

A1110 Assignment 8

Tejal Kulkarni
CS21BTECH11058

CBSE Probability Grade 12

May 19, 2022

Outline

1 Question

2 Solution

Question

Example 32: Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is atleast one defective egg.

Defining random variables

Let $X_i, 1 \leq i \leq N$ be N Bernoulli random variables. Then,

$$\Pr(X_i = k) = \begin{cases} 1 - p, & k = 0 \\ p, & k = 1 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Let Y be a random variable such that,

$$Y = \sum_{i=1}^{i=N} X_i \quad (2)$$

Moment generating Function of X

Using equation (1), the moment generating function of X_i can be given as,

$$M_Z(X_i) = \sum_{k=-\infty}^{k=\infty} z^{-k} P_X(k) \quad (3)$$

$$= P_X(0) + z^{-1} P_X(1) \quad (4)$$

$$= (1 - p) + pz^{-1} \quad (5)$$

Moment generating Function of X_i

Since all the X_i are independent and identically distributed, the moment generating function of Y is

$$M_Y(Z) = E(Z^{-Y}) = E(Z^{-\sum_{i=1}^{i=N} X_i}) \quad (6)$$

$$= \prod_{i=1}^{i=N} E(Z^{-X_i}) \quad (7)$$

$$= [(1 - p) + pz^{-1}]^N \quad (8)$$

$$= \sum_{k=0}^{k=N} z^{-k} \binom{N}{k} (1 - p)^{N-k} p^k \quad (9)$$

PMF and CDF of Y

The PMF of the Binomial random variable Y is

$$\Pr(Y = k) = \begin{cases} \binom{N}{k} (1-p)^{N-k} p^k, & 0 \leq k \leq N \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Therefore, the CDF of Y is given by,

$$\begin{aligned} F_Y(k) &= \sum_{i=-\infty}^{i=k} \Pr(Y = i) \\ &= \begin{cases} 0, & k < 0 \\ \sum_{K=0}^{K=k} \binom{N}{K} (1-p)^{N-K} p^K, & 0 \leq k < N \\ 1, & k \geq N \end{cases} \end{aligned} \quad (11)$$

Given Information

Now, let Y be the binomial random variable denoting the number of defective eggs drawn. Hence $N = 10$ and,

$$\text{Probability of success} = p = \frac{10}{100} = \frac{1}{10} \quad (12)$$

To find:

$$\Pr(\text{atleast one defective egg}) = \Pr(Y \geq 1) \quad (13)$$

Calculation of answer

Now,

$$\Pr(Y \geq 1) = \sum_{k=0}^{10} \Pr(Y = k) - \Pr(Y = 0) \quad (14)$$

Also,

$$\sum_{k=0}^{10} \Pr(Y = k) = 1 \quad (15)$$

$$\implies \Pr(Y \geq 1) = 1 - \Pr(X = 0) \quad (16)$$

Using (10),

$$\Pr(Y \geq 1) = 1 - {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} \quad (17)$$

$$= 1 - \frac{9^{10}}{10^{10}} = \boxed{0.651} \quad (18)$$

Graphs - PMF and CDF of Y

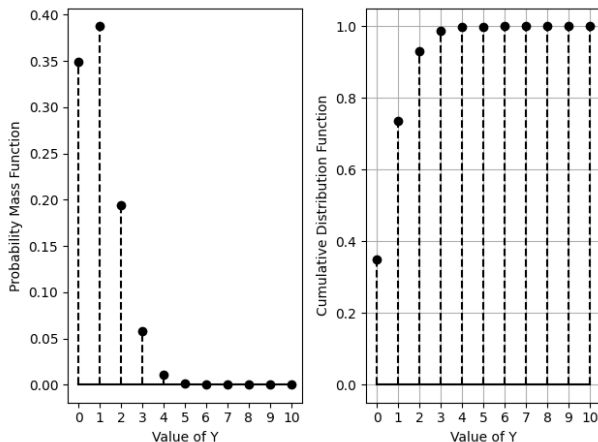


Figure: PMF and CDF for the given situation.