Applications of Catalan Numbers

Background :

Catalan numbers are defined using below formula:

$$C_n = \frac{(2n)!}{(n+1)!\,n!} = \prod_{k=2}^{n} \frac{n+k}{k} \qquad \text{for } n \geq 0$$

Catalan numbers can also be defined using following recursive formula.

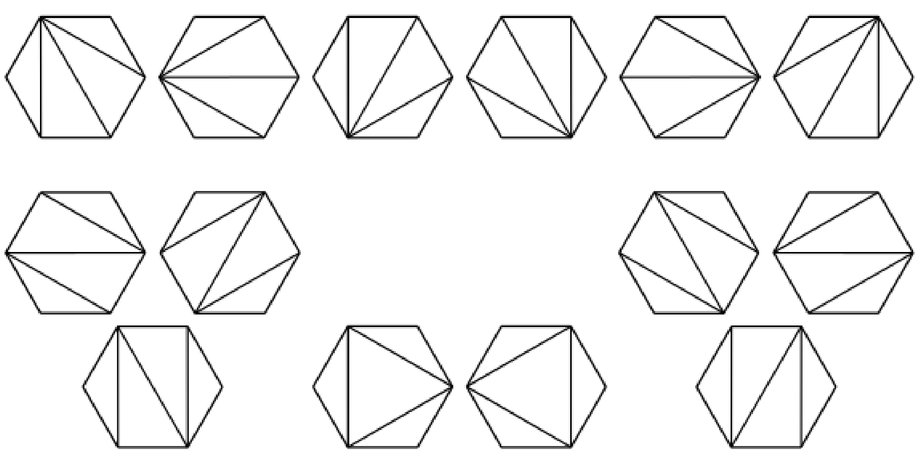
$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^{n} C_i\, C_{n-i} \quad \text{for } n \geq 0;$$

The first few Catalan numbers for n = 0, 1, 2, 3, … are **1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, …**

Refer this for implementation of n’th Catalan Number.

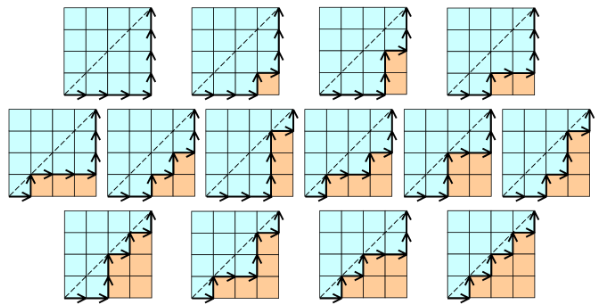
Applications :

1. Number of possible Binary Search Trees with n keys.
2. Number of expressions containing n pairs of parentheses which are correctly matched. For n = 3, possible expressions are ((())), ()(()), ()()(), (())(), (()()).
3. Number of ways a convex polygon of n+2 sides can split into triangles by connecting vertices.

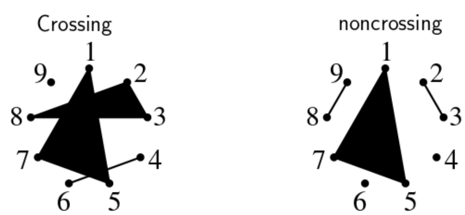


4. Number of full binary trees (A rooted binary tree is full if every vertex has either two children or no children) with n+1 leaves.
5. Number of different Unlabeled Binary Trees can be there with n nodes.

6. The number of paths with 2n steps on a rectangular grid from bottom left, i.e., (n-1, 0) to top right (0, n-1) that do not cross above the main diagonal.

