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EE-559

Homework-03

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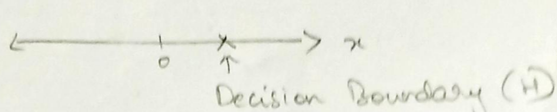
① ⓐ Non-Augmented

$D = 1$

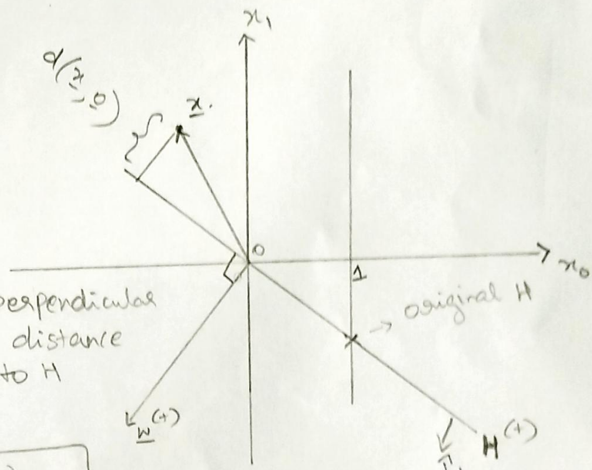
$g(x_1) = w_0 + w_1 x_1$ $\longrightarrow$



Augmented Space

$$\underline{x}^{(+)} = \begin{bmatrix} 1 \\ x_1 \end{bmatrix} \rightarrow x_0, \quad \underline{w}^{(+)} = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

$$g(\underline{x}^{(+)}) = \underline{w}^{(+)T} \underline{x}^{(+)} \longrightarrow$$



Arbitrary $\underline{x}$.

What is $d(\underline{x}, H)$

We know $d(\underline{x}, H) = d(\underline{x}, \underline{0}) \rightarrow$ perpendicular distance to H $- \, d(\underline{0}, H)$

Since H passes through $\underline{0} \rightarrow \boxed{d(\underline{0}, H) = 0}$

$$\therefore \; d(\underline{x}, H) = d(\underline{x}, \underline{0})$$

$$= \frac{\underline{w}^{(+)}}{\|\underline{w}^{(+)}\|} \cdot \underline{x}$$

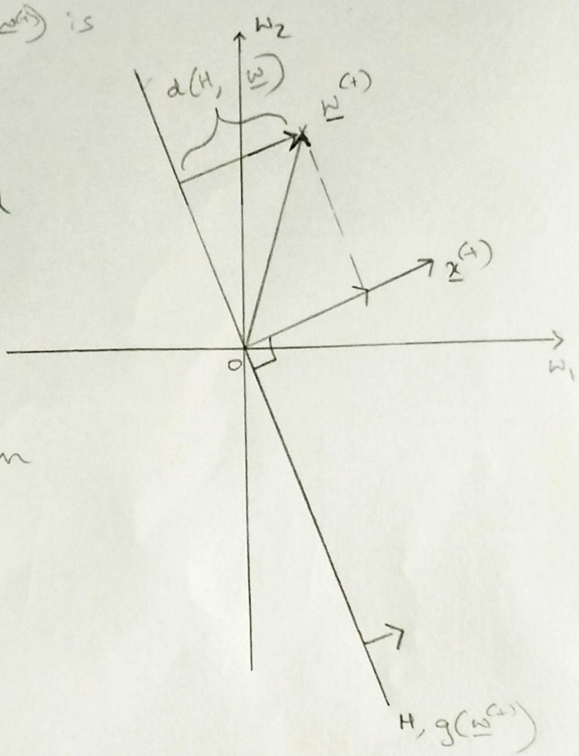
$$\boxed{d(\underline{x}, H) = \frac{g(\underline{x})}{\|\underline{w}^{(+)}\|}}$$

Here the distance would be negative as $\underline{x}^{(+)}$ is pointed opposite to $\underline{w}^{(+)}$ indicating $\underline{x}^{(+)}$ belongs to different class.

