In [66]:

```
import numpy as np
import scipy.integrate as evaluation_integral
import math
import matplotlib.pyplot as plot
```

In [67]:

```
n = int(input("Dimension = "))
# Define the lower limit & upper limit for the integrals
lower_limit = 0
upper_limit = math.pi
```

```
Dimension = 4
```

In [68]:

```
# Define a function that generates the base functions for the polynomial function
def base_function_generator(p):
 # Define a base function as a lambda function of x raised to the power p
    base_function = lambda x:x**p
    return base_function
```

In [69]:

```
# Create a list of base functions for the polynomial space up to degree n-1
b = []
for i in range(n):
    b.append(base_function_generator(i))
```

In [70]:

```
# Initialize the vector F, which contains the integrals of the product of each base func
F = np.zeros(n)
for i in range(n):
    F[i] = (evaluation_integral.quad(lambda x:(b[i](x)* math.sin(x)),lower_limit, upper_limit)[0]
```

In [71]:

```
K = np.zeros((n,n))
for i in range(n):
    for j in range(n):
        K[i,j] = (evaluation_integral.quad(lambda x:(b[i](x)*b[j](x)),lower_limit, upper_limit)[0
```

In [72]:

```
C = np.linalg.solve(K, F)
```

In [73]:

```
x = np.arange(lower_limit, upper_limit,0.01)
# Define the actual function to be approximated
y1 = np.sin(x)
```

In [74]:

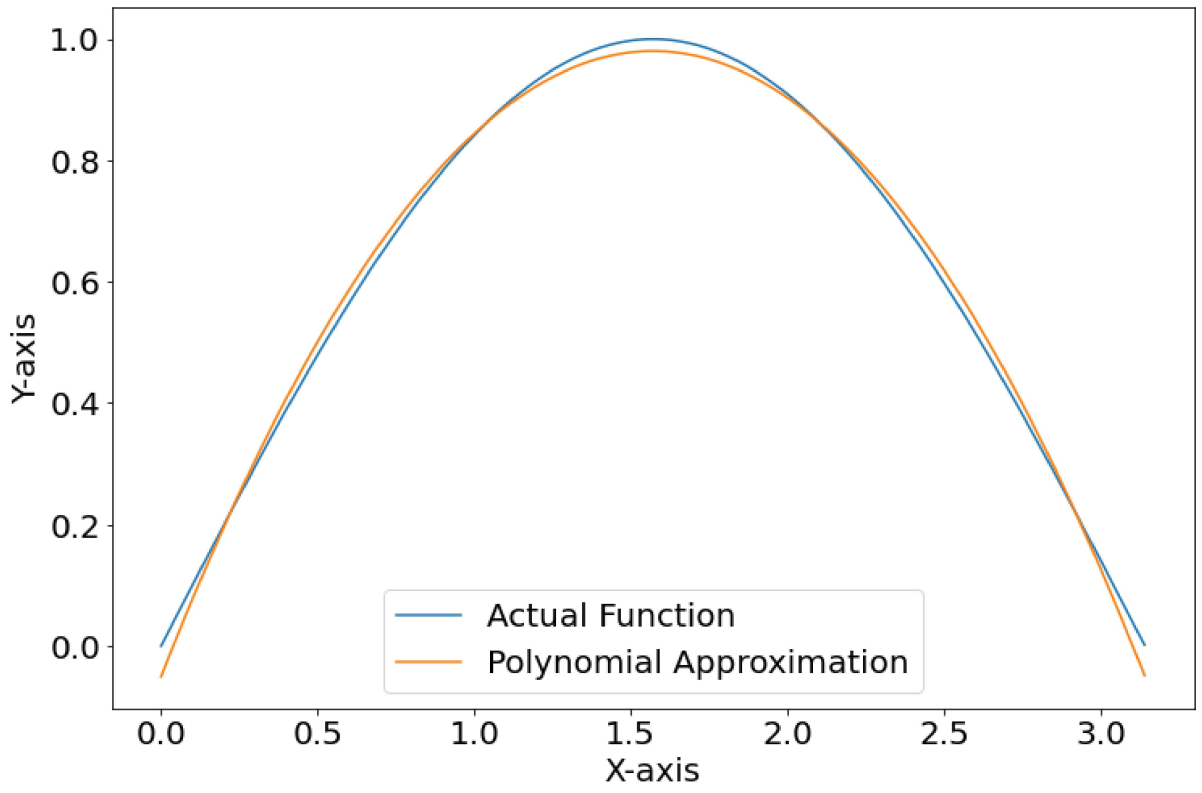
```
l = len(C)

# Creates an array y2 with the same size as the x array and initializes all of its elemen
y2 = np.zeros_like(x)

for i in range(n):
    y2 = C[i]*x**i + y2
```

In [75]:

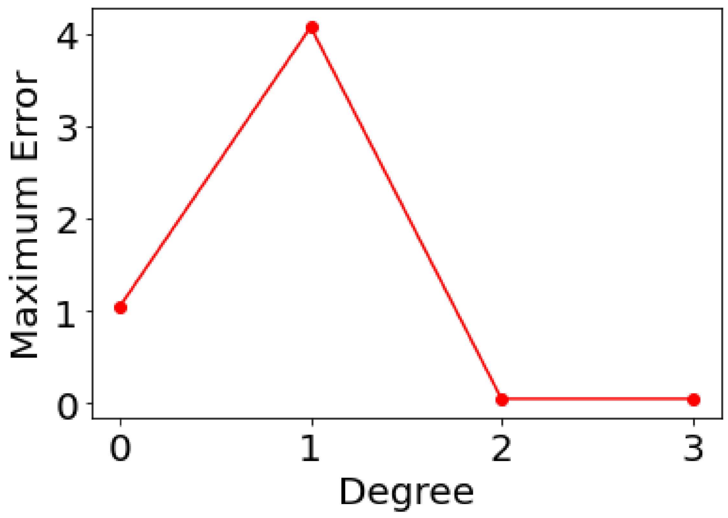
```
# Plot the actual function sin(x) and the polynomial approximation y2
plot.figure(figsize=(12,8))
plot.plot(x, y1, label="Actual Function")
plot.plot(x, y2, label="Polynomial Approximation")
plot.xlabel('X-axis', fontsize=20) # increase font size to 20
plot.ylabel('Y-axis', fontsize=20) # increase font size to 20
plot.tick_params(axis='both', which='major', labelsize=20) # increase font size to 20
Dimension = 4
# Add a legend to the plot
plot.legend(fontsize=20)
plot.show()
```



In [76]:

```
# plot the error for every degree polynomial
error = []
for i in range(n):
    y = np.zeros_like(x)
    for j in range(i+1):
        y += C[j]*x**j
    err = np.abs(y1-y)
    error.append(np.max(err))

plot.figure(figsize=(6,4))
plot.plot(range(n), error, 'ro-')
plot.xlabel('Degree', fontsize=20)
plot.ylabel('Maximum Error', fontsize=20)
plot.tick_params(axis='both', which='major', labelsize=20)
plot.show()
Dimension = 4
```



In [ ]:

In [ ]: