CHE636A HW-4

Date: March 8th, 2022
Due: March 17th, 2022 on Mookit

Question: Use the explicit method to calculate a numerical solution of the following differential equation:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = -u$$

where x goes from 0 to 10 and t goes from 0 to 0.85.

Initial and boundary conditions:

$$u(x,0) = e^{-x}, 0 \leq x \leq 10$$

$$u(0,t) = e^{t}, 0 \leq t \leq 0.85$$

$$\frac{du}{dx}(10,t) = -u(10,t), 0 \leq t \leq 0.85$$

The PDE with initial and boundary condition equations has the exact solution given by

$$u(x,t)=\exp(t-x)$$

Case 1: Use three-point central difference with dx=1/40

Case 2: Use one-point backward difference method for $\frac{du}{dx}$ and three-point central difference for $\frac{\partial^2 u}{\partial x^2}$ with dx=1/40.

Calculate the error at the final time for both cases mentioned above.