Divide and Conquer | Set 6 (Tiling Problem)

Given a n by n board where n is of form 2^k where k >= 1 (Basically n is a power of 2 with minimum value as 2). The board has one missing cell (of size 1 x 1). Fill the board using L shaped tiles. A L shaped tile is a 2 x 2 square with one cell of size 1×1 missing.

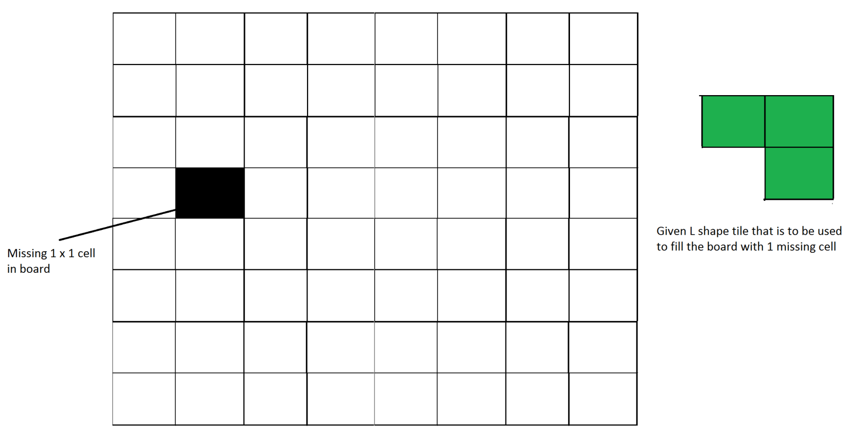

Figure 1: An example input

This problem can be solved using Divide and Conquer. Below is the recursive algorithm.

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// n is size of given square, p is location of missing cell
Tile(int n, Point p)

1) Base case: n = 2, A 2 x 2 square with one cell missing is nothing
   but a tile and can be filled with a single tile.

2) Place a L shaped tile at the center such that it does not cover
   the n/2 * n/2 subsquare that has a missing square. Now all four
   subsquares of size n/2 x n/2 have a missing cell (a cell that doesn't
   need to be filled).  See figure 2 below.

