* Stag Hunt

		2	
		S	H
1	S	2,2	0,1
	H	1,0	1,1

(S,H)
(H,S) } Not NE

*(S,S)
*(H,H) } NE

* If NE is at (S,S) then better for both the players

* Altruism

$m_i(a)$: money recieved by doing action a

$u_i(a) = m_i(a) + \alpha m_j(a)$

$\alpha > 0$: Altruism

$\alpha = 0$: Selfish

(Players not selfish ⇒ no longer ordinal payoffs)

	C	NC
C	1,1	3,0
NC	0,3	2,2

If $\alpha = 1$

	C	NC
C	2,2	3,3
NC	3,3	4,4

Not PD game

	C	NC
C	$1+\alpha$, $1+\alpha$	$3, 3\alpha$
NC	$3\alpha, 3$	$2+2\alpha$, $2+2\alpha$

To be PD : $2+2\alpha > 1+\alpha \Rightarrow \alpha > -1$

$3 > 2+2\alpha \Rightarrow \alpha < 1/2$

$2+2\alpha > 3\alpha \Rightarrow \alpha < 2$

At $\alpha = 1/2$ All four Action profiles NE PD if $\alpha < 1/2$

★ Co-ordination Game

	B	O
B	2,2	0,0
O	0,0	1,1

NE (B,B) , (O,O)

Pareto inferior state may prevail.

★ Querty Paradox

Most used letters e,a,r,s on left still same Keyboard design followed. (Pareto inferior NE)

★ Focal point

It indicates to everyone which NE to choose
(In BOS the focal point is where they went on their previous date.)

★ It is not true that all zero sum games have no NE.

★ Focal point is the NE which is more likely to occur among all NE

★ Game of Chicken (Hawk & Dove game)

Players: 2

Actions $A_1 = A_2 = \{A, P\}$ — P: Passive, A: Agressive

Pref: $u_1(A,P) > u_1(P,P)$

$u_1(P,A) > u_1(A,A)$

$u_1(P,A) < u_1(P,P)$

$\Rightarrow u_1(A,P) > u_1(P,P) > u_1(P,A) > u_1(A,A)$

★ Strict Nash Equilibrium:

Deviation causes a neccessary drop in payoff.

→ a^* is SNE if $\forall i$

$u_i(a_i^*) > u_i(a_i', a_{-i}^*)$ for $a_i' \in A_i$

and $a_i' \neq a_i^*$

NE

SNE

	x	y
x	1,1	0,1
y	1,1	3,2

(x,x) – NE

(y,y) – NE & SNE

★ Best Response function

$B_i(a_{-i}) = \{a_i \in A_i : u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \; \forall a_i' \in A_i\}$

a_{-i} : action profile of players other than i

→ a^* is NE if $\forall$ player i

$a_i^* \in B_i(a_{-i}^*)$

★ PD

	C	NC
C	$1^*, 1^*$	$3^*, 0$
NC	$0, 3^*$	$2, 2$

NE (C,C)

★ BOS

	B	O
B	$2^*, 1^*$	$0, 0$
O	$0, 0$	$1^*, 2^*$

NE (B,B)
(O,O)

★ MP

	H	T
H	$1^*, -1$	$-1, 1^*$
T	$-1, 1^*$	$1^*, -1$

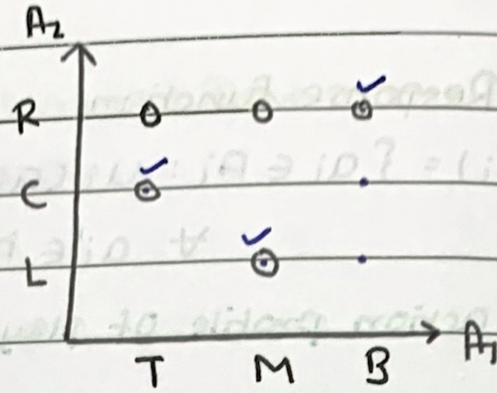
NE : X

★ Stag Hunt

	S	H
S	$2^*, 2^*$	$0, 1$
H	$1, 0$	$1^*, 1^*$

NE (S,S)
(H,H)

*	L	C	R
T	2,2	1*,3*	0*,1
M	3*,1*	0,0	0*,0
B	1,0*	0,0*	0*,0*



$\bullet : A_2$ } Best Response Function.
$0 : A_1$

★ $B_1(L) = \{M\}$ ★ $B_2(T) = \{C\}$
$B_1(C) = \{T\}$ $B_2(M) = \{L\}$
$B_1(R) = \{T, M, B\}$ $B_2(B) = \{L, C, R\}$

★ NE : (M, L) ; (T, C) , (B, R)

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★ Dividing Money

→ Rs 10 to be divided among 2 players P_1, P_2

$A_1 = A_2 = \{0, 1, 2, \ldots, 10\}$

If $a_1 = a_2$ each gets Rs 5

$a_1 + a_2 \leq 10$ each gets what they claimed

$a_1 + a_2 > 10$ smaller gets what claimed (x)

& other gets $(10 - x)$

★ $B_1(0) = \{10\}$

$B_1(1) = \{9, 10\}$

$B_1(2) = \{8, 9, 10\}$

$B_1(3) = \{7, 8, 9, 10\}$

$B_1(4) = \{6, 7, 8, 9, 10\}$

$B_1(5) = \{5, 6, 7, 8, 9, 10\}$

$B_1(6) = \{5, 6\}$

$B_1(7) = \{6\}$

$B_1(8) = \{7\}$

$B_1(9) = \{8\}$

$B_1(10) = \{9\}$

★ NE:

