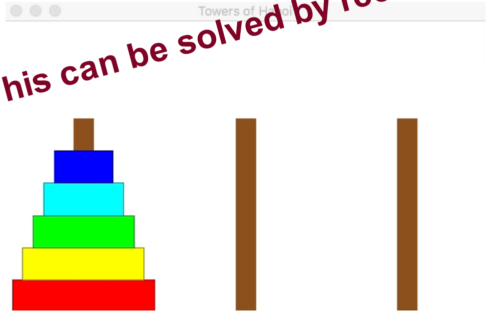
Recursion

Instructor: Jeeho Ryoo

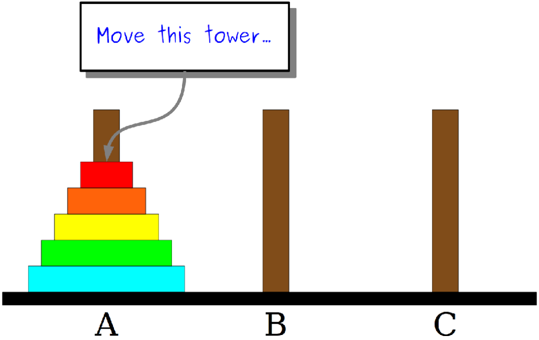
The Towers of Hanoi Puzzle



This can be solved by recursion!

Towers of Hanoi

Here is the way the game is played:



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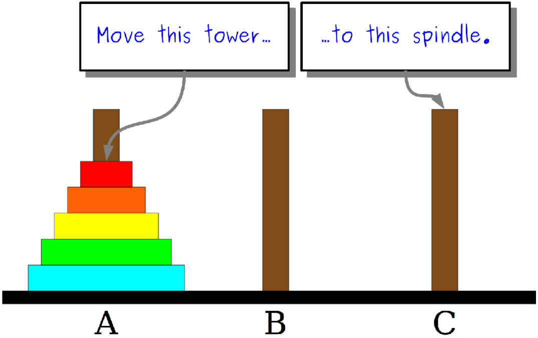

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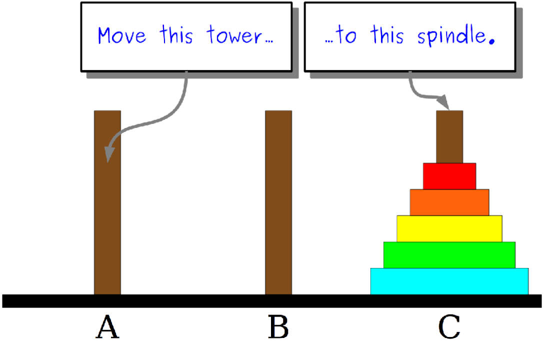

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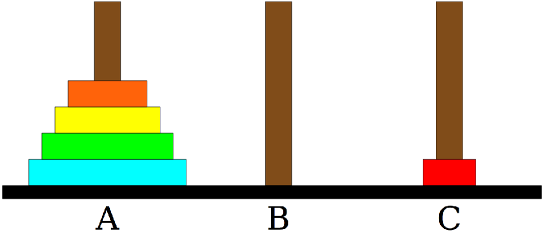

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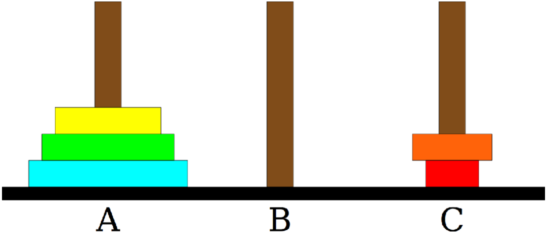

Towers of Hanoi

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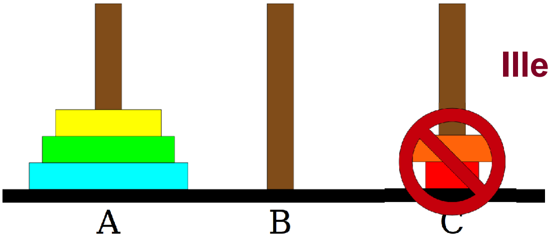

Illegal move!

