

# Magnetic Levitation Transportation

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## *Abstract*

The purpose of this report is to examine the newly developing field of magnetic levitation and its applications in transportation. The magnetic levitation train will be analyzed as well as what makes it the new evolving mode of transportation. The “maglev” train’s predecessors will be briefly mentioned, as well as their drawbacks and how the maglev system attempts to remedy those problems.

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# 1 Introduction

People have always needed to move around to different places within a certain time. Whether it be for work or to travel from one end of a country to another, trains have always been a fast and efficient way for people to travel. When John Fitch developed the steam locomotive in 1792, the train quickly became the most popular form of transportation. Trains have changed industry, as well as create it, transported cargo and people, and condense spatial and temporal distances<sup>1</sup>. People were able to get from one place to another in a much faster time than with other traditional forms of transportation.

People have also developed trains further over time to be faster and more efficient. Steam trains turned into electric trains. These electric trains then became bullet trains utilizing a high speed rail.

Whether or not a maglev or high speed rail system should be implemented lies in the situation. There is no point of having a maglev rail system within a single city. The cost of the magnets needed to power the rail is very expensive and as well as that, the distance within a city is too short to utilize the maglev system's full potential. Maglev trains almost double the speed of the current bullet trains. To reach from one end of a city to another, the maglev train would not be able to even reach its maximum speed before it reaches its terminal destination, thereby making it inefficient.

This is why maglev train systems are used for high volume inter-city transportation. These train systems are getting a lot of attention from military and private contractors. Although these trains are exclusively built in Europe (specifically Germany) and Japan, many places around the world will soon have magnetic levitation trains transporting people from one place to another. Below are some pros and cons of this new train system.

Pros	Cons
Extremely fast	Very expensive to build
Quiet operation (much less noise)	Not compatible with standard rails
Uses less energy than standard trains	
Efficient and simple to maintain	

The main drawback of the maglev system is how much time, money, and effort is needed to implement these systems. Magnets, in addition to the standard components of various metals, are required to build maglev rails, already putting the maglev train system at a severe disadvantage with regard to cost. As previously aforementioned, the maglev system is only utilized to its full potential when it can reach its maximum speed. After all, that is the main reason why people wish to build this kind of rail system. The maglev train system can only be built along a great distance and this makes it *much* more expensive than building a train with a standard or even bullet rail. However, the train does have many benefits if the proper investment is provided. It can even successfully become the successor

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<sup>1</sup>This is known as the time-space compression phenomenon

of short distance air travel. Also, the train does not touch the tracks in a maglev system. This makes maintenance much simpler when compared to a traditional train system. Now that it has been established that maglev trains are superior to traditional trains in regard to high speed transportation, what makes a maglev train the way it is can be analyzed.

## 1.1 The Magnetic Levitation Train

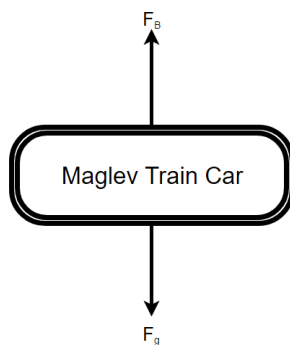
Magnetic Levitation (or “maglev”, derived from **magnetic levitation**) is essentially a train that operates via the science of electromagnetism. Magnetic levitation trains are what they are due to three core principles, listed below.

- Levitation
- Propulsion
- Guidance

### 1.1.1 Introduction to Levitation

Each train levitates above path, propelled along a path, and guided in said path by magnetism. There are two kinds of magnetic levitation rails, and each rail slightly alters the core principles of magnetic levitation. These rail forms will be discussed in the next section. Levitation simply refers to the train car not making contact with the track. This involves a magnetic force overcoming the force of gravity exerted by the maglev train car and its contents. A free body diagram of this is shown below, where  $F_B$  is the magnetic force exerted by the tracks onto the train car and  $F_g$  is the force of gravity exerted by the train car and onto the track:

Figure 1: Maglev Levitation FBD



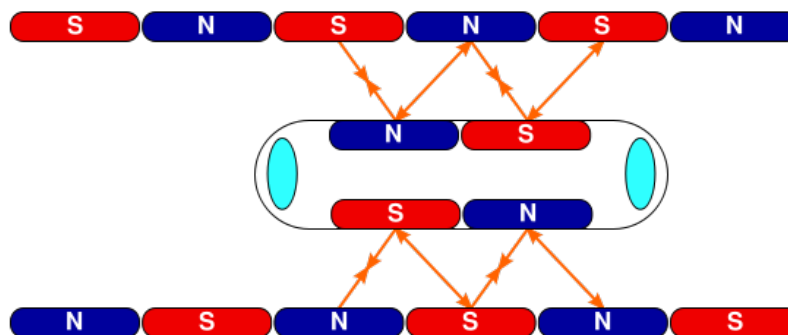
Maglev trains can operate at such high speeds because unlike a traditional train, maglev trains hover above a train, allowing them to ignore the resistance of friction. Friction between rails of a train and a track, despite the materials used, can significantly decrease the speeds of trains. Since the train hovers above a track, the only resistive forces these maglev trains

have to face is the resistive force of air resistance, a drag force. The effects of air resistance are further discussed in section 5. The actual physics and math behind the levitation of the train is discussed in section 2.

### 1.1.2 Introduction to Propulsion

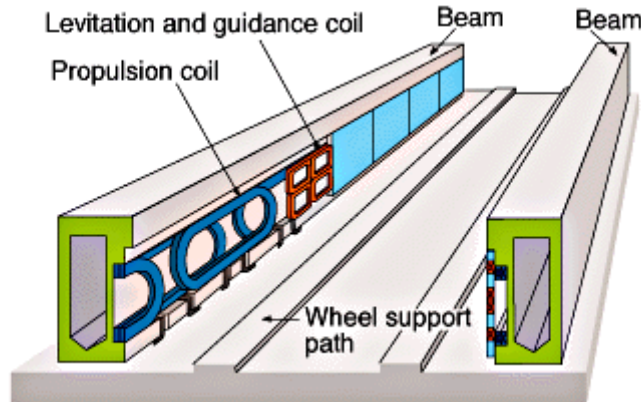
A train would not be what it is without some form of propulsion. The train needs to get from one place to another. There are a few ways how magnetic levitation trains can propel forward (or reverse - the same premise can be applied to slow the train down - known as magnetic braking). Maglev train systems, such as the ones in Japan, utilizing electrodynamic suspension, propel forward is using magnets on the side rails whose polarities are switched using an alternating current. Essentially, there are coils on the sides of the maglev train rails and magnets on the side of the train car. The magnets on the train car alternate in polarity. These coils have a current running through them, inducing an alternating polarity when the coils experience the changing magnetic field. These polarized coils then act as magnets due to the fact that a magnetic field is created. These coils react to the magnets on the sides of the trains. This illustrates the concept of propulsion in the electrodynamic suspension system (EDS - further discussed in the next section). These magnets in the side railing system alternate in polarity. The magnets on the train are then attracted to the magnets in front of them, and repelled from the magnets of opposite polarity behind them. As the train moves forward, towards the magnets it is attracted to (of opposite polarity) and away from the magnets it repels (of the same polarity), the magnets that the train car is attracted to is behind the car. To avoid an oscillating cycle of the car moving back and forth, an alternating current is then used to reverse the magnetic polarity of the coils on the side rails. The train can then continually be attracted to the magnets on the side of the rail *in front* of the car, thereby resulting in electromagnetic propulsion. This is illustrated in the figure below.

