CS2050-2019-Fall Exam 3 Makeup

Tejas Rajamadam Pradeep

TOTAL POINTS

87 / 100

QUESTION 1

1 T/F Strong Induction 5 / 5

+ **0 pts** True

✓ + **5 pts** **False**

+ **0 pts** No answer

QUESTION 2

2 Even length bit strings recursive set 15 / 15

- **15 pts** No answer

✓ - **0 pts** **Correct answer**

- **5 pts** Function or sequence definition

Basis Step Option 1 (cap at -8)

- **2 pts** Basis Step missing one case
(aka no 00 □ S OR 01 □ S OR 10 □ S OR 11 □ S)

- **4 pts** Basis Step missing two cases
(00 □ S OR 01 □ S OR 10 □ S OR 11 □ S)

- **6 pts** Basis Step missing three cases
(only one of 00 □ S, 01 □ S, 10 □ S, 11 □ S)

- **8 pts** Basis step missing all four base cases
(00 □ S, 01 □ S, 10 □ S, 11 □ S)

- **2 pts** Included wrong base cases

- **4 pts** Incorrect "element of" notation in basis step

Basis Step Option 2 (cap at -8)

- **2 pts** Missing LHS: x □ S or equivalent

- **2 pts** Missing LHS: y □ S or equivalent

- **4 pts** Missing RHS: xy □ S or equivalent

- **4 pts** Extra incorrect RHS term

- **2 pts** Extra incorrect LHS term

- **2 pts** Not conditional

- **2 pts** Used or instead of and for LHS

- **4 pts** Incorrect element of notation

Recursive Step option 1 (cap at -8)

- **2 pts** Missing Recursive Step LHS: w □ S or equivalent

- **2 pts** Missing Recursive Step LHS: x □ S or equivalent

- **2 pts** Missing Recursive Step RHS: xw □ S or equivalent

- **2 pts** Not using conditional in recursive step

- **4 pts** Recursive step wasn't recursive

- **4 pts** extra incorrect recursive RHSs

- **2 pts** Used OR instead of AND in RHS

- **4 pts** Incorrect "element of" notation in recursive step

Recursive step Option 2 (max cap -8)

- **2 pts** Missing LHS: x □ Σ or equivalent

- **2 pts** Missing LHS: y □ Σ or equivalent

- **2 pts** Missing LHS: w □ S or equivalent

- **2 pts** Additional incorrect term on LHS

- **2 pts** Missing RHS: xyw □ S or equivalent

- **2 pts** Didn't use conditional

- **4 pts** Recursive step wasn't recursive

- **4 pts** Incorrect "element of" notation in recursive step

- **2 pts** Additional incorrect RHS term

Recursive step Option 3 (max cap -8)

- **4 pts** Missing LHS: w □ S or equivalent

- **1 pts** Missing RHS: w00 □ S or equivalent

- **1 pts** Missing RHS: w01 □ S or equivalent

- **1 pts** Missing RHS: w10 □ S or equivalent

- **1 pts** Missing RHS: w11 □ S or equivalent

- **2 pts** Not using a conditional

- **2 pts** Recursive RHS uses OR instead of AND

- **1 pts** Using variable without defining it (e.g. w)

- **3 pts** Solved with english alphabet and not for bitstrings

- **0 pts** Put lambda in alphabet along with 0 and 1

- **1 pts** Did not name set S

- **15 pts** Incorrect
- **2 pts** Other small notation error

QUESTION 3

3 Pigeonhole card dealing 5 / 5

✓ - **0 pts 25 cards**
- **5 pts** No answer
- **3 pts** ⌈x/4⌉ = 7 but incorrect number chosen for x
- **1 pts** Correct work but minor math error leads to incorrect answer
- **5 pts** Incorrect answer. Should be 25 cards
- **5 pts** Answered: "How many cards must be dealt to guarantee 7 cards of EVERY suit" (28)
- **5 pts** Answered: "How many cards must be dealt to guarantee 7 cards of A SPECIFIC suit" (46)

QUESTION 4

4 Cupcake counting 5 / 5

✓ - **0 pts 4 * 10 * 3 = 120 possible cupcakes**
- **5 pts** No answer
- **1 pts** Correct work but minor math error leads to incorrect answer
- **5 pts** Incorrect answer

