Assignment-0 :-

Part-A :-

1.1) $G_1(s) = \frac{10}{s+10}$

i) Pole: $s = -10$, $G_1(0) = \frac{10}{0+10} = 1$ (i.e, DC Gain = 0 dB)

ii) $G_1(s) = \frac{10}{10(1+\frac{s}{10})} = \frac{1}{(1+s/10)}$

$G_1(j\omega) = \frac{1}{(1+\frac{j\omega}{10})}$ $\Rightarrow$ Corner frequency $= 10$

* For $\omega < 10$:- $M = 1$, i.e, $M_{dB} = 0$

For $\omega > 10$:- $M = \frac{10}{j\omega} = \frac{-10j}{\omega}$ $\Rightarrow M_{dB} = 20 \log 10 - 20 \log \omega$

* ~~About of for $\omega < 10$~~ For denominator $10 + j\omega$:-

$\angle(10 + j\omega) = \tan^{-1}\left(\frac{\omega}{10}\right)$

$\angle G_1(j\omega) = -\tan^{-1}\left(\frac{\omega}{10}\right)$ {Because the denominator contributes negative phase}

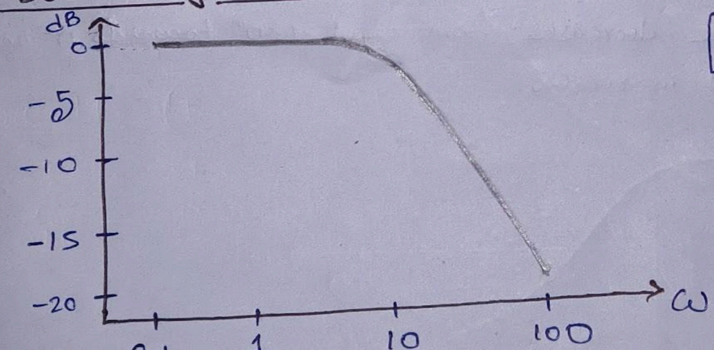
At $\omega = 0.1$, $\angle G_1 = -\tan^{-1}(0.01) \approx -0.57°$ (Approx 0°)

At $\omega = 10$, $\angle G_1 = -\tan^{-1}(1) = -45°$

At $\omega = 100$, $\angle G_1 = -\tan^{-1}(10) = -84.3°$

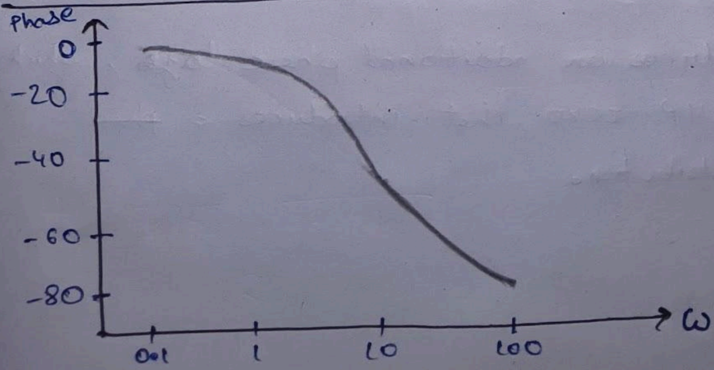
As frequency $\longrightarrow \infty$, $\angle G_1 \longrightarrow 90°$