Figure 2: EDS Maglev Propulsion



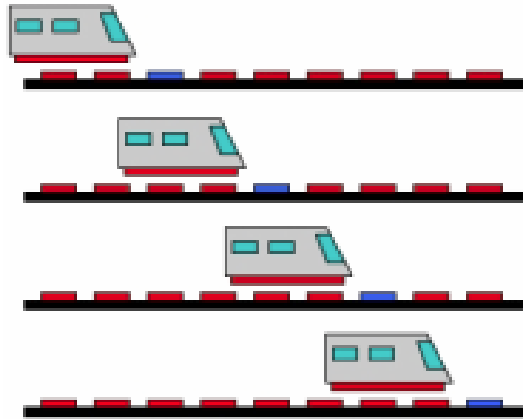
Before all of this, however, the train car is given an initial velocity with wheels on the bottom of the car. These wheels then retract and electromagnetic propulsion takes place. These wheels are also used in case of emergencies. These wheels also assist in stopping the maglev train. A diagram of the EDS rail system is illustrated below. Again, the specific rail types will be discussed in the next section.

Figure 3: EDS Rail System



As shown above, there are coils embedded within the side railings of the maglev system. Metal plates then cover the coils. These plates also serve as guidance rails. Parallel to the bottom of the train car lies the wheel support path. This is where the wheels are lodged, allowing for the train car to begin and end its journey. The same concept of alternating magnetic polarities can be used in a simplified maglev system by having the alternating current induced coils on the ground, under the train. This is illustrated below:

Figure 4: EDS Alternative Propulsion



### 1.1.3 Introduction to Guidance

Lastly, it is important for a maglev train to have some guidance. Without any form of guidance, the train would fall off of the track. In the EDS system, particularly, there are two guidance rails running parallel to the sides of the train. These guidance rails have a magnetic force that keeps the train in the center of the track. These forces would be equal in magnitude so that the train does not swerve or travel off-center.

## 1.2 Forms of Railing Systems

After many years of testing and experimenting, there have been two proper and efficient maglev railing systems that are used today. One is used in Germany and the other is used in Japan. These two are the only countries who have standardized maglev transportation. Germany has developed the TransRapid railing system, which utilizes electromagnetic suspension (EMS) whereas Japan uses the MLX railing system, which utilizes electrodynamic suspension (EDS) to levitate the train. Each railing system has its own benefits and drawbacks.

### 1.2.1 TransRapid Rail Systems

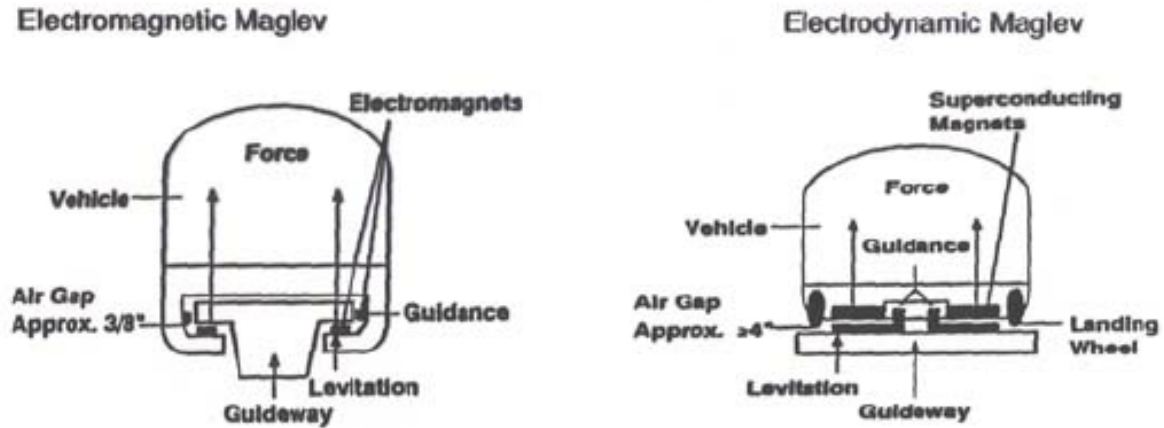
The TransRapid railing system utilizes EMS technology. In this railing system, the bottom of the train car is wrapped around the track like a C-shaped arm. Electromagnets are mounted in the part of the arm that is below the track. The electromagnets are attracted to the track and keep the vehicle hovering around the track. The strength of the magnetic field is continually altered by altering the current sent to electromagnets. The distance between the train and the track is very minimal, ranging from 10 to 15 millimeters. The TransRapid system is much simpler to implement and it does *not* require any initial velocity to sustain levitation. However, air resistance and other miscellaneous resistive forces can change the distances between the magnets on the train and the track. This causes some instabilities and minor turbulence at higher speeds. Complex stabilization systems, by altering the current that runs through the tracks, exist to counter these turbulence issues. There are still issues with turbulence in German TransRapid maglev systems, which makes this train system slightly problematic at extremely high speeds.

### 1.2.2 MLX Rail Systems

The MLX railing system utilizes EDS technology. In this railing system, magnets on the train induce currents in the guiding rails. These currents create magnetic fields which interact with the original field of the magnets. Levitation is supported by the repulsive force between the two fields. The magnets on the train are either electromagnets or an array of permanent magnets. The advantage of EDS systems is that they are naturally stable at high speeds and thus no feedback control is needed unlike with the TransRapid System. However, EDS systems have a major disadvantage. The train must be equipped with wheels because at slow speeds the induced currents are not strong enough to support levitation. This is shown in Fig. 3.

The difference between the TransRapid, utilizing EMS, and the MLX, utilizing EDS, railing systems is shown graphically below:

Figure 5: TransRapid (EMS) vs MLX (EDS) Railing Systems



## 2 Magnetic Levitation

A magnetic *levitation* train would not be what it is without levitating. After all, it is levitation and its removal of friction that allow it to reach some of its high speeds comparable to aircraft transportation. Although the two different railing systems utilize magnetism in different ways to achieve the “levitation” effect, they operate on similar premises. A TransRapid EMS system utilizes the law of attraction whereas an MLX EDS system uses the law of repulsion to “levitate” the maglev train car. Due to the fact that the TransRapid maglev system has a lot of fringing<sup>2</sup>, calculating lift force for it would be extremely complicated. Hence, the lift force for the MLX EDS maglev train system will be calculated instead, using a simplified model due to the fact that fringing still occurs with the EDS railing system at the edges of the train car and track.

