OR 7245 Network Analysis and Advanced Optimization

HELP TO CURE CANCER

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Goals

To maximize the radiation dosage over the tumor area and minimizing dosage over the critical area.

Description

We are given the above statement as the goal for which we need to find different approaches. The input data of the tumor area and critical area pixel location and the number of beams varies according to the patient and the machinery used. The upper limit for the minimum dosage over the critical area and the lower limit for the maximum dosage over tumor area is also defined by the doctors. There is one common objective of which we change the variables according to the approach we select. Q

Question 1:
Trying to find multipliers representing the beam intensities such that they satisfy the multiple objectives.

We were given two data sets; one was a data set and the other was pertaining to a patient and the proposed radiation treatment data. We ran our model on both these data sets. The model gave an infeasible solution.

MODEL:

$$minimize \sum_{m,n \in C} \sum_{i=1}^{M} X_i * target(i,m,n) - \sum_{m,n \in T} \sum_{i=1}^{M} X_i * target(i,m,n)$$

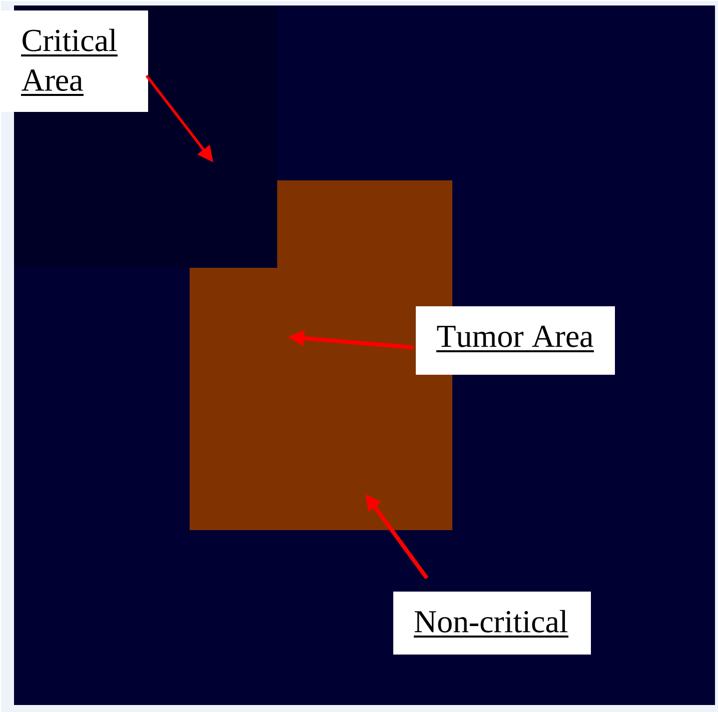
s.t

$$\sum_{i=1}^{M} X_i * target(i,m,n) \geq tumor\_lower \;\; \forall \, (m,n) \in T$$
$$\sum_{i=1}^{M} X_i * target(i,m,n) \leq critical\_upper \;\; \forall \, (m,n) \in C$$
$$x_i \;\; \geq 0 \; \forall \; i = 1 \ldots M$$
$$T = Tumor\ area$$
$$C = Critical\ area$$

In above model we are minimizing the dosage over critical area and the negative of over tumor area which is equivalent to maximizing.
The parameter $target(i, m. n)$ is every entry in the matrices which defines the critical and tumor area.
Variable X_i is the multiplier or intensity scalar for each beam pattern and determines the exact pattern of the.
Test data:

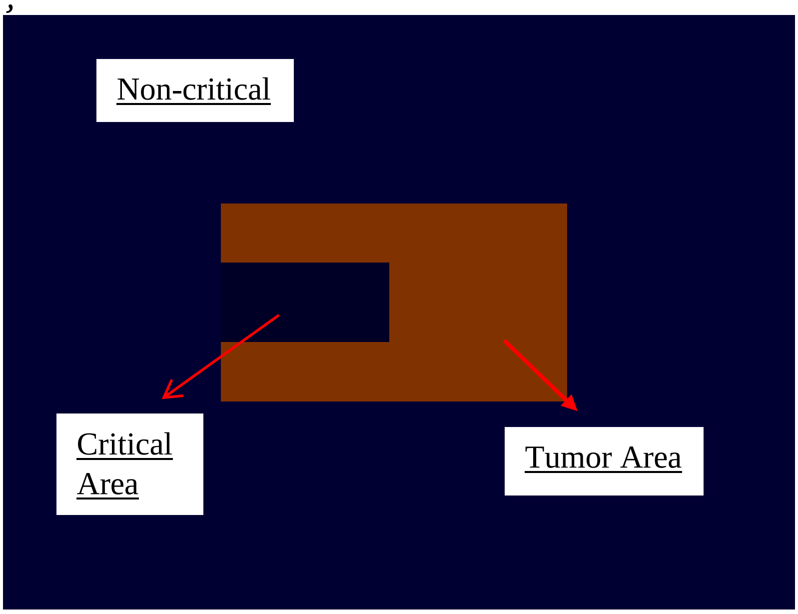
Output:
```
X [*] :=
1  0
2  0
3  0
4  0
5  0
;

dosage = 0
```



