Chase and Escape: The Mechanism Design of Pursuit-Evasion Games

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ABSTRACT

The purpose of this paper was to investigate the optimal strategies for both detectives and Mr. X in a simplified version of Scotland Yard board game, which varied based on the size of the map and the number of players. We analyzed the different scenarios and situations that could arise during gameplay and examined how these variables affected the gameplay strategies of both the detectives and Mr. X. Our results showed that the shortest path distance between players was the most significant factor in determining the likelihood of winning for either player. Specifically, we found that in most scenarios, Mr. X was likely to be caught by the detectives. However, we also proposed strategies for Mr. X to improve their chances of winning. Overall, this paper provides valuable insights into the mechanics of the simplified version of Scotland Yard board game and its gameplay strategies.

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1 INTRODUCTION

Scotland Yard is a popular board game that was released in 1983. The game is designed for 3-6 players, but essentially plays as a 2-player game where detectives work together to catch Mr.X. The game is played on a graph with numbered locations connected by four types of edges for transportation: taxi, bus, underground, and boat. Mr.X has a set of tickets and can use black tickets for any type of transportation, allowing him to keep his choice of transportation secret. He also has double move tickets that can be used to play an additional turn. However, Mr.X's position is only revealed on specific rounds, increasing the chances of being caught. The objective of the game is for detectives to locate and catch Mr.X before the 24-round limit.

To simplify the game of Scotland Yard and facilitate the task of finding optimal strategies for both players, it may be advantageous to begin by considering a minimalist version of the game. This involves reducing the number of variables involved in the game such as changing number of detectives involved, and gradually increasing their complexity to bring them closer to those found in the original game. By reducing the complexity of the game, it's easier to experiment various strategies as compared to, in the original game. Simplifying Scotland Yard makes it easier to predict optimal strategies, a task that is difficult in the original game due to the vast number of variables involved.

1.1 Contributions

- The result of the game, depends upon the shortest path between the Detectives and Mr.X initially allocated, i.e before the start of the game.
- The optimal strategy and the result of the game for both, the Detectives and Mr.X are independent of the size of the grid.

1.2 Related Work

The game of Hex : Hex is a two-player abstract strategy game played on a hexagonal grid of variable size. The goal of the game is to create a path connecting opposite sides of the board with stones of the same color, while simultaneously blocking your opponent from doing the same. Hex has been studied extensively in the context of game theory and computational complexity, and is known to be PSPACE-complete.

Game of Chomp: In Chomp, two players take turns eating squares from a rectangular grid of chocolate, with the top left square being poisoned. The goal is to force your opponent to eat the poisoned square. Chomp has been studied as an example of a game with a nontrivial winning strategy that is difficult to compute.

One of the outcomes of our study bears resemblance to the game of Chomp. Specifically, our study proposes a method to capture Mr. X, the evader in our pursuit-evasion game, to move towards a corner of the grid.

2 MODEL

The simplified game is setup on square grid of size "N" and as just like the original game, the two types of players

(1) The Detectives
(2) Mr.X

Here we have "A" number of detectives and only one Mr.X. Their initial position is randomly pre-allocated anywhere within the grid before the start of the game. The movements of both players are restricted to moving one step up, down, left, right anywhere within the grid in any particular round. The game begins with Mr.X making his move first, followed by each of the detectives making their move. When both the players complete one move each, a round is said to be completed.

The outcome of the game is determined are as follows

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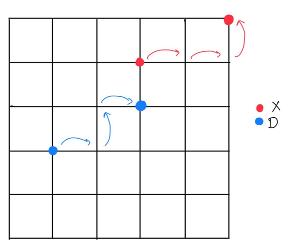
- The game is said to be won by the Detectives, if any one of them is in the same place as Mr. X.
- Otherwise if Mr.X manages to never land up in the same place as the Detectives, he wins i.e he manages to constantly be ahead of the Detectives.

