, Fertilizer Corporation of India, Delhi Cloth Mills, Tata Iron and Steel Co. etc.

A purpose of OR is to provide a rational basis for making decisions in the absence of complete information. OR can also be treated as science devoted to describing, understanding and predicting the behaviour of systems, particularly man-machine systems.

3. MEANING OF OR

Defining OR is difficult task as its boundaries and content are not yet fixed. It can be regarded as use of mathematical and quantitative techniques to substantiate the decision being taken. Further, it is multidisciplinary which takes tools from subjects like mathematics, statistics, engineering, economics, psychology etc. and uses them to score the consequences of possible alternative actions. Today it has become professional discipline that deals with the application of scientific methods to decision-making. Salient aspects related to definition stressed by various experts on the subject are as follows:

(*a*) Pocock stresses that OR is an applied science; he states "OR is scientific methodology-analytical, experimental, quantitative—which by assessing the overall implication of various alternative courses of action in a management system, provides an improved basis for management decisions''.

(*b*) Morse and Kimball have stressed the quantitative approach of OR and have described it as "a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control".

(*c*) Miller and Starr see OR as applied decision theory. They state "OR is applied decision theory. It uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough—going rationality in dealing with his decision problem".

(*d*) Saaty considers OR as tool of improving the quality of answers to problems. He say, "OR is the art of giving bad answers to problems which otherwise have worse answers".

Few other definitions of OR are as follows:

- "OR is concerned with scientifically deciding how to best design and operate man-machine system usually requiring the allocation of scare resources."

  – Operations Research Society, America

- "OR is essentially a collection of mathematical techniques and tools which in conjunction with

system approach, are applied to solve practical decision problems of an economic or engineering nature''.

– Daellenbach and George

- "OR utilizes the planned approach (updated scientific method) and an interdisciplinary team in order to represent complex functional relationships as mathematical models for the purpose of providing a quantitative analysis''.

– Thieraub and Klekamp

- "OR is a scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources."

– H.A. Taha

- "OR is a scientific approach to problem solving for executive management".

– H.M. Wagner

4. FEATURES OF OR

The significant features of operations research include the followings:

(*i*) **Decision-making.** Every industrial organisation faces multifacet problems to identify best possible solution to their problems. OR aims to help the executives to obtain optimal solution with the use of OR techniques. It also helps the decision maker to improve his creative and judicious capabilities, analyse and understand the problem situation leading to better control, better co-ordination, better systems and finally better decisions.

(*ii*) **Scientific Approach.** OR applies scientific methods, techniques and tools for the purpose of analysis and solution of the complex problems. In this approach there is no place for guess work and the person bias of the decision maker.

(*iii*) **Inter-disciplinary Team Approach.** Basically the industrial problems are of complex nature and therefore require a team effort to handle it. This team comprises of scientist/mathematician and technocrates. Who jointly use the OR tools to obtain a optimal solution of the problem. The tries to analyse the cause and effect relationship between various parameters of the problem and evaluates the outcome of various alternative strategies.

(*iv*) **System Approach.** The main aim of the system approach is to trace for each proposal all significant and indirect effects on all sub-system on a system and to evaluate each action in terms of effects for the system as a whole.
The interrelationship and interaction of each sub-system can be handled with the help of mathematical/analytical models of OR to obtain acceptable solution.

(*v*) **Use of Computers.** The models of OR need lot of computation and therefore, the use of computers becomes necessary. With the use of computers it is possible to handle complex problems requiring large amount of calculations.

The objective of the operations research models is to attempt and to locate best or optimal solution under the specified conditions. For the above purpose, it is necessary that a measure of effectiveness has to be defined which must be based on the goals of the organisation. These measures can be used to compare the alternative courses of action taken during the analysis.

5. PHASES OF OR STUDY

OR is a logical and systematic approach to provide a rational basis for decision-making. The phases of

OR must be logical and systematic. The various steps required for the analysis of a problem under OR are as follows:

Step I. Observe the Problem Environment

The first step of OR study is the observation of the environment in which the problem exists. The activities that constitute this step are visits, conferences, observations, research etc. with the help of such activities, the OR analyst gets sufficient information and support to proceed and is better prepared to formulate the problem.

