

189

3

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-: HAND WRITTEN NOTES:-

OF

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# ELECTRONICS & COMMUNICATION ENGINEERING

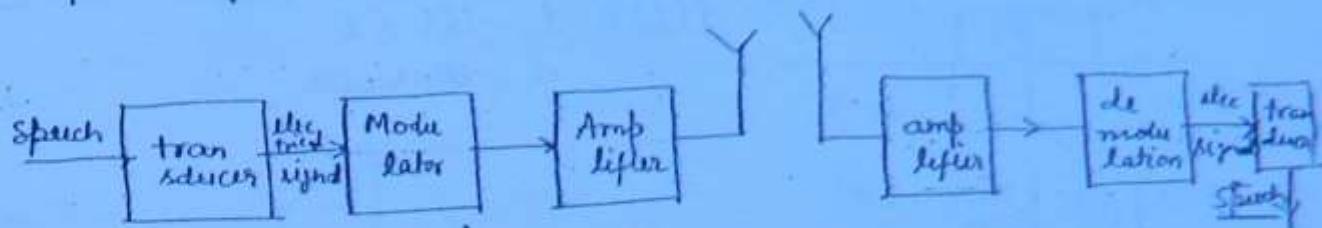
-: SUBJECT:-

# SIGNAL & SYSTEM

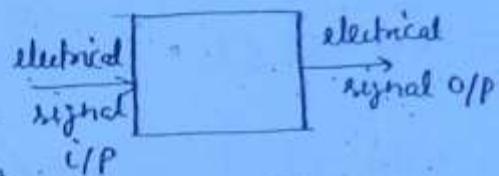
3

Speech signal  $\rightarrow$  300Hz  $\rightarrow$  3400Hz

(3)



\* SS  $\rightarrow$  we are concentrating to find response of system.



to find response of a system, we are following rules/tools-

1. Fourier series
2. Fourier transforms
3. Laplace transforms
4. Z-transforms

Audio signal  $\rightarrow$  20Hz - 20KHzVideo signal  $\rightarrow$  0 - 5MHzData signal  $\rightarrow$  range is based on application.

Signal  $\rightarrow$  is a quantity having associated information with it.

is a function of time.

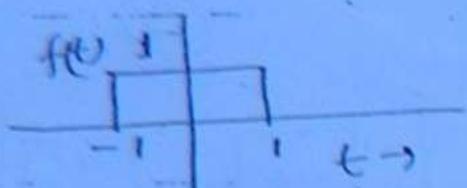
We deal with electrical signal which are voltages or currents, which are both functions of time. In general signal is a function time  $f(t)$ . (electrical)

But signal is not always a function of time.

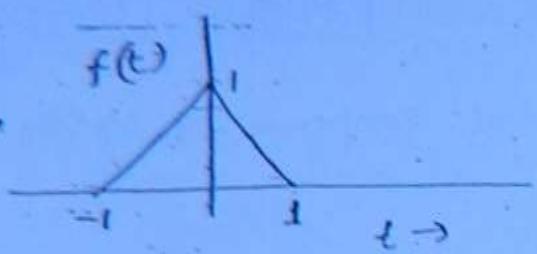
Collect still frame and play back to it in video signal (normally at 24 frames/second we play back the still frames).

In motion picture signal is three dimensional  $f(x, y, t)$ .

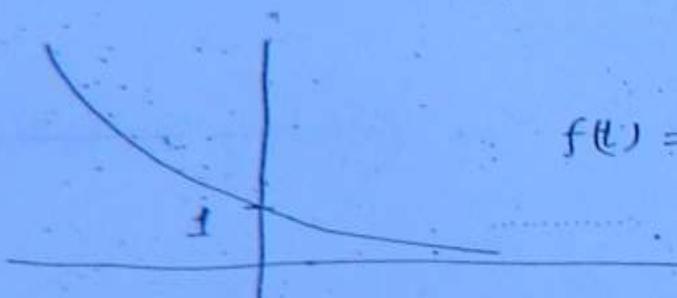
(4)



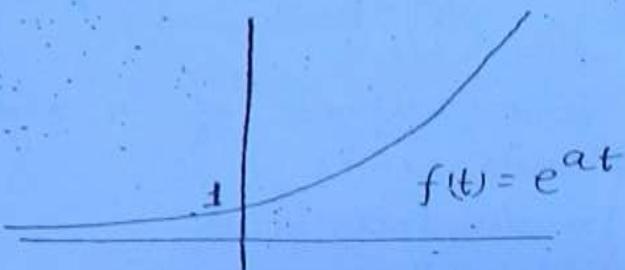
$$f(t) = \begin{cases} 1 & -1 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$



