Semi-Supervised Classification Using K-Means Classification and FixMatch

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Abstract

Semi-supervised classification is a very important problem. There are many situations in which it is simply too costly or time-consuming to label all data. However, getting additional information from the unlabeled data can often be beneficial to the model that is being trained.

Here, we outline an approach to semi-supervised classification leveraging supervised learning, K-means classification to ensure separability of the latent space distribution, and FixMatch loss to ensure consistent labeling across the unlabeled data. We will then analyze the training metrics after autotuning hyperparameters to get a deeper idea of what occurs during training.

The FixMatch loss function was inspired by FixMatch: Simplifying Semi-Supervised Learning with Consistency and Confidence, a paper released in 2020. Here, we extend this approach by using K-means classification during training in conjunction with the FixMatch loss on the unlabeled dataset. The model will be trained on the CIFAR-10 dataset

Motivation

The idea behind this approach is to learn using supervised learning, but also leverage the unlabeled data to get extra information.

Now, define D_l to be the distribution of labeled data and D_u to be the distribution of unlabeled data.

Let $f_{ heta}$ be our model that we are training

Let $z_{ heta}(x) \in \mathbb{R}^{100}$ be the logits from the second to last layer of $f_{ heta}$

Let $p_{\theta}(y|x)$ for $x \sim D$ be the probability of predicting class y given x under parameters θ .

(NOTE: One can think of $p_{ heta}$ and $f_{ heta}$ as being equivalent)

Supervised Loss L_{sup}

Our supervised loss will be cross-entropy on the labeled data:

$$L_{sup} = \mathbb{E}_{(x,y) \sim D_{sup}} \left[-\log p_{ heta}(y|x)
ight] = \mathbb{E}_{(x,y) \sim D_{sup}} \left[-\sum_{i=1}^K y_i \log p_{ heta}(y_i|x)
ight]$$

This ensures the network predicts ground truth labels where they exist

FixMatch Consistency Loss $L_{auqment}$

We want to ensure predictions don't change under augmentations. hence, we generate a pseudo-label from a weakly augmented image and enforce consistency on a strongly augmented image

Let $x\sim D_u$ Let x_{weak} be the view of x after weak augmentation Let x_{strong} be the view of x after strong augmentation

This motivates the following process:

1. Compute

$$\hat{y} = \langle \operatorname{argmax}_i p_{\theta}(i|x_{weak}) \rangle$$

2. If $\max_i p_{\theta}(i|x_{weak}) \geq au$, treat \hat{y} as true and penalize:

$$-\log p_{ heta}(\hat{y}|x_{strong})$$

Hence, we get the following term:

$$L_{augment} = -\mathbb{E}_{x \sim D_u} \left[\mathbb{I}\left(\max_i p_{ heta}(i|x_{weak}) \geq au
ight) \log p_{ heta}(\hat{y}|x_{strong})
ight]$$

K-means Clustering Loss L_{KMeans}

We want to incentivise z_{θ} to follow the cluster assumption. That is, when we cluster $z_{theta}(x)$ for all $x \sim D_u$, we want to ensure that decision boundaries lie in low density regions in \mathbb{R}^{100} .

This motivates the following process:

- 1. Fit K-means on all the logits $\{z_{ heta}(x)|x\in D_u\}$ to get centroids $\{C_j\}$
- 2. Assign each x to its nearest centroid $C_{k(x)}$ and then penlize the square distance from $z_{\theta}(x)$ to x's respective centroid:

$$L_{KMeans} = \mathbb{E} \|z_{ heta}(x) - C_{k(x)}\|_2^2$$

In theory, this incentivizes $z_t heta$ to output a representation of x that naturally form tight, well-separated clusters.

Final loss function

Combining the above gives our training objective:

$$\min_{ heta} L(heta) = \min_{ heta} \lambda_{sup} L_{sup} + \lambda_{augment} L_{augment} + \lambda_{KMeans} L_{KMeans}$$

External Libraries

Many libraries used in our implementation would be considered as "standard" in Computer Vision projects or courses:

