\nearrow An arithmetic function f(n) : N \rightarrow C is multiplicative if for any relatively prime n, m \in N: f(mn) = f(m)f(n).

The following are further examples of well-known multiplicative functions.

- $\mu(n)$, the Möbius function -
- $\rightarrow e(n) = 1 \longrightarrow n=1$ • $e(n) = \delta_{1,n}$, the Dirichlet identity –
- I(n) = 1 for all $n \in \mathbb{N}$
- id(n) = n for all $n \in \mathbb{N}$

| | 1 | |
|--------|------|---------------------------|
| H(U) = | ٥ | $\longrightarrow p^2 \ln$ |
| | (-1) | |

6(u) = 0 -

$$\longrightarrow F(n) = \sum_{d \mid n} f(d)$$

| f(d) | 7 | e | I | Ы |
|------|---|---|---|---|
| F(n) | 9 | I | τ | 6 |

* Möbius Function

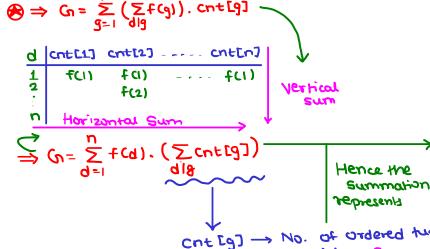
 \rightarrow Thm: $f(n) \rightarrow$ multiplicative $\iff F(n) = \sum_{d \mid n} f(d)$ is multiplicative

*Möbius Inversion Theorem:

$$\longrightarrow F(n) = \sum_{d \mid n} F(d) \implies f(n) = \sum_{d \mid n} \mu(d) F(\frac{n}{d})$$

> Code to find Möbius func of 1 to N

> Once We have the values of f(n)



* General Steps to Solve:

- 1. Let the problem be to find $G = \sum_{i=1}^n \sum_{j=i+1}^n h(gcd(i,j))$, here h(n) should be a multiplicative function.
 - For example if the problem was to find $G = \sum_{i=1}^n \sum_{j=i+1}^n gcd(i,j)^3$, then the function h() will be $h(n) = n^3$.
- 2. Re-write the equation like this: $G = \sum_{g=1}^n h(g) * cnt[g] \longrightarrow igoplus$ Where cnt[g] = number of pairs (i, j) such that gcd(i, j) = g, (1 <= i < j <= n).
 - 3. Find the function f(n), such that $h(n) = \sum_{d|n} f(d)$. This can be done using mobius inversion formula and sieve.

— In case of an given array and to find:

- 1) No of coprime pairs
- Sum of gcd for all pairs

for (int i = 1; i <= N; i++) {</pre> for (int j = i; $j \le N$; j += i) { f[j] += h[i] * mobi[j/i];}

No of ordered tuples 3 3 When property applied to the tuple we get a multiple of d.

Cnt [g] -> No. of ordered tuples
satisfying some property given in question.

* Typical Questions:

ZZ h(property) = G