★ Dominated Actions and Dominating Actions

	Traffic Jam	No Traffic Jam
Boat	1/4	1/4
Bus	$1\frac{1}{2}$	1

Payoff $\propto$ 1/time

Time in hrs

→ Bus is strictly dominated by the boat

→ Boat is dominating (strictly) over Bus

★ Def:

For player i action a_i' is strictly dominated by the action a_i'' $(a_i', a_i'' \in A_i)$

if $u_i(a_i', a_{-i}) < u_i(a_i'', a_{-i})$ for all possible a_{-i}

a_{-i}

	x	y	z
a	1	0	2
b	2	1	4
c	1	2	3

Payoff of player 1

★ b dominating over (strictly dominating) action a

★ A strictly dominated action is not played in a Nash Equilibrium

	C	NC
C	1,1	3,0
NC	0,3	2,2

★ C strictly dominates over NC for both the players

NE : (C, C)   strictly dominated actions : NC , NC

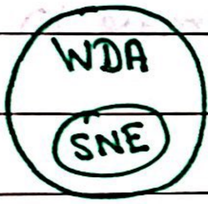
* Def:

For player i, action a_i' is weakly dominated by action a_i'' ($a_i', a_i'' \in A_i$)

if $u_i(a_i', a_{-i}) \leq u_i(a_i'', a_{-i})$ for all possible a_{-i}

and $u_i(a_i', a_{-i}) < u_i(a_i'', a_{-i})$ for atleast one a_{-i}.

* A weakly dominated action can be played in a NE.
* A weakly dominated action cannot be played in strict NE.



* Example where only NE action is NOT Weakly Dominated

	A	B
A	2, 2	1, 3
B	0, 0	1, 1 → NE

P1 : B is weakly dominated by A (Check rows)

P2 : A is weakly dominated by B (Check columns)

* Voting: (No abstain allowed i.e. all vote)

→ n is odd: no. of people

More people support A than B

Action profile n element vector

i is a supporter of A:

* leave i aside

Case 1:

① There is a tie

② Voting for A is strictly preferable to voting for B

Case 2:

① A or B wins by 2 or more votes (# n-1 is even)

In each case i is indifferent of voting A or B

(Hence B is weakly dominated by A for supporter of A)

(Hence A is weakly dominated by B for supporter of B)

* ∃ a equilibrium where no player uses a weakly dom action.

Everyone votes for prefered candidate.

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* Maximin Method (Zero Sum Game)

The more you get the less the other player gets.

* If P1 selects a row P2 tries to minimize
If P_2 selects a column P_1 tries to maximise

If Max of Min == Min of Max
then it is NE

	L	M	R	Min
T	2	5	13	2
H	6	5.6	10.5	5.6
L	6	4.5	1	1
B	10	3	-2	-2
Max	10	5.6	13	

→ P1's payoff

$u_{p_1}(a) = -u_{p_2}(a)$

∴ (H,M) is NE

$$\max_{a_{1i}} \left(\min_{a_{2j}} u_1(a_{1i}, a_{2j}) \right) = 5.6$$

$$\min_{a_{2j}} \left(\max_{a_{1i}} u_1(a_{1i}, a_{2j}) \right) = 5.6$$

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★ Iterated elimination of strictly dominated actions

	L	M	R
U	1,0	1,2	0,1
~~D~~	~~0,3~~	~~0,1~~	~~2,0~~

③ ① ②

For 2 : M dominates strictly over R
For 1 : U strictly dom over D
For 2 : M strictly dom over L

③ ② ①
P_2 knows that P_1 knows that P_2 is rational

★ In such a case all NE obtained : (U,M) : NE
★ BOS no dominated action nothing gets eliminated.

★ Dominance Solvability
Suppose all weakly dominated actions of each player are eliminated at each stage. If all players are indifferent between all the action profiles that survive the game is called dominance solvable.