Actual Data:

```
X [*] :=
  1 0    14 0    27 0    40 0    53 0    66 0    79 0    92 0   105 0   118 0
  2 0    15 0    28 0    41 0    54 0    67 0    80 0    93 0   106 0   119 0
  3 0    16 0    29 0    42 0    55 0    68 0    81 0    94 0   107 0   120 0
  4 0    17 0    30 0    43 0    56 0    69 0    82 0    95 0   108 0   121 0
  5 0    18 0    31 0    44 0    57 0    70 0    83 0    96 0   109 0   122 0
  6 0    19 0    32 0    45 0    58 0    71 0    84 0    97 0   110 0   123 0
  7 0    20 0    33 0    46 0    59 0    72 0    85 0    98 0   111 0   124 0
  8 0    21 0    34 0    47 0    60 0    73 0    86 0    99 0   112 0   125 0
  9 0    22 0    35 0    48 0    61 0    74 0    87 0   100 0   113 0   126 0
 10 0    23 0    36 0    49 0    62 0    75 0    88 0   101 0   114 0
 11 0    24 0    37 0    50 0    63 0    76 0    89 0   102 0   115 0
 12 0    25 0    38 0    51 0    64 0    77 0    90 0   103 0   116 0
 13 0    26 0    39 0    52 0    65 0    78 0    91 0   104 0   117 0
;
```



Question 2:

Trying to find multipliers representing the beam intensities such that they satisfy the multiple objectives while trying to accommodate the range of values with feasibility.

MODEL:

$$minimize \sum_{x,y\,\in C}\sum_{i=1}^{M} X_i * target(i,m,n) + \sum_{x,y\,\in C} B(m,n) + \sum_{x,y\,\in T} A(m,n) - \sum_{x,y\,\in T}\sum_{i=1}^{M} X_i * target(i,m,n)$$

s.t

$$\sum_{i=1}^{M} X_i * target(i,m,n) = tumor\_lower\ - A(m,n)\ \forall\ (m,n) \in T$$

$$\sum_{i=1}^{M} X_i * target(i,m,n) = critical\_upper\ + B(m,n)\ \ \forall\ (m,n) \in C$$

$$T = Tumor\ area$$

$$C = Critical\ area$$

$$x_i\ \ \geq 0\ \forall\ i = 1 \ldots M$$

To overcome the infeasibility, we introduce two new variables to relax the limits. These two new variables are defined over the tumor and critical area such that they are positive. We will try to minimize the summation of all such critical and tumor variables respectively.

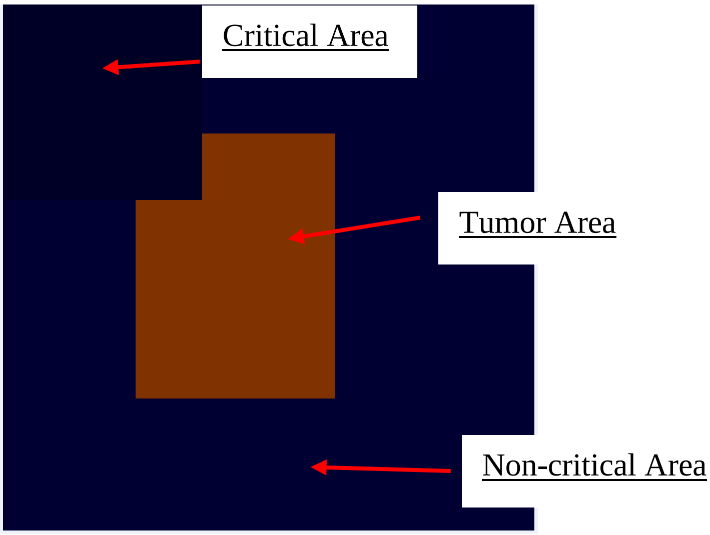
Test Data;