- torch / torchvision: The main libraries for PyTorch. These provide the ability to create transformations and our own CNNs. This is extensively used for manipulating our tensors, providing loaders for our training and testing process, and to perform training & computations on CUDA. torch.utils.dataset also provides us with the dataset used for training
- sklearn: Used for efficient KMeans classification on CPU
- **numpy**: Used to manipulate tensors from **torch**, helps convert PyTorch tensors to a form that **sklearn** can understand.
- matplotlib: Used specifically for the plt command, which plots an image. This is used to show the metrics from training.
- random: Implementing label noise requires randomly selecting indexes to flip the labels.
- tqdm: A library that displays a loading bar and approximates the time remaining per training epoch

```
In []: import torch
import torch.nn as nn
import torch.nn.functional as F
from tqdm import tqdm

from torch.utils.data import DataLoader, Subset
import matplotlib.pyplot as plt
from torchvision import datasets, transforms

from sklearn.cluster import KMeans
import numpy as np
```

```
import random
```

c:\Users\tejas_1n\anaconda3\envs\myenv\lib\site-packages\tqdm\auto.py:21: Tq
dmWarning: IProgress not found. Please update jupyter and ipywidgets. See ht
tps://ipywidgets.readthedocs.io/en/stable/user_install.html
from .autonotebook import tqdm as notebook tqdm

Here are instructions for installing these libraries:

```
{bash}
conda create -n cv-env python=3.8
conda activate cv-env
conda install pytorch torchvision torchaudio cudatoolkit=11.3
-c pytorch
conda install scikit-learn numpy matplotlib tqdm
```

Download the Dataset using torch.utils.dataset

Files already downloaded and verified Files already downloaded and verified

Generate Trainloaders

Let D be the training set we are sampling from

Here, we partition our training set into D_u and D_l . Our function takes an input M, the number of unlabeled points to contain in the dataset.

If M=0, then D_u is empty (hence D_u is returned as None)

If M = |D|, then all of our points are unlabeled, implying D_l is empty (hence D_l is returned as None)

Otherwise, we shuffle and partition the training set such that $|D_u|=M$ and $|D_l|=|D|-M$

```
In [3]: def generate trainloaders(trainset, batch size label=64, batch size unlabel=
            indices = list(range(len(trainset)))
            random.shuffle(indices)
            labeled indices = indices[M:]
            unlabeled indices = indices[:M]
            # No unlabeled data
            if M == 0:
                train unlabeled loader = None
                trainloader = DataLoader(Subset(trainset, labeled indices), batch si
                                                   shuffle=True, num workers=2)
            # No labeled data
            elif M == len(trainset):
                train unlabeled loader = DataLoader(Subset(trainset, unlabeled indic
                                                   shuffle=True, num workers=2)
                trainloader = None
            # Both labeled and unlabeled data
            else:
                train_unlabeled_loader = DataLoader(Subset(trainset, unlabeled indic
                                                   shuffle=True, num workers=2)
                trainloader = DataLoader(Subset(trainset, labeled indices), batch si
                                                   shuffle=True, num workers=2)
            return trainloader, train unlabeled loader
```

Model Class

Our network is a fairly standard CNN, with 2 convolutional layers and 3 fully connected layers.

Given some $x\sim D_u$, we may want to return $z_{ heta}(x)$ for K-means classification of unlabeled data.

```
In [4]: class SimpleCNN(nn.Module):
            def init (self):
                super(SimpleCNN, self). init ()
                self.conv1 = nn.Conv2d(3, 32, kernel size=3, padding=1)
                self.conv2 = nn.Conv2d(32, 64, kernel_size=3, padding=1)
                self.fc1 = nn.Linear(64 * 8 * 8, 512)
                self.fc2 = nn.Linear(512, 100)
                self.fc3 = nn.Linear(100, 10)
            def forward(self, x, return logits=False):
                x = F.relu(self.conv1(x))
                x = F.max pool2d(x, 2)
                x = F.relu(self.conv2(x))
                x = F.max pool2d(x, 2)
                x = x.view(-1, 64 * 8 * 8)
                x = F.relu(self.fc1(x))
                x logits = F.relu(self.fc2(x))
```

```
x = self.fc3(x_logits)
if return_logits:
    return x, x_logits
else:
    return x
```

Define augmentations

Given an $x \sim D_u$ this is what allows us to obtain x_{weak} and x_{strong}

```
In []: ### Select 3 random samples from the training set and
        ### apply the weak and strong transformations
        indices = random.sample(range(len(trainset)), 3)
        images, _ = zip(*[trainset[i] for i in indices])
        strong images = [strong transform(img) for img in images]
        weak images = [weak transform(img) for img in images]
        fig, axes = plt.subplots(3, 3, figsize=(5, 5))
        for i in range(3):
            # original image
            axes[i, 0].imshow(np.transpose(images[i].numpy(), (1, 2, 0)) * 0.5 + 0.5
            axes[i, 0].set title("Original")
            axes[i, 0].axis("off")
            # weakly transformed image
            axes[i, 1].imshow(np.transpose(weak_images[i].numpy(), (1, 2, 0)) * 0.5
            axes[i, 1].set title("Weak Transform")
            axes[i, 1].axis("off")
            # strongly transformed image
            axes[i, 2].imshow(np.transpose(strong images[i].numpy(), (1, 2, 0)) * 0.
            axes[i, 2].set title("Strong Transform")
            axes[i, 2].axis("off")
        plt.tight layout()
        plt.show()
```