Step II. Analyse and Define the Problem

In this step not only the problem is defined but also uses, objectives and limitations of the study that are stressed in the light of the problem. The end results of this step are clear grasp of need for a solution and understanding of its nature.

Step III. Develop a Model

The next step is to develop model, which is representation of same real or abstract situation. OR models are basically mathematical models representing systems, process or environment in form of equations, relationships or formulae. The activities in this step is to defining interrelationships among variables, formulating equations, using known OR models or searching suitable alternate models. The proposed model may be field tested and modified in order to work under stated environmental constraints. A model may also be modified if the management is not satisfied with the answer that it gives.

Step IV. Selection of Data Input

It is a established fact that without authentic and appropriate data the results of the OR models cannot be trusted. Hence, taping right kind of data is a vital step in OR process. Important activities in this step are analysing internal-external data and facts, collecting opinions and using computer data banks. The purpose of this step is to have sufficient input to operate and test the model.

Step V. Solution and Testing

In this step the solution of the problems is obtained with the help of model and data input. Such a solution is not implemented immediately and this solution is used to test the model and to find its limitations if any. If the solution is not reasonable or if the model is not behaving properly, updating and modification of the model is considered at this stage. The end result of this step is solution that is desirable and supports current organisational objectives.

Step VI. Implementation of the Solution

This is the last phase of the OR study. In OR the decision-making is scientific but implementation of decision involves many behavioural issues. Therefore, implementation authority has to resolve the behavioural issues, involving the workers and supervisors to avoid further conflicts. The gap between management and OR scientist may offer some resistance but must be eliminated before solution is accepted in totality. Both the parties should play positive role, since the implementation will help the organisation as a whole. A properly implemented solution obtained through OR techniques results in improved working conditions and wins management support.

6. OUTLINES OF OR MODELS

In OR the problem is expressed in the form of a model. Where, a model is a theoretical abstraction (approximation) of a real-life problem. It can be defined as a simplified representation of an operation

or a process in which only the basic aspects or the most important features of a typical problem under investigation are considered. OR analysts have given special impetus to the development and use of techniques like, linear programming, waiting line theory, game theory, inventory controls and simulation. In addition, some other common tools are non-linear programming, integer programming, dynamic programming, sequencing theory, Markov process, network scheduling—PERT and CPM, symbolic logic, information theory and utility/value theory. The list, of course, is not exhaustive. The detailed discussion on above will be presented in appropriate chapters, however, brief explanation of these is given below:

(i) Linear Programming (L.P.)

Linear programming is basically a constrained optimisation technique which tries to optimise some criterion within some constraints. It consists of an objective function which is some measure of effectiveness like profit, loss or return on investment and several boundary conditions putting restriction on the use of resources. Objective function and boundary conditions are linear in nature. There are methods available to solve a linear programming problem.

(ii) Waiting Line or Queuing Theory

This deals with the situation in which queue is formed or the customers have to wait for service or machines wait for repairmen and therefore concept of a queue is involved. If we assume that there are costs associated with waiting in line, and if there are costs of adding more service facilities, we want to minimize the sum of costs of waiting and the costs of providing service facilities. Waiting line theory helps to make calculations like number of expected member of people in queue, expected waiting time in the queue, expected idle time for the server, etc. These calculations then can be used to determine the desirable number of service facilities or number of servers.

(iii) Game Theory

It is used for decision-making under conflicting situations where there are one or more opponents. The opponents, in game theory, are called players. The motives of the players are dictomized. The success of one player tends to be at the cost of others and hence they are in conflict. Game theory models, a conflict situation arises and helps to improve the decision process by formulating appropriate strategy.

(iv) Inventory Control Models

These models deal with the quantities which are either to be purchased or stocked since each factor involves cost. The purchase and material managers are normally encounter such situations. Therefore, inventory models provide rational answer to these questions in different situations of supply and demand for different kind of materials. Inventory control models help managers to decide ordering time, reordering level and optimal ordering quantity. The approach is to prepare a mathematical model of the situation that expressed total inventory costs in terms of demand, size of order, possible over or under stocking and other relevant factors and then to determine optimal order size, optimum order level etc. using calculus or some other technique.

