A1110 Assignment 2

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Q21(b): A manufacturer's marginal cost function is $\frac{500}{\sqrt{2x+25}}$. Find the cost involved to increase production from 100 units to 300 units.

Solution:

TABLE I
TABLE WITH INPUT AND OUTPUT VARIABLES, THEIR SYMBOLS, THEIR FORMULAE AND VALUES:

Description	Symbol	Formula	Value
No of units	x	-	-
Initial units	a	-	100
Final units	b	-	300
Marginal Cost function	F(x)	$\frac{500}{\sqrt{2x+5}}$	-
Cost to increase production from a to b	C	$\int_a^b F(x)\,dx$	?

From the table we have cost involved to increase production from a units to b units

$$\text{C} = \int_a^b F(x)\,dx \tag{1}$$

where F(x)dx is the cost involved in changing production by dx units. Hence for a to b we integrate F(x) from a to b units

$$\implies \text{C} = \int_a^b \frac{500}{\sqrt{2x+25}}\,dx \tag{2}$$

$$= 500\int_a^b \frac{1}{\sqrt{2x+25}}\,dx \tag{3}$$

Let $2\text{x} + 25 = \text{t}$, $2\text{dx} = \text{dt}$

$$\implies \text{C} = 500\int_{2a+25}^{2b+25} \frac{1}{\sqrt{\text{t}}}\,\frac{\text{d}t}{2} \tag{4}$$

$$= \frac{500}{2}\int_{2a+25}^{2b+25} \frac{1}{\sqrt{\text{t}}}\,\text{dt} \tag{5}$$

$$= \frac{500}{2}\left[2\sqrt{\text{t}}\right]_{2a+25}^{2b+25} \tag{6}$$

$$= 500\left[\sqrt{\text{t}}\right]_{2a+25}^{2b+25} \tag{7}$$

$$= 500(\sqrt{2b+25} - \sqrt{2a+25}) \tag{8}$$

Now a = 100 units and b = 300 units

$$\implies \text{C} = 500(\sqrt{2\times 300+25} - \sqrt{2\times 100+25}) \tag{9}$$

$$= 500\times(\sqrt{625} - \sqrt{225}) \tag{10}$$

$$= 500\times(25-15) = 5000 \tag{11}$$

$$\implies \text{C} = \text{Rs.}5000 \tag{12}$$

Hence cost involved to increase production from 100 units to 300 units is $\boxed{\text{Rs.}5000}$.