Innlevering 2 TMA4240 Sander Lindberg

Oppgave 1.

a) $F(x) = 1 - \exp\left(-\frac{x^2}{2\alpha}\right)$

$F'(x) = f(x) =$ Sannsynlighetstettheten.

$f(x) = -u' \cdot \exp\left(-\frac{x^2}{2x}\right)$

$= -\left(-2x\frac{1}{2\alpha}\right) \cdot \exp\left(-\frac{x^2}{2x}\right)$

$= \underline{\underline{\frac{x \cdot \exp\left(-\frac{x^2}{2x}\right)}{\alpha}}}$

Maximum når $f'(x) = 0$:

$$f'(x) = \frac{1}{\alpha}\left(x' e^{-\frac{x^2}{2\alpha}} + x\left(u' \cdot e^{u}\right)\right)$$

$$= \frac{1}{\alpha}\left(e^{-\frac{x^2}{2x}} + x\left(\frac{-x \cdot e^{u}}{\alpha}\right)\right)$$

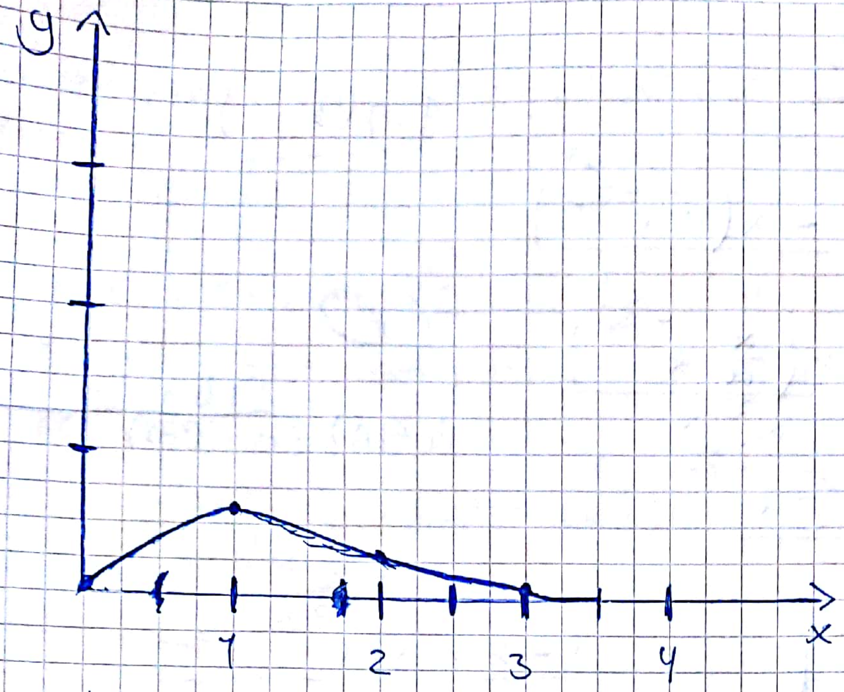
$$= \frac{e^{-\frac{x^2}{2\alpha}} - \frac{x^2 e^{-\frac{x^2}{2\alpha}}}{\alpha}}{\alpha} = \frac{(-x^2+\alpha)\, e^{-\frac{x^2}{2\alpha}}}{\alpha^2}$$

$$\frac{(-x^2+\alpha)\, e^{-\frac{x^2}{2\alpha}}}{\alpha^2} = 0$$

$x^2 = \alpha$

$\underline{\underline{x = \sqrt{\alpha}}}$

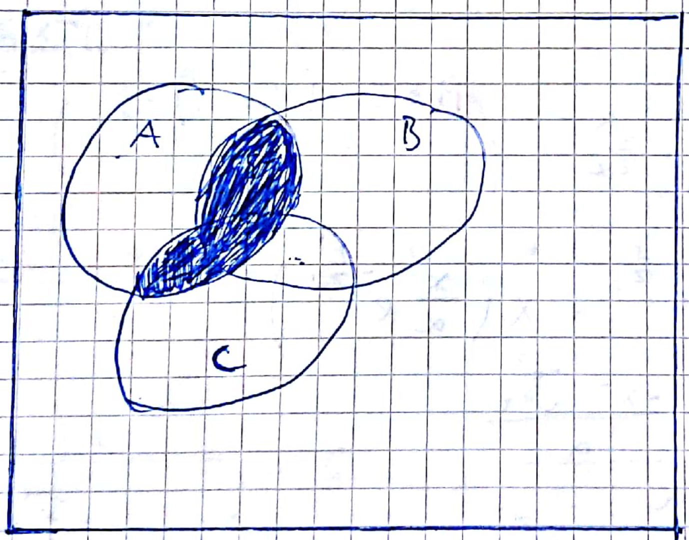
$f(x)$ har maximum når $\underline{\underline{x = \sqrt{\alpha}}}$



center $\alpha = 1$

Toppunkt $= (1, f(\sqrt{\alpha}))$

b)



$D = A \cap (B \cup C)$

D er skravert.

