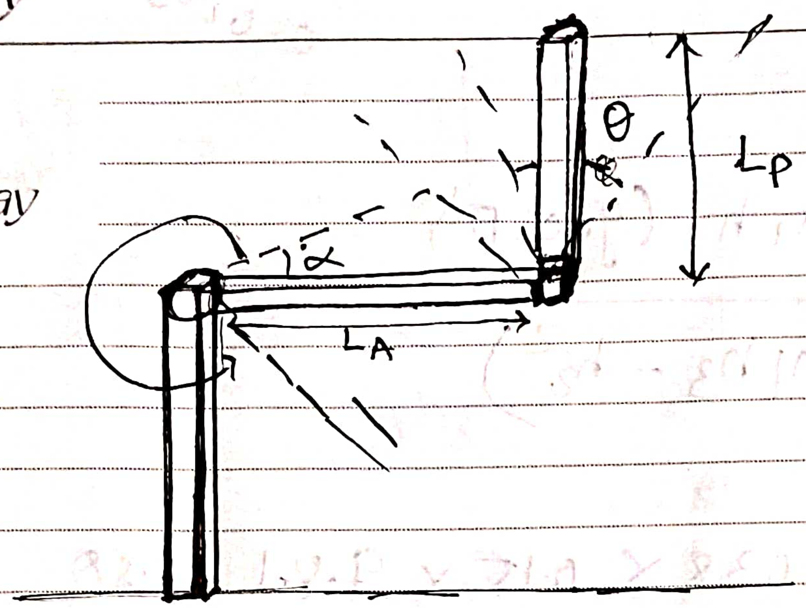
Task 1A (E-Yantra)

February

12

Saturday



Mass of The rotary arm $= 0.25$ kg $= M_A$

Total length of rotary arm $= 0.2$ m $= L_A = K$

length from pivot/motor To C.M of rotary arm $= 0.1$ m $= l_A$

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Sunday

Mass of The pendulum $= M_P = 0.5$ Kg

Total length of pendulum Arm $= L_P = 0.3$ m $= \phi$

length from pivot To C.M of pendulum Arm $= l_P = 0.15$ m $= B$

Arm angular displacement $= \alpha$

Pend angular displacement $= \theta$

$v_x = (K\dot{\alpha} - l_p \cos\theta (\dot{\theta}))$ | $v_y = -l_p \dot{\theta} \sin\theta$

2022

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Monday

$$T.KE_{Total} = KE_{ARM(ROT)} + KE_{(Pend)(ROT)} + KE_{Pend(Trans)}$$

$$= \frac{1}{2} J_{eq} \dot{\alpha}^2 + \frac{1}{2} J_{pend} \dot{\theta}^2 + \frac{1}{2} m_p (V_{pend})^2$$

$$= \frac{1}{2} J_{eq} \dot{\alpha}^2 + \frac{1}{2} J_{pend} \dot{\theta}^2 + \frac{1}{2} m_p \left[K\dot{\alpha} - l_p \cos\theta (\dot{\theta})\right]^2 + \frac{1}{2} m_p \left[-l_p \dot{\theta} \sin\theta\right]^2$$

$$= \frac{1}{2} J_{eq} \dot{\alpha}^2 + \frac{1}{2} J_{pen} \dot{\theta}^2 + \frac{1}{2} m_p \left[K^2 \dot{\alpha}^2 + l_p^2 \cos^2\theta \, \dot{\theta}^2 - 2K\dot{\alpha} \, l_p \cos\theta \, \dot{\theta}\right] + \frac{1}{2} m_p \left[l_p^2 \dot{\theta}^2 \sin^2\theta\right]$$

$$= \frac{1}{2} J_{eq} \dot{\alpha}^2 + \frac{1}{2} J_{pen} \dot{\theta}^2 + \frac{1}{2} m_p \left[K^2 \dot{\alpha}^2 + l_p^2 \dot{\theta}^2 - 2K\dot{\alpha} l_p \dot{\theta} \cos\theta\right]$$