Original



Weak Transform



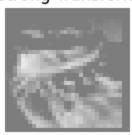
Strong Transform



Original



Weak Transform



Strong Transform





Training

For our train function, we pass in:

Datasets: trainset and testset

Transforms: weak_transform, strong_transforms

TAU: Which is the threshold for pseudo labels (default 0.95)

M: The number of unlabeled points in the trainset

epochs: Number of iterations to run the training loop

The main idea of the training loop is as follows:

- 1. For each batch in trainloader_labeled, we caluclate L_{sup} .
- 2. For each batch B in trainloader_unlabeled, we get x_{weak} and x_{strong} for all $x \in B$
- 3. We then calculate the KMeans loss over the whole dataset.

In []: torch.autograd.set_detect_anomaly(True)

def train(trainset=trainset, testset=testset, weak transform=weak transform,

```
strong transform=strong transform, TAU=0.95, run kmeans=True, M=2
# Initialize new model, optimizer, and loss function
model = SimpleCNN()
optim = torch.optim.Adam(model.parameters(), lr=0.001)
criterion = nn.CrossEntropyLoss()
# Check if GPU is available and move model and loss function to GPU if s
device = 'qpu' if torch.cuda.is available() else 'cpu'
if device == 'gpu':
    model = model.cuda()
    criterion = criterion.cuda()
# Generate the trainloaders for labeled and unlabeled data
# M is the number of unlabeled data points to use for training
# If M is 0, only labeled data will be used for training
trainloader labeled, trainloader unlabeled = generate trainloaders(trai
testloader = DataLoader(testset, batch size=64, num workers=2)
labeled losses = []
unlabeled augmentation losses = []
kmeans losses = []
valid loss = []
valid accuracies = []
for epoch in tqdm(range(epochs)):
    ### Labeled data training ###
    if trainloader labeled:
        for i, (labeled x, labeled y) in enumerate(trainloader labeled):
            labeled x = labeled x.to(device)
            labeled y = labeled y.to(device)
            z labeled, = model(labeled x, return logits=True)
            L_sup = w_labeled * criterion(z_labeled, labeled y)
            labeled losses.append(L sup.item())
            if i == 0:
                optim.zero grad()
            L sup.backward()
    if trainloader unlabeled:
        all logits = torch.tensor([]).view(0, 100).to(device)
        ### Unlabeled data augmentation ###
        for i, data in enumerate(trainloader unlabeled):
            # get current batch of unlabeled data and logits
            unlabeled x batch, = data # ignore labels
            _, logits_unlabeled = model(unlabeled_x batch, return logits
            all logits = torch.cat((all logits, logits unlabeled), 0) #
```

```
# transform the unlabeled data using weak and strong augment
        x weak, x strong = weak transform(unlabeled x batch), strong
        # get the final outputs for the weak and strong augmented da
        z_weak, _ = model(x_weak, return logits=True)
        z strong, = model(x strong, return logits=True)
        # If the model is confident in its guess for z weak
        # (i.e., the max logit is above TAU), use that as a pseudo-l
        # and compute the consistency loss between z strong and the
        pseudo values, = torch.max(nn.Softmax(dim=1)(z weak.detach
        pseudo labels = (pseudo values >= TAU).float().to(torch.long
        corresponding strong transforms = z strong[pseudo labels]
        # Treat the pseudo labels as the ground truth for the strong
        # and compute the consistency loss
        L consistency = criterion(corresponding strong transforms, p
        loss overall = L_consistency * w_augment
        unlabeled augmentation losses.append(L consistency.item())
        loss overall.backward()
    ### K-means clustering ###
    if run kmeans:
        logits cpu = all logits.detach().cpu().numpy()
        with torch.no grad():
            kmeans = KMeans(n clusters=10, n init=10).fit(logits cpu
            pseudo labels = torch.tensor(kmeans.labels ).to(torch.ld
            cluster centers = torch.tensor(kmeans.cluster centers ).
        # Compute the K-means loss using the cluster centers and the
        L kmeans = w kmeans * torch.mean((all logits - cluster cente
        kmeans losses.append(L kmeans.item())
        L kmeans.backward()
optim.step()
### Validation on the test set ###
with torch.no grad():
    total correct = 0
    total loss = 0
    for i, (test x, test y) in enumerate(testloader):
        test x.to(device)
        test y.to(device)
        z test = model(test x, return logits=False)
        z cats = torch.argmax(z test, dim=1)
        total correct += torch.sum(z cats == test y).item()
        total loss += criterion(z test, test y).item()
    valid loss.append(total loss)
    valid accuracies.append(total correct / len(testset))
```

```
torch.save(model.state_dict(), f'models/SSL_model_{M}_{w_kmeans}.pth')
return labeled_losses, unlabeled_augmentation_losses, kmeans_losses, val
```

