(xxi) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

1. $n=1$

$LS=\frac{1}{2}\quad PS=\frac{1}{2}$

$a=1$, $b=2$, $c=1$ $\quad D=0 \quad = -1$

2. $V(k)\Rightarrow V(k+1)$

$V(k): \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{k(k+1)}=\frac{k}{k+1}$

$V(k+1): \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{(k+1)}{(k+2)}$

$LS=\frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)\cancel{(k+1)}}{\cancel{(k+1)}(k+2)}=\frac{k+1}{k+2}=PS$

(xxii) $\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\cdots+\frac{1}{(n+3)(n+4)}=\frac{n}{4(n+4)}$

1. $n=1\quad LS=\frac{1}{20}\quad PS=\frac{1}{20}\quad LS=PS$

2. $V(k)\Rightarrow V(k+1)$

$a=1$, $b=5$, $c=4$ $\quad \frac{-5\pm 3}{2} \to -1,\ -4$

$V(k+1): \frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\ldots+\frac{1}{(k+3)(k+4)}+\frac{1}{(k+4)(k+5)}=\frac{k+1}{4(k+5)}$

$LS=\frac{k}{4(k+4)}+\frac{1}{(k+4)(k+5)}=\frac{k(k+5)+4}{4(k+4)(k+5)}=\frac{k^2+5k+4}{4(k+4)(k+5)}=\frac{(k+1)\cancel{(k+4)}}{4\cancel{(k+4)}(k+5)}=\frac{k+1}{4(k+5)}=PS$

(xvi) $1\cdot 4+2\cdot 7+3\cdot 10+\cdots+n\cdot(3n+1)=n(n+1)^2$

$1,\ 4,\ 4 \quad 0 \quad \frac{-4}{2}=-2$

1. $n=1\quad LS=4\quad PS=2^2=4$

2.

$V(k+1): 1\cdot 4+2\cdot 7+3\cdot 10+\ldots+k\cdot(3k+1)+(k+1)(3k+4)=(k+1)(k+2)^2$

$LS=PS$

$LS=k(k+1)^2+(k+1)(3k+4)=(k+1)\left(k(k+1)+(3k+4)\right)=(k+1)(k^2+4k+4)=(k+1)(k+2)^2=PS$