QUESTION 5

5 Mismatched socks 5 / 5

- **5 pts** No answer

✓ - **0 pts 9 socks**
- **5 pts** Incorrect answer. Answer should be 9
- **3 pts** ⌈x/8⌉ = 2 but incorrect value for x chosen

QUESTION 6

6 Matched socks 5 / 5

- **5 pts** No answer

✓ - **0 pts 4 socks**
- **5 pts** Incorrect answer. Answer should be 4
- **3 pts** ⌈x/3⌉ = 2 but wrong value chosen for x

QUESTION 7

7 Length 256 bit strings 10 / 10

- **10 pts** No answer
- **10 pts** Wrong Answer

✓ - **0 pts 2^252+2^252 - 2*2^248 (or -2^249)**
- **5 pts** Treated the problem like inclusive or (aka had one -2^248)
- **5 pts** Wrong double counting term (aka not -2^248)
- **2 pts** Wrong simplification
- **5 pts** Correct Work, Major problem in final answer

QUESTION 8

8 Non UGA license plate 2 / 10

- **10 pts** No answer
- **0 pts** 26^6 - (4*26^3) + 1
- **5 pts** Didn't account for rearranging UGA (aka forgot to multiply by 4)
- **5 pts** Didn't allow letters to be used more than once
- **1 pts** Adjustment off for case of UGAUGA (removing that twice within the 4(26^3) so need a +1 to adjust)
- **10 pts** Wrong answer

✓ - **8 pts Incorrect exclusion term (not 4*26^3)**

QUESTION 9

9 Pigeonhole initials 5 / 5

- **5 pts** No answer

✓ - **0 pts 2705 or (26*26*4+1)**
- **5 pts** Incorrect answer
- **1 pts** Correct work but incorrect answer
- **3 pts** Ceiling (x/26^2) = 5 but incorrect/no value for x

QUESTION 10

10 Odd length binary strings of only 1 recursive set 10 / 10

- **10 pts** No answer

✓ - **0 pts Correct answer**
- **5 pts** Function or sequence definition

Basis Step (cap at -4)

- **4 pts** Basis Step missing 1 ∈ S
- **2 pts** Included wrong base cases

Recursive Step (cap at -6)

- **3 pts** Missing Recursive Step LHS: w S
- **3 pts** Missing Recursive Step RHS: 11w S
- **2 pts** Not using conditional in recursive step
- **4 pts** Recursive step was not recursive
- **6 pts** Recursive step was not recursive and incorrect
- **6 pts** incorrect

- **1 pts** Using variable without defining it (e.g. x)
- **3 pts** Solved with english alphabet and not for bitstrings
- **3 pts** Used sigma-star instead of S
- **0 pts** Put lambda in alphabet along with 0 and 1
- **1 pts** Did not name set S
- **10 pts** Incorrect Answer

QUESTION 11

11 Username Counting **5 / 10**

- **10 pts** No answer
- **0 pts** 31^6-26^6+31^7-26^7
- **5 pts** Forgot to include the length 6 OR length 7 case
(aka 31^6-26^6 OR 31^7-26^7)
- **7 pts** Didn't make sure both cases had at least one special symbol (Subtract 26^6 and 26^7)
- **10 pts** Incorrect answer
- **1 pts** Math error when simplifying
- **5 pts** undercounting, such as saying a specific spot(s) must be a special symbol (e.g. the special symbol is at the start of the string). This problem calls for inclusion/exclusion to include those having anything anywhere, but then explude those with zero special symbols.

