CHE636A Homework-2
Due date: Jan 29th, 2022 at 5 pm on Mookit

Question 1: Solve the following equation using explicit finite difference method and answer the questions given in part (i) & (ii). The initial condition, boundary condition and exact solution of this equation are given below.

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

where $0<x<1$, $t>0$

$T=0$, $x=0$, $t>0$
$T=0$, $x=1$, $t>0$
$T=100\sin(2\pi x)$, $t=0$, $0\leq x\leq 1$

The exact analytical solution of this problem is

$$T(x,t) = 100e^{-4\pi^2 t}\sin(2\pi x)$$

Solve this problem using explicit finite difference method as discussed in the class. Plot temperature profiles obtained from exact solute (use dots/circles 'o' to show exact solution) and approximate solution as a function of x **after 0.04 seconds**.

Part 1(a): Using N=20, solve for the following cases (Hint: choose dt accordingly) and show the plot of temperature profile as a function of x (after 0.04 seconds as mentioned above).

(i) $\frac{\Delta t}{\Delta x^2} = 0.25$

(ii) $\frac{\Delta t}{\Delta x^2} = 0.75$

Part 1(b): Further, keeping $\frac{\Delta t}{\Delta x^2} = 0.25$, study the impact of changing N on the RMSE. Take N=10, 20, 30 & 40 and obtain the RMSE (Hint: Note, dt should be changed for each case such that the condition $\frac{\Delta t}{\Delta x^2} = 0.25$ is satisfied). RMSE is defined as shown below:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(T_i^{exact} - T_i^{approximate})^2}{N}}$$

where T_i^{exact} and $T_i^{approximate}$ are the exact and approximate solution, respectively, at some x and N is the number of nodes.

Question 2: Solve the previous problem with the following initial and boundary conditions. The exact solution using these conditions is also given below. Solve this problem for time **t=0.5 seconds**

$\frac{\partial T}{\partial x} = 0$, $x=0$, $t>0$
$T=0$, $x=1$, $t>0$
$T = 100\cos(\frac{\pi}{2}x)$, $t=0$, $0\leq x\leq 1$

The exact analytical solution of this problem is

$$T(x,t) = 100\cos\left(\frac{\pi}{2}x\right)\exp\left(-\frac{\pi^2}{4}t\right)$$

Assignment 2

CHE-636A

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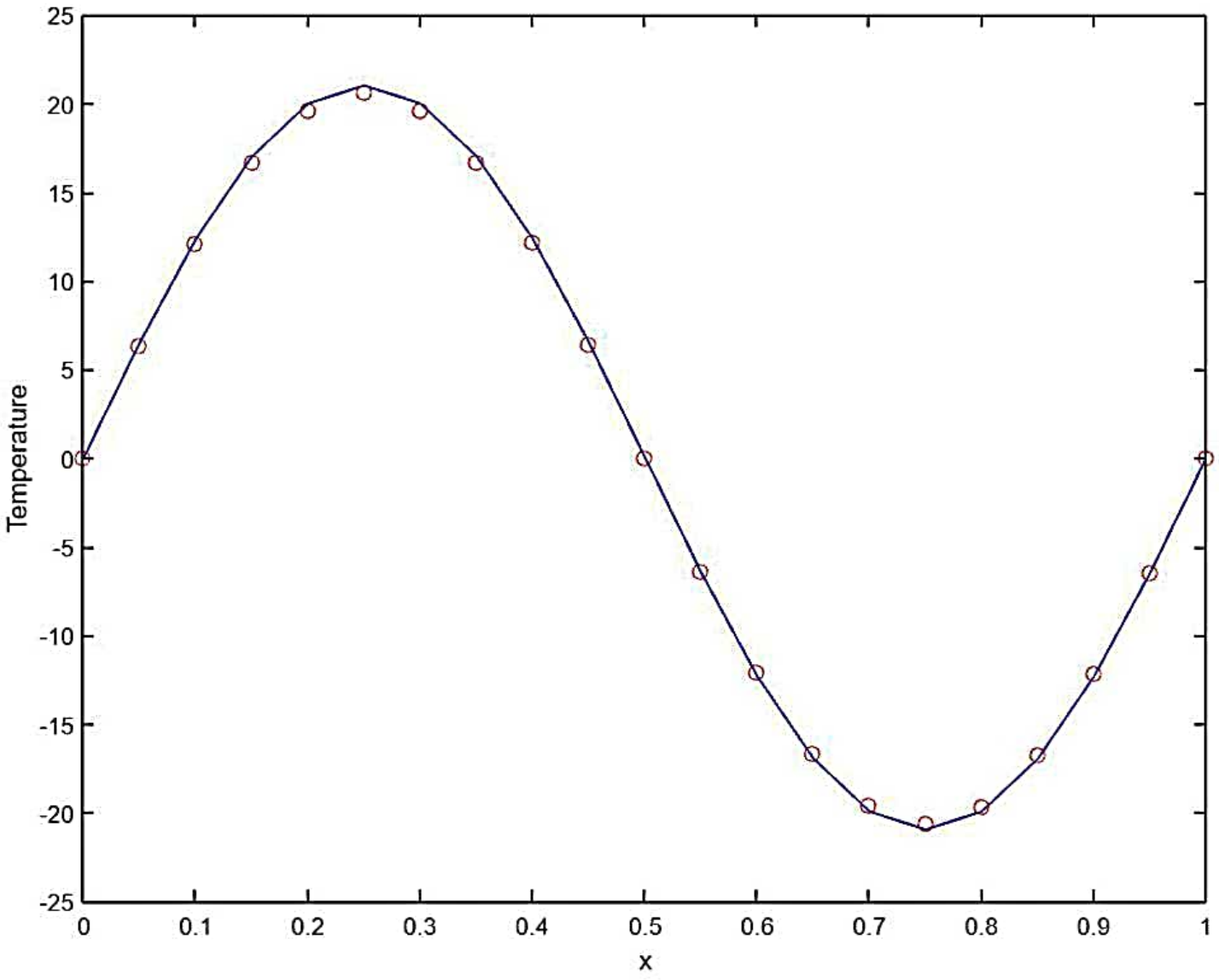
Question-1 part(a):-

