INFLUENTIAL BILLBOARD SLOT SELECTION PROBLEM

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This is the material point that will be delivered in the presentation.

PROBLEM STATEMENT

Given m billboard slots and n tags, select k slots from m and l tags from n to maximize an influence function, which returns a positive value for two subsets. The goal is to find the best subsets of slots and tags, considering constraints on m, n, k, and l.

$$pr(t, \mathcal{BS}|\mathcal{H}^{'}) = 1 - \prod_{b_i \in \mathcal{BS}} (1 - pr(t, b_i|\mathcal{H}^{'}))$$

SUBMODULARITY

SET FUNCTIONS?

Definition 1.1 (Discrete derivative) For a set function $f : 2^V \to \mathbb{R}$, $S \subseteq V$, and $e \in V$, let $\Delta_f(e \mid S) := f(S \cup \{e\}) - f(S)$ be the *discrete derivative* of f at S with respect to e.

Where the function f is clear from the context, we drop the subscript and simply write $\Delta(e \mid S)$.

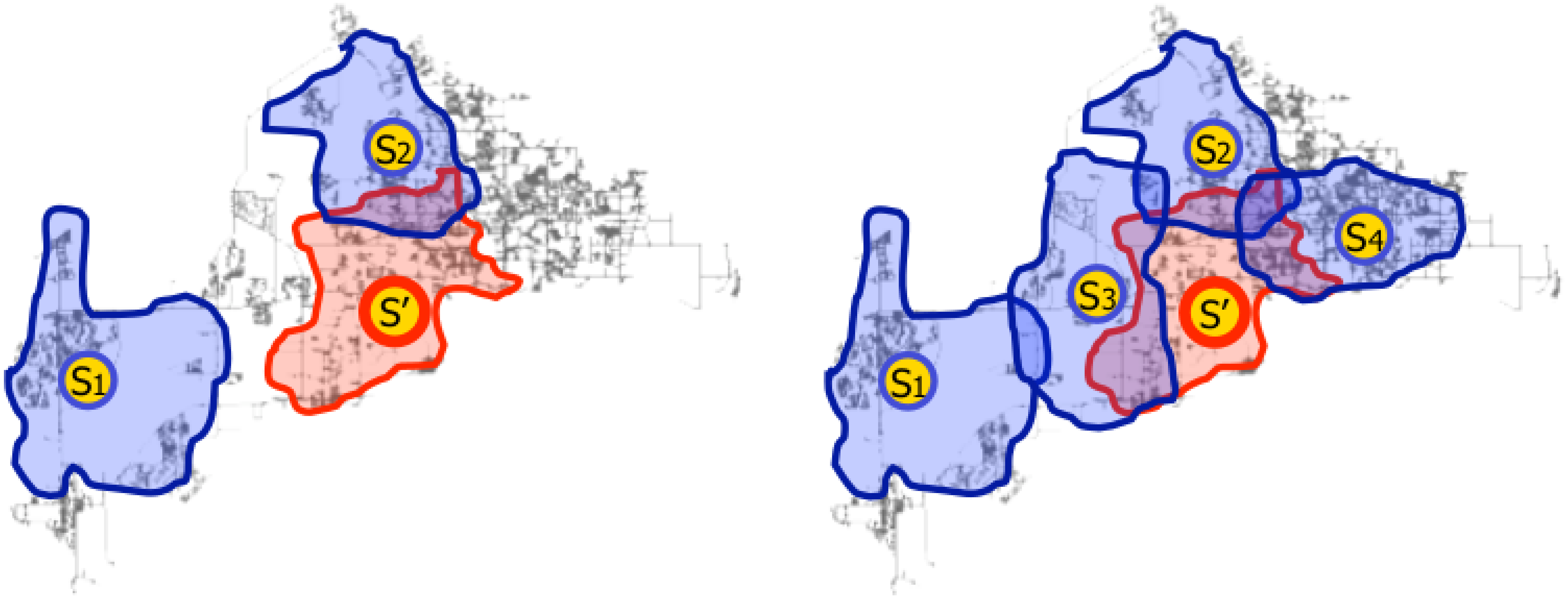
Definition 1.2 (Submodularity) A function $f : 2^V \to \mathbb{R}$ is *submodular* if for every $A \subseteq B \subseteq V$ and $e \in V \setminus B$ it holds that

$$\Delta(e \mid A) \geq \Delta(e \mid B).$$

Equivalently, a function $f : 2^V \to \mathbb{R}$ is *submodular* if for every $A, B \subseteq V$,

$$f(A \cap B) + f(A \cup B) \leq f(A) + f(B).$$

SUBMODULARITY



(a) *Adding* s' *to set* $\{s_1, s_2\}$ (b) *Adding* s' *to superset* $\{s_1, \ldots, s_4\}$

BISUBMODULARITY

Definition 4 (Simple Bisubmodularity). $f : 2^{2V} \to \mathbb{R}$ *is simple bisubmodular iff for each* $(A, B) \in 2^{2V}$, $(A', B') \in 2^{2V}$ *with* $A \subseteq A'$, $B \subseteq B'$ *we have for* $s \notin A'$ *and* $s \notin B'$:

$$f(A + s, B) - f(A, B) \geq f(A' + s, B') - f(A', B'),$$
$$f(A, B + s) - f(A, B) \geq f(A', B' + s) - f(A', B').$$

