

②

$$A = \begin{pmatrix} d & 2 & 1 \\ 2 & d & 1 \\ 1 & 2 & 1 \end{pmatrix} \quad \left| \begin{array}{ccc|c} d & 2 & 1 & 1 \\ 2 & d & 1 & 0 \\ 1 & 2 & 1 & 0 \end{array} \right| = d^2 + 4 + 2 - (d + 2d + 4) = d^2 - 3d + 2 = (d-2)(d-1)$$

$d \neq 2 \quad d \neq 1 \quad d \in \mathbb{R} \setminus \{1, 2\}$

③

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{pmatrix} \xrightarrow{+} \begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{pmatrix} \xrightarrow{\cdot (-1/2)} \begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \quad B^{-1} = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix} \xrightarrow{+} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix} \xrightarrow{\cdot (-1)} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & -2 \\ -3 & 6 \end{pmatrix} \quad \left| \begin{array}{cc|c} 1 & -2 & 1 \\ -3 & 6 & 0 \end{array} \right| = 0 \Rightarrow \text{NEEXISTUE } C^{-1}$$

$$(AB)^{-1} = B^{-1} \cdot A^{-1}$$

$$(AB)^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix} = \begin{pmatrix} -4 + 3/2 & 2 - 1/2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -5/2 & 3/2 \\ 2 & -1 \end{pmatrix}$$

$$(AC)^{-1} = \text{NEEXISTUE}$$

④

$$|A^{-1}| = \frac{1}{|A|} \quad |A^T| = |A| \quad |AB| = |A| \cdot |B| \quad |A \cdot A^{-1}| = |E| = 1$$

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 2 & 0 \end{pmatrix} = 4 + 4 - (6 + 4) = -2$$

$$|A \cdot A^{-1}| = 1$$

$$|A^T \cdot A| = |A| \cdot |A| = 4$$

⑤

$$|A^{-1} \cdot A^T| = |A \cdot A^{-1}| = 1$$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} = 1 + 2 - (0) = 3$$

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{3}$$