$\Rightarrow$ **Bode Magnitude Plot :-**

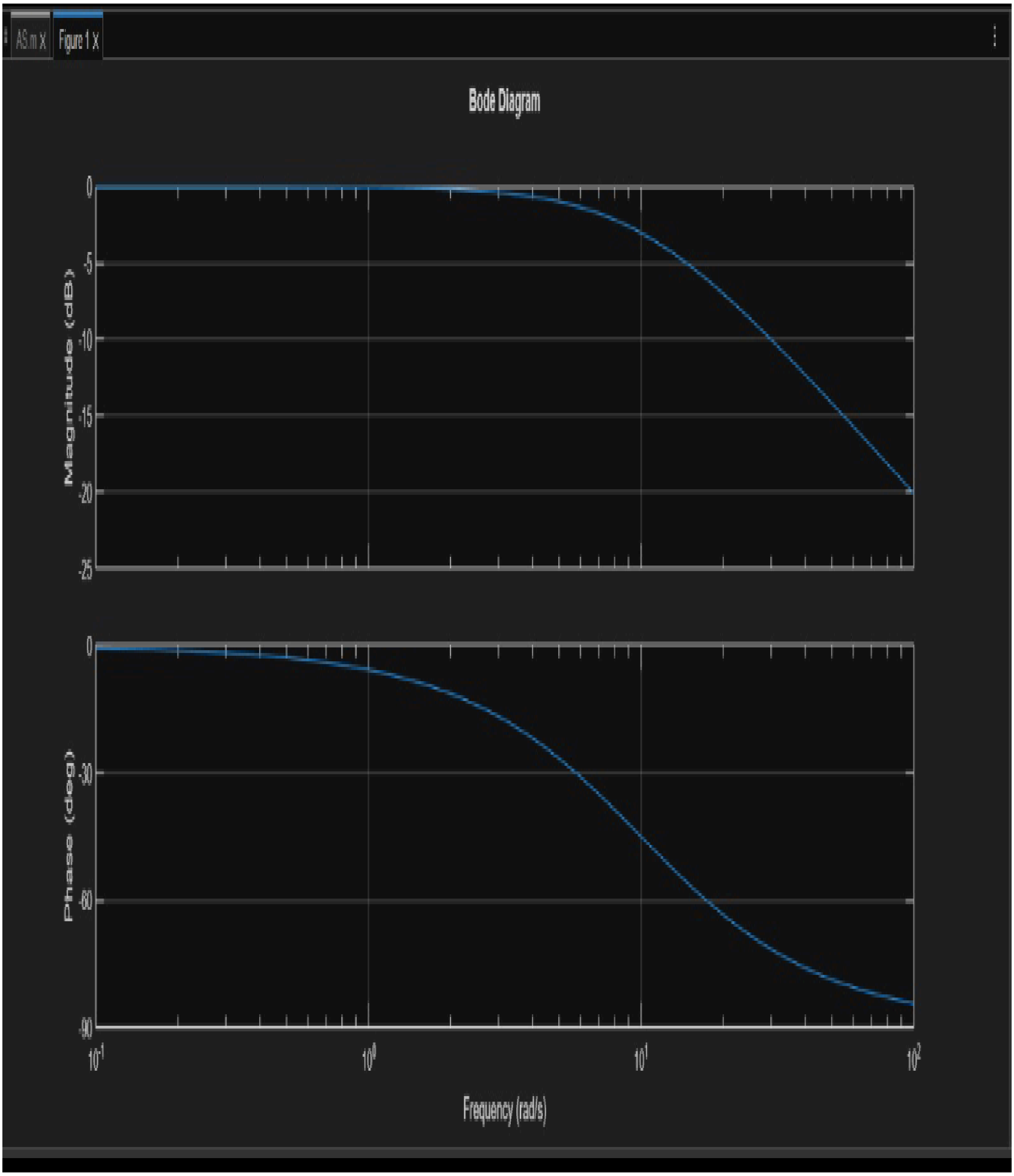


$\left[G_1 = \frac{10}{s+10}\right]$

$\Rightarrow$ **Bode Phase Plot :-**



$\left[G_1(s) = \frac{10}{s+10}\right]$



AS.m X
Figure 1 X
Bode Diagram
Magnitude (dB)
0
-5
-10
-15
-20
-25
Phase (deg)
0
-30
-60
-90
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

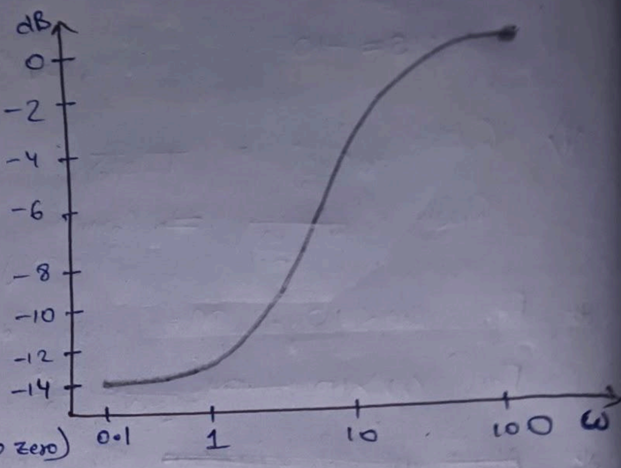
1.2) $G_2(s) = \frac{s-2}{s+10}$

i) Zero: $s = 2$, Pole: $s = -10$, $G_2(0) = -0.2$ (DC Gain ≈ -14 dB)

ii) <u>Bode Magnitude</u> :-

$$G_2(s) = \frac{2\left(\frac{s}{2}-1\right)}{10\left(\frac{s}{10}+1\right)}$$

$$G_2(j\omega) = \frac{0.2\left(\frac{j\omega}{2}-1\right)}{\left(\frac{j\omega}{10}+1\right)}$$



Corner frequency = 2, 10

For $\omega < 2$, $M = -0.2$

$M_{dB} \approx -14$ dB

For $\omega \in (2, 10)$, $M = 0.1 j\omega$

$M_{dB} = 20 \log \omega$ (due to zero)

For $\omega > 10$, $M = 1$

$M_{dB} = 0$

⇒ <u>Bode Phase Plot</u> :-

Numerator:- $\angle(j\omega - 2) = \tan^{-1}\left(\frac{\omega}{-2}\right) = \pi - \tan^{-1}\left(\frac{\omega}{2}\right)$

