A Comparative study of controllers for stabilizing a Rotary Inverted Pendulum

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Abstract

This paper describes comparative study of various controllers on Rotary Inverted Pendulum (RIP). PID, LQR, FUZZY LOGIC and H∞ controllers are tried on RIP in MatLab Simulink. The same four controllers have been tested on test bed of RIP system the controllers are compared from various aspects. The controllers in simulink are compared with the controllers in real time.

Keywords

Fuzzy Logic, H∞, LQR, PID, RIP

1.Introduction

A typical unstable non-linear Inverted Pendulum system is often used as a benchmark to study various control techniques in control engineering. Analysis of controllers on RIP illustrates the analysis in cases such as control of a space booster rocket and a satellite, an automatic aircraft landing system, aircraft stabilization in the turbulent air-flow, stabilization of a cabin in a ship etc. RIP is a test bed for the study of various controllers like PID controller, LQR controller, and fuzzy controller. A normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright, this can be done either by applying a torque at the pivot point, by moving the pivot point horizontally as part of a feedback system.

In this paper controllers are developed that keep the pendulum upright without any oscillations. The model is simulated using the MATLAB application. The paper is organized as follows. Section 2 deals with the modeling of the system, Section 3 discusses the control techniques PID, LQR, Fuzzy Logic and H infinity controllers, Section 4 gives the test bed results, Section 5 discusses the conclusion drawn from the analysis of these controllers in simulink and on test bed.

1.Modelling of Rotary Inverted Pendulum

The system, as shown in Fig. 1, consists of a vertical pendulum, a horizontal arm, a gear chain, and a servomotor which drives the pendulum through the gear transmission system. The rotating arm is mounted on the output gear of the gear chain. An encoder is attached to the arm shaft to measure the rotating angle of the arm. At the end of the rotating arm there is a hinge instrumented with an encoder. The pendulum is attached to the hinge.
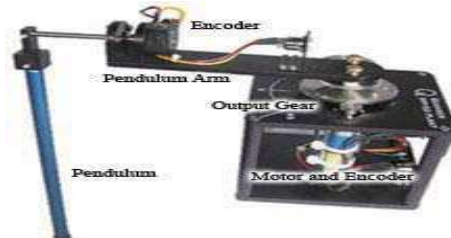
Fig.1. Rotary Inverted Pendulum system

The inverted pendulum is shown in Fig. 2, with its physical parameters α and θ are employed as the generalized coordinates to describe the inverted pendulum system. The pendulum is displaced with a given α while the arm rotates with an angle of θ.
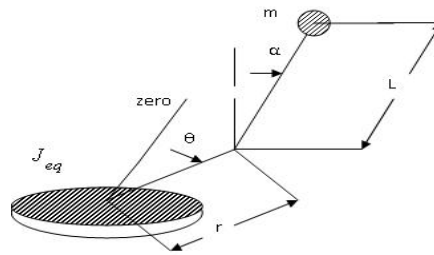

Fig.2. Figure showing physical parameters to be measured

Using the Lagrangian method [1], the equation of Rotary Inverted Pendulum is as follows:

$$(J + mr^2)\ddot{\theta} + mLr\sin(\alpha)\,\dot{\alpha}^2 - mLr\cos(\alpha)\ddot{\alpha} = T - B\dot{\theta} \qquad (1)$$

$$\frac{4}{3}mL^2\ddot{\alpha} - mLr\cos(\alpha)\ddot{\theta} - mgL\sin\alpha = 0 \qquad (2)$$

Where Torque T is given as, $\quad T = \eta_m\eta_g K_t K_g \dfrac{V - K_g K_m\dot{\theta}}{R_m}$

Table I

PHYSICAL PARAMETERS OF THE SYSTEM

Parameter	Description	Value(SI)
J	Moment of inertia at the load	0.0033
m	Mass of pendulum arm	0.1250
r	Rotating arm length	0.2150

L	Length to pendulum's center of mass	0.1675
g	Gravitational constant	9.81
B	Viscous damping coefficient	0.0040
K_t	Motor torque constant	0.0077
K_a	SRV02 system gear ratio	70
K_m	Back-EMF constant	0.0077
R_m	Armature resistance	2.6
η_m	Motor efficiency	0.69
η_a	Gear efficiency	0.90