① ⓑ Weight Space

Here $g(\underline{w}^{+}) = 0$ if $\underline{x}^{(+)}$ is $\perp$ H

$\underline{x}^{(+)}$ is in direction where $g(\underline{w}^{(+)})$ is positive.



$d(H, \underline{w}^{(+)})$ can be obtained by projecting $\underline{w}^{(+)}$ on $\underline{x}^{(+)}$

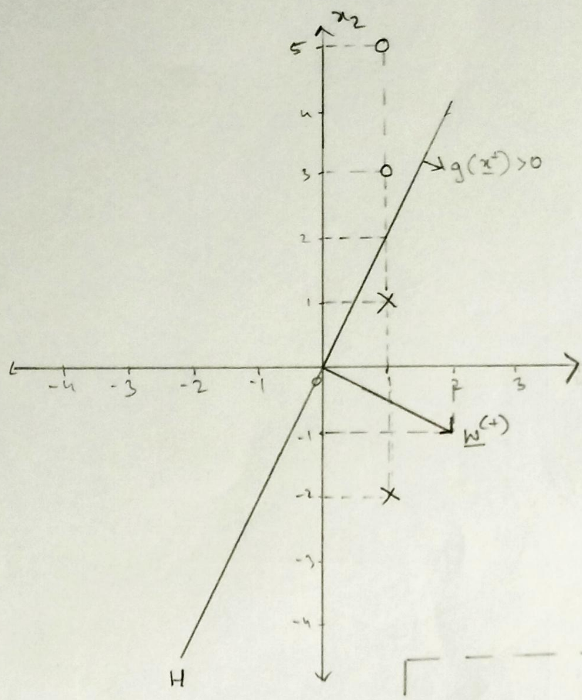
$\therefore$ By the Formula of projection of vectors

$$\boxed{d(H, \underline{w}^{(+)}) = \frac{\underline{x}^{(+)}}{\|\underline{x}^{(+)}\|} \cdot \underline{w}^{(+)}}$$

② $\underline{x}_1^{(1)} = (1, -2)$ $x_1^{(2)} = (1, 3)$

$\underbrace{\underline{x}_2^{(1)} = (1, 1)}_{X}$ $\underbrace{\underline{x}_2^{(2)} = (1, 5)}_{O}$

ⓐ



$\underline{w}^{(+)} = (2, -1)$

ⓑ $g\underline{x}_1^{(1)}(\underline{w}) = 1w_1 - 2w_2 = 0$

$(0,0), (2,1)$

$g\underline{x}_1^{(1)}([1,0]) = 1 - 0 > 0$ (good)

$g\underline{x}_2^{(1)}(\underline{w}) = 1w_1 + 1w_2 = 0$

$(0,0)$ $(1,-1)$

$gx_2^{(1)}([1,0]) = 1 + 0 > 0$ (good)

$gx_1^{(2)}(\underline{w}) = w_1 + 3w_2 = 0$

$(0,0)$ $(-3,1)$

$gx_1^{(2)}([1\ 0]) = 1 + 0 > 0$ (not good)