### 2.1 Overcoming the Force of Gravity

As shown in Fig. 1, the train will levitate when the magnetic force exerted on the train overcomes the force of gravity. To keep the train steadily hovering over the MLX track, the magnetic force must equal the force of gravity so that there will be vertical equilibrium. Although the core principle of electrodynamic suspension is simple, calculating it is a bit of a challenge since there are quite a few factors to be considered. As previously aforementioned, there are magnetic coils under the train embedded within the track. These coils have an electric current running through them, inducing a magnetic field and therefore creating  $F_B$ . When calculating the lift force of the maglev train, eddy currents<sup>3</sup> are ignored.

First, the energy stored in a magnetic field from a solenoid must be calculated. With the

<sup>2</sup>Fringing is the effect of edges on physical phenomena. The mass, volume, and magnetic field distribution is not constant near edges so it complicates calculations

<sup>3</sup>These are localized electric currents induced in conductors by varying magnetic fields. Eddy currents are what provide propulsion to the maglev train car



energy stored in the magnetic field found, the magnetic force exerted by the coil is simple to calculate. The change in energy of the coil dependent on the current, which is the change in charge over time, is shown below as an integral of the cross product of the voltage and current with respect to time.

$$W = \int_0^t (v \times I) dt \quad (1)$$

Faraday's Law gives the voltage to be:

$$\varepsilon = V = -N \frac{d\Phi}{dt} \quad (2)$$

However, the negative sign can be neglected due to the fact the *change* is positive. Eq. 2 can be substituted into Eq. 1 to attain:

$$W = \int_0^t (N \frac{d\Phi}{dt} \times I) dt \quad (3)$$

The cross product of  $a \times b$  is equal to  $|a||b| \cos \theta$ . Since the cosine of the angle between the magnetic field induced by the coil and the current is  $0^\circ$ ,  $a \times b = ab$ . Therefore:

$$W = \int_0^t (NI \frac{d\Phi}{dt}) dt \quad (4)$$

The integral can then be rewritten to be in terms of the magnetic flux induced by the coil  $\Phi$ .

$$W = \int_0^\Phi (NI) d\Phi \quad (5)$$

To simplify the aesthetics of this equation, the product of  $N$ , the number of turns in the coil, and  $I$ , the current being run through the coil, can be written as  $F_B$ , the electromotive force of the coil. As well as that, the magnetic field strength,  $B$ , can be expressed as the electromotive force,  $F_B$  divided by the length around a turn of the coil,  $L_f$ . Therefore  $N * I = H * L_f$ . These relationships are shown below.

$$N * I = F_m \quad (6)$$

$$B = \frac{F_m}{L_f} \quad (7)$$

$$\Rightarrow N * I = B * L_f \quad (8)$$

$$W = \int_0^\Phi (H * L_f) d\Phi \quad (9)$$

Flux,  $\Phi$ , is defined as the cross product of cross sectional area of the coil,  $A_x$ , and the magnetic flux density,  $H$ . The derivative of  $\Phi$  can be taken (it is important to note that  $A_x$  stays constant and only the flux density will change) and can be substituted into Eq. 9. The cross product quantity can be eliminated because the angle between the area vector and the flux is  $0^\circ$ , making the cosine of that angle 1, allowing the cross product of  $A_x$  and  $H$  to simply be the product of the two quantities.

$$\Phi = A_x \times H \quad (10)$$

$$\Rightarrow d\Phi = A_x \times dH \quad (11)$$

$$W = \int_0^H (H * L_f)(A_x dH) \quad (12)$$

This gives the total energy in the coil. However, the energy density must be found to attain the magnetic force induced by the coil. This can be found by dividing Eq. 12 by the volume of the coil.

$$W_d = \int_0^H \frac{(H * L_f)(A_x dH)}{(L_f)(A_x)} \quad (13)$$

$$W_d = \int_0^H (B) dH \quad (14)$$

Let's assume that the magnetic field is uniform for the sake of simplicity. It can then be implied that:

$$B = \frac{H}{\mu_0} \quad (15)$$

This can then be substituted into Eq. 13.

$$W_d = \int_0^H \frac{H}{\mu_0} dH \quad (16)$$

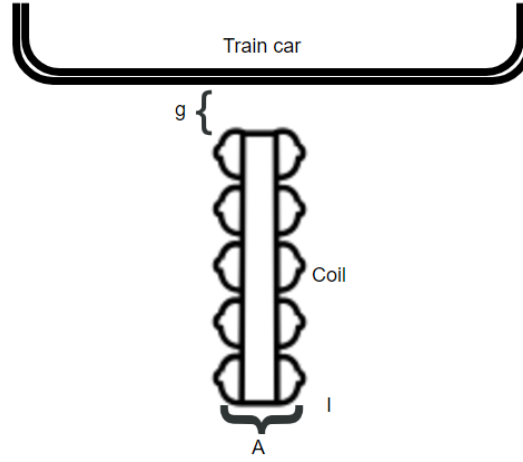
Integrating yields:

$$W_d = \frac{B^2}{2\mu_0} \quad (17)$$

Where the work done is in joules. It is also important to note that this is very similar to the formula for the energy stored in an inductor with inductance,  $L$ .

$$W_L = \frac{LI^2}{2} \quad (18)$$

Figure 6: Coil Diagram



$$\frac{B^2}{2\mu_0} \sim \frac{LI^2}{2} \quad (19)$$

Let's have the coil be on the bottom of the train, on the track, while the maglev train car is hovering above the coil by a gap distance,  $g$ , while having a cross sectional area,  $A$ . The relationship between voltage and the field energy will be used to obtain the force exerted by the coil onto the maglev train car. The fact that the force is the derivative of work with respect to the distance traveled by the force will also be used. The diagram of the coil, the maglev train car, and respective variables used are illustrated in the diagram below.