Solving the equations (1), (2) and (3) and values from the Table I, state space model is formed which is written as,

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 39.32 & -14.52 & 0 \\ 0 & 81.78 & -13.98 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 25.54 \\ 24.59 \end{bmatrix} V$$

The state space parameters [2] are written as,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 39.32 & -14.52 & 0 \\ 0 & 81.78 & -13.98 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 25.54 \\ 24.59 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \qquad D = 0;$$

1.Controller Design

3.1 PID Controller

Proportional, integral derivative are the controllers whose output, a control variable (CV), is generally based on the error (e) between some user-defined set point (SP) and some measured process variable (PV).
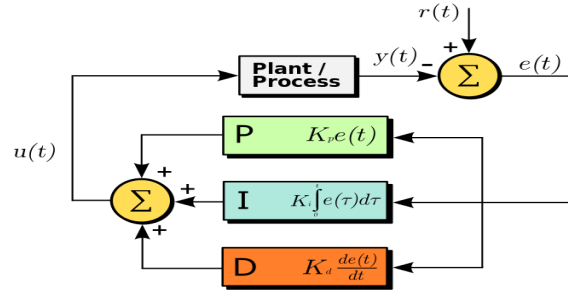
Fig.3. Schematic diagram for the closed loop system with force as a disturbance

The transfer function of PID is written as [3]

$$K(s) = K_p + \frac{K_i}{s} + K_d s$$

Manual tuning method is used to determine the gains [4].

K_i and K_d are set to zero. Then, K_p is increased until the output of the loop oscillates, after obtaining optimum K_p value, it is set to approximately half of that value for a "quarter amplitude decay" type response. Then, K_i is increased until any offset is corrected in sufficient time for the process. However, too much K_i will cause instability. Finally, K_d is increased, until the loop is acceptably quick to reach its reference after a load disturbance [5].
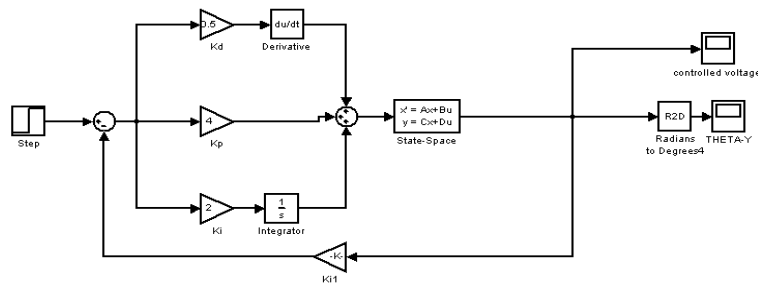

Fig.4. Simulink diagram of system using PID controller

Tuning the gain values K_p, K_i, K_d with 4, 2, 0.5 respectively along with a negative feedback as shown in the simulink model of Fig.4, the position of pendulum gets stabilized as shown in Fig 5.
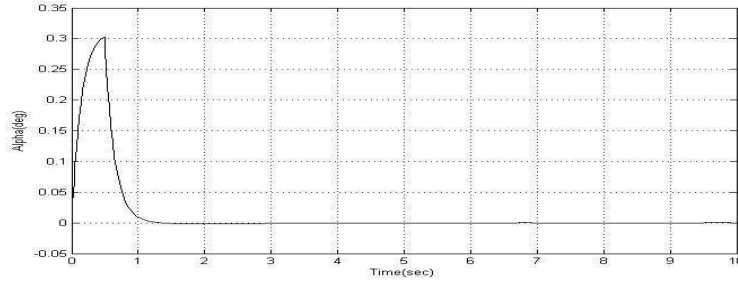
Fig.5 Variation of pendulum angle alpha (deg) with time (sec)

From Fig 5, it is observed that using PID control pendulum angle becomes zero within 1 second.

