Design of Linear Quadratic Regulator for Rotary Inverted Pendulum Using LabVIEW

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Abstract—**The Rotary Inverted Pendulum (RIP) is highly nonlinear, multivariable and unstable system. The design of controller for such a system is a challenging task. The main objective of this paper is to design a controller which balances the pendulum in the vertically upright position in spite of small disturbances using optimization based Linear Quadratic Regulator (LQR) technique. Here the system response is observed for different values of positive semi definite weighing matrix and also tested for upright mode of the pendulum. Finally, the experimental and LabVIEW based simulation results for arm and pendulum response are described.**

***Keywords*— LQR (Linear Quadratic Regulator), LabVIEW (Laboratory Virtual Instrumentation Engineering Workbench),** **Rotary Inverted Pendulum (RIP), PID**

I. Introduction

Rotary Inverted pendulum (RIP) system has been one of the benchmark problems in the control system. There are numerous applications of RIP system in practical engineering field such as in stabilizing the rocket during take-off, future transport vehicles like Segway, jack packs etc. The RIP is a captivating subject in the control system due to its intrinsic non linearity [9] which also makes it a suitable subject for teaching and practical demonstration of modern control techniques.

The most commonly used control techniques such as PID approach; full state feedback and LQR technique require adequate knowledge of the system for obtaining the desired performance. The inceptive step in the designing of the system is to obtain a reliable mathematical model which is quite complex to describe using differential equations. So, the design of controller for such nonlinear system is challenging task. Several researches such as Control of an Inverted pendulum using MODE- based optimized LQR controller [1], modeling and control of a Rotary Inverted Pendulum [2] and Control of the Inverted Pendulum using state feedback control [8] have been reported using MATLAB simulink interface.

The most commonly used technique is PID based controller which consists of three control parameters proportional, integral and derivative. The tuning of these parameters to achieve desired behavior is not an easy task. Also the classical technique, transfer function has certain limitations. It is suitable only for linear time invariant system and also does not give any information about the internal behavior of the system. These limitations are overcome using state variable approach. In this paper the pendulum is balanced in the upright position using LQR technique by tuning one or more design parameters while the same system is balanced using Ackermann's formula in [3] using LabVIEW.

The inverted pendulum, which is shown in fig.1, is energized by a 24 volt DC motor. The L shaped arm of the pendulum is mounted on the shaft of the motor which results in the rotation of the arm with the pendulum connected at the end. The system is equipped with optical encoders and tachometers to measure the pendulum angle α and arm angle θ respectively. So, the key attributes of system are it has durable DC motor, accurate and stiff machine components, inbuilt power amplifier, high resolution optical encodes and shield to protect system from external environment. The RIP system has two equilibrium points but one is stable and other unstable. When the rod is vertical downward the system is stable while when the rod is vertical and pointing upward then the system is unstable. In this paper LQR controller is applied to balance the pendulum in the upright position using LabVIEW based real time software.

LabVIEW offers unparalleled integration with different hardware devices and it has hundreds of inbuilt libraries for the control of these devices. It also offers proper analysis and visualization of data obtained from system by creating virtual instruments (VIs). LabVIEW programs called "Virtual Instruments" (VIs) are created using icons instead of conventional text-based codes [6]. The considerable advantage of LabVIEW is that it is fast and also facilitates the updating of parameters in a couple of second. LabVIEW can also be used for the control of different machines [4].

II. Mathematical Modelling of Rotary Inverted Pendulum

Mathematical models of a system can be achieved by applying the laws which is governing the physical nature of the different components present in the system. In this paper the model of RIP is obtained by using Euler-Langrange equations. For the designing of the controller of mechanical or electromechanical, linear or non-linear systems, Euler –

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Langrange equations has already been proven to be useful tool [7].

