Homework
Set 2

Exercise 1
Exercises 2.2

19.

c) $((\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)) \Leftrightarrow p \wedge q$

Statement	Reasons
$(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)$	
$\neg(\neg p \vee \neg q) \vee (p \wedge q \wedge r)$	$s \rightarrow t \Leftrightarrow \neg s \vee t$
$(\neg\neg p \wedge \neg\neg q) \vee (p \wedge q \wedge r)$	De Morgan
$(p \wedge q) \vee (p \wedge q \wedge r)$	Law of double negation
$(p \wedge q)$	absorption laws $(p = (p \wedge q),\ q = r)$

20.

$$(p \wedge (\neg r \vee q \vee \neg q)) \vee ((r \vee t \vee \neg r) \wedge \neg q).$$

$$\Updownarrow$$

$$(p \wedge (\neg r \vee T_0)) \vee ((T_0 \vee t) \wedge \neg q)$$

$$\Updownarrow$$

$$(p \wedge T_0) \vee (\neg q \wedge T_0)$$

$$\Updownarrow$$

$$\underline{(p \vee \neg q)}$$

Exercise 3

Exercises 2.3

2.

b)

p	q	$\neg p$	$\neg q$	$p \to q$	$(p \to q) \wedge \neg q$	$((p \to q) \wedge \neg q) \to \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

d)

p	q	r	$p \to r$	$q \to r$	$p \vee q$	$(p \to r) \wedge (q \to r)$	$(p \vee q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

$((p \to r) \wedge (q \to r)) \to ((p \vee q) \to r)$
T
T
T
T
T
T
T
T

Exercise 4

exercises 2.3

8.

Steps	Reasons
1) $\neg s \land \neg u$	Premise
2) $\neg u$	1) + rule of conjuctive simplication
3) $\neg u \to \neg t$	Premise
4) $\neg t$	2) + 3) + Modus ponens
5) $\neg s$	1) + rule of conjuctive simplification
6) $\neg s \land \neg t$	4) + 5) + rule of conjunction
7) $r \to (s \lor t)$	Premise
8) $\neg(s \lor t) \to \neg r$	7) + $(r \to (s \lor t)) \Leftrightarrow (\neg(s \lor t) \to \neg r$
9) $(\neg s \land \neg t) \to \neg r$	8) + DeMorgan
10) $\neg r$	6) + 9) + Modus Ponens
11) $(\neg p \lor q) \to r$	Premise
12) $\neg r \to \neg(\neg p \lor q)$	11) + $((\neg p \lor q) \to r) \Leftrightarrow (\neg r \to \neg(\neg p \lor q$
13) $\neg r \to (p \land \neg q)$	12) + DeMorgan + double Negation
14) $p \land \neg q$	10) + 13) + Modus Ponens
15) $\therefore p$	14) + rule of Conjunctive Simplification.

~~Exer~~

Exercise 5

Exercises 2.4

8)

e) True

f) The solution for $P(x)$ $(x^2 - 8x + 15 = 0)$

$= x = 5 \vee x = 3$.

Therefore $\forall x\,(\neg q(x) \to \neg r(x))$ is true, because $\neg q(x)$ is even numbers and the solutions for $r(x)$ is odd numbers.

g) True.

h) False. $P(4) = -1 \not\to r(x)$

$q(x)$ is odd means $q(x)$ can be -1

which does not ~~that~~ imply $R(x)$.

Exercise 6
Exercises 2.4

12)

a)

~~ii) $\forall x P(1, y)$ = False. for $y \neq -1$ or $y \neq 1$~~

iii) True

iv) $\forall x P(x, 0)$ = False if $x = 0$

vi) $\forall y \exists x\ P(x, y)$ = True

vii) $\exists y \forall x\ P(x, y)$ = ~~If~~ false if $x = 0$

viii) $\forall x \forall y [P(x, y) \land P(y, x) \to (x = y)]$

= False. For example -3 and 3.

c)

i) $\forall x \exists y\ P(x, y)$ = True (if 0 is not positive)

ii) $\forall y \exists x\ P(x, y)$ = True (if 0 is not positive)

iii) $\exists x \forall y\ P(x, y)$ = True ($x = 1$)

iv) $\exists y \forall x\ P(x, y)$ = False (let $x = 2y$)