CLASS-12
CHAPTER-6
Application of Derivatives

EXERCISE - 6.3

Short Answer (S.A.)

1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is propotional to the surface. prove that the radius is decreasing at a constant rate

2. If the area of a circle increases at a uniform rate.then prove that perimeter varies inversely as the radius.

3. A kite is maving horizontally at a height 151.5 meters .If the speed of kite is 10m/s, how fast is the string being let out when the kite is 250m away from the boy who is flying the kite? The heignt of boy is 1.5m.

4. Two men A and B start with velocities V at the some time from the junction of two roads inclined at 45°.to each other.If they travel by different roads find the rate at which they are being seperated

5. Find an angle θ,0 <0<$\frac{\pi}{2}$, which increases twice as fast as its sine.

6. Find the approximate value of $(1,999)^5$

7. Find the approximate value of metal in a hollow spherical shell whose internal and externat radi are 3 cm and 3.0005 cm ,respectively.

8. A man,2m tall,walks at the rate or $1\frac{2}{3}$ m/s towards a street light which is $5\frac{2}{3}$ mabove the ground .At what rate is the tip of his shadow moving ? At what rate is the length of the shadow changing when he is $3\frac{1}{3}$- m from the bag of the light?

9. A swimming pool is to be drained for cleaning If **L** represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and **L**=$200(10-t)^2$. How fast is the water running out at the end af 5 seconds? what is the average rate at which the water flows out during the first 5 seconds?

10. The volume of a cube increases at a constant rate. prove that the increase in its surface area varies inversety as the length of the side.

11. x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the area of second square with respect to the area of first square.

12. Find the condition that the curves $2x = y^2$ and 2xy=k intersect orthogonally.

13. prove that the curves xy=4 and $x^2 + y^2 = 8$ touch each other.

14. Find the co-ordinates of the point on the curve $\sqrt{x} + \sqrt{y}$=4 ot which tongent is equally inclined to the oxes.

15. Find the angle of intersection of the curves $y = 4 - x^2$ and $y = x^2$

16. prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touch each each other ot the point (1,2).

17. Find the equation of the normal lihes to the cure $3x^2 - y^2 = 8$ which are paralle to the line x+3y=4.

18. At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to the y-axis?

19. show that the line $\frac{x}{a} + \frac{y}{b} = 1$ .touches the curve $y = be - \frac{x}{a}$ at the point where the curve intersects the axis of y

20. show that f (x)=2x+cot1x +log $\cot^{-1}(\sqrt{1+x^2} - x)$ is intersects in R

21. show that for a >1,f(x)=3 sinx-cosx 2ax+b is decreasing in r.

22. show that f (x) $= \tan^{-1}(sinx + cosx)$ is an increasing function in$(0, \frac{\pi}{4})$

23. At what point ,the slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is maximum? Also find the maximum slope.

24. prove that f (x)=sinx+$\sqrt{3}$ cosx has maximum value at $x = \frac{\pi}{6}$

Long Answer (L.A)

25. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$

26. Find the points of local maximal, local minima and the points of inflection of the function f (x)=$x^5 - 5x4 + 5x^2 - 1$. Also find the corresponding local maximam and local minimum values

27. A telephone company in a town has 500 subscribers on its and collects fixed charges of RS 300/- per subscriber per year. The company proposes to increase the annual subscription and it is belived that fog every increase of Re1/. one subsariber will discontinue the serice. find what increase will bring maximum profit?

28. If the straight line x casoc+y sno oc=P touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,then prove that $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha = P^2$.

29. An open box with square base is to be made of a given quantity of card board of area c^2. show that the maiximum volume of the box is $\frac{c^2 3}{6\sqrt{3}}$cubic units.

30. Find the bimensions of the rectangle of perimeter 36cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also find the maximum voiume.

31. lf the sum of the surface areas of cube and a sphere is constant, what is the ratio of on edge of the cube to the diameter of the sphere when the sum of their volumes is minimum?

32. AB is a diameter of a circle and c is any point on the circle show that the area of $\triangle$ABC is maximum, when it is isosceles,

33. A metal box with a square base and vertical sides is to contain $1024cm^3$ The material for the top and bottom casts $RS5/cm^2$ and the material for the sides costs Rs2.50/cm^2.Find the least cost of the box.