$gx_2^{(2)}(\underline{w}) = w_1 + 5w_2 = 0$

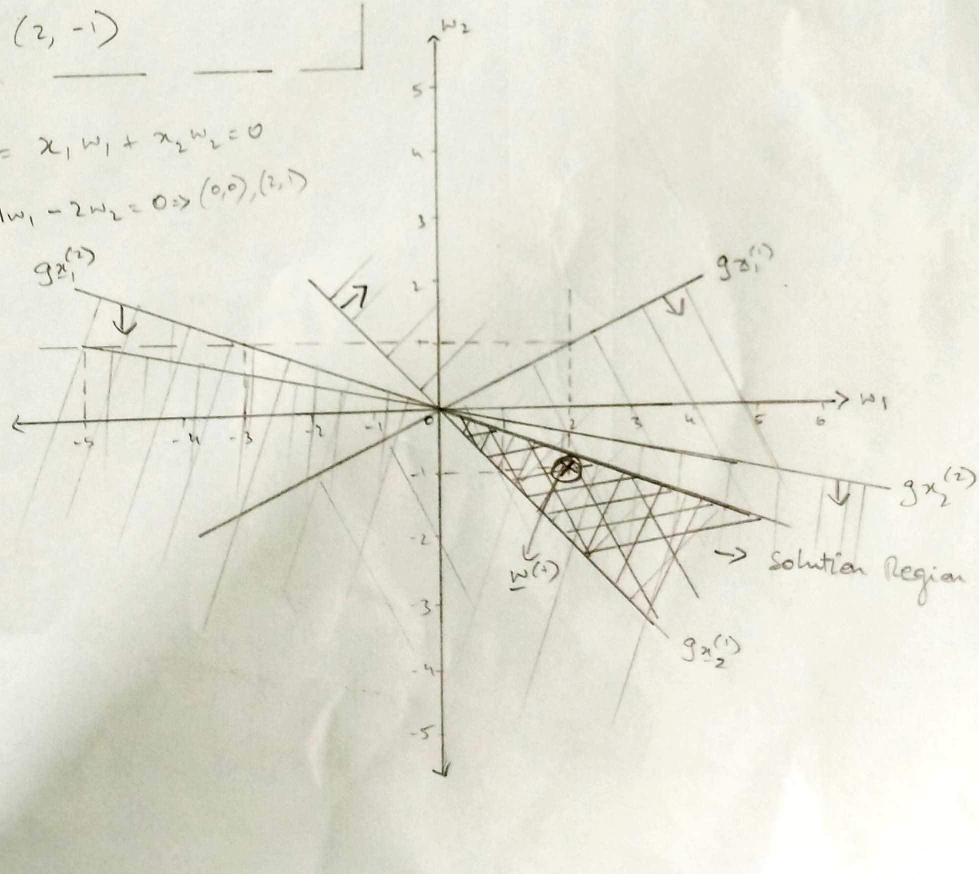
$(0,0)$ $(-5,1)$

$gx_2^{(2)}([1\ 0]) = 1 + 0 > 0$ (not good)

ⓒ $\underline{w}^{(+)}$ is in the solution region.

ⓑ $g(\underline{w}^{(+)}) = x_1 w_1 + x_2 w_2 = 0$

$g\underline{x}_1^{(1)}(\underline{w}^{(+)}) = 1w_1 - 2w_2 = 0 \Rightarrow (0,0), (2,1)$



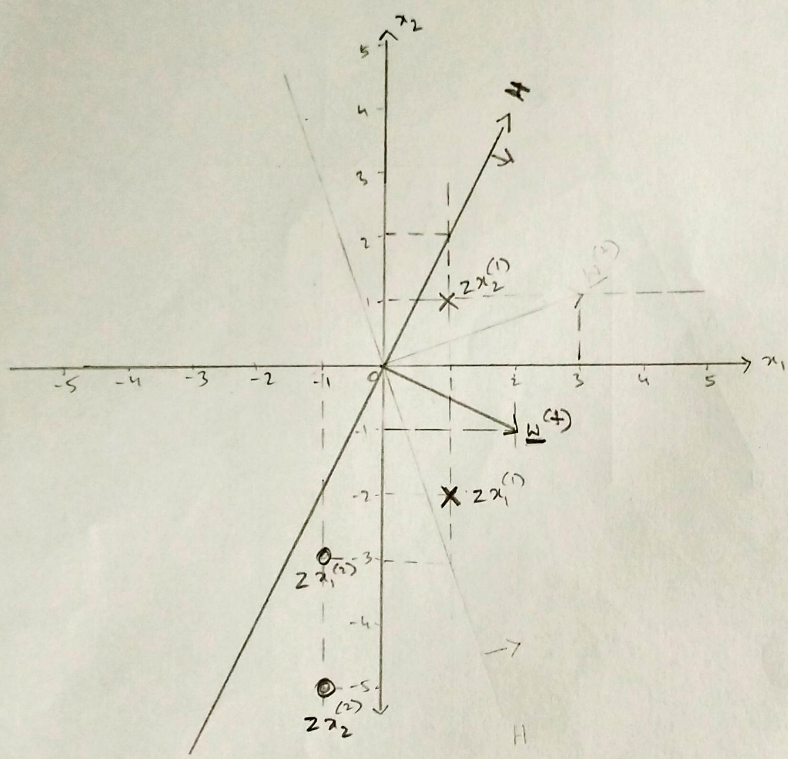
2 (a) $\underline{x}_1^{(1)} = (1,-2)$ $\quad \underline{x}_1^{(2)} = (1,3)$