3.2 LQR Controller

LQR is a method in modern control theory that uses state-space approach to analyze a system like inverted pendulum. The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where system dynamics are described by a set of linear differential equations and the cost is described by quadratic functions which are called LQ problem [6]. The goal of such problem is to find an optimal control that minimizes a quadratic cost functional associated with a linear system.
A system is expressed in state variable form as,

$$\dot{x} = Ax + Bu \tag{3}$$

The initial condition is x (0). Assuming here all the states measurable and seek to find a state-variable feedback (SVFB) control

$$u = -Kx + v \tag{4}$$

To design a SVFB that is optimal, an Index called *performance index* (PI) is used and is given by,

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt \tag{5}$$

Substituting the SVFB control equation in equation (1) yields

$$J = \frac{1}{2}\int_0^\infty x^T (Q + K^T R K)x \, dt \tag{6}$$

The objective in optimal design is to select the SVFB K that *minimizes* the performance index J. Solving equations(3) to (6), equation (7) is obtained,

$$A^T P + PA + Q + K^T R K - K^T B^T P - PBK = 0 \tag{7}$$

taking, $K = R^{-1}B^T P$

it gives, $$A^T P + PA + Q - PBR^{-1}B^T P = 0$$

It is a *matrix quadratic equation* that is solved to get the value of auxiliary matrix P. After getting the value of matrix P, SVFB gain K is determined.

Gain value is found as,

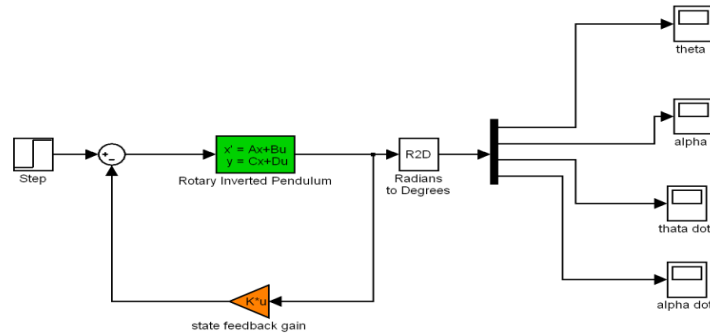$$K = [-1.13 \ 14.5 \ -1.44 \ 2.46]$$

Fig.6 Simulink diagram of system using LQR control

By substituting the SVFB gain into the system and implemented in Simulink model of Fig.6, system gets stabilized. Fig 7 is the response of the pendulum position of the system.
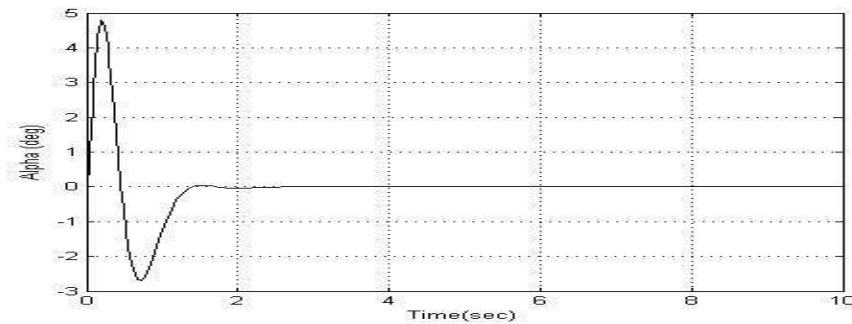

Fig. 7 Variation of pendulum position α (in deg) w.r.t time (in sec)

From Fig 7, it is observed that using LQR control, pendulum angle becomes zero within 1.5 seconds. Rotary Inverted Pendulum is stabilized at 34 degrees, and arm velocity becomes zero within 2 seconds.

Hence, Rotary Inverted Pendulum is stable within 1.5 seconds.

3.3 Fuzzy Logic Controller (FLC)

FLC provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information [7]. FLC's approach to control problems mimics how a person makes decisions. Fuzzy control describes the algorithm for process control as a fuzzy relation between information about the condition of the process to be controlled, x and y and the input for the process. The control algorithm is given in the IF - THEN expression such as

if x is small and y is big then z is medium.
if x is big and y is medium, then z is big.

6

The input and output variables as shown in FIS editor in Fig.8 are quantized into several modules or fuzzy subsets and the appropriate labels are assigned in this controller [8].
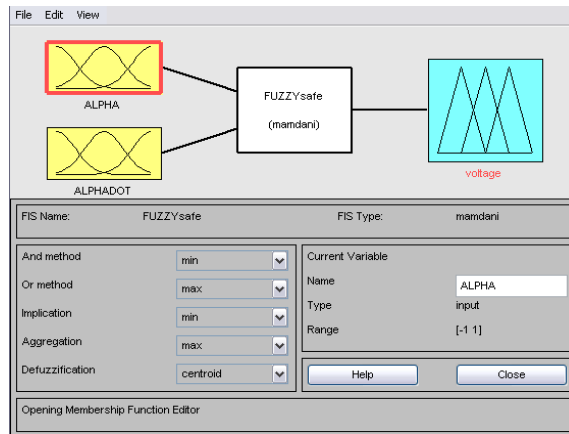

Fig. 8 FIS editor with two input and one output

There is FLC to control pendulum angle. Five fuzzy subsets have been taken to quantize each fuzzy variable for FLC as shown in Table II.