Equivalently, $\forall (A, B), (A', B') \in 2^{2V}$,

$$f(A, B) + f(A', B') \geq f(A \cup A', B \cup B') + f(A \cap A', B \cap B')$$

PROPOSED SOLUTIONS

INCREMENTAL GREEDY

STOCHASTIC GREEDY

CELF GREEDY

INCREMENTAL GREEDY

while $|\mathcal{S}| \neq k$ **do**

$\quad u^* \longleftarrow \underset{u \in \mathbb{BS} \setminus \mathcal{S}}{argmax}\ I(\mathcal{S} \cup \{u\}) - I(\mathcal{S});$

$\quad \mathcal{S} \longleftarrow \mathcal{S} \cup \{u^*\};$

end

Return $\mathcal{S}$;

Theorem 1.5 (Nemhauser et al. 1978) *Fix a nonnegative monotone submodular function $f : 2^V \rightarrow \mathbb{R}_+$ and let $\{S_i\}_{i \geq 0}$ be the greedily selected sets defined in Eq. (2). Then for all positive integers k and ℓ,*

$$f(S_\ell) \geq \left(1 - e^{-\ell/k}\right) \max_{S:|S| \leq k} f(S).$$

In particular, for $\ell = k$, $f(S_k) \geq (1 - 1/e) \max_{|S| \leq k} f(S)$.

INCREMENTAL GREEDY ON BISUBMODULAR FUNCTIONS

3.2 Coordinate-wise Maximization

Simple bisubmodular functions can also be maximized using a coordinate-wise procedure. Consider

$$\begin{aligned} \max_{A,B} \quad & f(A,B) \\ \text{subject to} \quad & (A,B) \in 2^{2V}, |A| \leq k_1, |B| \leq k_2. \end{aligned}$$

If f is simple then it suffices to solve the following pair of submodular optimizations:

$$\begin{aligned} A^* &= \operatorname*{argmax}_{A \subseteq V : |A| \leq k_1} f(A, \emptyset), \\ B^* &= \operatorname*{argmax}_{B \subseteq V : |B| \leq k_2} f(A^*, B), \end{aligned}$$

STOCHASTIC GREEDY

Algorithm 1 STOCHASTIC-GREEDY

Input: $f : 2^V \to \mathbb{R}_+, k \in \{1, \ldots, n\}$.
Output: A set $A \subseteq V$ satisfying $|A| \leq k$.

1: $A \leftarrow \emptyset$.
2: **for** $(i \leftarrow 1;\ i \leq k;\ i \leftarrow i + 1)$ **do**
3: $\quad R \leftarrow$ a random subset obtained by sampling s random elements from $V \setminus A$.
4: $\quad a_i \leftarrow \operatorname{argmax}_{a \in R} \Delta(a|A)$.
5: $\quad A \leftarrow A \cup \{a_i\}$
6: **return** A.

Theorem 1. *Let f be a non-negative monotone submoduar function. Let us also set $s = \frac{n}{k} \log \frac{1}{\epsilon}$. Then* STOCHASTIC-GREEDY *achieves a $(1 - 1/e - \epsilon)$ approximation guarantee in expectation to the optimum solution of problem (1) with only $O(n \log \frac{1}{\epsilon})$ function evaluations.*

COST EFFECTIVE LAZY FORWARD (CELF) GREEDY

CELF exploits the sub-modularity property of the spread function, which implies that the marginal spread of a given node in one iteration of the Greedy algorithm cannot be any larger than its marginal spread in the previous iteration.

EXPERIMENTAL DATASETS

NEW YORK TRAJECTORY DATASET

Publicly Available

LAMAR BILLBOARD DATASET

Crawled from LAMAR Website

LOS ANGELES TRAJECTORY DATASET

Publicly Available

EXPERIMENTAL RESULTS

CELF GREEDY

Results of Lamar Dataset					
No. of Slots	K=25	K=50	K=100	K=150	K= 200
Time Taken in Sec	61485.57943	61854.74085	62689.64432	63214.32614	63785.55542
Result	353.7404895	369.5898047	394.9954823	417.6126929	437.559641

STOCHASTIC GREEDY

ε=0.01

Results of Lamar Dataset					
No. of Slots	K=25	K=50	K=100	K=150	K= 200
Time Taken in Sec	194298.4948	193833.562	194488.2101	194766.0431	194082.7281
Result	353.3561835	368.7431633	390.7733548	415.4038416	434.214302

ε=0.1

Results of Lamar Dataset					
No. of Slots	K=25	K=50	K=100	K=150	K= 200
Time Taken in Sec	126879.7828	126276.4872	125925.3747	124943.1319	126094.7665
Result	328.8263501	315.0778075	377.0743121	376.2403809	415.3912859

CONCLUSION

1. On the experimental dataset, it is evident that CELF Greedy consumes significantly less time compared to both Incremental Greedy and Stochastic Greedy.
2. While the worst-case scenario for CELF Greedy is comparable to Incremental Greedy, experimental results demonstrate that CELF Greedy performs notably better.
3. Stochastic Greedy is independent of the values of k and l, showcasing its consistent behavior across various parameter settings.

PROPOSED BASELINE ALGORITHMS

1. TOP K - TOP L
2. TOP K - RANDOM
3. RANDOM - TOP L
4. MAXIMUM COVERAGE

REFERENCES

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