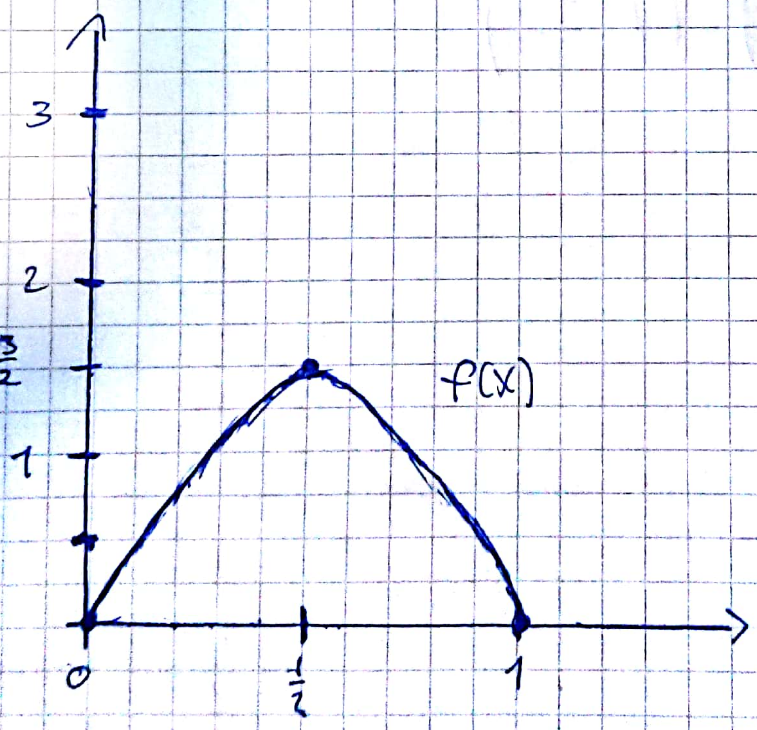
Øving 4 Sander Lindberg.

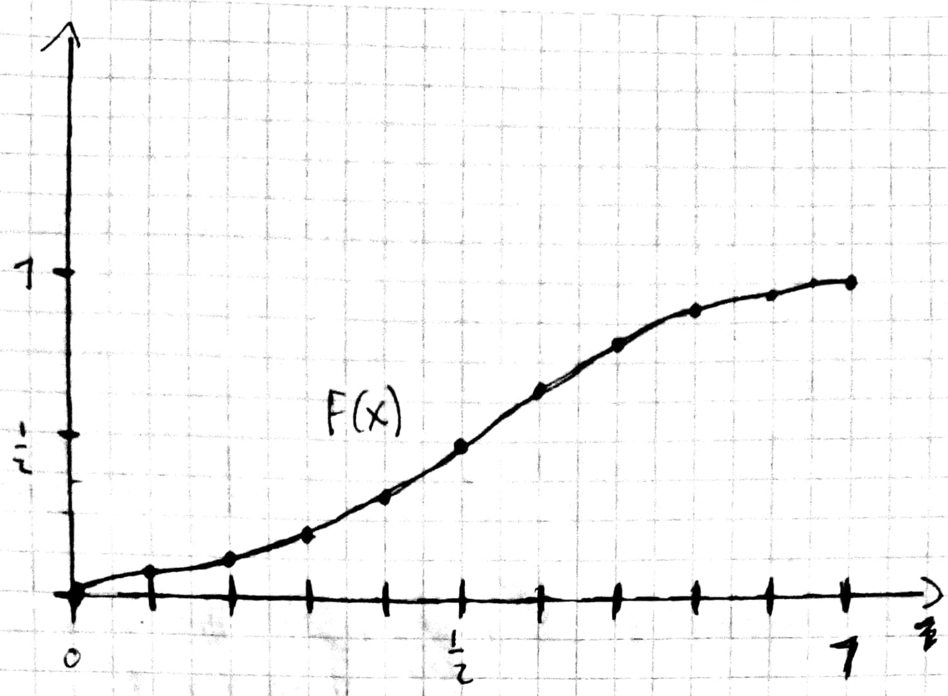
ST0103 2012 H

Oppgave 7)