Towers of Hanoi

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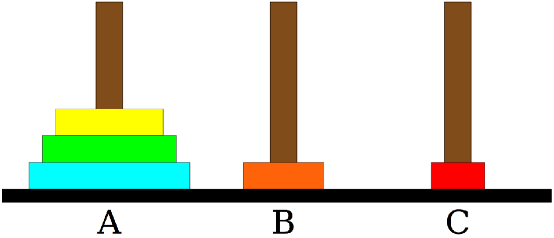

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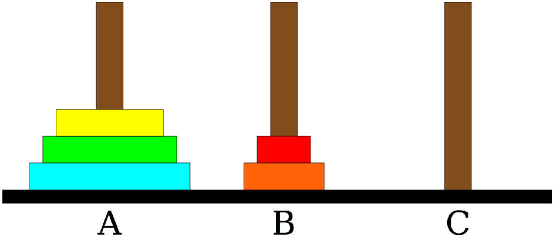

Towers of Hanoi

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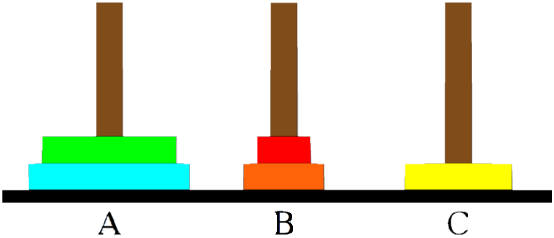


etc.

What is Recursion?

Recursion:

A problem solving technique in which problems are solved by reducing them to smaller problems *of the same form.*

Why Recursion?

1. Great style
2. Powerful tool
3. Master of control flow

Recursion In Programming

In programming, recursion simply means that a function will call itself:

```
int main() {
   main();
   return 0;
}
```

SEG FAULT!

(this is a terrible example, and will crash!)

main() isn't supposed to call itself, but if we do write this program, what happens?

We'll get back to programming in a minute...

Recursion In Real Life

Recursion

- How to solve a jigsaw puzzle recursively ("solve the puzzle")
  - Is the puzzle finished? If so, stop.
  - Find a correct puzzle piece and place it.
  - Solve the puzzle

ridiculously hard puzzle

Recursion In Real Life

Let's recurse on *you*.

How many students total are directly behind you in your "column" of the classroom?

Rules:

1. You can see only the people directly in front and behind you. So, you can't just look back and count.
2. You *are* allowed to ask questions of the people in front / behind you.

How can we solve this problem *recursively*?

Recursion In Real Life

Answer:

1. The first person looks behind them, and sees if there is a person there. If not, the person responds "0".
2. If there is a person, repeat step 1, and wait for a response.
3. Once a person receives a response, they add 1 for the person behind them, and they respond to the person that asked them.

In C

Every recursive algorithm involves at least **two** cases:

- base case: The simple case; an occurrence that can be answered directly; the case that recursive calls reduce to.

- recursive case: a more complex occurrence of the problem that cannot be directly answered, but can be described in terms of smaller occurrences of the same problem.

Three Musts of Recursion 🔑

1. Your code must have a case for all valid inputs

2. You must have a base case that makes no recursive calls

3. When you make a recursive call it should be to a simpler instance and make forward progress towards the base case.

More Examples!

The power() function:

Write a recursive function that takes in a number (*x*) and an exponent (*n*) and returns the result of x^n

Powers

$$x^0 = 1$$

$$x^n = x \cdot x^{n-1}$$

Powers

- Each previous call waits for the next call to finish (just like any function).

```
printf(power(5, 3));
```

```
// first call: power (5, 3)
// second call: power (5, 2)
// third call: power (5, 1)
// fourth call: power (5, 0)
int power(int x, int exp) {
    if (exp == 0) {
        return 1;
    } else {
        return x * power(x, exp - 1);
    }
}
```

Powers

- Each previous call waits for the next call to finish (just like any function).

```
printf(power(5, 3));
```

```
// first call: power (5, 3)
```

```
// second call: power (5, 2)
```

```
// third call: power (5, 1)
```

```
// fourth call: power (5, 0)
int power(int x, int exp) {
    if (exp == 0) {
        return 1;
    } else {
        return x * power(x, exp - 1);
    }
}
```

This call returns 1

Powers

- Each previous call waits for the next call to finish (just like any function).

```
printf(power(5, 3));
```

```
// first call: power (5, 3)
// second call: power (5, 2)
// third call: power (5, 1)
int power(int x, int exp) {
    if (exp == 0) {
        return 1;
    } else {
        return x * power(x, exp - 1);
    }
}
```

equals 1 from call

this entire statement returns 5 * 1

Powers

- Each previous call waits for the next call to finish (just like any function).

```
printf(power(5, 3));
```

```
// first call: power (5, 3)
```

```
// second call: power (5, 2)
int power(int x, int exp) {
    if (exp == 0) {
        return 1;
    } else {
        return x * power(x, exp - 1);
    }
}
```

equals 5 from call

this entire statement returns 5 * 5

Powers

- Each previous call waits for the next call to finish (just like any function).

```
printf(power(5, 3));
```

```
// first call: power (5, 3)
int power(int x, int exp) {
    if (exp == 0) {
        return 1;
    } else {
        return x * power(x, exp - 1);
    }
}
```

equals 25 from call

this entire statement returns 5 * 25

the original function call was to this one, so it returns 125, which is 5^3

Faster Method!

```
int power(int x, int exp) {
    if(exp == 0) {
        // base case
        return 1;
    } else {
        if (exp % 2 == 1) {
        // if exp is odd
            return x * power(x, exp - 1);
        } else {
            // else, if exp is even
            int y = power(x, exp / 2);
            return y * y;
        }
    }
}
```