$\underbrace{\underline{x}_2^{(1)} = (1,1)}_{\times}$ $\quad \underbrace{\underline{x}_2^{(2)} = (1,5)}_{o}$

$z^{(i)} = \begin{cases} +1, & \text{if } h=1 \\ -1 & \text{if } h=2 \end{cases}$

$\therefore \quad z^{(1)}\underline{x}_1^{(1)} = (1,-2) \qquad z^{(2)}\underline{x}_1^{(2)} = (-1,-3)$

$z^{(1)}\underline{x}_2^{(1)} = (1,1) \qquad z^{(2)}\underline{x}_2^{(2)} = (-1,-5)$



(c) $g(\underline{w}^{(+)}) = \underline{w}^{(+)} z \underline{x}^{(+)}$

$g_{z\underline{x}_1^{(1)}}(\underline{w}^{(+)}) = w_1 - 2w_2 = 0$

$(0,0) \quad (2,1)$

$g_{z\underline{x}_1^{(1)}}([1\ 0]) = 1 - 0 > 0$ (good)

$g_{z\underline{x}_1^{(2)}}(\underline{w}^{(+)}) = -w_1 - 3w_2 = 0$

$(0,0) \quad (-3,1)$

$g_{z\underline{x}_1^{(2)}}([1\ 0]) = -1 + 0 < 0$ (not good) (we are talking about reflected points)

