ILG-4 (REV)

III. SADA

1. Nechť $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$. Určete $|A|, |-A|$ a $|2A|$.

Výsledky: $|A| = 11, |-A| = 11, |2A| = 44$.

$|A| = \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} = 8-(-3) = 11$ $\quad |-A| = \begin{vmatrix} -2 & -3 \\ 1 & -4 \end{vmatrix} = 8-(-3) = 11$ $\quad |2A| = \begin{vmatrix} 4 & 6 \\ -2 & 8 \end{vmatrix} = 32-(-12) = 44$

2. Nechť $A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$. Určete $|A|$.

Výsledky: $|A| = -9$.

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 3 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix} = (6+6+2)-(4+1+18) = 14-23 = -9$$

$$\begin{matrix} 1 & 2 & 2 \\ 3 & 2 & 1 \end{matrix}$$

3. Nechť $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 4 & 4 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 4 & 6 & 2 \end{pmatrix}$,

$D = \begin{pmatrix} 2 & 3 & 2 \\ 3 & 2 & 4 \\ 2 & 2 & 6 \end{pmatrix}$, $E = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 6 & 8 & 5 \end{pmatrix}$.

(a) Určete: $|A|, |B|, |C|, |D|, |E|$.

(b) Určete: $|AB|$.

Výsledky: (a) $|A| = -9, |B| = -18, |C| = 0, |D| = -18, |E| = -9$, (b) 162.

$$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 2 & 2 & 3 \end{vmatrix} = (12+6+12)-(4+8+27) = 30-39 = \underline{-9}$$

$$\begin{matrix} 2 & 3 & 1 \\ 3 & 2 & 2 \end{matrix}$$

$/\cdot 2$

$$|B| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 4 & 4 & 6 \end{vmatrix} \Rightarrow |B| = 2|A| = \underline{-18}$$

$/\cdot 2$

$$|C| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 4 & 6 & 2 \end{vmatrix} = (8+18+24)-(8+24+18) = \underline{0}$$

$$\begin{matrix} 2 & 3 & 1 \\ 3 & 2 & 2 \end{matrix}$$

$$|D| = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 4 \\ 2 & 2 & 6 \end{vmatrix} \Rightarrow |D| = 2 \cdot |A| = -18$$

$$|E| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 6 & 8 & 5 \end{vmatrix} = (20+24+36)-(12+32+45) = 80-89 = \underline{-9}$$

$$\begin{matrix} 2 & 3 & 1 \\ 3 & 2 & 2 \end{matrix}$$

$$|AB| = |A| \cdot |B| = -9 \cdot (-18) = \underline{162}$$

4. Nechť $|A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$. Určete $|A^T A|$.

Výsledky: 144.

$$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = (12+6+3)-(2+4+27) = 21-33 = -12 \quad |A^T| = |A| = -12 \quad |A^T A| = |A| \cdot |A| = \underline{\underline{144}}$$

$$\begin{matrix} 2 & 3 & 1 \\ 3 & 2 & 1 \end{matrix}$$

5. Určete $c \in \mathbb{R}$ tak, aby:

$$\begin{vmatrix} c & 2 & 1 \\ 2 & c & 1 \\ 1 & 2 & 1 \end{vmatrix} = 0.$$

Výsledky: $c \in \{1, 2\}$.

$a = 1$, $b = -3$, $c = 2$ $\quad 9-8=1 \quad \frac{3\pm 1}{2} \begin{matrix} \nearrow 2 \\ \searrow 1 \end{matrix}$

$$\begin{vmatrix} c & 2 & 1 \\ 2 & c & 1 \\ 1 & 2 & 1 \end{vmatrix} = (c^2+4+2)-(c+2c+4) = c^2-3c+2 = 0$$

$$c \in \{1; 2\}$$

$$\begin{matrix} c & 2 & 1 \\ 2 & c & 1 \end{matrix}$$

$a = -1$, $b = 8$, $c = -12$ $\quad 64-48=16 \quad \frac{-8\pm 4}{-2} \begin{matrix} \nearrow 2 \\ \searrow 6 \end{matrix}$

6. Nechť $A = \begin{pmatrix} a & 2 & 2 \\ 3 & 2 & a \\ 1 & 1 & 3 \end{pmatrix}$. Určete všechna $a \in \mathbb{R}$ tak, aby $|A| = -4$.

