13.10 IDM - 4 (REV)

III SADA

1. Dokážte, že pre každé nenulové prirodzené číslo n platí:

(i) $1 + 2 + \cdots + n = \frac{1}{2}n(n+1)$

(1) $n = 1$

$LS = 1$

$PS = \frac{1}{2} 1 (2) = 1$

(2) $V(k+1)$

$V(k+1)$: $1 + 2 + \ldots + k + (k+1) = \frac{1}{2}(k+1)(k+2)$ $\qquad k^2 - 4ac = \frac{9}{4} - 4 \cdot \frac{1}{2} \cdot 1 = \frac{9}{4} - \frac{8}{4} = \frac{1}{4}$

$LS = \frac{1}{2}k(k+1) + (k+1) = \frac{1}{2}k^2 + \frac{1}{2}k + k + 1 = \frac{1}{2}k^2 + \frac{3}{2}k + 1 =$ NEVIM $\qquad \frac{-\frac{3}{2} \pm \frac{1}{2}}{1} \begin{matrix} -1 \\ -2 \end{matrix}$

(ii) $1 + 3 + 5 + \cdots + (2n-1) = n^2$

(1) $n = 1$

$LS = PS$

(2) $V(k) \Rightarrow V(k+1)$

$V(k+1)$: $1 + 3 + 5 + \ldots + (2k-1) + (2k+1) = (k+1)^2$ $\qquad 4 - 4 = 0$, $\frac{-2}{2} = -1$

$LS = k^2 + (2k+1) = k^2 + 2k + 1 = (k+1)(k+1) = (k+1)^2 = P$

(iii) $2 + 3 + 4 + \cdots + (3k+2) = \frac{1}{2} \cdot (3k+1) \cdot (3k+4)$

(1) $n = 1$

$LS = 2 + 3 + 4 + 5 = 14$

$PS = \frac{1}{2}(4)(7) = 14$ $\qquad LS = PS$

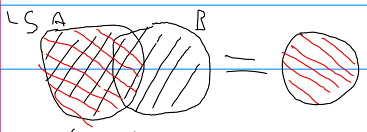
(2) $V(k) \Rightarrow V(k+1)$

$V(k+1)$: $2 + 3 + 4 + \ldots + (3k+2) + (3k+3) + (3k+4) + (3k+5) = \frac{1}{2}(3k+4)(3k+7)$

$LS = \frac{1}{2}(3k+1)(3k+4) + (3k+\overset{+1}{3}) + (3k+4) + (3k+\overset{-1}{5}) = \frac{1}{2}(3k+1)(3k+4) + 3(3k+4) = (3k+4)\left(\frac{1}{2} \cdot (3k+1) + 3\right) = \frac{1}{2}(3k+4)(3k+1+6) = PS$

1. Zistite, či pre ľubovolné množiny A, B platia nasledujúce rovnosti. V prípade, že rovnosť platí, dokážte ju (obrázok nie je dôkaz). V opačnom prípade nájdite vhodný protipríklad.

(a) $A \cap (A \cup B) = A$,



$A \cap (A \cup B) \Rightarrow x \in A \wedge x \in (A \cup B) \Rightarrow x \in A \wedge (x \in A \vee x \in B) \Rightarrow (x \in A \wedge x \in A) \vee (x \in A \wedge x \in B) \Rightarrow x \in A$?

(c) $A \setminus B = (A \cup B) \cap B$,