Examination in subject
SIF8010 Algorithms and Datastructures
Friday 11. December 1998, hrs. 0900-1500

Contact under examination: Arne Halaas, phone 73 593442.
Allowed: All types of calculators. All printed and handwritten material.
Answering boxes: All answers shall be written into boxes. Add extra sheets / calculations if you believe those are important for your answer, and write "see additional sheets" beside the box.
Requirements: "Pass" is required both for the ordinary and for the exercise questions.
Remember: Fill in "student nr." on top of each page.

PROBLEM 1.

Given a program segment consisting of the following double, nested loop:

```
while N > 0 do
        S1;
        for k := 1 to N do S2;
        N := N div 5;
```

Here S1and S2 are sentences demanding, respectivily, K1 and K2 time units. We assume $N = 5^m$. The operation "div" expresses integer division (f.x..: 4 div 5 = 0).

(a) Find an exact expression T(N) for the time consuption of the program segment. Show your calculations.

Answer: 10%

PROBLEM 2.

You shall here decide if the following claim, as you understand it, is true or false.

" Any sorting algorithm that moves elements just 1 position at a time (to a neighbouring position) must have time complexity at least $\Omega(n^2)$ "

(a) Give a justified answer where you also explain how you, eventually, will specify the claim.

Answer: 10%

PROBLEM 3.

Given an integer array A[1..n], where a = A[1], b = A[n], and $|A[i]-A[i+1]| < 2$ for all i where $0 < i < n$.
Describe shortly an efficient algorithm to find an index j such that A[j] =z, where integer z is a value in the interval [min(a,b), max(a,b)].

(a) Will it always be possible to find a z as described above? (Explain.)

Answer: 5%

(b) Sketch your algorithm:

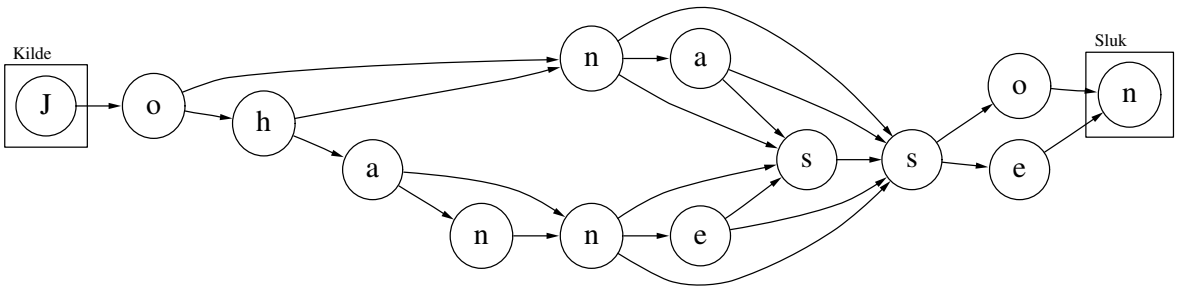
Answer: 10%

(c) What is the maximum number of value comparisons in your algorithm? (Explain.)

Answer: 5%

PROBLEM 4.

Given the below asyclic graph (DAG), G = (V,E), where there is just 1 node (source) without any incoming edges and just 1 node (sink) without any outgoing edges.



(a) Find the number of different paths from source to sink in the graph above. We here say that two paths are different only if they are not identical.

Answer: 5%

(b) Describe an efficient algorithm that finds the number of different paths in DAG with 1 source node og 1 sink node.

Answer: 10%

(c) Find the time complexity for your proposed algorithm (b). Show your calculations.

Answer: 5%

(d) Give a good idea for applications of DAGs that keep / generate related names, like in the above graph.

Answer: 5%

PROBLEM 5

Given the following problem related to a general graph G and the integer k:

Problem F(G,k): "Given an undirected graph G = (V,E). Decide if the nodes in G can be colored with k colours such that neighbouring nodes do not have the same colour."
It is known that F(G,3) is NP-complete.

(a) Give arguments showing that the problem F(G,3) belongs to the class NP.

Answer: 5%

(b) Give arguments showing that the problem F(G,2) is not NP-complete.

Answer: 5%

PROBLEM 6, Problems related to the exercises.

Exercise 1, Sorting:

(a) Under which conditions is Bubble-sort better that Heap-sort? (Short list.)

Answer: 3%

(b) Under which conditions is Heap-sort better than Bubble-sort? (Short list.)

Answer: 3%

Exercise 3, Shortest paths:

(c) Which datastructures are used in your programs? (Give general descriptions / names.)

Answer: 2%

(d) What is the time complexity of your program? (Short explanation.)

Answer: 3%

(e) How is your time complexity affected by the datastructures you are using?

Answer: 3%

(f) Which datastructures might have been replaced in your program with the effect that your program would get a worse time complexity? (Short explanation.)

Answer: 3%

Exercise 5, String matching:

(g) Show the Π– table for the searched pattern ”GACGATAGA”.

Answer: 4%

(h) Show how the KMP-algorithm searches through the text ”TGAGACGACGATAGA” with the pattern in (g).

Answer: 4%