Dynamic Small Cell Clustering and Non-Cooperative Game-Based Precoding Design for Two-Tier Heterogeneous Networks With Massive MIMO

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***Abstract*— In this paper, we investigate the dynamic small cell (SC) clustering strategy and their precoding design problem for interference coordination in two-tier heterogeneous networks (HetNets) with massive MIMO (mMIMO). To reduce interference among different SCs, an interference graph-based dynamic SC clustering scheme is proposed. Based on this, we formulate an optimization problem as design precoding weights at macro base station (MBS) and clustered SCs for maximizing the downlink sum rate of SC users (SUs) subject to the power constraint of each SC BS (SBS), while mitigating inter-cluster, eliminating inter-tier, intra-cluster and multi-macro user (MU) interference. To eliminate the inter-tier and multi-MU interference simultaneously, we propose a clustered SC block diagonalization precoding scheme for the MBS. Next, each SU's precoding vector at clustered SCs is designed as the product of the following two parts. The first part is designed with singular value decomposition to remove the intra-cluster interference. The second part is designed to coordinate the inter-cluster interference for maximizing the downlink sum rate of SUs, which is a non-convex optimization problem and difficult to solve directly. A non-cooperative game-based distributed algorithm is proposed to obtain a suboptimal solution. Meanwhile, we prove the existence and uniqueness of Nash equilibrium for the formed game. Finally, simulation results verify the effectiveness of our proposed schemes.**

***Index Terms*— Massive MIMO, heterogeneous network, cluster, non-cooperative game, nash equilibrium.**

I. Introduction

TO SATISFY the demand for the ever-increasing wireless data traffic, two strategies, namely massive multiple-input multiple-output (mMIMO) and densification through small cells (SCs), have been investigated for spectral- and energy-efficient future wireless networks [1], [2]. In mMIMO [3], [4], each base station (BS) is equipped with a large number of antennas to serve multiple single-antenna users with the same time-frequency resource. Densification is envisioned with SCs having short-range coverage through microcells, picocells and femtocells in order to achieve several technical advantages such as low transmit power [5], high signal to noise power ratio (SNR) [6], and dense spectrum reuse [7]. Therefore, a combination of these two strategies can form a two-tier mMIMO-heterogeneous network (mMIMO-HetNet) [8], [9].

In two-tier mMIMO-HetNet, interference coordination is one of the main challenges [10]–[13]. In particular, macro users (MUs) and SC users (SUs) usually share the same time-frequency resource to improve the spectral efficiency, which requires robust interference coordination scheme. In [10], a precoding scheme has been proposed for mMIMO to suppress the impact on neighboring users (victims) through cooperation between macro BS (MBS) and SC BSs (SBSs). Adhikary *et al.* [11] exploit the directionality of the channel vectors in mMIMO to obtain the spatial blanking, where the transmission energy is concentrated only in certain directions to create transmission opportunities for SBSs. User-cell association schemes, [12] and [13], are proposed to decrease the interference and improve the throughput of the system. However, the above mentioned works are limited with the fixed transmit power at SBSs. Power allocation and precoding design at SBSs are not considered at all, which restricts the performance of the system having un-efficient resource allocation.

Optimal power allocation in two-tier HetNet is very challenging, if not impossible, due to the large overhead and high complexity of computing. A game-based distributed power control strategy is considered to coordinate the interference [14]–[20]. Kang *et al.* [14] assume that the MBS can protect itself by pricing the interference from SUs and a Stackelberg game is formulated to study the joint utility maximization of the MUs and SUs subject to a maximum tolerable interference power constraint at MBS. Wang *et al.* [15] formulate a Stackelberg game between MBS and SBSs, where the MBS acts as a leader and all SBSs act as follower. By setting the interference price at MBS, the SBS can determine its transmit power to achieve its optimal utility. In [16],

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the energy-efficient resource allocation problem in HetNet cognitive radio networks with femtocells as a Stackelberge game is studied. The authors present a gradient based iteration algorithm to obtain the Stackelberg equilibrium. The authors in [17] develop a multi-operator multi-user Stackelberg game to analyze the interaction between multiple operators and users in unlicensed spectrum. A non-cooperative power allocation game is proposed in [18] to coordinate interference and improve the total downlink throughput of edge users. Similarly, Al-Zahrani and Yu [19] develop a transmit power adaptation method with a non-cooperative game to reduce the inter-cell interference while in [20] two rate maximization and gradient-norm minimization-based non-cooperative games are studied. However, the above mentioned works only consider the single antenna systems. The extension to multiple-antenna systems is not trivial. There has been a few works considering MIMO-based non-cooperative game [21], [22] among players, where each player (i.e., BS) is equipped with multiple antennas. However, each player has only a role of a BS, which limits interference coordination capability among SCs, especially for ultra-dense SCs.

For interference coordination,[1] unlike previous works, in this paper we investigate a new SC clustering strategy and their precoding designs for maximizing downlink sum rate of SUs in two-tier mMIMO-HetNet. The main contributions are summarized as follows:

- To reduce the interference among SCs, an interference graph-based dynamic SC clustering scheme is proposed, where SCs are grouped into multiple SC clusters according to their interference channel strength. On this basis, the SUs' signals are jointly designed in each cluster.
- To achieve joint interference coordination mentioned above, we formulate an optimization problem to design precoding weights at MBS and clustered SCs for maximizing the downlink sum rate of SUs subject to per-SBS power constraint. Precoding weights at MBS are designed to eliminate the multi-MU and inter-tier interference, while precoding weights at clustered SCs are designed to cancel the intra-cluster interference and mitigate inter-cluster interference.
- To eliminate multi-MU and inter-tier interference simultaneously, we propose a clustered SC block diagonalization (CSBD) precoding scheme for MBS. Specifically, we use the singular value decomposition (SVD) to find the null space of the inter-tier interference channels. Following this, the zero forcing (ZF) downlink precoding weight matrix for MUs is projected onto the above null space to simultaneously cancel the multi-MU and inter-tier interference.
- To cancel the intra-cluster interference and coordinate inter-cluster interference, the precoding vector of each SU at clustered SCs is designed as the product of the following two parts. The first part is designed with SVD to remove the intra-cluster interference. The second part is designed to coordinate the inter-cluster interference for maximizing the downlink sum rate of SUs. This is a non-convex optimization problem that is difficult to directly solve. We propose a cluster-based non-cooperative game and develop a distribute algorithm to obtain a suboptimal solution. Finally, we prove the existence and uniqueness of the Nash equilibria (NE) for the formed game.

[1]In this paper, we involve four classes interference in downlink mMIMO-HetNet using SC clustering: Inter-cluster interference indicates the interference among SUs' signals belonging to different clusters, intra-cluster interference indicates the interference among SUs' signals belonging to the same cluster, and multi-MU interference indicates the interference among MUs' signals, inter-tier interference indicates the interference from MBS to SUs.

The notations of this paper are as follows: $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively. $\|\cdot\|$ denotes the Euclidean norm, and $\mathbb{E}\{\cdot\}$ represents the expectation operator. $\mathrm{var}\{\cdot\}$ is the variance operator, and $\mathbf{I}_n$ and $\mathbf{0}_n$ means the $n \times n$ identity matrix and all-zero matrix, respectively. $[\cdot]^+$ denotes the $\max\{0, \cdot\}$, $\mathrm{Tr}(\mathbf{\Phi})$ is the trace and $\mathrm{Diag}(\mathbf{a})$ represents a diagonal matrix with the main diagonal given by $\mathbf{a}$. $\mathbb{C}^{x \times y}$ denotes the space of $x \times y$ complex matrix.

II. Related Works

In literature [23]–[26], there have been some studies about SC clustering for interference coordination. *Zhou et al.* [23] propose a graph-based approach combining SC clustering and user clustering for mitigating interference among SCs. According to the downlink signal-to-interference-plus-noise ratio (SINR) user receives, SCs are grouped into multiple clusters. A rate loss-based low-complexity algorithm for SC clustering is proposed by *Seno et al.* [24]. Each SBS's transmit power must be decided in advance for the above clustering schemes, which are inapplicable in our study. *Hong et al.* [25] investigate the BSs clustering and precoding design for partially coordinated transmission to maximize the utility of the system. Since one user can be served by different BS clusters in their scheme, the intra-cluster interference can not be cancelled completely. A dynamic greedy algorithm for cooperative BSs clustering is proposed in [26], but the number of BSs in each cluster must be known in advance. *Fan et al.* [27] propose a distance-based SBS clustering scheme. Although the proposed algorithm is simple, it is not appropriate for time-varying environment. In fact, [23]–[26] consider the clustering at user level, namely the cluster is formed by considering the real-time interference between users and BS, which is superior than clustering at SC level (e.g., [27]).

Several precoding schemes for interference coordination in MIMO system have also been investigated in literature [28]–[31]. For example, *Zhang et al.* [28] study the precoding design optimization problem for maximizing the weighted sum rate of all users in multi-cell system. Since the multi-user interference can be eliminated by using proposed BD precoding technique, the original problem can be transformed into convex optimization problem. Similarly, in [29] and [30], the authors design effective precoder to eliminate the multi-user interference so that the original problem can be transformed into convex optimization problem. Then, the optimal precoder can be obtained by using interior-point method or convex optimization toolbox directly. *Niu et al.* [31] propose a joint interference alignment and

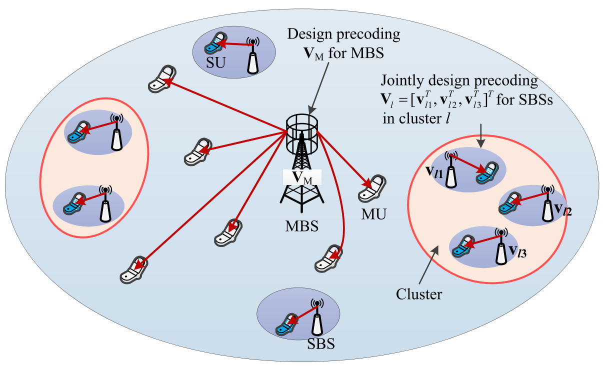


Fig. 1. System model for small cluster-based two-tier downlink mMIMO-HetNet.

power allocation problem for reducing the intra- and inter-tier interference, which is solved by some simply linear algorithms. Although the linear algorithm has low complexity, it cannot be used in non-convex problems such as ours due to per-SBS power constraint and interference among clusters.

