Q4 $\rightarrow$ It's Hero Time

Let's do Grandpa

1 Introduction

Grandpa Max is tied up on the road while an unmanned car is moving directly toward him. Since Ben is already transformed into Upgrade and cannot change form, he must rely on engineering principles to stop the vehicle. One of the most important safety systems in automobiles is the braking system, which allows a vehicle to reduce speed or come to a complete stop by converting kinetic energy into other forms, mainly heat.

2 Braking Systems in Automobiles

Braking systems are designed to slow down or stop vehicles by producing friction or by recovering energy. Different types of braking systems are used depending on the vehicle design and application.

2.1 Mechanical and Hydraulic Brakes

Mechanical brakes use rods and cables to transmit force from the brake pedal to the wheels. They are simple but require large effort and provide limited braking performance. Modern vehicles mainly use hydraulic braking systems based on Pascal's law. When the brake pedal is pressed, hydraulic pressure is transmitted through brake fluid to the calipers, which push brake pads against rotating discs and slow the vehicle efficiently.

2.2 Disc and Drum Brakes

Disc brakes use a caliper to clamp brake pads onto a rotating disc attached to the wheel. The friction generated converts kinetic energy into heat and reduces speed. Disc brakes offer good heat dissipation and stable performance at high speeds. Drum brakes work by pressing brake shoes outward against a rotating drum. They are economical and provide good braking torque at low speeds but suffer from poor cooling and brake fade.

2.3 Advanced Braking Technologies

Modern automobiles include systems such as the Anti-lock Braking System (ABS), which prevents wheels from locking and maintains steering control during emergency braking. Electric and hybrid vehicles use regenerative braking, where the motor operates as a generator to recover energy while slowing down. Some vehicles also use brake-by-wire systems, where braking commands are transmitted electronically for precise and fast control.

3 Part B: Brake Configurations

In most mass-market vehicles, disc brakes are used at the front wheels while drum brakes are used at the rear. This configuration is adopted mainly due to load transfer during braking, thermal performance, cost, and stability considerations.

When a vehicle decelerates, dynamic weight transfer shifts a large portion of the load to the front axle. The normal force on the front wheels increases, allowing them to generate a higher braking force according to

$$F_b = \mu N, \tag{1}$$

where μ is the coefficient of friction and N is the normal force. Since the front wheels carry more load during braking, they are responsible for nearly $60\% - 70\%$ of the total braking effort.

Disc brakes are preferred at the front because they provide superior heat dissipation. During braking, kinetic energy is converted into thermal energy,

$$E_k = \frac{1}{2}mv^2, \tag{2}$$

which must be removed efficiently to avoid brake fade. Disc brakes are exposed to airflow and cool faster than drum brakes, making them ideal for high-load and repeated braking conditions experienced by the front wheels.

At the rear, braking loads are smaller due to reduced normal force. Drum brakes are sufficient to handle this load and offer advantages such as lower cost, compact packaging, and built-in parking brake mechanisms. Additionally, drum brakes provide a self-energizing effect, where friction helps increase braking force with less input effort.

Using disc brakes at the front and drum brakes at the rear ensures a balance between performance, safety, thermal stability, and manufacturing cost. This configuration delivers strong braking where it is needed most while maintaining efficiency and affordability for mass-market vehicles.

4 Part C: May the Force Be With You – Part I

The vehicle is traveling at an initial speed of $v_0 = 100$ kmph $= 27.78$ m/s. Grandpa Max is located at a distance of $s = 45$ m. To avoid collision, the vehicle must come to rest within this distance.

4.1 Required Deceleration

Using the kinematic relation,

$$v^2 = u^2 + 2as, \tag{3}$$

and setting final velocity $v = 0$, we obtain

$$0 = u^2 + 2as, \tag{4}$$

which gives the required deceleration as

$$a = -\frac{u^2}{2s}. \tag{5}$$

Substituting values,

$$a = -\frac{(27.78)^2}{2 \times 45} = -8.57 \text{ m/s}^2. \tag{6}$$

4.2 Total Braking Force

The total braking force required is

$$F_b = ma. \tag{7}$$

For a vehicle mass of $m = 900$ kg,

$$F_b = 900 \times 8.57 = 7713 \text{ N}. \tag{8}$$

4.3 Weight Transfer Effect

During braking, load is transferred to the front axle. The longitudinal weight transfer is approximated by

$$\Delta W = \frac{mah}{L}, \tag{9}$$

where h is the center of gravity height and L is the wheelbase. Assuming typical values $h = 0.5$ m and $L = 2.5$ m,

$$\Delta W = \frac{900 \times 8.57 \times 0.5}{2.5} = 1543 \text{ N}. \tag{10}$$

This increases front braking capability and reduces rear load.

4.4 Brake Factor and Wheel Force

For drum brakes, the brake factor BF accounts for self-energizing effects:

$$F_{eff} = BF \cdot F_{act}. \tag{11}$$

Assuming a brake factor of $BF = 1.3$, the required actuation force per wheel is

$$F_{wheel} = \frac{F_b}{4 \cdot BF}. \tag{12}$$

Thus,

$$F_{wheel} = \frac{7713}{4 \times 1.3} = 1484 \text{ N}. \tag{13}$$

Hence, each wheel must generate approximately 1.48 kN of braking force to stop the vehicle in time.

5 Part D: May the Force Be With You – Part II

Now the vehicle is traveling on a circular path of radius $R = 100$ m while braking.

5.1 Centripetal Force Requirement

The centripetal acceleration required to maintain the circular path is

$$a_c = \frac{v^2}{R}. \tag{14}$$

Substituting,

$$a_c = \frac{(27.78)^2}{100} = 7.72 \text{ m/s}^2. \tag{15}$$

The corresponding lateral force is

$$F_c = ma_c = 900 \times 7.72 = 6948 \text{ N}. \tag{16}$$

5.2 Combined Braking and Cornering

When braking while cornering, tires must satisfy the friction ellipse constraint,

$$\left(\frac{F_x}{\mu N}\right)^2 + \left(\frac{F_y}{\mu N}\right)^2 \leq 1, \tag{17}$$

where F_x is longitudinal braking force and F_y is lateral force.

Thus, some tire friction capacity is already used for turning, reducing available braking force.

5.3 Effective Braking Force

The total force demand becomes

$$F_{total} = \sqrt{F_b^2 + F_c^2}. \tag{18}$$

Substituting,

$$F_{total} = \sqrt{(7713)^2 + (6948)^2} = 10384 \text{ N}. \tag{19}$$

Including brake factor,

$$F_{wheel,circ} = \frac{F_{total}}{4 \cdot BF}. \tag{20}$$

$$F_{wheel,circ} = \frac{10384}{4 \times 1.3} = 1997 \text{ N}. \tag{21}$$

5.4 Interpretation

While braking in a straight line requires approximately 1.48 kN per wheel, braking while cornering at a radius of 100 m increases the demand to nearly 2.0 kN per wheel. This occurs because the tires must simultaneously generate lateral and longitudinal forces, reducing the available braking margin and increasing actuation requirements.