Eq. 17 gives the relationship for the energy density. It will be assumed that the magnetic field inside of the air gap is uniform. This means that the total field energy will be attained by multiplying the field density by the volume of the field.

$$V = g * A \quad (20)$$

It is important to note that  $V$  is **not** the voltage, but rather it is the volume of the gap area between the coil and the maglev train.

$$W_d = \frac{B^2}{2\mu_0} \quad (17 \text{ revisited})$$

$$W = \frac{H^2}{2\mu_0} * (V) \quad (21)$$

$$W = \frac{H^2}{2\mu_0} * (g * A) \quad (22)$$

The force exerted on the maglev train car is the derivative of work with respect to the distance the force, which in this case is the force exerted by the magnetic field. This relationship is shown below.

$$F = \frac{dW}{dg} \quad (23)$$

$$F = \frac{H^2 A}{2\mu_0} \quad (24)$$

Now, the magnetic field density, H, must be found. It can be assumed that, with uniform field density:

$$H = \frac{F_m}{L_f} \quad (25)$$

$$H = \mu_0 * B \quad (26)$$

$$H = \frac{F_m * \mu_0}{g} \quad (27)$$

Substituting Eq. 27 into Eq. 24 yields:

$$F = \frac{F_m^2 * \mu_0 * A}{(2g)^2} \quad (28)$$

Resubstituting  $F_m$  to be  $N * I$ , the final equation for the magnetic field force,  $F_B$  exerted by a coil with a current, I, running through it is:

$$F = \frac{(N * I)^2 * \mu_0 * A}{(2g)^2} \quad (29)$$

The Principle of Superposition<sup>4</sup> applies to magnetic fields, and since there are multiple coils underneath the maglev train embedded within the track, the total magnetic field strength,  $F_{total}$  can be expressed in the equation below:

$$F_{total} = \sum_{x=1}^n F \quad (30)$$

It is important to note that ‘n’ is the number of coils under the maglev train whereas ‘N’ is the number of turns per coil. It is important to note that a computer directly controls and toggles the current flowing through certain coils so that only the ones *directly below* the maglev train affect its levitation. Therefore, there is no magnetic field induced by a coil affecting the train at some displaced angle. Since F in Eq. 30 directly calls on Eq. 29,

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<sup>4</sup>This principle states that said quantities are subject to summation if they add onto each other

substitution can yield the final equation for the magnetic field strength due to multiple coils with a constant current running through them on the maglev train above them.

$$F_{total} = \sum_{x=1}^n \frac{(N * I)^2 * \mu_0 * A}{(2g)^2} \quad (31)$$

The magnetic field strength,  $F_{total}$  must overcome the force of gravity,  $F_g$ , which comprises of the maglev train car and all of its components. Once the train has hovered above the ground, the current in the coils can be reduced until  $F_{total} = F_g$  so that the car remains in vertical equilibrium so that it may be propelled forward safely.

## 2.2 Earnshaw's Theorem and Superconductivity

Earnshaw's Theorem states that it is not possible to achieve static and stable levitation using any combination of magnetic and/or electric point charges. In this scenario, static and stable levitation is a constant distance sustained between the coil producing a magnetic field due to a current running through it and a maglev train. Mathematically speaking, a scalar magnetic field  $V(x, y, z)$  obeying the Laplace Equation<sup>5</sup> shown below does *not* have any local maxima or minima.

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad (32)$$

This theorem can be proven with some multivariable calculus. Any local minimum or maximum of  $V(x, y, z)$  that does not lie on the boundary must be a stationary point. These "stationary points" are where  $\nabla V$  vanishes. However, not every "stationary point" is a minimum or maximum. Some are saddle points<sup>6</sup>. Saddle points are derived from a matrix of second derivatives evaluated at a stationary point, as shown below.

$$\mathcal{M} = \begin{bmatrix} \frac{\partial^2 V}{\partial x \partial x} & \frac{\partial^2 V}{\partial x \partial y} & \frac{\partial^2 V}{\partial x \partial z} \\ \frac{\partial^2 V}{\partial y \partial x} & \frac{\partial^2 V}{\partial y \partial y} & \frac{\partial^2 V}{\partial y \partial z} \\ \frac{\partial^2 V}{\partial z \partial x} & \frac{\partial^2 V}{\partial z \partial y} & \frac{\partial^2 V}{\partial z \partial z} \end{bmatrix} \quad (33)$$

Since this matrix is a symmetric  $3 \times 3$  matrix, it has 3 eigenvalues. Call these eigenvalues  $M_1$ ,  $M_2$ , and  $M_3$ . If all three of these eigenvalues are positive, then the stationary point is a minimum. If all three eigenvalues are negative, then the stationary point is a maximum. If some eigenvalue(s) are positive while other(s) are negative, than the stationary point being evaluated is a saddle point. The eigenvalues are the sum of the diagonal elements, also referred to as the matrix's "trace"<sup>7</sup>, in matrix  $\mathcal{M}$  denoted in Eq. 33.

$$\Delta V = \mathcal{M}_{trace} \quad (34)$$

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<sup>5</sup>A second order partial differential equation whose solutions are harmonic functions

<sup>6</sup>A point in which its derivatives of orthogonal function components equal zero

<sup>7</sup>This is the sum of the main diagonal of a matrix

$$\mathcal{M}_{trace} = \mathcal{M}_{xx} + \mathcal{M}_{yy} + \mathcal{M}_{zz} \quad (35)$$

$$\mathcal{M}_{trace} = M_1 + M_2 + M_3 \quad (36)$$

Since the magnetic field must obey the Laplace Equation in which  $\Delta V = 0$ , as shown in Eq. 32, the trace of the matrix and therefore the sum of the eigenvalues must be zero.

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad (32 \text{ revisited})$$

$$\Delta V = \mathcal{M}_{trace} = 0 \quad (32 \text{ revisited})$$

$$\mathcal{M}_{trace} = M_1 + M_2 + M_3 \quad (36 \text{ revisited})$$

$$M_1 + M_2 + M_3 = 0 \quad (37)$$

The sum of these eigenvalues is zero is impossible with all of them being either positive or negative. This means that that these points are not minima or maxima, but rather they are saddle points. Since these points are neither minima or maxima, stable equilibrium cannot be sustained. There must be either minima or maxima to have a stable magnetic potential,  $V$ . Since  $V$  is not stable, the magnetic field strength  $B$  cannot be stable either. Therefore, the force exerted by the coils onto the maglev train car is not constant. There will always be some variant changes in the field strength, affecting the gap distance between the coils on the track and the maglev train car, causing minor turbulence.

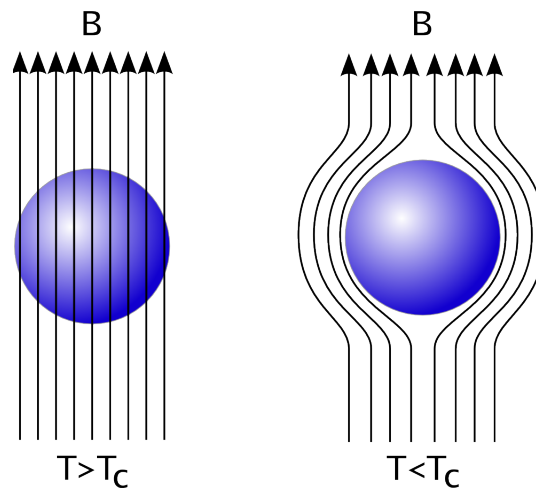
This can be countered, however, with feedback systems and diamagnetism. The Transrapid system with EMS technology utilizes feedback systems to counter turbulence whereas the MLX system with EDS technology utilizes pure diamagnetism to counter turbulence.