Testing

For reproducibility purposes. Given a model and a dataset, return the total loss over the dataset and the proportion of correct predictions

```
In [41]:

def test(model, testloader, device, criterion=nn.CrossEntropyLoss()):

with torch.no_grad():
    total_correct = 0
    total_loss = 0

for i, (test_x, test_y) in enumerate(testloader):
        test_x.to(device)
        test_y.to(device)
        z_test = model(test_x, return_logits=False)
        z_cats = torch.argmax(z_test, dim=1)
        total_correct += torch.sum(z_cats == test_y).item()
        total_loss += criterion(z_test, test_y).item()

print(f"Proportion of correct predictions: {total_correct / len(test_print(f"Total_loss: {total_loss}")
```

Metrics

Plot L_{sup} , $L_{augment}$, L_{KMeans} over all iterations. Note that we calculate L_{sup} , $L_{augment}$ once per the whole batch, but we calculate L_{KMeans} once over the entire training set. We also plot the validation loss and accuracy per epoch

```
In [8]: def plot metrics(labloss, augloss, kloss, validloss, validcorrect):
            fig, axes = plt.subplots(1, 5, figsize=(25, 5))
            # Plot labeled loss
            axes[0].plot(np.arange(len(labloss)), labloss, label="Labeled Loss")
            axes[0].set title("Labeled Loss")
            axes[0].set xlabel("Iterations")
            axes[0].set ylabel("Loss")
            axes[0].legend()
            # Plot augmentation loss
            axes[1].plot(np.arange(len(augloss)), augloss, label="Augmentation Loss"
            axes[1].set title("Augmentation Loss")
            axes[1].set xlabel("Iterations")
            axes[1].set ylabel("Loss")
            axes[1].legend()
            # Plot K-means loss
            axes[2].plot(np.arange(len(kloss)), kloss, label="K-means Loss")
```

```
axes[2].set title("K-means Loss")
axes[2].set xlabel("Iterations")
axes[2].set ylabel("Loss")
axes[2].legend()
# Plot validation loss
axes[3].plot(np.arange(len(validloss)), validloss, label="Validation Los
axes[3].set_title("Validation Loss")
axes[3].set xlabel("Epochs")
axes[3].set ylabel("Loss")
axes[3].legend()
# Plot validation accuracy
axes[4].plot(np.arange(len(validcorrect)), validcorrect, label="Validati
axes[4].set title("Validation Accuracy")
axes[4].set xlabel("Epochs")
axes[4].set ylabel("Accuracy")
axes[4].legend()
plt.tight layout()
plt.show()
# Report maximum and final validation accuracy
max valid accuracy = max(validcorrect)
final valid accuracy = validcorrect[-1]
print(f"Maximum Validation Accuracy: {max valid accuracy}")
print(f"Final Validation Accuracy: {final valid accuracy}")
```

Ablation Studies

Here, we do 2 main experiments with the training.