(v) Simulation

It is basically data generating technique, where sometimes it is risky, cumbersome, or time consuming to conduct real study or experiment to know more about situation or problem. The available analytical methods cannot be used in all situations due to large number of variables or large number of interrelationships among the variables and the complexity of relationship, it is not possible to develop an analytical model representing the real situation. Some times, even building of model is possible but its

solution may not be possible. Under such situations simulation is used. It should be noted that simulation does not solve the problem by itself, but it only generates the required information or data needed for decision problem or decision-making.

(vi) Non-Linear Programming

These models may be used when either the objective function or some of the constraints are not linear in nature. Non-linearity may be introduced by such factors as discount on price of purchase of large quantities and graduated income tax etc. Linear programming may be employed to approximate the non-linear conditions, but the approximation becomes poorer as the range is extended. Non-linear methods may be used to determine the approximate area in which a solution lies and linear methods may be used to obtain a more exact solution.

(vii) Integer Programming

This method can be used when one or more of the variables can only take integer values. Examples are the number of trucks in a fleet, the number of generators in a power house and so on. Approximate solutions can be obtained without using integer programming methods, but the approximation generally becomes poorer as the number becomes smaller. There are techniques to obtain solution of integer programming problems.

(viii) Dynamic Programming

This is a method of analyzing multistage decision processes, in which each elementary decision is dependent upon those preceding it as well as upon external factors. It drastically reduces the computational efforts otherwise necessary to analyze results of all possible combinations of elementary decisions.

(ix) Sequencing Theory

This is related to waiting line theory and is applicable when the facilities are fixed, but the order of servicing may be controlled. The scheduling of service or the sequencing of jobs is done to minimize the relevant costs and time.

(x) Markov Process

It is used for decision-making in situations where various states are defined. The probability of going from one state to another is known and depends on the present state and is independent of how we have arrived at that state. Theory of Markov process helps us to calculate long run probability of being in a particular state (steady state probability), which is used for decision-making.

(xi) Network Scheduling—PERT and CPM

These techniques are used to plan, schedule and monitor large projects such as building construction, maintenance of computer system installation, research and development design etc. The technique aims at minimizing trouble spots, such as, delays, interruptions and production bottlenecks, by identifying critical factors and coordinating various parts of overall job/project. The project/job is diagrammatically represented with the help of network made of arrows representing different activities and interrelationships among them. Such a representation is used for identifying critical activities and critical path. Two basic techniques in network scheduling are Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM). CPM is used when time taken by activities in a project are known for sure and PERT is used when activities time is not known for sure—only probabilistic estimate of time is available to the users.

(xii) Symbolic Logic

It deals with substituting symbols for words, classes of things or functional systems. Symbolic logic involves rules, algebra of logic and propositions. There have been only limited attempts to apply this technique to business problems; however has had extensive application in the design of computing machinery.

(xiii) Information Theory

Information theory is an analytical process transferred from the electrical communications field to operations research. It seeks to evaluate the effectiveness of information flow within a given system. Despite its application mainly to communication networks, it has had a indirect influence in simulating the examination of business organizational structures with a view to improving information or communication flow.

(xiv) Utility/Value Theory

It deals with assigning numerical significance to the worth of alternative choices. To date, this has been only a concept and is in the stage of elementary model formulation and experimentation and can be useful in decision-making process.

7. SCOPE OF OPERATIONS RESEARCH

As presented in the earlier paragraphs, the scope of OR is not only confined to any specific agency like defence services but today it is widely used in all industrial organisations. It can be used to find the best solution to any problem be it simple or complex. It is useful in every field of human activities, where optimisation of resources is required in the best way. Thus, it attempts to resolve the conflicts of interest among the components of organization in a way that is best for the organisation as a whole. The main fields where OR is extensively used are given below, however, this list is not exhaustive but only illustrative.

(i) National Planning and Budgeting

OR is used for the preparation of Five Year Plans, annual budgets, forecasting of income and expenditure, scheduling of major projects of national importance, estimation of GNP, GDP, population, employment and generation of agriculture yields etc.

(ii) Defence Services

Basically formulation of OR started from USA army, so it has wide application in the areas such as: development of new technology, optimization of cost and time, tender evaluation, setting and layouts of defence projects, assessment of "Threat analysis", strategy of battle, effective maintenance and replacement of equipment, inventory control, transportation and supply depots etc.

