A1110 Assignment 5

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Exercise 16.3 Q8: Three coins are tossed once. Find the probability of getting:

(i) 3 heads
(ii) 2 heads
(iii) atleast 2 heads
(iv) atmost 2 heads
(v) no head
(vi) 3 tails
(vii) exactly two tails
(viii) no tail
(ix) atmost two tails

Solution: Let the Bernoulli random variable $X \in \{0, 1\}$ where $X = 0$ denotes occurrence of head(success) and $X = 1$ denotes occurrence of tail(failure) for a single coin toss.For a fair coin,

$$\Pr(X = 0) = p = \frac{1}{2} \tag{1}$$

$$\Pr(X = 1) = q = \frac{1}{2} \tag{2}$$

Let the Binomial random variable $Y \in \{0, 1, 2, 3\}$ denote the number of heads. We can express this as a binomial distribution,

$$\Pr(Y = k) = {}^{n}C_k (\mathrm{p})^k (\mathrm{q})^{n-k} \tag{3}$$

where $k \in \{0, 1, 2, 3\}$ and n = 3 for 3 coins. By (1) and (2),

$$\Pr(Y = k) = {}^{3}C_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{3-k} \tag{4}$$

From Table 1,

S.No.	Event	Description
1	Pr(Y = 3)	3 Heads
2	Pr(Y = 2)	2 Heads
3	Pr(Y $\geq$ 2)	Atleast 2 Heads
4	Pr(Y $\leq$ 2)	Atmost 2 Heads
5	Pr(Y = 0)	No head
6	Pr(Y = 0)	3 Tails
7	Pr(Y = 1)	Exactly 2 Tails
8	Pr(Y = 3)	No Tail
9	Pr(Y $\geq$ 1)	Atmost 2 Tails

TABLE 1

(i) 3 heads:

$$\Pr(Y = 3) = {}^{3}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 \tag{5}$$

$$= \frac{1}{8} \tag{6}$$

(ii) 2 heads:

$$\Pr(Y = 2) = {}^{3}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 \tag{7}$$

$$= \frac{3}{8} \tag{8}$$

(iii) atleast 2 heads:

$$\Pr(Y \geq 2) = \Pr(Y = 2) + \Pr(Y = 3) \tag{9}$$

$$= {}^{3}C_3 \left(\frac{1}{2}\right)^3 + {}^{3}C_2 \left(\frac{1}{2}\right)^3 \tag{10}$$

$$= \frac{1}{2} \tag{11}$$

(iv) atmost 2 heads:
$\Pr(Y \leq 2)$

$$= \Pr(Y = 2) + \Pr(Y = 1) + \Pr(Y = 0) \tag{12}$$

$$= {}^{3}C_2 \left(\frac{1}{2}\right)^3 + {}^{3}C_1 \left(\frac{1}{2}\right)^3 + {}^{3}C_0 \left(\frac{1}{2}\right)^3 \tag{13}$$

$$= \frac{7}{8} \tag{14}$$

(v) no head:

$$\Pr\left(Y=0\right) = {}^{3}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 \tag{15}$$
$$= \frac{1}{8} \tag{16}$$

(vi) 3 tails:

$$\Pr\left(Y=0\right) = {}^{3}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 \tag{17}$$
$$= \frac{1}{8} \tag{18}$$

(vii) exactly two tails:

$$\Pr\left(Y=1\right) = {}^{3}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 \tag{19}$$
$$= \frac{3}{8} \tag{20}$$

(viii) no tail

$$\Pr\left(Y=3\right) = {}^{3}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 \tag{21}$$
$$= \frac{1}{8} \tag{22}$$

(ix) atmost two tails:
$\Pr\left(Y\geq 1\right)$

$$= \Pr\left(Y=1\right) + \Pr\left(Y=2\right) + \Pr\left(Y=3\right) \tag{23}$$
$$= {}^{3}C_1\left(\frac{1}{2}\right)^3 + {}^{3}C_2\left(\frac{1}{2}\right)^3 + {}^{3}C_3\left(\frac{1}{2}\right)^3 \tag{24}$$
$$= \frac{7}{8} \tag{25}$$

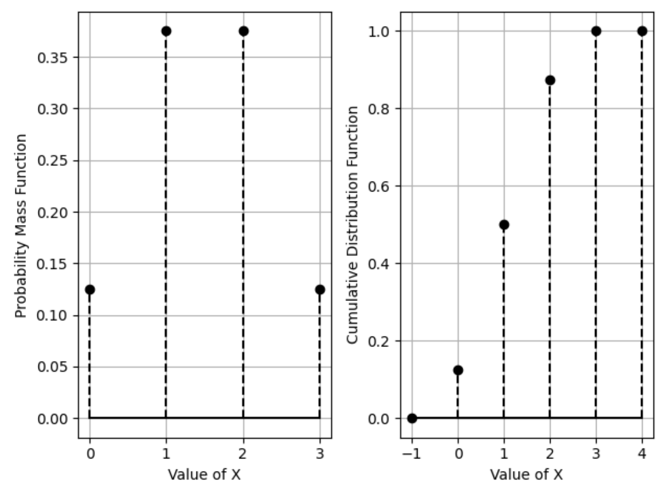

Fig. 1: Plot of PMF(left) and CDF(right)