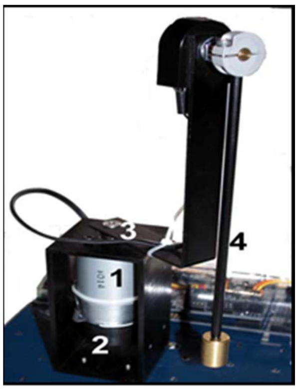


Fig .1 Rotary Inverted Pendulum Model

1. DC motor 2.Encoder 3. Arm 4.Pendulum

Fig. 2(a) depicts the free body diagram pendulum as one rigid body at the center of mass. Fig. 2(b) shows the pendulum system in the upright position.

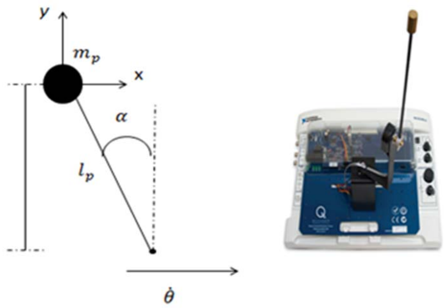


Fig.2. Free Body diagram and Pendulum motion

The following list of parameters used in the modelling of the system illustrated in table 1.

TABLE I. NOMENCLATURE OF SYSTEM PARAMETERS

Symbol	Description
θ	Arm angle (radians)
α	Pendulum angle (radians)
k	Length of arm pivot
l_p	Length of pendulum from center of mass
m_p	Mass of pendulum assembly
J_{CM}	Pendulum moment of inertia
V_x	Pendulum velocity in x – direction
V_y	Pendulum velocity in y - direction

The position coordinates of the pendulum relative to its base coordinate system xyz is

$$x_p = -l_p \sin \alpha(t) \tag{1}$$

$$y_p = \quad l_p \cos \alpha\,(t) \tag{2}$$

The velocity components of the pendulum are found by taking derivation of (1) and (2) at the centre of mass

$$v_p = -l_p \cos \alpha(\dot{\alpha})\,\hat{x} - l_p \sin \alpha(\dot{\alpha})\hat{x} \tag{3}$$

The pendulum also rotates with the rotating arm in the x coordinate at a rate given by:

$$V_{arm} = \mathrm{k}\,\dot{\theta} \tag{4}$$

The x and y velocity components from (3) and (4) are expressed as:

$$V_x = \ k\dot{\theta} - l_p \cos \alpha(\dot{\alpha}) \tag{5}$$

$$V_y = -\, l_p \sin \alpha(\dot{\alpha}) \tag{6}$$

After obtaining the velocity components, the dynamic equations for the system can be obtained using Euler Langrange equations. It uses the concept of energy to deliver different state space representations which is given by:

$$\frac{d}{dt}\left(\frac{dL}{d\theta}\right) - \frac{dL}{d\theta} = T_{out} - B_{eq}\dot{\theta} \tag{7}$$

$$\frac{d}{dt}\left(\frac{dL}{d\theta}\right) - \frac{dL}{d\alpha} = -\,B_p\dot{\alpha} \tag{8}$$

Where, L is the Langrangian function resulting from the difference of kinetic energy and potential energy.

Potential Energy: The total potential energy is only due to gravity and expressed as:

$$\mathrm{U}\,(\alpha) = m_p\,\mathrm{g}\,l_p \cos \alpha \tag{9}$$

Kinetic Energy: The KE arises from the moving hub T_{HUB}, the velocity of the point mass in the x direction T_{V_x} and y direction T_{V_Y} and the pendulum rotating about its center of mass T_{pen}. The T_{total} can be expressed as:

$$T_{total} = T_{HUB} + T_{v_x} + T_{v_y} + T_{pen} \tag{10}$$

$$T_{total} = \tfrac{1}{2}J_{eq}\dot{\theta}^2 + \tfrac{1}{2}m_p{v_x}^2 + \tfrac{1}{2}m_p{v_y}^2 + \tfrac{1}{2}J_{CM}\alpha^2 \tag{11}$$