Výsledky: $c \in \{2, 6\}$.

$$\begin{vmatrix} a & 2 & 2 \\ 3 & 2 & a \\ 1 & 1 & 3 \end{vmatrix} = (6a+6+2a)-(4+a^2+18) = 8a+6-4-a^2-18 = -a^2+8a-16 = -4 \quad a \in \{2; 6\}$$

$$-a^2+8a-12 = 0$$

$$\begin{matrix} a & 2 & 2 \\ 3 & 2 & a \end{matrix}$$

7. Vypočítejte determinant matice Y, víme-li, že $|X| = -1$, kde

$$X = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 3 \\ a & 2b & a \end{pmatrix}, \quad Y = \begin{pmatrix} 1 & 4 & 2 \\ 1 & 3 & 1 \\ a & a & b \end{pmatrix}, \quad a, b \in \mathbb{R}.$$

Výsledky: $|Y| = -\frac{1}{2}$.

$$|X| = \begin{vmatrix} 1 & 0 & 2 \\ 1 & 2 & 3 \\ a & 2b & a \end{vmatrix} \begin{matrix} 1 & 0 & 2 \\ 1 & 2 & 3 \end{matrix} = (2a+4b)-(4a+6b) = -2a-2b = -1$$

$$|Y| = \begin{vmatrix} 1 & 4 & 2 \\ 1 & 3 & 1 \\ a & a & b \end{vmatrix} \begin{matrix} 1 & 4 & 2 \\ 1 & 3 & 1 \end{matrix} = (3b+2a+4a)-(6a+a+4b) = -a-b$$

$$-2a-2b=-1 \;\Downarrow\; -a-b = -\frac{1}{2}$$

$$|Y| = -\frac{1}{2}$$

9. Určete determinanty matic A a B.

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & 1 & 0 \\ -1 & 2 & -4 & 2 \\ 1 & -4 & 1 & 1 \\ -2 & 2 & 1 & 0. \end{pmatrix}$$

Výsledky: vysl. $|A| = 5$, $|B| = -6$.

$$|A| = \begin{vmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} = (-1)^{1+1}\cdot 2 \cdot \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} \begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \end{matrix} + (-1)^{1+2}\cdot 1 \cdot \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} \begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{matrix} = 2\cdot(8-4) + (-1)\cdot(4-1) = 8-3 = \underline{5}$$

2. MOŽNOSŤ

$$|A| = \begin{vmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -3 & -2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 1 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -5 \end{vmatrix} = -(-5) = \underline{\underline{5}}$$

$$|B| = \begin{vmatrix} 1 & -2 & 1 & 0 \\ -1 & 2 & -4 & 2 \\ 1 & -4 & 1 & 1 \\ -2 & 2 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -2 & 0 & 1 \\ 0 & -2 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & -2 & 3 & 0 \\ 0 & 0 & -3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \underline{\underline{-6}}$$

12. Určete všechna $c \in \mathbb{R}$, pro která má soustava:

$$\begin{aligned} cx + 2y + z &= 1 \\ 2x + cy + z &= 2 \\ x + 2y + z &= 3 \end{aligned}$$

(a) právě jedno řešení.
(b) alespoň jedno řešení.

Výsledky: (a) $c \in \mathbb{R} \setminus \{1, 2\}$, (b) $c \in \mathbb{R} \setminus \{1, 2\}$.

$a = 1$
$b = -3$
$c = 2$

$$\frac{3 \pm 1}{2} \to 2, \; 1$$

a) $D \neq 0 \Rightarrow$ 1 RIEŠENIE

$$\begin{vmatrix} c & 2 & 1 \\ 2 & c & 1 \\ 1 & 2 & 1 \end{vmatrix} \begin{matrix} c & 2 & 1 \\ 2 & c & 1 \end{matrix} = (c^2+4+2)-(c+2c+4) = c^2-3c+2 \qquad c \in \{1, 2\}$$

b) TO ISTÉ (1 RIEŠENIE Z 1 RIEŠENIE)

13. Uveďte příklad hodnot $c, p, r, s \in \mathbb{R}$, pro které má soustava:

$$\begin{aligned} cx + 2y + z &= p \\ 2x + cy + z &= r \\ x + 2y + z &= s \end{aligned}$$

(a) právě dvě řešení.
(b) alespoň dvě řešení.