Table II

STANDARD LABELS OF QUANTIZATION

Linguistic Term	Label
Negative big	NB
Negative small	NS
Zero	ZE
Positive small	PS
Positive big	PB

Depending upon the range of alpha and alpha_dot, controlled voltage is decided. If alpha is NB and alpha_dot is PB then according to the rule base shown in Table III, the voltage applied to the system is zero.

Table III

FUZZY RULE BASE

α \ $\dot{\alpha}$	NB	NS	Z	PS	PB
NB	PB	PB	PB	PS	Z
NS	PB	PB	PS	Z	NS
Z	PB	PS	Z	NS	NB
PS	PS	Z	NS	NB	NB
PB	Z	NS	NB	NB	NB

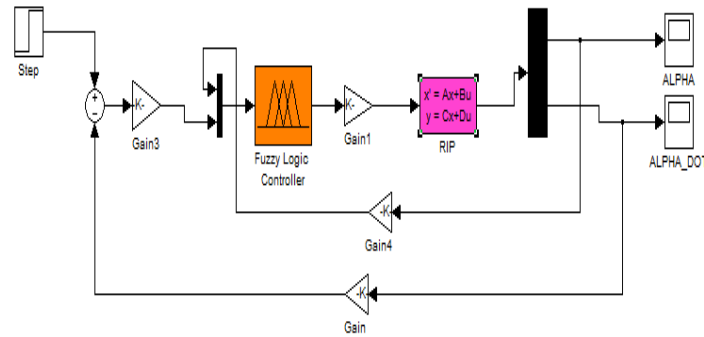Fig. 9 and Fig.10 shows the simulink model and simulation result of the system using fuzzy logic controller:

Fig. 9. Simulink diagram of system for fuzzy logic controller
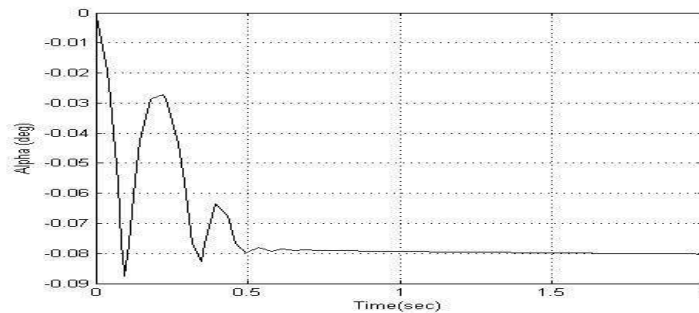

Fig. 10. Variation of pendulum angle with time (sec)

From results it is concluded that using fuzzy logic controller Rotary Inverted pendulum is stabilized within 0.5 sec.

3.4 H infinity controller

In order to achieve robust performance or stabilization, the H-Infinity control method is used. The H∞ name derives from the fact that mathematically the problem may be set in the space H∞,

which consists of all bounded functions that are analytic in the right-half complex plane [9]. H∞ method is also used to minimize the closed loop impact of a perturbation depending on the problem formulation the impact will be measured in terms of either stabilization or performance.

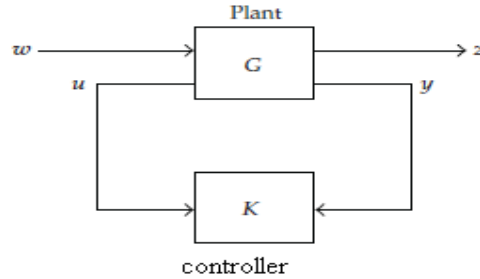This problem is defined by the configuration of Fig 11.

Fig.11 Generalized plant

The "plant" is a given system with two inputs and two outputs. It is often referred to as the generalized plant [9]. The signal w is an external input and represents driving signals that generate disturbances, measurement noise, and reference inputs. The signal u is the control input. The output z has the meaning of control error and ideally should be zero. The output y, finally, is the observed output and is available for feedback.

