A1110 Assignment 12

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Outline

Question

Problem 12-19: Find the maximum entropy estimate $S_{MEM}(w)$ and the line-spectral estimate of a process $\mathrm{x}[n]$ if

$$R[0] = 13 \tag{1}$$
$$R[1] = 5 \tag{2}$$
$$R[2] = 2 \tag{3}$$

Solution

By Levinson's algorithm,

$$a_1^1 = K_1 = \frac{R[1]}{R[0]} \tag{4}$$

$$P_1 = \left(1 - K_1^2\right) R[0] \tag{5}$$

$$P_N = \left(1 - K_N^2\right) P_{N-1} \tag{6}$$

Hence,

$$P_0 = R[0] = 13 \tag{7}$$

$$a_1^1 = K_1 = \frac{5}{13} \tag{8}$$

$$P_1 = \frac{144}{13} \tag{9}$$

Also,

$$P_{N-1}K_N = R[N] - \sum_{k=1}^{N-1} a_k^{N-1} R[N-1] \tag{10}$$

$$a_N^N = K_N \tag{11}$$

$$a_k^N = a_k^{N-1} - K_N a_{N-k}^{N-1} \tag{12}$$

where $1 \leq k \leq N-1$
This gives,

$$P_1 K_2 = R[2] - a_1^1 R[1] \tag{13}$$

$$\implies K_2 = \frac{1}{144} \tag{14}$$

$$\implies a_2^2 = \frac{1}{144} \tag{15}$$

By (12),

$$a_1^2 = a_1^1 - K_2 a_1^1 \tag{16}$$

$$\implies a_1^2 = \frac{55}{144} \tag{17}$$

By (6),

$$P_2 = \left(1 - K_2^2\right) P_1 \tag{18}$$

$$\implies P_2 = \frac{1595}{144} \tag{19}$$

Hence,

$$S_{MEM}\left(w\right) = \frac{1595 \times 144}{\left|144 - 55e^{-jwT} - e^{-j2wT}\right|^2} \tag{20}$$

The Yule-Walker's equations are given by,

$$R[0] + a_1 R[1] + \dots + a_N R[N] = P_N \tag{21}$$

$$R[1] + a_1 R[0] + \dots + a_N R[N-1] = 0 \tag{22}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \tag{23}$$

$$R[N] + a_1 R[N-1] + \dots + a_N R[0] = 0 \tag{24}$$

The correlation matrix D_{N+1} is given by,

$$D_{N+1} = \begin{bmatrix} R_{yy}[0] - q & R_{yy}[1] & \cdots & R_{yy}[N] \\ R_{yy}[1] & R_{yy}[0] - q & \cdots & R_{yy}[N-1] \\ \cdots & \cdots & \cdots & \cdots \\ R_{yy}[N] & R_{yy}[N-1] & \cdots & R_{yy}[0] - q \end{bmatrix}$$

To find q_0,

$$\begin{vmatrix} 13-q & 5 & 2 \\ 5 & 13-q & 5 \\ 2 & 5 & 13-q \end{vmatrix} = 0 \tag{25}$$

This gives $q_0 = 14 - \sqrt{51} \simeq 6.86$. Inserting the modified data in the Yule Walker's equations we obtain,

$$a_1^2 \simeq 4.07 \tag{26}$$

$$a_2^2 = -1 \tag{27}$$

Also,

$$E_N(z) = 1 - a_1^N z^{-1} - \ldots - a_N^N z^{-N} \tag{28}$$

$$\implies E_2(z) = 1 - 4.07z^{-1} + z^{-2} \tag{29}$$

$$z_{1,2} = e^{\pm j0.62} \tag{30}$$

By (29) we get,

$$\varepsilon[n] = x[n] - a_1^N x[n-1] - \ldots - a_N^N x[n-N] \tag{31}$$

Solving this we get,

$$R_L[m] = 6.86 \times \delta[m] + 3.07 \cos 0.62m \tag{32}$$

$$S_L(m) = 6.86 + \frac{2\pi}{T} \times 3.07 \left[\delta(w - 0.62) + \delta(w + 0.62)\right] \tag{33}$$