III. System Model and Problem Formulation

In this section, we first briefly describe the system model. Then, we formulate SC clustering and precoding design optimization problem for maximizing the downlink sum rate of SUs under per-SBS power constraint.

A. System Model

A two-tier downlink mMIMO-HetNet system is considered as shown in Fig. 1, which is composed of a macro cell (MC) and J overlaid SCs ($\mathbf{J}=\{1,2,\ldots,J\}$). We assume that K_{M} single-antenna MUs are served by the central MBS equipped with M antennas ($M \gg K_{\mathrm{M}}$),[1] and each SBS associated with N antennas serves K_{S} single-antenna SUs ($N \geq K_{\mathrm{S}}$).[2] The MC and all SCs share the overall spectrum while users (MUs and SUs) are served with the same time-frequency resource. In this case, there exists interference among SCs. Cooperative downlink transmission among SCs eliminates the inter-SC interference. However, transmission data must be exchanged and shared among SBSs, which needs huge backhaul overhead. SC clustering approach is effective in decreasing the required overhead, where the transmission data are only shared within each cluster. To harvest the benefits of the mMIMO antennas, we assume that time division duplex (TDD) protocol is applied with perfect channel state information (CSI) at MBS and SBSs [12], [13].

We assume that all SCs are grouped into C clusters, where each SU is served by all SBSs belonging to the same cluster. Let C_l denote the number of SCs in the lth cluster with the total number of SUs and antennas across all SBSs in the lth cluster denoted as $K_l = C_l K_{\mathrm{S}}$ and $N_l = C_l N$, respectively. Meanwhile, we assume that $((O_l-1)N+1)$th to $(O_l N)$th antennas are taken as the N SBS antennas in the O_lth SBS in the lth cluster with $O_l = 1,2,\ldots,C_l$. Similarly, the indices of SUs in the O_lth SBS in the lth cluster can be denoted as the $((O_l-1)K_{\mathrm{S}}+1)$th to $(O_l K_{\mathrm{S}})$th. A summary of key notations is presented in Table I.

[1]We assume that BSs (i.e., SBS and MBS) decides their serving users according to the user association scheme in [7] and [32].

[2]In this paper, we assume that the number of SUs in each SC is the same. However, the number of SUs in each SC may differ, and our proposed scheme can be extended cover this scenario by using arbitrary number of SUs as long as the number of SBSs antennas is larger than the number of SUs in each cluster.

TABLE I
Summary of Key Notations

Notations	Descriptions
J	Number of SCs
K_{M}	Number of MUs
M	Number of MBS antennas
K_{S}	Number of SUs in each SC
C	Number of clusters
C_l	Number of SBSs in the lth cluster
K_l	Number of SUs in the lth cluster
N_l	Number of antennas across all SBSs in the lth cluster
O_l	Index of SBS in the lth cluster
P_{0k}	The transmit power for the kth MU
P	The maximum transmit power of each SBS

The received signal by the kth SU in the lth cluster can be expressed as:

$$
\begin{aligned}
y_{lk} &= \sum_{i=1}^{C}\sum_{j=1}^{K_i}\mathbf{h}_{ilk}\mathbf{v}_{ij}x_{ij} + \sum_{m=1}^{K_{\mathrm{M}}}\mathbf{h}_{0lk}\mathbf{v}_{0m}x_{0m} + n_{lk} \\
&= \underbrace{\mathbf{h}_{llk}\mathbf{v}_{lk}x_{lk}}_{\text{Desired signal}} + \underbrace{\sum_{j\neq k}^{K_l}\mathbf{h}_{llk}\mathbf{v}_{lj}x_{lj}}_{\text{Intra-cluster interference}} + \underbrace{\sum_{i\neq l}^{C}\sum_{j=1}^{K_i}\mathbf{h}_{ilk}\mathbf{v}_{ij}x_{ij}}_{\text{Inter-cluster interference}} \\
&\quad + \underbrace{\sum_{m=1}^{K_{\mathrm{M}}}\mathbf{h}_{0lk}\mathbf{v}_{0m}x_{0m}}_{\text{Inter-tier interference}} + \underbrace{n_{lk}}_{\text{Noise}}, \qquad (1)
\end{aligned}
$$

where $\mathbf{h}_{ilk} \in \mathbb{C}^{1\times N_i}$ and $\mathbf{h}_{0lk} \in \mathbb{C}^{1\times M}$, respectively, denote the downlink channel from all C_i SBSs and the MBS to the kth SU in the lth cluster. $\mathbf{v}_{ij} \in \mathbb{C}^{N_i\times 1}$ and $\mathbf{v}_{0m} \in \mathbb{C}^{M\times 1}$, respectively, denote the precoding vector for the jth SU in the ith cluster and the mth MU. x_{ij} and x_{0m} denote the transmit signals of the jth SU in the ith cluster and the mth MU, respectively. We assume $\mathbb{E}[|x|^2]=1$ and n_{lk} is an independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) defined as $\mathcal{CN}(0,\ \delta^2)$.

Similarly, the received signal at the kth MU can be expressed as follows:

$$
\begin{aligned}
y_{0k} &= \sum_{m=1}^{K_{\mathrm{M}}}\mathbf{h}_{00k}\mathbf{v}_{0m}x_{0m} + \sum_{i=1}^{C}\sum_{j=1}^{K_i}\mathbf{h}_{i0k}\mathbf{v}_{ij}x_{ij} + n_{0k} \\
&= \underbrace{\mathbf{h}_{00k}\mathbf{v}_{0k}x_{0k}}_{\text{Desired signal}} + \underbrace{\sum_{m\neq k}^{K_{\mathrm{M}}}\mathbf{h}_{00k}\mathbf{v}_{0m}x_{0m}}_{\text{Intra-tier interference}} + \underbrace{\sum_{i=1}^{C}\sum_{j=1}^{K_i}\mathbf{h}_{i0k}\mathbf{v}_{ij}x_{ij}}_{\text{Inter-tier interference}} \\
&\quad + \underbrace{n_{0k}}_{\text{Noise}}, \qquad (2)
\end{aligned}
$$

where $\mathbf{h}_{00k} \in \mathbb{C}^{1\times M}$ and $\mathbf{h}_{i0k} \in \mathbb{C}^{1\times N_i}$ denote the downlink channel from the MBS and all C_i SBSs in the ith cluster to the kth MU, respectively.

Following this, the constraints for eliminating inter-tier interference from MBS to SUs and intra-cluster interference are given as:

$$\mathbf{h}_{0lk}\mathbf{v}_{0m} = 0, \quad \forall l,\ k,\ m = \{1, \ldots, K_{\mathrm{M}}\}, \tag{3a}$$
$$\mathbf{h}_{llk}\mathbf{v}_{lj} = 0, \quad \forall k \neq j,\ l = \{1, \ldots, C\}, \tag{3b}$$

Therefore, the received signal by the kth SU in the lth cluster can be rewritten as:

$$y_{lk} = \mathbf{h}_{llk}\mathbf{v}_{lk}x_{lk} + \sum_{i \neq l}^{C}\sum_{j=1}^{K_i}\mathbf{h}_{ilk}\mathbf{v}_{ij}x_{ij} + n_{lk}, \tag{4}$$

and its rate can be expressed as follows:

$$R_{lk} = \log_2\left(1 + \frac{\mathbf{h}_{llk}\mathbf{v}_{lk}\mathbf{v}_{lk}^H\mathbf{h}_{llk}^H}{\sum\limits_{i \neq l}^{C}\sum\limits_{j=1}^{K_i}\mathbf{h}_{ilk}\mathbf{v}_{ij}\mathbf{v}_{ij}^H\mathbf{h}_{ilk}^H + \delta^2}\right). \tag{5}$$

B. Problem Formulation

Since total transmit antennas in each cluster comes from more than one SBS, the per-SBS power constraint is expressed as follows:

$$\sum_{k=1}^{K_l}\mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{v}_{lk}\mathbf{v}_{lk}^H) \leq P, \quad \forall l,\ O_l = \{1, \ldots, C_l\}, \tag{6}$$

with

$$\mathbf{B}_{O_l} \triangleq \mathrm{Diag}(\underbrace{0, \ldots, 0}_{(O_l-1)N}, \underbrace{1, \ldots, 1}_{N}, \underbrace{0, \ldots, 0}_{(C_l-O_l)N}). \tag{7}$$

Next, we formulate the optimization problem to maximize the downlink sum rate of SUs as follows[3]:

$$\max_{\{\mathbf{V}_1, \ldots, \mathbf{V}_C\}, \mathbf{V}_{\mathrm{M}}} \sum_{l=1}^{C}\sum_{k=1}^{K_l} R_{lk} \tag{8a}$$
$$\text{s.t. } \mathbf{h}_{0lk}\mathbf{v}_{0m} = 0, \quad \forall l,\ k,\ m = \{1, \ldots, K_{\mathrm{M}}\}, \tag{8b}$$
$$\mathbf{h}_{llk}\mathbf{v}_{lj} = 0, \quad \forall k \neq j,\ l = \{1, \ldots, C\}, \tag{8c}$$
$$\sum_{k=1}^{K_l}\mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{v}_{lk}\mathbf{v}_{lk}^H) \leq P, \quad \forall l,\ O_l, \tag{8d}$$

where $\mathbf{V}_l = [\mathbf{v}_{l1}^T, \ldots, \mathbf{v}_{lK_l}^T]^T$ and $\mathbf{V}_{\mathrm{M}} = [\mathbf{v}_{01}^T, \ldots, \mathbf{v}_{0K_{\mathrm{M}}}^T]^T$.