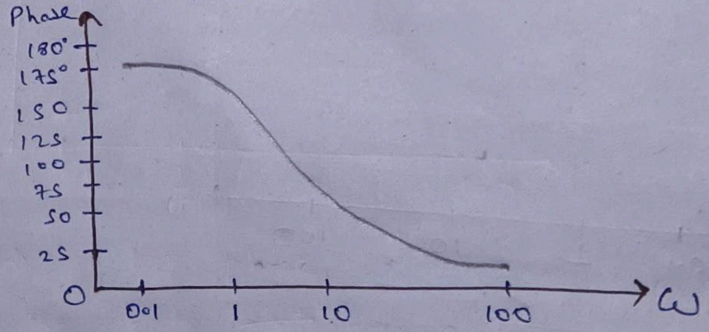
Denominator:- $\angle(j\omega + 10) = \tan^{-1}\left(\frac{\omega}{10}\right)$

$$\angle G_2(j\omega) = 180^\circ - \left(\tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{10}\right)\right)$$

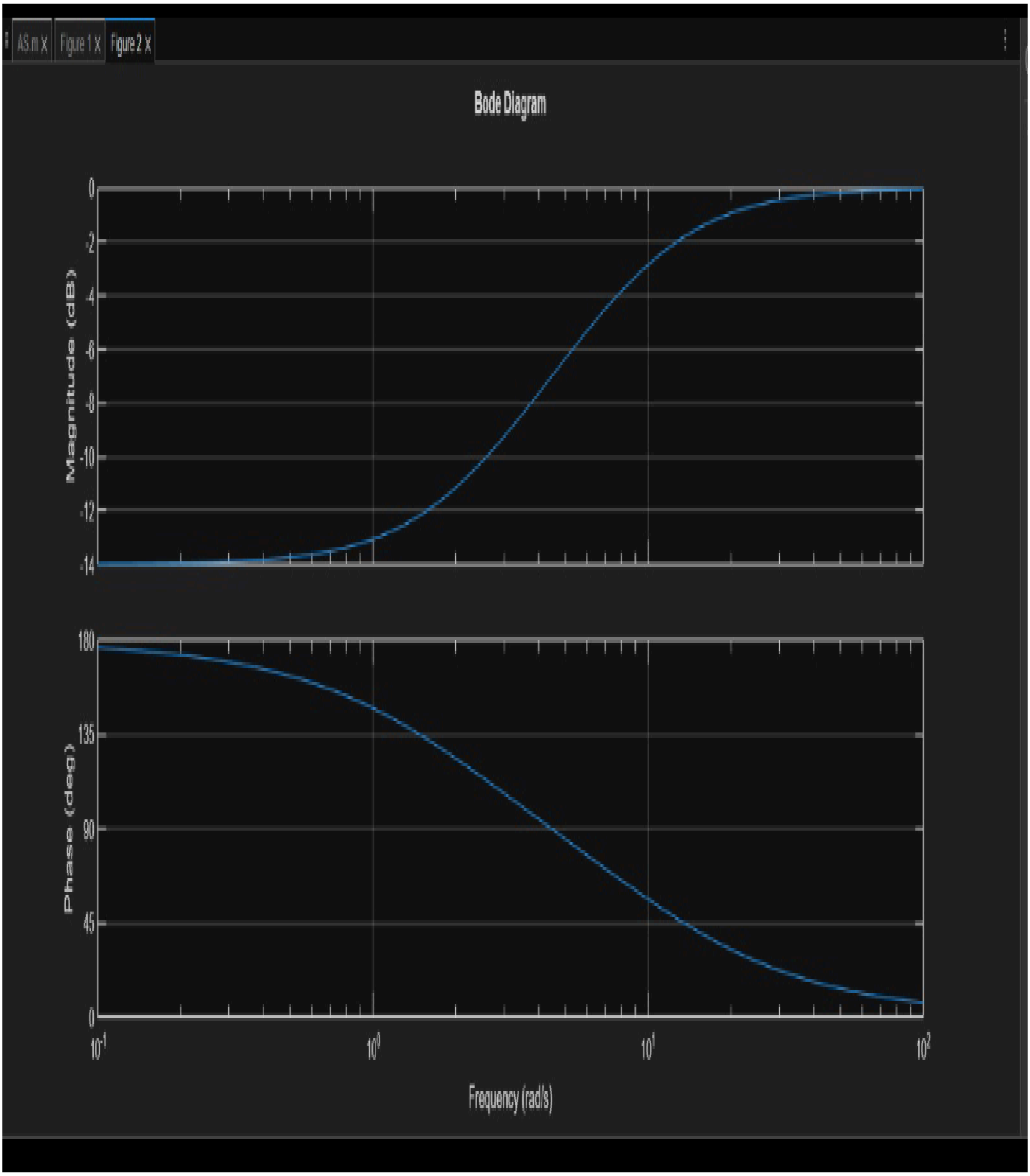
⇒ At Low ω ⇒ DC Gain is negative.

