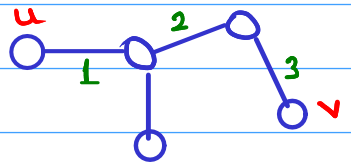
* Degree Centrality → Degree of the node
  - If weighted network $\sum w_i$

  ↓ Local measure

* Closeness Centrality (CC):



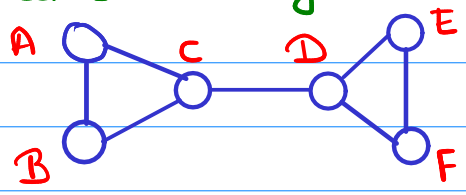
$d(u,v)$ = no. of hops ↓ Length of shortest path bw u & v

For node x:

$$CC(x) = \sum_{y \in V(G)} \frac{1}{d(x,y)}$$

Used for Information Spreading

* Betweenes Centrality (BC):



* BC (x) = No. of shortests path passing through node x.

→ BC(C) = 6 → AD, AE, AF, BD, BE, BF

→ Destruct a network remove node with max BC (# terrorists)

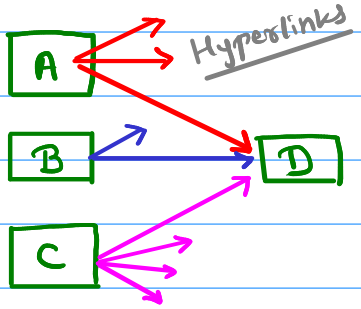
* Eigen Centrality (EC) → Influence of neighbours

$A_{n \times n}$ → Adjacency matrix of Network

Principal Eigen Vector (PEV) → $[x_1, x_2 \ldots x_n]^T$ (Eigen Vector with max mag)

Node with max x_i has max EC

→ Varient of EC : Google PageRank (HITS didn't become popular)



- P(x): Page Rank of Page x

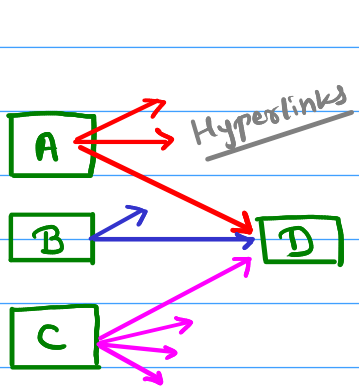
$$P(D) = \frac{P(A)}{3} + \frac{P(B)}{2} + \frac{P(C)}{4}$$

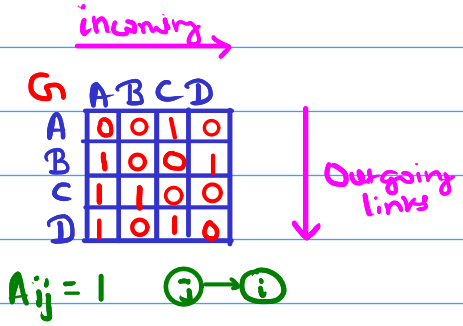
$$* P(c) = \sum_{x \to c} \frac{P(x)}{Out(x)}$$

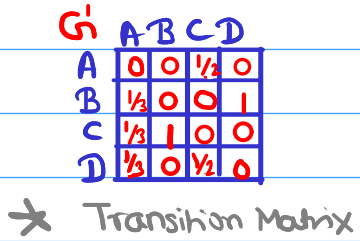
* P(x) depends on → indegree & Page rank of neighbours

Update Page Ranks → Stop when process converges

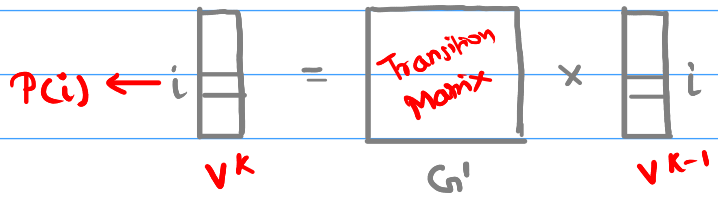
* Like a Random Walk







* Transition Matrix



* k → no. of iter

* $V^k = (G')^{k-1} V^0$

↳ Base Case (Initial value vector of PR)

* $V^0 \rightarrow [x_1\ x_2 \ldots x_n]^T \ni \sum x_i = 1$ (Stochastic vector)

When $V^k = V^{k-1} \Rightarrow$ Convergence say V^*

$\therefore V^* = G' V^*$

↳ Eigen vector of G'