	L	M ③	R ②
U	2,2	8,1	1,0
~~D~~	~~2,1~~	~~2,2~~	~~0,2~~

①

1 : U WDom D
2 : M WDom R
2 : L WDom M

	L ①	M	R ②
U	1,1	1,1	0,0
D	0,0	1,2	1,2

2 : M WDom L
2 : M WDom R

★ Hence it is not Dominance Solvable

classmate

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★ Rationalizable Actions

	C_1	C_2	C_3	C_4
R_1	$0, 7^*$	$2, 5$	$7^*, 0$	$0, 1$
R_2	$5, 2$	$3^*, 3^*$	$5, 2$	$0, 1$
R_3	$7^*, 0$	$2, 5$	$0, 7^*$	$0, 1$
R_4	$0, 0^*$	$0, -2$	$0, 0^*$	$10^*, -1$

$R_1 = B_1(C_3)$
$C_1 = B_2(R_1)$
$R_3 = B_1(C_1)$
$C_3 = B_2(R_3)$
$R_1 = B_1(C_3)$

Cycle found here
Hence R_1, R_3, C_1, C_3 are rationalisable

★ NE is always rationalisable

$R_2 = B_1(C_2)$ & $C_2 = B_1(R_2)$ ⇒ R_2, C_2 rationalisable

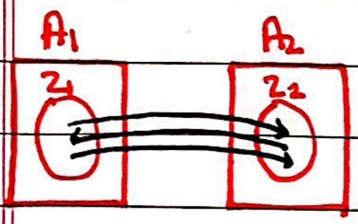
$R_4 = B_1(C_4)$ But C_4 is not a best response of player ever } R_4, C_4 not rationalisable.

★ Rationalisable actions in strategic game

The action a_i^* of Player i in a strategic game is rationalisable if for each player j ∃ a set Z_j of actions ∋

(a) Z_i contains a_i^*

(b) For every player j, every action a_j in Z_j is a best response to only list of actions in Z_{-j}



★ Symmetric Game & Symmetric Equilibria.

2 players

$A_1 = A_2$

$u_1(a_1, a_2) = u_2(a_2, a_1)$

	X	Y
X	α, α	Δ, δ
Y	δ, Δ	β, β

$u_1(a, a) = u_2(a, a)$ (IF $a_1 = a_2 = a$)

→ Symm Equilibrium

a^*: NE is a symm Nash Eq. if for both players

$A_1 = A_2$ & $\underline{a_1^* = a_2^*}$

($\therefore$ SNE can only be located on diagonal!

★ A non symmetric game can have a symmetric equilibrium

→ If NE of symm game is off diagonal then it is difficult to justify why those actions were played.

	X	Y
X	0,0	1,1
Y	1,1	0,0

NE: (X,Y), (Y,X)

★ Applications of Nash Equilibrium:

① Cournot Model

Firms: Players

Actions: Produces quantities $q_i (\geq 0)$

Preferences: maximise profit π_i

Profit = Total Revenue − Total cost

$= q_i \times \text{Price} - \underline{c_i(q_i)}$

cost function as a function of q_i

$[c_i'(q_i) > 0]$

$\pi_i = q_i P - c_i(q_i)$

★ Demand Function (Demand here is Q^d)

$Q^d = F(P)$ $(F'(P) < 0$ ∵ Q^d inversely related to Price$)$

$F^{-1}(Q^d) = f(Q^d) = P$ $(f'(Q^d) < 0)$

In equi $Q^s = \sum_{i=1}^{n} q_i$

$\therefore P = f(Q^d)$

$= f\left(\sum_{i=1}^{n} q_i\right)$ (∵ At equi $Q^s = Q^d = Q$) (Supply = Demand)

$\therefore \pi_i = q_i f(q_1 + q_2 + \dots + q_n) - c_i(q_i)$

$f(Q)$

↓ Inverse demand function.

