# A1110 Assignment 8

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# Outline

1. Question

2. Solution

# Question

**Example 32:** Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is atleast one defective egg.

## Defining random variables

Let $X_i, 1 \leq i \leq N$ be N Bernoulli random variables. Then,

$$\Pr\left(X_i = k\right) = \begin{cases} 1-p, & k=0 \\ p, & k=1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Let Y be a random variable such that,

$$Y = \sum_{i=1}^{i=N} X_i \tag{2}$$

# Moment generating Function of $X_i$

Using equation (1), the moment generating function of $X_i$ can be given as,

$$M_Z(X_i) = \sum_{k=-\infty}^{k=\infty} z^{-k} P_X(k) \tag{3}$$

$$= P_X(0) + z^{-1} P_X(1) \tag{4}$$

$$= (1-p) + pz^{-1} \tag{5}$$

# Moment generating Function of Y

Since all the $X_i$ are independent and identically distributed, the moment generating function of $Y$ is

$$M_Y(Z) = E(Z^{-Y}) = E(Z^{-\sum_{i=1}^{i=N} X_i}) \tag{6}$$

$$= \prod_{i=1}^{i=N} E(Z^{-X_i}) \tag{7}$$

$$= \left[(1-p) + pz^{-1}\right]^N \tag{8}$$

$$= \sum_{k=0}^{k=N} z^{-k} (\binom{N}{k} (1-p)^{N-k} p^k) \tag{9}$$

# PMF and CDF of Y

The PMF of the Binomial random variable $Y$ is

$$\Pr\left(Y = k\right) = \begin{cases} \binom{N}{k}(1-p)^{N-k}p^k, & 0 \le k \le N \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

Therefore, the CDF of $Y$ is given by,

$$\begin{aligned} F_Y\left(k\right) &= \sum_{i=-\infty}^{i=k} \Pr\left(Y = i\right) \\ &= \begin{cases} 0, & k < 0 \\ \sum_{K=0}^{K=k} \binom{N}{K}(1-p)^{N-K}p^K, & 0 \le k < N \\ 1, & k \ge N \end{cases} \end{aligned} \tag{11}$$

# Given Information

Now,let Y be the binomial random variable denoting the number of defective eggs drawn.Hence $N = 10$ and,

$$\text{Probability of success} = p = \frac{10}{100} = \frac{1}{10} \tag{12}$$

To find:

$$\Pr(\text{atleast one defective egg}) = \Pr(Y \geq 1) \tag{13}$$

# Calculation of answer

Now,

$$\Pr(Y \geq 1) = \sum_{k=0}^{10} \Pr(Y = k) - \Pr(Y = 0) \tag{14}$$

Also,

$$\sum_{k=0}^{10} \Pr(Y = k) = 1 \tag{15}$$

$$\implies \Pr(Y \geq 1) = 1 - \Pr(X = 0) \tag{16}$$

Using (10),

$$\Pr(Y \geq 1) = 1 - {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} \tag{17}$$

$$= 1 - \frac{9^{10}}{10^{10}} = \boxed{0.651} \tag{18}$$

# Graphs - PMF and CDF of Y

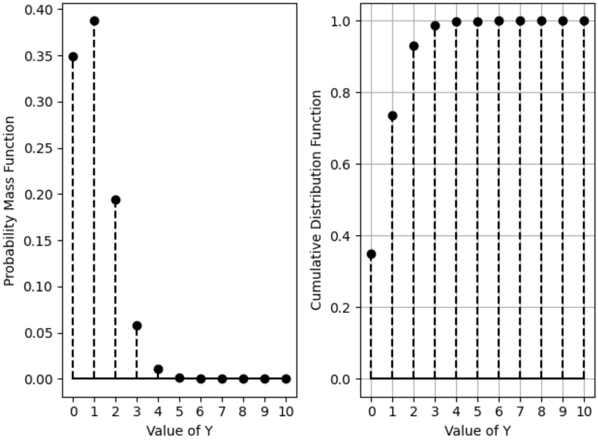


Figure: PMF and CDF for the given situation.