MA 471: Lab Assignment 02

Due on Monday, August 14, 2017

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Problem 1

R Code :

```
library(fBasics)
A <- read.table("d-csp0108.txt", header=TRUE)
n = nrow(A)

K_C = kurtosis(A[,2])[1]
K_SP = kurtosis(A[,3])[1]
cat(paste("Kurtosis of C = ",K_C,"\n"))
cat(paste("Kurtosis of SP = ",K_SP,"\n"))

shape = 2
scale = 1
maxs = c()
sample_sizes = c(20,40,100,200)
for(i in 1:4)
{
  maxs = c(maxs,max(rweibull(sample_sizes[i],shape,scale)))
}
cat("Maximums = ")
cat(maxs)

L <- function(beta, theta, X)
{
  n = length(X)
  d1 = sum(log(X))
  d3 = X^beta
  d3 = vector(,n)
  for(i in 1:n)
    d3[i] = X[i]^beta
  d3 = sum(d3)
  return(n*log(beta) + n*beta*log(theta) + (beta-1)*d1 - (theta^beta)*d3 )
}

L_beta <- function(beta,X)
{
  n = length(X)
  d1 = log(X)
  d3 = X^beta
  d2 = sum(d1*d3)
  d1 = sum(d1)
  d3 = sum(d3)
  return((n/beta) - (n*d2/d3) + d1)
}

L_beta_prime <- function(beta,X)
{
  n = length(X)
  d1 = log(X)
```

```
  d3 = X^beta
  d2 = d1*d3
  d4 = (d1*d1)%*%d3 # scalar
  d2 = sum(d2)
  d3 = sum(d3)
  return( -(n/(beta^2)) -n*( (d2^2 - d3*d4)/(d3^2) ) )
}

newton_raphson <- function(f,f_prime,X,tol=1e-5,x0=1,N=100)
{
  i=1; x1=x0
  p = numeric(N)
  while (i<=N)
  {
    df.dx = (f(x0+tol,X)-f(x0,X))/tol#f_prime(x0,X)
    x1 = x0 - (f(x0,X)/df.dx)
    p[i] = x1
    i=i+1
    if(abs(x1-x0)<tol)
      break
    x0=x1
  }
  return(x0)
}

est_param <- function(X)
{
  # beta0 = uniroot(function(x) L_beta(x,X),lower = 1, upper = 5, tol = 1e-5)$root
  beta0 = newton_raphson(L_beta,L_beta_prime,X)
  d3 = sum(X^beta0)
  theta0 = (length(X)/d3)^(1/beta0)
  return(c(beta0,theta0))
}
cat("\nEstimates of beta and theta\n")
for(i in 1:4)
{
  est = est_param(rweibull(sample_sizes[i],shape,scale))
  cat(est[1]," ",est[2],"\n")
}
```

Explanation :

PDF of Weibull Distribution :

$$f(x) = \begin{cases} \beta\theta^\beta x^{\beta-1}e^{-(\theta x)^\beta} & \text{if } x > 0, \theta > 0, \beta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Generating n random variables from a givel Weibull distribution, the Log-likelihood function will be

$$L(\theta, \beta) = n\log\beta + (\beta - 1)\sum_{i=1}^{n}\log x_i - \theta^\beta \sum_{i=1}^{n} x_i^\beta$$

$$\hat{\beta} = \underset{\beta}{\text{argmax}}\{\max_{\theta}\{\log L(\theta, \beta)\}\}$$

$$\frac{\partial L(\theta,\beta)}{\partial \theta} = \frac{n\beta}{\theta} - \sum_{i=1}^{n} x_i^{\beta} \beta \theta^{\beta-1} = 0$$

$$\theta = (\frac{n}{\sum_{i=0}^{n} x_i^{\beta}})^{1/\beta}$$

Putting the expression for θ, we get

$$L(\beta) = n\log\beta + n\log n - n\log(\sum_{i=0}^{n} x_i^{\beta}) + (\beta-1)\sum_{i=1}^{n}\log x_i$$

$$\frac{\partial L(\beta)}{\partial \beta} = \frac{n}{\beta} - n\frac{\sum_{i=1}^{n} x_i^{\beta}\log x_i}{\sum_{i=0}^{n} x_i^{\beta}} + \sum_{i=0}^{n}\log x_i = 0$$

The above equation is to be numerically solved using Newton-Raphson method. Below is the output of the program and we see that it was possible to get the estimates of the parameters.

```
Kurtosis of C =  72.0408944995996
Kurtosis of SP =  10.2350025967748
Maximums = 1.753254 2.541142 2.427241 2.652432
Estimates of beta and theta
2.17458   1.129585
2.135453   0.9107949
1.929853   1.010101
2.030664   0.9887918
```

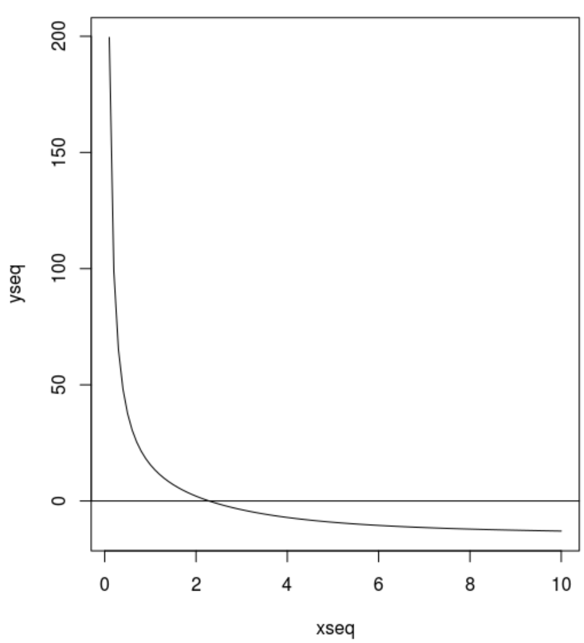

Figure 1: $L(\beta)$ vs β