★ $n = 1$: Monopoly

$n = 2$: Duopoly

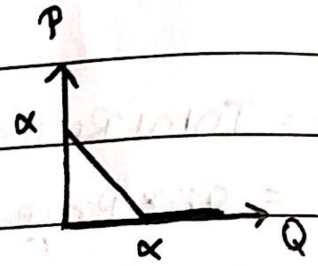
($n > 1$ small n) : Oligopoly

$n \to \infty$: Perfect competition

★ Duopoly : $C_i(q_i) = cq_i$, $c > 0$ (Both firms same cost per unit production

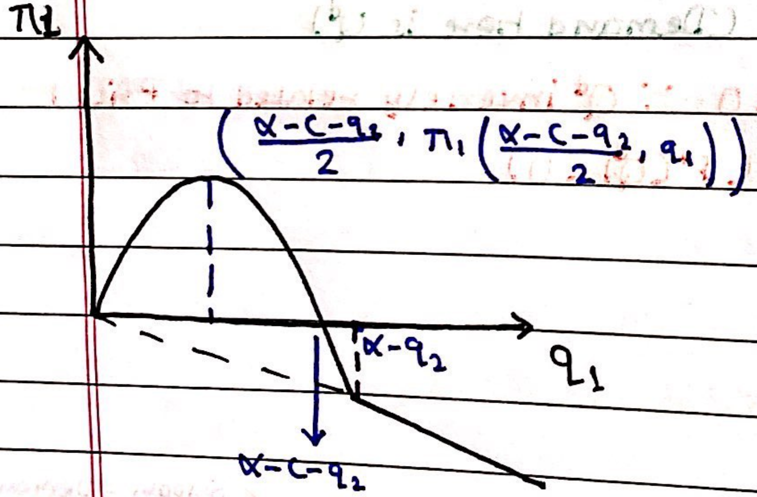
★ Inverse Demand Function:

$$P = f(Q) = \begin{cases} \alpha - Q & \alpha \geq Q \\ 0 & \alpha < Q \end{cases}$$



★ α : willingness of customers to pay

$$\pi_1(q_1, q_2) = \begin{cases} q_1(\alpha - (q_1 + q_2)) - cq_1 & \alpha \geq q_1 + q_2 \\ = -cq_1 & \alpha < q_1 + q_2 \end{cases}$$



$\star$ $B_1(q_2) = \max(\pi_1(q_1, q_2))$

$\therefore\ q_1(-1) + (\alpha - c - q_2 - q_1) = 0$

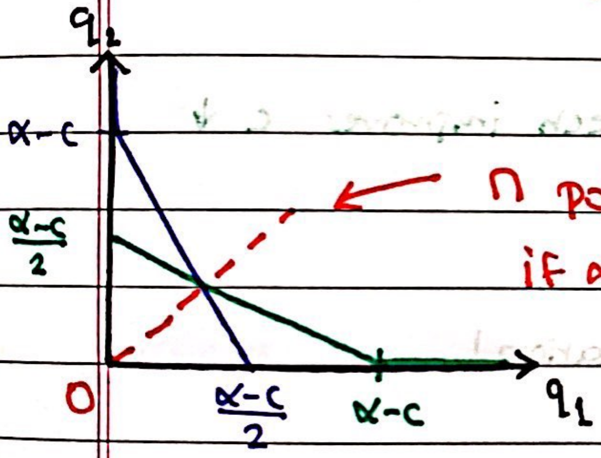
$\Rightarrow q_1^* = B_1(q_2) = \frac{\alpha - c - q_2}{2}$ if $q_2 \le \alpha - c$

$= 0$ $\qquad q_2 > \alpha - c$

III'ly $q_2^* = B_2(q_1)$

$= \frac{\alpha - c - q_1}{2}$ if $q_1 \le \alpha - c$

$= 0$ $\qquad q_1 > \alpha - c$



Π point along the line.

if $\alpha \uparrow$ graph transposes along the line.