To solve the above problem (8), we consider the following three steps. The first step is to design a SC clustering scheme so that all SCs form multiple clusters as shown in Sect. IV. The second step in Sect. V designs precoding at MBS to eliminate inter-tier interference, namely (8b). The last step in Sect. VI designs precoding at each cluster to maximize the downlink sum rate of SUs, namely (8a), (8c) and (8d).

[3]In mMIMO-HetNet, MBS is mainly used to guarantee the backward compatibility with traditional cellular networks and support the seamless coverage with low bits rates [33], [34]. The SBSs are mainly deployed to provide high bit rates in special zones. Thus, in this paper, we focus on solving the SUs sum rate maximization problem and do not consider the MUs rate optimization problem, where the equal power allocation for MUs is assumed for simplicity.

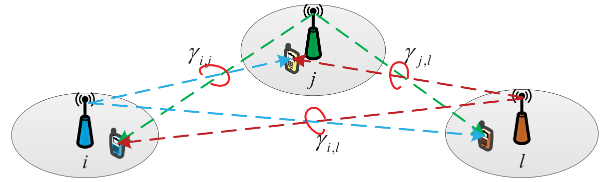
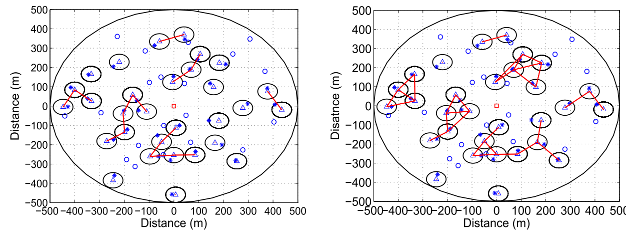


Fig. 2. An example for interference graph.



(a) γ_{th}=-100dB (b) γ_{th}=-105dB

Fig. 3. An example for SC clustering with different γ_{th}.

IV. SC Clustering Scheme for Interference Coordination

We define the average interference channel strength between two SCs i, j as follows:

$$\gamma_{i,j} = \frac{1}{NK_{\mathrm{T}}}\sum_{k=1}^{K_{\mathrm{S}}}\left(\left\|\bar{\mathbf{h}}_{ijk}\right\| + \left\|\bar{\mathbf{h}}_{jik}\right\|\right), \qquad i,\ j = \{1, \ldots, C\}, \tag{9}$$

where K_{T} denotes the total number of SUs in SCs i and j ($K_{\mathrm{T}} = 2K_{\mathrm{S}}$). $\bar{\mathbf{h}}_{ijk} \in \mathbb{C}^{1\times N}$ denotes the downlink interference channel from the ith SC to the kth SU in the jth SC. $\gamma_{i,j}$ in (9) represents the potential average interference strength level between SCs i and j. For example, a larger $\gamma_{i,j}$ denotes higher interference and vice versa. Therefore, two SCs form a cluster when $\gamma_{i,j}$ is high.

The potential interference relationship among all SCs can be constructed as the interference graph. First, we set an interference threshold γ_{th} that is used to determine whether two SCs should form a cluster. We assume three SCs in Fig. 2 denoted as i, j, l. If we have $\gamma_{i,j} \geq \gamma_{th}$, $\gamma_{j,l} \geq \gamma_{th}$ and $\gamma_{i,l} < \gamma_{th}$, according to the above definition, the SC j will belong to two different clusters. For simplifying the problem and coordinating more interference among SCs, the SCs i, j and l will form one cluster under the above situation. Based on the above analysis, an interference graph can be constructed as an undirected graph $G(\mathbf{J}, \mathbf{E})$ in SBSs, where $\mathbf{J}$ vertices denote all SCs and $E(u, v)$ edges stand the potential interference between SCs u and v, $\forall u, v \in \mathbf{J}$.

Fig. 3 illustrates the SC clustering results with different γ_{th} having the same randomly located SCs. It can be clearly observed from the figure that higher interference threshold leads to smaller-size clusters with fewer SCs. This reduces the information exchange within each cluster, which decreases the backhaul overhead and system latency. On the contrary, lower interference threshold leads to larger-size clusters with more SCs. This increases information exchange within each cluster, which increases the backhaul overhead and system latency. Therefore, in practice, the interference threshold can be determined according to the required criteria, e.g., the system sum rate maximization, delay and/or backhaul overhead minimization. In this paper, the interference threshold is empirically selected to maximize the system sum rate for simplicity. We note here that information exchange is only done between SBSs within each cluster as indicated by $\mathbf{h}_{llk}$ and x_{lk} in (4). The information exchange among users is not needed.

The SC clustering scheme is presented in Algorithm 1. We first compute $\gamma_{i,j}$ between SCs i and j (in line 4). Next, two SCs are decided whether to form a cluster or not based on γ_{th} (in lines 5-6). When SC i and j form a cluster according to the above scheme, if any one of them has belonged to other cluster, SCs i, j and that cluster members will reform one cluster (in lines 7-9). The above procedure is repeated until all SCs are clustered.

Algorithm 1 Interference Graph-Based Dynamic SC Clustering Scheme

Input : J, γ_{th}, $n = 1$.
Output: $\mathbf{C}$.
1 **for** $i = 1 : J$ **do**
2   $\mathbf{C}\{n\} = \{i\}$.
3   **for** $j = 1 : J$ $(i \neq j)$ **do**
4   Compute $\gamma_{i,j}$ according to (9).
5   **if** $\gamma_{i,j} \geq \gamma_{th}$ **then**
6   The SCs i and j form a new cluster, namely $\mathbf{C}\{n\} = \mathbf{C}\{n\} \cup \{j\}$.
7   **if** The SC i or j has belonged to any other cluster n' **then**
8   The SCs i, j and their all cluster members form a new cluster, namely $\mathbf{C}\{n\} = \mathbf{C}\{n\} \cup \mathbf{C}\{n'\}$, $n = n - 1$.
9   **end if**
10   **end if**
11   **end for**
12   $n = n + 1$.
13 **end for**
14 **Note:** Here, $\mathbf{C}$ should be a set consisting of several subsets. Each subset indicates a cluster and its elements represent SCs' index. Therefore, $\mathbf{C}\{n\}$ denotes the nth subset in set $\mathbf{C}$.

V. CSBD Precoding Design for MBS

To eliminate the inter-tier interference from MBS to SUs and multi-MU interference simultaneously, we propose a CSBD precoding scheme for MBS.

First, we define the inter-tier interference channels from MBS to SUs as follows:

$$\mathbf{H}_{\text{in}} = [\mathbf{h}_{011}^T, \ldots, \mathbf{h}_{01K_1}^T, \ldots, \mathbf{h}_{0C1}^T, \ldots, \mathbf{h}_{0CK_C}^T]^T. \quad (10)$$

where $\mathbf{H}_{\text{in}} \in \mathbb{C}^{JK_{\text{S}} \times M}$. To obtain the null space of the inter-tier interference channels Null($\mathbf{H}_{\text{in}}$), we apply the classical SVD to the matrix $\mathbf{H}_{\text{in}}$, yielding

$$\mathbf{H}_{\text{in}} = \mathbf{U}\Sigma\mathbf{V}^H, \quad (11)$$

where $\mathbf{U} \in \mathbb{C}^{JK_{\text{S}} \times JK_{\text{S}}}$ denotes the left-singular-vector matrix, $\mathbf{V} \in \mathbb{C}^{M \times M}$ denotes the right-singular-vector matrix, and $\Sigma \in \mathbb{C}^{JK_{\text{S}} \times M}$ denotes the singular values as follows:

$$\Sigma = \begin{bmatrix} \widehat{\Sigma}_r & \mathbf{0}_{r \times (M-r)} \\ \mathbf{0}_{(JK_{\text{S}}-r) \times r} & \mathbf{0}_{(JK_{\text{S}}-r) \times (M-r)} \end{bmatrix}. \quad (12)$$

where $r = \text{rank}(\mathbf{H}_{\text{in}})$ is the rank of $\mathbf{H}_{\text{in}}$, and $\widehat{\Sigma}_r = \text{Diag}\{\sigma_1, \ldots, \sigma_{\text{r}}\}$.

Therefore, the null space of $\mathbf{H}_{\text{in}}$ can be found by spanning the columns of $\mathbf{V}$ as follows:

$$\widehat{\mathbf{V}} = [\mathbf{v}_{r+1}, \mathbf{v}_{r+2}, \ldots, \mathbf{v}_M]. \quad (13)$$

Note that there is a constraint condition for the existence of $\widehat{\mathbf{V}} \in \mathbb{C}^{M \times (M-r)}$, namely the number of SUs must be lower than the number of MBS antennas ($JK_S \leq M$). Then, we have the following:

$$\mathbf{H}_{\text{in}}\widehat{\mathbf{V}} = \mathbf{0}. \quad (14)$$

According to (14), we can find that the inter-tier interference can be cancelled completely if a column vector for each MU is randomly chosen from $\widehat{\mathbf{V}}$, but it may cause serious multi-MU interference. To eliminate the inter-tier and multi-MU interference simultaneously, we first define the projection matrix $\widetilde{\mathbf{V}}_{\text{in}}$ based on the null space Null($\mathbf{H}_{\text{in}}$) as follows:

$$\widetilde{\mathbf{V}}_{\text{in}} = \widehat{\mathbf{V}}\widehat{\mathbf{V}}^H. \quad (15)$$

