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Gruppe 3.

Oppgave 7.

a)

$$\int_1^3 2 - x + 2\sin(x) - x\cos(x)\,dx$$

Midtpunktreglen:

$$\int_a^b f(x)dx \approx M_n = \delta\left(f(c_1) + f(c_2) + \dots + f(c_n)\right)$$

$$\delta = \frac{b-a}{n}, \quad c_k = a + \delta\left(k - \tfrac{1}{2}\right)$$

Oppgaven gir $a = 1$, $b = 3$ og $N = 4$

Dette gir

$$\delta = \frac{3-1}{4} = \frac{1}{2}$$

$$c_k = 1 + \tfrac{1}{2}\left(k - \tfrac{1}{2}\right), \quad c_1 = \frac{5}{4}$$

$$c_2 = \frac{7}{4}$$

$$c_3 = \frac{9}{4}$$

$$c_4 = \frac{11}{4}$$

$$f\left(\tfrac{5}{4}\right) = 2 - \tfrac{5}{4} + 2\sin\left(\tfrac{5}{4}\right) - \tfrac{5}{4}\cos\left(\tfrac{5}{4}\right) \approx 2,254$$

$$f\left(\tfrac{7}{4}\right) \approx 2,53$$

$$f\left(\tfrac{9}{4}\right) \approx 2,72$$

$$f\left(\tfrac{11}{4}\right) \approx 2,55$$

legger disse sammen og får $\underline{10.06}$

$$M_n = \tfrac{1}{2} \cdot 10.06 \approx \underline{\underline{5}}$$

Oppgave 1

b) $f'(x) = 2\cos(x) - (\cos(x) - x\sin(x)) - 1$

$f'(x) = 2\cos(x) - \cos(x) + x\sin(x) - 1$

$= \cos(x) + x\sin(x) - 1$

$f''(x) = -\sin(x) + (\sin(x) + x\cos(x))$

$f''(x) = \underline{\underline{+x\cos(x)}}$

c) Siden $x \in [1, 3]$ vil $K \geq |\cos(1)|$

eller $K \geq |3\cos(3)|$:

$|3\cos(3)| \leq K \Rightarrow |-2,97| \leq K \Rightarrow K \geq 2,97$

$|\cos(1)| \leq K \Rightarrow |0,54| \leq K \Rightarrow K \geq 0,54$

K er derfor større eller lik $\underline{2.97}$

d) $\left| \int_1^3 f(x)dx - M_4 \right| \leq \frac{K(b-a)^3}{24n^2}$

$\Downarrow$

$\left| \int_1^3 f(x)dx - 5 \right| \leq \frac{2.97(3-1)^2}{24 \cdot 4^2} = \underline{\underline{\frac{99}{1600}}}$

e) $\frac{K(3-1)^2}{24n^2} = \frac{1}{100}$

$4K = \frac{24n^2}{100}$

$\frac{400K = 24n^2}{24}$

$n^2 = \frac{50K}{3}$, setter $K = 2,97$

$n = \sqrt{\frac{50 \cdot 2,97}{3}} = 7,03.$

Må ta 8 steg for å være sikker på en

Feil $\leq \frac{1}{100}$

Opgave 2:

a) $$\int_0^1 x^2 + \sqrt{x} + 2x \, dx = \left[\frac{1}{3}x^3 + \frac{2}{3}x^{\frac{3}{2}} + x^2\right]_0^1$$
$$= \frac{1}{3} + \frac{2}{3} + \frac{3}{3} = \underline{\underline{2}}$$

b) $$\int_{-1}^{1} \frac{e^x - e^{-x}}{2} \, dx = \frac{1}{2}\int_{-1}^{1} e^x - e^{-x} \, dx$$
$$= \frac{1}{2}\left[e^x + e^{-x}\right]_{-1}^{1} =$$
$$= \frac{1}{2}\left(\cancel{e^1} + \cancel{e^{-1}} - \cancel{e^{-1}} + \cancel{e^1}\right) = \underline{\underline{0}}$$

c) $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x), \quad f(x) = \begin{cases} \sin(x), & x \leq 0 \\ 4x, & x > 0 \end{cases}$$

$$\int_{-\frac{\pi}{2}}^{0} \sin(x) \, dx + \int_0^{\frac{\pi}{2}} 4x \, dx = \left[-\cos(x)\right]_{-\frac{\pi}{2}}^{0} + \left[2x^2\right]_0^{\frac{\pi}{2}}$$

$$\left(-\cos(0) + \cos\left(\frac{\pi}{2}\right)\right) + \left(2 \cdot \left(\frac{\pi}{2}\right)^2 + 0\right) \approx \underline{\underline{3.935}}$$

opgave 3:

$$\int_0^{\frac{\pi}{2}} e^{ax} + \sin(bx)\,dx = \left[\frac{e^{ax}}{a} \cancel{+} - \frac{\cos(bx)}{b}\right]_0^{\frac{\pi}{2}}$$

$$= \left(\frac{e^{\frac{a\pi}{2}}}{a} - \frac{\cos(\frac{b\pi}{2})}{b} - \left(\frac{e^0}{a} - \frac{\cos(0)}{b}\right)\right)$$

$$= \frac{e^{\frac{a\pi}{2}}}{a} - \frac{\cos(\frac{b\pi}{2})}{b} - \frac{1}{a} + \frac{1}{b}$$

$$= \frac{e^{\frac{a\pi}{2}} - 1}{a} - \frac{\cos(\frac{b\pi}{2}) + 1}{b}$$