Game Theory

Theory of Rational Choice

The decision maker chooses the best action acc. to her pref. among the actions available to her.

① Set A of actions ② Preference

$a_1, a_2, a_3 \in A$

Ranking

R_1	R_2
a_2	a_1, a_3
a_1	a_2
a_3	

Consistency

$\succ_i$ (Preferred by i)

$\sim_i$ (indifferent by i)

→ If $a_1 \succ_i a_2$ and $a_2 \succ_i a_3$

Then $a_1 \succ_i a_3$

→ If $a_1 \sim_i a_2$ and $a_2 \sim_i a_3$

Then $a_1 \sim_i a_3$

★ Payoff Function / Utility Function

$F : A \rightarrow \mathbb{R}$ (maps actions to real numbers)

→ Principle of ordinal pref followed by TORC

Hence absolute value doesn't matter.

For i $a \succ_i b \Leftrightarrow u_i(a) > u_i(b)$

$u_i(\,)$: payoff function

★ Person P:

C1: {a, b} : always chooses a

C2: {a, b, c} : chooses a or b

Violates Theory of rational choice (# not consistent)

★ Cardinal Functions makes the absolute value matter.

★ In game Theory payoff function not determined by action of one person alone.

* Actions:

For player i he has A_i set of actions. Any generic action is denoted by $a_i \in A_i$

* Action Profile:

$a_1 \in A_1, a_2 \in A_2, \ldots, a_i \in A_i$

$(a_1, a_2, \ldots, a_n)$: Action Profile

$(a_i, -a_i)$ $-i$ = other than i

Players rank these Action Profiles

* Strategic Games: (Simultaneous move game)

① Set of players : $n \geq 2$

② For each player set of actions

$A_1, A_2, \ldots, A_n$

③ Preferences are defined over action profile

$A_1 = \{a_1, a_2\}$; $A_2 = \{b_1, b_2, b_3\}$

Then no. of action profiles are 6.

$u_i : A_1 \times A_2 \times \ldots \times A_n \longrightarrow \mathbb{R}$

map a real number to an action profile.

$u_i(a_1, a_2, \ldots, a_n)$

Type 1: 1 Prisoner's Dilemma:

Robbery

2 criminals

	P_1	P_2
Both confess	3	3
None confess	1	1
P_1 confess only	0	5
P_2 confess only	5	0

$A_1 = A_2 = \{C, NC\}$

$u_{P_1}(C, NC) > u_{P_1}(NC, NC) > u_{P_1}(C, C) > u_{P_1}(NC, C)$

3 2 1 0

$u_{P_2}(NC, C) > u_{P_2}(NC, NC) > u_{P_2}(C, C) > u_{P_2}(C, NC)$

3 2 1 0

* Payoff Matrix / Table

Player 2

Player 1	C	NC
C	1, 1	3, 0
NC	0, 3	2, 2

* If both co-operate (NC, NC) then one might deviate to confess.

* FREE RIDING: Taking advantage of other person

② Duopoly

Players : 2 Products / seller

Actions : $\{P_H, P_L\}$

Preferences :

$u_1(P_L, P_H) > u_1(P_H, P_H) > u_1(P_L, P_L) > u_1(P_H, P_L)$

★ Both ↑ prices prefered as ↑ profit margin.

1 \ 2	P_H	P_L
P_H	1000, 1000	-200, 1200
P_L	1200, -200	500, 500

★ If both co-operate to set high price one might deviate to set low price. (#TORC)

$(P_H, P_H) = (NC, NC)$

③ Joint Project

Players 2

Actions : {WH, GO}

GO = Goof off

WH = Work Hard

1 \ 2	WH	GO
WH	2, 2	0, 3
GO	3, 0	1, 1

★ You are better off when you goof off.

(WH, WH) = (NC, NC)

Scanned with CamScanner

Date
Page

(*) Prisoner's Dilema can be applied in many real li
situations

(4) Arm's Race
2 countries
Actions = { Build Bomb (B), Refrain (R) }

	B	R
B	1,1	3,0
R	0,3	2,2

(R,R) ≈ (NC,NC)

(5) Common property
Players : N villagers with sheeps
Actions : {HG, LG}

HG = High grazing
LG = Low grazing

→ All sheeps ↑ graze then ∃ some problem for regeneration of grass

Preferences:

$u_1(HG, LG, \ldots, LG) > u_1(LG, LG, \ldots, LG) >$
$u_1(HG, HG, \ldots, HG) > u_1(LG, HG, \ldots)$

Similar to prisoner's Dilema

If all go for LG then I freeride over them to choose HG

Type 2: 1 BATTLE OF SEXES

Players : Husband (H) , Wife (W)

Actions : {Boxing (B), Opera (O)}

Preferences :

Husband likes (B,B) over (O,O) opp for wife.

$$\underset{2}{u_H(B,B)} > \underset{1}{u_H(O,O)} > \underbrace{u_H(B,O) = u_H(O,B)}_{\text{If they are not together they donot enjoy}} \quad 0$$

$$\underset{2}{u_W(O,O)} > \underset{1}{u_W(B,B)} > \underset{0}{u_W(B,O)} = \underset{0}{u_W(O,B)}$$

W

H	B	O
B	2,1	0,0
O	0,0	1,2

★ Here co-operation is better for them

But which of (B,B) , (O,O) is better is not clear

→ BoS ∈ Coordination Game

★ Pure Co-ordination

	X	Y
X	1,1	0,0
Y	0,0	1,1

★ Assurance

	X	Y
X	1,1	0,0
Y	0,0	2,2

Date

Page

★ Pareto Improvement

Increase payoff of atleast one player without reducing payoff of the other player.

