

# A1110 Assignment 10

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# Outline

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# Question

**Example 6-21:** Suppose  $x$  and  $y$  are independent uniformly distributed random variables in the interval  $(0, \theta)$ . Define  $z = \min(x, y)$ ,  $w = \max(x, y)$ . Determine  $f_{zw}(z, w)$ .

# Solution

Both  $z$  and  $w$  vary in the interval  $(0, \theta)$ . Thus,

$$F_{zw}(z, w) = 0 \quad (1)$$

if  $z < 0$  or  $w < 0$

$$F_{zw}(z, w) = P\{z \leq z, w \leq w\} \quad (2)$$

$$= P\{\min(x, y) \leq z, \max(x, y) \leq w\} \quad (3)$$

We must consider 2 cases:  $w \geq z$  and  $w < z$  as shown in the figure

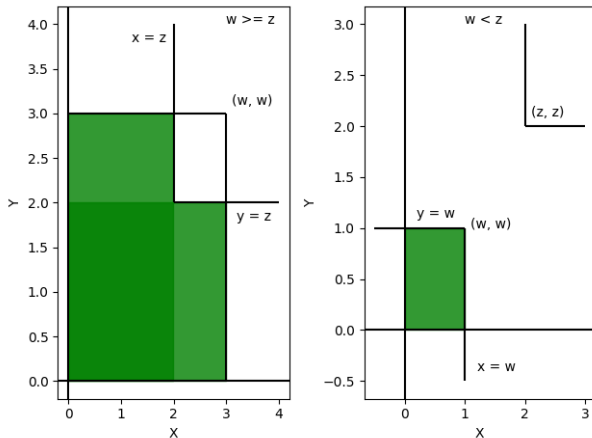


Figure: a)  $w \geq z$  and b)  $w < z$

Case 1:  $w \geq z$

$$F_{zw}(z, w) = F_{xy}(z, w) + F_{xy}(w, z) - F_{xy}(z, z) \quad (4)$$

Case 2:  $w < z$

$$F_{zw}(z, w) = F_{xy}(w, w) \quad (5)$$

with

$$F_{xy}(x, y) = F_x(x) F_y(y) \quad (6)$$

$$= \frac{x}{\theta} \times \frac{y}{\theta} \quad (7)$$

$$= \frac{xy}{\theta^2} \quad (8)$$

we obtain,

$$F_{zw}(z, w) = \begin{cases} (2wz - z^2) / \theta^2, & 0 < z < w < \theta \\ w^2 / \theta^2, & 0 < w < z < \theta \end{cases} \quad (9)$$

Thus,

$$f_{zw}(z, w) = \begin{cases} 2/\theta^2, & 0 < z < w < \theta \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

By equation (10),

Case 1:  $0 < z < \theta$

$$f_z(z) = \int_z^\theta f_{zw}(z, w) dw \quad (11)$$

$$= \frac{2}{\theta} \left(1 - \frac{z}{\theta}\right) \quad (12)$$

Case 2:  $0 < w < \theta$

$$f_w(w) = \int_0^w f_{zw}(z, w) dz \quad (13)$$

$$= \frac{2w}{\theta^2} \quad (14)$$