$\therefore (q_1^*, q_2^*) = \left(\frac{\alpha - c}{3}, \frac{\alpha - c}{3}\right)$

$Q^* = \frac{2(\alpha - c)}{3}$

$P^* = \frac{1}{3}(\alpha + 2c)$

$\pi_1^* = \frac{(\alpha - c)^2}{9}$

$\pi_2^* = \frac{(\alpha - c)^2}{9}$

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★ If $c_i = q_i^2$

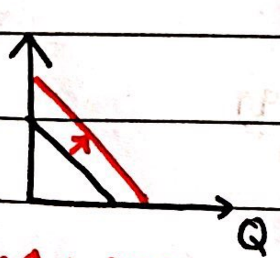
and $P = 2 - q_1 - 2q_2$

$\therefore q_1 = (1-q_2)/2$ if $q_2 \leq 1$

$= 0$ else $q_2 > 1$

★ $P = \alpha - Q \quad Q \leq \alpha$

$= 0 \quad Q > \alpha$



$\alpha \uparrow \Rightarrow$ Demand $\uparrow$ ($\alpha\uparrow$: exogenous rise in demand)

★ c depends on technology as tech improves $c \downarrow$

$\Rightarrow q_i^* \uparrow, P^* \downarrow, \pi_i^* \uparrow, Q^* \uparrow$

★ Collusion (Joint profit maximization)

$Q = q_1 + q_2$

Joint profit $\pi = PQ - cQ$

$\pi = Q(\alpha - Q) - cQ$

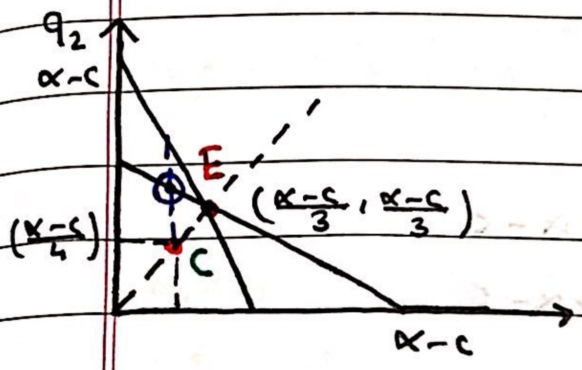
$\therefore Q^* = \dfrac{\alpha - c}{2}$

$\therefore q_1^* = q_2^* = \dfrac{\alpha - c}{4}$

$P^* = \dfrac{\alpha + c}{2}$ $\left(\because \alpha > c \therefore \dfrac{\alpha + c}{2} > \dfrac{\alpha + 2c}{3} \Rightarrow \text{Price has gone up}\right)$

$\pi_1^* = \pi_2^* = \dfrac{\pi}{2} = \dfrac{(\alpha - c)^2}{8}$

* Above situation similar to Prisoner's Dilemma.



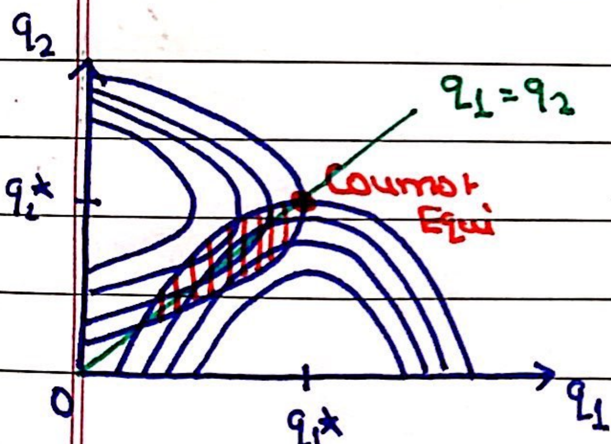
$C : (\frac{\alpha - c}{4}, \frac{\alpha - c}{4})$

$C \notin$ Best response function of any player

$\therefore$ One of the players tends to Freeride

Hence one tends to produce $3/8(\alpha - c)$ (q_2) given than q_1 produces $(\alpha - c)/4$

* Iso profit Curves:



* Suppose:

$\pi_1 = q_1(\alpha - c - q_2) - q_1^2$

If $q_2 \downarrow$, $\pi_1 \uparrow$

$z = f(x, y)$

$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$

$d\pi_1 = 0$

$\Rightarrow \frac{dq_1}{dq_2} = \frac{\alpha - c - q_2 - 2q_1}{q_1}$

at q_1^* : $\frac{dq_1}{dq_2} = 0$

Before q_1^* : $\frac{dq_1}{dq_2} > 0$ & $q_1 > q_1^* \Rightarrow \frac{dq_1}{dq_2} < 0$

i.e $q_1 < q_1^*$

★ Maximise the Market Share (q_i/Q)