(iii) Industrial Establishment and Private Sector Units

OR can be effectively used in plant location and setting finance planning, product and process planning, facility planning and construction, production planning and control, purchasing, maintenance management and personnel management etc. to name a few.

(iv) R & D and Engineering

Research and development being the heart of technological growth, OR has wide scope for and can be applied in technology forecasting and evaluation, technology and project management, preparation of tender and negotiation, value engineering, work/method study and so on.

(v) Business Management and Competition

OR can help in taking business decisions under risk and uncertainty, capital investment and returns, business strategy formation, optimum advertisement outlay, optimum sales force and their distribution, market survey and analysis and market research techniques etc.

(vi) Agriculture and Irrigation

In the area of agriculture and irrigation also OR can be useful for project management, construction of major dams at minimum cost, optimum allocation of supply and collection points for fertilizer/seeds and agriculture outputs and optimum mix of fertilizers for better yield.

(vii) Education and Training

OR can be used for obtaining optimum number of schools with their locations, optimum mix of students/teacher student ratio, optimum financial outlay and other relevant information in training of graduates to meet out the national requirements.

(viii) Transportation

Transportation models of OR can be applied to real life problems to forecast public transport requirements, optimum routing, forecasting of income and expenses, project management for railways, railway network distribution, etc. In the same way it can be useful in the field of communication.

(ix) Home Management and Budgeting

OR can be effectively used for control of expenses to maximize savings, time management, work study methods for all related works. Investment of surplus budget, appropriate insurance of life and properties and estimate of depreciation and optimum premium of insurance etc.

8. ROLE OF OR IN ENGINEERING AND SCIENCE

The information available in science has been used to develop engineering. Whatever available in the engineering is based on basic fundamentals of science. With the growth of technology the practitioners faced many challenges to improve the product and market it efficiently. OR has emerged to help every one to improve their performance and produce items at optimum cost.

Since the modern problems are complex in nature and modern technology is “knowledge based” and “skill intensive”. The knowledge possessed by single individual or group is limited and OR is a team effort, specialists from all relevant disciplines participate in it, to find out best possible solution of the problem under the given environmental conditions.

It has been described that OR is a systematic method of stating the problem in clear terms, collecting facts and data, analysing them and then reaching certain conclusions in the form of solutions to the problem, which further can be tested and verified for its optimality in most of the cases.

OR is a team effort have been used with the existence of mankind. But OR is a systematic approach using only scientific methods/techniques to find solution which distinguishes OR from team efforts past or present.

OR is being effectively used in areas such as: production planning and inventory control, transportation, military operations and weapon system development, personnel management, social services, health services, communication systems, computer networks and information system to name but a few—the problems they pose with ever increasing rate are similarly formulated, can be identified by several features they have in common and, last but not least, can be solved by similar methods. These problems are therefore conveniently grouped under the common heading of OR problems.

Under OR study an objective is defined which may have alternative solutions. A decision has to be made based upon choosing from a set of possible alternatives. Each choice offers its own advantages and disadvantages, so that in complex situation the decision maker might not be able to make a preferable option at once and quickly decide why he should prefer one alternative over the other one. To clarify the situation and compare the alternatives in several aspects, OR suggests a series of mathematical operations. Their aim is to analyze the situation critically and thus prepare a decision for those bearing the responsibility for a final choice.

During implementation the operations research team should prepare a detailed instructions for those who have to implement the solution. Translating solution into operating procedure should indicate who should do, what and when. The facilities and information required should also be clearly specified and persons concerned should be taken into confidence for obtaining their cooperation for the success of the program.

9. SIMPLIFICATION OF OR MODELS

The more complex, expensive and large in scale the designed system is, the less allowable in it are wilful decisions and the more gain in importance scientific methods which, when implemented, provide an estimate of each decision's consequences, help discard the unallowable versions and recommend the most successful ones. They help in assessing whether the available information is adequate to prepare a correct decision and, if not, then indicate what data should be additionally collected. It would be extremely risky to be guided solely by intuition *i.e.*, experience and common sense. Modern science and technology evolve so fast that the experience may simply not have been acquired. The calculations that make the process of decision-making easier are the subject matter of operations research.

