

# Homework Set 2

## Exercise 1

### Exercises 2.2

19.

$$c) ((\neg P \vee \neg Q) \rightarrow (P \wedge Q \wedge R)) \Leftrightarrow P \wedge Q$$

$$(\neg P \vee \neg Q) \rightarrow (P \wedge Q \wedge R) \quad \begin{array}{l} \text{Reasons} \\ \hline \text{Statement} \end{array}$$

$$\neg(\neg P \vee \neg Q) \vee (P \wedge Q \wedge R) \quad S \rightarrow t \Leftrightarrow \neg S \vee t$$

$$(\neg\neg P \wedge \neg\neg Q) \vee (P \wedge Q \wedge R) \quad \text{De Morgan}$$

$$(P \wedge Q) \vee (P \wedge Q \wedge R) \quad \text{Law of double negation}$$

$$(P \wedge Q) \quad \text{absorption laws}$$

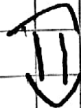
$$(P = (P \wedge Q), Q = R)$$

20.

$$(P \wedge (\neg R \vee Q \vee \neg Q)) \vee ((R \vee t \vee \neg R) \wedge \neg Q)$$



$$(P \wedge (\neg R \vee T_0)) \vee ((T_0 \vee t) \wedge \neg Q)$$



$$(P \wedge T_0) \vee (\neg Q \wedge T_0)$$



$$(P \vee \neg Q)$$

# Exercise 3

## Exercises 2.3

2. b)

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \neg q$	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

d)

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$p \vee q$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$(p \vee q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

$$\underline{((p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)}$$

T  
T  
T  
T  
T  
T  
T  
T

# Exercise 4

## exercises 2.3

8.

Steps	Reasons
1) $\neg S \wedge \neg U$	Premise
2) $\neg U$	1) + rule of conjunctive simplification
3) $\neg U \rightarrow \neg t$	Premise
4) $\neg t$	2) + 3) + Modus ponens
5) $\neg S$	1) + rule of conjunctive simplification
6) $\neg S \wedge \neg t$	4) + 5) + rule of conjunction
7) $r \rightarrow (S \vee t)$	Premise
8) $\neg(S \vee t) \rightarrow \neg r$	7) + $(r \rightarrow (S \vee t)) \Leftrightarrow (\neg(S \vee t) \rightarrow \neg r)$
9) $(\neg S \wedge \neg t) \rightarrow \neg r$	8) + De Morgan
10) $\neg r$	6) + 9) + Modus Ponens
11) $(\neg P \vee q) \rightarrow r$	Premise
12) $\neg r \rightarrow \neg(\neg P \vee q)$	11) + $((\neg P \vee q) \rightarrow r) \Leftrightarrow (\neg r \rightarrow \neg(\neg P \vee q))$
13) $\neg r \rightarrow (P \wedge \neg q)$	12) + De Morgan + double Negation
14) $P \wedge \neg q$	10) + 13) + Modus Ponens
15) $\therefore P$	14) + rule of conjunctive simplification.

~~Exer~~

Exercise 5

Exercises 2.4

8)

e) True

f) The solution for  $P(x)$  ( $x^2 - 8x + 15 = 0$ )  
 $= x = 5 \vee x = 3$ .

Therefore  $\forall x (\neg q(x) \rightarrow \neg r(x))$  is  
true, because  $\neg q(x)$  is even numbers  
and the solutions for  $r(x)$  is odd  
numbers.

g) True.

h) False.  $P(4) = -1 \not\rightarrow r(x)$

$q(x)$  is odd means  $q(x)$  can be  $-1$   
which does not ~~imply~~ imply  $R(x)$ .



Exercise 6

Exercises 2.4

2)

a)

~~ii)  $\forall x \exists y P(x, y) = \text{False}$  for  $y \neq 1$  or  $y \neq 4$~~

iii) True

iv)  $\forall x P(x, 0) = \text{False}$  if  $x = 0$

vi)  $\forall y \exists x P(x, y) = \text{True}$

vii)  $\exists y \forall x P(x, y) = \text{False}$  if  $x = 0$

viii)  $\forall x \forall y [P(x, y) \wedge P(y, x) \rightarrow (x = y)]$   
= False. For example -3 and 3.

c)

i)  $\forall x \exists y P(x, y) = \text{True}$  (if 0 is not positive)

ii)  $\forall y \exists x P(x, y) = \text{True}$  (if 0 is not positive)

iii)  $\exists x \forall y P(x, y) = \text{True}$  ( $x = 1$ )

iv)  $\exists y \forall x P(x, y) = \text{False}$  (let  $x = 2y$ )