February

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February

15 Tuesday

$\boxed{L = T - V}$ | $V = m_p g \, l_p \cos\theta$

$$L = \frac{1}{2} J_{eq} \dot{\alpha}^2 + \frac{1}{2} J_{pen} \dot{\theta}^2 + \frac{1}{2} m_p \left[K^2 \dot{\alpha}^2 + l_p^2 \dot{\theta}^2 - 2 K \dot{\alpha} l_p \cos\theta \, \dot{\theta} - 2 g \, l_p \cos\theta \right]$$

$$\frac{\partial L}{\partial \theta} = \boxed{\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} = \text{input}}$$

State-variable.

Lagrange's Equation

So, Here, state variable's are

$[\theta, \dot{\theta}, \alpha, \dot{\alpha}] = X$ = State-Matrix.

Pend-pos, Pend-vel, Arm-pos, Arm-vel

$$\Rightarrow \frac{\partial L}{\partial \theta} = \frac{1}{2} m_p \left[-2k\dot{\alpha} l_p \dot{\theta} (-\sin\theta) - 2g l_p (-\sin\theta) \right]$$

$$\boxed{\frac{\partial L}{\partial \theta} = m_p l_p \sin\theta \left[k\dot{\alpha}\dot{\theta} + g \right]} \quad - 1$$

$$\frac{\partial L}{\partial \dot{\theta}} = J_{pen}\dot{\theta} + \frac{1}{2} m_p \left[-2k\dot{\alpha} l_p \cos\theta + 2 l_p^2 \dot{\theta} \right]$$

$$\boxed{\frac{\partial L}{\partial \dot{\theta}} = J_{pen}\dot{\theta} + m_p l_p \left[l_p\dot{\theta} - k\dot{\alpha}\cos\theta \right]} \quad - 2$$

$$\boxed{\frac{\partial L}{\partial \alpha} = 0} \quad - 3$$

$$\frac{\partial L}{\partial \dot{\alpha}} = J_{eq}\dot{\alpha} + \frac{1}{2} m_p \left[2k^2\dot{\alpha} - 2k l_p \cos\theta\, \dot{\theta} \right]$$

$$\boxed{\frac{\partial L}{\partial \dot{\alpha}} = J_{eq}\dot{\alpha} + m_p k \left[k\dot{\alpha} - l_p \cos\theta\, \dot{\theta} \right]} \quad - 4$$

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February

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Thursday

langrange's Equations are.

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\alpha}}\right] - \frac{\partial L}{\partial \alpha} = \tau \quad - (3)$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}}\right] - \frac{\partial L}{\partial \theta} = 0 \quad - (4) \text{ (under actuated)}$$

eq (4) & (3) in (5)

$$\frac{d}{dt}\left[J_{eq}\dot{\alpha} + m_p K\left[K\dot{\alpha} - l_p \cos\theta\,\dot{\theta}\right]\right] - 0 = T$$

$$\Rightarrow \boxed{T = J_{eq}\ddot{\alpha} + m_p K\left[K\ddot{\alpha} - l_p\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\right]} \quad (6)$$

Eq ① & ② in ⑥

$$\frac{d}{dt}\left[J_{pen}\dot{\theta} + m_p l_p [l_p\dot{\theta} - K\dot{\alpha}\cos\theta]\right] - m_p l_p \sin\theta [K\dot{\alpha}\dot{\theta} + g] = 0 \quad -⑥$$

$$J_{pen}\ddot{\theta} + m_p l_p \left[l_p\ddot{\theta} - K[\ddot{\alpha}\cos\theta - \dot{\alpha}\dot{\theta}\sin\theta]\right] = m_p l_p \sin\theta [K\dot{\alpha}\dot{\theta} + g] \quad -⑦$$

TO linearize The system following assumptions are made.

$$\cos\theta = 1, \quad \sin\theta = \theta, \quad [\dot{\alpha}, \dot{\theta}, \dot{\alpha}^2, \dot{\theta}^2] = 0$$

so, ⑦ & ⑥ becomes.

