Mid – II

40 Marks

Part-A

1. a) $\sin C + \sin D =$ CO2

   b) $\cos C + \cos D =$ CO2

   c) ~~Find~~ Write the General Solution formulas for the functions $\sin\theta$ and $\cos\theta$ CO2

   d) $\tan^{-1}x + \tan^{-1}y =$ CO2

2. Express $(3-4i)(7+2i)$ in terms of $a+ib$ CO2

3. Find the intercepts made by a straight line $x+5y-10=0$ CO3

4. write the polar or Modulus Amplitude form of $-1-\sqrt{3}i$ CO2

5. Find the equation of straight line passing through $(3,-4)$ and Parallel to the line $x+7y+1=0$ CO3

Part-B

6 a) Prove that $\dfrac{\sin 5\theta + \sin\theta}{\cos 5\theta + \cos\theta} = \tan 3\theta$ CO2

(or)

b) Solve $2\cos^2\theta - 3\cos\theta + 1 = 0$ CO2

7 a) Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{4}$ CO2

(or)

b) Solve the triangle ABC with $a=2, b=2\sqrt{3}, C=$[illegible] CO2

8 a) Find the angle b/w the lines $2x-y+3=0$ and $x+y-2=0$

(or)

b) Find the equation of a straight line passing through $(-2,-5)$ and perpendicular to $7x+2y-1=0$.

Mid - II

40 marks

part - A:

1. a) $\sin C + \sin D = 2\sin\left(\frac{(C+D)}{2}\right) \cdot \cos\left(\frac{(C-D)}{2}\right)$

b) $\cos C + \cos D = 2\cos\left((C+D)/2\right) \cdot \cos\left((C-D)/2\right)$.

c) Write the General solution formulas for the functions $\sin\theta$ and $\cos\theta$

Sol

General solution formulas:

General solution for sine

$\sin(\theta) = \sin(\alpha)$ when $\theta = n\pi + (-1)^n * \alpha$, where $n \in z$ (integers) and $\alpha \in [0, \pi]$

General solution for cosine

$\cos(\theta) = \cos(\alpha)$ when $\theta = 2n\pi \pm \alpha$, where $n \in z$ (integers) and $\alpha \in [0, \pi]$.

d) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$

2) Express $(3-4i)(7+2i)$ in terms of $a+ib$.

Sol

$= (3-4i)(7+2i)$

$= 21+6i-28i-8i^2$

$= 21+6i-28i-8(-1)$ $(i^2=-1)$

$= 21+(-22i)+8$

$= 29-22i$

Here $a=29$, $b=-22$

3) Find the intercepts made by a straight line $x+5y-10=0$.

Sol: Given, $x+5y-10=0$

$x+5y=10$

Divide with 10.

$\frac{x}{10}+\frac{\cancel{5}y}{\cancel{10}_2}=\frac{10}{10}$

$\frac{x}{10}+\frac{y}{2}=1$

$\frac{x}{10}+\frac{y}{(2)}=1$

Here x-intercept $=10$

y-intercept $=2$

4) Write the polar or modulus Amplitude form of $-1-\sqrt{3}i$.

A. Let $z=-1-\sqrt{3}i$

compare with $x+iy$

$x=-1,\ y=-\sqrt{3}$

$r=\sqrt{x^2+y^2}$

$=\sqrt{(-1)^2+(-\sqrt{3})^2}$

$=\sqrt{1+3}=\sqrt{4}=2$

$\boxed{r=2}$

$\alpha=\tan^{-1}\left(\left|\frac{y}{x}\right|\right)$

$=\tan^{-1}\left(\left|\frac{-\sqrt{3}}{-1}\right|\right) \Rightarrow \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$\alpha=\frac{\pi}{3}$

The point $p(-1, -\sqrt{3})$ lies in III^{rd} quadrant

$$\theta = -(\pi - \alpha)$$

$$= -\left(\pi - \frac{\pi}{3}\right)$$

$$= -\left(\frac{3\pi - \pi}{3}\right)$$

$$\boxed{\theta = -\frac{2\pi}{3}}$$

modulus amplitude form

$$z = r(\cos\theta + i\sin\theta)$$

$$z = 2\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$$

$$= 2\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$$ //

5) Find the equation of straight line passing through $(3,-4)$ and parallel to the line

$x + 7y + 1 = 0$

sol: Given $P(x_1, y_1) = (3, -4)$

Given line is $x + 7y + 1 = 0$

compare with $ax + by + c = 0$, $a = 1$, $b = 7$, $c = 1$

The equation of the required line is

$$a(x - x_1) + b(y - y_1) = 0$$

$$1[x - 3) + 7(y - (-4))] = 0$$

$$x - 3 + 7(y + 4) = 0$$

$$x - 3 + 7y + 28 = 0$$

$$\boxed{x + 7y + 25 = 0}$$

6) a) prove that $\dfrac{\sin 5\theta + \sin\theta}{\cos 5\theta + \cos\theta} = \tan 3\theta$

