CS 2050 Exam 2 - Fall 2019 October 18

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Taking this exam signifies you are aware of and in accordance with the Academic Honor Code of Georgia.

Fairness matters - Do not open this exam until instructed to do so.

- There are no breaks during this exam.
- Signing signifies you are aware of and in accordance with the **Academic Honor Code of Georgia Tech** and the **Code of Conduct**.
- When time is called you are to stop writing immediately. No exceptions.
- You are not allowed to speak to other students during the exam unless that student is a TA proctor.
- Do your best to keep your answers covered.
- You are to uphold the honor and integrity expected of you.
- Books, notes, laptops, cell phones, smart watches, etc. are NOT allowed.
- Calculators are not allowed in CS2050.

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Points:	10	10	35	10	30	5	100
Score:							

[5] 1. Give the definition of $A \times B$ (the Cartesian product of A and B) using set builder notation. Assume A and B are sets. Your solution should contain no set operations other than "is an element-of" written as the symbol $\in$.

$A \times B =$

$$A \times B = \{ (a, b) \mid a \in A \wedge b \in B \}$$

[5] 2. Given sets named A and B, prove $A - B = B - A$ is always true (that is that set subtraction is a commutative operation) or use a proof by counter-example to show that set subtraction is not commutative.

Using Counter Example.

let $A = \{1, 2, 3\}$

$B = \{3\}$

$A - B = \{1, 2\}$

$B - A = \{\ \}$

Conclusion :- hence using counter example, I have shown that the statement $A - B = B - A$ is not always true.

[5] 3. Show your work computing gcd(56, 20) using the Euclidean algorithm. (We want to see 1) all of the intermediate results and also 2) the final integer answer.) No credit for answers not demonstrating the correct use of the Euclidean algorithm or simply giving the final integer result.

$$\gcd(56,20) = \gcd(20, 56 \bmod 20)$$
$$= \gcd(20, 16)$$
$$= \gcd(16, 20 \bmod 16)$$
$$= \gcd(16, 4)$$
$$= \gcd(4, 16 \bmod 4)$$
$$= \gcd(4, 0) = \underline{4}$$

hence $\gcd(56,20) = \underline{4}$

[5] 4. Give the least common multiple (lcm) of the following. Leave your answer in prime factorized form.
$lcm(3^2 \cdot 5^5 \cdot 7^3, 2^2 \cdot 3^1 \cdot 5^5 \cdot 11^3) =$

$$lcm(3^2 \cdot 5^5 \cdot 7^3, 2^2 \cdot 3^1 \cdot 5^5 \cdot 11^3) = 3^2 \cdot 5^5 \cdot 7^3 \cdot 2^2 \cdot 11^3$$

[5] 5. Assume you have n pieces of data to process. If an algorithm tends to repeatedly cut the data in half each time and pursue the processing with only one of the halves as it continues (and it never comes back to process the other half), what is the correct Big O to describe the behavior of the algorithm? (Give the smallest, most descriptive Big O, that describes the behavior.) Circle your answer.

(a) $O(1)$
b (b) $O(log\ n)$
(c) $O(n)$
(d) $O(n^2)$
(e) $O(2^n)$

[5] 6. If an algorithm needs to compare every piece of data with every other piece of data (and assume that there is no way to prevent this behavior), what is the correct Big O to describe the behavior of the algorithm given n pieces of data? (Give the smallest, most descriptive Big O, that describes the behavior.) Circle your answer.

(a) $O(1)$
(b) $O(log\ n)$
(c) $O(n)$
d (d) $O(n^2)$ ✓
(e) $O(2^n)$

[5] 7. Within an inductive proof proving $P(n)$ is true for all positive integers, what exactly is the inductive hypothesis? Circle your answer.

(a) inductive principle
(b) the statement that $P(k) \rightarrow P(k+1)$ is true
(c) the outcome that $P(k+1)$ is true
d (d) the assumption that $P(k)$ is true ✓

[5] 8. If $\mathbb{P}(S) = \{\}$, then what is true? Circle your answer.

(a) $S = \{\}$ $S = \phi = \{\}$
(b) $S = \{0\}$
(c) $S = \{\{\}\}$ $P = \{\{\}\}$
d (d) there has been an error because {} cannot be the powerset of any set

[5] 9. If $S = \{1, 1, 2, 2, 5\}$, then what is the size of $\mathbb{P}(S)$? Circle your answer.

(a) 5
(b) 3
c (c) 2^3
(d) 2^5
(e) none of the above

[5] 10. Imagine a function f mapping elements of set A to elements of set B. The cardinality of set A is 10. The cardinality of set B is 20. What do we know about this function f? Circle your answer.

(a) f is one-to-one
(b) f cannot be one-to-one
e (c) f is onto
(d) f cannot be onto
(e) none of the above

[5] 11. Imagine a function f mapping elements of set A to elements of set B. The cardinality of set A is 10. The cardinality of set B is 10. What do we know about this function f? Circle your answer.

