A1110 Assignment 3

Tejal Kulkarni
CS21BTECH11058

April 2022
CBSE Probability Grade 9

Exercise 15.1 Q4: Three coins are tossed simultaneously 200 times with the following frequencies of different items:

Outcome	3 Heads	2 Heads	1 Head	No Head
Frequency	23	72	77	28

TABLE 1

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.
Solution: Let the random variable $X \in \{0, 1, 2, 3\}$ denote the number of heads in the coin-tossing experiment. Now,

$$\Pr(X = i) = \frac{n(X = i)}{\sum_{i=0}^{3} n(X = i)} \tag{1}$$

where $i \in \{0, 1, 2, 3\}$ and n(X = i) is the frequency of getting i heads. Also,

$$\text{Number of times 3 coins were tossed} = 200 \tag{2}$$

$$\implies \sum_{i=0}^{3} n(X = i) = 200 \tag{3}$$

And from Table 1,

$$n(X = 2) = 72 \tag{4}$$

$$\therefore \Pr(X = 2) = \frac{72}{200} \tag{5}$$

$$= \frac{36}{100} = 0.36 \tag{6}$$

Hence, the probability of 2 heads coming up is $\boxed{0.36}$.

We have,

$$\Pr(X = 0) = \frac{28}{200} = 0.14 \tag{7}$$

$$\Pr(X = 1) = \frac{77}{200} = 0.385 \tag{8}$$

$$\Pr(X = 2) = \frac{72}{200} = 0.36 \tag{9}$$

$$\Pr(X = 3) = \frac{23}{200} = 0.115 \tag{10}$$

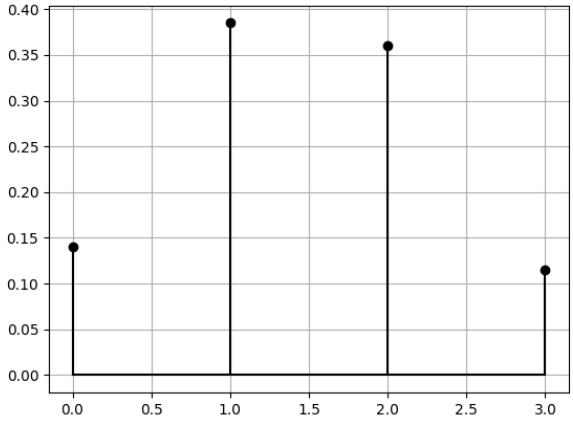

Fig. 1: Plot of PMF using above data

Now considering fair coins: Let probability of getting a head be a success and equal to p and probability of getting a tail be a failure and equal to q where $p + q = 1$. We can express this as a binomial distribution

$$\sum_{i=0}^{n} \Pr(X = i) = \sum_{i=0}^{n} {}^{n}C_i (\text{p})^i (1 - \text{p})^{n-i} \tag{11}$$

where $n = 3$ for 3 coins. Therefore,

$$\Pr(X = i) = {}^{3}C_i (\text{p})^i (\text{q})^{3-i} \tag{12}$$

For fair coins,

$$\mathrm{p} = \frac{1}{2} \tag{13}$$

$$\therefore \mathrm{q} = \frac{1}{2} \tag{14}$$

Therefore,

$$\Pr\left(X = 0\right) = {}^{3}C_{0} \left(\frac{1}{2}\right)^{0} \left(\frac{1}{2}\right)^{3} = \frac{1}{8} \tag{15}$$

$$\Pr\left(X = 1\right) = {}^{3}C_{1} \left(\frac{1}{2}\right)^{1} \left(\frac{1}{2}\right)^{2} = \frac{3}{8} \tag{16}$$

$$\Pr\left(X = 2\right) = {}^{3}C_{2} \left(\frac{1}{2}\right)^{2} \left(\frac{1}{2}\right)^{1} = \frac{3}{8} \tag{17}$$

$$\Pr\left(X = 3\right) = {}^{3}C_{3} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{0} = \frac{1}{8} \tag{18}$$

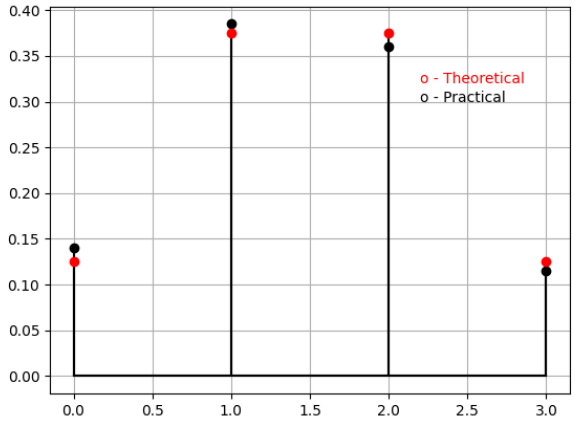

Fig. 2: Comparison of theoretical and practical PMF plots