

Öving 7

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Öppgave 5.25)

$$P(Z < 0) = \underline{0,5}$$

$$P(Z < 1,22) = \underline{0,8888}$$

$$P(1,13 < Z < 2,45) = P(Z < 2,45) - P(Z < 1,13) = 0,9929 - 0,8708$$

$$~~P(2,87 < Z < 1,14) = P(Z < 1,14) - P(Z < 2,87) = 0,1227~~$$

$$P(-0,87 < Z < 7,71) = P(Z < 7,71) - P(Z < -0,87) \\ = 0,8665 - 0,1422 = \underline{0,6743}$$

Öppgave 5.27)

$$X \sim n(X; 180, 8)$$

$$P(X < 167) = P\left(Z < \frac{167 - 180}{8}\right)$$

$$= P(Z < -1,625) = \underline{0,0526} \quad (\text{antake } Z < -1,62)$$

$$P(X > 195) = 1 - P\left(Z < \frac{195 - 180}{8}\right) = 1 - P(Z < 1,88)$$

$$= 1 - 0,9699 = \underline{0,0301}$$

$$P(175 < X < 182) = P\left(Z < \frac{182 - 180}{8}\right) - P\left(Z < \frac{175 - 180}{8}\right)$$

$$= P(Z < 0,25) - P(Z < -0,63)$$

$$= 0,5987 - 0,2643 = \underline{0,3344}$$

Opgave 5.32)

$$T \sim N(T; 24,8, 2,2)$$

~~$$X = 24,8$$
$$2,2$$
$$3,09$$~~

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$$2,2$$
$$3,09$$~~

$$P(Z > \frac{x - 24,8}{2,2}) =$$

Må finne x slik at $\frac{x - 24,8}{2,2}$ gir en

Verdi slik at $P(Z > \frac{x - 24,8}{2,2}) = 0,07$

Ser i tabellen at $\frac{x - 24,8}{2,2}$ må være $-3,09$

$$\frac{x - 24,8}{2,2} = -3,09$$

$$x - 24,8 = -6,798$$

$$x = 18,002 \approx \underline{\underline{18^\circ \text{C}}}$$

Oppgave 5.32)

Y er normalfordelt.

Siden de er uavhengige er $E[Y] = E[X_1] + E[X_2] + E[X_3] + E[X_4]$
 $= 4 + 5 + 7 + 9 = \underline{\underline{25}}$

Samme med variansen:

$$\text{Var}[Y] = 1 + 1 + 1 + 1 = \underline{\underline{4}}$$

$$P(Y \leq 20) = P\left(Z \leq \frac{20 - 25}{2}\right) = P\left(Z \leq -\frac{5}{2}\right) \\ = P(Z \leq -2,5) = \underline{\underline{0,0062}}$$

ST0203 2015H 3)

a) $X \sim N(X; 9, 2)$

$$Y \sim N(Y; 9, 2)$$

$$P(X > 10) = 1 - P(Z \leq 0,5) = \cancel{0,6915} \quad \cancel{0,6915} \\ = 1 - 0,6915 = \underline{\underline{0,3085}}$$

$$P\left(\frac{1}{2}(X+Y) > 10\right):$$

$\frac{1}{2}(X+Y)$ er gjennomsnittet. Forventningen til denne er
gitt ved $E\left(\frac{1}{2}(X+Y)\right) = \frac{1}{2}E(X+Y) = \frac{1}{2}(E[X] + E[Y])$
 $= \frac{2 \cdot 9}{2} = \underline{\underline{9}}$

Variansen er $\text{Var}\left(\frac{1}{2}(X+Y)\right) = \left(\frac{1}{2}\right)^2 \text{Var}(X+Y)$
 $= \frac{1}{4} \cdot (\text{Var}(X) + \text{Var}(Y)) = \frac{2+2}{4} = \underline{\underline{1}}$

$$P\left(\frac{1}{2}(X+Y) > \frac{10-9}{\sqrt{1}}\right) = P\left(\frac{1}{2}(X+Y) > 0,707\right) = 1 - P(X \leq 0,707) \\ = 1 - 0,7611 = \underline{\underline{0,24}}$$

b)

$$P(10 < X < Y)$$

Siden X og Y er uafhængige og har samme fordeling \mathcal{U} , har de også lik sannsynlighet.