Accordingly, we can project ZF precoding matrix for MUs onto the null space Null($\mathbf{H}_{\text{in}}$) and obtain the final precoding matrix as:

$$\begin{aligned} \mathbf{W}_{\text{CSBD}} &= \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\right)^H \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\right)^H\right)^{-1} \\ &= \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\right)^H \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\widetilde{\mathbf{V}}_{\text{in}}^H\mathbf{H}_{\text{M}}^H\right)^{-1} \\ &= \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\right)^H \left(\mathbf{H}_{\text{M}}\widehat{\mathbf{V}}\widehat{\mathbf{V}}^H\widehat{\mathbf{V}}\widehat{\mathbf{V}}^H\mathbf{H}_{\text{M}}^H\right)^{-1} \\ &= \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\right)^H \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H\right)^{-1}, \end{aligned} \quad (16)$$

where $\mathbf{H}_{\text{M}} = [\mathbf{h}_{001}^T, \mathbf{h}_{002}^T, \ldots, \mathbf{h}_{00K_{\text{M}}}^T]^T$ denotes the multi-MU downlink channel matrix. Meanwhile, the necessary condition for the existence of $\mathbf{W}_{\text{CSBD}}$ is $JK_S + K_{\text{M}} \leq M$, namely the number of SUs and MUs should be lower than the number of MBS antennas. Next, we provide the proof for the above necessary condition. For the i.i.d. Rayleigh fading channel, rank($\mathbf{H}_{\text{in}}$) should be JK_{S}. Thus, $\widehat{\mathbf{V}}$ is a $M \times (M\text{–}JK_{\text{S}})$ matrix, where rank($\widehat{\mathbf{V}}$) $= M\text{–}JK_{\text{S}}$ and rank($\widetilde{\mathbf{V}}_{\text{in}}$) $\leq M\text{–}JK_{\text{S}}$. In addition, since $\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H$ is a $K_{\text{M}} \times K_{\text{M}}$ matrix, the condition for existence of $(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H)^{-1}$ is rank$(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H) = K_{\text{M}}$. Due to rank$(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H) \leq \min\{M - JK_{\text{S}}, K_{\text{M}}\}$, we have $M - JK_{\text{S}} \geq K_{\text{M}}$, namely, $JK_{\text{S}} + K_{\text{M}} \leq M$.

According to the obtained precoding $\mathbf{W}_{\text{CSBD}}$, we have the following:

$$\begin{aligned} \mathbf{H}_{\text{M}}\mathbf{W}_{\text{CSBD}} &= \mathbf{H}_{\text{M}}\left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\right)^H \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H\right)^{-1} \\ &= \mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H\right)^{-1} = \mathbf{I}, \quad (17a) \\ \mathbf{H}_{\text{in}}\mathbf{W}_{\text{CSBD}} &= \mathbf{H}_{\text{in}}\left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\right)^H \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H\right)^{-1} \\ &= \mathbf{H}_{\text{in}}\widehat{\mathbf{V}}\widehat{\mathbf{V}}^H\mathbf{H}_{\text{M}}^H \left(\mathbf{H}_{\text{M}}\widetilde{\mathbf{V}}_{\text{in}}\mathbf{H}_{\text{M}}^H\right)^{-1} = \mathbf{0}, \quad (17b) \end{aligned}$$

where (17a) and (17b), respectively, illustrate that the multi-MU and inter-tier interference can be cancelled simultaneously similarly to [35].

The precoding vector for the kth MU can be written as:

$$\mathbf{v}_{0k} = \frac{\sqrt{P_{0k}}\mathbf{W}_{\mathrm{CSBD}}^{k}}{\left\|\mathbf{W}_{\mathrm{CSBD}}^{k}\right\|}, \tag{18}$$

where $\mathbf{W}_{\mathrm{CSBD}}^{k}$ is the kth column vector of $\mathbf{W}_{\mathrm{CSBD}}$, and P_{0k} denotes the transmit power for the kth MU.

In addition, when $JK_S + K_{\mathrm{M}} \leq M$, the ZF precoding can be also used to cancelled the multi-MU and inter-tier interference directly. For example, we define the following channel matrix:

$$\mathbf{H}_{\mathrm{ZF}} = [\mathbf{h}_{001}^{T}, \mathbf{h}_{002}^{T}, \ldots, \mathbf{h}_{00K_{\mathrm{M}}}^{T}, \mathbf{h}_{011}^{T}, \ldots, \mathbf{h}_{01K_1}^{T}, \ldots, \mathbf{h}_{0K_1}^{T}, \ldots, \mathbf{h}_{0CK_C}^{T}]^{T}. \tag{19}$$

On this basis, we obtain the ZF precoding as follows:

$$\mathbf{W}_{\mathrm{ZF}} = \mathbf{H}_{\mathrm{ZF}}^{H}\left(\mathbf{H}_{\mathrm{ZF}}\mathbf{H}_{\mathrm{ZF}}^{H}\right)^{-1}. \tag{20}$$

The precoding vector for the kth MU can be written as:

$$\mathbf{v}_{0k} = \frac{\sqrt{P_{0k}}\mathbf{W}_{\mathrm{ZF}}^{k}}{\left\|\mathbf{W}_{\mathrm{ZF}}^{k}\right\|}, \tag{21}$$

where $\mathbf{W}_{\mathrm{ZF}}^{k}$ is the kth column vector of $\mathbf{W}_{\mathrm{ZF}}$, and P_{0k} denotes the transmit power for the kth MU.

However, for ZF precoding, we find that $\mathbf{H}_{\mathrm{ZF}}\mathbf{H}_{\mathrm{ZF}}^{H}$ is a $(JK_{\mathrm{S}}+K_{\mathrm{M}}) \times\ (JK_{\mathrm{S}}+K_{\mathrm{M}})$ matrix. Accordingly, we need to inverse a high dimension matrix $((JK_{\mathrm{S}}+K_{\mathrm{M}}) \times\ (JK_{\mathrm{S}}+K_{\mathrm{M}}))$ in (20) with high complexity, especially for a lager JK_{S} (i.e., number of SUs). In contrast, for our proposed CSBD precoding, we only need to inverse a low dimension matrix $\mathbf{H}_{\mathrm{M}}\widetilde{\mathbf{V}}_{\mathrm{in}}\mathbf{H}_{\mathrm{M}}^{H}$ $(K_{\mathrm{M}} \times K_{\mathrm{M}})$, reducing the computational complexity.

VI. Non-Cooperative Game-Based Precoding Design for Clustered SCs

The precoding vector of the kth SU in the lth cluster is designed as the product as follows:

$$\mathbf{v}_{lk} = \mathbf{T}_{lk}\mathbf{s}_{lk}, \tag{22}$$

where $\mathbf{T}_{lk}$ is used to remove the intra-cluster interference, and $\mathbf{s}_{lk}$ is designed to coordinate the inter-cluster interference for maximizing the downlink sum rate of SUs.

We first define the intra-cluster interference channels of the kth SU in the lth cluster as follows:

$$\mathbf{H}_{lk} = [\mathbf{h}_{ll1}^{T}, \ldots, \mathbf{h}_{ll(k-1)}^{T}, \mathbf{h}_{ll(k+1)}^{T}, \ldots, \mathbf{h}_{llK_l}^{T}]^{T}, \tag{23}$$

where $\mathbf{H}_{lk} \in \mathbb{C}^{(K_l-1)\times N_l}$. Then, we can obtain the null space of the interference channel matrix $\mathbf{H}_{lk}$ by SVD of $\mathbf{H}_{lk}$. The $\mathbf{h}_{llk}$ is mutually independent for any k and we obtain:

$$\mathbf{H}_{lk} = \mathbf{U}_{lk}\,\Sigma_{lk}\left[\mathbf{V}_{lk}\widetilde{\mathbf{V}}_{lk}\right]^{H}, \tag{24}$$

where $\widetilde{\mathbf{V}}_{lk} \in \mathbb{C}^{N_l\times\ (N_l-K_l+1)}$ denotes the orthogonal basis of the null space of $\mathbf{H}_{lk}$, namely $\mathbf{H}_{lk}\widetilde{\mathbf{V}}_{lk} = 0$ and $\widetilde{\mathbf{V}}_{lk}^{H}\widetilde{\mathbf{V}}_{lk} = \mathbf{I}$. Thus, we have $\mathbf{T}_{lk} = \widetilde{\mathbf{V}}_{lk}$. Following this, the original problem (8) can be transformed as:

$$\max_{\{\boldsymbol{\Phi}_{lk}\}} \sum_{l=1}^{C}\sum_{k=1}^{K_l}\log_2\left(1+\frac{\mathbf{h}_{llk}\mathbf{T}_{lk}\boldsymbol{\Phi}_{lk}\mathbf{T}_{lk}^{H}\mathbf{h}_{llk}^{H}}{\sum\limits_{i\neq l}^{C}\sum\limits_{j=1}^{K_i}\mathbf{h}_{ilk}\mathbf{T}_{ij}\boldsymbol{\Phi}_{ij}\mathbf{T}_{ij}^{H}\mathbf{h}_{ilk}^{H}+\delta^2}\right) \tag{25a}$$

$$\text{s.t.} \sum_{k=1}^{K_l}\mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{T}_{lk}\boldsymbol{\Phi}_{lk}\mathbf{T}_{lk}^{H}) \leq P, \quad \forall l, O_l, \tag{25b}$$

$$\mathrm{rank}\,(\boldsymbol{\Phi}_{lk}) = 1, \tag{25c}$$

where $\boldsymbol{\Phi}_{lk} = \mathbf{s}_{lk}\mathbf{s}_{lk}^{H} \in \mathbb{C}^{(N_l-K_l+1)\times\ (N_l-K_l+1)}$. In this paper we consider the single-antenna SU and $\mathbf{s}_{lk}$ is a $(N_l - K_l + 1) \times 1$ vector, so we have (25c).