(O,B) → (B,B) ✓

(B,B) → (O,O) ✗

(C,C) → (NC,NC) ✓ ((NC,NC) is pareto superior to (C,C))

Type 3: ① Matching Pennies

2 players

$A_1 = A_2 = \{H, T\}$

Player 1 wins if pennies match else player 2 wins

(Loser gives winner 1 rupee)

$$\underbrace{u_1(H,H) = u_1(T,T)}_{1} > \underbrace{u_1(H,T) = u_1(T,H)}_{-1}$$

$$u_2(H,T) = u_2(T,H) > u_2(H,H) = u_2(T,T)$$

→ Also known as ZERO SUM game

1 \ 2	H	T
H	1,-1	-1,1
T	-1,1	1,-1

Pure conflict

Strictly competitive Game

No pareto improvement

classmate
Date ________
Page ________

② Cricket

2 players : Batsman, Bowler

Bowler : {Fast (F), Slower (S)}

Batsman: {Fast (F), Slower (S)}

Batsman like player 1 in matching pennies

he wins if he expected correct type of ball.

Type 4: ① Stag Hunt

n Players (Hunters) → all are partners

Action: {Stag (S), Heir (H)}

Purse the Stag or go after the Heir that comes in their way

$\frac{1}{n}$ Stag > 1 heir

If $n = 2$

$u_1(S,S) > u_1(H,S) = u_1(H,H) > u_1(S,H)$

gets half the stag — gets heir — gets heir — Left alone to purse the stag

$u_2(S,S) > u_2(S,H) = u_2(H,H) > u_2(H,S)$

2 — 1 — 0

★ Varient of Prisoner's Dilemma Game

		2: S	H
1	S	2,2	0,1
	H	1,0	1,1

Pareto improvement

* Solution Concept

(i) Nash Equilibrium.

→ Theory of Rational Choice

In game theory what is best for me depends on other players.

→ Beliefs of players regarding other players actions are correct

(Beliefs)

* Expectations are formed from experiences and expectations are co-ordinated.

Suppose 1 takes action a given 2 takes action b
|||ly 2 takes action b given that 1 takes action

$$\begin{array}{ccc} 1 & & 2 \\ a & \longleftarrow & b \\ a & \longrightarrow & b \end{array}$$

(Hence equilibrium)

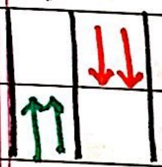
* Equilibrium is a steady state stable outcome.

★ Social Convention:

Other people have been behaving in a particular way and hence we take action according to that hence it reinforces actions of other people.

Traffic

A



B

★ $u_i(a)$

→ a = action profile (vector of actions)

→ a^* is a Nash Equilibrium if for each player i playing a_i^* is best compared to all other actions of i (according to her pref) given that any other player j is playing a_j^*

eg: $a^* = (a_1^*, a_2^*)$

given that 2 is taking a_2^* then a_1^* is the best action of player 1 III'r for player 2 a_2^* is the best action given player 1 is taking a_1^*.

* a^* is NE if for each i

$u_i(a_i^*, a_{-i}^*) = u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ for every $a_i \in A_i$

A_i = set of actions of player i

$-i$ = players other than i

→ n players & each with m actions

$n(m-1)$ equations

* value not compared to itself.

* Unilateral deviation from * actions is unprofita

→ Every game doesn't have NE

→ Every finite game have atleast one NE

→ Multiple Nash equilibria possible

* Prisoners Dilemma

1 \ 2	NC	C
NC	2,2	0,3
C	3,0	1,1

(NC, NC) X (not NE)

→ If player 2 plays NC one can deviate to C

(C,NC) X (not NE) → If player 1 play player 2 devi

(NC,C) X (not NE)

* (C,C) is NE

* Pareto Superior State maynot be a NE.

(NC, NC) Pareto Superior to (C, C)
× NE ✓ NE

* Joint Project: (GO, GO)
Duopoly: (P_L, P_L)
Arm's Race: (B, B)
} NE

Common Property: (HG, HG) } NE

* Battle of Sexes:

		2	
		B	O
1	B	2,1	0,0
	O	0,0	1,2

(B, O)
(O, B)
} Not NE

* (B, B) : NE
* (O, O) : NE

Wife goes to Opera
Husband tends to deviate

* Matching Pennies:

		2	
		H	T
1	H	1,-1	-1,1
	T	-1,1	1,-1

(H, H)
(T, T)
(H, T)
(T, H)
} Not NE

★ Stag Hunt

		2: S	2: H
1	S	2,2	0,1
	H	1,0	1,1

$\left.\begin{array}{l}(S,H)\\(H,S)\end{array}\right\}$ Not NE

$\left.\begin{array}{l}\star(S,S)\\\star(H,H)\end{array}\right\}$ NE

★ If NE is at (S,S) then better for both the player