Where, J_{eq} is equivalent inertia of the hub and J_{CM} is the inertia of the pendulum about its center of mass. After substituting the equation of V_x and V_y in (11), T_{total} becomes:

$$T_{total} = \tfrac{1}{2}J_{eq}\dot{\theta}^2 + + \tfrac{1}{2}m_p\left(k\dot{\theta} - l_p \cos \alpha(\dot{\alpha})\right)^2 + \tfrac{1}{2}m_p(-l_p \sin \alpha(\dot{\alpha}))^2 + \tfrac{1}{2}{J_{cm}}^{\dot{\alpha}^2} \tag{12}$$

The Langrangian function is expressed as:

$$L = T_{total} - U \tag{13}$$

Substituting (13) in (7) and (8) and linearizing about $\alpha = 0$ and equations are represented as:

$$J_{eq}\ddot{\theta} + m_p k^2 \dot{\theta} - m_p l_p \mathrm{k} \ddot{\alpha} = T_{out} - B_a \dot{\theta} \tag{14}$$

$$J_{cm}\,\ddot{\alpha} + m_p\, {l_p}^2 \ddot{\alpha} - m_p\, l_p\, \mathrm{k}\, \ddot{\theta} - m_p \mathrm{g}\, l_p \alpha = -B_p \dot{\alpha} \tag{15}$$

The net output torque of the DC motor which acts on the load is given as:

$$T_{OUT} = \frac{K_e}{R_m}(V_m - K_m\,\dot{\theta}) \tag{16}$$

After combining the equations derived above, the state space representation of the system is:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{qs}{A} & -\frac{rB}{A} & 0 \\ 0 & \frac{ps}{A} & -\frac{qB}{A} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{rKe}{RmA} \\ \frac{qKe}{RmA} \end{bmatrix} V_m \tag{17}$$

Output equation is given by:

$$\mathrm{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_m \tag{18}$$

Here, $p = J_{eq} + m_p k^2$, $q = m_p l_p k$, $r = J_{CM} + m_p l_p^2$, $s = m_p g l_p$, $A = pr - s^2$, $B = \frac{K_e K_m}{R_m}$

The values of different parameters [5] tabulated in table 2.

TABLE II. SYSTEM CONFIGURATION PARAMETERS

Symbol	Description	Value
K_e	Motor Torque Constant	0.02797
R_m	Armature Resistance of Motor	3.30
K_m	Back Electromotive force constant	0.02797
J_{eq}	Equivalent Moment of inertia	$1.77e^{-4}$

After substituting the values of the different parameters, the final state space model of the system is obtained as:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 77.3881 & -0.572113 & 0 \\ 0 & 83.4513 & -0.243399 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 17.1806 \\ 7.30929 \end{bmatrix} V_m \tag{19}$$

The output equation is same as given in (18). So, the mathematical given in (18) and (19) is used for the designing of a controller.

III. CONTROLLER DESIGN

A. Controller requirements:

Inverted pendulum is highly nonlinear system and must be actively balanced to remain upright. The open loop analysis of system is conducted using LabVIEW. Fig. 3 shows the block diagram of the open loop system with mathlab script which facilitates direct import of MATLAB function into LabVIEW. From fig. 4 following observations are made, the poles of the system lie in the right half of the s-plane, hence the system is unstable. There are different techniques by which system poles can be placed anywhere in the s-plane but there are certain conditions that the system must be met for the design of controller:

a) The system should be completely controllable.
b) The state variables (θ, α) must be measurable and available for feedback.
c) Control Input (V_m) should be unconstrained.