⇒ RHP zero contributes negative phase, the pole also contributes negative phase.

⇒ So total phase decreases from about 180° towards 0° and Lower as ω increases.



iv) A RHP zero introduces an additional phase lag, which in contrast to a LHP zero, that introduces a ~~phas~~ positive phase contribution.



AS.m X
Figure 1 X
Figure 2 X
Bode Diagram
Magnitude (dB)
0
-2
-4
-6
-8
-10
-12
-14
Phase (deg)
180
135
90
45
0
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

1.3) $G_3(s) = \dfrac{100}{s^2+10s+100}$

i) $s^2+10s+100 = 0$ (For finding poles)

$$s = \frac{-10 \pm \sqrt{100-400}}{2} = -5 \pm j5\sqrt{3} \approx -5 \pm j8.67$$

Poles :- $s = -5 + 8.67j$, $s = -5 - 8.67j$

DC Gain :- $G_3(0) = 1 \Rightarrow 0\,dB$.

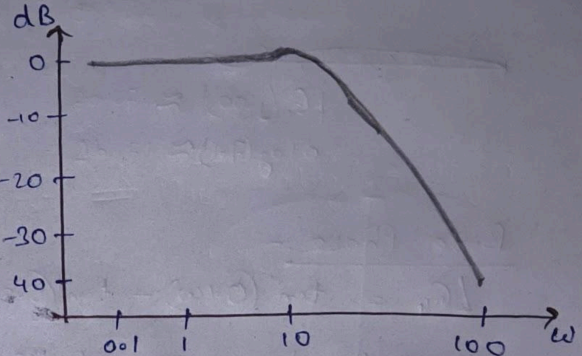
ii) Bode Magnitude :-

Natural frequency :- $\omega_n = \sqrt{100} = 10$ rad/s

For $\omega < 10$, $M_{dB} = 0$

For $\omega > 10$, slope $= -40$ dB

because, $G_3(j\omega) \approx \dfrac{100}{s^2}$



Bode Phase :-

$$G_3(j\omega) = \frac{100}{(j\omega)^2 + 10(j\omega) + 100} = \frac{100}{(100-\omega^2) + 10\omega j} \qquad \left[(j\omega)^2 = -\omega^2\right]$$

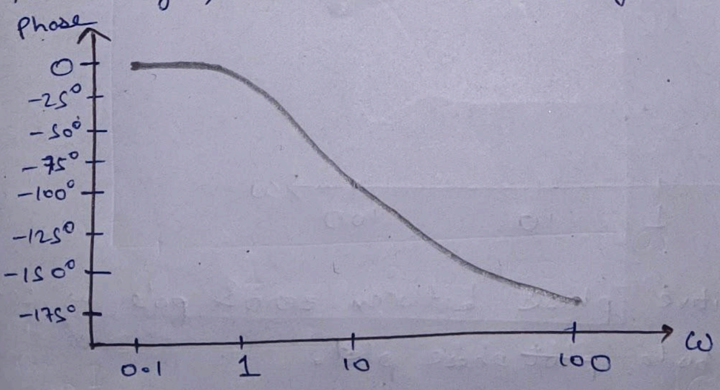
$\angle G_3(j\omega) = \angle$numerator $- \angle$denominator