Factorial

```
n! = n * (n-1) * (n-2) * (n-3) * … * 1
```

- Write a function that computes and returns the factorial of a provided number, recursively (no loops).
  - e.g. **`factorial(4)`** should return **`24`**
  - You should be able to compute the value of any non-negative number. (**`0! = 1`**).

Factorial

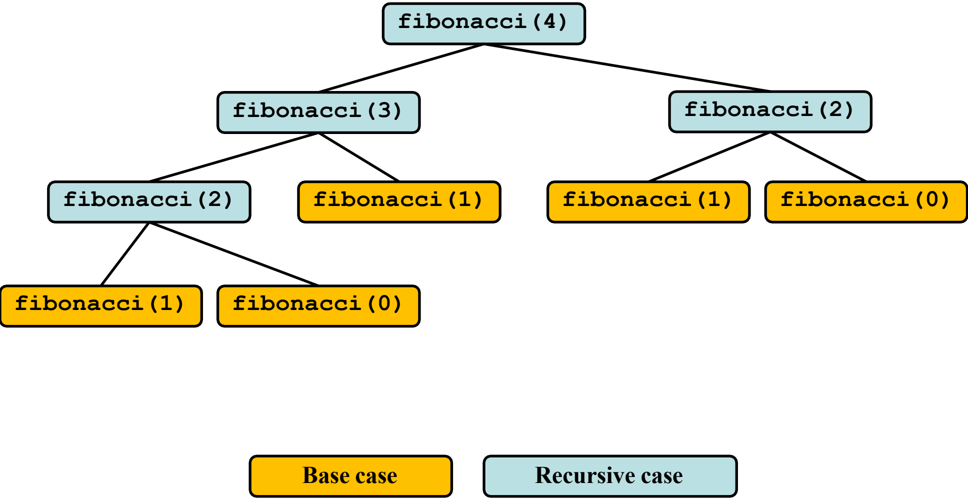
```
// Returns  n!, or 1  * 2 * 3 * 4 * ...  *n.
// Assumes n >= 0

int factorial(int n) {
    if (n < 0) {
        throw "illegal negative n";
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n - 1);
    }
}
```

Fibonacci

`0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...`

- The Fibonacci sequence starts with 0 and 1, and each subsequent number is the sum of the two previous numbers.
- Write a function that computes and returns the nth Fibonacci number, recursively (no loops).
  - e.g. **`fibonacci(6)`** should return **`8`**

Recursive Tree



Fibonacci

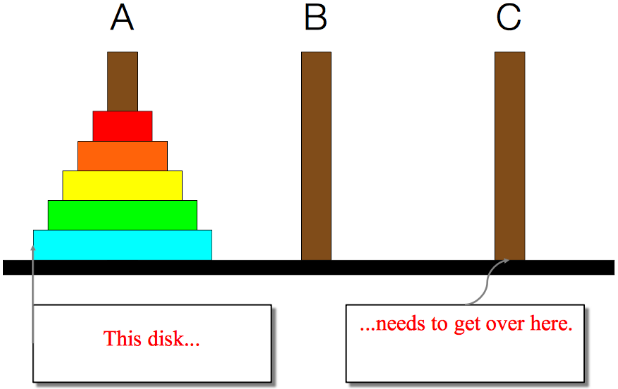
```
// Returns the i’th Fibonacci number in the sequence
// (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …)
// Assumes i >= 0.
int fibonacci(int i) {







}
```

Fibonacci

```
// Returns the i'th Fibonacci number in the sequence
// (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …)
// Assumes i >= 0.
int fibonacci(int i) {
    if (i < 0) {
        throw "illegal negative index";
    else if (i == 0) {
        return 0;
    } else if (i == 1) {
        return 1;
    } else {
        // recursive case
        return fibonacci(i-1) + fibonacci(i-2);
    }
}
```

Back to Towers of Hanoi

This is a hard problem to solve iteratively, but can be done recursively (though the recursive insight is not trivial to figure out)