TransRapid train systems, involving feedback control systems which gauge the position of objects to automatically adjust magnetic field strength, are used in near all commercial maglev train systems in Germany. The position of the train in space relative to the track can be detected and fed into a control system that can vary the strength of electromagnets that are acting on the train car. The strength of the magnets can then be adjusted by tweaking the current sent to the magnets. The feedback system would simply weaken the strength of the magnet whenever the object approaches it and strengthen when it moves away, thereby maintaining a safe distance between the track and the maglev train. However, at higher speeds, the tracking of the train's position and feeding it into the control systems becomes much more complicated because the sensors located below the train have a much shorter time to track the train's position. The faster the train moves, the less time it spends above one sensor. That, combined with the seemingly minute delay of the control systems collecting, analyzing, and reacting to the data, cause some inaccuracies in adjusting the

magnet strengths of the proceeding magnets. This allows for some turbulence to occur at higher speeds with the TransRapid system.

MLX train systems, involving pure diamagnetism, are used in all commercial train systems in Japan. Diamagnetism is the phenomena in which materials that are diamagnetic are repelled by magnets. Pure diamagnetism is not possible because of Earnshaw's Theorem, however, superconductors are perfectly diamagnetic since it expels all magnetic fields. Regular conductors take in *some* magnetic fields, whereas superconductors don't. The difference between a regular conductor and a superconductor is shown below, with the regular conductor being on the left of the diagram and the superconductor being on the right of the diagram.

Figure 7: Regular Conductor vs. Superconductor



Superconductors expel all field lines, which allows for a constant magnetic field which counters Earnshaw's Theorem, which is known as the Meissner Effect. Regular conductors, such as regular electromagnets, become superconductors when they reach their critical temperature (or as close to it as possible). When this happens, two things occur.

1. The electrical resistance of the material drops to zero
2. The material becomes a perfect diamagnet

When superconductors were first discovered, they only became superconductors from regular conductors at the temperature of liquid helium<sup>8</sup>, making it extremely impractical and expensive. Having magnets at a temperature of liquid helium, or 4K, would require expensive and vast laboratory setups. However, research discovered that densely oxidized magnets reached critical temperature at the temperature of liquid nitrogen<sup>9</sup>. This is much more realistic being

<sup>8</sup>A superfluid whose temperature is 4K or -269°C

<sup>9</sup>Nitrogen in liquid state whose temperature is -196°C

that liquid nitrogen itself can be purchased in bulk by manufacturers to be used for maglev train systems, such as the MLX train system in Japan. In MLX maglev train systems, liquid nitrogen tanks are placed on the train rails to cool the magnets to such a low temperature that they become superconductors. This means that the magnets emit a constant magnetic field, countering Earnshaw's theorem. This means that at high speeds, control systems are not needed for the MLX train system to operate with minimal turbulence. This does mean, however, that MLX maglev train systems are much more expensive to implement due to the extra cost of liquid nitrogen being factored in.

### 3 Magnetic Propulsion

As previously afroementioned, in the MLX train system in Japan, there are wheels under the train that retract to give the train an initial velocity. Similar to a plane taking off, it is easier for a maglev train to levitate using electrodynamic suspension rather than vertically levitating immediately at zero velocity. As the maglev train is moving at an acceleration, or an increasing velocity, a magnetic force can constantly be applied to the train allowing it to slowly levitate. This is easier and require much less power than an immediate levitation while the maglev train is standing still. Once the maglev train is levitated, electromagnetism can be used to propel it forward.

#### 3.1 Linear Induction and Synchronous Motors

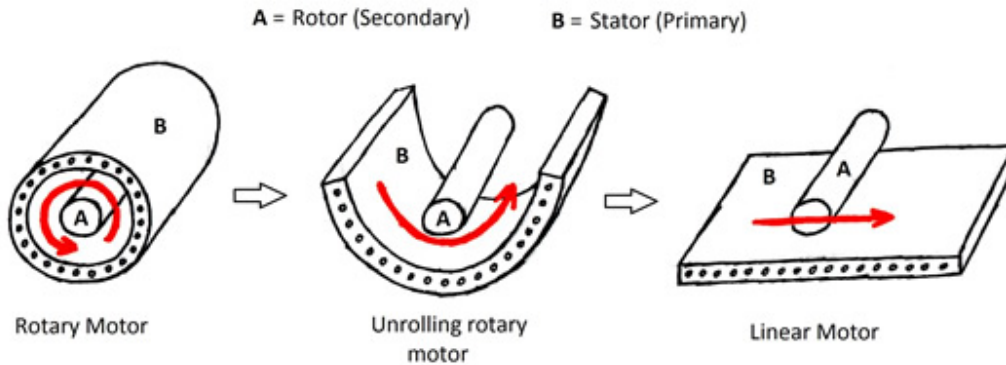
This directly ties into linear induction motors and linear synchronous motors. To reach its own "takeoff velocity" before it levitates, the MLX train system utilizes a linear induction motor. There are three different kinds of motors:

1. Rotary Motor
2. Uni-rolling Rotary Motor
3. Linear Motor

It is important to note that the rotor, denoted by the letter 'A', is the axle that rotates and the stator, denoted by the letter 'B', is the part of the motor that stays stationary and typically causes the rotor to rotate. The linear *induction* motor (or a linear synchronous motor - LSM - since there are independent magnets placed along the track) uses an induced current to produce motion. The rotor is the axle attached to wheels under the maglev train whereas the stator is the track underneath it which contains the wheel support path (refer to Fig. 3). The stator has a current running through it and this generates a magnetic field that travels down its length, increasing as the rotor travels farther downwards. This causes an acceleration, which quickly brings the maglev train to its own "takeoff" velocity. However, this motor can only produce a force while the rotor is above the stator. Once the rotor has reached the end of the stator, it stops moving. At this point, though, it becomes irrelevant because the wheels will have already been retracted due to the fact that the train is levitating



Figure 8: Different kinds of motors



above the track. The wheel support rails still remain under the train for emergency purposes for when wheels are needed in a surprise landing.

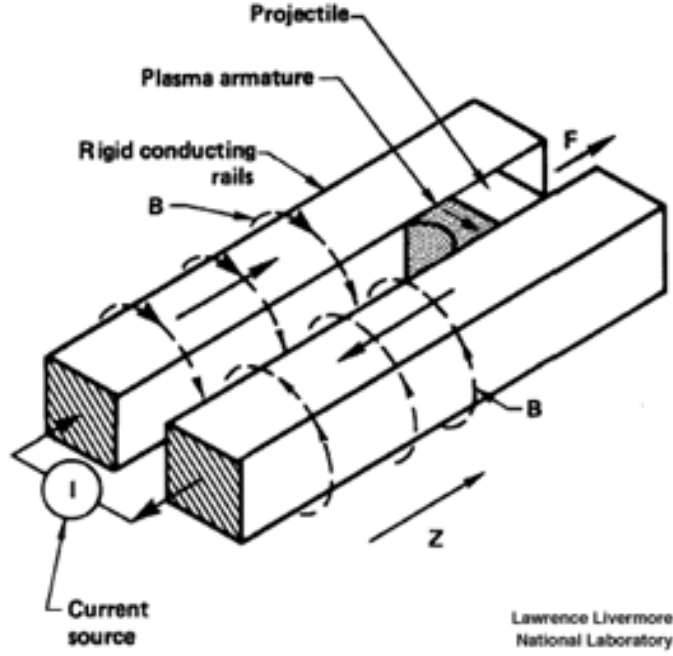
### 3.2 Electromagnetic Propulsion

The math and physics behind electromagnetic propulsion is difficult to analyze because it involves accounting for the speed at which the alternating current flows, delays when changing the polarity of the magnets beside the train to propel it forward, summing up the vertical component of the magnetic forces that propel the train forward, with air resistance being taken into account. All of this requires high caliber mathematical computing systems which, still, have significant inaccuracies. However, a model for a maglev train can be simplified with the model of a rail gun.