Next, we define $\widetilde{\mathbf{h}}_{llk} = \mathbf{h}_{llk}\mathbf{T}_{lk}$, $\widetilde{\mathbf{h}}_{ilj} = \mathbf{h}_{ilk}\mathbf{T}_{ij}$ and the final optimization problem can be written as:

$$\max_{\{\boldsymbol{\Phi}_{lk}\}} \sum_{l=1}^{C}\sum_{k=1}^{K_l}\widetilde{R}_{lk} \tag{26a}$$

$$\text{s.t.} \sum_{k=1}^{K_l}\mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{T}_{lk}\boldsymbol{\Phi}_{lk}\mathbf{T}_{lk}^{H}) \leq P, \quad \forall l, O_l, \tag{26b}$$

$$\mathrm{rank}\,(\boldsymbol{\Phi}_{lk}) = 1, \tag{26c}$$

where $\widetilde{R}_{lk} = \log_2\left(1+\frac{\widetilde{\mathbf{h}}_{llk}\boldsymbol{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^{H}}{\sum_{i\neq l}^{C}\sum_{j=1}^{K_i}\widetilde{\mathbf{h}}_{ilj}\boldsymbol{\Phi}_{ij}\widetilde{\mathbf{h}}_{ilj}^{H}+\delta^2}\right)$. We note here that (26) is a non-convex optimization problem due to the non-concave objective function (26a) and the rank-one constraint (26c). Problem in (26) is very difficult to be directly solved even through a centralized algorithm. On this basis, we devise a distributed algorithm based on the non-cooperative game. Finally, we prove the existence and uniqueness of the NE for the formulated non-cooperative game and propose iterative algorithm to obtain NE solution.

A. The Formulated Non-Cooperative Game Model

With the certain price, the non-cooperative game for the cluster player is defined as:

$$\mathcal{G} = \left\{\mathcal{C}, \{\boldsymbol{\Phi}_l\}_{l\in\mathcal{C}}, \{U_l\,(\mathbf{m}, \boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{-l})\}\right\}, \tag{27}$$

where $\mathcal{C} = \{1, \ldots, C\}$ is the set of all clusters; $\boldsymbol{\Phi}_l = [\boldsymbol{\Phi}_{l1}^{T}, \ldots, \boldsymbol{\Phi}_{lK_l}^{T}]^{T}\,(l \in \mathcal{C})$ denotes the precoding matrix of the lth cluster; $U_l\,(\mathbf{m}, \boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{-l})$ is the utility function of the lth cluster; $\mathbf{m} = [m_1, \ldots, m_{\mathrm{C}}]$ denotes the interference price for clusters; $\boldsymbol{\Phi}_{-l} = [\boldsymbol{\Phi}_1^{T}, \ldots, \boldsymbol{\Phi}_{l-1}^{T}, \ldots, \boldsymbol{\Phi}_{l+1}^{T}, \ldots, \boldsymbol{\Phi}_{\mathrm{C}}^{T}]^{T}$ is precoding matrix of other $(C - 1)$ clusters. We define the utility function as follows:

$$\begin{aligned} &U_l\,(\mathbf{m}, \boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{-l}) \\ &\quad= \sum_{k=1}^{K_l}\widetilde{R}_{lk} - \sum_{k=1}^{K_l}L_{lk}(\boldsymbol{\Phi}_{lk}) \\ &\quad= \sum_{k=1}^{K_l}\widetilde{R}_{lk} - \sum_{k=1}^{K_l}\sum_{i\neq l}^{C}\sum_{j=1}^{K_i}m_i\mathbf{h}_{lij}\mathbf{T}_{lk}\boldsymbol{\Phi}_{lk}\mathbf{T}_{lk}^{H}\mathbf{h}_{lij}^{H}, \end{aligned} \tag{28}$$

where L_{lk} denotes the interference imposed by the precoding vector of the kth SU in the lth cluster to all SUs in other $(C-1)$ clusters.

From (28), we find that the second term of the utility function accounts for the cost due to generated interference to other clusters, which discourages the lth cluster from maximizing its own sum rate selfishly. However, when the price vector $\mathbf{m} = \mathbf{0}$, the cluster will maximize its own sum rate uniquely.

Therefore, for the cluster player l ($l \in \mathcal{C}$), we solve the following problem:

$$\max_{\{\mathbf{\Phi}_l\}} U_l\left(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l}\right) \tag{29a}$$

$$\text{s.t.} \sum_{k=1}^{K_l} \mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H) \leq P, \quad \forall O_l, \tag{29b}$$

$$\mathrm{rank}\left(\mathbf{\Phi}_{lk}\right) = 1, \quad \forall k. \tag{29c}$$

Problem (29) includes the non-convex constraint (29c). To overcome the obstacle of non-convexity, we first reformulate the problem without the rank-one constraint and then, the obtained closed-form solution can be guaranteed to be rank-one. Following this, we formulate the optimization problem as follows:

$$\max_{\{\mathbf{\Phi}_l\}} U_l\left(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l}\right) \tag{30a}$$

$$\text{s.t.} \sum_{k=1}^{K_l} \mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H) \leq P, \quad \forall O_l. \tag{30b}$$

Definition of NE : A strategy profile $\mathbf{\Phi} = [\mathbf{\Phi}_1^T, \ldots, \mathbf{\Phi}_C^T]^T$ is a NE, if $\mathbf{\Phi}_l$ is the best response to $\mathbf{\Phi}_{-l}$ for every player l. Formally, the strategy profile $\mathbf{\Phi}_l$ is an NE if

$$U_l\left(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l}\right) \geq U_l\left(\mathbf{m}, \mathbf{\Phi}_l', \mathbf{\Phi}_{-l}\right), \quad l \in \{1, \ldots, C\}, \tag{31}$$

where $\mathbf{\Phi}_l'$ is an arbitrary profile of player l in strategy space. Next, we have the following theorem:

Theorem 1: There exists a NE for the non-cooperative game in (27).

Proof: Please refer to Appendix A for proof.

B. The Solution of the Non-Cooperative Game

Before proving the uniqueness of NE, we first solve the optimization problem (30), where a cluster obtains its best response for given other clusters' action. Since we have proved that the objective function in (30) is concave and the constraint is a convex set w.r.t. $\mathbf{\Phi}_l$, (30) is a convex optimization problem and can be solved using standard convex optimization techniques, e.g., the interior point method [36] and standard determinant maximization (MAXDET) software [37]. However, our interest is to design an algorithm for solving (30), which is based on dual method due to the zero gap between problem (30) and its dual [36].

The Lagrange dual function of (30) is defined as:

$$g(\boldsymbol{\mu}_l) = \max_{\mathbf{\Phi}_l \succeq \mathbf{0}} L\left(\mathbf{\Phi}_l, \boldsymbol{\mu}_l\right), \tag{32}$$

where

$$\begin{aligned} L\left(\mathbf{\Phi}_l, \boldsymbol{\mu}_l\right) &= \sum_{k=1}^{K_l}\left(\log_2\left(1 + \frac{\widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk} + \delta^2}\right) - \sum_{i\neq l}^{C}\sum_{j=1}^{K_i} m_i \mathbf{h}_{lij}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H\mathbf{h}_{lij}^H\right) \\ &\quad + \sum_{O_l=1}^{C_l}\mu_{O_l}\left(P - \sum_{k=1}^{K_l}\mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H)\right) \\ &= \sum_{k=1}^{K_l}\left(\log_2\left(1 + \frac{\widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk} + \delta^2}\right) - \sum_{i\neq l}^{C}\sum_{j=1}^{K_i} m_i \mathbf{h}_{lij}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H\mathbf{h}_{lij}^H \right. \\ &\quad \left. - \sum_{O_l=1}^{C_l}\mu_{O_l}\mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H)\right) + \sum_{O_l=1}^{C_l} u_{O_l}P, \end{aligned} \tag{33}$$

and $\boldsymbol{\mu}_l = [\mu_1, \ldots, \mu_{C_l}]$ a vector of dual variables each associated with one corresponding power constraint given in (30b). Accordingly, the dual optimization problem is as follows:

$$\min_{\boldsymbol{\mu}_l \succeq \mathbf{0}} g(\boldsymbol{\mu}_l). \tag{34}$$

It is obvious that (34) is convex and satisfies the Slater's condition [36], so the dual gap between the optimal objective value of (30) and that of (34) is zero. Therefore, we can solve (34) to obtain the optimal value of (30). The subgradient method [36] can be used to minimize $g(\boldsymbol{\mu}_l)$, and the dual variables $\boldsymbol{\mu}_l$ are updated as follows:

$$\mu_{O_l}(n+1) = \left[\mu_{O_l}(n) + \zeta(n)\left(P - \sum_{k=1}^{K_l}\mathrm{Tr}(\mathbf{B}_{O_l}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H)\right)\right]^+, \tag{35}$$

where $\zeta(n)$ is the diminishing step size, and n is the iterative index.

In addition, solving dual problem (34) involves determining the optimal $\mathbf{\Phi}_l$ at given dual variables $\boldsymbol{\mu}_l$. Next, we focus on solving $\mathbf{\Phi}_l$ for fixed $\boldsymbol{\mu}_l$. We find that problem (32) can be divided into K_l independent subproblems and each only involves $\mathbf{\Phi}_{lk}$. Since $\sum_{O_l=1}^{C_l} u_{O_l}P$ is a constant for fixed $\boldsymbol{\mu}_l$, for the kth SU in the lth cluster, the corresponding subproblem can be expressed as:

$$\max_{\mathbf{\Phi}_{lk} \succeq \mathbf{0}} \log_2\left(1 + \frac{\widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk} + \delta^2}\right) - \sum_{i\neq l}^{C}\sum_{j=1}^{K_i} m_i \mathbf{h}_{lij}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H\mathbf{h}_{lij}^H - \mathrm{Tr}(\mathbf{B}_\mu\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H), \tag{36}$$

where $\mathbf{B}_\mu = \sum_{O_l=1}^{C_l}\mu_{O_l}\mathbf{B}_{O_l}$.