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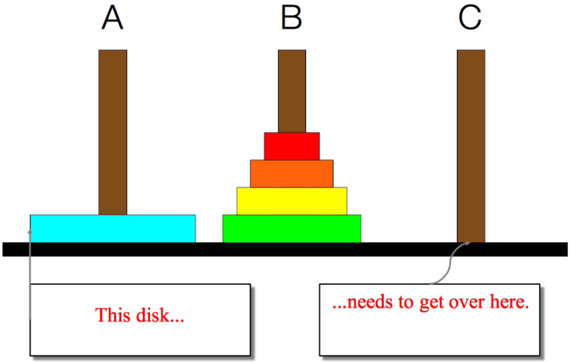

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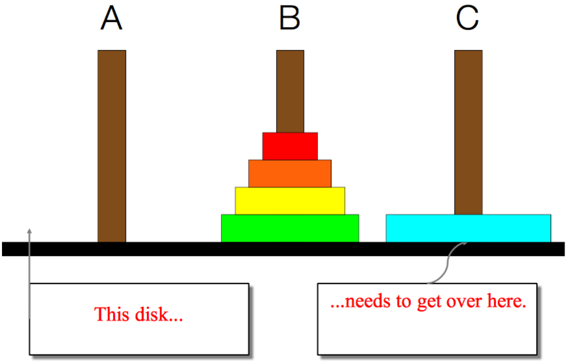

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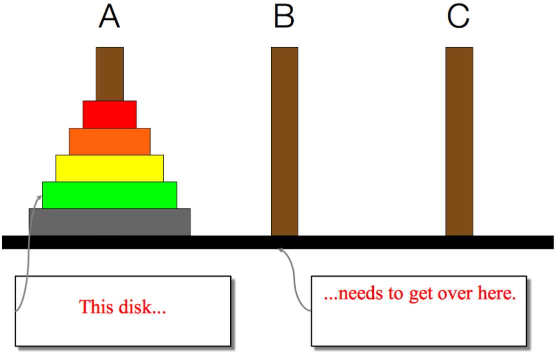

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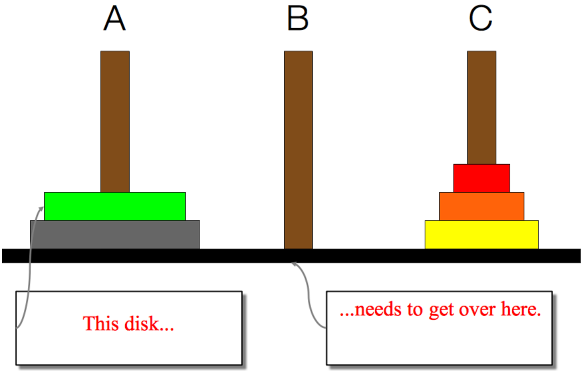

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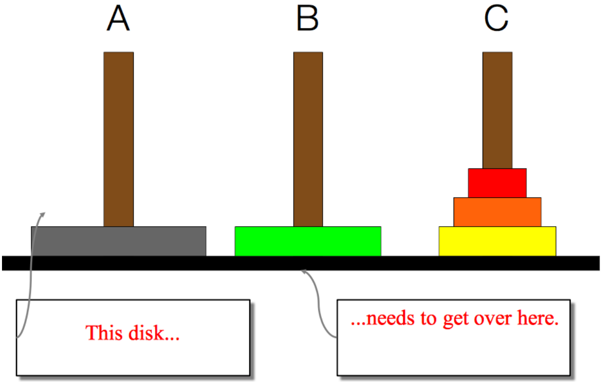

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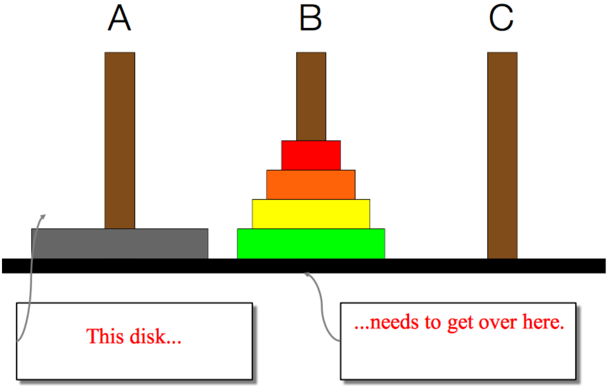


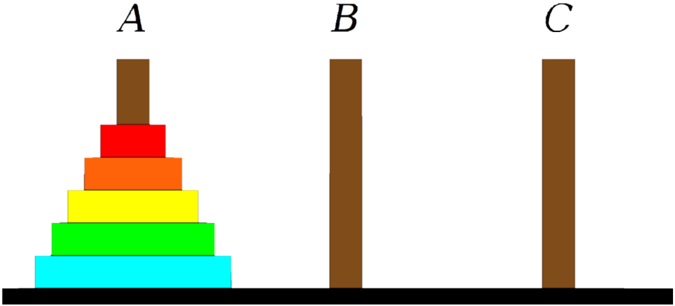
Back to Towers of Hanoi

- We need to find a very simple case that we can solve directly in order for the recursion to work.
- If the tower has size one, we can just move that single disk from the source to the destination.
- If the tower has more than one, we have to use the auxiliary spindle.