3) Solve the problem recursively for following four. Let p1, p2, p3 and
   p4 be positions of the 4 missing cells in 4 squares.
   a) Tile(n/2, p1)
   b) Tile(n/2, p2)
   c) Tile(n/2, p3)
   d) Tile(n/2, p3)
```

The below diagrams show working of above algorithm

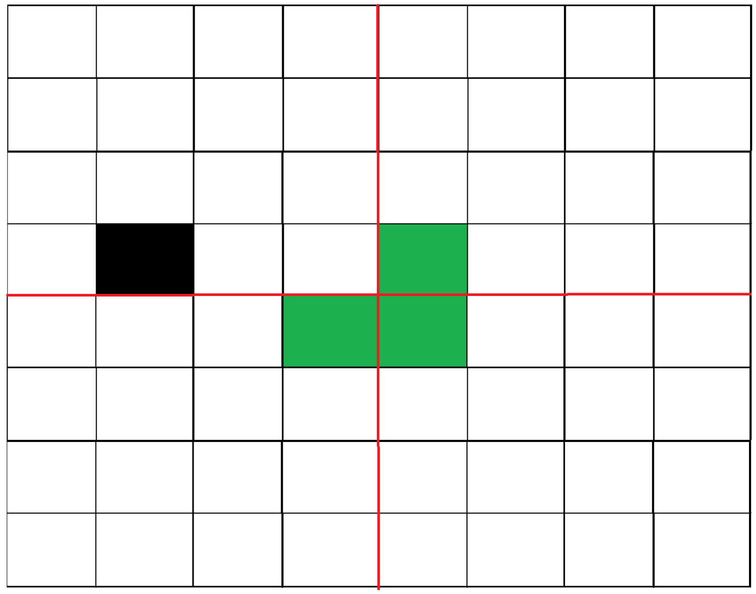

Figure 2: After placing first tile

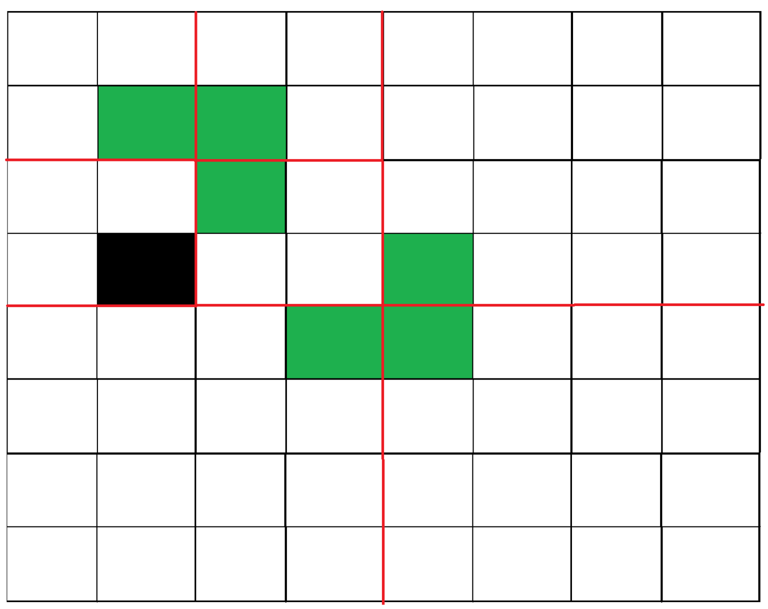

Figure 3: Recurring for first subsquare.

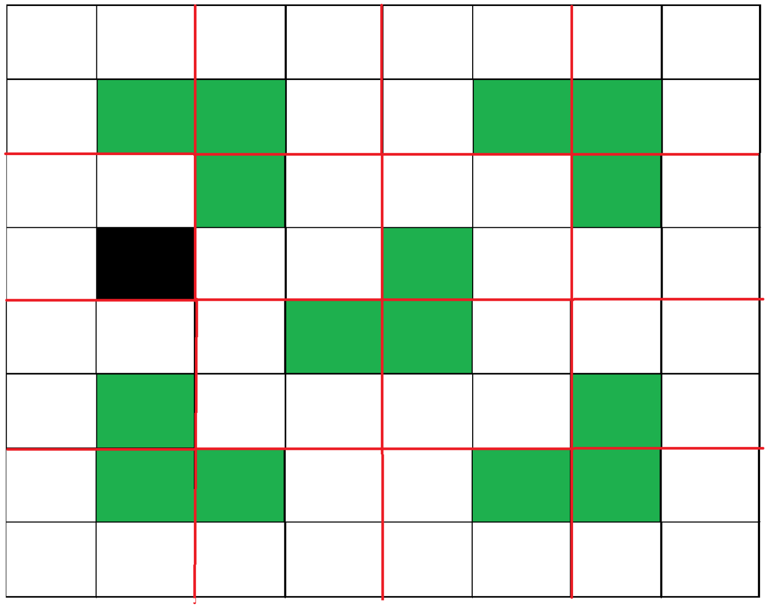

Figure 4: Shows first step in all four subsquares.

Time Complexity:

Recurrence relation for above recursive algorithm can be written as below. C is a constant.

T(n) = 4T(n/2) + C

The above recursion can be solved using Master Method and time complexity is $O(n^2)$

How does this work?

The working of Divide and Conquer algorithm can be proved using Mathematical Induction. Let the input square be of size 2^k x 2^k where k >=1.

Base Case: We know that the problem can be solved for k = 1. We have a 2 x 2 square with one cell missing.

Induction Hypothesis: Let the problem can be solved for k-1.

Now we need to prove to prove that the problem can be solved for k if it can be solved for k-1. For k, we put a L shaped tile in middle and we have four subsqures with dimension 2^{k-1} x 2^{k-1} as shown in figure 2 above. So if we can solve 4 subsquares, we can solve the complete square.

References:

http://www.comp.nus.edu.sg/~sanjay/cs3230/dandc.pdf

This article is contributed by **Abhay Rathi**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.