Výsledky: (a) nelze, (b) pro $c = 1$ jakékoli hodnoty, kde $p = s$; pro $c = 2$ jakékoli hodnoty, kde $p = r$.

$D = c^2 - 3c + 2 \Rightarrow c \in \{1, 2\}$

a) NEJDE

b) $c = 1$

$$\begin{aligned} x + 2y + z &= p \\ 2x + y + z &= r \\ x + 2y + z &= s \end{aligned} \qquad p = s$$

$c = 2$

$$\begin{aligned} 2x + 2y + z &= p \\ 2x + 2y + z &= r \\ x + 2y + z &= s \end{aligned} \qquad p = r$$

15. Soustavy rovnic s parametrem c z druhé sady cvičení řešte pomocí Cramerova pravidla.

a) $$\begin{array}{rcrcrcr} x & + & cy & + & 4z & = & 2 \\ x & + & y & + & cz & = & 9 \\ 2x & + & y & + & cz & = & 4 \end{array}$$

$$\begin{vmatrix} 1 & c & 4 \\ 1 & 1 & c \\ 2 & 1 & c \end{vmatrix} = (c+4+2c^2)-(8+c+c^2) = c^2-4 = (c-2)(c+2)$$
$$\begin{array}{ccc} 1 & c & 4 \\ 1 & 1 & c \end{array}$$

$|A| = (c-2)(c+2)$

$$|A_x| = \begin{vmatrix} 2 & c & 4 \\ 9 & 1 & c \\ 4 & 1 & c \end{vmatrix} = (2c+36+4c^2)-(16+2c+9c^2) = -5c^2+20 = -5(c-2)(c+2)$$
$$\begin{array}{ccc} 2 & c & 4 \\ 9 & 1 & c \end{array}$$

$$x = \frac{|A_x|}{|A|} = \frac{-5\cancel{(c-2)}\cancel{(c+2)}}{\cancel{(c-2)}\cancel{(c+2)}} = \underline{-5}$$

$$|A_y| = \begin{vmatrix} 1 & 2 & 4 \\ 1 & 9 & c \\ 2 & 4 & c \end{vmatrix} = (9c+16+4c)-(72+4c+2c) = 7c-56 = 7(c-8)$$
$$\begin{array}{ccc} 1 & 2 & 4 \\ 1 & 9 & c \end{array}$$

$$y = \frac{7(c-8)}{(c-2)(c+2)}$$

$$|A_z| = \begin{vmatrix} 1 & c & 2 \\ 1 & 1 & 9 \\ 2 & 1 & 4 \end{vmatrix} = (4+2+18c)-(4+9+4c) = 14c-7 = 7(2c-1)$$
$$\begin{array}{ccc} 1 & c & 2 \\ 1 & 1 & 9 \end{array}$$

$$z = \frac{7(2c-1)}{(c-2)(c+2)}$$

$$\left[-5;\ \frac{7(c-8)}{c^2-4};\ \frac{7(2c-1)}{c^2-4}\right]$$

$$\begin{array}{c} -1 \\ 7 \\ -10 \end{array} \qquad \frac{-7\pm3}{-2} \begin{array}{l} 5 \\ 2 \end{array}$$

b) $$\begin{array}{rcrcrcr} 2x & + & cy & + & 4z & = & c \\ cx & + & 2y & + & 3z & = & 3c-1 \\ x & + & y & + & z & = & 2c \end{array}$$

$$\begin{vmatrix} 2 & c & 4 \\ c & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = (4+4c+3c)-(8+6+c^2) = -c^2+7c-10 = (c-5)(c-2)$$
$$\begin{array}{ccc} 2 & c & 4 \\ c & 2 & 3 \end{array}$$