A railgun is a relatively simple device that uses the flow of current through a loop to generate a Lorentz force<sup>10</sup> on a projectile, thereby accelerating it to high velocities. In this case, a projectile can be a maglev train with a switch connected to a source of current being the normal source of current for a maglev train. Rather than account for the alternating current that changed the polarity of magnets to propel the maglev train car forward, it will be assumed that the magnetic force exerted on the train is similar to that of a rail gun. That way, accounting for the alternating current speed and any other delays can be neglected. The two, however, are actually comparable in their structures and physics operating it. Although railguns are in fact dangerous and primarily used in military and astronomical fields, the physics principles behind a railgun can be used to better explain a simplified magnetic levitation train model. A diagram of a railgun is shown on the next page.

<sup>10</sup>A force exerted by a magnetic field on a moving electric charge

Figure 9: Railgun Diagram



For the sake of simplicity, the maglev train will be referred to as a “projectile” as if it were a small round material in a railgun setup. The force of the projectile will be determined first by finding the the instantaneous induced magnetic field at any point. This must be calculated as a function of the loop geometry and current sent through the parallel rails. This is given by the Biot-Savart Law, shown below as:

$$d\vec{B} = \frac{\mu_0 I d\vec{L} \times \hat{r}}{4\pi r^2} \quad (38)$$

$$\vec{B}(t) = \frac{\mu_0 I(t)}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} \quad (39)$$

As well as that, Ampere’s Law should be considered, shown below.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad (40)$$

Or more formally:

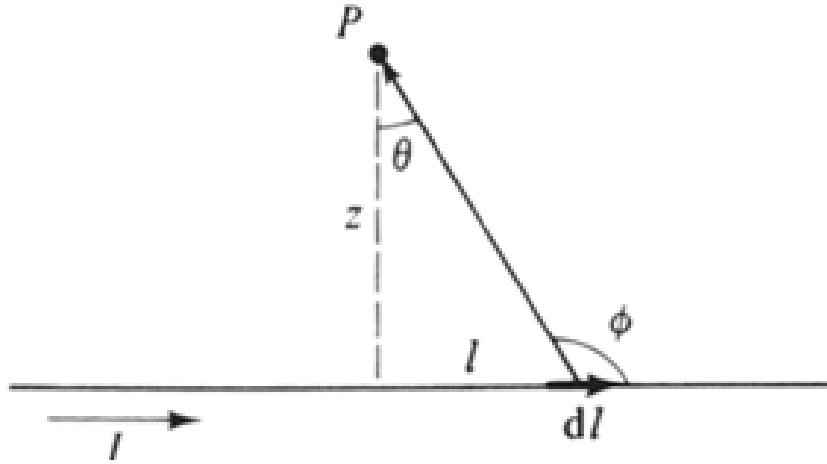
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (41)$$

Since the electric field changes with time, as the projectile travels along the track, there will be some contribution to the B field by the displaced current. For the sake of simplicity, this will be neglected and this situation will be treated as if a constant current is being supplied to the tracks. A formal equation for  $B(t)$  must be calculated, which can be done by splitting up the integral of the Biot-Savart Law. This can be split up into four parts<sup>11</sup>:

- The two parallel rails
- The connection between the rails caused by a switch
- The projectile itself

A rectangular loop of dimension  $L \times W$  with a current,  $I$ , running through it can be used to model the railgun. A diagram of this is shown below.

Figure 10: Biot-Savart Law Diagram



It is important to note that the vectors  $\vec{l}$  and  $\vec{z}$  create a right triangle, whose hypotenuse,  $h$ , is shown by the equation below:

$$h = \sqrt{z^2 + l^2} \quad (42)$$

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<sup>11</sup>This entire derivation was done with the assistance of Griffiths' *Introduction to Electrodynamics* - see sources

The magnetic field, B, at a point, P, dependent on the angle,  $\phi$ , can be expressed as:

$$\vec{B} = \frac{\mu_0 \vec{I}}{4\pi} \int_{\theta_1}^{\theta_2} \left( \frac{\cos^2 \theta}{s^2} \right) \left( \frac{s}{\cos^2 \theta} \right) \cos \theta d\theta \quad (43)$$

$$\vec{B} = \frac{\mu_0 \vec{I}}{4\pi s} \int_{\theta_1}^{\theta_2} \cos \theta d\theta \quad (44)$$

$$\vec{B} = \frac{\mu_0 \vec{I}}{4\pi s} (\sin \theta_2 - \sin \theta_1) \quad (45)$$

With Eq. 45, the magnetic field components of the two rails, the switch connecting the two rails, and the projectile can be expressed as:

$$\vec{B}_{rail1} = \frac{\mu_0 \vec{I}}{4\pi} \frac{1}{y} \left( \frac{L-x}{\sqrt{(L-x)^2 + y^2}} + \frac{x}{\sqrt{x^2 + y^2}} \right) \quad (46)$$

$$\vec{B}_{rail2} = \frac{\mu_0 \vec{I}}{4\pi} \frac{1}{W-y} \left( \frac{x}{\sqrt{x^2 + (W-y)^2}} - \frac{x-L}{\sqrt{(L-x)^2 + (W-y)^2}} \right) \quad (47)$$

$$\vec{B}_{proj} = \frac{\mu_0 \vec{I}}{4\pi} \frac{1}{L-x} \left( \frac{W-y}{\sqrt{(W-y)^2 + (L-x)^2}} + \frac{y}{\sqrt{y^2 + (L-x)^2}} \right) \quad (48)$$

$$\vec{B}_{switch} = \frac{\mu_0 \vec{I}}{4\pi} \frac{1}{x} \left( \frac{y}{\sqrt{y^2 + x^2}} - \frac{W-y}{\sqrt{x^2 + (W-y)^2}} \right) \quad (49)$$

Being that the total magnetic field,  $\vec{B}$ , invokes the principle of superposition:

$$\vec{B}_{total}(I, x, y, L, W) = \vec{B}_{rail1} + \vec{B}_{rail2} + \vec{B}_{proj} + \vec{B}_{switch} \quad (50)$$