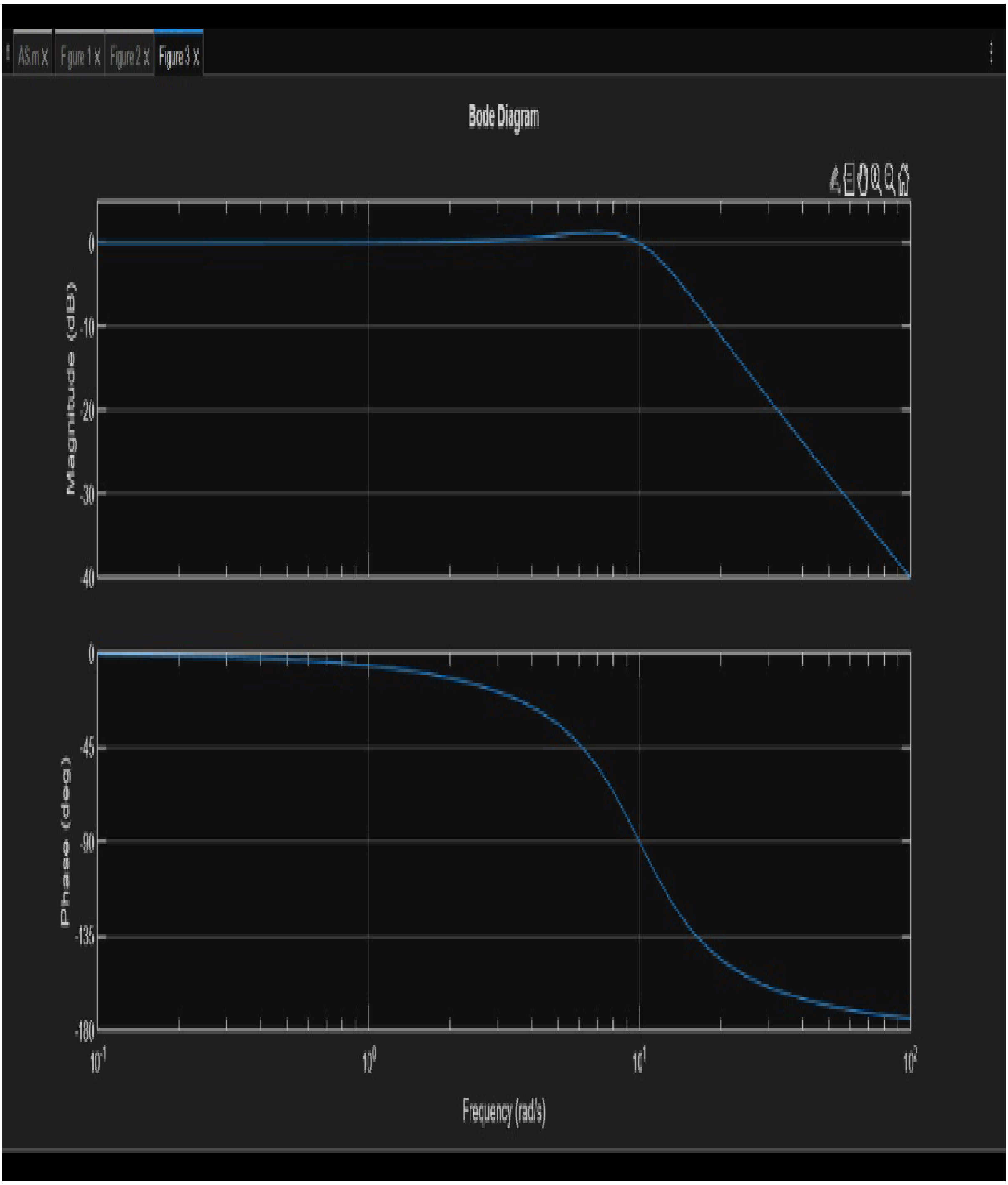
$$\angle G_3 = 0 - \tan^{-1}\left(\frac{10\omega}{100-\omega^2}\right)$$

At very low $\omega \Rightarrow 100-\omega^2 \approx 100 \Rightarrow$ imaginary part small, i.e, angle $\to 0°$
Phase $= 0°$

At $\omega = 10$, phase $\approx -90°$

At ω = high , $100-\omega^2 \lll 0$, angle of denominator $\to 180°$
Phase $= -180°$




AS.m X
Figure 1 X
Figure 2 X
Figure 3 X
Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

1.4) $G_4(s) = \dfrac{0.1s+1}{0.01s+1} = \dfrac{\left(\frac{s}{10}+1\right)}{\left(\frac{s}{100}+1\right)}$; $G_4(j\omega) = \dfrac{\left(\frac{j\omega}{10}+1\right)}{\left(\frac{j\omega}{100}+1\right)}$

i) Zero :- $s=-10$, Pole :- $s=-100$.

ii) Bode Magnitude :-

Corner frequency = 10, 100.

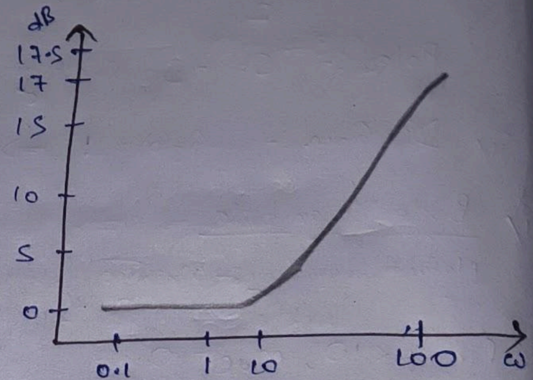
For $\omega < 10$, $M_{dB} = 0$

For $\omega \in (10, 100)$, Slope = 20 dB/decade (zero)

~~For $\omega > 100$, M_{dB}~~

$|G(j100)| \approx 7.106$

$20\log(7.1) \approx 17$ dB



Bode Phase :-

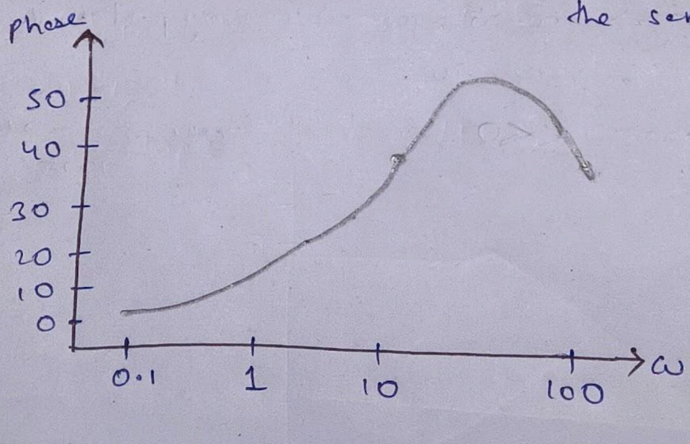
$\angle G_4 = \tan^{-1}(0.1\omega) - \tan^{-1}(0.01\omega)$