$|A| = (c-5)(c-2)$

$$\begin{array}{c} 3 \\ -4 \\ -4 \end{array} \qquad 16+48=64 \qquad \frac{4\pm8}{6} \begin{array}{l} 2 \\ -\frac{2}{3} \end{array}$$

$$|A_x| = \begin{vmatrix} c & c & 4 \\ 3c-1 & 2 & 3 \\ 2c & 1 & 1 \end{vmatrix} = (2c+12c-4+6c^2)-(16c+3c+3c^2-c) = 3c^2-4c-4 = (c-2)\left(c+\frac{2}{3}\right)$$
$$\begin{array}{ccc} c & c & 4 \\ 3c-1 & 2 & 3 \end{array}$$

$$x = \frac{\cancel{(c-2)}\left(c+\frac{2}{3}\right)}{(c-5)\cancel{(c-2)}} = \underline{\frac{c+\frac{2}{3}}{c-5}}$$

$$\begin{array}{c} 7 \\ -15 \\ 2 \end{array} \qquad 225-56=169 \qquad \frac{15\pm13}{14} \begin{array}{l} 2 \\ \frac{1}{7} \end{array}$$

$$|A_y| = \begin{vmatrix} 2 & c & 4 \\ c & 3c-1 & 3 \\ 1 & 2c & 1 \end{vmatrix} = (6c-2+8c^2+3c)-(12c-4+12c+c^2) = 7c^2-15c+2 = (c-2)\left(c-\frac{1}{7}\right)$$
$$\begin{array}{ccc} 2 & c & 4 \\ c & 3c-1 & 3 \end{array}$$

$$y = \frac{\cancel{(c-2)}\left(c-\frac{1}{7}\right)}{(c-5)\cancel{(c-2)}} = \underline{\frac{c-\frac{1}{7}}{c-5}}$$

$$|A_z| = \begin{vmatrix} 2 & c & c \\ c & 2 & 3c-1 \\ 1 & 1 & 2c \end{vmatrix} = (8c+c^2+3c^2-c)-(2c+6c-2+2c^3) = -2c^3+4c^2-c+2$$
$$\begin{array}{ccc} 2 & c & c \\ c & 2 & 3c-1 \end{array}$$

$$z = \frac{-2c^3+4c^2-c+2}{(c-5)(c-2)}$$

c) $$\begin{array}{rcrcrcr} cx & + & 2y & + & z & = & c \\ 2x & + & 3y & + & 2z & = & 3c \\ x & + & y & + & z & = & 2c \end{array}$$

$$\begin{vmatrix} c & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 1 \end{vmatrix} = (3c+2+4)-(3+2c+4) = c-1$$
$$\begin{array}{ccc} c & 2 & 1 \\ 2 & 3 & 2 \end{array}$$

$|A| = c-1$

$$c=1 \quad \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 3 & 2 & 3 \\ 1 & 1 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x+2y+z=1 \qquad z=t$

$-y=1 \to y=-1$

$x-2+z=1$

$x+z=3$

$x=3-t$

$$|A_x| = \begin{vmatrix} c & 2 & 1 \\ 3c & 3 & 2 \\ 2c & 1 & 1 \end{vmatrix} = (3c+3c+8c)-(6c+6c+4c) = 0$$
$$\begin{array}{ccc} 3c & 2 & 1 \\ 2c & 3 & 2 \end{array}$$

$$x = \frac{0}{c-1} = \underline{0}$$

$$|A_y| = \begin{vmatrix} c & c & 1 \\ 2 & 3c & 2 \\ 1 & 2c & 1 \end{vmatrix} = (3c^2+4c+2c)-(3c+4c^2+2c) = -c^2+c = -c(c-1)$$
$$\begin{array}{ccc} c & c & 1 \\ 2 & 3c & 2 \end{array}$$

$$y = \frac{-c\cancel{(c-1)}}{\cancel{(c-1)}} = -c$$

$$|A_z| = \begin{vmatrix} c & 2 & c \\ 2 & 3 & 3c \\ 1 & 1 & 2c \end{vmatrix} = (6c^2+2c+6c)-(3c+3c^2+8c) = 3c^2-3c = 3c(c-1)$$
$$\begin{array}{ccc} c & 2 & c \\ 2 & 3 & 3c \end{array}$$

$z = 3c$

$[0; -c; 3c$

IV SADA

1. Najděte inverzní matici k matici $\begin{pmatrix} 2 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{pmatrix}$, pokud taková matice existuje.

Výsledky: $\begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & -2 & 2 \end{pmatrix}$.

