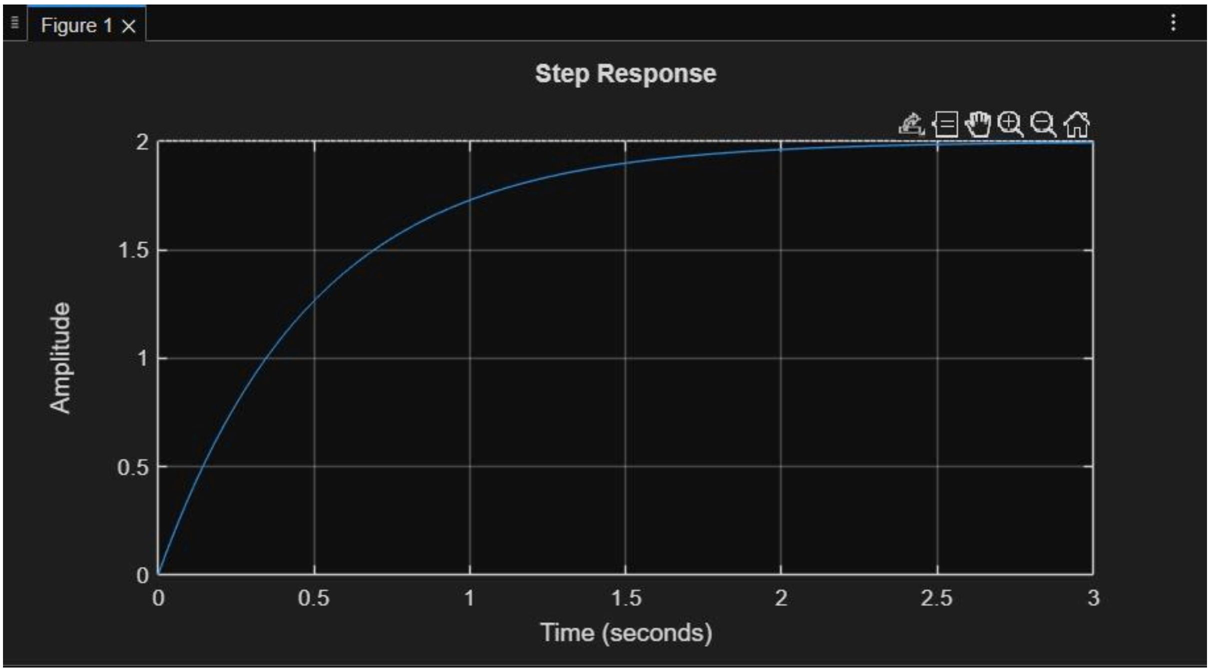

Assignment-1 :-

1) Consider the first-order plant :- $G(s) = \frac{4}{s+2}$

$\Rightarrow$ ~~i) Time constant (τ)~~

For a unit step input $R(s) = \frac{1}{s}$,

$$Y(s) = G(s)\,R(s) = \frac{4}{s(s+2)}$$

In time domain :- $y(t) = 2\,(1-e^{-2t})$

ii) * Time constant $(\tau) = \frac{1}{a} = \frac{1}{2} = 0.5s$

* Rise time $(t_r) = 2.2\,\tau = 2.2 \times \frac{1}{2} = 1.1s$

* Settling time $(t_s) = \frac{4}{a} = \frac{4}{2} = 2s$

* Final value :- $y_{ss} = \lim_{s\to 0} s\,Y(s) = \lim_{s\to 0} \frac{4}{s+2} = 2$

* Steady-state error :- $e_{ss} = 1 - y_{ss} = 1-2 = -1$

iii) $y_{ss} = \lim_{s\to 0} s\cdot G(s)\cdot\frac{1}{s} = \lim_{s\to 0} \frac{4}{s+2} = 2$ (Matlab's value & Theory's value matched)

2) $G(s) = \frac{10}{s(s+5)}$

i) Number of integrator = 1. (i.e, 1 pole at origin (s=0))
So, Type-1 system.

ii) We know that a Type-1 system has zero steady-state error to a unit-step input.

iii) Final value will be exactly 1 because there is no steady-state error.

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3) Given, $t_s < 1.2s$, $e_{ss} = 0.1$

i) * $t_{ss} \approx \frac{4}{a} < 1.2 \Rightarrow a > \frac{40}{12} = 3.33 \Longrightarrow \boxed{a=4}$

* $e_{ss} = \frac{1}{1+K} = \frac{1}{10} \Longrightarrow 10 = 1+K \Longrightarrow \boxed{K=9}$

ii) $G_{new}(s) = \frac{K}{s+a} = \frac{9}{s+4}$

iii) * It should be faster then Q1, because in Q3 pole moved more frther in LHP.

* $y_{ss} = \lim_{s\to 0} s \cdot G(s) \cdot \frac{1}{s} = \lim_{s\to 0} \frac{9}{s+4} = \frac{9}{4} = 2.25$

Higher then Q1.

4) $G(s) = \frac{3}{s+1}$, $t_s < 2$, $M_p < 10\%$, $y_{ss} = 0.8$

Controller :- $C(s) = K(s+z)$

i) Let $z = 3$, then $C(s) = K(s+3)$

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

$C(0) = K(z)$
$G(0) = 3$

* $y_{ss} = T(0) = \frac{C(0)G(0)}{1 + C(0)G(0)} = \frac{3K}{1+3K} = 0.8$

$$K = \frac{4}{3}$$

* $M_p = 100 \cdot e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} < 10\ \%$

$$0.1 = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\ln 0.1 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \implies \zeta \approx 0.59.$$

So, $\zeta \geq 0.6$ for $M_p < 10\%$.

ii) $T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{K(s+3)\frac{3}{s+1}}{1 + \frac{3K(s+3)}{(s+1)}} = \frac{4(s+3)}{5(s+1)+8}$

$$T(s) = \frac{4s+12}{5s+13}$$

iii) * Adding zero increases overshoot.

* Increasing K will increase y_{ss}
* Response will be faster

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5) $r(t) = t$ (unit ramp), i.e, $R(s) = \frac{1}{s^2}$

i) Type 0 system

ii) For unit ramp input & type 0 system, there is infinite error.

iii) $e_{ss} = \lim_{s\to 0} s\left(\frac{1}{s^2} - T(s)\frac{1}{s^2}\right) = \lim_{s\to 0} \frac{1-T(s)}{s}$

$$e_{ss} = \frac{1-T(0)}{0} = \frac{1 - 12/13}{0} = \frac{1/13}{0} = \infty.$$

iv) Adding the zero $(s+z)$ does not help ramp tracking, as it does not change the system type, so it remains poor.