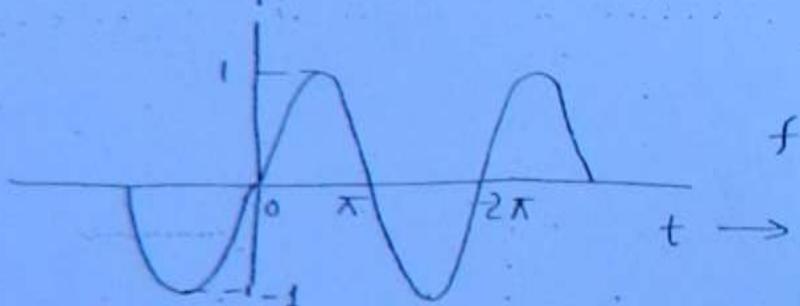
$$\begin{aligned} f(t) &= -1 \leq t \leq 0 \\ &= -t+1 \quad 0 \leq t \leq 1 \end{aligned}$$



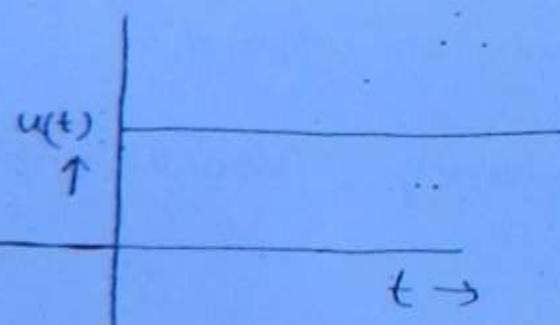
$$f(t) = e^{-|t|}$$



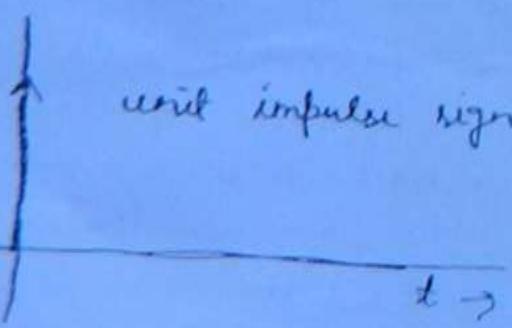
$$f(t) = e^{at}$$



$$f(t) = \sin t$$



$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



unit impulse signal

existence is split second  
effect is anomalous.

$$\begin{aligned} \delta(t) &= 0 \quad t \neq 0 \\ &= \infty \quad t=0 \quad \text{very large} \\ &\neq 0 \quad t=0 \quad (\infty) \end{aligned}$$

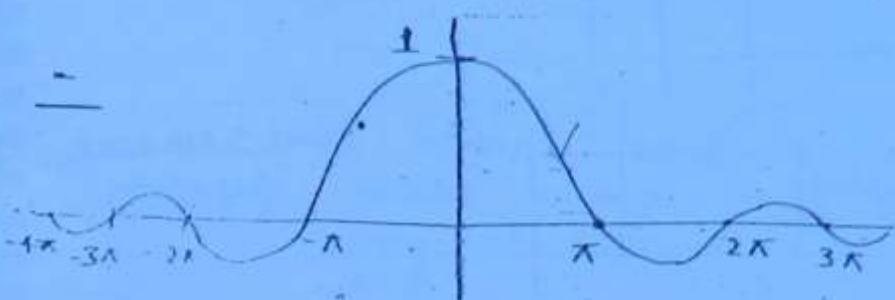
$$\text{rect} \cdot \frac{(t-1)}{2}$$

3/4

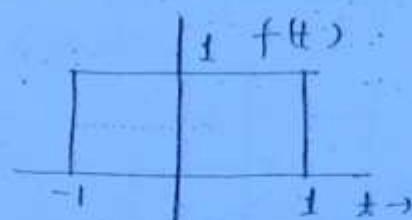
$$\text{rect}(t+3/4)$$

$$\int_{-\infty}^{\infty} f(t) dt = 1 \rightarrow \int_{-\infty}^{\infty} \delta(t) dt = 1$$

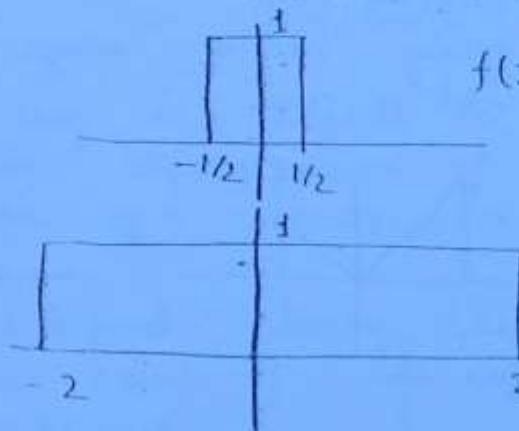
(5)



