Homework exercises set 3

Exercise 1

Exercises 3.1

6)

a) $A \subseteq B \cap B \subseteq A \rightarrow$ True

b) $A \subseteq C \cap C \subseteq A \rightarrow$ True

c) $B \subseteq C \cap C \subseteq B \rightarrow$ True

d) $D \nsubseteq E \rightarrow$ False

e) $D \subseteq F \cap F \subseteq D \rightarrow$ True

f) $E \nsubseteq F \rightarrow$ False

Exercise 2

Exercises 2.2

6)

$(A \subseteq B, \ C \subseteq D)$

a) $x \in (A \cap C) \rightarrow (x \in A \cap x \in C) \rightarrow (x \in B \cap x \in D)$

$\rightarrow x \in (B \cap D) \rightarrow \underline{A \cap C \subseteq B \cap D}$

$(A \subseteq B, \ C \subseteq D)$

$x \in (A \cup C) \rightarrow (x \in A \cup x \in C) \rightarrow (x \in B \cup x \in D)$

$\rightarrow x \in (B \cup D) \rightarrow \underline{A \cup C \subseteq B \cup D}$

Exercise 3

Exercises 3.2

7)

b) False, if $A=\{1\}$, $B=\{2\}$ and $C=\{1,2\}$

$A \cup C = B \cup C$, but $A \neq B$

d) If $x \in A$ and $x \in C$ then $x \notin A \Delta C$

$\Rightarrow x \notin B \Delta C \Rightarrow x \in B$, because $A \Delta C = B \Delta C$.

if $x \in A$ and $x \notin C$, then $x \in A \Delta C$

$\Rightarrow x \in B \Delta C \Rightarrow x \in B$, Because $A \Delta C = B \Delta C$

this implies that $A \subseteq B$ and $B \subseteq A \Rightarrow \underline{\underline{A = B}}$

Exercise 4

Exercises 3.2

16)

	Steps	Reasons
	$(A \cap B) \cup (B \cap ((C \cap D) \cup (C \cap \bar{D})))$	
$=$	$(A \cap B) \cup (B \cap (C \cap (D \cup \bar{D})))$	Distributive laws
$=$	$(A \cap B) \cup (B \cap (C \cap U))$	Inverse ~~Identity~~ laws
$=$	$(A \cap B) \cup (B \cap C)$	Identity laws
$=$	$(B \cap A) \cup (B \cap C)$	Commutative law
$=$	$B \cap (A \cup C)$	Distributive law.

Exercise 5

Exercises 2.5

8)

a) Let $\forall x\, P(x) = \text{true}$, then $\forall x P(x) \vee \forall x\, q(x) = \text{true}$.

then for all c in $P(c)$ $P(c) \vee q(c)$ is also true

therefore $\forall x\,(P(x) \vee q(x))$ is true and

$\forall x\, P(x) \vee \forall x\, q(x) \rightarrow \forall x[P(x) \vee q(x)]$.

b) If $P(x): x > 0$ and $q(x): x < 0$.

Then $\forall x\, P(x) \vee \forall x\, q(x)$ is false, while

$\forall x\,[P(x) \vee q(x)]$ is true.

Exercise 6

Exercises 2.5

10)

Steps	reasons
4) $P(a) \vee q(a)$	1) + RUS
5) $q(a)$	3) + 4) + rule of disjunctive syllogism
6) $\forall x\,[\neg q(x) \vee r(x)]$	Premise
7) $\neg q(a) \vee r(a)$	6) + RUS
8) $q(a) \rightarrow r(a)$	7) + $\neg q \vee r \Leftrightarrow q \rightarrow r$
9) $r(a)$	5) + 8) + Modus ponens
10) $\forall x\,[s(x) \rightarrow \neg r(x)]$	Premise
11) $s(a) \rightarrow \neg r(a)$	10) + RUS
12) $r(a) \rightarrow \neg s(a)$	11) + $r \rightarrow \neg s \Leftrightarrow \neg\neg s \rightarrow \neg r \rightarrow s \rightarrow \neg r$
13) $\neg s(a)$	12) + 9) + Modus ponens.

Exercise 7:

a	b	c	$(A \land B)$	$(A \land B) \to C$	$A \to C$	$B \to C$	$(A \to C) \lor (B \to C)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	F	F	T
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

We see that $(A \land B) \to C$ have the same truth table as $((A \to C) \lor (B \to C))$, therefore, $((A \land B) \to C) \Leftrightarrow ((A \to C) \lor (B \to C))$.