Output:

```
X [*] :=
1  0
2  0
3  0
4  0
5  0
;

dosage = 0
```

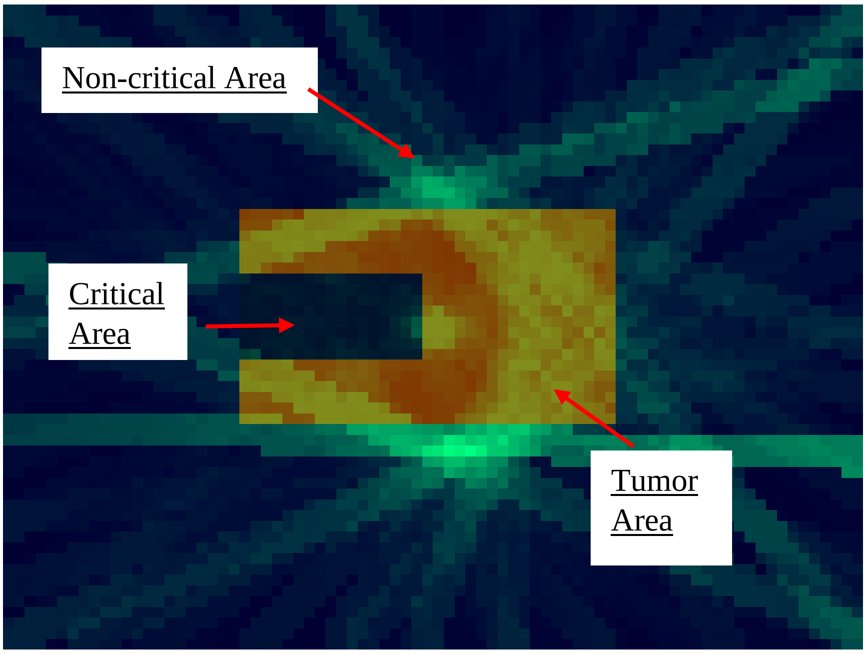
Visual Output:



Actual Data:

```
X [*] :=
  1  0           33  0.308441     65  3.6441       97  0.471012
  2  0           34  0            66  1.12096      98  1.6694
  3  0           35  2.06121      67  0            99  0.139688
  4  0           36  0.454205     68  0.299376    100  0.0766539
  5  2.68613     37  0.687599     69  3.75402     101  0.464318
  6  0.161672    38  0            70  1.2393      102  0
  7  0.356887    39  0            71  3.42235     103  0.426541
  8  0.273762    40  1.33707      72  3.21319     104  0
  9  0           41  1.51244      73  0.683713    105  0.370985
 10  0           42  0.820899     74  0.566803    106  0.455215
 11  2.04092     43  0.26536      75  0           107  0.761615
 12  0.658716    44  0.350996     76  0.63572     108  0
 13  0.165364    45  0.471888     77  6.3483      109  0.757702
 14  0.0188686   46  0.617809     78  0           110  0
 15  2.06017     47  0            79  0.294219    111  0.137583
 16  0           48  1.1572       80  0           112  0.827166
 17  0           49  0.351935     81  0           113  1.01864
 18  0.0846922   50  0            82  0           114 12.1517
 19  0.30581     51  1.74028      83  0           115  0.883539
 20  0           52  0            84  0           116  1.15576
 21  2.85745     53  0            85  0           117  0.0889129
 22  0           54  0.627046     86  0.347375    118  0
 23  1.77178     55  0.286086     87  0           119  0.0775345
 24  0           56  0            88  0           120  0
 25  0           57  0.868638     89  0.158884    121  0
 26  0.111006    58  0            90  2.02511     122  1.41011
 27  2.95747     59  0            91  0.418875    123  0
 28  0           60  0            92  6.81471     124  1.25166
 29  2.00609     61  0            93  0.890358    125  0
 30  0.254125    62  0.122989     94  0           126  0
 31  0.470935    63  0            95  0
 32  0           64  0            96  0
;
critical_dosage = -871.72
```

Visual Output:



We can see that the maximum beams pass through the tumor area. However there are quite a few beams passing through the non-critical area. This might not be always desirable. We will discuss in the further models how can we reduce the dosage over the non-critical area.

Question 3:

In this question we have penalised the border area between the critical and non-critical area.