a) $F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{x} 6t(1-t)\,dt$

$= 6\int_{0}^{x} t - t^2\,dt = 6\left[\frac{t^2}{2} - \frac{t^3}{3}\right]_0^x = \underline{\underline{3x^2 - 2x^3}}$





b) $P(X > \frac{1}{2}) = 1 - P(X \le \frac{1}{2}) = 1 - F(\frac{1}{2}) = 1 - \frac{1}{2} = \frac{1}{2}$

$$P(X > \tfrac{3}{4} \mid X > \tfrac{1}{2}) = \frac{P(X \ge \frac{3}{4} \cap X > \frac{1}{2})}{P(X > \frac{1}{2})}$$

$$P(X > \tfrac{3}{4} \cap X > \tfrac{1}{2}) = P(X > \tfrac{3}{4})$$

$$= 1 - P(X \le \tfrac{3}{4}) = 1 - F(\tfrac{3}{4})$$

$$= \frac{5}{32}$$

$$P(X > \tfrac{3}{4} \mid X > \tfrac{1}{2}) = \frac{\frac{5}{12}}{\frac{1}{2}} = \frac{5}{16}$$

c) $E(X) = \int_0^1 x(6x(1-x))\,dx = 6\int_0^1 x^2 - x^3\,dx = 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$

$$\mathrm{Var}[X] = E[X^2] - E[X]^2$$

$$= \int_0^1 x^2(6x(1-x))\,dx - \left(\tfrac{1}{2}\right)^2 = 6\int_0^1 x^3 - x^4\,dx - \left(\tfrac{1}{2}\right)^2$$

$$= 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 - \left(\tfrac{1}{2}\right)^2 = \frac{3}{10} - \left(\tfrac{1}{2}\right)^2 = \frac{1}{20}$$

Oppgave 3.36)

80 kunder

7 misfornøyde

12 tilfeldig utvalgt

Her blir det antall ~~mulige~~ gunstige delt på antall ~~gunstige~~ mulige

P(nøyaktig en misfornøyd) = $\frac{\binom{7}{1}\binom{73}{11} \text{ gunstige}}{\binom{80}{12} \text{ mulige}} = \underline{\underline{0,413}}$

P(Mer enn 2 misfornøyde) = P(X > 2)

$= 1 - P(X \leq 2)$

$= 1 - \frac{\binom{7}{0}\cdot\binom{73}{12}}{\binom{80}{12}} - \frac{\binom{7}{1}\binom{73}{11}}{\binom{80}{12}} - \frac{\binom{7}{2}\binom{73}{10}}{\binom{80}{12}}$

$= \underline{\underline{0,065}}$

Oppgave 3.47)

Igjen anntall gunstige delt på mulige:

$$\left.\begin{array}{l} \text{gunstige} = P(365, 50) \\ \text{mulige} = 365^{50} \end{array}\right\} \quad \frac{P(365,50)}{365^{50}} \approx 0,03$$

$$P(\text{minst } 2) = 1 - P(\text{ingen}) = 1 - 0,03 = \underline{0,97}$$

Oppgave 4.1)

Utfallsrom = $\{KK, KM, MK, MM\}$

Verdimengde (Hvor mange kron) $V_x = \{0, 1, 2\}$

Sannsynlighetsfordeling:

x	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$$E[X] = \sum_x x f(x) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{2}{4} + 2 \cdot \frac{1}{4} = \underline{\underline{1}}$$

Oppgave 4.2)

$V_X = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

Jeg kan se at fordelingen er symmetrisk.

