Homework set 9 Sander Lindberg

Exercise 1

Exercises 1 (suppl)

16)

b) $\left(\frac{x}{2}+y-3z\right)^5$

$\binom{3+5-1}{5} = \binom{7}{5} = \underline{\underline{21}}$

c) $\left(\frac{1}{2}+1-3\right)^5 = \left(-\frac{3}{2}\right)^5 = \underline{\underline{-\frac{243}{32}}}$

Exercise 2

Exercises 18

b) $x_1+x_2+x_3 \leq 6$

$\pm\ x_1+x_2+x_3 \leq k = \binom{3+k-1}{k} = \underline{\binom{k+2}{k}}$

$x_4+x_5 \leq 15-k$

$x_4+x_5+x_6 = 15-k \quad , x_4, x_5, x_6 \geq 0 , \quad 1 \leq i \leq 6$

$= \binom{3+15-k-1}{15-k} = \underline{\binom{17-k}{15-k}}$

Total number $= \underline{\underline{\sum_{k=0}^{6} \binom{17-k}{15-k}\binom{k+2}{k}}}$

Exercise 3

Exercises 28 b)

From $x=1$ to $x=5$ we can go 4 Horizontal moves

From $y=2$ to $y=9$, there are 7 vertical moves.

We can rearrange them in $\frac{11!}{4!\cdot 7!}$ ways.

$(11!) = (4+7)!$

$(4!) =$ Horizontal

$(7!) =$ vertical.

Now we introduce a diagonal move.

Since the diagonal goes one to the right and one up, we can get to $(5,9)$ wit 0, 1, 2, 3 or 4 diagonals.

$$\left.\begin{array}{l}
0: \frac{11!}{0!\,4!\,7!} \\[6pt]
1: \frac{10!}{1!\,6!\,3!} \\[6pt]
2: \frac{9!}{2!\,5!\,2!} \\[6pt]
3: \frac{8!}{3!\,4!\,1!} \\[6pt]
4: \frac{7!}{4!\,3!\,0!}
\end{array}\right\} \quad \sum_{i=0}^{4} \frac{(11-i)!}{i!\,(7-i)!\,(4-i)!} = \underline{\underline{2241}}$$

Exercise 4

Exercises 2 (Summ)

7) a) $(\neg P \lor \neg q) \land (F_0 \lor P) \land P$

b) $(\neg P \lor \neg q) \land (F_0 \lor P) \land P$

~~$(\neg P \lor \neg q) \land (F_0 \lor P) \land P$~~ ~~= DeMorgan~~

$(\neg P \lor \neg q) \land P \land P$ -> Identity Law.

$(\neg P \lor \neg q) \land P$ -> Idempotent Law.

$(P \land \neg P) \lor (P \land \neg q)$ -> Distributive Law.

$F_0 \lor (P \land \neg q)$ -> Inverse Law

$(P \land \neg q)$ -> Identity Law.

$\therefore$

Exercize 5

Exercise 2 (suppl)

10)

$$\left[(p \to q) \land [(q \land r) \to s] \land r\right] \to (p \to s)$$

1) $(q \land r) \to s$ Premise

2) $r \to (q \to s)$ $(q \land R) \to s \iff r \to (q \to s)$

3) r Premice

4) $q \to s$ 2) + 3) + Modus Ponens

5) $p \to q$ Premise

6) $p \to s$ 4) + 5) + law of syllogism.

Exercise 6

Exercises sect 3 (suppl)

4)

a) if we pick k elements from A $(0 \leq k \leq m)$ and $(r-k)$ elements from B $(0 \leq r-k \leq n)$, where $(m+n)$ is $\{x_1, \ldots, x_n\} \cup \{y_1, \ldots, y_m\}$ and $A = \{x_1, \ldots, x_n\}$ and $B = \{y_1, \ldots, y_m\}$

the number of subsets $A \cup B$ is $\binom{m+n}{r} = \binom{m}{0}\binom{n}{r} + \binom{m}{1}\binom{n}{r-1} + \binom{m}{2}\binom{n}{r-2} + \ldots + \binom{m}{r}\binom{n}{0} = \sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k}$

b) Show that $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$

I use what I found out in a).

replacing m with n and r with n, I get:

$$\binom{n+n}{n} = \binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k}$$

$\binom{n}{k}\binom{n}{n-k} = \binom{n}{k}^2$, therefore $\underline{\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2}$

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Exercise 7

Suppl sect. 3

9) $A, B, C \subseteq U$

To show that $(A \cap B) \cup C = A \cap (B \cup C)$

I suppose it is true, and $x \in C$.

this implies that $x \in (A \cap B) \cup C \Rightarrow x \in A \cap (B \cup C) \subseteq A$.

$x \in A$ and $C \subseteq A$. I then suppose $C \subseteq A$:

$y \in (A \cap B) \cup C$

$y \in A \cap B$ or $y \in C$

$y \in (A \cap B) \cup (A \cap C) \Rightarrow y \in A \cap (B \cup C)$.

Since $C \subseteq A$ and $y \in C \Rightarrow y \in A$.

$y \in A \cap (B \cup C) \rightarrow (A \cap B) \cup C \subseteq A \cap (B \cup C)$.

$z \in A \cap (B \cup C)$.

$(A \cap B) \cup (A \cap C) \subseteq (A \cap B) \cup C$.

This implies that $(A \cap B) \cup C = A \cap (B \cup C)$

Exercise 8.

Suppl. sect 4

6)

d) $S_n = \frac{(n+1)!-1}{(n+1)!}$

Basis step $(n=1)$

$$\frac{1}{2!} = \frac{(1+1)!-1}{(1+1)!} = \frac{2!-1}{2!} = \frac{1}{2!}$$

assuming it holds for $n=k$ and ~~k~~ $n=k+1$:

$$S_k + \frac{k+1}{(k+2)!} = \frac{(k+2)!-1}{(k+2)!}$$

$$\frac{(k+1)!-1}{(k+1)!} + \frac{k+1}{(k+2)!} \overset{?}{=}$$

$$= \frac{(k+2)!\left((k+1)!-1\right) + (k+1)!(k+1)}{(k+2)!\,(k+1)!}$$

$$= \frac{(k+2)!(k+1)! - (k+2)! + (k+1)!(k+1)}{(k+2)!\,(k+1)!}$$

$$= \frac{(k+2)!(k+1)! - (k+2)(k+1)! + (k+1)!(k+1)}{(k+2)!\,(k+1)!}$$

$$= \frac{(k+1)!\left((k+2)! - (k+2) + (k+1)\right)}{(k+2)!\,(k+1)!} = \frac{(k+2)!-1}{(k+2)!}$$

q.E.D.

Exercise 9

Suppl. sect 4

7)

b) Basis step $(n=0)$:

$0^3 + (1)^3 + (2)^3 = 9$ which is divisable by 9.

assuming it holds for $n=k$ and $n=k+1$:

$(k+1)^3 + (k+2)^3 + (k+3)^3$

$(k+1)^3 + (k+2)^3 + (k^3 + 9k^2 + 27k + 27)$

$(k^3 + (k+1)^3 + (k+2)^3) + (9(k^2 + 3k + 3))$

The first summand is divisable by 9, because of the hypothesis.

The second summand is also divisable by 9, because it gets multiplied by 9.

Therefore $(k+1)^3 + (k+2)^3 + (k+3)^3$ is divisable by 9. q.E.D