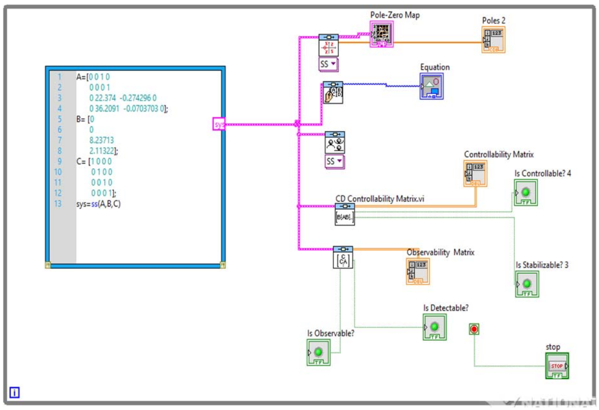


Fig.3 Open Loop Block Diagram of System

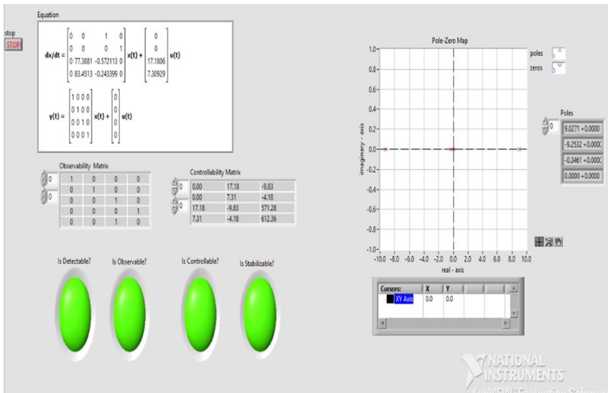


Fig.4 Analysis of open loop system

Fig. 4 shows that the system is controllable and observable.

B. Overview of Linear Quadratic Regulator Formulation

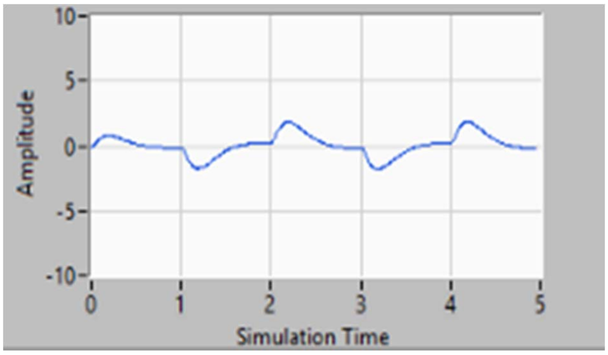
The objective of the LQR controller is to compute the feedback gain matrix K_{LQR} that minimizes the quadratic cost function J_{LQR} given in (20) and keep the pendulum in the vertically upright state in spite of the disturbance.

$$J_{LQR} = \int_0^{\infty} (\hat{x}^T Q \hat{x} + \hat{u}^T R \hat{u}) \quad (20)$$

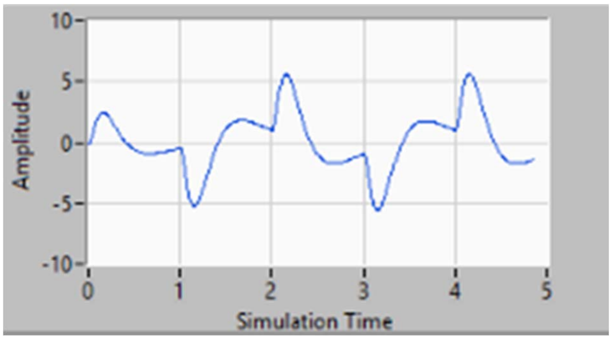
The control input signal is given as $u = -K_{LQR}\, x$ and Q and R are positive definite and semi definite weighing matrix respectively. In general, as indicated in the (20), the matrices Q and R affect the overall performance of the system. So the parameters of Q and R are carefully chosen for the successful design of LQR controller.

Fig. 5(a) and (b) shows pendulum response for

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } Q = \begin{bmatrix} 15 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(a)



(b)

Fig.5 Pendulum Response for different values of weighing matrix

The value of optimal gain (K_{LQR}) also increases from [-1 60.51 -1.39 7.92] to [-3.87 79.30 -2.36 10.55], this shows that when the value of Q increases then pendulum deflects more from its vertical position as shown in fig. 5 because control input will work harder to minimize moment of inertia and the value of optimal gain becomes larger.