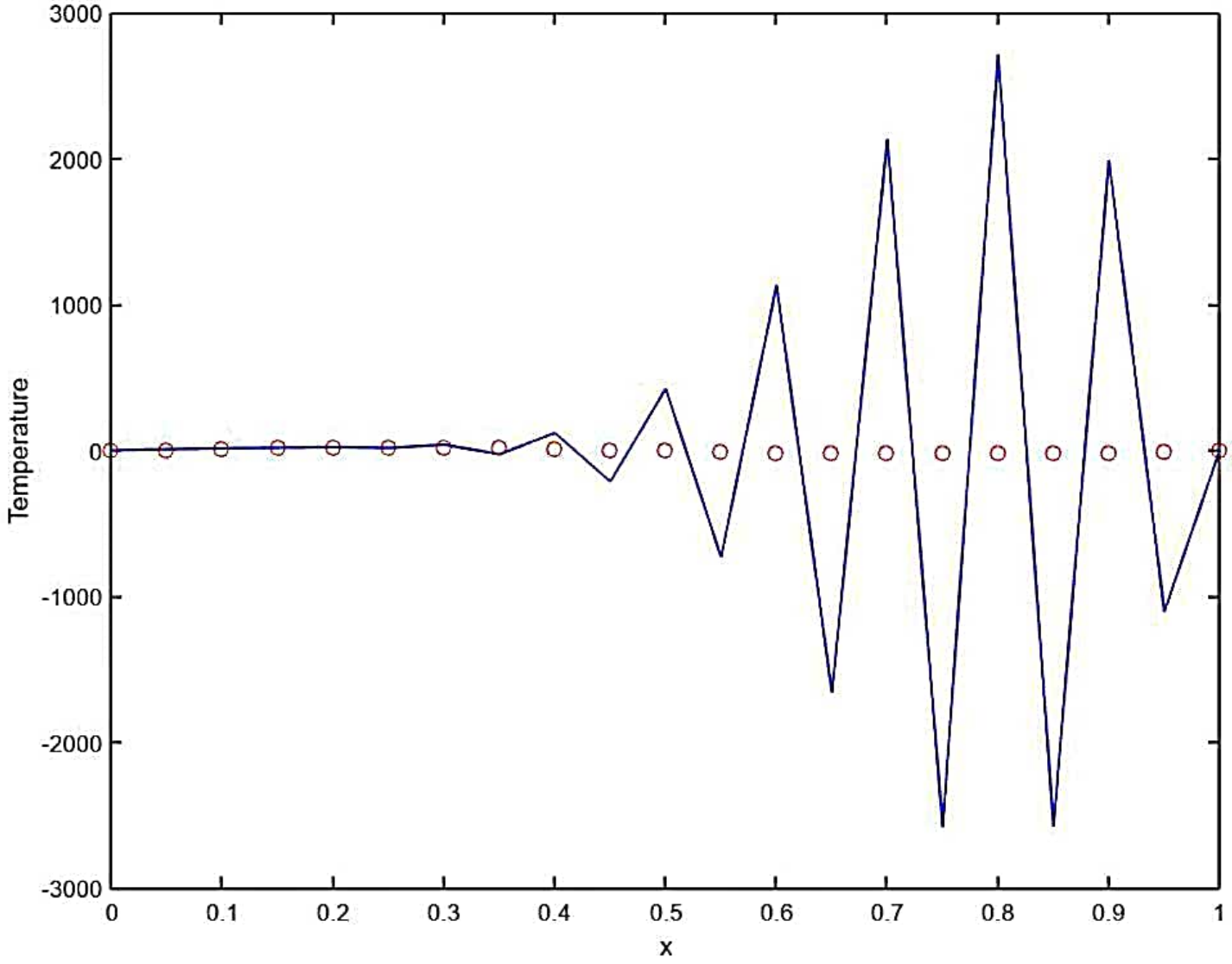
```
clear
clc
L = 1; %length
t_final = 0.04;

N=20; %change value of N as given in problem
dx = L/N;
dt = 0.25*dx*dx; %This is condition to find dt.
m = t_final/dt;
i_max = round(m,0);
x=linspace(0,L,N+1);
t=linspace(0,t_final,i_max);

T_e = zeros(1,N+1); %getting values by excat solution at t=0.04
T_e(1,N+1)=0;
for j = 1:N
T_e(1,j) = 100*exp(-4*3.14*3.14*t_final)*sin(6.28*dx*(j-1));
end

T=zeros(i_max,N+1); %getting analytical value
T(:,1)=0; %this is B.C.
T(:,N+1)=0; %this is B.C.

for j=2:N
T(1,j)= 100*sin(6.28*dx*(j-1));
end

for i=2:i_max
     for j =2:N
         T(i,j)= T(i-1,j)+(0.25)*(T(i-1,j+1)-2*T(i-1,j)+T(i-1,j-1));
     end
end
plot(x,T(end,:),'b')
hold on
plot(x,T_e,'or')
xlabel('x')
ylabel('Temperature')
hold off

sigma = zeros(1,N+1);
for j=1:N+1
sigma(1,j) = (T_e(1,j)-T(end,j))^2;
end
RMSE = sqrt(sum(sigma)/N);
```

The graphs are shown below.

