# Lunar Scout: Task 1A Mathematical Modeling - Theory (Part 3)

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## Mathematical Modeling of a system

## Controller Design

So far we have discussed the basics of state space analysis. In this section, we will discuss different types of controller design.

Consider the State Space Equations of a system:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

This system can be represented in form of a block diagram as follows:

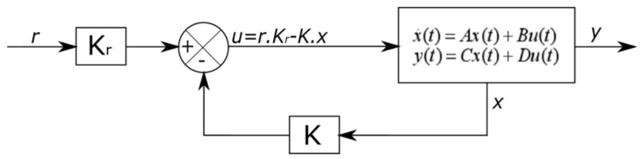


Figure 4: System block diagram

Here

r - Reference point

Kr - Gain by which reference is multiplied

x - State Vector

y -Output Vector

u - Input to system

K - Gain by which input is multiplied.

In this system we have taken the state vector x, multiplied that with some gain matrix K and fed that as feedback input to the system.

Skip to main content

The State Equation for the system can be written as follows:

$$\begin{aligned} \dot{x} &= Ax + B(rK_r - Kx) \\ => \dot{x} &= Ax - BKx + BrK_r \\ => \dot{x} &= \underbrace{(A - BK)}_{\text{New State Matrix}}\, x + BrK_r \end{aligned}$$

The new state matrix (A-BK) defines the dynamics of the system where -Kx is fed as input. The system stability can be calculated by finding the eigenvalues of the (A-BK) matrix.

## Linear Quadratic Regulator (LQR)

So far we have seen that if we have a system which is controllable, then we can place its eigenvalues anywhere in the left half plane by choosing appropriate gain matrix K. But the main question is where should we place our eigenvalues?

Till now we have only discussed about the stability of the system. But nowhere have we asked that what is our performance measure?
In this section, we'll see how to optimize the value of gain matrix K to get the desired performance measure from the system.

Linear Quadratic Regulator is a powerful tool which helps us choose the K matrix according to our desired response. Here we use a cost function.

$$J = \int_0^\infty (x^T Q x + u^T R u)$$

where, Q and R are positive semi-definite diagonal matrices (positive semi-definite matrices are those matrices whose all the eigenvalues are greater than or equal to zero). Also to remind you that for a diagonal matrix, the diagonal entries are its eigenvalues. x and u are the state vector and input vector respectively.

Let us say that you have your system with four states and one input.

Skip to main content

Then $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ and input *u*. Let

$$Q = \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & Q_4 \end{bmatrix}$$

Then
$x^T Q x + u^T R u = Q_1 x_1^2 + Q_2 x_2^2 + Q_3 x_3^2 + Q_4 x_4^2 + Ru^2$
. Thus, you may see that the system taken here is our usual system represented as

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

The controller is of the form *u = -Kx* which is a **Linear** controller and the underlying cost function is **Quadratic** in nature and hence the name **Linear Quadratic Regulator**.
With a careful look at the integrand of the cost function J, we may observe that each Qi are the weights for the respective states xi.

So, the trick is to choose weights Qi for each state xi so that the desired performance criteria is achieved. Greater the state objective is, greater will be the value of Q corresponding to the said state variable. We can choose R = 1 for single input system. In case we have multiple inputs, we could use similar arguments for weighing the inputs as well.

In case of inverted pendulums, we know that angle with the vertical and the angular velocity is very important and hence the weights corresponding to them should be more as compared to linear position and linear velocity .

LQR minimizes this cost function J based on the chosen matrices Q and R. Its a bit complicated to find out matrix K which minimizes this cost function. This is usually done by solving Algebraic Riccatti Equation (ARE). We'll not go into details of how to

Skip to main content

solve ARE, as it is not required in our tasks. There is inbuilt **lqr** command in octave to find K matrix. What is required to be done is to choose the Q and R matrix appropriately to get the desired performance.

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**Now that you have understood the theory, let's move on to implementation of Task 1A**
**Click on the link given below.**

# Task 1A - Practical

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Lunar Scout: Task 1A - Theory Part 2 Quiz

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