CHE636A Homework-1
Due date: Jan 22nd, 2022 at 5 pm on Mookit

Question 1: Using Taylor series expansion approach, obtain the order of the following expressions of second derivate:

(a) $\frac{\partial^2 f}{\partial x^2} = \frac{f_i - 2f_{i-1} + f_{i-2}}{(\Delta x)^2}$

(b) $\frac{\partial^2 f}{\partial x^2} = \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta x)^2}$

Question 2: sin(x) function can be approximated using the following infinite series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Write a program that approximates sin(x) using the infinite series mentioned above. The program should allow as many terms to be incorporated as desired. For example, a use may take 2, 3 and 4 terms as shown below:

$$\sin(x) = x - \frac{x^3}{3!}$$
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

Using the program, obtain the error as a function of number of terms taken (maximum up to 8 terms that is $x^{15}/15!$). Test your program for x=pi/3 and x=pi/2 and prepare a table similar to one shown below:

Number of terms	Actual value	Computed value	%Error
2			
3			
4			
5			

where $\%\ error = \frac{Actual\ value - Computed\ value}{Actual\ value} \times 100$

Question 3: Piecewise continuous functions are often used to describe the relationship between dependent and independent variable when a single function cannot describe this relationship. Write a program for the piecewise continuous function given below:

$$f(x) = 11t^2 - 5t \quad for\ 0 \le t \le 10$$
$$f(x) = 1100 - 5t \quad for\ 10 < t \le 20$$
$$f(x) = 50t + 2(t-20)^2 \quad for\ 20 < t \le 30$$
$$f(x) = 1520\exp(-0.2(t-30)) \quad for\ t > 30$$
$$f(x) = 0\ otherwise$$

Use this program to plot f(x) vs t for $-5 \le t \le 50$

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Q-1 a) taylor's series expansion for $f(x-\Delta x)$ about x,

$$f(x-\Delta x) = f(x) - \Delta x \frac{\partial f}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 f}{\partial x^2} - \frac{(\Delta x)^3}{3!}\frac{\partial^3 f}{\partial x^3} + \dots \quad (1)$$

→ Now similar way for $f(x-2\Delta x)$ we can write it as below,

$$\therefore f(x-2\Delta x) = f(x) - (2\Delta x)\frac{\partial f}{\partial x} + \frac{(2\Delta x)^2}{2!}\frac{\partial^2 f}{\partial x^2} - \frac{(2\Delta x)^3}{3!}\frac{\partial^3 f}{\partial x^3} + \dots \quad (2)$$

→ let's do eqⁿ (2) - 2x eqⁿ (1) so we get,

$$f(x-2\Delta x) - 2f(x-\Delta x) = -f(x) + \frac{2(\Delta x)^2}{2!}\frac{\partial^2 f}{\partial x^2} - (\Delta x)^3\frac{\partial^3 f}{\partial x^3} + \dots$$

$$\therefore \frac{f(x-2\Delta x) - 2f(x-\Delta x) + f(x)}{(\Delta x)^2} = \frac{\partial^2 f}{\partial x^2} - \Delta x\frac{\partial^3 f}{\partial x^3} + \dots$$

$$\therefore \frac{\partial^2 f}{\partial x^2} = \frac{f(x) - 2f(x-\Delta x) + f(x-2\Delta x)}{(\Delta x)^2} + O(\Delta x)^1$$

→ above eqⁿ can be written as,

$$\frac{\partial^2 f}{\partial x^2} = \frac{f_i - 2f_{i-1} + f_{i-2}}{(\Delta x)^2} + O(\Delta x)$$ ⇒ Hence proved,

where, $f_i = f(x)$, $f_{i-1} = f(x-\Delta x)$, $f_{i-2} = f(x-2\Delta x)$

→ the above eqⁿ represent's the backward difference approximation for the 2nd derivative of function $f(x)$ & it's order is $O(\Delta x)^1$.

Q-1-B) → taylor's series expansion for $f(x+\Delta x)$,

$$f(x+\Delta x) = f(x) + \Delta x \frac{\partial f}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 f}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 f}{\partial x^3} + \cdots \quad (3)$$

→ Now eqⁿ (1) + eqⁿ (3),

$$\therefore f(x-\Delta x) + f(x+\Delta x) = 2f(x) + \frac{2(\Delta x)^2}{2!}\frac{\partial^2 f}{\partial x^2} + \frac{2(\Delta x)^4}{4!}\frac{\partial^4 f}{\partial x^4} + \cdots$$

$$\therefore \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{(\Delta x)^2} = \frac{\partial^2 f}{\partial x^2} + \frac{2(\Delta x)^4}{4!}\frac{\partial^4 f}{\partial x^4} + \cdots$$

$$\therefore \frac{\partial^2 f}{\partial x^2} = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{(\Delta x)^2} + O(\Delta x)^2$$

⇒ above eqⁿ can be written as,

$$\frac{\partial^2 f}{\partial x^2} = \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta x)^2} + O(\Delta x)^2,$$ Hence proved,

→ the above eqⁿ represents the central difference approximation for 2nd derivative of function $f(x)$ & it's order is $O(\Delta x)^2$

```
clear
clc
x = pi/3;
digits(10)
Actual_Value = vpa(sin(x));
n = 2;
while( n < 9 )
     y = zeros(1,n);
for i = 1:n
  y(i) = (-1)^(i+1)*x^(2*i-1)/factorial(2*i-1);
end
Computed_value = vpa(sum(y));
Error = (Actual_Value-Computed_value)*100/Actual_Value;
table(n,Actual_Value,Computed_value,Error)
n = n + 1;
end
```

```
clear
clc
x = pi/2;
digits(10)
Actual_Value = vpa(sin(x));
n = 2;
while( n < 9 )
     y = zeros(1,n);
for i = 1:n
  y(i) = (-1)^(i+1)*x^(2*i-1)/factorial(2*i-1);
end
Computed_value = vpa(sum(y));
Error = (Actual_Value-Computed_value)*100/Actual_Value;
table(n,Actual_Value,Computed_value,Error)
n = n + 1;
end
```

Matlab gives empty spaces in between answer so i have provided all answer as table below.