✓ **- 5 pts overcounting, such as trying to say any index could be a special symbol but not accounting for duplicates. This problem calls for inclusion/exclusion to include those having anything anywhere, but then exclude those with zero special symbols.**

- **10 pts** Answered for "exactly one symbol" instead of "at least one symbol"
- **10 pts** Incorrect Answer: 5 * (31^5) + 5 * (31^6)
- **10 pts** Added factorials to correct solution, making it incorrect
- **10 pts** Did correct exclusion, but started with (26^6 + 26^7) instead of (36^6 + 36^7)

QUESTION 12

12 Product of primes strong induction **15 / 15**

✓ **- 0 pts All Correct**

- **15 pts** All Incorrect
- **15 pts** Used Math Induction
- **2 pts** Used a predicate (e.g. P(n)) without defining it or defined incorrectly
- **1 pts** Didn't define/incorrect domain of n
- **1 pts** Incorrect variable in predicate definition
- **2 pts** Used a predicate as a non-boolean
- **1 pts** Didn't use/Incorrect P(2) as a base case
- **2 pts** Incorrect/Missing basis step conclusion
- **4 pts** Missing / Incorrect Assumption (should only assume "j P(j)")
- **1 pts** Doesn't explicitly say "assume", "premise", "hypothesis"
- **2 pts** Used same variable for predicate declaration and inductive step
- **1 pts** Wrong inductive hypothesis but never assumed
- **1 pts** Incorrect domain/bounds for j
- **1 pts** Incorrect domain/bounds for k
- **4 pts** Didn't use P(r) and P(s) to reach P(k+1) by making r*s=k+1
- **2 pts** not stating "P(something) is true by the Inductive Hypothesis"
- **1 pts** Incorrect/Missing conclusion for inductive step
- **1 pts** Concludes j (P(j) → P(k+1)) instead of j P(j) → P(k+1)
- **8 pts** Invalid inductive step
- **1 pts** Overall Conclusion: Missing/Incorrect "basis step is true"
- **1 pts** Overall Conclusion: Missing/Incorrect "inductive step is true"
- **1 pts** Overall Conclusion: Missing/Incorrect "P(n) is

true"

- 1 pts Overall Conclusion: Missing/Incorrect "Strong induction"

- 4 pts Did inductive step for a specific value of k, rather than an arbitrary k

CS 2050 Exam 3 - MAKEUP

November 19, 2019

Name (print clearly): Tejas Rajamadam Pradeep

Taking this exam signifies you are aware of and in accordance with the Academic Honor Code of Georgia.

Fairness matters - Do not open this exam until instructed to do so.

- There are no breaks during this exam.
- Signing signifies you are aware of and in accordance with the **Academic Honor Code of Georgia Tech** and the **Code of Conduct**.
- When time is called you are to stop writing immediately. No exceptions.
- You are not allowed to speak to other students during the exam unless that student is a TA proctor.
- Do your best to keep your answers covered.
- You are to uphold the honor and integrity expected of you.
- Books, notes, laptops, cell phones, smart watches, etc. are NOT allowed.
- Calculators are not allowed in CS2050.

Page	Points	Score
2	30	
3	30	
4	25	
5	15	
Total:	100	

[5] 1. True/False. A Strong Induction Proof's inductive hypothesis is $P(k)$, where k represents an arbitrary single value from the same domain as n. False

[15] 2. Write a recursive set definition for the set of bit strings having even length of 2 or more. (So legal lengths are 2, 4, 6, 8, and so on.)

You are forbidden to use Σ^*. Your alphabet is $\Sigma = \{0, 1\}$.

Name the set S.

Let S be the set of bit strings with even length

Base Case:

01 ∈ S

10 ∈ S

11 ∈ S

00 ∈ S

Recursive step:

Let x ∈ Σ, y ∈ Σ, if w ∈ S, then xwy ∈ S

[5] 3. There are 52 cards in a deck consisting of 13 ranks and 4 suits. How many cards must be dealt to guarantee that 7 cards of the same suit have been dealt? (Like 7 hearts for example.)

Give your answer in the form of an integer.

13 spades, 13 diamonds, 13 clubs, 13 hearts.

no. of pegions = x

no. of pigeon holes = 4

$\lceil \frac{x}{4} \rceil = 7$ $x = 4 \cdot 6 + 1 = 25$

[5] 4. Your cupcake factory makes 4 kinds of cake, 10 kinds of icing, and 3 types of sprinkles. How many different cupcake combinations are possible if a cupcake is defined as one type of cake with one type of icing with one type of sprinkles?