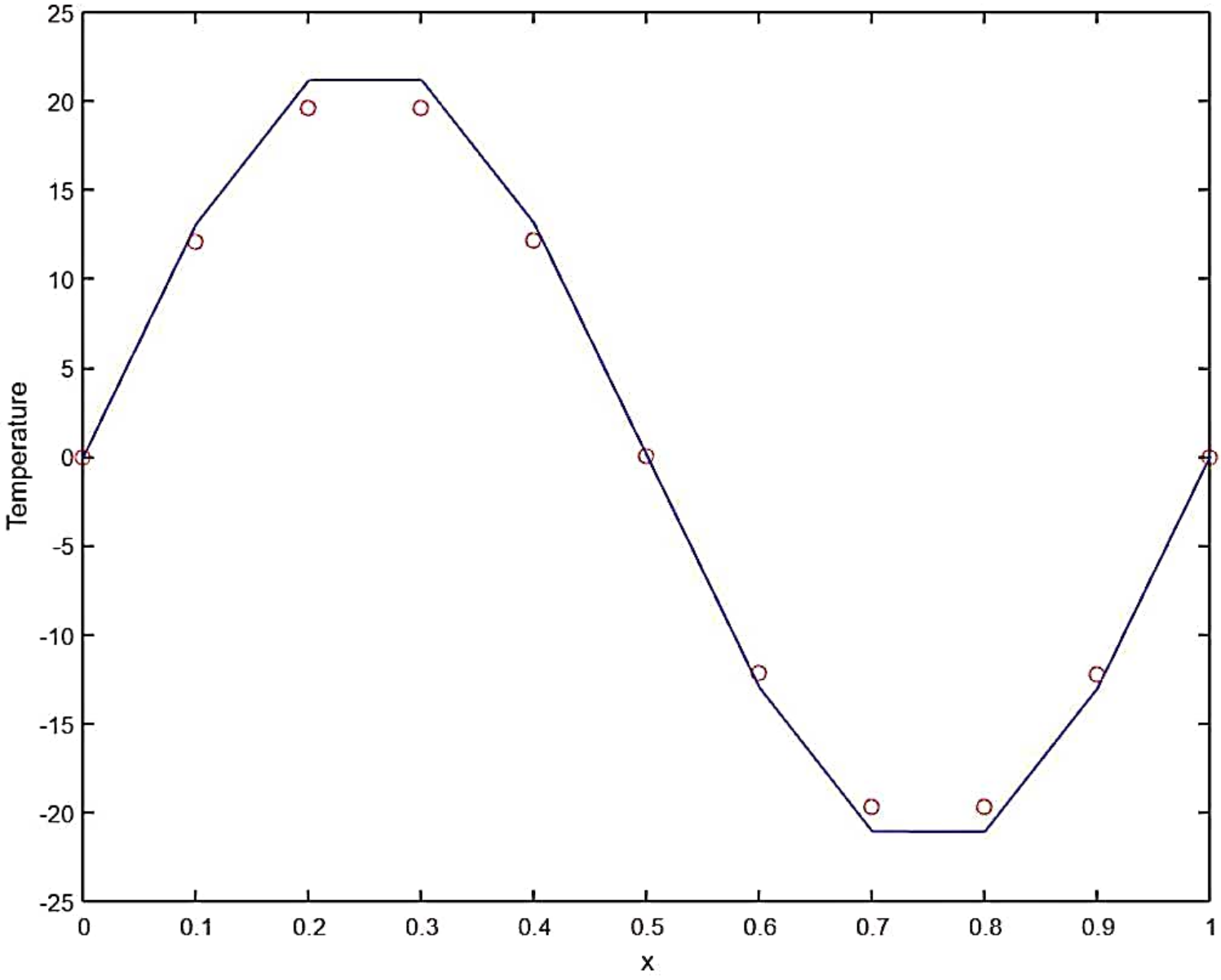
For N=20 & dt = 0.25* dx^2



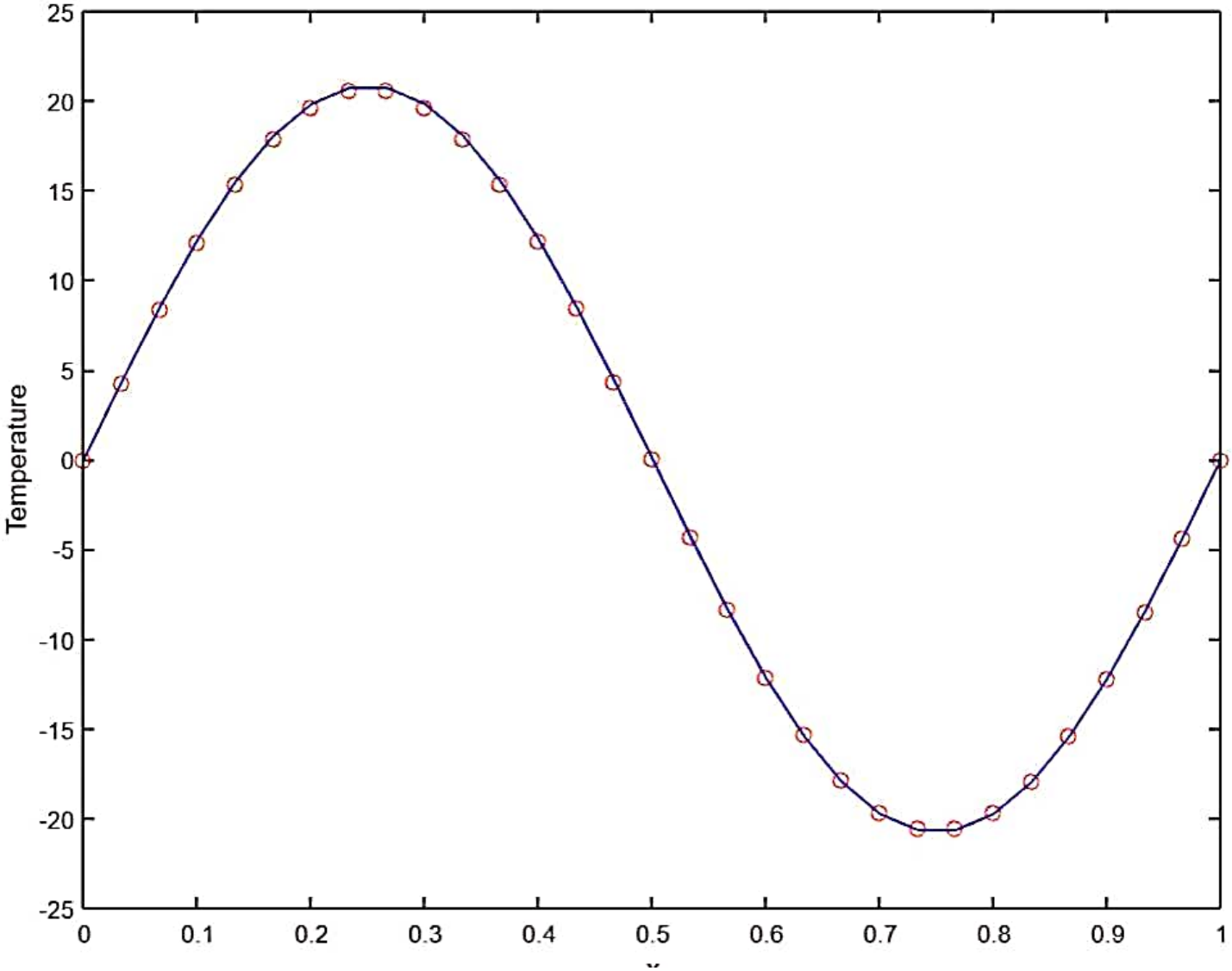
For N=20 & dt=0.75*dx^2



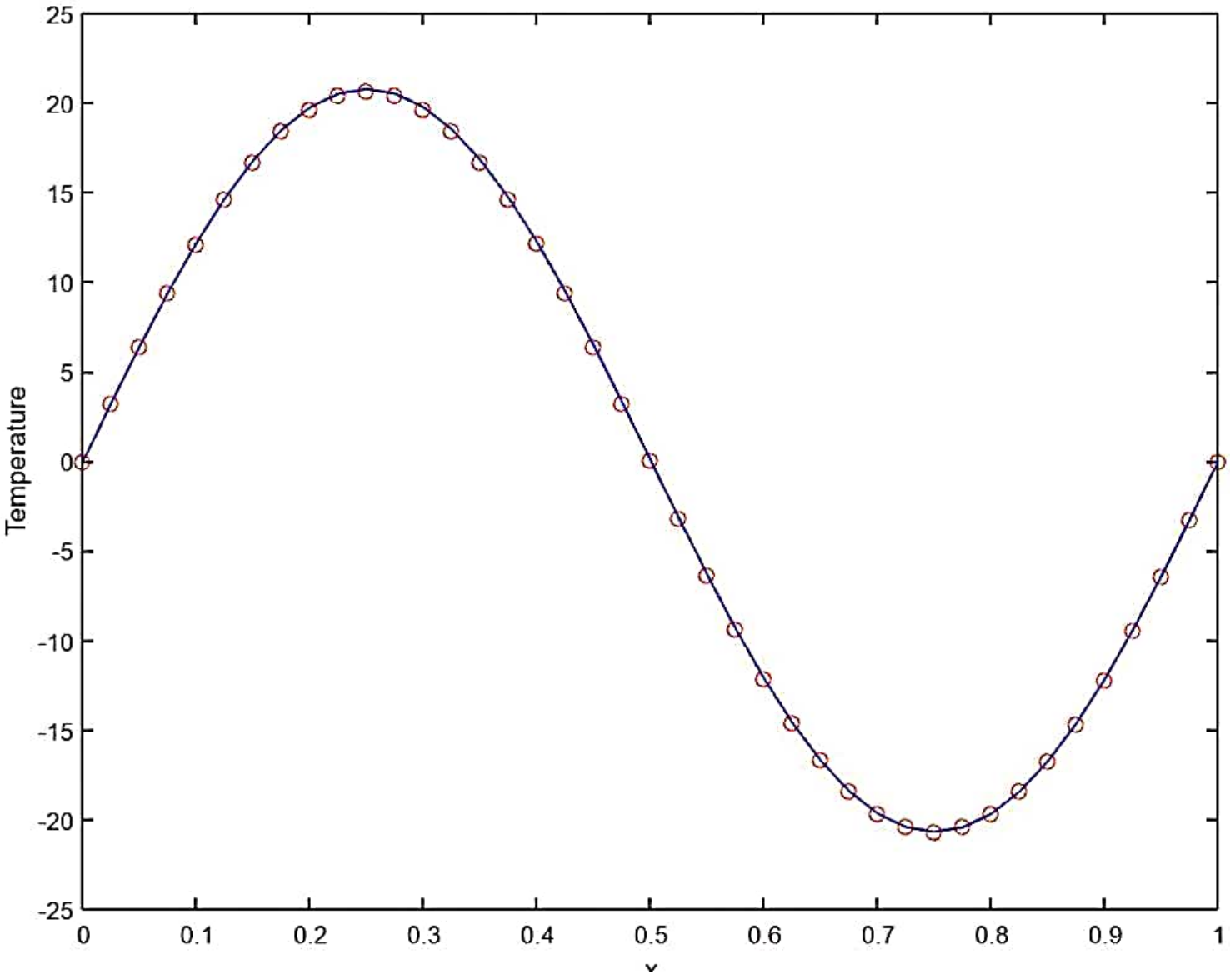
For N=10


25
20
15
10
5
0
-5
-10
-15
-20
-25
Temperature
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

For N=30


Temperature
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

For N=40


25
20
15
10
5
0
-5
-10
-15
-20
-25
Temperature
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

