ME 314 Final Project

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1 Setup

This project was an attempt to roughly model a passive-orthotic system for the upper limb, inspired by a system found in a paper that came out of the Ability Lab [1], the use of which is meant to help stroke patients open their hand. The model system I ended up using was a 6-linked system shown below. Arrows indicate input forces, where red signifies forces coming out of the orthotic from internal springs, and green arrows coming out of the patients hand. The two-link system at the top of the system correspond to a simplified version of the 4-joint parallelogram system used in the paper. The remaining joints represent the parts corresponding to the upper arm, with the leftmost link corresponding to the forearm, and the rightmost link corresponding to the two end joints in the finger. The middle two joints correspond to the hand, and the joint at the based of the finger, respectively. Lengths are given in meters, and measurements were taken from my own arm for the model. Positive angles correspond to rotations in the counter-clockwise direction. The simulation runs for 10 seconds, and is meant to show the process of the hand opening as a result of the red spring forces, despite the forces at the finger actively pulling those joints inward. The system is designed so that the fingers will lock at an angle of 0, and the wrist will lock at an angle of $\frac{\pi}{6}$.

1.1 Lagrange Equations

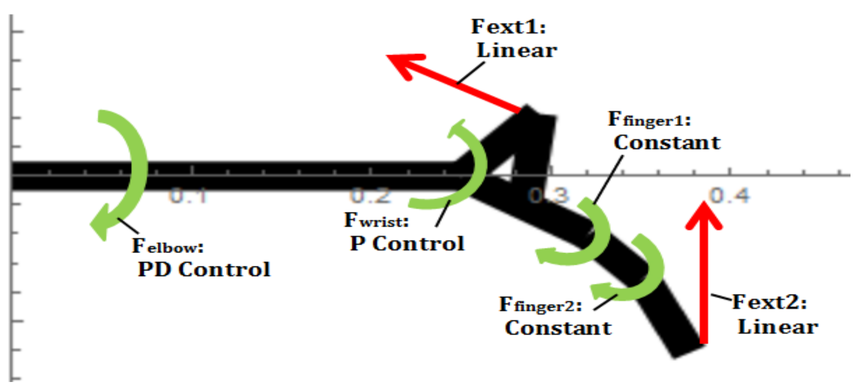

Figure 1: Diagram of the Setup With Input Forces

Each object was modeled as a simple rectangle, and the system in 2-dimensional. Kinetic energy was calculated for each body from the equation:

$$KE = \Sigma \frac{1}{2} V^T \mathcal{I} V$$

1.2 Constraint

the end of the 2-link triangle system mounted to the wrist is constrained to lie on the midpoint of the hand. Using the notation that g4 is the rotation from the based of the elbow to the end link of the triangle, and g2 is the transformation to the hand (both w.r.t the center of mass), this means that:

$$g4.\begin{bmatrix} 1 & 0 & l4/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} == g2.\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Which gives 2 constraints: 1 for conserving the x-coordinate, and 1 for the y coordinate, of the linkage:

$$\Phi x == \frac{1}{2} * l2 * Cos(\theta 1 + \theta 2) - l3 * Cos(\theta 1 + \theta 3) - l4 * Cos(\theta 1 + \theta 3 + \theta 4)$$

$$\Phi y == \frac{1}{2} * l2 * Sin(\theta 1 + \theta 2) - l3 * Sin(\theta 1 + \theta 3) - l4 * Sin(\theta 1 + \theta 3 + \theta 4)$$

1.3 Input Forces

There were a number of input forces on the hand that guided the system upward: otherwise, everything would have just fallen down!

1. $F_{ext\theta1}$: This is an input force from the arm that kept the elbow in place. The elbow muscle is assumed to be functional and is thus modeled as a PD controller: $F_{ext\theta1} = -k0 * (\theta 1 + \theta 1')$

2. F_{wrist}: This is a force meant to model the user's force from the wrist. It is assumed to be somewhat week and have less precise control, and thus is modeled as a P controller: $F_{wrist} = Min[k1 * \theta 2, 0]$. Unfortunately for our simulated user, he is too weak to maintain the hand in the upright position once he gets past equillibria, so his hand tends to ossilate around a small negative angle (or it would, given enough time)

3. $F_{finger1}$ and $F_{finger2}$: Input forces from the finger joints of the user. They are assumed to be involuntary and a constant: $F_{finger1} = k2, F_{finger2} = k3$

4. $F_{ext\theta3}$ Is the force from the orthotic device at the joint in the wrist. The torque acts on the first link that shares it's joint origin with the wrist. This force is modeled as a torque that acts on the angle between the top link and the elbow; force is transmitted to the hand via a change in the constraint forces. The force always acts in the -x direction, and is linear wrt the x position of the link, as well as the moment arm of the force from the base: $F_{ext\theta3} = k1 * l3 * Sin(\theta 3) * Cos(\theta 3)$

5. $F_{ext\theta5}$ Force from the orthotic device on the finger joint. Based on the paper, the input force is constant in the y direction regardless of the y displacement of the finger. Since it acts through a moment arm from the tip of the finger to the base, it is defined as: $F_{ext\theta5} = k2 * (l5 * Cos(\theta 5) + l6 * Cos(\theta 5 + \theta 6))$

6. $F_{ext\theta6}$ Similar to the above but acts upon the rightmost joint in the finger: $F_{ext\theta5} = k3 * l6 * Cos(\theta 6)$

1.4 Plastic Impacts/Conditional Constraints

The idea of the system was to model the joints as 'locking' once they reached their limit points (0 for the finger joints and $\frac{\pi}{6}$ for the wrist joint). Solve these equations as 'impacts' does no good; the dynamics simply set the velocities of the preceding joints to 0 when solved for, momentarily stopping the system, which then continuous past these limits. Instead, they were modeled as 'plastic' impacts. Since these impact locations are defined by simple linear functions of a single angle, the plastic impact updates are satisfied by simply setting the given angle's velocity to 0. That is, for:

$$\Phi n = \theta n(t)$$

$$D\Phi(\theta n) * \theta n'(t) == 0 \quad at \quad t = \tau$$

$$\theta n'(\tau^+) = 0$$

Once impact occurs, a new system needs to be modeled with an added constraint force for the respective joint. For this simulation, $\theta 5 \quad and \quad \theta 6$, the angles representing the fingers, both experience impact, but the wrist falls short. The systems switches over the course of the simulation as such:

$$\text{Starting System} \rightarrow \text{System with } \phi 2 = \theta 5 \rightarrow \text{System with } \phi 2 = \theta 5 \text{ and } \phi 3 = \theta 6$$

Since the forces are large enough for the fingers that the torques pulling the finger in the positive direction are neccessarily greater than the antagonistic forces, seperation from the force constraints didn't occur.

1.5 Running the Code

The code is divided into 4 cells and, on my machine, should take 10 minutes to run

1. Quit[]. This should clear the Kernel

2. This section is the part where the bulk of the calculations are performed and helper functions are defined
3. This section is when NDSolve is called for each part of the system. It is fast.
4. This section splices together the 3 sets of interpolating functions and produces an animation (using Manipulate[]) for the System from 0 to 10. It takes 10 seconds to run.

Bibliography

[1] S. Ates et al. "SCRIPT passive orthosis: design and technical evaluation of the wrist and hand orthosis for rehabilitation training at home". In: *IEEE Int Conf Rehabil Robot* 2013 (June 2013), p. 6650401.