$g_{z\underline{x}_2^{(1)}}(\underline{w}^{(+)}) = w_1 + w_2 = 0$

$(0,0) \quad (1,-1)$

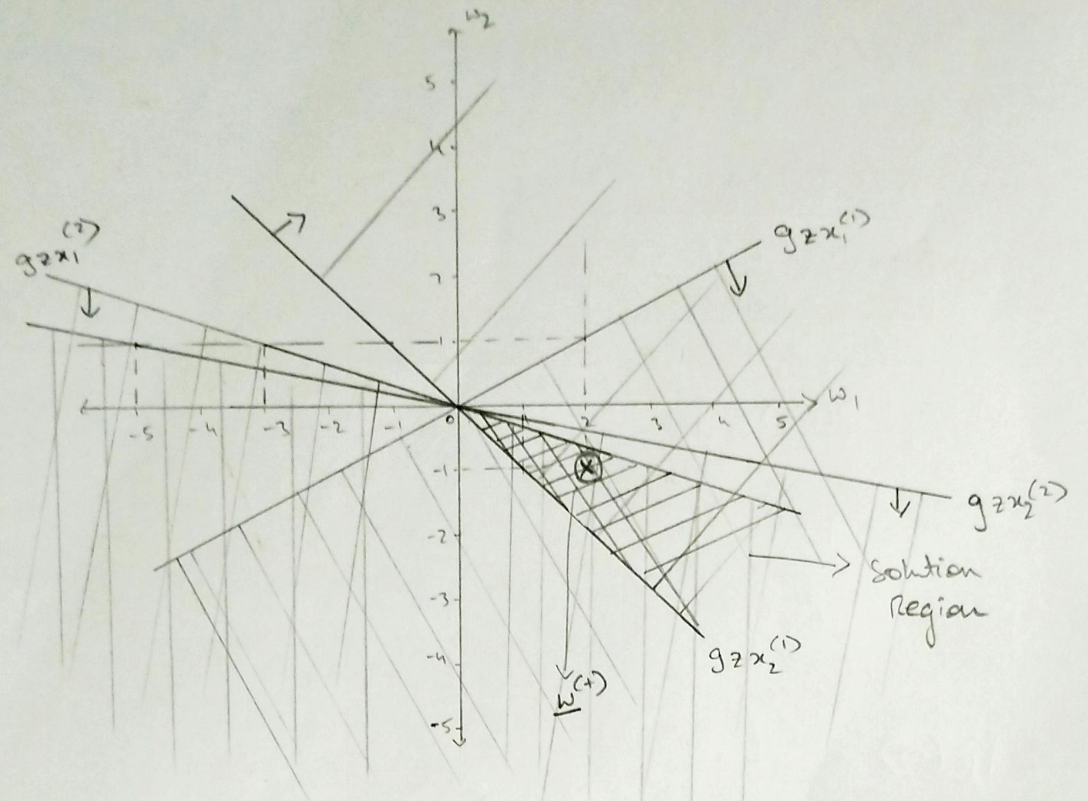
$g_{z\underline{x}_2^{(1)}}([1\ 0]) = 1 + 0 > 0$ (good)

$g_{z\underline{x}_2^{(2)}}(\underline{w}^{(+)}) = -w_1 - 5w_2 = 0$

$(0,0) \quad (-5,1)$

$g_{z\underline{x}_2^{(2)}}([1\ 0]) = -1 + 0 < 0$ (not good)

(2) (e)



(f) $\underline{w}^{(*)}$ is in solution region.

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3 (a) $p(m) = O(m) \rightarrow q(m) = m \; ; \; p(m) = 2m + 1000$

Proof

For $a = 1$

$(2m + 1000) \leq 1 \cdot m$ can't be true for large $\underline{m}$.

Larger value of a is needed.

$\underline{a = 3}$

$(2m + 1000) \leq 3 \cdot m$ will be true for $m_0 \geq 1000$

$\boxed{a = 3, \; m_0 = 1000} \rightarrow 1000 \leq 3m - 2m$

(b) $p(m) = \Omega(m) \rightarrow q(m) = m, \; p(m) = 2m + 1000$

For $b = 1$,

$1 \cdot m \leq 2m + 1000$ is satisfied f for $m_1 = 0$

$\boxed{b = 1, \; m_1 = 0}$

3 (c) Yes,

Since $p(m) = O(m)$ &

$p(m) = \Omega(m)$

$\therefore \boxed{p(m) = \Theta(m)}$

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(4) (a) Given, $p(m) = p_1(m) + p_2(m) + p_3(m)$

$$p_k(m) = O(q_k(m)) \quad , \; k = 1, 2, 3$$

Prove, $p(m) = O(q_1(m) + q_2(m) + q_3(m))$

$$p_k(m) = O(q_k(m))$$

$$\Rightarrow \quad p_k(m) \leq a_k q_k(m) \quad \text{for } m > m_k$$

$$\therefore \; p(m) = p_1(m) + p_2(m) + p_3(m) \leq$$

$$a_1 q_1(m) + a_2 q_2(m) + a_3 q_3(m) \text{ for } m > m_k$$

$$\Rightarrow \boxed{p(m) = O(q_1(m) + q_2(m) + q_3(m))}$$

(b) Given, $p(m) = p_1(m) + p_2(m) + p_3(m)$

$$p_k(m) = \Omega(q_k(m)) \quad , \; k = 1, 2, 3$$

$$\Rightarrow \; b_k q_k(m) \leq p_k(m) \quad \text{for} \quad m > m_k$$

$$b_1 q_1(m) + b_2 q_2(m) + b_3 q_3(m) \leq$$

$$p_1(m) + p_2(m) + p_2(m) = p(m)$$

4 (b) $\therefore \quad p(m) = \Omega\left(q_1(m) + q_2(m) + q_3(m)\right)$

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(5) (a) $P(m) = m^2 \log_2 m + \dfrac{2m^3}{\log_2 m} + (\log_2 m)^3$