→ $(5,5)$; $(5,6)$

→ $(6,5)$; $(6,6)$

★ Synergic Relationship

$u_1(a_1, a_2) = a_1(a_2 - a_1)$

$u_2(a_1, a_2) = a_2(1 - a_1 - a_2)$

$a_1, a_2 \geq 0$

→ $B_1(a_2) \star \max(u_1(a_1, a_2))$

$$\left.\begin{aligned}\frac{\partial u_1(a_1,a_2)}{\partial a_1} = -2a_1 + a_2 = 0\end{aligned}\right\}\text{First order Condition}$$

$$\Rightarrow \boxed{a_1 = a_2/2}$$

$$\left.\frac{\partial^2 u_1(a_1,a_2)}{\partial a_1} = -2 < 0\right\}\text{Second order Condition}$$

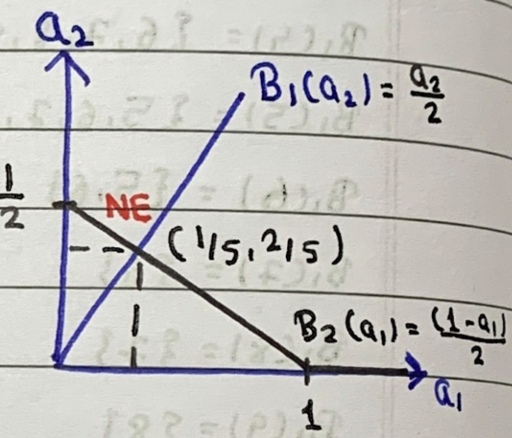
Hence maxima

→ $B_2(a_1) = \max(u_2(a_1, a_2))$

$$\frac{\partial u_2(a_1,a_2)}{\partial a_2} = (1 - a_1) - 2a_2 = 0$$

$$\Rightarrow \boxed{a_2 = \frac{1}{2}(1 - a_1)}$$

$$\frac{\partial^2 u_2}{\partial a_2} = -2 < 0 \quad \text{Hence maxima}$$



$NE = (a_1^*, a_2^*) = (1/5, 2/5)$

★ Synergic relation for P1 as P2 puts more effort P1 also does.

* Players 1 & 2

$w_i \geq 0$ (wealth)

$w_i \geq c_i \geq 0$ → Contribution for public good

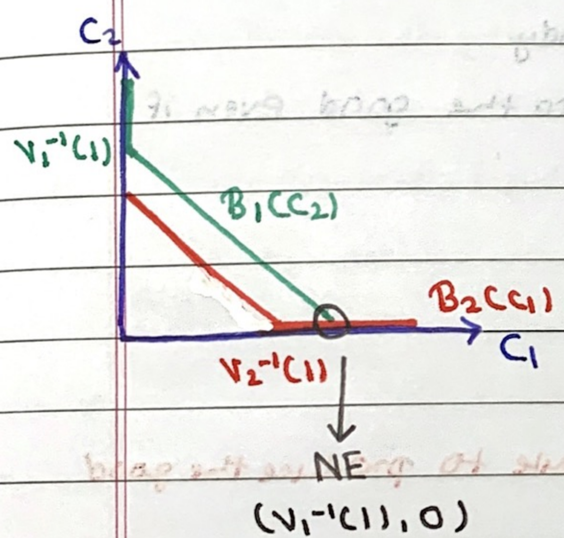
$$u_i(c_1, c_2) = \underbrace{v_i(c_1 + c_2)}_{\text{Public good}} + \underbrace{w_i - c_i}_{\text{money left (private)}}$$

v_i strictly concave func.

$B_1(c_2)$ is when $c_1 + c_2 = v_1^{-1}(1)$

$c_1 = B_1(c_2) = v_1^{-1}(1) - c_2$

|||'y $B_2(c_1) = v_2^{-1}(1) - c_1$



∵ c_2 cant be negative

∴ $B_2(c_1) = c_2 = 0$ for $c_1 > v_2^{-1}(1)$

|||'y for $B_1(c_2) = c_1 = 0$

for $c_2 > v_1^{-1}(1)$

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★ Stag Hunt

n hunters, m hunters are required to hunt the stag

$m > 1$, k is a value such that $\frac{1}{k}$ Stag > 1 Hare &

$\frac{1}{k+1}$ Stag < 1 Hare

→ Total no. of NE : $\binom{n}{k} + 1$

(H, H, H, . . . H) : NE

$\left.\begin{array}{l}\text{k go after stag}\\\text{and rest after hare}\end{array}\right\}$ NE

★ Public Goods

① Your use doesn't restrict anybody

② Nobody can restrict you to use the good even if you have not contributed

→ n people

$A_i = \{C, NC\}$

C ↓ Contribute, NC ↳ Not Contribute

atleast k people must contribute to produce the good

NE : $\binom{n}{k} + 1$

Good produced — Contribute a / Not Contribute b

Good Not produced — Contribute c / Not Contribute d

$u_i(b) > u_i(a) > u_i(d) > u_i(c)$

P3 = C

P1 \ P2	C	NC
C	5,5,5	3,6,3
NC	6,3,3	4,4,1

P3 = NC

P1 \ P2	C	NC
C	3,3,6	1,4,4
NC	4,1,4	2,2,2

$\therefore$ NE is (NC, NC, NC)