Numerator has a zero at 10 rad/s $\rightarrow$ gives positive phase

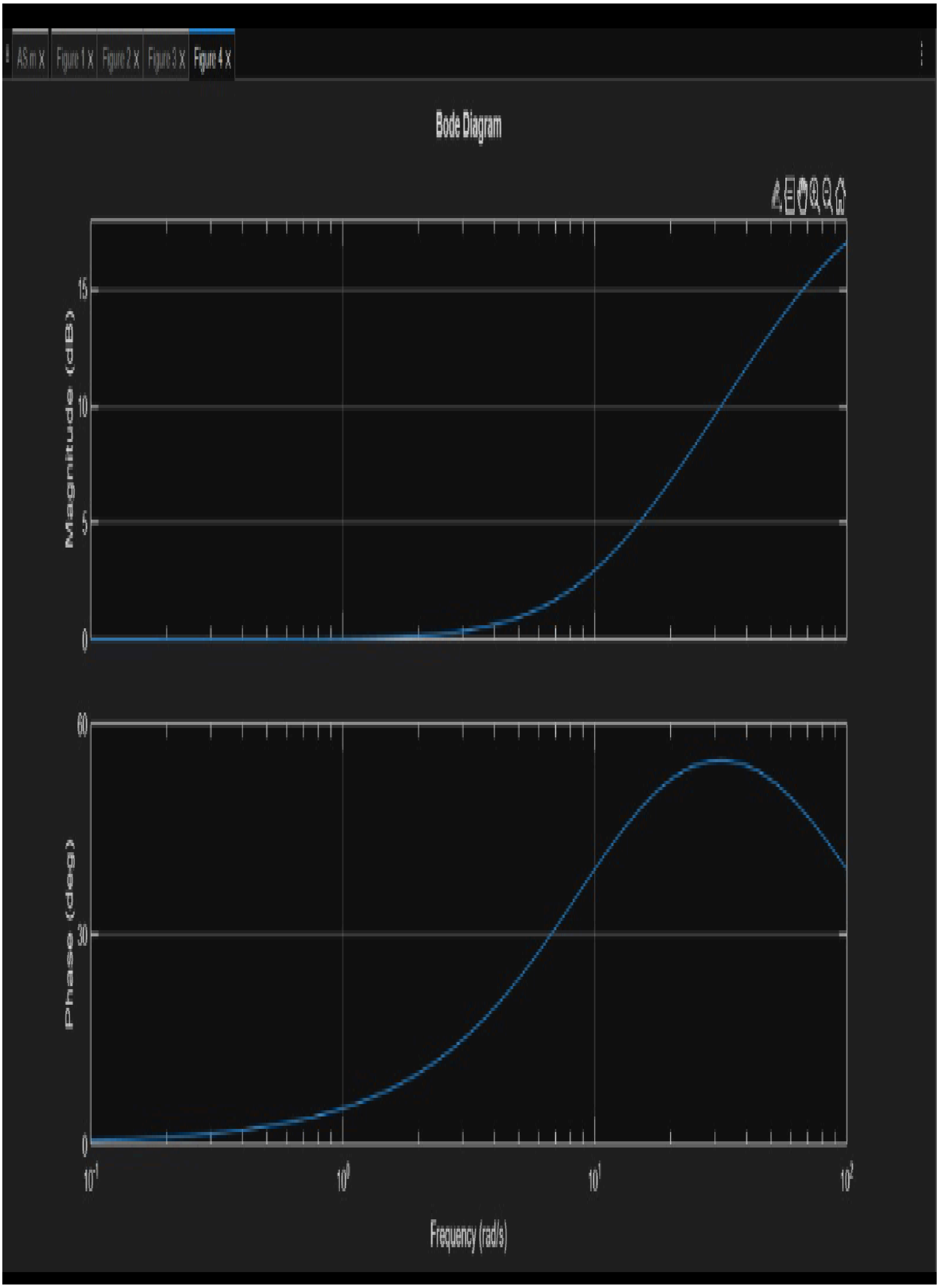
Denominator has a pole at 100 rad/s $\rightarrow$ gives negative phase.

At $\omega = 0.1$, $\angle G_4 \approx 0.51$
At $\omega = 10$, $\angle G_4 \approx 39.3$
At $\omega = 100$, $\angle G_4 \approx 39.4$
At $\omega = 50$, $\angle G_4 \approx 52.12$

$\rightarrow$ These means between $10 < \omega < 100$, the phase goes up & then comes back at the same point.



