Project: Machine Learning

ME5201 – Computational Methods in Engineering

Due date: 10 November 2023

Problem: To measure the velocity of the sound wave propagation in a given fluid medium.

Setup: Assume the fluid at rest is a region on the XY-plane and let

$p(x, y, t)$= acoustic pressure of the fluid from equilibrium at position (x, y) and time t.

Under the ideal assumptions of the fluid (e.g., uniform fluid density and uniform bulk modulus), one can write the linear two-dimension sound wave equation problem in a homogeneous fluid medium, which may be written as follows:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right)$$

We assume that the fluid lies over the rectangular region $R = [0, a]x\ [0, b]$ and has fixed edges. The boundary conditions can be expressed as follows:

$$p(0, y, t) = p(a, y, t) = 0 \qquad 0 \leq y \leq b, t > 0$$

$$p(x, 0, t) = p(x, b, t) = 0 \qquad 0 \leq x \leq a, t > 0$$

How the initial disturbance is created in the fluid media and set into the motion of the fluid can be done via the initial conditions.

$$p(x = 0, y = b/2, t) = f(x, y, t) \qquad (x, y) \in R, \qquad t > 0$$

Now, the sound wave equation with initial disturbance can be expressed as follows:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) + f(x, y, t) \qquad \textbf{(1)}$$

Here, c is the positive constant, which is the function of bulk modules and fluid density.

$$c = \sqrt{\frac{K}{\rho}}$$

X, Y are Cartesian coordinates.

<u>**Goal:**</u> To solve the 2-D sound wave equation (1) in fluid subject to the boundary and initial conditions as given above.

The solution of this equation is typically obtained analytically or using numerical techniques such as Finite Difference Method, Finite Element method, etc. The sound wave equation must be solved to measure the fluid velocity for changes in fluid properties, such as density and bulk modules.

However, our idea is to use the machine learning (ML) approach to obtain the equivalent solution of the sound wave equation in fluid media using linear regression and Feedforward neural networks (FNN).

As we know, the ML model needs a good amount of the training dataset to fit the linear regression or train the FNN model.

Training dataset generation: We have used the Finite Element Method for solving the sound wave equation in the fluid media. The dimension of the fluid media used is $R = [0, 50\ mm] x\ [0, 200\ mm]$. The fluid properties, such as density and bulk modules, are randomly varied in a range to create 1000 FE simulation models. The fluid densities are considered in the range of 999.8 kg/m^3 to 13550 kg/m^{3,} and the bulk modulus is in the range of 0.92e+09 Pa to 2.3e+09 Pa. We have solved these (1000) FE models by giving a sinusoidal wave function as the initial disturbance to set the sound wave in motion, and the reflected sound wave from the boundary (x=a) is collected at the probe location (refer to Fig.1).

As we know, the velocity could be calculated by using the given formula:

$$\boldsymbol{Velocity}\ \left(\frac{\boldsymbol{m}}{\boldsymbol{s}}\right) = \frac{\boldsymbol{the\ total\ distance\ sound\ wave\ traveled}}{\boldsymbol{time\ it\ took\ (to\ and\ fro)}}$$

In our case, the sound wave traveled distance is 100 mm (50 mm forward and 50 mm backward to reach the probe location), the time taken to reach the probe location is 7.65e-05 sec, and the calculated velocity is 1307 m/s.