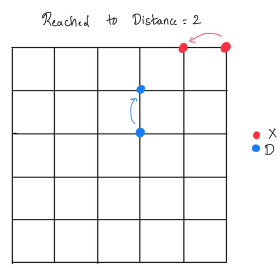
Now, let's consider the bottom-left corner of the grid of size N as the origin, then the coordinates of the top-right corner become (N, N). Suppose we consider a one-Detective case, say the coordinates of him are (X_1, Y_1) and coordinates of Mr.X are (X_2, Y_2).

The shortest path distance is $|X_1 - X_2| + |Y_1 - Y_2|$. After a move by X, its position becomes any of $(X_2 - 1, Y_2)$, $(X_2 + 1, Y_2)$, $(X_2, Y_2 - 1)$, $(X_2, Y_2 + 1)$ and similar can be said for the detective. Therefore the shortest path distance after both X and the detective move is one of $|X_1 - X_2| + |Y_1 - Y_2| - 2$, $|X_1 - X_2| + |Y_1 - Y_2|$, $|X_1 - X_2| + |Y_1 - Y_2| + 2$.

Therefore, the shortest path distance stays even (or odd) if it's even (or odd) initially. This property is called **Grid Property** and is used in our problem.

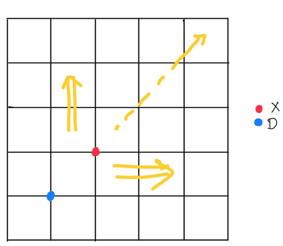
Result 1: The initial shortest distance if even, gets reduced to 2 eventually.





The Assumption: If possible, Mr.X always tries to increase the distance between itself and the detective.

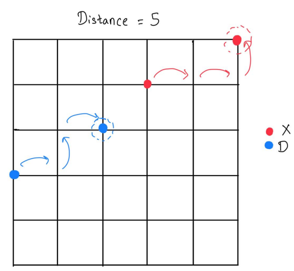
If Mr.X always tries to increase the distance between itself and the detective, Mr.X moves in a constant direction and therefore gets pushed to an edge. As the detective tries to decrease the distance between himself and Mr.X, it compensates for the movement of Mr.X. Therefore the shortest path distance remains constant. By assumption, Mr.X moves toward the corner which is farther among the two possible corners of the edge. Therefore, Mr.X must end up in a corner. No matter what position to which Mr.X moves, the distance gets reduced by 2. Until Mr.X reaches a corner, the distance remains constant. The next time it reaches a corner, it again gets reduced by 2 and the process continues. Thus, this result holds with this assumption.

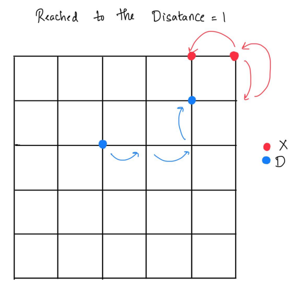


If Mr.X doesn't always try to increase the distance between himself and the detective, Mr.X can move in a direction that results in the distance getting reduced by 2. Every time, Mr.X moves in a direction as opposed to the previous assumption, the distance gets reduced by 2. Therefore, even if the assumption is not taken, the distance eventually gets reduced to 2 and is even faster than with the assumption taken.

Result 2: The initial shortest distance if odd, gets reduced to 1 eventually.

The explanation given for Result 1 also explains this result.

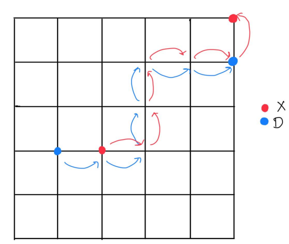




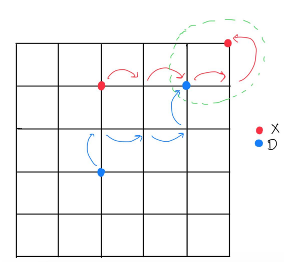
3 MAIN RESULTS

3.1 Case 1: One Detective only

3.1.1 Shortest Path is Odd. As from the above section whenever the initial distance is odd, the distance between them eventually becomes one. Since Mr.X makes his move ahead of the Detective, therefore he always manages to succeed the Detective by one edge. Thus irrespective of various strategies by the Detective, Mr.X always wins!