The total magnetic field, substituting Eq. 46 - 49 into Eq. 50, yields:

$$\vec{B}_{total} = \frac{\mu_0 \vec{I}(t)}{4\pi} \left[ \frac{\sqrt{x^2 + y^2}}{xy} + \frac{\sqrt{(L(t)-x)^2 + y^2}}{(L(t)-x)y} + \frac{\sqrt{x^2 + (W-y)^2}}{x(W-y)} + \frac{\sqrt{(L(t)-x)^2 + (W-y)^2}}{(L(t)-x)(W-y)} \right] \quad (51)$$

An analytical expression for the Lorentz Force exerted by the projectile is as shown below:

$$\vec{F}_B(I, L, W) = I(t) \int dy \times \vec{B}_{proj}(I, y, L, W) \quad (52)$$

Since the projectile does not produce any contribution to the B field passing through itself, this component of the expression for B can be ignored. Therefore, this term can be eliminated and evaluating at the projectile length,  $x = L$ :

$$\vec{B}_{arm}(I, y, L, W) = \frac{\mu_0 \vec{I}}{4\pi} \left[ \frac{\sqrt{L^2(t) + y^2}}{L(t)y} + \frac{\sqrt{L^2(t) + (W - y)^2}}{L(t)(W - y)} \right] \quad (53)$$

Combining Eq. 53 with Eq. 52 yields:

$$\vec{F}_B(I, L, W) = \left[ \frac{\mu_0 L(t) I^2(t)}{\pi} \right] \left[ \frac{W^2 + L^2(t) - L(t)\sqrt{W^2 + L^2(t)}}{\sqrt{W^2 + L^2(t)}} \right] \hat{x} \quad (54)$$

According to Newton's Second Law of Motion, the acceleration of any projectile would be:

$$\vec{F} = m\vec{a} \quad (55)$$

$$\vec{a} = \frac{\vec{F}}{m} \quad (56)$$

The expression of the Lorentz magnetic force on the projectile can be simplified by creating the term  $L_1$ , the inductance gradient of the loop, reducing the expression to:

$$F_B = \frac{L_1 \cdot I(t)^2}{2} \quad (57)$$

All that would be needed is to divide Eq. 56 by the mass of the projectile to obtain its acceleration. This is shown below:

$$\vec{a} = \frac{L_1 \cdot I(t)^2}{2m} \quad (58)$$

This same equation can be used in the simplified model of an MLX maglev train shown earlier. Eq. 56 shows that an increase in current will directly increase the magnetic Lorentz force exerted on the projectile thereby increasing its acceleration. As well as that, any resistance added to the rails drastically decreases the acceleration. That is why MLX train rails involve the use of liquid nitrogen to make the rails superconductors. As previously aforementioned, superconductors have zero resistance. The use of liquid nitrogen allows the rails to have little to no resistance, allowing for the maglev train to reach its maximum potential speed possible.

To brake the maglev train, it is simple. All that needs to be done is to flip the polarities of the motors. The initial velocity of the high speed maglev train will soon decrease because magnets to the rear of the train are attracting it whereas magnets in front of the train are repelling it. The train can decelerate until it reaches a steady halt. For the MLX maglev system, the wheels soon deployed underneath the train, further assisting the train in slowing down due to the introduction of a frictional force in the system between the wheels and the track.

## 4 Magnetic Guidance

The maglev train concept is a marvel in itself, however, without the proper safety precautions, many fatalities can result. Many engineers and architects suspected this so many regulatory measures and systems were implemented to ensure the safety of the new magnetic levitation train systems.

### 4.1 Operation Control Systems

Operation Control Systems (OCS) can maintain distances between tracks and train cars in TransRapid Systems and even MLX systems. Although the OCS plays a bigger role in TransRapid systems due to its susceptibility to turbulence at higher speeds, the OCS for MLX railways still serves a paramount purpose to the safety of magnetic levitation railway travel. The OCS functions comprise of:

- Ensure safe movement of maglev trains
- Ensure safe route
- Ensure safe separation of maglev train cars and rails
- Authorize vehicle movement
- Ensure detection and management of emergency situations
- Handle emergency situation

But most importantly, the OCS roles include:

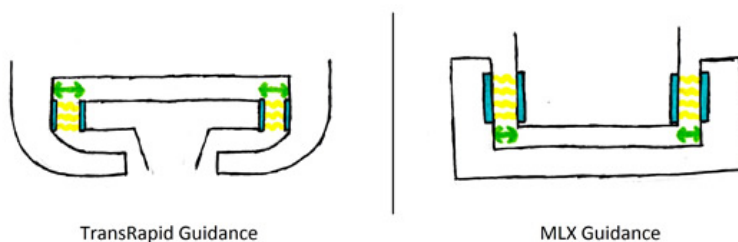
- Ensure safe speed
- Ensure safe separation of maglev train cars and rails

The OCS manages to do the last two tasks using mainly automated computer systems. However, these systems can be manually overridden in emergency situations. For the most part, though, computer algorithms take care of the maglev micromanagement with an overseer simply taking watch over the processes. The speed of the maglev train cars was proven earlier to be directly related to the magnetic field strength and Lorentz force which is directly manipulated by a varying current. This current is controlled by a central computer in a rail station. The guidance to prevent the maglev train from hitting the rails, however, is much more complicated.

## 4.2 Track Guidance Systems

The guideway system depends on the track being examined. The MLX guidance system and TransRapid guidance system both use the laws of magnetic repulsion and attraction - like polarities repel each other and opposite polarities attract each other. The TransRapid guidance system features magnets on the sides of the guide rails on which the train wraps around. They repel and attract guiding magnets on the train. The MLX guidance system has magnets on the sides of the rails in which the train “sits” in. These magnets also repel and attract guidance magnets on the train. The magnet strengths are adjusted during turns and turbulence along with the levitating magnets to ensure that the train does *not* crash into or bump the guidance rails. As previously aforementioned, a computer takes care of this by monitoring the position of the train in relation to the center of the track, and adjusts the magnetic field strength of the guiding magnets accordingly. A diagram of the guidance systems for each kind of track is included below.