C. Controller Objectives

The objective of controller system design is to balance the system in the upright position with the following specifications:

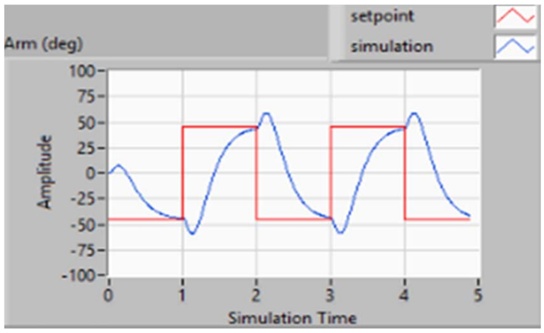
i. The pendulum angle should not be more than $\pm 8°$ when the step input is applied to the system , $|\alpha| \leq \pm 8°$

ii. The control signal V_m should be less than 12.5 V, $|V_m| \leq 12.5$ V

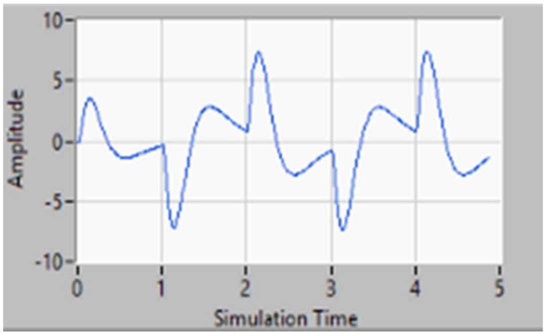
iii. The arm peak time must be less than 0.75s, $t_p \leq$ 0.75s.

The challenges of this control problem arises from the conflicting nature of maintaining an inverted pendulum in upright position as well as satisfying the constraints imposed by specifications (i) to (iii) represent a non-trivial performance goal.

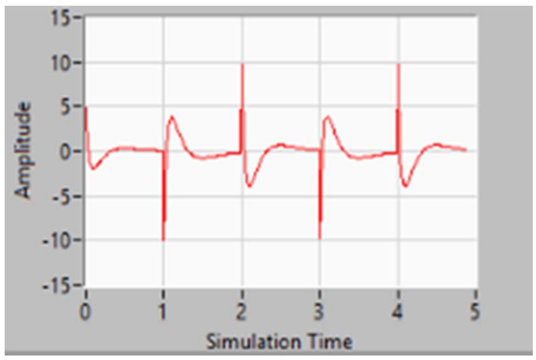
Fig. 6(a), (b) and (c) meet the constraints specified in (i) to (iii).



(a)



(b)



(c)

Fig.6 (a) Arm response (b) Pendulum Response and (c) Control input signal of System in Upright Position.

The weighing matrix is obtained as

$$Q = \begin{bmatrix} 40 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And R = 1, the following gain is generated which minimizes the cost function J_{LQR} is given as:

$$K = [-6.32 \quad 81.2 \quad -2.76 \quad 10.87]$$

And the poles of the system after balancing are obtained as:

-14.026, -6.5317 + i 5.0839, -6.5317 – i 5.0839 and - 5.9536.

IV. Conclusion

This paper presented the application of LQR control for regulation and stabilization of rotary inverted pendulum. The system using LQR controller remains stable even in the case of model approximation errors and also insensitive to the parameter variations. From the simulation results it is seen that the LQR controller not only maintains the pendulum in the upright position but also maintain stability at the same position. So the system can successfully be handled by using LQR technique. The advantage of using LabVIEW over MATLAB simulink is that it provides programming environment which is used to develop measurements, test and controls using graphical icons and wires. It allows easy monitoring, control, and data acquisition with easy hardware integration. It also provides facility of doing complex calculations using mathlab script.

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