$= P_1(m) + P_2(m) + P_3(m)$

$P_1(m) = m^2 \log_2 m$, We know $m > \log_2 m$

Let $a=1$;

$m^2 \log_2 m \leq 1 \cdot m^3$ for m_0

$\therefore \boxed{P_1(m) = O(m^3)}$

$P_2(m) = \dfrac{2m^3}{\log_2 m}$

$\Rightarrow \dfrac{m^3}{\log_2 m} \leq 1 \cdot m^3$; As $\log_2 m \geq 1$

$\Rightarrow \boxed{P_2(m) = O(m^3)}$

$P_3(m) = (\log_2 m)^3$

$(\log_2 m)^3 \leq 1 \cdot m^3$, as $m > \log_2 m$

$\Rightarrow \boxed{P_3(m) = O(m^3)}$

Using Proof from Q (4)

$P(m) = O(m^3 + m^3 + m^3) = O(3m^3) = O(m^3)$

(b) (5) $P(m) = \underbrace{m^2 \log_2 m}_{P_1(m)} + \underbrace{\frac{2m^3}{\log_2 m}}_{P_2(m)} + \underbrace{(\log_2 m)^3}_{P_3(m)}$

$\boxed{b = 1}$

$P_1(m) = m^2 \log_2 m$

$m^2 \log m \geq 1 \cdot m^3$ is not satisfied for any m

as, $\boxed{m^2 \log m < m^3}$

$\therefore \boxed{P_1(m) \neq \Omega(m^3)}$

$P_1(m) < m^3$ $\boxed{P_1(m) = \Omega(m^2)}$

$P_2(m) = \frac{2m^3}{\log_2 m}$

$\frac{2m^3}{\log_2 m} \geq 1 \cdot m^3$ is not satisfied for any m

as $\boxed{\log_2 m > 1}$

$\boxed{P_2(m) \neq \Omega(m^3)}$

$P_2(m) < m^3$ $\boxed{P_2(m) = \Omega(m^2 \log_2 n)}$

$P_3(m) = (\log_2 m)^3$

$(\log_2 m)^3 \geq 1 \cdot m^3$ not possible as

$\boxed{m > \log_2 m}$

$\boxed{P_3(m) \neq \Omega(m^3)}$

$P_3(m) < m^3$ $\boxed{P_3(m) = \Omega((\log_2 m)^2)}$

$P(m) = P_1(m) + P_2(m) + P_3(m) \leq 3m^3$ for all m

$P(m) \leq 3m^3$ for all m

⑤ ⑥

$\therefore P(m) > m^3$ is not possible

$$\therefore \boxed{P(m) \neq \Omega(m^3)}$$

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⑥ Serial Computer

$C = 2$; $N_1 = N_2 = \frac{N}{2}$; D

ⓐ Compute each mean vector ->

$$\underline{\mu}_1 = \frac{1}{N_1} \sum_{i=1}^{N/2} \underline{x}^{(1)} \quad ; \quad \underline{\mu}_2 = F_2 \sum_{i=1}^{N_2} \underline{x}^{(2)}$$

-> F_1

Multiplications -> DC

Divisions -> 1

Additions -> $\left(\frac{N}{2} - 1\right) DC$

Total Time Complexity, $T = DC\,T_M + T_D + \left(\frac{N}{2} - 1\right) DC\,T_A$