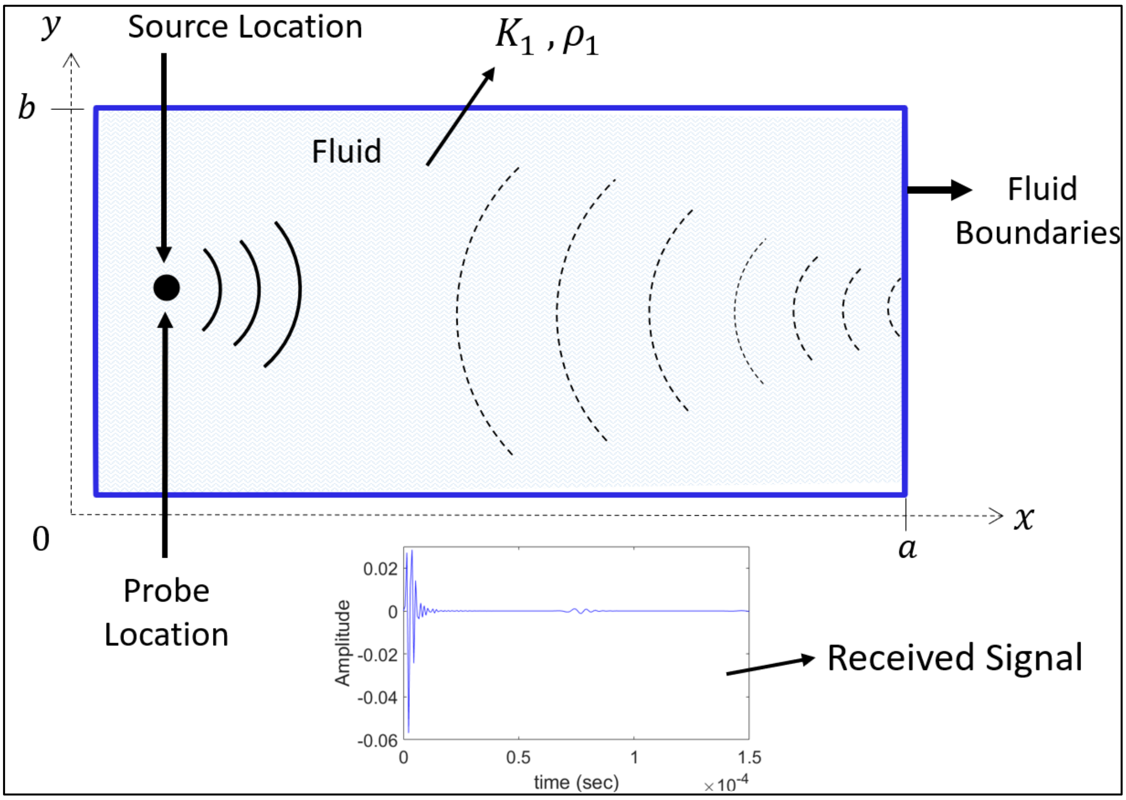
The training dataset is available in a tabular form given to you as supplement to this document, circulated through Google Drive. The document contains the input dataset of various density and bulk moduli and the measured output velocity.

Questions:

A person collects a set of data (n = 1000 observations) containing a single predictor (sound velocity in fluid) and two input parameters (fluid density and bulk modules), a quantitative response. The total 1000 dataset would be divided into 70% for training, 20% for testing, and 10% for validation of the trained model.

(1) ***Suppose that the true relationship between input and output parameters is linear*. One can train the linear and cubic regression models using the dataset provided. Considered the training residual to be Mean Square Error (MSE) for linear regression, and also trained the MSE using cubic regression. Once the model is trained, feed the input dataset from the validation and get the velocity from the ML model. We can expect the residual between <u>(A. linear and B. cubic regression)</u> one of them will be lower than the other or if there is not enough information to tell from the validation dataset. Please Justify your answer.**

(2) Suppose the true relationship between input and output parameters is not linear, but we don't know how far it is from linear. Consider the training MSE using the Feedforward neural networks (FNN). Once the FNN model is trained, use the same validation dataset (used in question 1) for calculating the velocity from the FNN model. Now, compare <u>the (C.) residual from FNN with Linear and cubic regression output</u>. We would expect one of the residuals to be lower than the others or would be expected to be the same, or is there not enough information to tell from the dataset given? Justify your answer. (note: Used the validation dataset for computing the residuals).



y
Source Location
K1 , ρ1
b
Fluid
Fluid
Boundaries
0
x
a
Probe
Location
0.02
0
-0.02
-0.04
-0.06
Amplitude
0
0.5
1
1.5
time (sec)
×10-4
Received Signal

Figure 1