In the first experiment, we adjust the size of the unlabeled dataset, M. We use the following values:

```
1. M=0 (Hence, all datapoints are labeled)
2. M=50
3. M=5000
4. M=40000
5. M=50000 (Hence all datapoints are unlabeled)
```

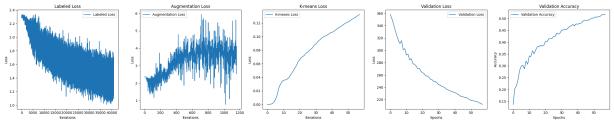
In the second experiment, we adjust the w_kmeans with respect to w_augment and w_labeled. This experiment tries to understand what happens when we emphasize w_kmeans more vs less, and how does this affect the kmeans loss over time. We use the following values

```
    w_kmeans=0.025, w_augment=0.25, w_labeled=1
    w_kmeans=0.25, w_augment=0.25, w_labeled=1
    w_kmeans=100, w_augment=0.125, w_labeled=0.5
```

Notice in the fist experiment, w_kmeans is small compared to w_augment and w_labeled. In the last experiment, w_kmeans is large compaired to w_augment and w_labeled

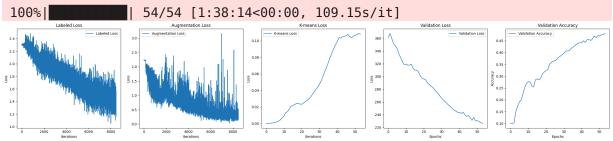
Experiment 1 - Adjusting M

```
device = 'gpu' if torch.cuda.is available() else 'cpu'
In [39]:
         model = SimpleCNN()
         testloader = DataLoader(testset, batch size=64, num workers=2)
In [ ]: lab0, aug0, kmeans0, validloss0, validcorrect0 = train(M=0, epochs=35)
         plot metrics(lab0, aug0, kmeans0, validloss0, validcorrect0)
         ## Reproduce validation results:
         # model.load state dict(torch.load(f'models/SSL model 0 0.25.pth'))
         # test(model, testloader, device)
                        | 35/35 [36:30<00:00, 62.60s/it]
                                       S 0.00
                                                       ğ 300
        Maximum Validation Accuracy: 0.4551
        Final Validation Accuracy: 0.4551
In [ ]: lab50, aug50, kmeans50, validloss50, validcorrect50 = train(M=50, epochs=31)
         plot metrics(lab50, aug50, kmeans50, validloss50, validcorrect50)
         ## Reproduce validation results:
         # model.load state dict(torch.load(f'models/SSL model 50 0.25.pth'))
         # test(model, testloader, device)
                         31/31 [31:09<00:00, 60.30s/it]
        100%
                                                                       0.30
        Maximum Validation Accuracy: 0.4504
        Final Validation Accuracy: 0.4479
In []: lab5000, aug5000, kmeans5000, validloss5000, validcorrect5000 = train(M=5000)
         plot metrics(lab5000, aug5000, kmeans5000, validloss5000, validcorrect5000)
         ## Reproduce validation results:
         # model.load state dict(torch.load(f'models/SSL model 5000 0.25.pth'))
         # test(model, testloader, device)
                       | 58/58 [1:04:58<00:00, 67.21s/it]
        100%|
```



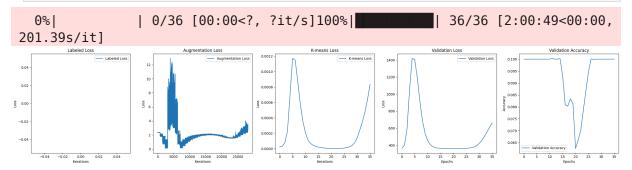
Maximum Validation Accuracy: 0.5177 Final Validation Accuracy: 0.5177

In []: lab40000, aug40000, kmeans40000, validloss40000, validcorrect40000 = train(P
plot_metrics(lab40000, aug40000, kmeans40000, validloss40000, validcorrect40
Reproduce validation results:
model.load_state_dict(torch.load(f'models/SSL_model_40000_0.25.pth'))
test(model, testloader, device)



Maximum Validation Accuracy: 0.4809 Final Validation Accuracy: 0.4809

test(model, testloader, device)



Maximum Validation Accuracy: 0.1003 Final Validation Accuracy: 0.1

Experiment 2 - Adjusting Loss Weights

In []: lab_xsmall, aug_xsmall, kmeans_xsmall, validloss_xsmall, validcorrect_xsmall
 plot_metrics(lab_xsmall, aug_xsmall, kmeans_xsmall, validloss_xsmall, validc