(a) f is one-to-one
(b) f cannot be one-to-one
(c) f is onto
(d) f cannot be onto
(e) none of the above

e

[10] 12. Use the Principle of Mathematical Induction to prove

$P(n) : \sum_{i=1}^{n}(2i-1) = n^2$. Prove $P(n)$ is true for all $n \in \mathbb{Z}^+$.

Basis Step: Show that $P(1)$ is true

LHS

$\sum_{i=1}^{1}(2i-1) = 2(1) - 1 = 1$

RHS

$n^2 = 1^2 = 1$

LHS = RHS

I have show that the ~~bas~~ basis step is true, ie $P(o)$ is true.

Inductive step: Show that $P(k) \longrightarrow P(k+1)$, $k \in \mathbb{Z}^+$

1. $P(k)$ — assume $P(k)$
2. $\sum_{i=1}^{k}(2i-1) = k^2$ — def'n of $P(k)$
3. $1+3+\ldots+2k-1 = k^2$ — def'n of Σ operator
4. $1+3+\ldots+2k-1+2(k+1)-1 = k^2+(2(k+1)-1)$ — Adding $2(k+1)-1$ to LHS and RHS
5. $1+3+\ldots 2k-1+2(k+1)-1 = k^2+2k+1$ — Simplyfying math
6. $1+3+\ldots+2k-1+2(k+1)-1 = k^2+k+k+1$ — Spliting $2k$ into $k+k$
7. $1+3+\ldots+2k-1+2(\cancel{m}k+1)-1 = k(k+1)+(k+1)$ — Factoring k from k^2+k
8. $1+3+\ldots+2k-1+2(k+1)-1 = (k+1)(k+1)$ — Factoring $(k+1)$ on RHS
9. $\sum_{i=1}^{k+1} 2i-1 = (k+1)(k+1)$ — def'n of Σ
10. $\sum_{i=1}^{k+1} 2i-1 = (k+1)^2$ — Simplyfying $(k+1)(k+1)$ to $(k+1)^2$
11. $P(k+1)$ — def'n of $P(k+1)$

Hence ~~I~~ Assume $P(k)$, I have shown $P(k+1)$ to be true, hence the Inductive step is true.

Conclusion: - I have shown both basis step and inductive step to be true, hence, using the Principle of Mathematical Induction, I have shown $P(n)$ to be true.

[5] 13. Suppose the universal set $U = \{a, c, g, t\}$. $S = \{g, t\}$. Write $\bar{S}$ (the complement of S) using list notation.

$\bar{S} = U - S = \{a, c\}$

[5] 14. What are the terms a_1, a_2, a_3 of the sequence $\{a_n\}$, where $a_n = 3^n$, $n \in Z^+$?

$a_n = 3^n$

$a_1 = 3$

$a_2 = 3^2 = 9$

$a_3 = 3^3 = 27$

[5] 15. You are given the following function information. What is true? Circle your answer.

$f : \mathbb{Z} \to \mathbb{Z}$, where $f(x) = x^2$.

(a) f is not a function ✓

(b) f is one-to-one and onto ×

(c) f is one-to-one, but not onto ×

(d) f is not one-to-one, but it is onto

e (e) f is not one-to-one, and it is not onto

[5] 16. If A, B, and C are sets such that $A \cup C = B \cup C$ then we know $A = B$. Circle your answer.

(a) the statement is true $A = \{1, 2\}$ $C = \{2, 3\}$

(b) the statement is false $B = \{1\}$

[5] 17. For all sets, S, $\{\} \subset S$. Circle your answer.

(a) the statement is true

(b) the statement is false

(c) cannot be determined

[5] 18. If a, b, c, d are four positive integers such that $a \mid b$ and $b \mid c$, then $a \mid cd$. Circle your answer.

(a) the statement is true $b = k_1 a$

(b) the statement is false $c = k_2 b = k_3 a$

(c) it cannot be determined $cd = k_3 ad$

[5] 19. If the original phrase is "hello", what is resultant phrase when encrypted by the Caesar cipher?
For your reference, the English alphabet is given here. abcdefghijklmnopqrstuvwxyz

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

h - $(p+3) \bmod 26 = (7+3) \bmod 26 = 10 \bmod 26 = 10$ - k

e - $(p+3) \bmod 26 = (4+3) \bmod 26 = 7 \bmod 26 = 7$ - h

l - $(p+3) \bmod 26 = (11+3) \bmod 26 = 14 \bmod 26 = 14 = o$

l - $(p+3) \bmod 26 = (11+3) \bmod 26 = 14 \bmod 26 = 14 = o$

o - $(p+3) \bmod 26 = (14+3) \bmod 26 = 17 \bmod 26 =$ r

encrypted version of hello, using a Caesar cipher is khoor

This page provides extra space if you need it. Clearly mark any question that has its answer here.

Do not open this exam until instructed to do so.

$\{1,2\} \Rightarrow P(S) = \{\{\}, \{1,2\}, \{1\}, \{2\}\} = 2^2$

$\{1,1\} = P(S) = \{\{\}, \{1,1\}, \{1\}\} = 3$

$\{1,2,3\} = \{\{\}, \{1,2,3\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$

$\{1,1,2\} = \{\{\}, \{1\}, \{2\}, \{1,2\}\}$