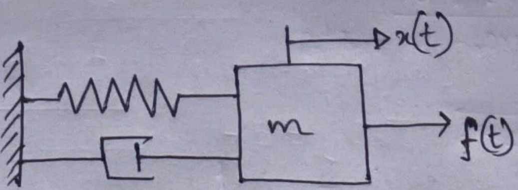
iv) G_4 adds positive phase between zero & pole as seen from the bode ~~plot~~ phase plot.



AS.m X
Figure 1 X
Figure 2 X
Figure 3 X
Figure 4 X
Bode Diagram
Magnitude (dB)
15
10
5
0
Phase (deg)
60
30
0
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

Part B :-

B.1) 1)



=> Externd Force = $f(t)$, Spring force = $-c\,x(t)$, damping force = $-d\,\frac{dx(t)}{dt}$

=> $$m\frac{d^2x(t)}{dt^2} = f(t) - c\,x(t) - \cancel{d}\; d\frac{dx(t)}{dt}$$

Ans => $$\boxed{m\frac{d^2x(t)}{dt^2} + d\frac{dx(t)}{dt} + c\,x(t) = f(t)}$$

2) Given, $$f(t) = m\frac{d^2x(t)}{dt^2} + d\frac{dx(t)}{dt} + c\,x(t)$$

Let, $x(t) \rightleftharpoons X(s)$ & $f(t) = F(s)$. Using Laplace Transform:-

=> $$F(s) = m\cdot s^2X(s) - ms\,X(0^-) - X'(0^-) + ds\,X(s) - d\,X(0^-) + c\,X(s)$$

As initial conditions are zero. So,

$$F(s) = ms^2X(s) + dsX(s) + cX(s)$$

Ans => $$\boxed{F(s) = (ms^2+ds+c)\,X(s)}$$

3) $$\boxed{G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2+ds+c}}$$

B.2) 1) Given, $m = 1\,kg$, $d = 4\ N\cdot s/m$, $c = 16\ N/m$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2+4s+16}$$

2) For poles :-

$$s^2+4s+16=0$$

$$s = \frac{-4 \pm \sqrt{16-64}}{2} = \frac{-4 \pm \sqrt{-48}}{2} = \frac{-4\pm\sqrt{48}\,j}{2}$$

$$s = -2 \pm 2\sqrt{3}\,j$$

~~Poles:- $s = -2 + 2\sqrt{4}\,j$~~

~~$s = -2 - 24j$~~

Ans => Poles:- $s = -2 + 2\sqrt{3}\,j$

$s = -2 - 2\sqrt{3}\,j$

3) Bode ~~Magnitude~~ Phase Plot :-

$\omega_n^2 = 16 \Rightarrow \omega_n = 4$ rad/s $\qquad (j\omega)^2 = -\omega^2$

$$G(s) = \frac{1}{s^2+4s+16}$$

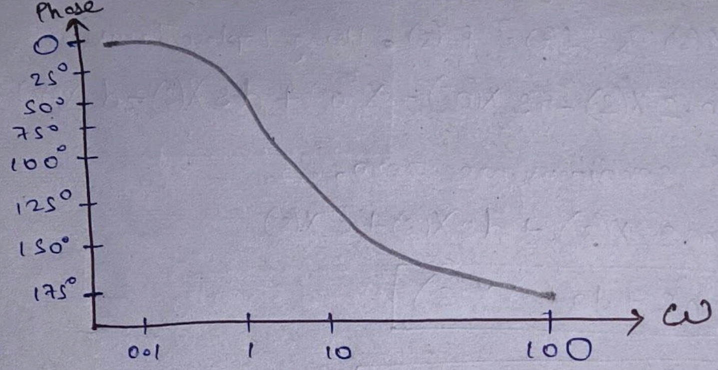
$$G(j\omega) = \frac{1}{(j\omega)^2+4j\omega+16} = \frac{1}{(16-\omega^2)+4j\omega}$$

$$\angle G = -\tan^{-1}\left(\frac{4\omega}{16-\omega^2}\right).$$

At low frequency, imaginery part small, phase $\approx 0^\circ$

At high frequency, $16-\omega^2$ becomes negative, phase $\rightarrow -180^\circ$

At mid frequency (around $\omega \approx 4$), $\angle G \approx -90^\circ$



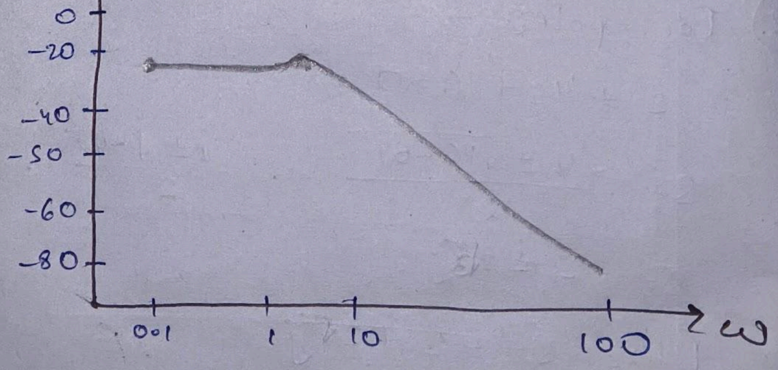
Bode Magnitude Plot :-

At $\omega = 0$, $G = \frac{1}{16}$

$M_{dB} = 20\log\frac{1}{16} \approx -24$ dB.