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February

Equ 6

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Saturday

$$J_{eq}\ddot{\alpha} + m_p k\left[k\ddot{\alpha} - l_p(\ddot{\theta} - 0)\right] = T$$

$$\Rightarrow J_{eq}\ddot{\alpha} + m_p k^2\ddot{\alpha} - m_p l_p k\ddot{\theta} = T$$

$$\boxed{(J_{eq} + m_p k^2)\ddot{\alpha} - m_p l_p k\ddot{\theta} = T} - 8$$

Eq 7

$$J_{pen}\ddot{\theta} + m_p l_p\left[l_p\ddot{\theta} - k[\ddot{\alpha} - 0]\right] = m_p l_p g \sin\theta$$

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Sunday

$$J_{pen}\ddot{\theta} + m_p l_p^2\ddot{\theta} - m_p l_p k\ddot{\alpha} = m_p l_p g \sin\theta$$

$$\boxed{(J_{pen} + m_p l_p^2)\ddot{\theta} - m_p l_p k\ddot{\alpha} - m_p l_p g\sin\theta = 0} - 9$$

$$(J_{eq} + m_p k^2)\ddot{\alpha} - m_p l_p k \ddot{\theta} = T$$

$$(J_{pen} + m_p l_p^2)\ddot{\theta} - m_p l_p k \ddot{\alpha} = m_p l_p g \sin\theta$$

let

$$h_1 = J_{eq} + m_p k^2$$

$$h_2 = m_p l_p$$

$$h_3 = J_{pen} + m_p l_p^2$$

$$h_1 \ddot{\alpha} - h_2 \ddot{\theta} k = T \quad - (10)$$

$$h_3 \ddot{\theta} - h_2 k \ddot{\alpha} = h_2 g \sin\theta \quad - (11)$$

$$\Rightarrow h_3 \ddot{\theta} = h_2 k \ddot{\alpha} + h_2 g \sin\theta$$

$$\ddot{\theta} = \frac{h_2}{h_3}\left[k\ddot{\alpha} + g\sin\theta\right] \rightarrow \text{in } (10)$$

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S 6 13 20 27
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T 3 10 17 24
F 4 11 18 25
S 5 12 19 26

$$h_1\ddot{\alpha} - h_2 k\left[\frac{h_2}{h_3}\left[\ddot{\alpha}k + g\sin\theta\right]\right] = T$$

$$h_1\ddot{\alpha} - \frac{h_2^2 k^2 \ddot{\alpha}}{h_3} - \frac{h_2^2 k g \sin\theta}{h_3} = T$$

$$\ddot{\alpha}\left[\frac{h_3 h_1 - h_2^2 k^2}{h_3}\right] = \frac{h_2^2 k g\sin\theta}{h_3} + T$$

$$\ddot{\alpha} = \frac{h_2^2 k g \sin\theta}{(h_3 h_1 - h_2^2 k^2)} + \frac{h_3 T}{(h_3 h_1 - h_2^2 k^2)}$$

$$\Rightarrow \boxed{\ddot{\alpha} = \frac{h_2^2 k g \sin\theta}{A} + \frac{h_3 T}{A}} \quad -(12)$$

in (10)

$$A = h_3 h_1 - h_2^2 k^2 \quad -(13)$$

$$h_1 \ddot{\alpha} - T = h_2 \ddot{\theta} K$$

$$h_2 \ddot{\theta} K = h_1 \left[\frac{h_2^2 K g \sin\theta + h_3 T}{A} \right] - T$$

$$\cancel{K} \ddot{\theta} = \frac{h_1 h_2^{\cancel{2}} \cancel{K} g \sin\theta}{\cancel{h_2} \cancel{K} A} + \frac{h_1 h_3 T}{h_2 \cancel{K} A} - \frac{T}{h_2 \cancel{K}}$$