Derfor blir

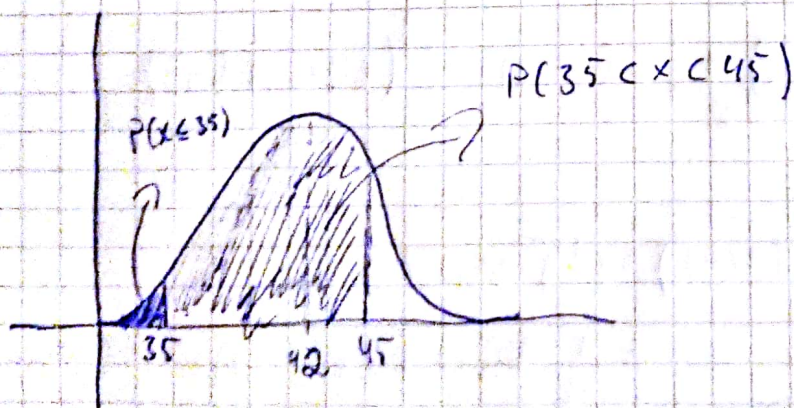
$$\begin{aligned} P(10 < X < Y) &= \frac{1}{2} P(X > 10, Y > 10) = \frac{1}{2} P(X > 10) \cdot P(Y > 10) \\ &= \frac{1}{2} P\left(Z > \frac{10-9}{2}\right) \cdot P\left(Z > \frac{10-9}{2}\right) \\ &= \frac{1}{2} \left(1 - P\left(Z \leq \frac{1}{2}\right)\right)^2 = \frac{1}{2} (1 - 0,7617)^2 \\ &= \frac{1}{2} \cdot 0,3085^2 = \underline{\underline{0,048}} \end{aligned}$$

ST0703 2016H 1)

a) $X \sim N(X; 42, 4)$

$$P(X < 35) = P\left(Z \leq \frac{35-42}{4}\right) = P(Z \leq -1,75) = 0,0407$$

$$\begin{aligned} P(35 < X < 45) &= P\left(Z \leq \frac{45-42}{4}\right) - P(Z \leq -1,75) \\ &= P(Z \leq 0,75) - 0,0407 \\ &= 0,7734 - 0,0407 = \underline{\underline{0,7333}} \end{aligned}$$



$$5) \bar{X} \sim N(42, \sigma)$$

$$\sigma = \sqrt{\frac{Var(X_1) + Var(X_2) + Var(X_3) + Var(X_4)}{4}}$$

$$\sigma = \frac{\sqrt{4 \cdot 4^2}}{4} = \frac{\sqrt{64}}{4} = \frac{8}{4} = \underline{2}$$

$$\bar{X} \sim N(42, 2)$$

$$P(\bar{X} < 35) = P(Z \leq \frac{35 - 42}{2}) = P(Z \leq -3,5) = \underline{\underline{0,0002}}$$

Hver år de fire prøvene har en succes
sandsynlighed på $P(X=1) = 0,7333$ (fra a)).

Jeg skal følge X ut av N med succes p ,

Jeg kender den binomiske fordeling:

$$\begin{aligned} P(X \geq 3) &= P(X=3) + P(X=4) \\ &= \binom{4}{3} \cdot 0,7333^3 \cdot (1-0,7333)^1 + \binom{4}{4} \cdot 0,7333^4 \\ &= \underline{\underline{0,710}} \end{aligned}$$

ST0703 2017H 2)

$$\begin{aligned} \text{a) } & P(\mu - 2\sigma < y < \mu + 2\sigma) \\ &= P(y < \mu + 2\sigma) - P(y < \mu - 2\sigma) \\ &= P\left(z < \frac{(\mu + 2\sigma) - \mu}{\sigma}\right) - P\left(z < \frac{(\mu - 2\sigma) - \mu}{\sigma}\right) \\ &= P(z < 2) - P(z < -2) = 0,9772 - 0,0228 = \underline{\underline{0,9544}} \end{aligned}$$

$$\text{b) } P(y < 0) = 0,5, \quad P(y < 1) = 0,6915$$

$$\frac{0 - \mu}{\sigma} = 0 \quad \frac{1 - \mu}{\sigma} = 0,5$$

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$$\mu = \underline{\underline{0}} \rightarrow \frac{1 - 0}{\sigma} = 0,5$$

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$$\frac{1 - \frac{1}{2}\sigma}{\frac{1}{2}} \rightarrow \sigma = 2 \Rightarrow \sigma^2 = \underline{\underline{4}}$$

$$\underline{\underline{\mu = 0 \text{ og } \sigma^2 = 4}}$$