Det vil si at $E[X]$ ligger i midten, som er

$$\frac{10+11}{2} = \underline{\underline{10,5}}$$

Siden fordelingen er symmetrisk er også halvparten av fordelingen på venstre siden av midten og den andre halvparten på høyre. Det vil si at $P(X<11)$ (som er den ene halvparten) er $\underline{\underline{\frac{1}{2}}}$

Oppgave 4.3)

$V_y = \{0, 1, 2, 3\}$

Fordeling:

y	0	1	2	3
$P(Y=y)$	$\left(\frac{5}{6}\right)^3$	$\frac{3}{6}\cdot\left(\frac{5}{6}\right)^2$	$\frac{5}{2}\cdot\left(\frac{1}{6}\right)^2$	$\left(\frac{1}{6}\right)^3$

Oppgave 4.4)

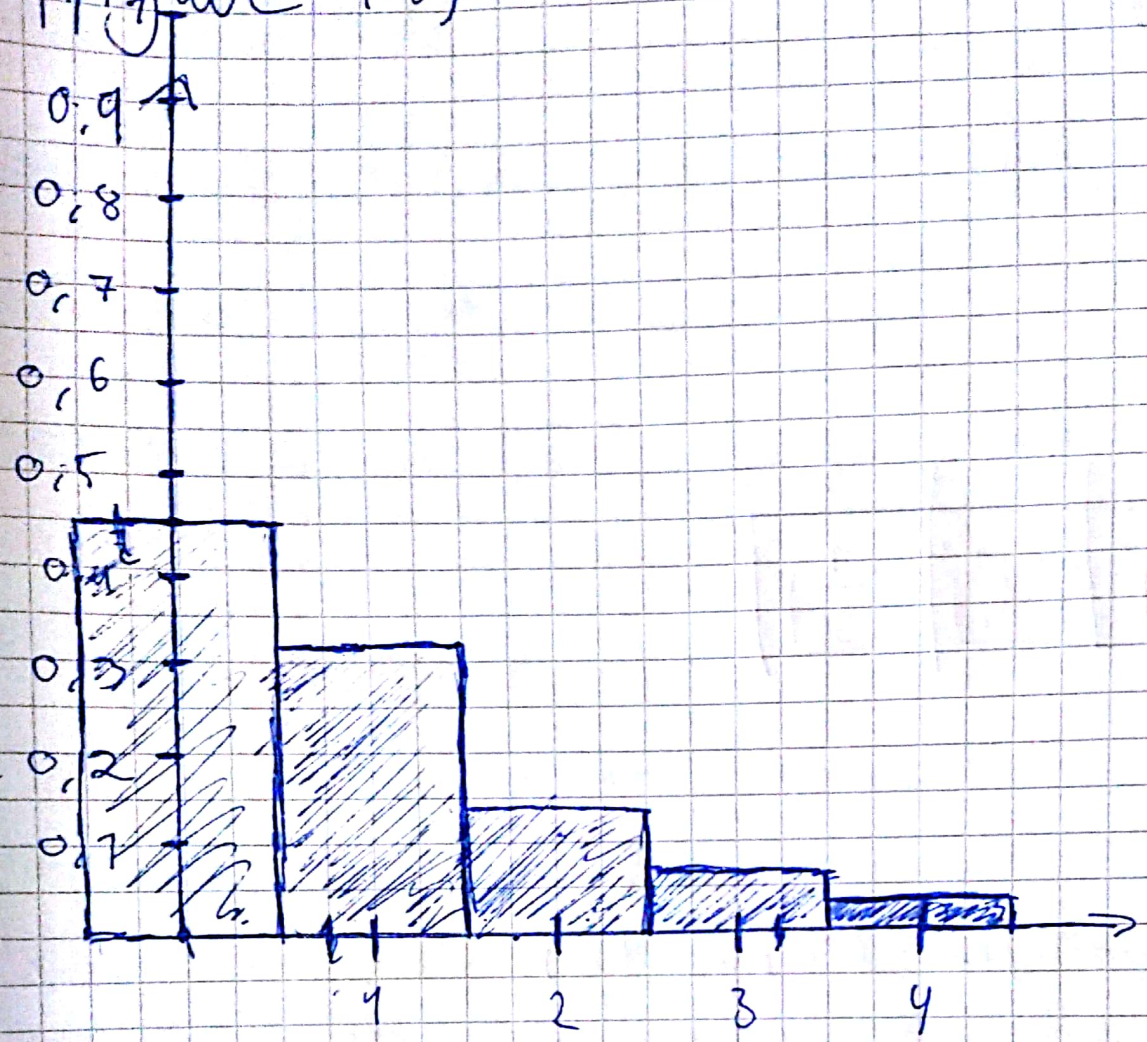
Z = antall trekninger (1 nr uke)

