CS 2050 Exam 3 - MAKEUP

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Taking this exam signifies you are aware of and in accordance with the Academic Honor Code of Georgia.

Fairness matters - Do not open this exam until instructed to do so.

- There are no breaks during this exam.
- Signing signifies you are aware of and in accordance with the **Academic Honor Code of Georgia Tech** and the **Code of Conduct**.
- When time is called you are to stop writing immediately. No exceptions.
- You are not allowed to speak to other students during the exam unless that student is a TA proctor.
- Do your best to keep your answers covered.
- You are to uphold the honor and integrity expected of you.
- Books, notes, laptops, cell phones, smart watches, etc. are NOT allowed.
- Calculators are not allowed in CS2050.

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4	25	
5	15	
Total:	100	

[5] 1. True/False. A Strong Induction Proof's inductive hypothesis is $P(k)$, where k represents an arbitrary single value from the same domain as n. False

[15] 2. Write a recursive set definition for the set of bit strings having even length of 2 or more. (So legal lengths are 2, 4, 6, 8, and so on.)

You are forbidden to use Σ^*. Your alphabet is $\Sigma = \{0, 1\}$.

Name the set S.

Let S be the set of bit strings with even length

Base Case:

$01 \in S$

$10 \in S$

$11 \in S$

$00 \in S$

Recursive step:

Let $x \in \Sigma$, $y \in \Sigma$. if $w \in S$, then $xwy \in S$

[5] 3. There are 52 cards in a deck consisting of 13 ranks and 4 suits. How many cards must be dealt to guarantee that 7 cards of the same suit have been dealt? (Like 7 hearts for example.)

Give your answer in the form of an integer.

13 spades, 13 diamonds, 13 clubs, 13 hearts.

no. of pegions = x

no. of pigeon holes = 4

$\left\lceil \frac{x}{4} \right\rceil = 7$ $x = 4 \cdot 6 + 1 = 25$

[5] 4. Your cupcake factory makes 4 kinds of cake, 10 kinds of icing, and 3 types of sprinkles. How many different cupcake combinations are possible if a cupcake is defined as one type of cake with one type of icing with one type of sprinkles?

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

cupcake: $\frac{4}{\text{icing}}$ $\frac{10}{\text{cake}}$ $\frac{3}{\text{sprinkle}}$

Combinations $4 \cdot 10 \cdot 3 = 120$ types of cupcakes

[5] 5. You own 8 single blue socks, 6 single orange socks, and 4 single green socks. (Meaning 4 pairs of blue, 3 pairs of orange, and 2 pairs of green.) If these socks are mixed in a drawer, without looking, what is the minimum number of socks you need to guarantee you have a MISMATCHED pair of socks? (Assume you want to wear two socks that do NOT match each other.)

Give your answer in the form of an integer.

~~minimum~~

worst case :- 8 blue socks + 1 green sock

= 9 socks

[5] 6. You own 8 single blue socks, 6 single orange socks, and 4 single green socks. (Meaning 4 pairs of blue, 3 pairs of orange, and 2 pairs of green.) If these socks are mixed in a drawer, without looking, what is the minimum number of socks you need to guarantee you have a MATCHED pair of socks? (Assume you want to wear two socks that match each other.)

Give your answer in the form of an integer.

no. of pigeons = x
no. of pigeonholes = 3

$\left\lceil \frac{x}{3} \right\rceil = 2$

$x = 3(1) + 1 = 4$ socks

4 socks are needed to garuntee a match.

[10] 7. How many bit strings of length 256 start with 0011 or end with 1100 but never both?

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

no. that start with 0011 = 2^{252}

no. that end with 1100 = 2^{252}

no. of both = 2^{248}

No. that either start with 0011 or end with 1100, but not both = $2^{252} + 2^{252} - 2(2^{248})$

$= 2^{253} - 2^{249}$

[10] 8. Assume vehicle license plates (tags) are exactly 6 uppercase letters long using letters from the alphabet A-Z (26 letters). Assuming letters can be used more than once, how many unique tags are possible if the substring UGA cannot appear within the string of length 6?

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

Total combinations of letters = 26^6.

Total combination with UGA = {UGA---, -UGA--, --UGA-, ---UGA}

Total combination with out the substring UGA = $26^6 - 4$

$$\sum_{i=0}^{5} 5^{6-i} \cdot \frac{6!}{(6-i)!} \cdot 26^{i} + \sum_{i=0}^{6} 5^{7-i}$$

[5] 9. Let's pretend everyone has a first-name and a last-name (like Frank Zappa, who has the initials F.Z.). Assume each part of a name starts with an uppercase letter A-Z (26 letters). How many different people are required to guarantee at least 5 people have the same first-name, last-name initials?

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

Possible first name initials = 26
Possible last name initials = 26 } – Total combinations $= 26^2$

No. of pegions = x
No. of pigeon holes = 26^2

$\left\lceil \frac{x}{26^2} \right\rceil = 5$

$x = 4 \cdot 26^2 + 1$

[10] 10. Give a recursive set definition for the set of binary strings of odd length that consist of all ones. Name the set S. You are forbidden to use Σ^* or any other usage of the star operator. Your alphabet is $\Sigma = \{0, 1\}$.

Let S be the set of all binary strings of odd length, containing all ones.

Base case:

$1 \in S$

Recursive step:

if $w \in S$, then $1w1 \in S$

[10] 11. Imagine everyone has a username built using letters and special symbols having length 6 or 7. If a username must contain one or more special symbols, how many unique usernames are possible?

