Homework set 4

Exercise 1
Exercises 15.7

1)

c) $wx + \bar{y} + yz$

$= 1 \cdot 1 + 1 + 0 \cdot 0 = 1 + 1 + 0 = \underline{1}$

d) $(wx + y\bar{z}) + w\bar{y} + \overline{(w + y)(\bar{x} + y)}$

$= (wx + y\bar{z}) + w\bar{y} + (\bar{w} + \bar{y})(x + \bar{y})$

$= (1 \cdot 1 + 0 \cdot 1) + 1 \cdot 1 + (0 + 1)(1 + 1)$

$= 1 + 0 + 1 + 1 \cdot 1 = \underline{1}$

2) a) $x + xy + w = x(y + 1) + w = x + w$

$= 1 + w.$ ~~~~~~~~ This expression is always 1.

b) ~~~~~~~~ $xy + w = y + w$

This expression is 1 when y or w is 1.

That is : $(y = 1, w = 1) \cup (y = 1, w = 0)$

$\cup (y = 0, w = 1)$.

Exercise 2

Exercises 15.7

11)

a) $xy + (x+y)\bar{z} + y$

$= y + xy + \bar{z}(x+y)$

$= y + \bar{z}x + \bar{z}y$

$= y + \bar{z}y + \bar{z}x$

$= \underline{\underline{y + \bar{z}x}}$

12)
$$X + \bar{X}y = 0 \implies X = 0 \cap y = 0$$
$$\bar{X}y = \bar{X}Z = 1 \cdot 0 = 1 \cdot Z \implies Z = 0$$
$$\bar{X}y + \bar{X}\bar{Z} + Zw = \bar{Z}w$$
$$= 1 \cdot 0 + 1 \cdot 1 + 0 \cdot w = 1 \cdot w \implies w = 1$$

$$\underline{X = y = Z = 0 \quad , \quad w = 1}$$

Exercise 3

i) $(x + \bar{y}) \cdot (x + \bar{z}) \cdot (y + \bar{x})$

ii) $(x + y + \bar{z}) \cdot (x + \bar{y} + z)$

iii) $x + y + (x \cdot 1 \cdot (z + 7))$

$\quad = x + y + (x (z + 7))$

$\quad = x + y + (xz + x)$

Exercise 4:

i) $x \bar{x} y z = x \bar{x} \cdot y z = 1 \cdot 0 \cdot y z = 0 \cdot y z = \underline{\underline{0}}$

ii) $x y \cdot z y = y \cdot y (x z) \quad | \quad y \cdot y = y$

$\quad y (x z) = \underline{\underline{y \times z}}$

Exercise 5

Exercises 3.2

4)

a) i) True   iv) True

  ii) False   v) True

  iii) False   vi) False

  b) i) E   iv) D

    ii) B   v) ~~ZEzzn~~ Z - A

    iii) D   vi) E

Exercise 6

Exercises 3.2

17)

b) ~~$(A \cap B) \cup (B \cap ((A (B \cup D)))$~~

$(A \cap B) \cup (A \cap B \cap \bar{C} \cap D) \cup (\bar{A} \cap B)$

$= (A \cap B) \cup (\bar{A} \cap B)$ (Absorption Law

$$A = (A \cap B)$$
$$B = (\bar{C} \cap D))$$

$= (A \cup \bar{A}) \cap B$

$= \cancel{~~~~} \cup \cap B = \underline{\underline{B}}$

d) $\bar{A} \cup \bar{B} \cup (A \cap B \cap \bar{C})$

$= \overline{A \cap B} \cup (A \cap B \cap \bar{C})$

$= (\overline{(A \cap B)} \cup (A \cap B)) \cap ((\overline{A \cap B}) \cup \bar{C})$

    ↗

This is always true

$= \overline{(A \cap B)} \cup \bar{C} = \underline{\bar{A} \cup \bar{B} \cup \bar{C}}$

Exercise 7

$$(P \wedge (\neg S \vee q \vee \neg q)) \vee ((S \vee t \vee \neg S) \wedge \neg q)$$

$$\Downarrow$$

$$(P \wedge (\neg S \vee T_0)) \vee ((\neg S \vee t \vee T_0) \wedge \neg q)$$

$$\Downarrow$$

$$(P \wedge T_0) \vee (\neg q \wedge T_0)$$

$$\Downarrow$$

$$\underline{\underline{P \vee \neg q}}$$