$V_Z = [0, \infty\rangle$

$P(Z=0) = \frac{7}{34}$

D.v.s $P(Z=z) = \left(1 - \frac{7}{34}\right)^{z-1} \cdot \frac{7}{34}$

Oppgave 4.5)



$$E[X] = \sum_x x f(x) = 0 \cdot 0{,}45 + 1 \cdot 0{,}37 + 2 \cdot 0{,}14 + 3 \cdot 0{,}07 + 4 \cdot 0{,}03$$
$$= \underline{\frac{23}{25}}$$

$$E[X^2] = \sum_x x^2 f(x) = \frac{99}{50}$$

$$Var[X] = E[X^2] - E[X]^2 = \frac{99}{50} - \left(\frac{23}{25}\right)^2 = \underline{\underline{\frac{1417}{1250}}}$$

$$P(X > 2) = 1 - P(X \leq 2) = 1 - 0{,}45 - 0{,}37 - 0{,}14 = \underline{\underline{\frac{1}{10}}}$$

$$P(X < 2) = P(X \leq 1) = 0{,}45 + 0{,}37 = \underline{\underline{\frac{14}{25}}}$$

Scanned by CamScanner

Oppgave 4.6)

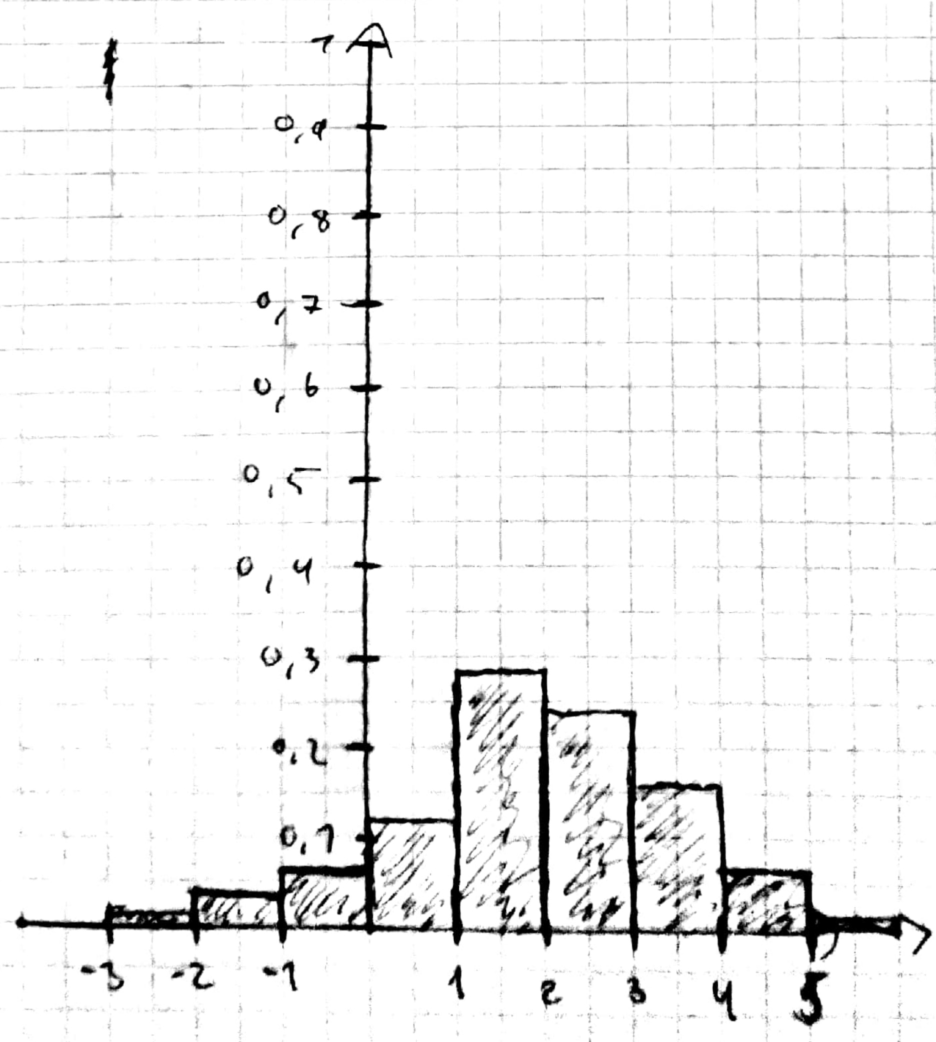
y	-3	-2	-1	0	1	2	3	4	5
F(y)	0,04	0,05	0,11	0,23	0,51	0,75	0,91	0,98	1