The augmented plant is formed by accounting for the weighting functions W_1, W_2, and W_3 as shown in the Fig 12.
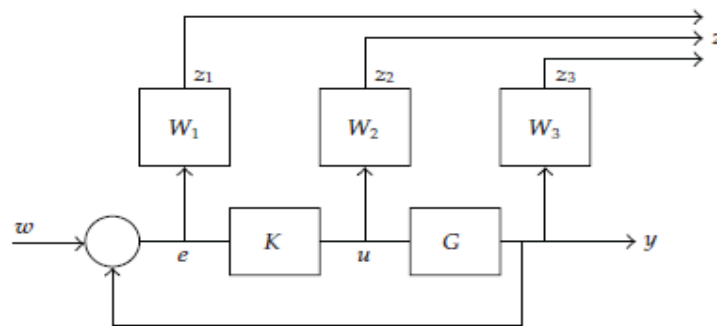

Fig.12. Plant with weighting functions for H∞ design

Choosing weights W_1, W_2 and W_3 as,

$$W_1 = 0.99*(s+50)/(s+.001);$$
$$W_2 = 1;$$
$$W_3 = 10*(s+50)/(s+500);$$

Using weights, controller in the form of state space form is obtained,

$$A = \begin{bmatrix} -0.001 & 0 & 0 & 0 & 0 \\ 8.356e-025 & -500 & 82.33 & 0 & 0 \\ 6.685e-024 & -3.829e-007 & -15.08 & 0 & 1 \\ -110.9 & 0.6872 & -425.9 & 48.48 & -84.08 \\ -106.7 & 0.6616 & -454 & 46.67 & -80.95 \end{bmatrix};$$

$$B = \begin{bmatrix} 11.11 \\ -3.961 \\ -23.69 \\ -42.23 \\ -178.7 \end{bmatrix};$$

$$C = \begin{bmatrix} -2.763 & 0.01 & -10.92 & 1.57 & -2 \end{bmatrix};$$

$$D = [0];$$

The new state space model which is obtained is implemented in simulink diagram (Fig.13) as a feedback and pendulum angle alpha is observed in Fig.14.
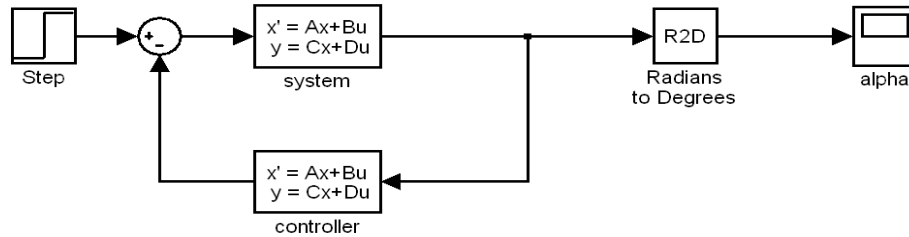

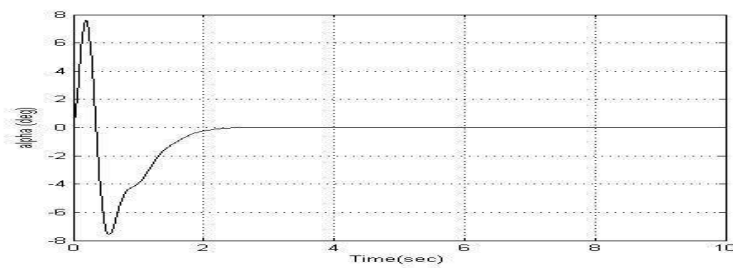Fig.13. Simulink diagram of system using H infinity control

Fig.14 Variation of pendulum angle with time (sec)