MODEL:

$$minimize \sum_{x,y\in C}\sum_{i=1}^{M} X_i * target(i,m,n) + \sum_{x,y\in C} B(m,n) + \sum_{x,y\in T} A(m,n) - \sum_{x,y\in T}\sum_{i=1}^{M} X_i * target(i,m,n) + q * \sum_{m,n\in NCB}\sum_{i=1}^{M} X_i * target(i,m.n)$$

s.t

$$\sum_{i=1}^{M} X_i * target(i,m,n) = tumor\_lower - A(m,n)\ \forall\ (m,n) \in T$$

$$\sum_{i=1}^{M} X_i * target(i,m,n) = critical\_upper + B(m,n)\ \forall\ (m,n) \in C$$

$$\sum_{i=1}^{M} X_i * target(i,m,n) \le critical\_upper\ \forall\ (m,n) \in NCB$$

$$x_i \ \ge 0\ \forall\ i = 1 \dots M$$

$$T = Tumor\ area$$
$$C = Critical\ Area$$
$$NCB = Non - critical\ border$$
$$q = penalty\ assigned\ to\ th\ e\ non - critical\ border\ area$$

In this model we defined the border area between the critical and non-critical area. We minimized the variables over the border area, We penalise the non-critical area border in the objective function. We decide the value of the penalty according to the risk associated with dosage over the area.

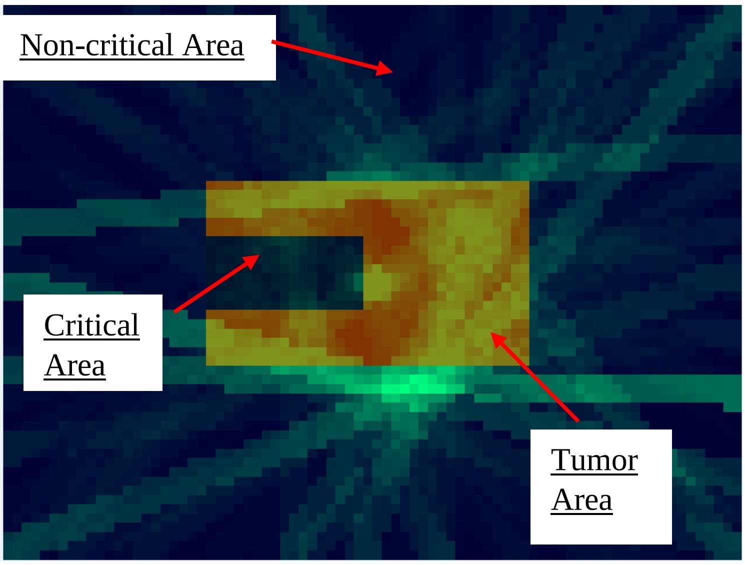
Output:

```
X [*] :=
  1  0           33  0           65  3.1273        97  0
  2  0           34  0           66  1.62282       98  7.4361
  3  0           35  1.944       67  0.00760839    99  0
  4  0           36  0.758449    68  0            100  0
  5  5.25524     37  0           69  0.644417     101  0.842775
  6  0.930957    38  0.0235188   70  1.8677       102  0
  7  0           39  0.366123    71  0.327777     103  0
  8  0.0356945   40  1.09549     72  3.36858      104  0
  9  0           41  0.972329    73  1.21423      105  0.0042744
 10  0           42  1.44506     74  0            106  0.55133
 11  1.06475     43  0           75  0            107  0.653429
 12  0.862396    44  0.694015    76  1.18578      108  0
```

13	0.475694	45	0.917239	77	1.00974	109	0.402458
14	0.0778524	46	0.722294	78	1.58774	110	0
15	0.560718	47	0	79	0	111	0
16	0	48	0.678515	80	0.0585956	112	0.493742
17	0.856757	49	0	81	0	113	0.445627
18	0	50	0.571158	82	0	114	10.8386
19	0.556753	51	2.27789	83	0	115	0.515331
20	0	52	0	84	0	116	0.476795
21	1.31416	53	0.108533	85	0.398594	117	0
22	0	54	1.07505	86	0.0267812	118	0
23	0.662288	55	0.101225	87	0	119	0.1237
24	0	56	1.18854	88	0.500073	120	0
25	0.343439	57	0.716938	89	0	121	0
26	0.462607	58	0	90	1.32865	122	0
27	2.3984	59	0.888426	91	0	123	0
28	0	60	0	92	0	124	0.938789
29	2.99209	61	0.575156	93	6.68951	125	0
30	0.487896	62	0	94	1.30855	126	0.450341
31	0	63	0	95	0		
32	0	64	0	96	0		

;

total = -530.531



Question 4:

The above model can be improved based on various variables or parameters. We thought about a few enhancements that can be done.