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

cupcake: $\underset{\text{icing}}{\underline{4}}$ $\underset{\text{cake}}{\underline{10}}$ $\underset{\text{sprinkle}}{\underline{3}}$

Combinations $4 \cdot 10 \cdot 3 = \underline{120}$ types of cupcakes

[5] 5. You own 8 single blue socks, 6 single orange socks, and 4 single green socks. (Meaning 4 pairs of blue, 3 pairs of orange, and 2 pairs of green.) If these socks are mixed in a drawer, without looking, what is the minimum number of socks you need to guarantee you have a MISMATCHED pair of socks? (Assume you want to wear two socks that do NOT match each other.)

Give your answer in the form of an integer.

~~minimum~~

worst case :- 8 blue socks + 1 green ~~or~~ sock

= 9 socks

[5] 6. You own 8 single blue socks, 6 single orange socks, and 4 single green socks. (Meaning 4 pairs of blue, 3 pairs of orange, and 2 pairs of green.) If these socks are mixed in a drawer, without looking, what is the minimum number of socks you need to guarantee you have a MATCHED pair of socks? (Assume you want to wear two socks that match each other.)

Give your answer in the form of an integer.

no. of pigeons = x

no. of pigeonholes = 3

$\left\lceil \frac{x}{3} \right\rceil = 2$

$x = 3(1) + 1 = 4$ socks

4 socks are needed to garuntee a match.

[10] 7. How many bit strings of length 256 start with 0011 or end with 1100 but never both?

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

no. that start with 0011 = 2^{252}

no. that end with 1100 = 2^{252}

no. of both = 2^{248}

No. that either start with 0011 or end with 1100, but not both = $2^{252} + 2^{252} - 2(2^{248})$

$= 2^{253} - 2^{249}$

[10] 8. Assume vehicle license plates (tags) are exactly 6 uppercase letters long using letters from the alphabet A-Z (26 letters). Assuming letters can be used more than once, how many unique tags are possible if the substring UGA cannot appear within the string of length 6?

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

Total combinations of letters = 26^6.

Total combination with UGA = {UGA---, -UGA--, --UGA-, ---UGA}

Total combination with out the substring UGA = $26^6 - 4$

$$\sum_{i=0}^{5} 5^{6-i} \cdot \frac{6!}{(6-i)!} \cdot 26^{i} + \sum_{i=0}^{6} 5^{7-i}$$

[5] 9. Let's pretend everyone has a first-name and a last-name (like Frank Zappa, who has the initials F.Z.). Assume each part of a name starts with an uppercase letter A-Z (26 letters). How many different people are required to guarantee at least 5 people have the same first-name, last-name initials?

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

Possible first name initials = 26
Possible last name initials = 26 } Total combinations $= 26^2$

No. of pegions = x
No. of pigeon holes $= 26^2$

$\left\lceil \frac{x}{26^2} \right\rceil = 5$

$x = 4 \cdot 26^2 + 1$

[10] 10. Give a recursive set definition for the set of binary strings of odd length that consist of all ones. Name the set S. You are forbidden to use Σ^* or any other usage of the star operator. Your alphabet is $\Sigma = \{0, 1\}$.

Let S be the set of all binary strings of odd length, containing all ones.

Base Case:

$1 \in S$

Recursive step:

if $w \in S$, then $1w1 \in S$

[10] 11. Imagine everyone has a username built using letters and special symbols having length 6 or 7. If a username must contain one or more special symbols, how many unique usernames are possible?

There are 26 letters (A through Z) and 5 special symbols {+, –, !, \$, %}. A letter or special symbol can be used zero or more times.

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

1. Names of length 6

5 5 5 5 5 5 $\rightarrow 5^6$
+ (Accounting for position)
5 5 5 5 5 26 $6 \cdot 5^5 \cdot 26$
+
5 5 5 5 26 26 $6 \cdot 5 \cdot 5^4 \cdot 26^2$
+
5 5 5 26 26 26 $6 \cdot 5 \cdot 4 \cdot 5^3 \cdot 26^3$
+
5 5 26 26 26 26 $6 \cdot 5 \cdot 5^2 \cdot 26^4$
+
5 26 26 26 26 26 $6 \cdot 5 \cdot 26^5$