Question-1 part(b):-

By running code for different values of N, RMSE Value as below,

For N=10 , RMSE = 1.09

For N=20, RMSE = 0.2470

For N=30 , RMSE = 0.1003

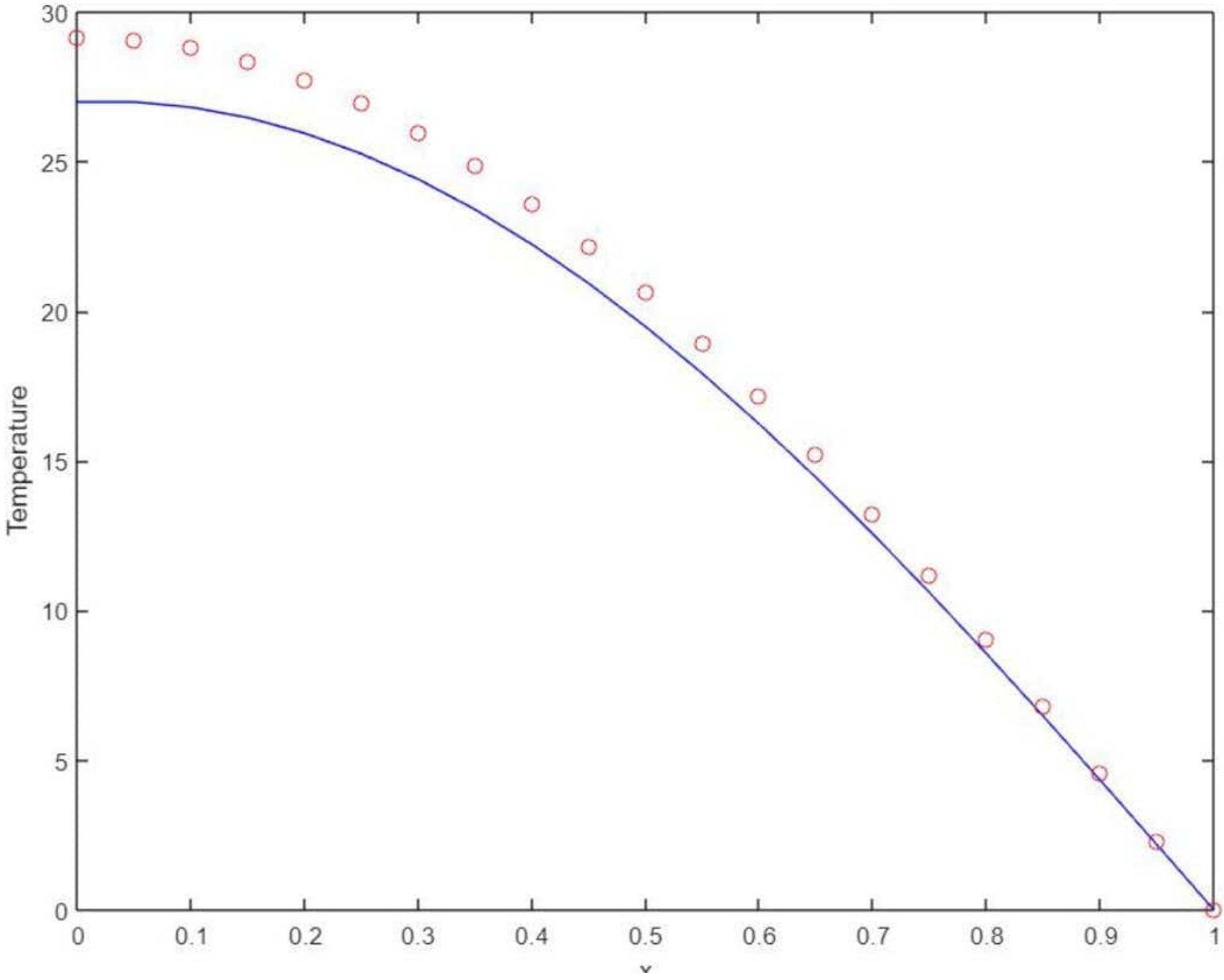
For N=40 , RMSE = 0.0535

Question-2 part(a):-

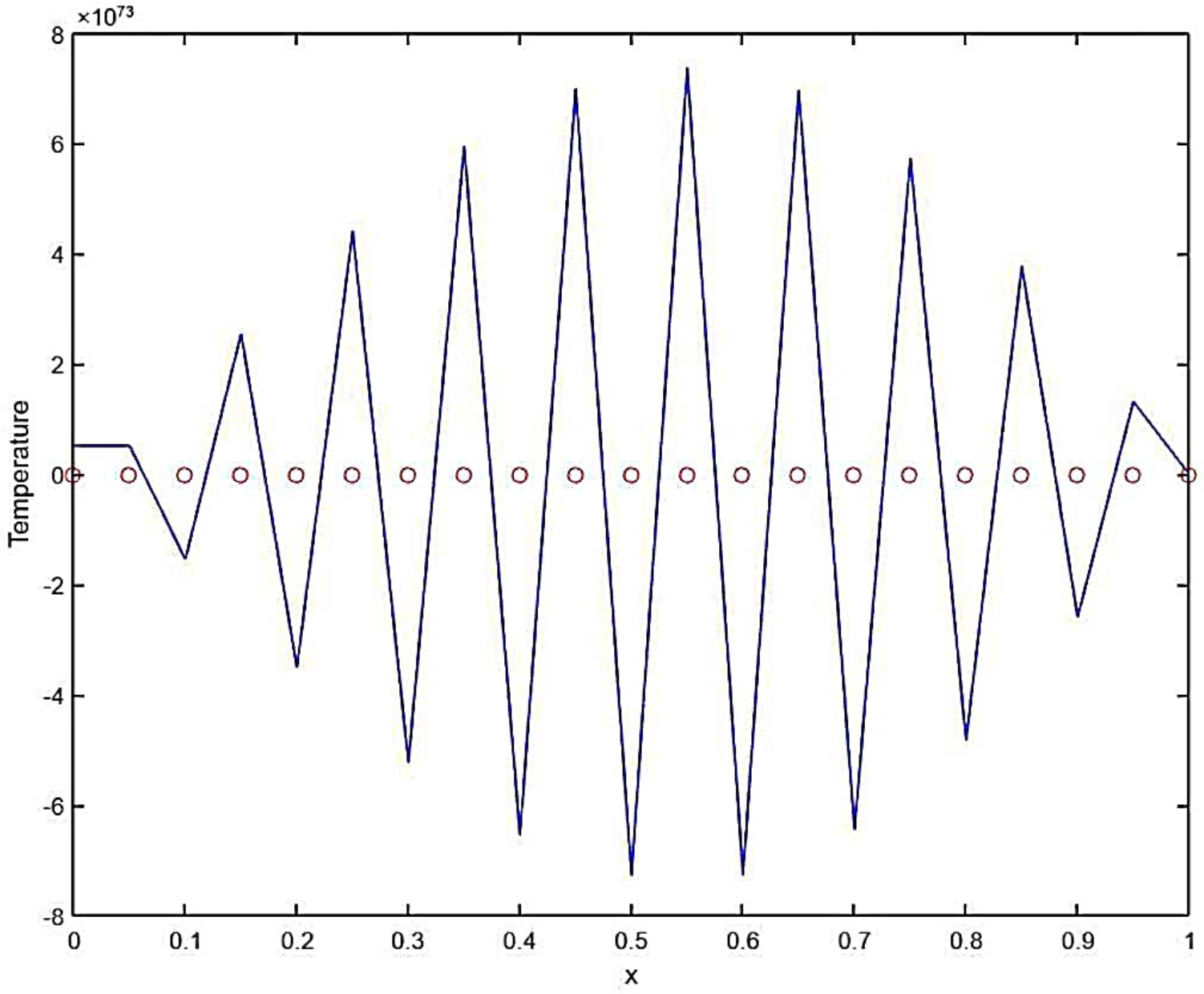
```
clear
clc
L = 1; %length
t_final = 0.5;

N=20; %change value of N as given in problem
dx = L/N;
dt = 0.25*dx*dx; %This is condition to find dt.
m = t_final/dt;
i_max = round(m,0);
x=linspace(0,L,N+1);
t=linspace(0,t_final,i_max);

T_e = zeros(1,N+1); %getting values by exact solution at t=0.5
T_e(1,N+1)=0;
for j = 1:N
T_e(1,j) = 100*exp(-0.25*3.14*3.14*t_final)*cos(0.5*3.14*dx*(j-1));
end

T=zeros(i_max,N+1);%now getting analyatical values
T(:,N+1)=0;%this is B.C.

for j=1:N
T(1,j)= 100*cos(0.5*3.14*dx*(j-1));
end

for i=2:i_max
     for j =2:N
         T(i,j)= T(i-1,j)+(0.25)*(T(i-1,j+1)-2*T(i-1,j)+T(i-1,j-1));
     end
      T(i,1)=T(i,2);%this is B.C.
end

plot(x,T(end,:),'b')
hold on
plot(x,T_e,'or')
xlabel('x')
ylabel('Temperature')
hold off

sigma = zeros(1,N+1);
for j=1:N+1
sigma(1,j) = (T_e(1,j)-T(end,j))^2;
end
RMSE = sqrt(sum(sigma)/N);
```

The graphs are shown below.

