A1110 Assignment 2

Tejal Kulkarni

CS21BTECH11058

April 2022

ICSE 12th 2018

Q21(b): A manufacturer's marginal cost function is $\dfrac{500}{\sqrt{2x+25}}$. Find the cost involved to increase production from 100 units to 300 units.

Solution:

TABLE I

TABLE WITH INPUT AND OUTPUT VARIABLES, THEIR SYMBOLS, THEIR FORMULAE AND VALUES:

Description	Symbol	Formula	Value
No of units	x	-	-
Initial units	a	-	100
Final units	b	-	300
Marginal Cost function	F(x)	$\dfrac{500}{\sqrt{2x+5}}$	-
Cost to increase production from a to b	C	$\int_a^b F(x)\,dx$	?

From the table we have cost involved to increase production from a units to b units

$$C = \int_a^b F(x)\,dx \tag{1}$$

where F(x)dx is the cost involved in changing production by dx units. Hence for a to b we integrate F(x) from a to b units

$$\implies C = \int_a^b \frac{500}{\sqrt{2x+25}}\,dx \tag{2}$$

$$= 500 \int_a^b \frac{1}{\sqrt{2x+25}}\,dx \tag{3}$$

Let,

$$2x + 25 = t \implies 2dx = dt \tag{4}$$

$$\implies C = 500 \int_{2a+25}^{2b+25} \frac{1}{\sqrt{t}}\,\frac{dt}{2} \tag{5}$$

$$= \frac{500}{2} \int_{2a+25}^{2b+25} \frac{1}{\sqrt{t}}\,dt \tag{6}$$

$$= \frac{500}{2} \left[2\sqrt{t} \right]_{2a+25}^{2b+25} \tag{7}$$

$$= 500 \left[\sqrt{t} \right]_{2a+25}^{2b+25} \tag{8}$$

$$= 500(\sqrt{2b+25} - \sqrt{2a+25}) \tag{9}$$

Now,

$$a = 100\text{units}, b = 300\text{units} \tag{10}$$

$$\implies C = 500(\sqrt{2 \times 300 + 25} - \sqrt{2 \times 100 + 25}) \tag{11}$$

$$= 500 \times (\sqrt{625} - \sqrt{225}) \tag{12}$$

$$= 500 \times (25 - 15) = 5000 \tag{13}$$

$$\implies C = \text{Rs.}5000 \tag{14}$$

Hence cost involved to increase production from 100 units to 300 units is $\boxed{\text{Rs.}5000}$.