Sol L.H.S $= \dfrac{\sin 5\theta + \sin\theta}{\cos 5\theta + \cos\theta}$

$$\left[\begin{array}{l} \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \\ \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \end{array}\right]$$

$$= \frac{\cancel{2}\sin\left(\frac{5\theta+\theta}{2}\right)\cancel{\cos\left(\frac{5\theta-\theta}{2}\right)}}{\cancel{2}\cos\left(\frac{5\theta+\theta}{2}\right)\cancel{\cos\left(\frac{5\theta-\theta}{2}\right)}}$$

$$= \frac{\sin\left(\frac{6\theta}{2}\right)}{\cos\left(\frac{6\theta}{2}\right)} = \frac{\sin 3\theta}{\cos 3\theta}$$

$$= \tan 3\theta$$

$$\therefore \frac{\sin 5\theta + \sin\theta}{\cos 5\theta + \cos\theta} = \tan 3\theta$$

Hence proved

b) Solve $2\cos^2\theta - 3\cos\theta + 1 = 0$

Sol

Given $2\cos^2\theta - 3\cos\theta + 1 = 0$

$2\cos^2\theta - 2\cos\theta - 1\cos\theta + 1 = 0$

$2\cos\theta(\cos\theta - 1) - 1(\cos\theta - 1) = 0$

$(\cos\theta - 1)(2\cos\theta - 1) = 0$

2, -2 -1

Case - 1

when $\cos\theta - 1 = 0$

$\cos\theta = 1$

$\cos\theta = \cos 0$

$\cos\theta = \cos\alpha$

$\boxed{\alpha = 0^\circ}$

G.S $\Rightarrow \theta = 2n\pi \pm \alpha$

$\theta = 2n\pi \pm 0$

$\theta = 2n\pi \longrightarrow (1)$

7b Given $a=2$, $b=2\sqrt{3}$, $c=4$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(2\sqrt{3})^2+(4)^2-(2)^2}{2(2\sqrt{3})(4)}$$

$$= \frac{12+16-4}{16\sqrt{3}}$$

$$\cos A = \frac{24}{16\sqrt{3}} = \frac{3\sqrt{3}}{2}$$

$$\cos A = \frac{\sqrt{3}}{2} \Rightarrow \cos A = \cos 30^\circ$$

$$\boxed{A=30^\circ}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos B = \frac{2^2+(4)^2-(2\sqrt{3})^2}{2(2)(4)}$$

$$= \frac{4+16-12}{16}$$

$$\cos B = \frac{8}{16} = \frac{1}{2}$$

$$\cos B = \frac{1}{2} \Rightarrow \cos B = \cos 60^\circ$$

$$\boxed{B=60^\circ}$$

W.K.T

$$A+B+C=180^\circ$$

$$30+60+C=180^\circ \Rightarrow C = 180-90$$

$$\boxed{C=90^\circ}$$

$\therefore A=30^\circ, B=60^\circ, C=90^\circ$

8)a) Find the angle between the lines $2x-y+3=0$ and $x+y-2=0$

Given lines are $2x-y+3=0$ & $x+y-2=0$

$$\tan\theta = |(m_1-m_2)/(1+m_1m_2)|$$

Line 1: $2x-y+3=0$

$$y = 2x+3$$

$$m_1 = 2$$

Line 2: $x+y-2=0$

$$y = -x+2$$

$$m_2 = -1$$

$$\tan\theta = |(2-(-1))/(1+2(-1))|$$

$$= |3/-1| = -3$$

$$\tan\theta = \sqrt{3}$$

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\theta = 60^\circ$$

8.b) Find the equation of a straight line passing through $(-2,-5)$ and perpendicular to $7x+2y-1=0$

Sol: Given $P(x_1,y_1) = (-2,-5)$

Given line is $7x+2y-1=0$

compare with $ax+by+c=0$

$a=7,\ b=2,\ c=-1$

The Eqn of the st.line is

$b(x-x_1)-a(y-y_1)=0$

$2(x+2)-7(y-(-5))=0$

$2(x+2)-7(y+5)=0$

$2x+4-7y-35=0$

$\boxed{2x-7y-31=0}$

8(a)

Ans Given lines are $2x-y+3=0$ and $x+y-2=0$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$

$a_1=2,\ b_1=-1,\ c_1=3$ and $a_2=1,\ b_2=1,\ c_2=-2$

The angles b/w two lines

$$\cos\theta = \frac{|a_1a_2+b_1b_2|}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}}$$

$$= \frac{|(2)(1)+(-1)(1)|}{\sqrt{(2)^2+(-1)^2}\sqrt{(1)^2+(1)^2}} = \frac{|2-1|}{\sqrt{4+1}\sqrt{1+1}}$$

$$\cos\theta = \frac{1}{\sqrt{5}\sqrt{2}}$$

$$\cos\theta = \frac{1}{\sqrt{10}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$$