T_M, T_A, T_D -> Constants

C -> Constant

$$T = D + 1 + (N-1)D$$

$$T = D[N - 1 + 1]$$

$$\boxed{T = ND}$$

$\therefore$ Total Complexity $O(\underline{ND})$

⑥ (a) Space Complexity

Algorithm o/p → $\mu_j^{(k)}$ [Serial machine]

- Initialize $(\underline{x}_{sum}^{(k)})_j = 0 \quad \forall j, k$
- For each k
  - For each j
    - For each i
      - Load $x_{ij}^{(k)}$
      - $(\underline{x}_{sum}^{(k)})_j = \underbrace{x_{ij}^{(k)} + (\underline{x}_{sum}^{(k)})_j}_{\text{2 space needed}}$
  - $\mu_j^{(k)} = \frac{1}{N_c} \cdot (\underline{x}_{sum}^{(k)})_j$
  - Output $\mu_j^{(k)}$

∴ Space Complexity = 2 ⇒ Constant ⇒ $O(1)$

(b) Classify M points

Algo.

- For j = 1, 2, ... M → $O(c)$ space
  - Store $\underline{E}$
  - For k = 1, 2, ... C → C-Assignments
    - Initialize $\underline{E}^{(k)} = 0$
    - For i = 1, 2, ... D
      - $e = x_{ij}^{(k)} - \mu_i^{(k)}$ → $MC(D-1)$ Addition
      - $e = e \times e$ → $MC(D-1)$ Multiplication
      - $E^{(k)} = e + E^{(k)}$ → $MC(D-1)$ Addition
    - $E^{(k)} = \sqrt{E^{(k)}}$ → MCD Multiplication
  - Minimum, result = $\text{Min}(\underline{E})$ → $O(c)$ Time, $O(1)$ space
  - Output result

6 (b)

<u>Time Complexity</u>

$$T = 2MC(D-1)\,T_A + MC(D-1)\,T_M + MCD\,T_M + C\overset{\text{Time for Assignments}}{T_{AS}} + \underbrace{C}_{\text{Sort}}$$

2, C, T_A, T_M can be discarded

$$T = 1M(D-1) + M(D-1) + MD$$

$$T = MD$$

$$\therefore \quad T = O(MD)$$

<u>Space Complexity</u>

$$S = \underbrace{C}_{\substack{\text{Store} \\ C \text{ values}}} + \underbrace{1}_{\substack{\text{Store} \\ e}} + \underbrace{1}_{\text{Sort}}$$

$$S = O(C)$$

Since C is Constant.

$$\boxed{S = O(1)}$$

(d) Same as B, but C is not constant can't be discarded.

<u>Time Complexity</u>

$$T = 2MC(D-1)\,T_A + MC(D-1)\,T_A + MCD\,T_M + C\,T_{AS} + C$$

2, T_A, T_M, T_{AS} can be dropped.

$$T = MC(D-1) + MC(D-1) + MCD + C + C$$

$$T = MCD + MCD + MCD + 2C$$

$$T = C\left[MD + MD + MD + 2\right]$$

$$T = 3MDC \Rightarrow \boxed{T = O(MDC)}$$

6 ⓑ <u>Space Complexity</u>

$S = C + 1 + 1$

$S = C$

$\therefore \boxed{S = O(C)}$

ⓒ Same as ⓐ but C is not constant.

<u>Time Complexity</u>

$T = DCT_M + T_D + \left(\frac{N}{2} - 1\right) DC\, T_A$

$T_M, T_D, T_A \rightarrow$ Constants

$T = DC + 1 + (N-1) DC$

$T = DC + NDC$

$T = DC\, [1 + N]$

$T = DCN$

$\therefore$ Time Complexity $= \underline{\underline{O(DCN)}}$

<u>Space Complexity</u>

Same as ⓐ, Only 2 Space Needed.

Constant Space

Hence $\underline{\underline{O(1)}}$