AI1110 Assignment 10

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CS21BTECH11058

May 2022
Papoullis Text Book

Example 6-21: Suppose x and y are independent uniformly distributed random variables in the interval $(0, \theta)$.Define $z = \min(x, y)$, $w = \max(x, y)$.Determine $f_{zw}(z, w)$.
Solution: Both z and w vary in the interval $(0, \theta)$.Thus,

$$F_{zw}(z, w) = 0 \tag{1}$$

if $z < 0$ or $w < 0$

$$F_{zw}(z, w) = P\{z \le z, w \le w\} \tag{2}$$
$$= P\{min(x, y) \le z, max(x, y) \le w\} \tag{3}$$

We must consider 2 cases: $w \ge z$ and $w < z$ as shown in the figure

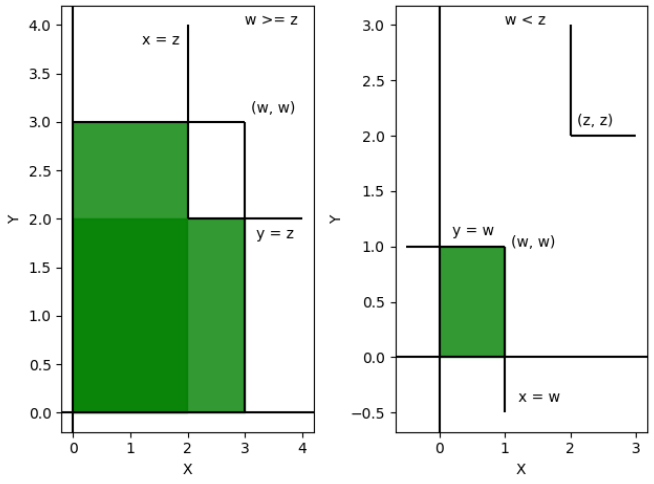

Fig. 1: a) $w \ge z$ and b) $w < z$

Case 1:$w \ge z$

$$F_{zw}(z, w) = F_{xy}(z, w) + F_{xy}(w, z) - F_{xy}(z, z) \tag{4}$$

Case 2: $w < z$

$$F_{zw}(z, w) = F_{xy}(w, w) \tag{5}$$

with

$$F_{xy}(x, y) = F_{xx}(x) F_y(y) \tag{6}$$
$$= \frac{x}{\theta} \times \frac{y}{\theta} \tag{7}$$
$$= \frac{xy}{\theta^2} \tag{8}$$

we obtain,

$$F_{zw}(z, w) = \begin{cases} (2wz - z^2)/\theta^2, & 0 < z < w < \theta \\ w^2/\theta^2, & 0 < w < z < \theta \end{cases} \tag{9}$$

Thus,

$$f_{zw}(z, w) = \begin{cases} 2/\theta^2, & 0 < z < w < \theta \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

By equation (10),
Case 1: $0 < z < \theta$

$$f_z(z) = \int_z^\theta f_{zw}(z, w)\, dw \tag{11}$$
$$= \frac{2}{\theta}\left(1 - \frac{z}{\theta}\right) \tag{12}$$

Case 2: $0 < w < \theta$

$$f_w(w) = \int_0^w f_{zw}(z, w)\, dz \tag{13}$$
$$= \frac{2w}{\theta^2} \tag{14}$$