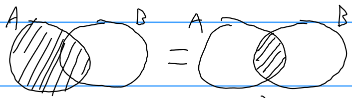
(f) $A \setminus (B \setminus A) = A \cap B$,

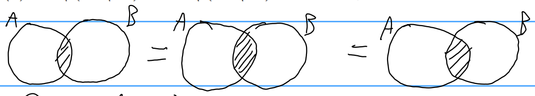


$A = \{1\} \quad B = \{2\}$

$LS = A \setminus (B \setminus A) = \{1\} \setminus \{2\} = \{1\} = A$

$PS = A \cap B = \{1\} \cap \{2\} = \emptyset$

(e) $A \setminus (A \setminus B) = B \setminus (B \setminus A) = A \cap B$,



① $A \setminus (A \setminus B) \subseteq A \cap B$

$\exists x: x \in A \setminus (A \setminus B) \Rightarrow x \in A \wedge x \notin (A \setminus B) \Rightarrow x \in A \wedge \neg(x \in A \wedge x \notin B) \Rightarrow x \in A \wedge (x \notin A \vee x \in B) \Rightarrow$

$\Rightarrow \cancel{(x \in A \wedge x \notin A)} \vee (x \in A \wedge x \in B) \Rightarrow x \in A \wedge x \in B \Rightarrow x \in A \cap B = PS$

SPOR

② $A \setminus (A \setminus B) \subseteq B \setminus (B \setminus A)$ $\quad x \in B \wedge x \notin (B \setminus A) \Rightarrow x \in B \wedge (x \notin B \vee x \in A) \Rightarrow \cancel{(x \in B \wedge x \notin B)} \vee (x \in B \wedge x \in A) = x \in B \cap A$

$\exists x: x \in A \setminus (A \setminus B) \Rightarrow x \in A \wedge x \notin (A \setminus B) \Rightarrow x \in A \wedge (x \notin A \vee x \in B) \Rightarrow x \in A \cap B$

(d) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

① $A \setminus (B \cap C) \subseteq (A \setminus B) \cup (A \setminus C)$

$x \in A \setminus (B \cap C) \Rightarrow x \in A \wedge x \notin (B \cap C) \Rightarrow x \in A \wedge \neg(x \in B \wedge x \in C) \Rightarrow x \in A \wedge (x \notin B \vee x \notin C) \Rightarrow$

$(x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C) \Rightarrow x \in (A \setminus B) \cup (A \setminus C) = PS$

② $(A \setminus B) \cup (A \setminus C) \subseteq A \setminus (B \cap C)$

$x \in (A \setminus B) \vee x \in (A \setminus C) \Rightarrow (x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C) \Rightarrow x \in A \wedge (x \notin B \vee x \notin C) \Rightarrow$

$x \in A \wedge x \notin (B \cap C) \Rightarrow x \in A \setminus (B \cap C) = PS$

(xii) $1 \cdot 2 + 2 \cdot 3 + \cdots + n \cdot (n+1) = \frac{1}{3} n(n+1)(n+2)$

① $n = 1$

$LS = 1 \cdot 2 = 2 \qquad PS = \frac{1}{3}(2)(3) = 2$

② $V(k) \Rightarrow V(k+1)$

$V(k+1): 1 \cdot 2 + 2 \cdot 3 + \ldots + k \cdot (k+1) + (k+1)(k+2) = \frac{1}{3}(k+1)(k+2)(k+3) = \frac{1}{3}(k^2+3k+2)(k+3) = \frac{1}{3}(k^3+6k^2+11k+6)$

$LS = \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2) = \frac{1}{3}(k^2+k)(k+2) + (k^2+3k+2) = \frac{1}{3}(k^3+3k^2+2k) + (3k^2+9k+6) = \frac{1}{3}(k^3+6k^2+11k+6)$ $LS = PS$

(xiii) $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n \cdot (n+1) \cdot (n+2) = \frac{1}{4} n(n+1)(n+2)(n+3)$ ① $n = 1$ $LS = 6$ $PS = \frac{1}{\cancel{4}}(2)(3)(\cancel{4}) = 6$