Figure 11: Guideways for Different Rails



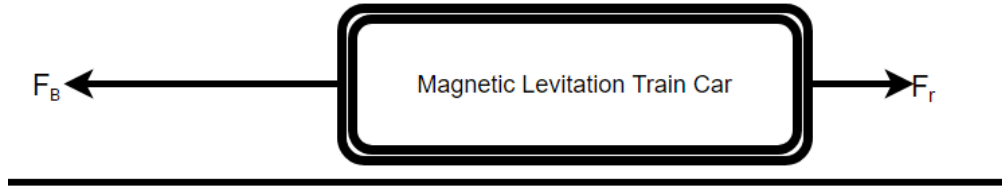
Guiding the trains is pretty simple, but guiding it when there is a power failure or some other accident is much difficult. Even though the TransRapid maglev train system cannot derail because of the fact that it is wrapped around the track and the MLX maglev train system also cannot derail because it is “sitting” inside the track, other accidents still can happen. Luckily, each kind of train is equipped with a back-up in case there is no current being sent through the rails. The TransRapid system has a special material coated on the bottom with a coefficient of sliding friction of 0.1. This reduces frictional heat produced by the drag between the train and the rails. The train is produced with non combustible materials so an emergency skid is safe. There are also on-board batteries which assist the guidance and levitation magnets in providing an eddy braking current to gently stop the train so that it may cruise to a steady stop. The MLX maglev train system has wheels, as previously aforementioned which can be deployed to assist in stopping the train. As well as that, the MLX maglev train cars have permanent magnets underneath its chassis. These magnets, still in motion from the velocity of the train before it lost power, can set up a current (along with the train’s back-up batteries) which is repelled by the permanent magnet.

Overall, magnetic levitation trains are a very safe and fast means of transportation that are easily comparable to air travel and other high speed forms of train travel.

## 5 Air Resistance and other Drawbacks

Maglev trains are undeniably fast due to the fact that they don't have a frictional force between the train and its tracks slowing it down. However, maglev trains, much like all other earth vehicles, are limited by air resistance. Air resistance is a drag force that affects objects as they move through the air. Air resistance is a second order differential equation dependent on an objects velocity, among other things. A free body diagram of the horizontal motion of the train car is shown below.

Figure 12: Maglev Horizontal FBD



An equation of motion can then be set up using Newton's Second Law of Motion. This assuming that the magnetic field propels the train car to give is *positive* acceleration.

$$\vec{F} = m\vec{a} \quad (55 \text{ revisited})$$

$$F_B - F_r = ma \quad (59)$$

Since acceleration is the derivative of velocity, Eq. 59 can be rewritten as is shown below.

$$a = \frac{dv}{dt} \quad (60)$$

$$\Rightarrow F_B - F_r = m \frac{dv}{dt} \quad (61)$$

$$\frac{dv}{dt} = \frac{F_B - F_r}{m} \quad (62)$$

This equation can be used to find the terminal velocity of the maglev train. However, air resistance isn't a hindrance to the speed of a maglev train since its acceleration is not capped to a certain constant unlike a motion in freefall, whose acceleration is constant at  $-9.81m/s^2$ . The current sent along the rails can be increased and this, in turn, can increase the acceleration of the train car. Eq. 62 was included to estimate the terminal velocity of



an experimental maglev set-up with a fixed current to see how a resistive drag force such as air resistance can affect it.

While on the topic, however, a maglev train's drag force, regardless, still hinders its maximum speed. The formula for air resistance is shown below:

$$F_r = kv^2 \quad (63)$$

Or in its expanded, more formal, form:

$$F_r = \frac{\rho C_D A}{2} v^2 \quad (64)$$

Where  $\rho$  is the density of the air the object moves through (in  $\text{kg/m}^3$ ),  $C_D$  is the drag coefficient,  $A$  is the cross sectional area of the object the air presses on ( $\text{m}^2$ ),  $v$  is the velocity the object is moving at ( $\text{m/s}$ ), and  $k$  is constant that collects the effects of density, drag, and area ( $\text{kg/m}$ ). The equation can be rewritten to solve for a maglev train's velocity, as is shown below.

$$F_r = \frac{\rho C_D A}{2} v^2 \quad (64 \text{ revisited})$$

$$v^2 = \frac{2F_r}{\rho C_D A} \quad (65)$$

$$v = \sqrt{\frac{2F_r}{\rho C_D A}} \quad (66)$$

This formula can be used to calculate the velocity of a maglev train. Keep in mind that the air density,  $\rho$ , varies depending on the area and altitude the train is being operated at.

## 6 Other Forms of Transportation

Although maglev trains are becoming the new mode of transportation, there are other forms of transportation that may soon emerge as “The New Maglev”. The HyperLoop, proposed by Elon Musk<sup>12</sup>, may soon beat the maglev train system at its own game. Although it may not involve the same principles as a maglev train, it still involves two key components:

- Levitation
- Propulsion

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<sup>12</sup>Business magnate, investor, engineer, and founder of Tesla and SpaceX

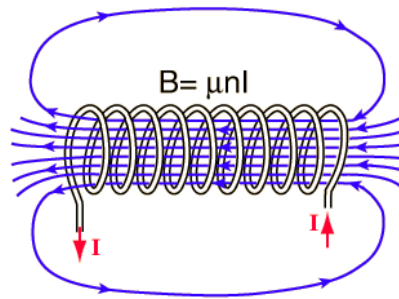
The Hyperloop is a capsule that travels through a narrow tube by a means of high speed propulsion. It is going to be designed to travel at over 1000 km/h (277.77 m/s). People travel inside of the tube from one destination to another. In this situation, levitation serves as a form of guidance. The actual Hyperloop is designed to either utilize air pressure and fans as a means of levitation and propulsion or a vacuum with electromagnetic suspension (EMS) as a means of levitation and propulsion instead. The Hyperloop itself, when Musk released rough blueprints for the engineering community to review and solve, is complicated. However, the principle behind it can be examined with a simple little “hyperloop” that can even be created at home.

A small AAA battery, some copper wiring malleable enough to bend into a spring (or a flexible spring made of conductive material), and two small round magnets are all that are needed. Attach the magnets to each end of the battery and then push the battery through the spring. The battery appears to move inside of the spring all by itself. This is possible because of the fact that a field is concentrated inside of a solenoid. The field inside a solenoid is shown using the equation below:

$$B = \mu_0 n I \quad (67)$$

$I$  is the current flowing through the solenoid,  $n$  is the number of turns in the solenoid, and  $\mu_0$  is the vacuum permeability of free space (also known as the magnetic constant). Contact from the battery with magnets to the conductive material of the solenoid allow for a current from the battery to pass through the solenoid. A circuit is formed just in the vicinity of the battery. The two magnets are automatically at the ends of the generated magnetic field, where the field is divergent, so a force is exerted on the magnets. This results in constant motion. A diagram of this is shown below:

Figure 13: Magnetic Field Inside Solenoid



Unfortunately, this isn't a realistic representation of the Hyperloop because the “capsule” is in contact with the “tube”. This, in real life, would lead to uncomfortable travel. And at over 1000 km/h, this can lead to death.

## 7 Conclusion

For now, however, this simple electric train is enough to demonstrate the concept of what a Hyperloop is supposed to do. Regardless, the magnetic levitation train and other forms of transportation are emerging rapidly and becoming popular by using recent discoveries in physical phenomena to *propel* humanity into the future of travel.

## 8 Sources

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*Physics is the science of all the tremendously powerful invisibility - of magnetism, electricity, gravity, light, sound, cosmic rays. Physics is the science of the mysteries of the universe.*

*How could anyone think it dull?*