$$\ddot{\theta} = \frac{h_1 h_2 g \sin\theta}{A} + T \left[\frac{h_1 h_3 - A}{h_2 K A} \right]$$

$$= \frac{h_1 h_2 g \sin\theta}{A} + T \left[\frac{\cancel{h_1 h_3} - \cancel{h_3 h_1} + h_2^{\cancel{2}} K^{\cancel{2}}}{\cancel{h_2 K} A} \right]$$

$$\boxed{\ddot{\theta} = \frac{h_1 h_2 g \sin\theta}{A} + \frac{h_2 K}{A} T} - (14)$$

$$\dot{x} = Ax + BU$$

$$Y = Cx + DU$$

$$x = \begin{bmatrix} \dot{\alpha} \\ \alpha \\ \dot{\theta} \\ \theta \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} \ddot{\alpha} \\ \dot{\alpha} \\ \ddot{\theta} \\ \dot{\theta} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{\partial \ddot{\alpha}}{\partial \dot{\alpha}} & \frac{\partial \ddot{\alpha}}{\partial \alpha} & \frac{\partial \ddot{\alpha}}{\partial \dot{\theta}} & \frac{\partial \ddot{\alpha}}{\partial \theta} \\ \frac{\partial \dot{\alpha}}{\partial \dot{\alpha}} & \frac{\partial \dot{\alpha}}{\partial \alpha} & \frac{\partial \dot{\alpha}}{\partial \dot{\theta}} & \frac{\partial \dot{\alpha}}{\partial \theta} \\ \frac{\partial \ddot{\theta}}{\partial \dot{\alpha}} & \frac{\partial \ddot{\theta}}{\partial \alpha} & \frac{\partial \ddot{\theta}}{\partial \dot{\theta}} & \frac{\partial \ddot{\theta}}{\partial \theta} \\ \frac{\partial \dot{\theta}}{\partial \dot{\alpha}} & \frac{\partial \dot{\theta}}{\partial \alpha} & \frac{\partial \dot{\theta}}{\partial \dot{\theta}} & \frac{\partial \dot{\theta}}{\partial \theta} \end{bmatrix}$$

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$$\frac{\partial \ddot{\alpha}}{\partial \dot{\alpha}} = 0 \quad \Big| \quad \frac{\partial \ddot{\alpha}}{\partial \alpha} = 0 \quad \Big| \quad \frac{\partial \ddot{\alpha}}{\partial \dot{\theta}} = 0$$

$$X = \frac{\partial \ddot{\alpha}}{\partial \theta} = \frac{h_2^2 k g \cos\theta}{(h_3 h_1 - h_2^2 k^2)} \rightarrow \text{from } (12)$$

$$\left[\frac{\partial \dot{\alpha}}{\partial \dot{\alpha}} = 1 \quad \Big| \quad \frac{\partial \dot{\alpha}}{\partial \alpha} = 0 \quad \Big| \quad \frac{\partial \dot{\alpha}}{\partial \dot{\theta}} = 0 \quad \Big| \quad \frac{\partial \dot{\alpha}}{\partial \theta} = 0 \right]$$

$$\frac{\partial \ddot{\theta}}{\partial \dot{\alpha}} = 0 \quad \Big| \quad \frac{\partial \ddot{\theta}}{\partial \alpha} = 0 \quad \Big| \quad \frac{\partial \ddot{\alpha}}{\partial \dot{\theta}} = 0$$

$$Y = \frac{\partial \ddot{\theta}}{\partial \theta} = \frac{h_1 h_2 g \cos\theta}{(h_3 h_1 - h_2^2 k^2)} \rightarrow \text{from } (14)$$