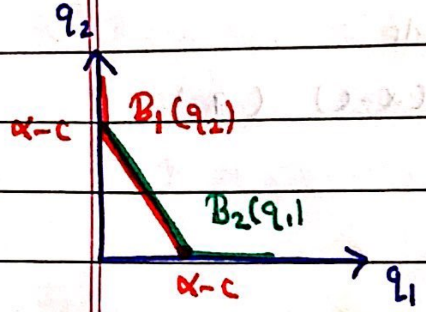
To maximise q_i/Q maximise q_i

subject to $\pi_i \geq 0$

If $\pi_i = q_i(\alpha - c - q_i - q_j) \geq 0$

$\Rightarrow \quad q_i = \alpha - c - q_j \qquad q_j \leq \alpha - c$

$\qquad\qquad = 0 \qquad\qquad q_j > \alpha - c$



∞ NE exists in this case

★ If $c_1 \neq c_2$

$C_i(q_i) = c_i q_i$

$\alpha > c_1 > c_2$

$P = \alpha - Q \qquad \alpha \geq Q$

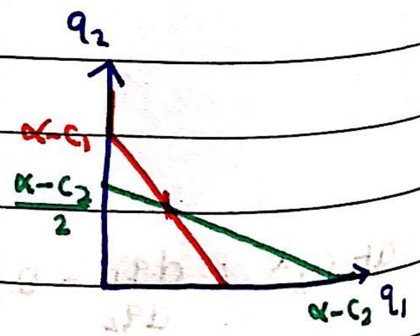
$\quad\; 0 \qquad\qquad \alpha < Q$

$q_1 = \dfrac{\alpha - c_1 - q_2}{2} \qquad \alpha \geq c_1 + q_2$

$\quad = 0 \qquad\qquad\qquad$ o.w

$q_2 = \dfrac{\alpha - c_2 - q_1}{2} \qquad \alpha - c_2 \geq q_1$

$\quad = 0 \qquad\qquad\qquad$ o.w



$c_1 > c_2$

$\Rightarrow q_1^* < q_2^*$

$$q_1^* = \frac{1}{3}(\alpha - 2c_1 + c_2)$$
$$q_2^* = \frac{1}{3}(\alpha - 2c_2 + c_1)$$
If $\alpha - c_1 > \frac{\alpha - c_2}{2}$

$$q_1^* = 0$$
$$q_2^* = \frac{\alpha - c_2}{2}$$
$\alpha - c_1 \leq \frac{\alpha - c_2}{2}$ (Corner soln fo BR func)

$$\boxed{c_1 \geq \frac{\alpha + c_2}{2}}$$

Oligopoly:

$$\pi_i(q_1, q_2, \ldots, q_i, \ldots, q_n) = q_i P - c q_i$$
$$= q_i(\alpha - q_1 - q_2 \ldots - q_n) - c q_i \quad \text{if } \alpha \geq \sum q_i$$
$$= 0 \quad \alpha < \sum q_i$$

FOC: $\alpha - c - q_1 - q_2 \ldots - 2q_i - \ldots - q_n = 0$

$\alpha - c - 2q_1 - q_2 \ldots - q_n = 0$

$\alpha - c - q_1 - 2q_2 \ldots - q_n = 0$

(1) - (2) $\Rightarrow q_1 = q_2$

$\therefore$ In equi $q_i = q_j$ for all i, j

$\therefore q_i^* = \frac{\alpha - c}{(n+1)}$

$\therefore p^* = \left(\frac{\alpha + nc}{n+1}\right)$ $\therefore \lim_{n \to \infty} p^* = \frac{\frac{1}{n}\alpha + c}{1 + \frac{1}{n}} = c$ (Perfect Competition)