Reproduce validation results:

test(model, testloader, device) 35/35 [50:14<00:00, 86.13s/it] Maximum Validation Accuracy: 0.4567 Final Validation Accuracy: 0.4567 In []: lab small, aug small, kmeans small, validloss small, validcorrect small = tr plot metrics(lab small, aug small, kmeans small, validloss small, validcorre ## Reproduce validation results: # model.load state dict(torch.load(f'models/SSL model 20000 0.25.pth')) # test(model, testloader, device) | 0/35 [00:00<?, ?it/s]100%| | 35/35 [50:18<00:00, 8 0%1 6.25s/it] 0.35 Maximum Validation Accuracy: 0.4577 Final Validation Accuracy: 0.4577 In []: lab large, aug large, kmeans large, validloss large, validcorrect large = tr plot metrics(lab large, aug large, kmeans large, validloss large, validcorre ## Reproduce validation results: # model.load state dict(torch.load(f'models/SSL model 20000 100.pth')) # test(model, testloader, device) 35/35 [48:35<00:00, 83.30s/it] 0.30 Maximum Validation Accuracy: 0.4754 Final Validation Accuracy: 0.4754

model.load state dict(torch.load(f'models/SSL model 20000 0.025.pth'))

Conclusions

Overall Conclusions

The supervised losses and the augmentation losses are quite noisy as we sampled one loss per batch. On the other hand, for K-means loss, and the validation losses and accuracy we sampled one per epoch. Hence, the latter 3 metrics were a lot more stable. In addition, almost every model had a validation accuracy of around 0.475, which is much better than random chance (0.1).

Furthermore, for almost every model, the validation losses seem to be continuing on a downward trend after we stopped implying that many of our models likely hadn't yet converged. This is also evident in the fact that the maximum validation accuracy for most experiments is equal to the final validation accuracy. Thus, more training is likely required to get the true model validation accuracy.

Conclusions - Experiment 1

In terms of the augmentation loss, as the size of the unlabeled set increased, the change in training loss decreased. That is, if the size of the unlabeled set was small, then augmentation loss tended to increase more. If the size of the unlabeled set was large, the augmentation loss tended to decrease.

This is likely due to the fact that only data points $x \in D_u$ such that $\max_i p_{\theta}(i|x_{weak}) >= \tau$ were accounted for in the loss. For smaller unlabeled dataset, there were far less (x_{weak}, x_{strong}) pairs for the model to learn from. Hence, the losses tended to increase.

Another interesting result was the fact that in all of these experiments, the validation accuracy was greater than 0.4. That is, the model got at least 40% of the data points right on almost all experiments except for the final one. In the final experiment M=50000=|D|. In this experiment, the model only got 10% of its predictions correct, on par with random chance.

The reason for the above is that while our model can learn to create representations of the $x\in D$ such that the predictions are consistent under augmentation and the representations form tight clusters, there is still no supervision. Hence, what we might think of as a plane could really be anything as far as the model is concerned as it has no ground truth to work off of. Hence, the model for such a training set partition (50000 unlabeled points) will not result in a working model without some form of ground truth.

Conclusions - Experiement 2

The main takeaway from this experiment was that as we increased the weight for the KMeans loss, the KMean loss curve tended to flatten out more. When w_kmean = 0.025, the KMean loss never flattened, and only continued to increase. When w_kmean = 0.25, the KMean loss flattened out around epochs 10-20 and increased. When w_kmean = 100 and w_labeled = 0.5, w_augment = 0.125, the KMeans loss actually flattened out.

As we increased w_kmean, the derivative updates tended to favour L_{KMean} . Thus, our KMean objective flattened out (but didn't decrease). Interestingly enough, the rate of decrease for L_{sup} remained the same for all 3 experiments. This is likely because the 2 losses tend to go hand in hand.

Let $x \in D_u$, if $z_{\theta}(x)$ gives us a tight, clustered representation of x. Then, labeled classification becomes easier as well.

Let us assume $z_{\theta}(x)$ gives us a tight, clustered representation of x. Let x_1, x_2 be 2 images. Assume that $z_{\theta}(x_1)$ and $z_{\theta}(x_2)$ are close together. That is $\|z_{\theta}(x_1) - z_{\theta}(x_2)\|_2^2 < \epsilon$. Then, they are likely in the same category. Hence, the final layer of f_{θ} will likely classify both x_1 and x_2 in the same category. Most of the time, this is what we want. Hence why the supervised loss continues to decrease even when we emphasize the kmeans loss.

Conclusions - Main Takeaways

- 1. Almost all models (except M=50000) performed better than random chance
- 2. Direction of augmented loss changes as M increases
- 3. KMean loss tends to flatten as you increase its contribution to the main loss

Future Work

None of the models above have been run to convergence. Running one model to convergence will likely take 3-4 hours, maybe less if using more sophisticated hardware.

In addition, playing around with the model architecture. Seeing if a deeper model, or even a less deep model, could potentially improve the performance of the network.