2. Names of length 7

5 5 5 5 5 5 5 $\rightarrow 5^7$
+
5 5 5 5 5 5 26 $6 \cdot 26 \cdot 5^6$
+
5 5 5 5 5 26 26 $6 \cdot 5 \cdot 26^2 \cdot 5^5$
+
5 5 5 5 26 26 26 $6 \cdot 5 \cdot 4 \cdot 26^3 \cdot 5^4$
+
5 5 5 26 26 26 26 $6 \cdot 5 \cdot 4 \times 5^3 \cdot 26^4$
+
5 5 26 26 26 26 26 $6 \cdot 5 \cdot 5^2 \cdot 26^5$
+
5 26 26 26 26 26 26 $6 \cdot 5 \cdot 26^6$

Total Combinations $\Rightarrow 5^6 + 6 \cdot 5^5 \cdot 26 + 6 \cdot 5 \cdot 26^2 + 6 \cdot 4 \cdot 5^4 \cdot 26^3 + 6 \cdot 5^3 \cdot 26^4 + 6 \cdot 5 \cdot 26^5 + 7 \cdot 6 \cdot 5 \times 26^4 + 5^3$
$+ 5^7 + 7 \cdot 26 \cdot 5^6 + 7 \cdot 6 \cdot 26^2 \cdot 5^5 + 7 \cdot 6 \cdot 5 \cdot 26^3 \cdot 5^4$
$+ 7 \cdot 6 \cdot 26^5 \cdot 5^2 + 7 \cdot 26^6 \cdot 5$

[15] 12. Write a strong induction proof to show that if n is a positive integer greater than 1, then n can be written as a product of primes. For example, 18 can be written as $2 * 3 * 3$. (Hint: For the inductive step, consider separately the case where $k + 1$ is a prime and the case where $k + 1$ is composite.)

A positive integer is prime if its only factors are 1 and itself (for example, the only factors of 5 are 1 and 5 so it is prime).

A positive integer is composite if it has a factor that is neither 1 nor itself (for example, the factors of 12 are 1, 2, 3, 4, 6, and 12, so it is composite).

Your proof must demonstrate your knowledge of strong induction and not use mathematical induction. Math induction as well as other non-strong-induction proof techniques will earn no credit.

Let $P(n)$ be the proposition that n can be written as a product of primes,
$n \in \mathbb{Z}$, $n \geq 1$

Basis Step:
Consider $P(2)$,
2 is a prime no., hence
2 can be written as a product of primes
hence $P(2)$ is true.

Hence I've shown my basis step to be true.

Inductive Hypothesis
$\forall j\, P(j)$, $j \in \mathbb{Z}$, $2 \leq j \leq k$, $k \geq 2$
$k \in \mathbb{Z}$

Inductive Step:
Show that $\forall j P(j) \longrightarrow P(k+1)$, $j \in \mathbb{Z}$, $2 \leq j \leq k$,
$k \in \mathbb{Z}$, $k \geq 2$
Assume $\forall j P(j)$ is true.
Case 1: $k+1$ is prime.
if $k+1$ is prime,
then $k+1$ can be written as a product of itself,
hence $P(k+1)$ is true.

Case 2: $k+1$ is composite,
$k+1$ can be represented
as $1 \cdot m \cdot n$, where $m \in \mathbb{Z}$, $m < k+1$
$n \in \mathbb{Z}$, $n < k+1$

continuation on other side

To be fair to all students, do not open this exam until instructed to start.

This page has no questions. Use it for overflow if needed.

$k+1 = 1 \cdot m \cdot n$

but sime $m < k+1$, $n < k+1$

$m \leq k$ def'n of $\leq$

$n \leq k$ def'n of $\leq$

hence $P(m)$ and $P(n)$ are true,

hemc both m an n can be represented as a product of primes,

hence $k+1 = 1 \cdot m \cdot n$

$= 1 \cdot$ (product of primes) $\cdot$ (product of primes)

hence $k+1$ can be represented as a product of primes,

hence $P(k+1)$ is true.

Hence I've show that the inductive step, $\forall j P(j) \rightarrow P(k+1)$, $j \in \mathbb{Z}, k \in \mathbb{Z}, 2 \leq j \leq k, k \geq 2$ is true.

Conclusion :- As the basis step and Inductive step are shown to be true, using the principle of strong induction, I've shown $P(n)$ to be true, for $n \in \mathbb{Z}$, $n > 1$