1.Test Bed Results

4.1 PID Controller

Real time results of alpha i.e. pendulum angle using PID control is shown in Fig.15. Here, controller is activated as soon as pendulum reaches to the reference position and pendulum is stabilized.
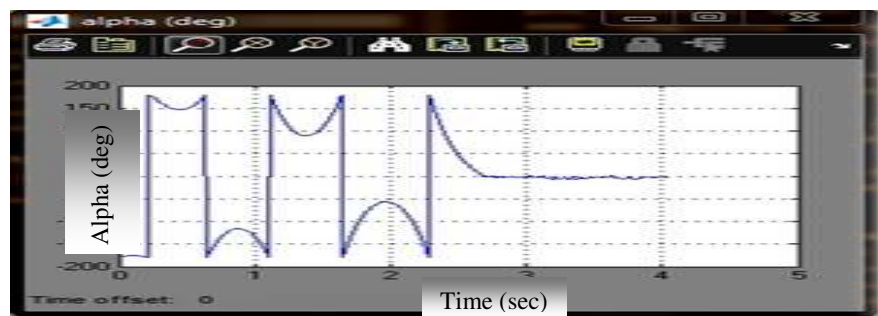

Fig.15 Real time result of system using PID control
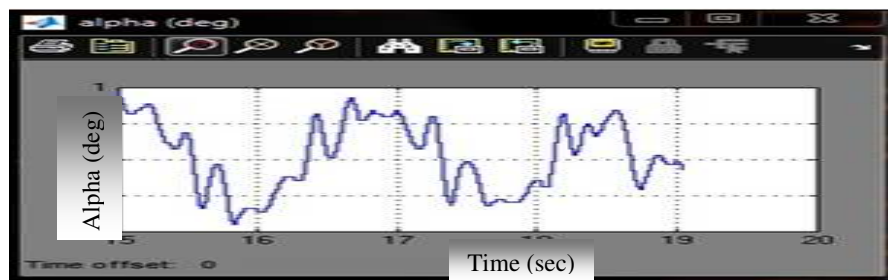
4.2 LQR Controller

Fig.16 Real time result of system using LQR control

Fig.16 shows the real time results of alpha i.e. pendulum angle and controlled voltage using Linear Quadratic Control. Here, pendulum is oscillating between 1 and -3 degrees which is approximately negligible.
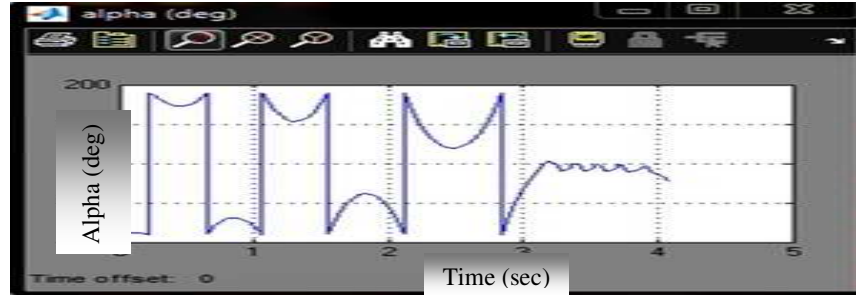
4.3 Fuzzy Logic Controller

Fig.17 Real time result of system using FUZZY LOGIC control

Fig.17 shows the real time results of alpha i.e. pendulum angle and controlled voltage using fuzzy logic controller. Here, controller is activated as soon as pendulum reaches to the reference position and pendulum is slightly oscillating.

1.Conclusion

5.1 COMPARISON OF SIMULATION RESULTS

In this section comparison of all the four controllers based on Simulation Results are discussed. Parameters compared are percentage peak overshoot and rise time as shown in Table IV.

Table IV
Comparison of controllers based on simulation results

Controller	Peak overshoot (%Mp)	Rise time (t_r)
PID	0.3	0.1
LQR	4.8	0.1
Fuzzy Logic	0.09	0.05
H∞	7.8	0.25

Comparing simulation results of all the four controllers, from the Table IV, it is concluded that, percentage peak overshoot is less in case of Fuzzy Logic controller as compared to other three controllers. Rise time is also less for this controller.

5.2 COMPARISON OF TEST BED RESULTS

In this section comparison of all the four controllers based on Test bed Results are discussed. Parameters compared are percentage peak overshoot and rise time as shown in Table V.

Table V : Comparison of controllers based on Test Bed results

Controller	Peak overshoot (%Mp)	Rise time (t_r)
PID	2	0.3
LQR	2.5	0.5
Fuzzy Logic	10	0.1
H∞	--	--

Comparing test bed results of PID, LQR and FUZZY LOGIC, from the Table V it is concluded that, percentage peak overshoot as well as rise time of response is less for LQR controller as compared to PID and Fuzzy Logic controller

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