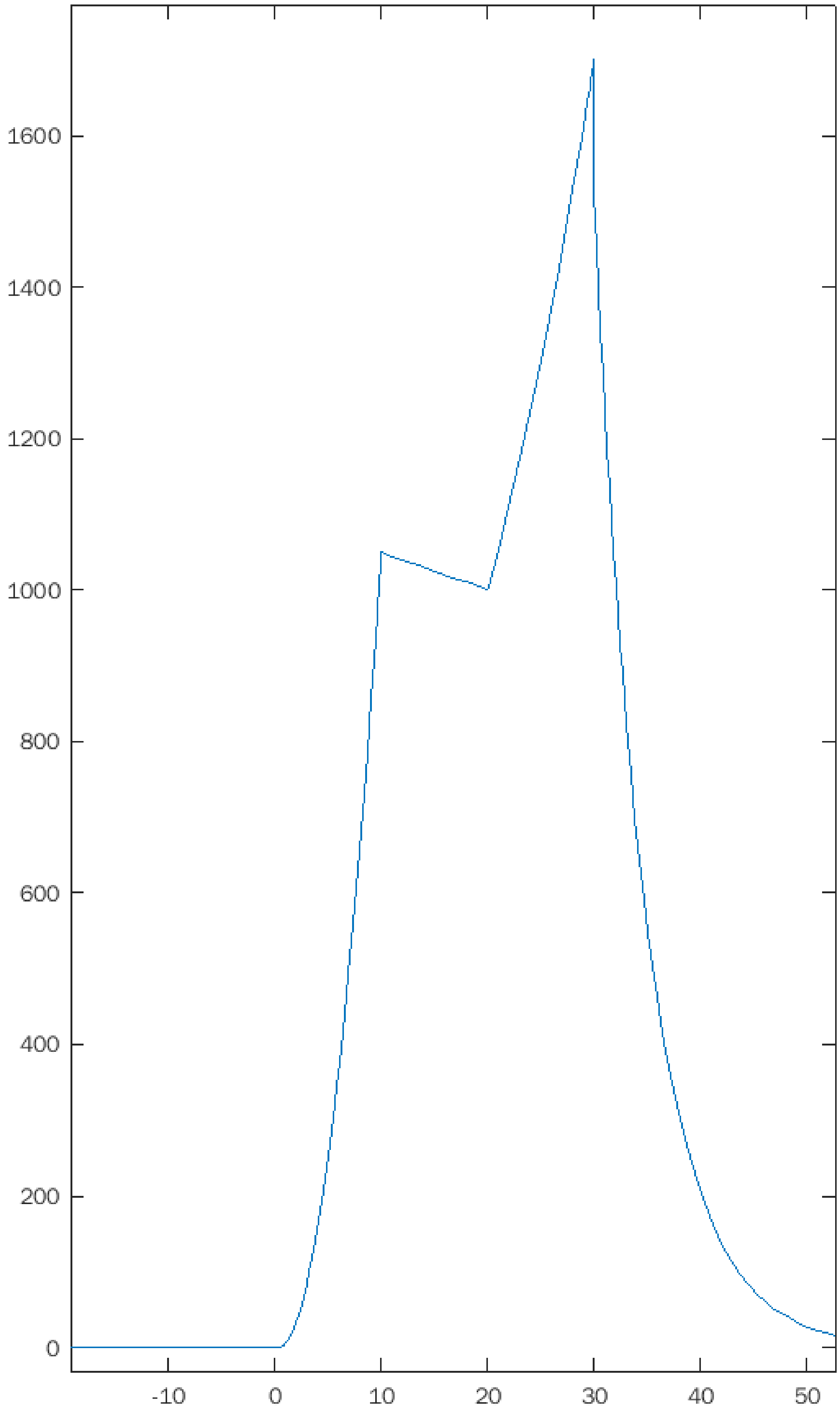
For X=pi/3

n	Actual_Value	Computed_value	%Error
2	0.8660254038	0.8558007816	1.180637678
3	0.8660254038	0.8662952838	-0.03116305841
4	0.8660254038	0.8660212717	0.0004771370503
5	0.8660254038	0.8660254451	-0.000004770684815
6	0.8660254038	0.8660254035	0.00000003359670225
7	0.8660254038	0.8660254038	-0.0000000001756376028
8	0.8660254038	0.8660254038	0.0

For X=pi/2

n	Actual_Value	Computed_value	Error
2	1.0	0.9248322293	7.516777071
3	1.0	1.004524856	-0.4524855535
4	1.0	0.9998431014	0.01568986005
5	1.0	1.000003543	-0.0003542584286
6	1.0	0.9999999437	0.000005625894905
7	1.0	1.000000001	-0.00000006627802751
8	1.0	1.0	0

```
clear
clc
syms y(t)
y(t) = piecewise(t<0         ,0, ...
                 10>=t>=0 ,11*t^2 - 5*t, ...
                 20>=t>10 ,1100 - 5*t, ...
                 30>=t>20 ,50*t + 2*(t-20)^2, ...
                 t>30      ,1520*exp(-0.2*(t-30)));
subs(y,t,[-5:1:50]);
fplot(y)
```



1600
1400
1200
1000
800
600
400
200
0
-10
0
10
20
30
40
50