At $\omega > 4$, the second order term dominates, i.e, $G \approx \frac{1}{s^2}$

So, Slope = -40 dB/decade



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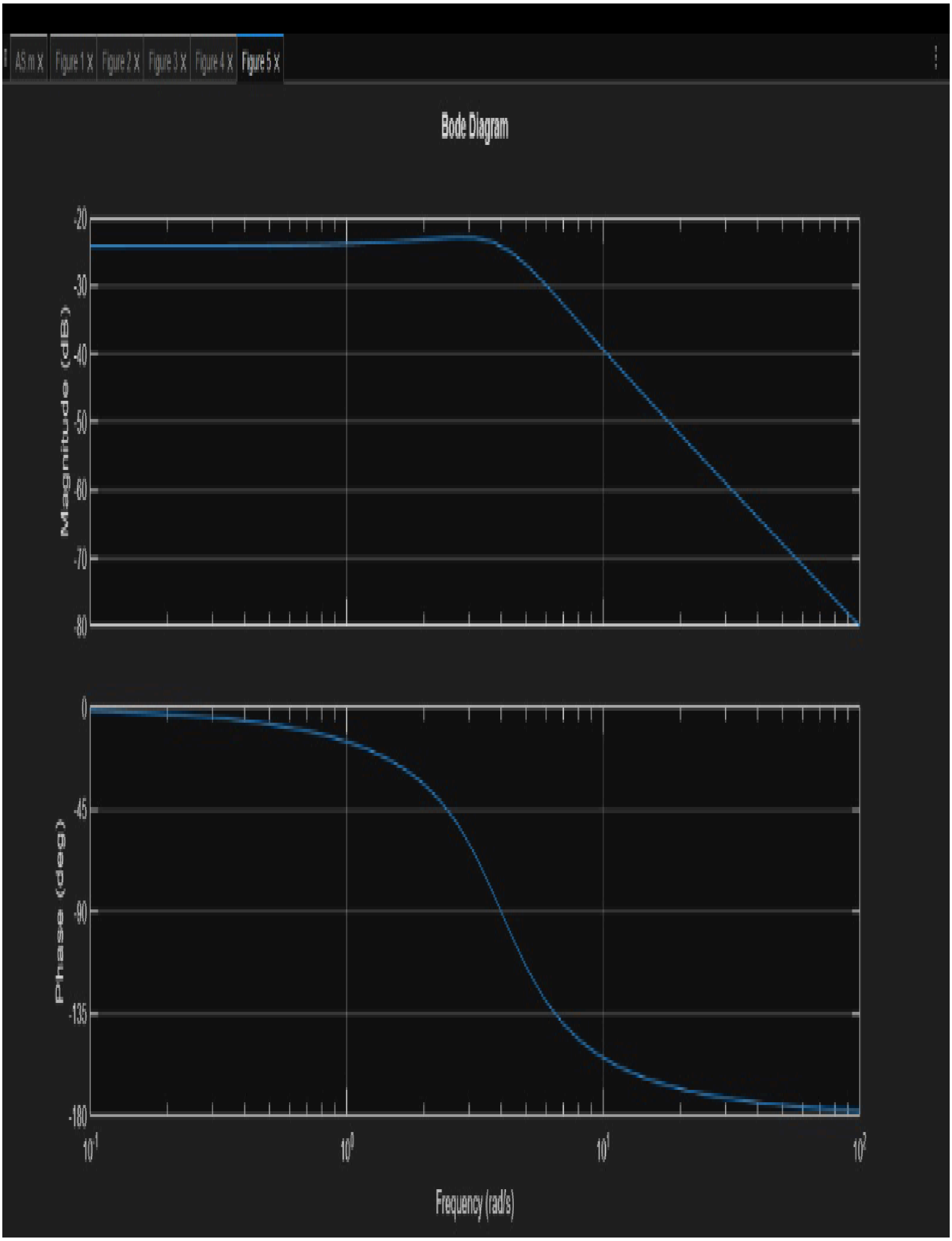
AS.m X
Figure 1 X
Figure 2 X
Figure 3 X
Figure 4 X
Figure 5 X
Bode Diagram
Magnitude (dB)
-20
-30
-40
-50
-60
-70
-80
Phase (deg)
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)