There are 26 letters (*A* through *Z*) and 5 special symbols {+, –, !, \$, %}. A letter or special symbol can be used zero or more times.

Give your answer in the form of an integer or as a formula using addition, subtraction, multiplication, division, exponentiation, ceiling and/or floor. Do not use C() nor P().

1. Names of length 6

5 5 5 5 5 5 → 5^6 +

5 5 5 5 5 26 $6 \cdot 5^5 \cdot 26$ + (Accounting for position)

5 5 5 5 26 26 $6 \cdot 5 \cdot 5^4 \cdot 26^2$ +

5 5 5 26 26 26 $6 \cdot 5 \cdot 4 \cdot 5^3 \cdot 26^3$ +

5 5 26 26 26 26 $6 \cdot 5 \cdot 5^2 \cdot 26^4$ +

5 26 26 26 26 26 $6 \cdot 5 \cdot 26^5$

2. Names of length 7

5 5 5 5 5 5 5 → 5^7 +

5 5 5 5 5 5 26 $6 \cdot 26 \cdot 5^6$ +

5 5 5 5 5 26 26 $6 \cdot 5 \cdot 26^2 \cdot 5^5$ +

5 5 5 5 26 26 26 $6 \cdot 5 \cdot 4 \cdot 26^3 \cdot 5^4$ +

5 5 5 26 26 26 26 $6 \cdot 5 \cdot 4 \times 5^3 \cdot 26^4$ +

5 5 26 26 26 26 26 $6 \cdot 5 \cdot 5^2 \cdot 26^5$ +

5 26 26 26 26 26 26 $6 \cdot 5 \cdot 26^6$

Total combinations $\Rightarrow 5^6 + 6 \cdot 5^5 \cdot 26 + 6 \cdot 5 \cdot 5^5 \cdot 26^2 + 6 \cdot 4 \cdot 5^4 \cdot 26^3 + 6 \cdot 5^3 \cdot 26^4 + 6 \cdot 5 \cdot 26^5 + 7 \cdot 6 \cdot 5 \cdot 26^3 \cdot 5^4 + 7 \cdot 6 \cdot 5 \times 26^4 + 5^3$
$+ 5^7 + 7 \cdot 26 \cdot 5^6 + 7 \cdot 6 \cdot 26^2 \cdot 5^5 + 7 \cdot 6 \cdot 5 \cdot 26^3 \cdot 5^4$
$+ 7 \cdot 6 \cdot 26^5 \cdot 5^2 + 7 \cdot 26^6 \cdot 5$

[15] 12. Write a strong induction proof to show that if n is a positive integer greater than 1, then n can be written as a product of primes. For example, 18 can be written as $2 * 3 * 3$. (Hint: For the inductive step, consider separately the case where $k + 1$ is a prime and the case where $k + 1$ is composite.)

A positive integer is prime if its only factors are 1 and itself (for example, the only factors of 5 are 1 and 5 so it is prime).

A positive integer is composite if it has a factor that is neither 1 nor itself (for example, the factors of 12 are 1, 2, 3, 4, 6, and 12, so it is composite).

Your proof must demonstrate your knowledge of strong induction and not use mathematical induction. Math induction as well as other non-strong-induction proof techniques will earn no credit.

Let P(n) be the proposition that n can be written as a product of primes, $n \in \mathbb{Z}$, $n \geq 1$

Basis Step,

Consider P(2),

2 is a prime no., hence
2 can be written as a product of primes
hence P(2) is true.

Hence I've shown my basis step to be true.

Inductive Hypothesis

$\forall j P(j)$, $j \in \mathbb{Z}$, $k \in \mathbb{Z}$, $2 \leq j \leq k$, $k \geq 2$

Inductive Step:

Show that $\forall j P(j) \longrightarrow P(k+1)$, $j \in \mathbb{Z}$, $2 \leq j \leq k$, $k \in \mathbb{Z}$, $k \geq 2$

Assume $\forall j P(j)$ is true.

Case 1: k+1 is prime.
if k+1 is prime,
then k+1 can be written as a product of itself,
hence P(k+1) is true.

Case 2: k+1 is composite,
k+1 can be represented
as $1 \cdot m \cdot n$, where $m \in \mathbb{Z}$, $m < k+1$
$n \in \mathbb{Z}$, $n < k+1$

continuation on other side

To be fair to all students, do not open this exam until instructed to start.

This page has no questions. Use it for overflow if needed.

$k+1 = 1 \cdot m \cdot n$

but sime $m < k+1$, $n < k+1$

$m \leq k$ def'n of $\leq$

$n \leq k$ def'n of $\leq$

hence $P(m)$ and $P(n)$ are true,

hemc both m an n can be represented as a product of primes.

hence $k+1 = 1 \cdot m \cdot n$

$= 1 \cdot$ (product of primes) $\cdot$ (product of primes)

hence $k+1$ can be represented as a product of primes,

hence $P(k+1)$ is true.

Hence I've show that the Inductive step, $\forall j P(j) \rightarrow P(k+1)$, $j \in \mathbb{Z}, k \in \mathbb{Z}, 2 \leq j \leq k$, $k \geq 2$ is true.

Conclusion: As the basis step and Inductive step are shown to be true, using the principle of strong induction, I've shown $P(n)$ to be true, for $n \in \mathbb{Z}$, $n > 1$