P(Sannsynlighet for at instrumentet fortsatt fungerer etter 2 år) $= P(D) = P(A \cap (B \cup C)) = P(A) \cdot P((B \cup C))$

$P(X \leq 2) = P(A) = P(B) = P(C) = 1 - (1 - e^{-\frac{2}{2}}) = e^{-2}$

$P(B \cup C) = e^{-2} + e^{-2} - (e^{-2})^2 = 0{,}25$

$P(D) = 0{,}25 \cdot e^{-2} = 0{,}034$

Oppgave 2

$$F(x) = \int_0^x \frac{1}{\theta}\,dx = \underline{\underline{\frac{x}{\theta}}}$$

$$P(X \leq 0{,}4) = \int_0^{0{,}4} \frac{1}{2}\,dx = \underline{\underline{0{,}2}}$$

Opgave 3

$X = \{-2, -1, 0, 1, 2\}$

$P(X \geq 0) = 0{,}5 + 0{,}2 + 0{,}1 = \underline{0{,}8}$

$P(X \geq 0 \mid X \leq 1) =$

$= \dfrac{f(0) + f(1)}{P(X \leq 1)} = \dfrac{0{,}5 + 0{,}2}{0{,}9}$

$= \underline{\underline{0{,}78}}$

$E[X] = (-2 \cdot 0{,}1) + (-0{,}1) + 0{,}2 + (2 \cdot 0{,}1) = \underline{\underline{0{,}1}}$

Oppgave 4:

$$g(x) = \sum_y f(x,y)$$

$$g(-1) = P(X=-1) = \frac{1}{6} + 2 \cdot \frac{1}{12} = \underline{\frac{1}{3}}$$

$$g(0) = P(X=0) = \frac{1}{6} + 2 \cdot \frac{1}{12} = \underline{\frac{1}{3}}$$

$$g(1) = P(X=1) = \underline{\frac{1}{3}}$$

$$h(y) = \sum_x f(x,y)$$

$$h(0) = \frac{1}{6} + \frac{1}{12} \cdot 2 = \underline{\frac{1}{3}}$$

$$h(1) = \frac{1}{6} + \frac{1}{12} \cdot 2 = \underline{\frac{1}{3}}$$

$$h(2) = \underline{\frac{1}{3}}$$

$$E(X) = -1 \cdot \frac{1}{6} + (-2) \cdot \frac{1}{12} + 2 \cdot \frac{1}{12} + \frac{1}{6} = \underline{0}$$

$$E(Y) = 2 \cdot \frac{1}{12} + \frac{1}{6} + 4 \cdot \frac{1}{12} + 2 \cdot \frac{1}{6} = \underline{1}$$

$$Var(X) = (-1-0)^2 \cdot \frac{1}{6} + 2 \cdot (-1-0^2) \cdot \frac{1}{12} + (1-0)^2 \cdot 2 \cdot \frac{1}{12} + (1-0)^2 \cdot \frac{1}{6}$$
$$= \underline{\underline{\frac{2}{3}}}$$

$$Var(Y) = (2-1)^2 \cdot 2 \cdot \frac{1}{12} + (2-1)^2 \cdot \frac{1}{6} = \underline{\underline{\frac{1}{3}}}$$

$$\cancel{Cov(x,y) = 2(-1-0)(0-1) \cdot \frac{1}{6} + (0-0)(0-1) \cdot \frac{1}{12} + (1-0)(0-1) \cdot \frac{1}{12}}$$
$$\cancel{+2(-1)}$$

$$cov(x,y) = \sum_x \sum_y (x - \mu_x)(y - \mu_y) f(x,y) = \underline{\underline{\frac{1}{6}}}$$

X og Y er ikke uavhengige, da $f(x,y) \neq g(x) \cdot h(y)$ for alle x, y

Oppgave 5:

a) $P(A) = P(A \cap B) + P(A \cap \bar{B}) = 0{,}05 + 0{,}15 = \underline{\underline{0{,}2}}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B) = 0{,}05 + 0{,}1 = 0{,}15$$

$P(A|B) = \dfrac{0{,}05}{0{,}15} = \underline{\underline{\dfrac{1}{3}}}$ = Olje på A etter olje på B

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{0{,}15}{0{,}85} = \underline{\underline{0{,}176}}$$

A og B er ikke uavhengige siden:

$$P(A|B) \neq P(A) \quad \text{og} \quad P(B|A) = \frac{1}{4} \neq P(B)$$

Oppgave 6:

$P(Y \geq y) = \left(\frac{k}{y}\right)^{\theta}$

Jeg vil vise at $f(y) = \frac{\theta k^{\theta}}{y^{\theta+1}}$

$F(y) = P(Y \leq y)$

$f(y) = F'(y)$

Siden jeg har den komplimentære til $P(Y \leq$, altså $P(Y \geq y)$, blir $F(y) = 1 - P(Y \geq y)$

$= 1 - \left(\frac{k}{y}\right)^{\theta}$

$f(y) = \left(1 - \left(\frac{k}{y}\right)^{\theta}\right)'$

$= -k^{\theta}\left(-\theta y^{-\theta-1}\right) = \underline{\underline{\frac{\theta k^{\theta}}{y^{\theta+1}}}}$ $\square$

$E[y] = \int_{k}^{\infty} y f(y)\,dy = \frac{\theta k}{\theta - 1}$

$Var[y] = E[y^2] - E[y]^2$

$E[y^2] = \int_{k}^{\infty} y^2 f(y)\,dy = \frac{\theta k^2}{\theta - 2}$

$Var[y] = \frac{\theta k^2}{\theta - 2} - \left(\frac{\theta k}{\theta - 1}\right)^2 = \frac{\theta k^2}{(\theta-2)(\theta-1)^2}$

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