Under the complex situations some of the models need a lot of computational efforts particularly in case of linear programming. Efforts should be made to simplify the situation and development of model so as to generate the optimal solution with minimum effort. The selection of model for a particular problem has its own bearings and availability of solution. Assumptions to be imposed on the model should be such that, it makes it possible to achieve desirable solution without affecting the constraints of the problem.

In most intricate cases, when scanning an operation and its outcome depend on a large number of intimately interrelated random factors, the analytical techniques fail altogether and the analyst has to employ Monto Carlo methods of statistical modelling. In this case a computer simulates the process of an operation development with all the random variables involved. This manipulation of the process yields an observation of one random operation run. One such realization gives no grounds for decision-making, but, once a manifold of them is collected after several runs, it may be handled statistically to find the process means and make inferences about the real system and how in the mean, it is influenced by initial conditions and controllable variables.

Both analytical and statistical models are widely implemented in OR. Each of the models possess its own advantages and disadvantages. The analytical models are more rough, but they yield more meaningful results. However, statistical models are more accurate, but are bulky, poorly analyzable, need more computational time and do not yield optimal results. Therefore, the analyst should make correct judgement to select either model depending upon requirements and situation of the problem.

10. DEVELOPMENT OF OR IN INDIA

OR being a new discipline started a bit late in India with its inception at Regional Research Laboratory at Hyderabad and at the same time a group was established in Defence Science Laboratory to solve the

problems of stores, purchase and planning. OR society was formed in 1953. Today OR subject is very popular and is being taught at graduation and post graduation level in all the university of the country. It is also being used in industrial establishment extensively to improve decision-making process.

11. COMPUTERS IN OR

As has been presented earlier that OR tries to find optimal solutions with multiple variables. In most of the cases a large number of iterations are required to reach optimal solution. Manually this task becomes time consuming and single mistake at any point can generate erroneous results. With the development of computers and P.C's this has reduced manual efforts considerably and solutions can be obtained in a short period of time and possibility of errors is also minimised considerably.

Storage of information/data is easy and faster with the use of computers because of its memory. The computational time requirements are also less and no paper work is required. Transfer of data from one place to another is also possible through net/computers. The reliability of solutions is also high. For the large size problems, where simulation was to be used, it was not possible to carry it out manually, which is now possible with the use of computers. To handle linear programming problem with multiple variables use to be cumbersome and time taking, can be done at wink of moment without any manual efforts.

12. LIMITATIONS OF OPERATIONS RESEARCH

OR has some limitations however, these are related to the problem of model building and the time and money factors involved in application rather then its practical utility. Some of them are as follows:

(*i*) **Magnitude of Computation.** Operations research models try to find out optimal solution taking into account all the factors. These factors are enormous and expressing them in quantity and establishing relationships among these require voluminous calculations which can be handled by computers.

(*ii*) **Non-Quantifiable Factors.** OR provides solution only when all elements related to a problem can be quantified. All relevant variables do not lend themselves to quantification. Factors which cannot be quantified, find no place in OR study. Models in OR do not take into account qualititative factors or emotional factors which may be quite important.

(*iii*) **Distance between User and Analyst.** OR being specialist's job requires a mathematician or statistician, who might not be aware of the business problems. Similarly, a manager fails to understand the complex working of OR. Thus there is a gap between the two. Management itself may offer a lot of resistance due to conventional thinking.

(*iv*) **Time and Money Costs.** When basic data are subjected to frequent changes, incorporating them into the OR models is a costly proposition. Moreover, a fairly good solution at present may be more desirable than a perfect OR solution available after sometime. The computational time increases depending upon the size of the problem and accuracy of results desired.

(*v*) **Implementation.** Implementation of any decision is a delicate task. It must take into account the complexities of human relations and behaviour. Sometimes, resistance is offered due to psychological factors which may not have any bearing on the problem as well as its solution.

EXERCISES

1. What is operations research and explain briefly its applications in industrial organizations?
2. What are the characteristics of operations research? Discuss.
3. Discuss the importance of OR in decision-making process.
4. Enumerate, with brief description, some of the important techniques used in OR.
5. Discuss the limitations of operation research.
6. Describe the various steps involved in OR study.
7. Discuss significance and scope of operation research.
8. Describe briefly the different phases of operation research.
9. Explain steps involved in the solution of OR problems.
10. Illustrate the importance of features in OR.