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{pmatrix} \quad A^{-1} = \left(\begin{array}{ccc|ccc} 2 & 2 & 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 2 & 2 & 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & -2 & -1 & 1 & 0 & -2 \\ 0 & -1 & -1 & 0 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & -2 \\ 0 & 0 & 1 & 1 & -2 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -1 & 2 & -1 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 2 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 2 \end{array}\right)$$

2. Určete všechna $d \in \mathbb{R}$, pro která existuje inverzní matice k matici $A = \begin{pmatrix} d & 2 & 1 \\ 2 & d & 1 \\ 1 & 2 & 1 \end{pmatrix}$.

Výsledky: $d \in \mathbb{R} \setminus \{1, 2\}$.

$$\begin{vmatrix} d & 2 & 1 \\ 2 & d & 1 \\ 1 & 2 & 1 \end{vmatrix} = (d^2+4+2)-(d+2d+4) = d^2-3d+2$$

$$\begin{matrix} d & 2 & 1 \\ 2 & d & 1 \end{matrix}$$

$\begin{matrix} 1 \\ -3 \\ 2 \end{matrix}$ $\quad \frac{3\pm1}{2} \begin{matrix} 2 \\ 1 \end{matrix}$

$d \in \mathbb{R} \setminus \{1, 2\}$

3. Jsou dány matice

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -2 \\ -3 & 6 \end{pmatrix}.$$

Najděte k nim matice inverzní (pokud existují). Dále najděte inverze (pokud existují) k maticím AB a AC.

Výsledky: $A^{-1} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$, C^{-1} neexistuje, $(AB)^{-1} = \begin{pmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & -1 \end{pmatrix}$, $(AC)^{-1}$ neexistuje.

$$|A| = -2 \quad A^{-1} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{array}\right) \quad A^{-1} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

$$|B| = 3 \quad A^{-1} = \left(\begin{array}{cc|cc} 0 & -1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right) \quad B^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

$|C| = 0 \Rightarrow C^{-1}$ NEEXISTUJE

$$(AB)^{-1} = B^{-1} \cdot A^{-1} \quad \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & -1 \end{pmatrix}$$

$(AC)^{-1}$ NEEXISTUJE

4. Nechť

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 2 & 0 \end{pmatrix}.$$

Určete $|AA^{-1}|, |A^TA|, |A^{-1}A^T|$.

Výsledky: $|AA^{-1}| = 1$, $|A^TA| = 4$, $|A^{-1}A^T| = 1$.

$$|A| = \begin{vmatrix} 2 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 2 & 0 \end{vmatrix} = 8 - 10 = -2$$

$$\begin{matrix} 2 & 2 & 1 \\ 2 & 3 & 1 \end{matrix}$$

$|AA^{-1}| = |E| = 1$

$|A^TA| = |A| \cdot |A| = 4$

$|A^{-1}A^T| = |A^{-1}A| = |E| = 1$

5. Jsou dány matice $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 2 & 2 & 3 \end{pmatrix}$.

(a) Určete: $|A|$
(b) Určete: $|A^{-1}|$
(c) Určete matici X tak, aby platilo:

$$AX = B.$$

Výsledky: a) 3, b) 1/3, c) $X = \begin{pmatrix} \frac{2}{3} & 2 & 2 \\ \frac{1}{3} & 1 & 0 \\ \frac{4}{3} & 0 & 1 \end{pmatrix}$.

a) $|A| = 3$

b) $|A^{-1}| = \frac{1}{3}$

c) $$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 2 & 2 & 3 \end{pmatrix} \quad \begin{pmatrix} \frac{2}{3} & 2 & 2 \\ \frac{1}{3} & 1 & 0 \\ \frac{4}{3} & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} 3 & 6 & 9 & \\ \end{array} \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 3 & 3 \end{pmatrix} \quad \begin{matrix} x_6 = 0 \\ x_9 = 1 \end{matrix}$$

$$\begin{array}{cccc} 1 & 4 & 7 & \\ \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & -1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 4 \end{array}\right) \quad \begin{matrix} x_1 + \frac{1}{3} = 1 & x_1 = \frac{2}{3} \\ x_4 + \frac{8}{3} = 3 & x_4 = \frac{1}{3} \\ 3x_7 = 4 & x_7 = \frac{4}{3} \end{matrix}$$

$$\begin{array}{cccc} 2 & 5 & 8 & \\ \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \end{array}\right) \sim \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 3 & 0 \end{pmatrix} \quad x_8 = 0$$