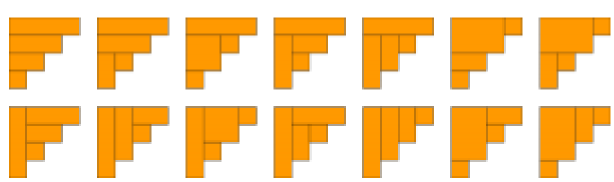


7. Number of ways to insert n pairs of parentheses in a word of n+1 letters, e.g., for n=2 there are 2 ways: ((ab)c) or (a(bc)). For n=3 there are 5 ways, ((ab)(cd)), (((ab)c)d), ((a(bc))d), (a((bc)d)), (a(b(cd))).
8. Number of noncrossing partitions of the set {1, …, 2n} in which every block is of size 2. A partition is noncrossing if and only if in its planar diagram, the blocks are disjoint (i.e. don't cross). For example, below two are crossing and non-crossing partitions of {1, 2, 3, 4, 5, 6, 7, 8, 9}. The partition {{1, 5, 7}, {2, 3, 8}, {4, 6}, {9}} is crossing and partition {{1, 5, 7}, {2, 3}, {4}, {6}, {8, 9}} is non-crossing.

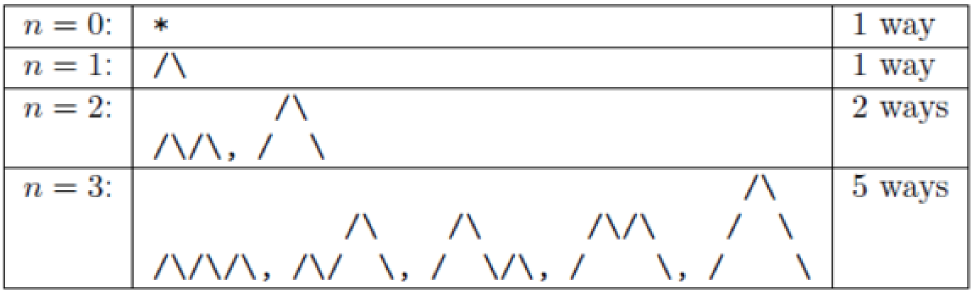


(Source : http://www4.ncsu.edu/~nreadin/papers/NCSUslides.pdf)

9. Number of Dyck words of length 2n. A Dyck word is a string consisting of n X's and n Y's such that no initial segment of the string has more Y's than X's. For example, the following are the Dyck words of length 6: XXXYYY XYXXYY XYXYXY XXYYXY XXYXYY.
10. Number of ways to tile a stairstep shape of height n with n rectangles. The following figure illustrates the case n = 4:



11. Number of ways to connect the points on a circle disjoint chords. This is similar to point 3 above.
12. Number of ways to form a "mountain ranges" with n upstrokes and n down-strokes that all stay above the original line.The mountain range interpretation is that the mountains will never go below the horizon.



Mountain Ranges

13. Number of stack-sortable permutations of {1, …, n}. A permutation w is called stack-sortable if S(w) = (1, …, n), where S(w) is defined recursively as follows: write w = unv where n is the largest element in w and u and v are shorter sequences, and set S(w) = S(u)S(v)n, with S being the identity for one-element sequences.
14. Number of permutations of {1, …, n} that avoid the pattern 123 (or any of the other patterns of length 3); that is, the number of permutations with no three-term increasing subsequence. For n = 3, these permutations are 132, 213, 231, 312 and 321. For n = 4, they are 1432, 2143, 2413, 2431, 3142, 3214, 3241, 3412, 3421, 4132, 4213, 4231, 4312 and 4321

Sources:

1. https://en.wikipedia.org/wiki/Catalan_number
2. http://mathworld.wolfram.com/CatalanNumber.html
3. http://www-groups.dcs.st-and.ac.uk/history/Miscellaneous/CatalanNumbers/catalan.html
4. http://www.mhhe.com/math/advmath/rosen/r5/instructor/applications/ch07.pdf
5. https://oeis.org/A000108

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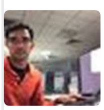

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Great Article summarising many applications of Catalan Number's at one Place!!
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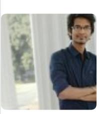

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