Problem 3:

Matrix Multiplication, Image Processing, and Photo Filters

The goal of this problem is to understand how matrix operations can be used to manipulate images and create different visual effects, just like filters image editing tools. You are required to apply various mathematical transformations to an image using matrix operations. Through this, you will explore fundamental concepts of image processing, including colour manipulation, grayscale conversion, colour filtering, and inversion.

Instructions:

- Open and load an image as a matrix.
- Perform multiple image transformations using matrix operations.
- Analyse the effects of these transformations.
- Implement code based on the mathematical explanations provided.
- Submit your final implementation file.

3 images

R	G
B	W

$\Rightarrow$ 2x2 pixel

$$\begin{bmatrix} (255,0,0) & (0,255,0) \\ (0,0,255) & (255,255,255) \end{bmatrix}$$

$2 \times 2 \times 3$

h w ch

Task 1: Understanding Digital Images as Matrices

- A digital image is represented as a 3D matrix (tensor) with dimensions **(height, width, colour channels)**.
- Each pixel in a **colour image** has **three values (R, G, B)** representing red, green, and blue intensity levels, ranging from 0 to 255.

Task 2: Extracting Image Information

- **Checking the image dimensions**: Extracting the height, width, and number of colour channels.
- **Extracting colour channels**: Splitting the image into separate **red, green, and blue** components and displaying each.

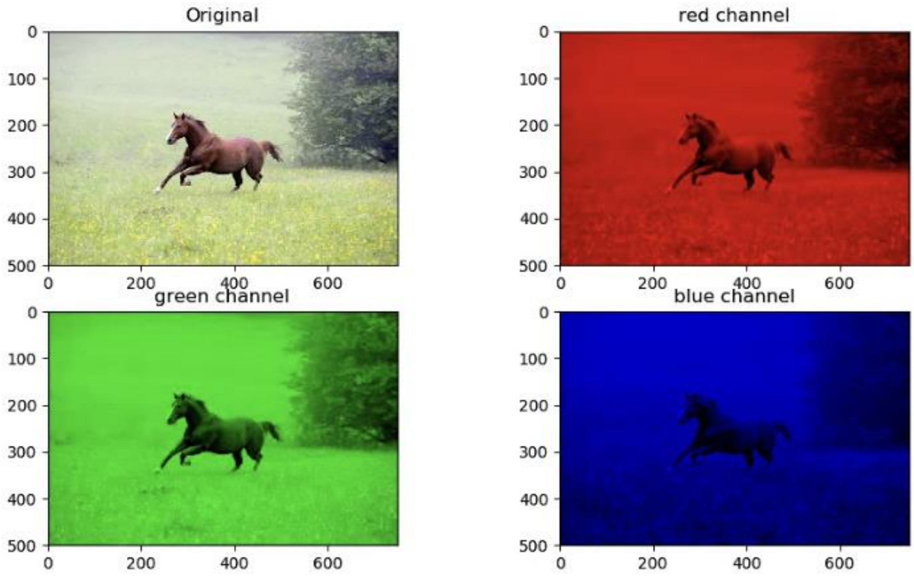

Image Transformations Using Matrix Multiplication

Key Concept: Matrix multiplication allows us to apply transformations to pixel values in an image.

Task 3:

Grayscale conversion: By averaging the R, G, and B values, we create a grayscale image where all pixels have equal R, G, and B intensities.

- Mathematically, we use a matrix multiplication:

$$\text{GrayMatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

- This ensures that each pixel's colour information is averaged to produce a black-and-white version.

Task 4:

Sepia filter: A filter that gives a brownish tint to an image, often seen in old photographs.

- The transformation matrix is:

$$\text{SepiaMatrix} = \begin{bmatrix} 0.393 & 0.769 & 0.189 \\ 0.349 & 0.686 & 0.168 \\ 0.272 & 0.534 & 0.131 \end{bmatrix}$$

Task 5:

Red Channel Extraction: To isolate the red component of an image, we use:

$$\text{RedMatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Removes all green and blue values, keeping only the red tones.

Task 6:

Colour Permutation: Swapping colour channels using:

$$\text{PermuteMatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- This shifts Red → Blue, Blue → Red, and Green stays the same.

Task 7:

Deleting a colour (e.g., removing green):

$$\text{DeleteGreenMatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- This keeps Red and Blue while setting Green to 0.

Image Inversion and Colour Adjustments

Task 8:

Colour Inversion: Subtracting the pixel values from 255 gives a negative effect.

$$\text{Inverted Image} = 255 - \text{Original Image}$$

Task 9:

Saturation Modification: Enhancing certain colours using:

$$\text{SaturateMatrix} = \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- This increases red and blue while muting green.

Task 10:

User-defined colour adjustments:

$$\text{UserMatrix} = \begin{bmatrix} 0.7 & 0.15 & 0.15 \\ 0.15 & 0.7 & 0.15 \\ 0.15 & 0.15 & 0.7 \end{bmatrix}$$

- This balances colours without completely removing any.

Task 11: Matrix Inverses and Reversibility

- Some transformations are **invertible** (we can undo them by applying the inverse matrix).
- The inverse of a matrix **M** is $\mathbf{M^{-1}}$, and applying it to the transformed image should restore the original.
- Some filters like **sepia** may not be perfectly reversible due to **information loss**.

What does the inverse transformation do with the colours?

Inverse Transformation Effect: It adjusts the colour values in such a way that, ideally, the resulting image resembles the original image more closely in terms of colour balance and intensity.

Purpose: The goal of applying the inverse transformation is to demonstrate whether the original image colours can be reasonably recovered after applying a colour adjustment filter (UserMatrix). This helps in understanding whether the transformation is reversible and how much information might have been altered or lost during the initial colour adjustment process.

Task 12:

Gamma Correction (Non-Linear Transformation)

- **Gamma transformation adjusts brightness**:

$$\text{Gamma1} = (\text{Image})^{0.9} + 30$$

  - Makes the image **brighter**.

$$\text{Gamma2} = (\text{Image})^{1.1} - 50$$

  - Makes the image **darker**.