* G' → Stochastic matrix ⇒ Principal Eigen value is 1

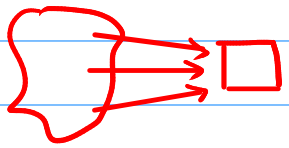
↳ irreducible & +ve semidefinite

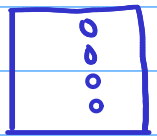
⇒ V^* → Eigen vector

↳ if $G'_{ij} = 0$, $\exists\ k \ni G'^k_{ij} > 0$

* In reality we might not have irreducible matrix

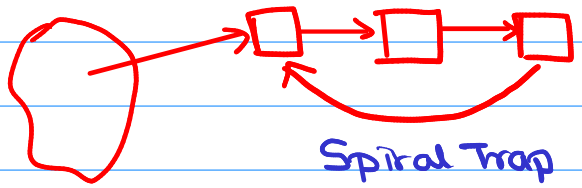
① Dangling Node:





→ Not a Stochastic matrix

② Cycle:



→ Google Page Rank avoids the above 2 problems

★ Google PageRank

$$P(x) = \frac{1-\delta}{n} + \delta \sum_{y \to x} \frac{P(y)}{Out(y)}$$

- $\frac{1-\delta}{n}$ → Start afresh (Complete Network)
- $\delta \sum_{y \to x} \frac{P(y)}{Out(y)}$ → Follow the Links (Original Network)

δ: Damping factor → Typically 0.85

$$G'' = (1-\delta)\underbrace{\begin{bmatrix} 1/n & 1/n & \cdots \\ \vdots & & \\ 1/n & \cdots & 1/n \end{bmatrix}}_{I} + \delta G'$$

★ $v^k = (G'')^{k-1} v^0$ Or $v^k = G'' v^{k-1}$

★ Personalized PageRank:

$$v^* = G'' v^*$$

$$\Rightarrow v^* = Iv^* + G'v^* = Iv^0 + G'v^*$$

$\hookrightarrow [1/n \ 1/n \ \cdots \ 1/n]^T : v^0$

$$\approx Ip + G'v^*$$

$\hookrightarrow$ Personalized vector

Make random walker biased to a domain

★ Rooted PageRank

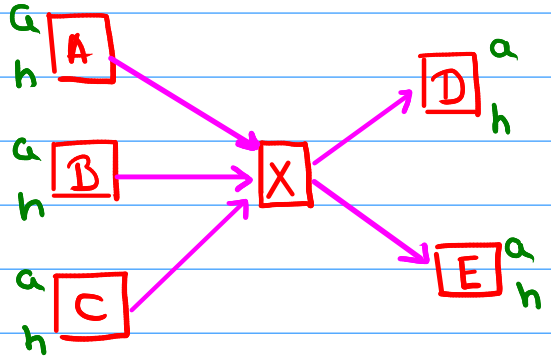
→ $p = [0 \ 0 \ 0 \ \cdots 1 \ \cdots 0]^T$

★ HITS (Exactly same as pagerank)

(Hyperlink Induced Topic Search)

a: Authority Score (how popular I am) (Recommend from others)

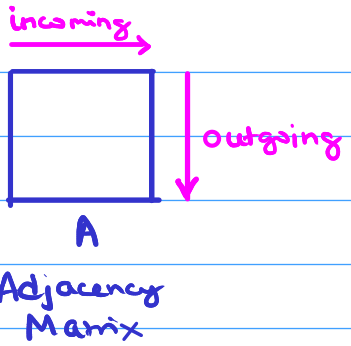
h: Hubness Score (high resources)



$a(x) = h(A) + h(B) + h(C)$

$h(x) = a(D) + a(E)$

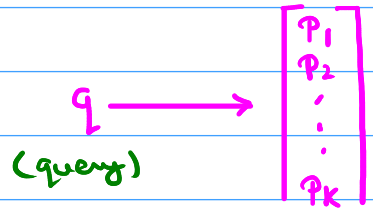
★ $a(x) = \sum_{y \to x} h(y)$ & $h(x) = \sum_{x \to y} a(y)$



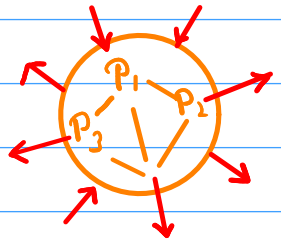
$\therefore \quad \bar{a} = A\bar{h} \quad \& \quad \bar{h} = A^T\bar{a}$

$\Rightarrow \bar{a} = (AA^T)\bar{a} \quad \& \quad \bar{h} = (A^TA)\bar{h}$

* $AA^T \rightarrow$ symmetric matrix & Positive Semi Def.
  ↓
  Eigen vectors orthogonal



(returned by web browser)



Expand by 1 more hop