3.1.2 Shortest Path is Even. As seen from Result 1, the shortest path distance gets reduce to 2. As we can see from the figure, X ends up in a corner and the detective on a position which is diagonally opposite to the corner.

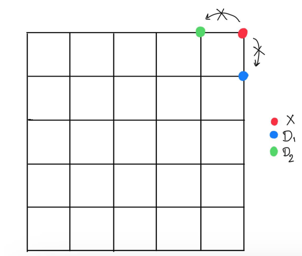


No matter what move X makes at this point, the detective can directly go to the position of X and hence X gets caught. Therefore X always loses, irrespective of its strategies.

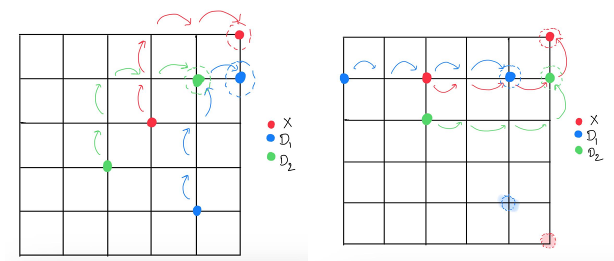
3.2 Case 2: Two Detectives

After the positions of detectives and Mr. X are randomly choosen initially, we find the shortest path distances from each of the detective to Mr. X and based on whether they are even or odd, they are divided into three categories as follows.

3.2.1 Odd-Odd. In this case, we see that any initial distances will eventually end up in a situation where X is trapped at a corner and and the immediate two positions that X can go to are occupied by the detectives. Therefore Mr. X loses.

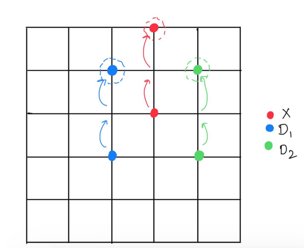


3.2.2 Odd-Even. As we observed in previous section, Mr. X is caught by the detective having even initial distance. It may seem obvious that the presence of other detective at an odd initial distance will help the other detective in catching X, but its not true in all cases. The detective at odd distance helps the other detective in some cases in pushing Mr. X towards a corner by cutting off some paths which result in Mr. X going to other corner. This can be clearly observed in the following figure.



Therefore Mr. X loses.

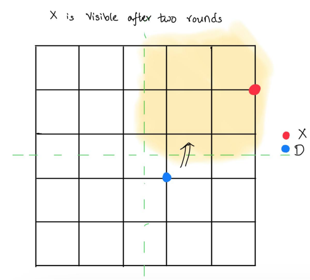
3.2.3 Even-Even. In this case, the detectives may not help each other at all as both try to compete for a same position(diagonally opposite to a corner) in which they can capture Mr. X. The special case observed here is where Mr.X getting caught on an edge rather than on a corner. This can be observed from the following figure.



Therefore Mr. X loses.

3.3 Case 3: One Detective only but X is visible after every two rounds

Let's breakdown the grid into four quadrants as shown below. Since X's position is first revealed only after two rounds, the detective tries to stay at position which allows him to move to any quadrant easily i.e roughly the center of the grid. When X's position is revealed, the detective moves towards that particular quadrant.



4 CONCLUSIONS

To bring this simplified game closer to the original game, we could consider multiple detectives and position of Mr. X visible after multiple rounds. In such cases, one way for the detectives could be such that one detective stays near the center and the other detectives evenly spread across the different quadrants until X's position is revealed. This is one such strategy, there could be many others too.

How can we make chances for X to win?: As we can see, there needs to be atleast one detective at an even initial distance from Mr. X in order to capture him(barring some exceptions like in section 3.2.1). So for Mr. X to have a chance, we could consider the method used in the original Scotland Yard game which is to limit the number of rounds. Therefore Mr. X has a chance of winning, by basically moving around the grid until he reaches the winning round.

Here are a few citations of journal articles [1–6] we referred to.

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