Homework set 7 Sander Lindberg.

Exercise 1:

Exercises 5.2

1)

c) this is a function Range = $\cancel{\mathbb{R}}$ $\mathbb{R}$

d) not a function, example both $(0,-1)$ and $(0,1)$ is in the relation.

e) $|R| > 5$, not a function

Exercise 2:

3) $A = \{1, 2, 3, 4\}$ $B = \{x, y, z\}$

a) $\{(1,x), (2,x), (3,x), (4,x)\}$, $\{(1,y), (2,y), (3,y)(4,y)\}$, $\{(1,z), (2,z), (3,z), (4,z)\}$, $\{(1,y), (2,y), (3,\cancel{x}), (4,z)\}$, $\{(1,z), (2,x), (3,y), (4,y)\}$.

b) there are 3^4 functions $f: A \to B$

c) 0, $|A| \not< |B|$

d) there are 4^3 functions $f: B \to A$.

e) $4 \cdot 3 \cdot 2 = \underline{24}$

f) 3^3

g) 3^2

h) 3^2

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Exercise 3

Exercises 5.2

5).

c) $A \cup C = \overline{\overline{A}} \cap \overline{\overline{C}} = A \cap C.$

$2x + 1 = x - 7 \Rightarrow \underline{x = -8}$

$A \cap C = (\cancel{+}8, -8-7) = \underline{\underline{(-8, -15)}}$

d) $\overline{B} \cup \overline{C} = \overline{B \cap C}$

$B \cap C =$

$3x = \cancel{7x}\ x - 7$

$\Downarrow$

$\frac{2x = -7}{2}$, $x = -\frac{7}{2}$

$\underline{y = 3 \cdot \left(-\frac{7}{2}\right) = -\frac{27}{2}}$

$\overline{B \cap C} = \left(-\frac{7}{2}, -\frac{27}{2}\right) = R^2 - \left\{\left(-\frac{7}{2}\right), \left(-\frac{27}{2}\right)\right\}$

everything else then $x = -\frac{7}{2}$ and $y = -\frac{27}{2}$

Exercice 4

exercizes 5.2

8)

a) True

b) False, $\lfloor 4,01 \rfloor = \underline{4}$, $\lceil 4,01 \rceil = \underline{5}$

c) True

d) False, $-\lceil 3,01 \rceil = \underline{-4}$, $\lceil -3,01 \rceil = \underline{-3}$

9) b) $\lfloor 7x \rfloor = 7$

$7x = 7.99999\ldots\ldots$

$x = \frac{8}{7}$

$\underline{\underline{x \in [1, \frac{8}{7})}}$

c) $\lfloor x+7 \rfloor = x + 7$

$\underline{\underline{x \in \mathbb{Z}}}$

d) $\lfloor x + 7 \rfloor = \lfloor x \rfloor + 7$

$\lfloor x \rfloor + \lfloor 7 \rfloor = \lfloor x \rfloor + 7$

$\lfloor x \rfloor + 7 = \lfloor x \rfloor + 7$

$\underline{\underline{x \in \mathbb{R}}}$

Excercise 5

Exercises 5.3

2)

b) $f(x) = 2x - 3$

Injective:

$f(x_1) = f(x_2)$

$2x_1 - 3 = 2x_2 - 3$

$\frac{2x_1 = 2x_2}{2}$

$x_1 = x_2 \Rightarrow$ Injective!

Surjective:

$y = 2x - 3$

$x = \frac{y - 3}{2}$, since $f: \mathbb{Z} \to \mathbb{Z}$, this is not surjective. $(y = 2 = \frac{2-3}{2} = -\frac{1}{2})$

the range $f(\mathbb{Z}) = \{x \mid x \text{ is odd}\}$

d) $f(x) = x^2$

Injective:

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$\pm x_1 = \pm x_2$, $\Rightarrow$ Not injective $(x_1 = -x_2)$

Surjective?:

$y = x^2$

$x = \sqrt{y}$ not surjective ($\sqrt{3}$ is not an integer).

range $f(\mathbb{Z}) = \{x^2 \mid x \in \mathbb{Z}^+\}$

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f) $f(x) = x^3$

Injective? :

$f(x_1) = f(x_2)$

$x_1^3 = x_2^3$

$\underline{x_1 = x_2} \Rightarrow$ Injective!

Surjective? :

$y = x^3$

$x = \sqrt[3]{y} \Rightarrow$ not surjective! $(\sqrt[3]{3} \notin \mathbb{Z})$.

range $= \{x^3 \mid x \in \mathbb{Z}\}$

Exercise 6

Exercises 5.3

3) $g: \mathbb{R} \to \mathbb{R}$

b) $g(x) = 2x - 3$

Injective? :

$g(x_1) = g(x_2)$

$2x_1 - 3 = 2x_2 - 3$

$\underline{x_1 = x_2}$ => Injective!

Surjective? :

$x = \frac{y-3}{2}$ => Surjective!

Since the function is defined for all real and rational numbers, this is surjective.

d) $g(x) = x^2$

Injective?

$\pm x_1 = -x_2$ => Not Injective!

Surjective? :

$x = \sqrt{y}$ Not surjective.

cannot have negative numbers.

$\text{range} = \{x^2 \mid x \in \mathbb{R}\}$

f) $g(x) = x^3$

Injective? :

$x_1 = x_2$ => Injective!

Surjective? :

$x = \sqrt[3]{y}$ => yes! Surjective.

Exercise 7

Exercises 5.3

9)

$A = \{1,2,3,4\}$, $B = \{1,2,3,4,5,6\}$

a) there are 6^4 functions. (n^m)

There are $\frac{n!}{(n-m)!} = \frac{6!}{2!}$ Injective functions.

There are 0 Surjective functions because $|A| \not\geq |B|$

b) ($A = \{1,2,3,4,5,6\}$, $B = \{1,2,3,4\}$)

There are 4^6 functions.

$|A| \not\leq |B|$, so there are no injective functions.

There are $4! \cdot S(6,4)$ Surjective functions.

this is $\sum_{k=0}^{4} (-1)^k \binom{4}{4-k} (4-k)^6 = \underline{1560}$