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Gruppe 3

Oppgave 1

$f(x) = x$

$g(x) = x^2$

$f(x) = g(x)$

$x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \quad , x = 1.$

arealet mellom er gitt ved $\int_0^1 (f(x) - g(x))\,dx$

Som gir $\int_0^1 x - x^2\,dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$

$$\left(\frac{1^2}{2} - \frac{1^3}{3} - \left(\frac{0^2}{2} - \frac{0^3}{3}\right)\right) = \frac{1}{2} - \frac{1}{3} = \underline{\underline{\frac{1}{6}}}$$

Oppgave 2

a) $B(t) = \int \frac{d}{dt} B(t)\, dt$

$B(t) = \int \cos\left(\frac{\pi}{12}t\right) dt$

Setter $u = \frac{\pi}{12}t$

$\frac{du}{dt} = \frac{\pi}{12} \Rightarrow dt = \frac{12du}{\pi}$

$\Rightarrow \frac{12}{\pi}\int \cos(u)\,du = \frac{12}{\pi}\sin\left(\frac{\pi}{12}t\right) + C$

Setter inn $B(0) = 100$ for å finne C:

$\frac{12}{\pi}\sin\left(\frac{\pi}{12}\cdot 0\right) + C = 100$

$\underline{C = 100}$

$$\underline{\underline{B(t) = \frac{12}{\pi}\sin\left(\frac{\pi}{12}t\right) + 100}}$$

B) $B(24) = \frac{12}{\pi}\sin\left(\frac{\pi}{12}\cdot 24\right) + 100$

$= \frac{12}{\pi}\sin(2\pi) + 100 = \underline{\underline{100}}$

c) gjennomsnitt $= \frac{1}{b-a}\int_a^b B(t)dt$

$\frac{1}{24-0}\int_0^{24} B(t)\,dt$

$\frac{1}{24}\int_0^{24} \frac{12}{\pi}\sin\left(\frac{\pi}{12}t\right) + 100\, dt = \frac{1}{24}\left[-\cos\left(\frac{\pi}{12}t\right) + 100t\right]_0^{24}$

$= \frac{1}{24}\left(-\cos(2\pi) + 100\cdot 24 - \left(-\cos(0) + 100\cdot 0\right)\right)$

$= \frac{1}{24}\left(-1 + 2400 + 1 + 0\right) = \frac{2400}{24} = \underline{\underline{100}}$

$B(t)_{avg} = \underline{\underline{100}}$

Oppgave 3

$$\frac{d}{dx}\int_{\sqrt{x}}^{x^2} u - \frac{1}{\sqrt{u}}\,du$$

Leibniz regel : $\frac{d}{dx}\int_{h(x)}^{g(x)} f(y)\,dy = f(g(x))\,g'(x) - f(h(x))\cdot h'(x)$

$g(x) = x^2$

$h(x) = \sqrt{x}$

Deriverer begge:

$g'(x) = 2x$

$h'(x) = \frac{1}{2\sqrt{x}}$

$f(g(x))\,g'(x) = \left(x^2 - \frac{1}{\sqrt{x^2}}\right)(2x) = \underline{2x^3 - 2}$

$f(h(x))\,h'(x) = \left(\sqrt{x} - \frac{1}{\sqrt{\sqrt{x}}}\right)\cdot \frac{1}{2\sqrt{x}} = \frac{1}{2} - \frac{1}{2x^{\frac{3}{4}}}$

$$\frac{d}{dx}\int_{\sqrt{x}}^{x^2} u - \frac{1}{\sqrt{u}}\,du = 2x - 2 - \frac{1}{2} + \frac{1}{2x^{\frac{3}{4}}} = \underline{\underline{2x - \frac{5}{2} - \frac{1}{2x^{\frac{3}{4}}}}}$$