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Gruppe 3. Bit

Oppgave 1:

$$\lim_{x \to 0} \frac{\sqrt{x^2+3} - \sqrt{3}}{x} \quad \Big| \cdot \sqrt{x^2+3} + \sqrt{3}$$

$$\lim_{x \to 0} \frac{\left(\sqrt{x^2+3} - \sqrt{3}\right)\left(\sqrt{x^2+3} + \sqrt{3}\right)}{x\left(\sqrt{x^2+3} + \sqrt{3}\right)}$$

$$= \lim_{x \to 0} \frac{x^{\cancel{2}}}{\cancel{x}\left(\sqrt{x^2+3} + \sqrt{3}\right)}$$

$$= \lim_{x \to 0} \frac{x}{\sqrt{x^2+3} + \sqrt{3}} = \frac{0}{\sqrt{3} + \sqrt{3}} = \underline{\underline{0}}$$

Oppgave 2:

a)

$$\lim_{x\to\infty} \frac{x^2+2x+3}{5x^2+3}$$

$$\lim_{x\to\infty} \frac{1+\frac{2}{x}+\frac{3}{x}}{5+\frac{3}{x}} = \frac{1+0+0}{5+0} = \underline{\underline{\frac{1}{5}}}$$

b) $$\lim_{x\to\sqrt{3}} \left(\sin(x^2-3)+2e^{x^2}-3x^4+2\right)$$

$$\lim_{x\to\sqrt{3}} \left(\sin((\sqrt{3})^2-3)+2e^{(\sqrt{3})^2}-3(\sqrt{3})^4+2\right)$$

$$\lim_{x\to\sqrt{3}} \left(0+2e^3-3\cdot 9+2\right) = \underline{\underline{2e^3-25}}$$

c) $$\lim_{x\to\pi} \left(\sin(x)\cdot\cos\left(\frac{1}{x-\pi}\right)\right)$$

$$-1 \le \cos\left(\frac{1}{x-\pi}\right) \le 1$$

D.V.S:

$$-\sin(x) \le \sin(x)\cos\left(\frac{1}{x-\pi}\right) \le \sin(x)$$

og $$\sin(x) \ge -\sin(x)\cos\left(\frac{1}{x-\pi}\right) \ge -\sin(x)$$

$$\lim_{x\to\pi} \sin(x) = 0$$

Skviseteoremet sier da at $\lim_{x\to\pi} \cos\left(\frac{1}{x-\pi}\right) = \underline{0}$

$\lim_{x\to\pi} \left(\sin(x)\cos\left(\frac{1}{x-\pi}\right)\right)$ er derfor $= \underline{\underline{0}}$

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Oppgave 3:

~~Kon~~ $f(x) = \begin{cases} x^2 + 4, & x \geq 0 \\ \frac{\sin(kx)}{x}, & x < 0 \end{cases}$

for at den skal være kontinuerlig i 0

må $\lim_{x \to 0^+} (x^2+4) = \lim_{x \to 0^-} \frac{\sin(kx)}{x}$.

finner $\lim_{x \to 0^+} (x^2+4)$:

$\lim_{x \to 0^+} (x^2+4) = 0^2 + 4 = \underline{\underline{4}}$

$\lim_{x \to 0^-} \frac{\sin(kx)}{x}$, Bruker L'Hôpital:

$\lim_{x \to 0^-} \frac{[\sin(kx)]'}{(x)'} = \frac{k\cos(kx)}{1}$

$\lim_{x \to 0^-} k\cos(k \cdot 0) = \underline{k}$

Siden $\lim_{x \to 0^+} (x^2+4) = \lim_{x \to 0^-} \frac{\sin(kx)}{x}$

Må $\underline{\underline{k = 4}}$