Next, we define the objective function of (36) as $\mathcal{L}(\mathbf{\Phi}_{lk})$ and then, we obtain:

$$
\begin{aligned}
&\mathcal{L}(\mathbf{\Phi}_{lk})\\
&= \log_2\left(1+\frac{\widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk}+\delta^2}\right)-\sum_{i\neq l}^{C}\sum_{j=1}^{K_i} m_i \mathrm{Tr}(\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H\mathbf{h}_{lij}^H\mathbf{h}_{lij}\mathbf{T}_{lk})\\
&\quad -\mathrm{Tr}(\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H\mathbf{B}_{\mu}\mathbf{T}_{lk})\\
&= \log_2\left(1+\frac{\widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk}+\delta^2}\right)\\
&\quad -\mathrm{Tr}\left(\mathbf{\Phi}_{lk}\left(\sum_{i\neq l}^{C}\sum_{j=1}^{K_i} m_i\mathbf{T}_{lk}^H\mathbf{h}_{lij}^H\mathbf{h}_{lij}\mathbf{T}_{lk}\right)\right)\\
&\quad -\mathrm{Tr}(\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H\mathbf{B}_{\mu}\mathbf{T}_{lk})\\
&= \log_2\left(1+\frac{\widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk}+\delta^2}\right)-\mathrm{Tr}(\mathbf{Z}_{lk}\mathbf{\Phi}_{lk}),
\end{aligned}
\tag{37}
$$

where $\mathbf{Z}_{lk}=\sum_{i\neq l}^{C}\sum_{j=1}^{K_i} m_i\mathbf{T}_{lk}^H\mathbf{h}_{lij}^H\mathbf{h}_{lij}\mathbf{T}_{lk}+\mathbf{T}_{lk}^H\mathbf{B}_{\mu}\mathbf{T}_{lk}$ and $\mathbf{Z}_{lk}\in\mathbb{C}^{(N_l-K_l+1)\times(N_l-K_l+1)}$. Meanwhile, for the above derivation, some equations are used, such as $\mathrm{Tr}(\mathbf{XY})=\mathrm{Tr}(\mathbf{YX})$ and $a\mathrm{Tr}(\mathbf{X})+b\mathrm{Tr}(\mathbf{Y})=\mathrm{Tr}(a\mathbf{X}+b\mathbf{Y})$. We then have the following theorem.

Theorem 2: For the problem in (36) to have a bounded objective value, matrix $\mathbf{Z}_{lk}$ should be positive definite.

Proof: Please refer to Appendix B for proof.

We rewrite $\mathbf{\Phi}_{lk}$ in its original form, i.e., $\Phi_{lk}=\mathbf{s}_{lk}\mathbf{s}_{lk}^H$. Accordingly, the problem (36) can be transformed as follows:

$$\max_{\mathbf{s}_{lk}\succeq\mathbf{0}}\log_2\left(1+\frac{\widetilde{\mathbf{h}}_{llk}\mathbf{s}_{lk}\mathbf{s}_{lk}^H\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk}+\delta^2}\right)-\mathbf{s}_{lk}^H\mathbf{Z}_{lk}\mathbf{s}_{lk}. \tag{38}$$

According to Cholesky decomposition [38], we have $\mathbf{Z}_{lk}=\hat{\mathbf{Z}}_{lk}\hat{\mathbf{Z}}_{lk}^H$ and $\hat{\mathbf{Z}}_{lk}$ is reversible due to $\mathbf{Z}_{lk}$ is positive definite. We define $\hat{\mathbf{s}}_{lk}=\mathbf{s}_{lk}^H\hat{\mathbf{Z}}_{lk}$ and then, (38) can be rewritten as follows:

$$\max_{\hat{\mathbf{s}}_{lk}\succeq\mathbf{0}}\log_2\left(1+\frac{\widetilde{\mathbf{h}}_{llk}\hat{\mathbf{Z}}_{lk}^{-H}\hat{\mathbf{s}}_{lk}^H\hat{\mathbf{s}}_{lk}\hat{\mathbf{Z}}_{lk}^{-1}\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk}+\delta^2}\right)-\hat{\mathbf{s}}_{lk}\hat{\mathbf{s}}_{lk}^H. \tag{39}$$

Next, we define $\mathbf{a}_{lk}=\widetilde{\mathbf{h}}_{llk}\hat{\mathbf{Z}}_{lk}^{-H}/||\widetilde{\mathbf{h}}_{llk}\hat{\mathbf{Z}}_{lk}^{-H}||$. It is clear that the optimal precoding $\hat{\mathbf{s}}_{lk}$ has the same direction with $\mathbf{a}_{lk}$, i.e., $\hat{\mathbf{s}}_{lk}=\sqrt{p_{lk}}\mathbf{a}_{lk}$, where p_{lk} needs to be optimized to maximize (39). Based on this, substituting $\hat{\mathbf{s}}_{lk}=\sqrt{p_{lk}}\mathbf{a}_{lk}$ into (39), we get the following optimization problem:

$$\max_{p_{lk}\succeq\mathbf{0}}\log_2\left(1+\frac{p_{lk}\alpha_{lk}}{\Xi_{lk}+\delta^2}\right)-p_{lk}. \tag{40}$$

where $\alpha_{lk}=||\widetilde{\mathbf{h}}_{llk}\hat{\mathbf{Z}}_{lk}^{-H}||^2$, and optimal p_{lk} can be obtained by the standard water-filling algorithm [36]:

$$p_{lk}=\left(\frac{1}{\ln 2}-\frac{\Xi_{lk}+\delta^2}{\alpha_{lk}}\right)^{+}. \tag{41}$$

Finally, we obtain the optimal precoding as follows:

$$\mathbf{s}_{lk}=\sqrt{p_{lk}}(\mathbf{a}_{lk}\hat{\mathbf{Z}}_{lk}^{-1})^H,\quad \text{and } \mathbf{\Phi}_{lk}=p_{lk}(\mathbf{a}_{lk}\hat{\mathbf{Z}}_{lk}^{-1})^H\mathbf{a}_{lk}\hat{\mathbf{Z}}_{lk}^{-1}. \tag{42}$$

Algorithm 2 Non-Cooperative Game-Based Precoding Design For Each Cluster

```
Form multiple SBS clusters according to Algorithm 1.
Given the price vector m, initialize the feasible precoding
  Φ^(0) = [Φ_1^(0), ..., Φ_C^(0)], set a counter n = 1.
repeat
    Design precoding for each cluster l ∈ {1, ..., C}.
    Initialize μ_l.
    repeat
        Compute Φ_lk (k = 1, ..., K_l) according to (42).
        Update dual varies according to (35).
    until μ_l converges;
    Obtain Φ^(n).
    Update n ← n + 1.
until ||Φ^(n) − Φ^(n−1)|| ≤ ξ, for some prescribed ξ;
Obtain the optimal precoding Φ^(n).
```

From (42), it is clear that $\mathbf{\Phi}_{lk}$ is a rank-one solution. Therefore, the solution of relaxed problem (30) is also the solution of the original problem (29) via our proposed algorithm. Based on the above results, we define the following theorem:

Theorem 3: There exists an unique NE for the non-cooperative game (27).

Proof: Please refer to Appendix C for proof.

C. NE Searching Algorithm

For SC clustering, the SBSs send SUs' CSI to MBS through high speed backhaul links. Next, the MBS executes Algorithm 1 according to a predefined interference threshold and then, communicates the final clustering information with an interference price of each cluster to SBSs through backhaul links. After forming the clusters, each cluster sets the initial feasible precoding (we assume that one of SBSs takes charge of the precoding design and denoted as SBS header (SBSH)). Since the SBSH has obtained all CSI through sharing among SBSs belonging to the same cluster, the precoding can be computed when SUs send back the received interference to SBSH. The SBSH will update precoding when SUs send back the updated interference, and the process is executed until convergence. We assume that the update of the interference price and precoding strategy among clusters are ideal synchronous. According to the above analysis, we find that information exchange is not needed among clusters. We summarize the distributed precoding design scheme for each cluster as Algorithm 2.

VII. Numerical Results

In this section, we provide numerical results to evaluate the performance of our proposed schemes. We consider a single MC with a radius of 500 meters, where the MBS is located at the center of the MC and MUs are randomly distributed in the MC. We assume that the MC-hole radius is 100 meters (all MUs and SUs do not locate in this area). We assume that the radius of each SC is 40 meters, where all SCs are randomly located within the MC but their coverage are not overlapped

TABLE II
SIMULATION PARAMETERS

Parameters	Value
Radius of MC	500 m
Radius of SC	40 m
Number of MUs	20
Number of SCs	20
Number of SUs each SC	2
Number of SBS antennas	2
Number of MBS antennas	500
Transmit power of MBS	46 dBm
Maximum transmit power of SBS	30 dBm
Pathloss between MBS and MU or SU	$27.3+39.1\log 10(d)$
Pathloss between SBS and MU or SU	$36.8+36.7\log 10(d)$
Downlink bandwidth	10 MHz
Noise power	-174 dBm/Hz

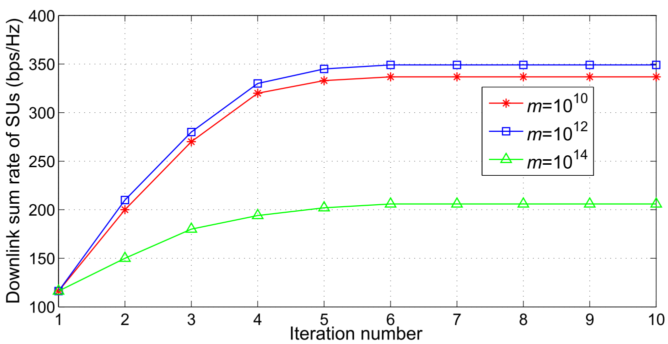


Fig. 4. Downlink sum rate of SUs versus iteration number.

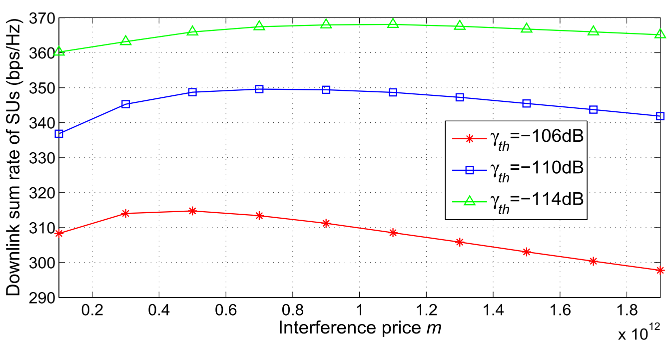


Fig. 5. Downlink sum rate of SUs versus interference price.

each other. The minimum distance between SUs and SBS is 5 meters. We assume that all prices for different clusters are the same for simplicity. Other related simulation parameters are listed in Table II. Examples of users and SCs distributions are given in Fig. 3.