34. The sum of the surface areas of a rectangular parallelopiped with sides x,2x and $\frac{x}{3}$ and a sphere is given to be constant. prove that the sum

of their volumes is minimum, if x is equal to three times the radius of the sphere Also find the minimum value of the sum of their volumes

Objective Type Questions

choose the correct answer from the given four options in each of the following questions 35 to 39 ;

35. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec The rate at which the area increases, when side is 10 cm is

    (a) $10cm^2/s$
    (b) $\sqrt{3}cm^2/s$
    (c) $10\sqrt{3}cm^2/s$
    (d) $\frac{10}{3}cm^2/s$

36. A ladder, 5 meter long standing on a horizontal floor , leans against a vertical wall.lf the top of the ladder slides downwards ot the rate ot the rate of 10cm/sec then the rate at the rate of 10c/sec the the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 metres from the wall is ;

    (a) $\frac{1}{10}$ radian/sec
    (b) $\frac{1}{20}$ redian/sec
    (c) 20 radian/sec
    (d) 10 radian/sec

37. The curve y=$x\frac{1}{5}$ has at(0,0)

    (a) a vertical tongent(parallel to y-axis)
    (b) ahorizontal tangent(parallel tox.axis)
    (c) an oblique tangent
    (d) an tangent

38. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallei to the line x+3y=8 is

(a) 3x-y=8

(b) 3x+y+8=0

(c) x+3y+8=0

(d) x+3y=0

39. If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthoonally at (1,1) ,then the value of ais

(a) 1

(b) 0

(c) -6

(d) 6

40. If $y = x^4 - 10$ and if x changes from 2+01,9a what is the change in y

(a) -32

(b) .032

(c) 5.68

(d) 5.969

41. The equation of tongent to the curvey $(1+x^2) = 2-x$, where it crosses x-axis

(a) x+5y=2

(b) x-5y=2

(c) 5x-y=2

(d) 5x+y=2

42. The points at which th tangents to the curve $y = x^3 - 12x + 18$ are parallel to x-axis are:

(a) (2,-2)(-2,-34)

(b) (2,34)(-2,0)

(c) (0,34),(-2,0)

(d) (2,2),(-2,34)

43. The tangent to the curve $y = e^2x$ at the point (0,1) meetsx-axis at:

(a) (0,1)

(b) $(\frac{1}{2}, 0)$

(c) (2,0)

(d) (0,2)

44. The slope of tangent to the curve $x = t^2 + 3t = 8y = 2t^2 - 2t - 5$ ot the point (2,-1) is:

(a) $\frac{22}{7}$

(b) $\frac{6}{7}$

(c) $\frac{-6}{7}$

(d) -6

45. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{6}$

46. The interval on which the function f(x)=$2x^3+9x^2+12x-1$ is decreasing is

-1,8

-2,-1

-8,-2

-1,1

47. Let the f : $\mathbf{R} \to \mathbf{R}$ be defined by f(x)=2x+ cosx, then f:

(a) has a minimum at x=π

(b) has a maximuum,at x=0

(c) is a decreasing function

(d) is an increasing function

48. y=$x(x-3)^2$ decreases for the value of x given by:

(a) 1<x

(b) x<0

(c) x>0

(d) 0<x<$\frac{3}{2}$

49. The function f(x)=$4sin^3x - 6sin^2x + 12sinx + 100$ is strictly

(a) increasing in$[,\frac{3}{2}]$

(b) decrea sing in$[-\frac{}{2},]$

(c) decreasing in$[-\frac{\pi}{2},\frac{\pi}{2}]$

(d) decreasing in$[0,\frac{\pi}{2}]$

50. which of the following functions is decreasing an$[0,\frac{pi}{2}]$

(a) $\sin 2x$

(b) $\tan x$

(c) $\cos x$

(d) $\cos 3x$

51. The function f (x)=tanx-x

(a) always increases

(b) always decreases

(c) never increases

(d) sometimes increases and sometimes decreases

52. if x is real ,the minimum value of $x^2 - 8x + 17$is

(a) -1

(b) 0

(c) 1

(d) 2

53. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0,9] is

(a) 126
(b) 0
(c) 135
(d) 160

54. The function f (x)=$2x^3 - 3x^2 - 12x + 4$ has

(a) two points of local maximum
(b) two points of local minimum
(c) one maxima and one minima
(d) no marima or minima

55. The maximum volue of sin x casx is

(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\sqrt{2}$
(d) $2\sqrt{2}$

56. At x =5$\frac{\pi}{6}$ f (x)=2 sin3x+3 cosx

(a) maximum
(b) minimum
(c) zero
(d) neither maximum nor minimum

57. maximum slope of the curve y=$-x^3 + 3x^2 + 9x - 27$ is

(a) 0
(b) 12
(c) 16
(d) 32

58. f (x)=x^2 has a stationary point at

(a) x=e

(b) x=$\frac{1}{e}$

(c) x=1

(d) x=$\sqrt{e}$

59. The maximum value of $[\frac{1}{x}]$ is

(a) e

(b) e^e

(c) $e^{\frac{1}{e}}$

(d) $\frac{1}{e}^{\frac{1}{e}}$

Fill in the blanks in each of the following exercisexs 60 to 64:

60. The curves $y = 4x^2 + 2x - 8$ and $y = x^3 - x + 13$ touch each other at the point is ____

61. The equation of normal to the curvey=tanx at (0,0) is ____

62. The values of a for which the function f (x) sinx-ax+b increases on R are ____

63. The function f (x)=$\frac{2x^2-1}{x^4}$,x>0 decreases in the interval is ____

64. The least value of the function f (x)=ax+$\frac{b}{x}$(a>0,b>0,x>0) is ____