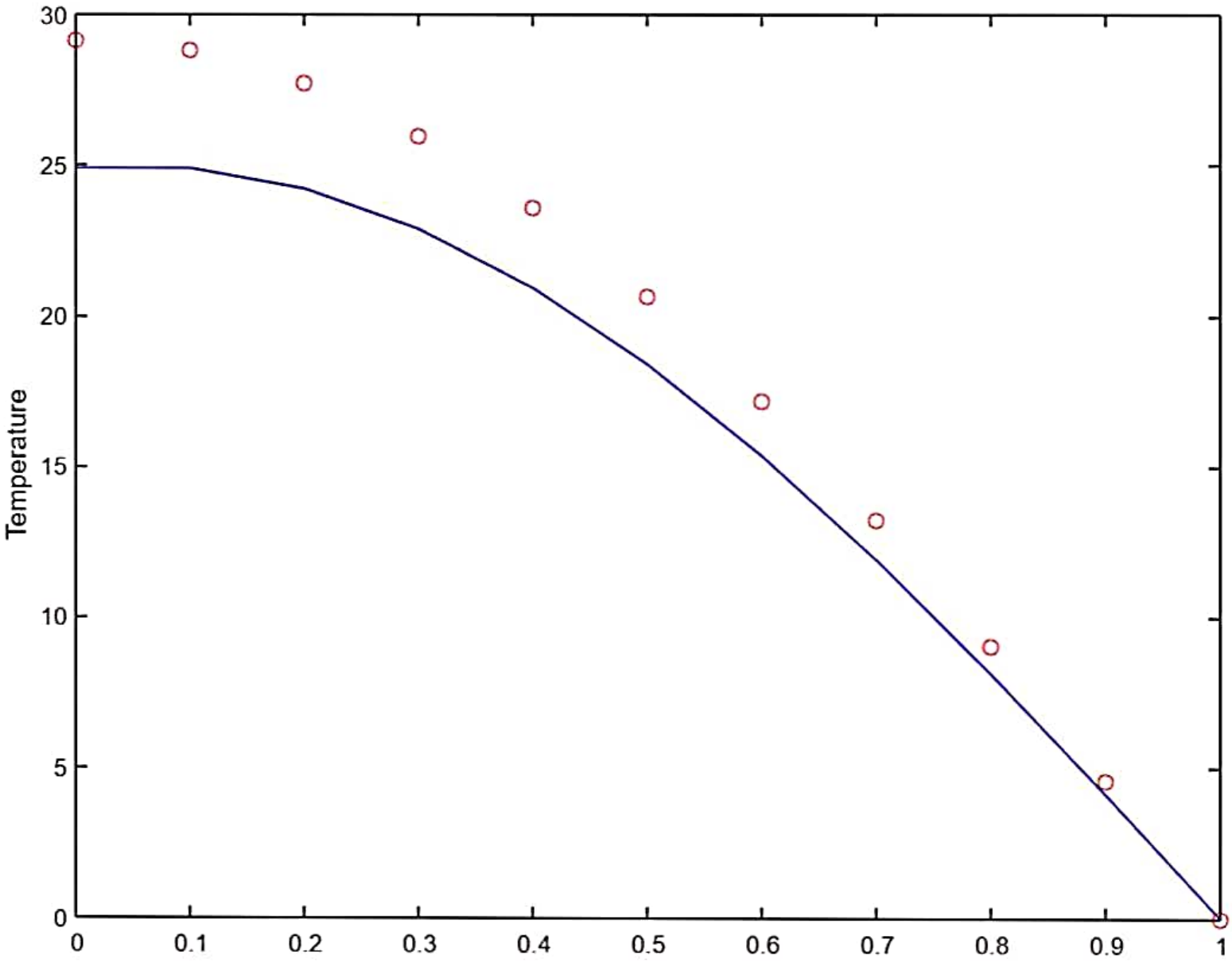
For N=20 & dt=0.25*dx^2



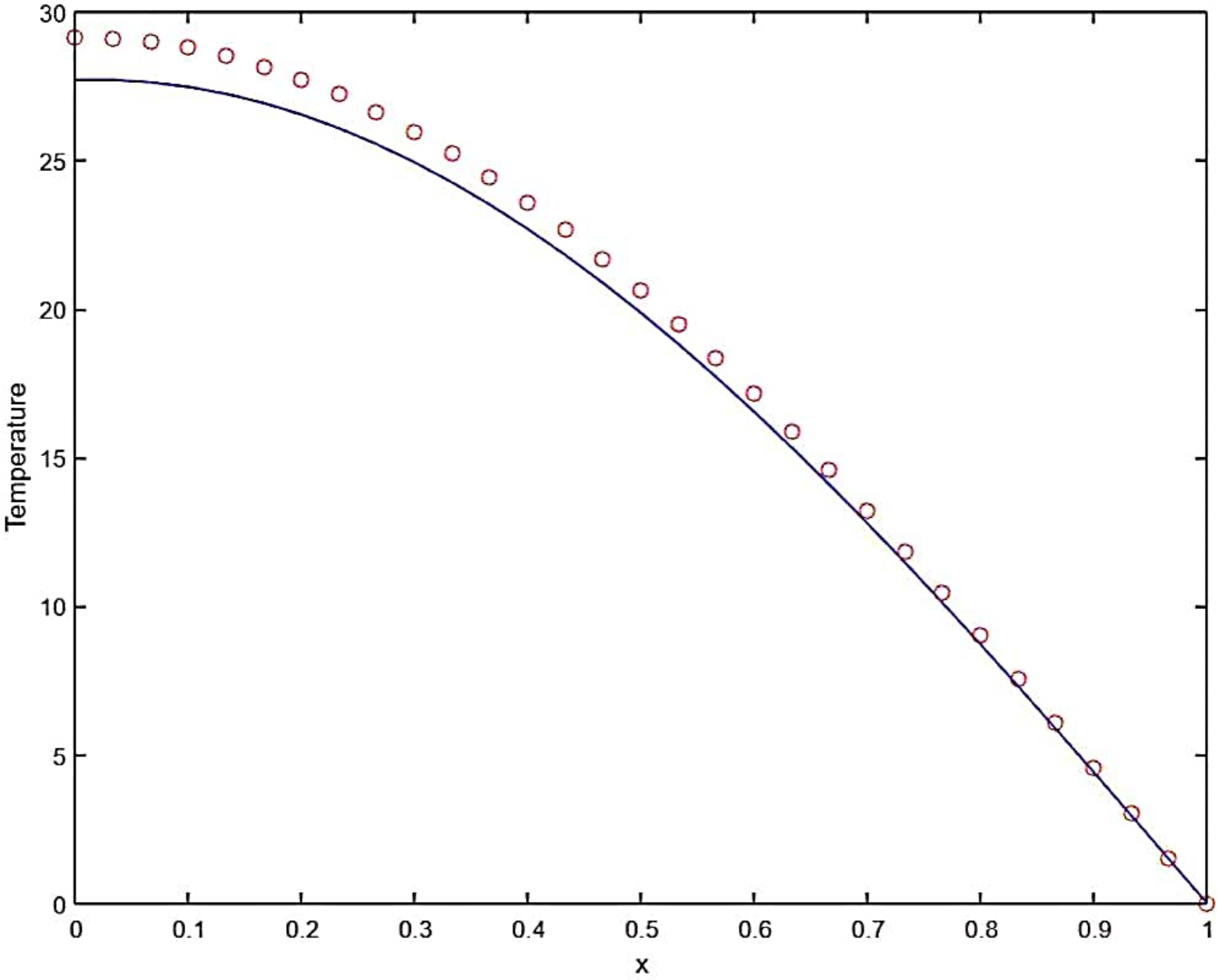
For N=20 & dt=0.75*dx^2



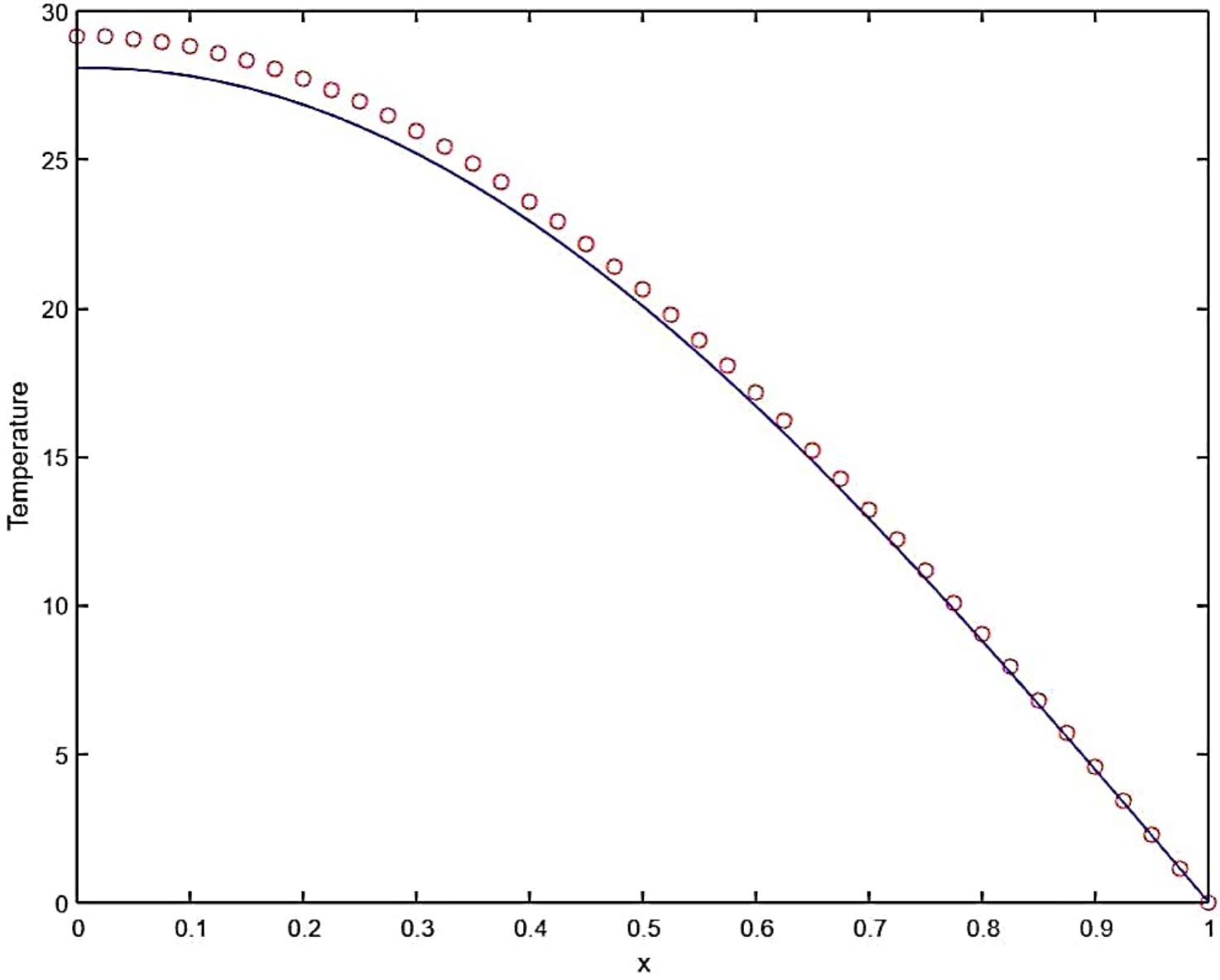
For N=10



For N=30



For N=40



Question-2 part(b):-

By running code for different values of N, RMSE Value as below,

For N=10 , RMSE = 2.7322

For N=20, RMSE = 1.3432

For N=30 , RMSE = 0.8953

For N=40 , RMSE = 0.6744