Fig. 4 shows the convergent speed under different interference prices, where $\gamma_{th} = -110$ dB. It is clearly found that the downlink sum rate of SUs is maximized after 6 iterations. The sum rate under different interference prices is different. To clearly analyze the impact of the interference price on the performance of the system, we plot Fig. 5 to show the downlink sum rate of SUs with the interference price m. It can be observed that the sum rate first increases and then decreases with m. In other words, there exits an optimal m for maximizing the sum rate under a certain γ_{th}. Therefore,

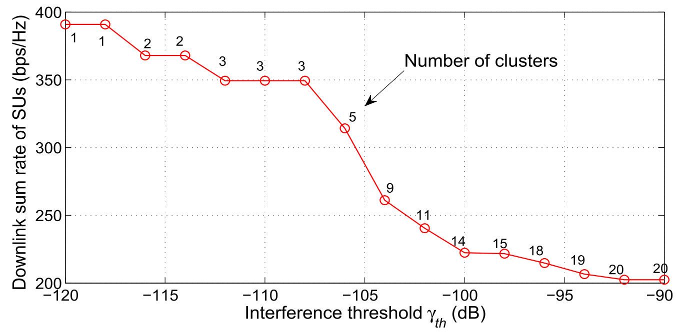


Fig. 6. Downlink sum rate of SUs versus interference threshold.

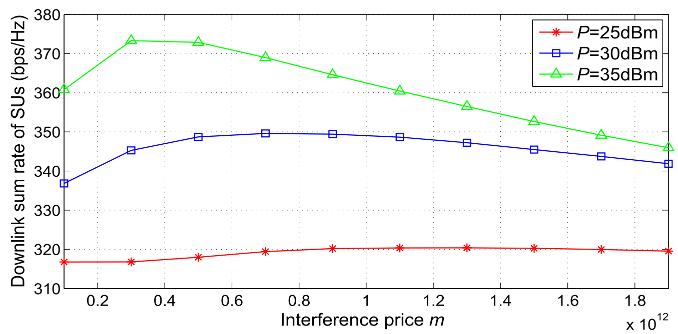


Fig. 7. Downlink sum rate of SUs versus interference price.

we can use some simple methods, e.g., one-dimension search, to find the optimal m for obtaining the maximum sum rate. Meanwhile, we can find that the sum rate increases with γ_{th} decreases. This is because more larger-size clusters are formed so that more interference are cancelled.

Fig. 6 shows the downlink sum rate versus interference threshold γ_{th}. Here, the one-dimension search is used to obtain the maximum sum rate under each interference threshold. For a low γ_{th}, i.e., $\gamma_{th} = -120$ dB, all SCs form one cluster. In this case, the interference among SUs will be cancelled completely, so that the sum rate is maximized. Number of clusters increases as γ_{th} increases, while the sum rate decreases. This is because the interference increases among SUs due to the increase of the number of clusters. When γ_{th} is between -92 dB and -90 dB, there are no pairs of SCs to form the same cluster, resulting in the lowest sum rate.

Fig. 7 plots the downlink sum rate versus m under different per-SBS transmit power, where $\gamma_{th} = -110$ dB. When the maximum transmit power of each SBS is high, as shown in Fig. 5, the sum rate first increases and then decreases with m. However, when the maximum transmit power of each SBS is low, the impact of m on the sum rate is slow. This is because for the former, each SBS is not allowed to fully transmit such large power for avoiding serious interference among clusters. On the contrary, for the later, each SBS can almost transmit overall power for improving sum rate.

Fig. 8 illustrates the downlink sum rate of SUs versus number of SUs in each SC, where $\gamma_{th} = -104$ dB and $N = 6$. It can be observed that the sum rate increases with the number of SUs, while the increased ratio is reduced. It is easy to understand that more SUs lead to higher gain gaps

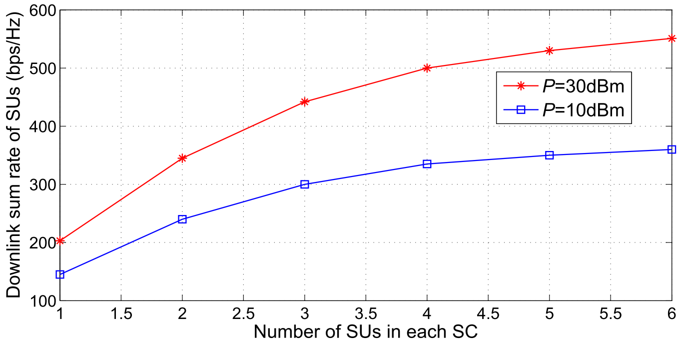


Fig. 8. Downlink sum rate of SUs versus number of SUs.

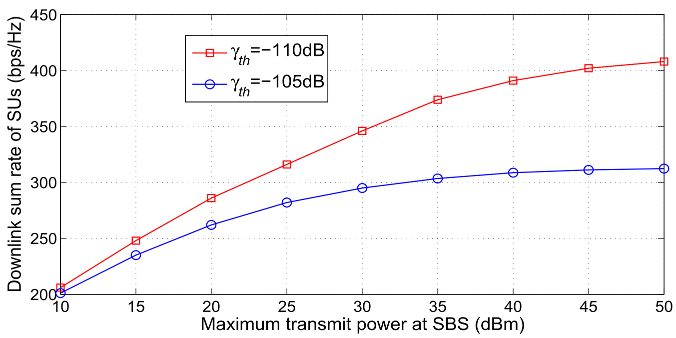


Fig. 9. Downlink sum rate of SUs versus transmit power.

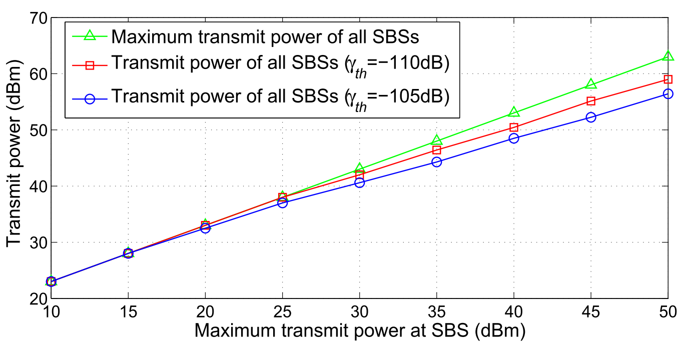


Fig. 10. Transmit power versus available transmit power each SBS.

and improve the sum rate. However, the increased sum rate is limited because the total transmit power is a constant.

Figs. 9 and 10 show the downlink sum rate and transmit power versus the maximum transmit power of each SBS, respectively. It is easy to understand that the sum rate increases with the maximum transmit power. However, the increase of the sum rate is limited due to interference among clusters, especially for a lower γ_{th}. The detailed reason can be found in Fig. 10. When the maximum transmit power of each SBS is low, the power will be fully transmitted due to weak interference among clusters. However, as the maximum transmit power increases, to avoid serious interference, the transmit power at each SBS is not allowed such high.

Fig. 11 shows the downlink sum rate of MUs versus number of MBS antennas. Here, "Conventional ZF precoding" denotes that only the ZF precoding is applied at MBS to eliminate the multi-MU interference, while the proposed CSBD precoding

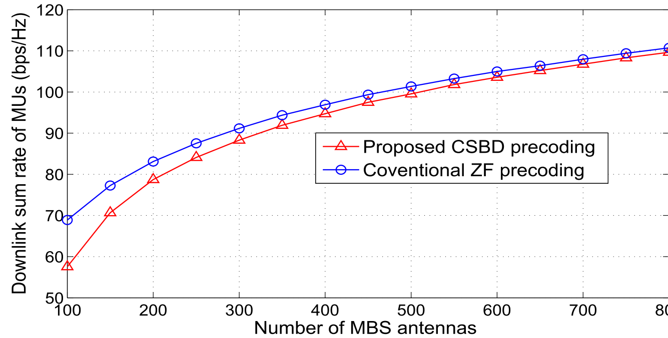


Fig. 11. Downlink sum rate of MUs versus number of MBS antennas.

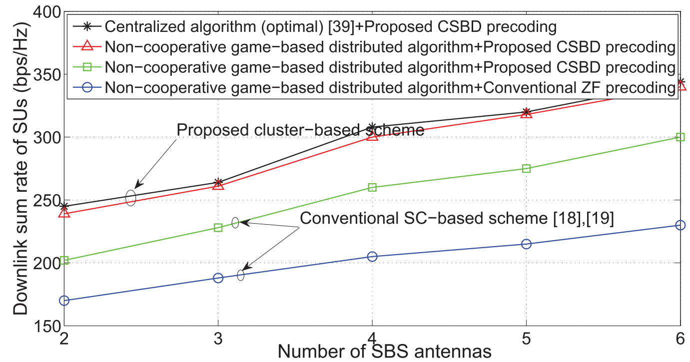


Fig. 12. Downlink sum rate of MUs versus number of SBS antenas.

scheme works to eliminate the inter-tier and multi-MU interference simultaneously. From this figure, we can find that the downlink sum rate of MUs with proposed CSBD precoding is always lower than that with ZF precoding, but the sum-rate gap is slight when the number of MBS antennas is larger. The reason is that when the proposed CSBD precoding is applied at MBS, the MBS needs to sacrifice some degree of freedom (DoF) to eliminate SUs' interference, which results in the decrease in the sum rate of MUs. However, for a large antennas at MBS, it is suitable to apply the proposed CSBD precoding for eliminating SUs' interference, which can effectively improve the downlink sum rate of SUs (as shown in Fig. 12). Meanwhile, the impact on the downlink sum rate of MUs is slight.