$E[y] = \sum_y y f(y) = \frac{29}{20}$

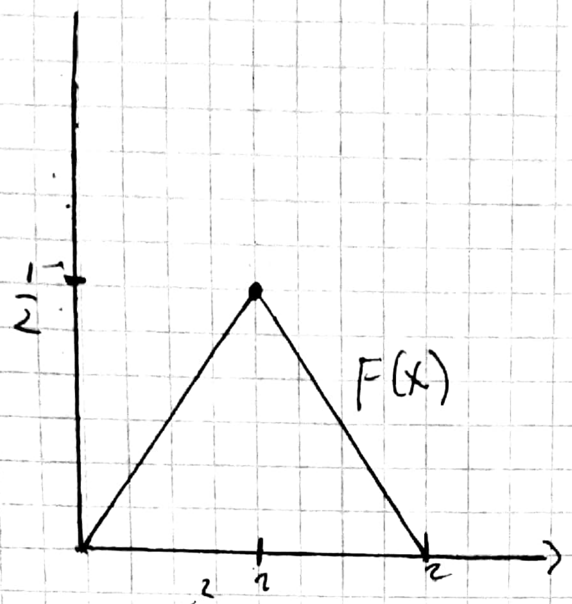
$E[y^2] = \sum_y y^2 f(y) = \frac{467}{100}$

$Var[y] = \frac{467}{100} - \left(\frac{29}{20}\right)^2 = \underline{\underline{\frac{257}{100}}}$

$P(0 < y \leq 4) = F(4) - F(0) = 0,75$



Oppgave 4.16)



$$F(x) = \begin{cases} \frac{x^2}{2}, & 0 \leq x \leq 1 \\ \frac{\cancel{(2-x)}^2}{2}, & 1 \leq x \leq 2 \end{cases}$$

$$P(X < \tfrac{1}{2}) = F(\tfrac{1}{2}) = \frac{0{,}5^2}{2} = \frac{1}{8}$$

$$P(\tfrac{3}{4} < X < \tfrac{3}{2}) = \int_{3/4}^{1} x\,dx + \int_{1}^{1{,}5} (2-x)\,dx = \underline{\underline{\frac{19}{32}}}$$

$$E[X] = \int_0^1 x \cdot x\,dx + \int_1^2 x(2-x)\,dx = \underline{\underline{1}}$$

$$E[X^2] = \int_0^1 x^2 \cdot x\,dx + \int_1^2 x^2(2-x)\,dx = \frac{7}{6}$$

$$V[X] = E[X^2] - E[X]^2 = \frac{7}{6} - 1 = \underline{\underline{\frac{1}{6}}}$$