$V(k+1): 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \ldots + k \cdot (k+1) \cdot (k+2) + (k+1)(k+2)(k+3) = \frac{1}{4}(k+1)(k+2)(k+3)(k+4)$

$LS = \frac{1}{4}k(k+1)(k+2)(k+3) + (k+1)(k+2)(k+3) = (k+1)(k+2)(k+3) \cdot (\frac{1}{4}k+1) = \frac{1}{4}(k+1)(k+2)(k+3)(k+4) = PS$

(xiv) $1 \cdot 2 + 2 \cdot 5 + \cdots + n \cdot (3n-1) = n^2(n+1)$

① $n=1$ $LS=2$ $PS=1(2)=2$

$a=1$, $b=3$, $c=2$ $3^2-4\cdot1\cdot2=1$ $\frac{-3\pm1}{2}$ -2, -1

② $V(k) \Rightarrow V(k+1)$

$V(k+1)$: $1\cdot2+2\cdot5+\ldots+k\cdot(3k-1)+(k+1)(3k+2)=(k+1)^2(k+2)$

$LS = k^2(k+1)+(k+1)(3k+2) = (k+1)(k^2+3k+2) = (k+1)(k+1)(k+2) = (k+1)^2(k+2) = PS$

(xv) $1 \cdot 3 + 3 \cdot 5 + \cdots + (2n-1) \cdot (2n+1) = \frac{1}{3}n(4n^2+6n-1)$

$4k^2+4+6k+6-1$

$4(k+1)^2+6(k+1)-1$

$4k^2+4$

$a=4$, $b=6$, $c=-1$ $36-4\cdot4\cdot(-1)=52$

① $n=1$ $LS=3$ $PS=\frac{1}{3}(4+6-1)=3$

② $V(k+1)$: $1\cdot3+3\cdot5+\ldots+(2k-1)(2k+1)+(2k+1)(2k+3)=\frac{1}{3}(k+1)(4k^2+6k+9)=\frac{1}{3}(4k^3+10k^2+15k+9)$

$LS=\frac{1}{3}k(4k^2+6k-1)+(2k+1)(2k+3)=\frac{1}{3}(4k^3+6k^2-k)+(\underset{12}{4k^2}+\underset{24}{8k}+\underset{9}{3})=\frac{1}{3}($ ✗

(xvi) $1 \cdot 4 + 2 \cdot 7 + 3 \cdot 10 + \cdots + n \cdot (3n+1) = n(n+1)^2$

① $n=1$ $LS=4$ $PS=2^2=4$ $LS=PS$

$a=1$, $b=4$, $c=4$ $16-4\cdot4=0$ $\frac{-4}{2}=-2$

②

$V(k+1)$: $1\cdot4+2\cdot7+3\cdot10+\ldots+k\cdot(3k+1)+(k+1)(3k+4)=(k+1)(k+2)^2$

$LS=k(k+1)^2+(k+1)(3k+4)=(k+1)(k\cdot(k+1)+3k+4)=(k+1)(k^2+4k+4)=(k+1)(k+2)(k+2)=(k+1)(k+2)^2=PS$

(ii) $\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}>\frac{13}{24}$

① $n=2$ $LS=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}=\frac{14}{24}>\frac{13}{24}$ ✓

② $V(k+1)$: $\frac{1}{k+1}+\frac{1}{k+2}+\ldots+\frac{1}{2k}+\frac{1}{2k+1}+\frac{1}{2k+2}>\frac{13}{24}$

$\frac{13}{24}+\frac{1}{2k+1}+\frac{1}{2k+2}>\frac{13}{24} \Rightarrow \frac{1}{2k+1}+\frac{1}{2k+2}>0 \Rightarrow \frac{2k+1+2k+2}{(2k+1)(2k+2)}=\frac{4k+3}{4k^2+6k+2}>0$