In Fig 12, we plot that the downlink sum rate of SUs versus number of SBS antennas, where $\gamma_{th} = -104$ dB. We compare the performance gain with the conventional SC-based non-cooperative game scheme, e.g., [18], [19]. It is clear that sum rate with our proposed cluster-based scheme is higher than that with conventional SC-based scheme. It means that it is necessary to form multiple clusters for interference cancellation, especially for ultra-dense SCs. In addition, to compare with the performance loss with optimal algorithm, we apply the centralized algorithm proposed in [39] to obtain the optimal solution. We can find the rate gap is slight between optimal centralized algorithm and our proposed distributed algorithm. Meanwhile, it can be observed that the proposed CSBD effectively reduces the inter-tier interference and improves the sum rate of the SUs.

VIII. Conclusions and Future Works

We have investigated the SC clustering and precoding design problems for the mMIMO-HetNet. An interference graph-based dynamic SC clustering scheme has been proposed in order to cooperative transmission among SBSs belonging to the same cluster. We designed precoding schemes for MBS to eliminate the inter-tier and multi-MU interference. Then, we presented the precoding design at clustered SCs as an optimization problem to maximize downlink sum rate of SUs under per-SBS power constraint. A non-cooperative game-based distributed algorithm was proposed to obtain a suboptimal solution. Simulation results show that proposed schemes effectively improve the downlink sum rate of SUs.

In this paper, we have considered single MC scenario. For multi-MC scenario, the proposed CSBD precoding can be applied to cancel the interference from MBS to SUs and MUs located adjacent MCs, especially for edged SUs and MUs. However, larger number of MBS antennas and more efficient cooperation among MCs are necessary. In addition, it is not realistic approach to directly extend the proposed game model to MC scenario, because it is difficult to obtain the overall CSI for each SC and the overheads of information exchange is also large. Therefore, the CSI, the information overhead and cooperation problem among SCs and MBSs should be considered when the non-cooperative game is designed in multi-MC scenario. The analysis for multiple MC scenarios is left as an interesting future work. In addition, channel estimation error affects the achievable system performance (e.g., rate and energy efficiency). The effect of imperfect CSI on the precoder design and related analysis is another interesting future work.

Appendix A
The Proof of *Theorem 1*

Proof: According to the Nash theorem [40], NE exists if the following conditions hold:1) The action space of each player is convex and compact. 2) The utility function $U_l(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l})$ is concave with respect to (w.r.t.) $\mathbf{\Phi}_l$.

According to (30b), we can easily get that the action space $\mathbf{\Phi}_l$ satisfies the above 1). Next, we still need to prove that $U_l(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l})$ is concave w.r.t. $\mathbf{\Phi}_l$.

$$\begin{aligned} &U_l(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l}) \\ &= \sum_{k=1}^{K_l} \widehat{U}_{lk}(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l}) = \sum_{k=1}^{K_l} \left(\widetilde{R}_{lk} - L_{lk}\right) \\ &= \sum_{k=1}^{K_l} \left(\log_2\left(1 + \frac{\widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk} + \delta^2}\right) \right. \\ &\qquad \left. - \sum_{j\neq l}^{C}\sum_{i=1}^{K_j} m_j \mathbf{h}'_{lji}\mathbf{T}_{lk}\mathbf{\Phi}_{lk}\mathbf{T}_{lk}^H\mathbf{h}'^H_{lji} \right), \end{aligned} \tag{43}$$

where $\Xi_{lk} = \sum_{i\neq l}^{C}\sum_{j=1}^{K_i} \widetilde{\mathbf{h}}_{ilj}\mathbf{\Phi}_{ij}\widetilde{\mathbf{h}}_{ilj}^H$. Then, we have:

$$\begin{aligned} &\frac{\partial^2 \widehat{U}_{lk}(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l})}{\partial^2 \mathbf{\Phi}_{lk}} \\ &\qquad = -\frac{\Xi_{lk} + \delta_2}{\ln 2}\frac{\widetilde{\mathbf{h}}_{lk}^H\widetilde{\mathbf{h}}_{lk}\left(\widetilde{\mathbf{h}}_{lk}^H\widetilde{\mathbf{h}}_{lk}\right)^H}{\left(\Xi_{lk} + \delta_2 + \widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}\widetilde{\mathbf{h}}_{llk}^H\right)^2} \preceq 0. \end{aligned} \tag{44}$$

According to (44), we can verify that $\widehat{U}_{lk}(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l})$ is concave w.r.t. $\mathbf{\Phi}_{lk}$ [36], so the utility function $U_l(\mathbf{m}, \mathbf{\Phi}_l, \mathbf{\Phi}_{-l})$ is concave w.r.t. $\mathbf{\Phi}_l$. Therefore, (27) is a concave game and there exists one NE.

Appendix B
The Proof of *Theorem 2*

Proof: It is clear that matrix $\mathbf{Z}_{lk}$ is a symmetric matrix. Therefore, we only need to prove that matrix $\mathbf{Z}_{lk}$ is a full-rank matrix. Next, we prove it by contradiction. We assume that $\mathbf{Z}_{lk}$ is not a full-rank matrix and then, we can always find a vector $\mathbf{q}_{lk} \in \mathbb{C}^{(N_l-K_l+1)\times 1}$ such that $\mathbf{Z}_{lk}\mathbf{q}_{lk} = \mathbf{0}$ and $\widetilde{\mathbf{h}}_{llk}\mathbf{q}_{lk} \neq \mathbf{0}$. On this basis, we assume the optimal $\mathbf{\Phi}_{lk}^* = x\mathbf{q}_{lk}\mathbf{q}_{lk}^H (x \geq 0)$. Substituting this optimal solution into (36), we have:

$$\begin{aligned} &\log_2\left(1 + \frac{\widetilde{\mathbf{h}}_{llk}\mathbf{\Phi}_{lk}^*\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk} + \delta^2}\right) - \mathrm{Tr}(\mathbf{Z}_{lk}\mathbf{\Phi}_{lk}^*) \\ &\qquad = \log_2\left(1 + \frac{x\widetilde{\mathbf{h}}_{llk}\mathbf{q}_{lk}\mathbf{q}_{lk}^H\widetilde{\mathbf{h}}_{llk}^H}{\Xi_{lk} + \delta^2}\right). \end{aligned} \tag{45}$$

Since $\widetilde{\mathbf{h}}_{llk}\mathbf{q}_{lk}\mathbf{q}_{lk}^H\widetilde{\mathbf{h}}_{llk}^H > 0$, the objective value is unbounded as x goes to infinity. Therefore, the original assumption that $\mathbf{Z}_{lk}$ not is a full-rank matrix is not correct. Furthermore, $\mathbf{Z}_{lk}$ is a positive definite matrix .

Appendix
The Proof of *Theorem 3*

Proof: The concept of the standard function is defined in [41] to prove the uniqueness of the NE for a non-cooperative game. Here, we define $\mathbf{\Phi}_{lk} = \mathbf{\Phi}_{lk}(\Xi_{lk})$, namely other parameters ($\mathbf{m}$ and $\boldsymbol{\mu}_l$) other than Ξ_{lk} can be taken as given constants. Next, we need to prove that $\mathbf{\Phi}_{lk}(\Xi_{lk})$ is a standard function with respect to (w.r.t.) Ξ_{lk}, which must hold following: 1) positive: $\mathbf{\Phi}_{lk}(\Xi_{lk}) \succeq \mathbf{0}$; 2) monotonic; 3) scalable: $\lambda\mathbf{\Phi}_{lk}(\Xi_{lk}) \succ \mathbf{\Phi}_{lk}(\lambda\Xi_{lk})$ for any $\lambda > 1$.

First, we rewrite the $\mathbf{\Phi}_{lk}$ as follows:

$$\mathbf{\Phi}_{lk}(\Xi_{lk}) = \left(\frac{1}{\ln 2} - \frac{\Xi_{lk} + \delta^2}{\alpha_{lk}}\right)^+ \Theta_{lk}, \tag{46}$$

where $\Theta_{lk} = (\mathbf{a}_{lk}\hat{\mathbf{Z}}_{lk}^{-1})^H\mathbf{a}_{lk}\hat{\mathbf{Z}}_{lk}^{-1}$.

From (46), it is clear that $\Theta_{lk} \succeq \mathbf{0}$ and $\mathbf{\Phi}_{lk}(\Xi_{lk})$ is a monotonically decreasing function w.r.t. Ξ_{lk}. Therefore, 1) and 2) hold. Next, we focus on proving 3).

$$\begin{aligned} &\lambda\mathbf{\Phi}_{lk}(\Xi_{lk}) - \mathbf{\Phi}_{lk}(\lambda\Xi_{lk}) \\ &= \lambda\left(\frac{1}{\ln 2} - \frac{\Xi_{lk}+\delta^2}{\alpha_{lk}}\right)^+\Theta_{lk} - \left(\frac{1}{\ln 2} - \frac{\lambda\Xi_{lk}+\delta^2}{\alpha_{lk}}\right)^+\Theta_{lk} \\ &= (\lambda - 1)\left(\frac{1}{\ln 2} - \frac{\delta^2}{\alpha_{lk}}\right)^+\Theta_{lk} \\ &\succ (\lambda - 1)\left(\frac{1}{\ln 2} - \frac{\Xi + \delta^2}{\alpha_{lk}}\right)^+\Theta_{lk} \\ &= (\lambda - 1)\mathbf{\Phi}_{lk}(\Xi_{lk}) \succeq \mathbf{0}. \end{aligned} \tag{47}$$

According to (47), it can be verified that the above 3) is held and $\mathbf{\Phi}_{lk}(\Xi_{lk})$ is a standard function w.r.t. Ξ_{lk}. Since a standard function will converge to a unique value [41], the NE for the non-cooperative game (27) is unique and we finish the proof.

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