We implement two of the above enhancement models.

Model 1:

Our objective here is to minimize the dose over the non-critical and critical areas and the priority for the tumor dose is the least. We do this considering the case of brain tumor where each cell/tissue is sensitive. Though the imaging techniques are highly advanced we might categorize the critical area as a non-critical area. We have placed weights in the objective function so the user can place a greater emphasis on either reducing non-critical or critical dose. Also, high dose sent to the patient could lead to patient complications, thus we add an upper limit to the tumor dose.

$$minimize \quad q * \sum_{x,y \in C} \sum_{i=1}^{M} X_i * target(i,m,n) + \sum_{x,y \in C} B(m,n)$$

$$+ w * \sum_{m,n \in NC} \sum_{i=1}^{M} X_i * target(i,m,n)$$

s.t.

$$\sum_{i=1}^{M} X_i * target(i,m,n) \geq tumor\_lower \ \forall (m,n) \in T$$

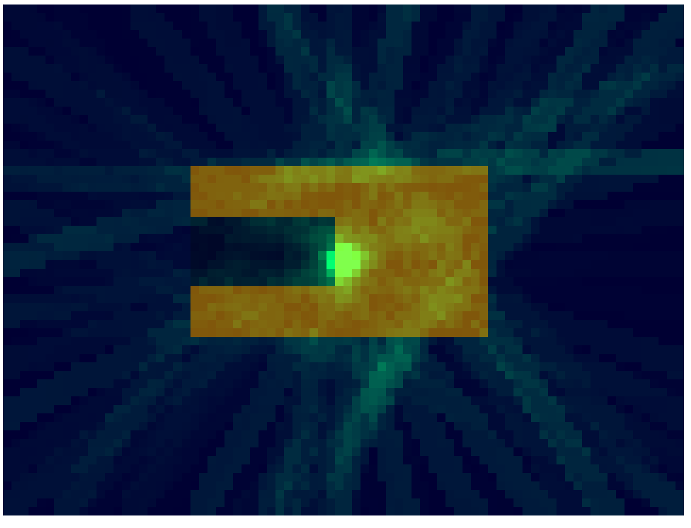
$$\sum_{i=1}^{M} X_i * target(i,m,n) \leq tumor\_upper \ \forall (m,n) \in T$$

$$\sum_{i=1}^{M} X_i * target(i,m,n) = critical_{upper} + B(m,n) \ \forall (m,n) \in C$$

$$x_i \geq 0 \ \forall \ i = 1 \dots M$$

$$T = Tumor\ area, \quad C = Critical\ area, \quad NC = Non-Critical\ area$$

$$q, w = weights$$



Model 2:

In this model we aim to place greater emphasis on beams passing through only the tumor and non-critical areas. This would increase the weight of a subset of these beams taking into consideration that they do not pass thrugh the critical area. To avoid high dosage through the non-critical area we place an upper bound on the dosage at the non-critical areas.

Objective:

$$minimize \sum_{x,y \in T} A(m,n) - \sum_{x,y \in T} \sum_{i=1}^{M} X_i * target(i,m,n)$$
$$+ \sum_{x,y \in C} \sum_{i=1}^{M} X_i * target(i,m,n) + \sum_{m,n \in NC} \sum_{i=1}^{M} X_i * target(i,m,n)$$
$$- q * \sum_{x,y \in SB} \sum_{i=1}^{M} X_i * target(i,m,n)$$

s.t.

$$\sum_{i=1}^{M} X_i * target(i,m,n) \le critical\_upper \;\; \forall\, (m,n) \in C$$
$$\sum_{i=1}^{M} X_i * target(i,m,n) \le w * critical\_upper \;\; \forall\, (m,n) \in NC$$
$$\sum_{i=1}^{M} X_i * target(i,m,n) = tumor\_lower - A(m,n)\; \forall\, (m,n) \in T$$
$$x_i \ge 0 \;\forall\, i = 1 \dots M$$
$$T = Tumor\ area,\;\; C = Critical\ Area,\;\; NCB = Non-critical\ area$$
$$q, w = weight/constant$$