1. What is a random variable in probability theory?

A **random variable** is a variable that takes on numerical values based on the outcomes of a random phenomenon.

- Example: Tossing a coin → Random variable XXX = 0 for tails, 1 for heads.

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2. What are the types of random variables?

- **Discrete Random Variable** – Takes **countable** values (e.g., number of students).
- **Continuous Random Variable** – Takes **infinite** values within a range (e.g., height, weight).

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3. What is the difference between discrete and continuous distributions?

- **Discrete Distribution**: Probability is assigned to individual values (e.g., binomial, Poisson).
- **Continuous Distribution**: Probability is spread over intervals (e.g., normal, uniform).

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4. What are probability distribution functions (PDF)?

- **PDF** gives the likelihood of a random variable taking a specific value (discrete) or lying within an interval (continuous).
- For continuous variables, the **area under the curve** represents the probability.

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5. How do cumulative distribution functions (CDF) differ from PDFs?

- **CDF** shows the probability that a variable is **less than or equal** to a value:
- **PDF** gives the **instantaneous** probability for a value or interval.
- CDF is the **integral** of the PDF (for continuous cases).

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6. What is a discrete uniform distribution?

- All outcomes have **equal probability**.
- Example: Rolling a fair die → P(1)=P(2)=...=P(6)=16P(1) = P(2) = ... = P(6) = \frac{1}{6}P(1)=P(2)=...=P(6)=61.

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7. What are the key properties of a Bernoulli distribution?

- Two possible outcomes: **Success (1)** and **Failure (0)**.
- Used for single binary trials.
- Mean = ppp, Variance = p(1−p)p(1 - p)p(1−p)

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8. What is the binomial distribution, and how is it used?

- Used to model **number of successes** in nnn independent Bernoulli trials.
- Parameters: nnn (trials), ppp (probability of success).
- Example: Tossing a coin 5 times and counting heads.

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9. What is the Poisson distribution and where is it applied?

- Models **number of events** occurring in a **fixed interval** of time or space.
- Example: Number of calls at a call center per hour.
- Parameter: λ\lambdaλ = average number of events.

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10. What is a continuous uniform distribution?

- Every value in a given interval has **equal probability**.
- PDF: f(x)=1b−af(x) = \frac{1}{b - a}f(x)=b−a1 for a≤x≤ba \leq x \leq ba≤x≤b

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11. What are the characteristics of a normal distribution?

- Bell-shaped and symmetric.
- Mean = Median = Mode.
- Defined by: **mean (μ)** and **standard deviation (σ)**.

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12. What is the standard normal distribution, and why is it important?

- A **normal distribution** with μ=0\mu = 0μ=0 and σ=1\sigma = 1σ=1.
- Used to compute probabilities and Z-scores for any normal variable.

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13. What is the Central Limit Theorem (CLT), and why is it critical?

- The **CLT** states that the **sampling distribution** of the sample mean approaches a **normal distribution** as sample size increases ($n \geq 30$), regardless of the population's shape.
- Crucial for making inferences.

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14. How does the CLT relate to the normal distribution?

- It allows us to **use the normal distribution** to approximate the behavior of the **sample mean** even if the population is not normal.

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15. What is the application of Z statistics in hypothesis testing?

- Z-statistic helps test hypotheses about population means (when σ is known).
- Used to determine how far a sample mean is from the population mean in standard deviation units.

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16. How do you calculate a Z-score, and what does it represent?

- Shows how many standard deviations a value XXX is from the mean μ\muμ.

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17. What are point estimates and interval estimates in statistics?

- **Point Estimate**: Single value estimate (e.g., sample mean).
- **Interval Estimate**: Range of values likely to contain the parameter (e.g., confidence interval).

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18. What is the significance of confidence intervals in statistical analysis?

- A **confidence interval (CI)** shows the range within which the population parameter is likely to lie with a certain confidence (e.g., 95%).

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19. What is the relationship between a Z-score and a confidence interval?

- Z-scores define the **margin of error** in confidence intervals.
  Example: For 95% CI, Z ≈ 1.96.

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20. How are Z-scores used to compare different distributions?

- Z-scores **standardize** different datasets, allowing comparison by showing relative position within each distribution.

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21. What are the assumptions for applying the Central Limit Theorem?

- Sample size n≥30n \geq 30n≥30
- Independent, identically distributed samples.
- Finite mean and variance.

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22. What is the concept of expected value in a probability distribution?

- The **expected value (E[X])** is the long-run average of a random variable.

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23. How does a probability distribution relate to the expected outcome of a random variable?

- The **expected outcome** is calculated from the **probability distribution**, showing what we expect on average after many trials.