$$\left[\frac{\partial \dot{\theta}}{\partial \dot{\alpha}} = 0 \quad \Big| \quad \frac{\partial \dot{\theta}}{\partial \alpha} = 0 \quad \Big| \quad \frac{\partial \dot{\theta}}{\partial \dot{\theta}} = 1 \quad \Big| \quad \frac{\partial \dot{\theta}}{\partial \theta} = 0 \right]$$

$$\Rightarrow \Rightarrow A = \begin{bmatrix} 0 & 0 & 0 & X \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Now, $h_1 = J_{eq} + m_p k^2$

$$J_{eq} = \frac{m_p l^2}{3} = \frac{0.25 \times (0.2)^2}{3}$$

$$= 0.0033$$

$$h_1 = 0.0033 + 0.5(0.2)^2$$

$$= 0.0033 + 0.02$$

$$\boxed{h_1 = 0.0233}$$

March

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Friday

$h_2 = m_p l_p$

$= 0.5 \times 0.15 = \boxed{0.075 = h_2}$

$h_3 = J_{pen} + m_p l_p^2$

$= \dfrac{M_p L_p^2}{12} + 0.5(0.15)^2$

$= \dfrac{0.5(0.3)^2}{12} + 0.5(0.15)^2$

$= 0.00375 + 0.001125$

$\boxed{h_3 = 0.015}$

$\dfrac{M_p l_p^2}{3} + 0.5(0.15)^2$

$= 0.015 + 0.001125$

$= 0.016125$

$$\Rightarrow x = \frac{(0.075)^2 \times 0.2 \times 9.8 \quad \cos(0/\pi)}{\left(\underset{0.0161}{0.015} \times 0.0233 - (0.075)^2 \times (0.2)^2\right)}$$

$$= \frac{0.011025}{\underset{\cancel{0.0003751}}{0.0003495 - 0.000225}} = \frac{0.011025}{\underset{\cancel{0.00015013}}{0.0001245}}$$

$$X = \frac{0.011025\,(\cos(0/\pi))}{0.0001245/0.00015013}$$

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Saturday

$$\boxed{X = 88.55} \;/\; -88.55 \;/\!/\; 73.43 \;/\; -73.43$$

$$Y = \frac{h_1 h_2 g \cos\theta}{A}$$

$$= \frac{0.0233 \times 0.075 \times 9.8 \times \cos(0/\pi)}{0.0001245}$$

Sunday

$$= \frac{0.0171255}{0.0001245/0.00015013} = 137.55 \,/$$

$$\boxed{Y = 137.55 \;/\; -137.55} = \pm 114.07$$

March

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March

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Monday

$$B = \begin{bmatrix} \dfrac{\partial \ddot{\alpha}}{\partial T} \\[2mm] \dfrac{\partial \dot{\alpha}}{\partial T} \\[2mm] \dfrac{\partial \ddot{\theta}}{\partial T} \\[2mm] \dfrac{\partial \dot{\theta}}{\partial T} \end{bmatrix}$$

$$\frac{\partial \ddot{\alpha}}{\partial T} = \frac{h_3}{A} = \frac{0.015 / 0.0161}{\underset{0.00015013}{0.0001245}} = \frac{120.48}{107.4}$$

$$\frac{\partial \dot{\alpha}}{\partial T} = 0$$

$$\frac{\partial \ddot{\theta}}{\partial T} = \frac{h_2 K}{A} = \frac{0.075 \times 0.2}{\underset{0.00015013}{0.0001245}}$$

$$= \frac{0.015}{0.0001245}$$

$$= 120.48 = \frac{\partial \ddot{\theta}}{\partial T}$$

$$\frac{\partial \dot{\theta}}{\partial T} = 0$$

$$\Rightarrow B = \begin{bmatrix} 120.48 \\ 0 \\ 120.48 \\ 0 \end{bmatrix} \quad \Bigg| \quad \begin{bmatrix} 107.73 \\ 0 \\ 99.9 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & -88.55/73.43 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -137.55/114.07 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$