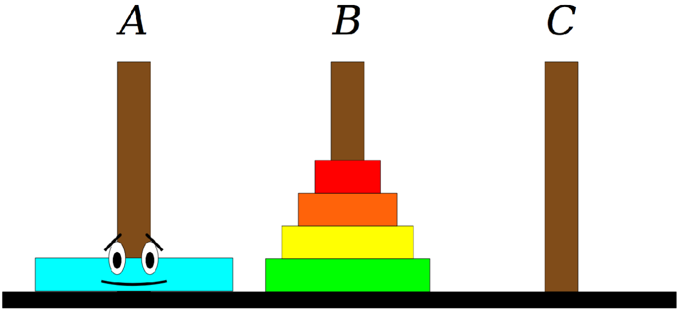
Back to Towers of Hanoi

- We can break the entire process down into very simple steps -- not necessarily easy to think of steps, but simple ones!

Back to Towers of Hanoi

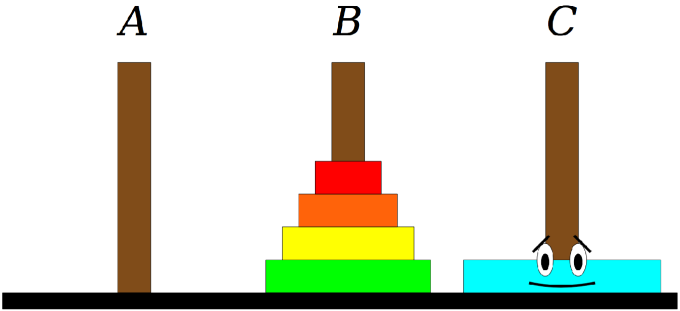


Back to Towers of Hanoi



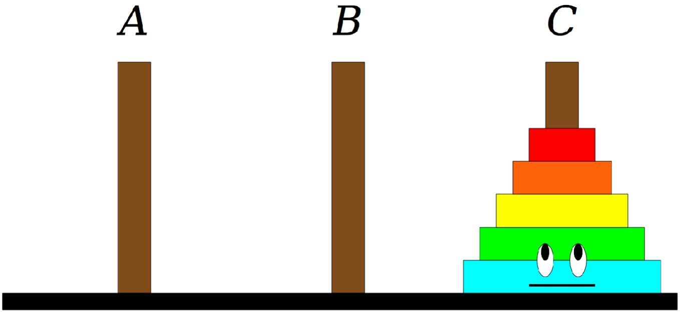
Step One: Move the four smaller disks from Spindle A to Spindle B.

Back to Towers of Hanoi



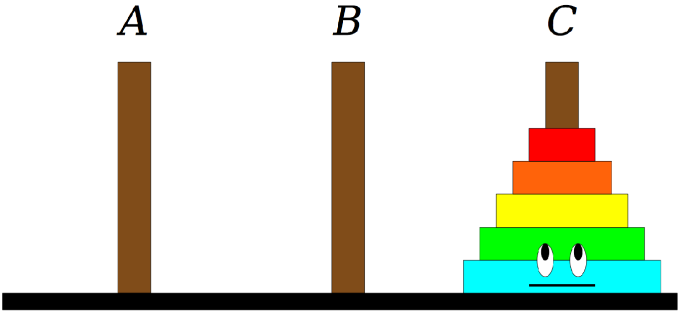
Step One: Move the four smaller disks from Spindle A to Spindle B.
Step Two: Move the blue disk from Spindle A to Spindle C.

Back to Towers of Hanoi



Step One: Move the four smaller disks from Spindle A to Spindle B.
Step Two: Move the blue disk from Spindle A to Spindle C.
Step Three: Move the four smaller disks from Spindle B to Spindle C.

Back to Towers of Hanoi



Repeat these steps at each stage!

Step One: Move the four smaller disks from Spindle A to Spindle B.
Step Two: Move the blue disk from Spindle A to Spindle C.
Step Three: Move the four smaller disks from Spindle B to Spindle C.