$$f(t) = \frac{\sin t}{t} = \text{Sa}(t) = \text{Sinc}(t/\pi)$$



$$f(t) = \text{rect}\left(\frac{t}{2}\right)$$

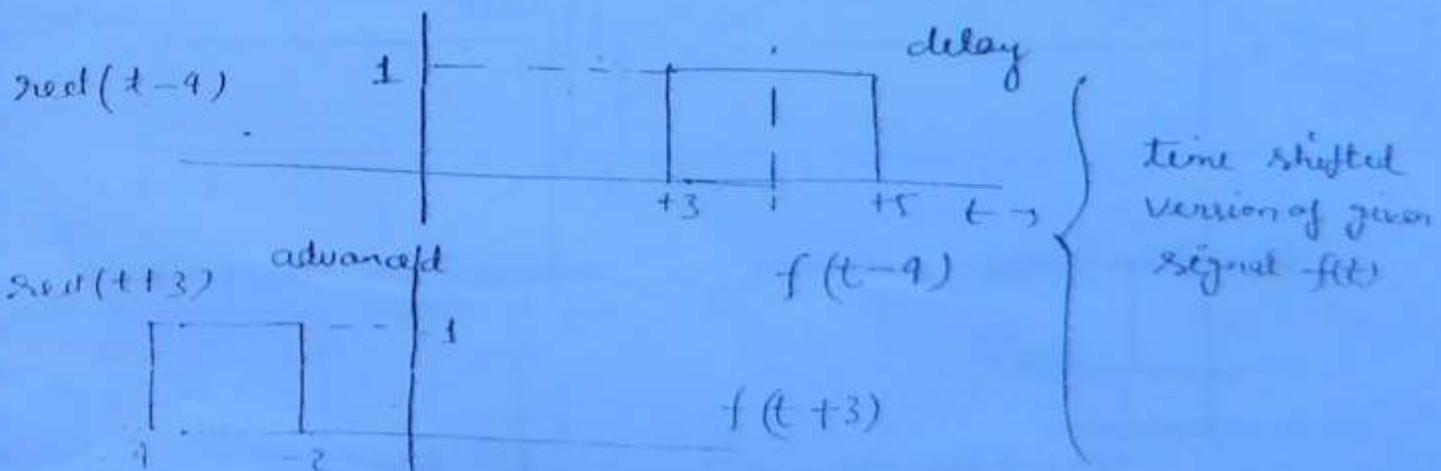


$$f(2t)$$

$$f(t) = \text{rect}(t)$$

time scaled version of  
f(t) (amplified  
time-axis  
change)

$$f\left(\frac{t}{2}\right) \quad f(t) = \text{rect}(t/4)$$



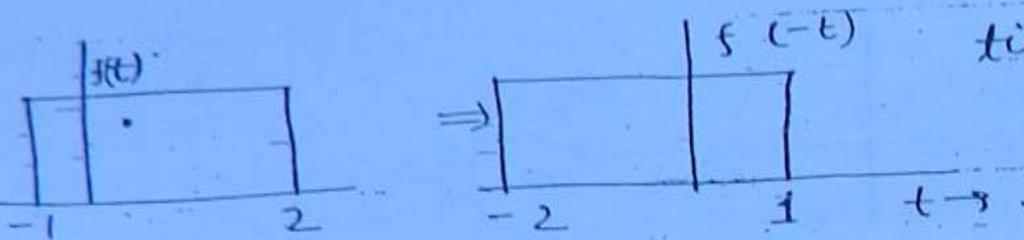
delay

time shifted  
version of given  
signal - f(t)

[(devide all time instant)]  $f(t)$  [in terms of t(at)]  $a > 1 \rightarrow \text{compression}$   
 $0 < a < 1 \rightarrow \text{expansion}$

$f(t) \xrightarrow{\text{subtract } T \text{ from all instant}}$  advanced (left)  
 $\xrightarrow{u(t-T)}$   
 $f(t-T) \xrightarrow{\text{add } T \text{ to all instant}}$  delayed (right)

(6)

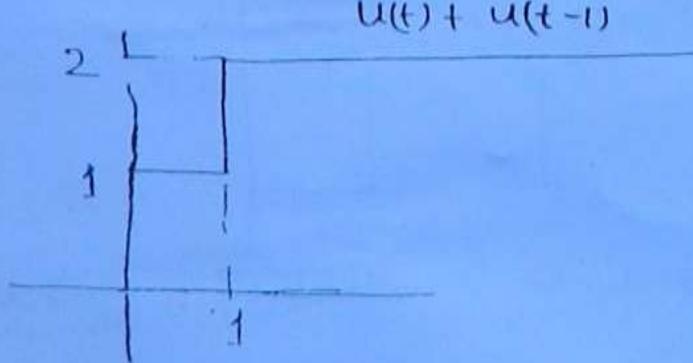
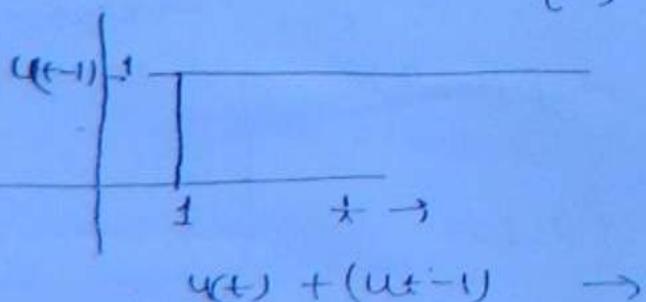
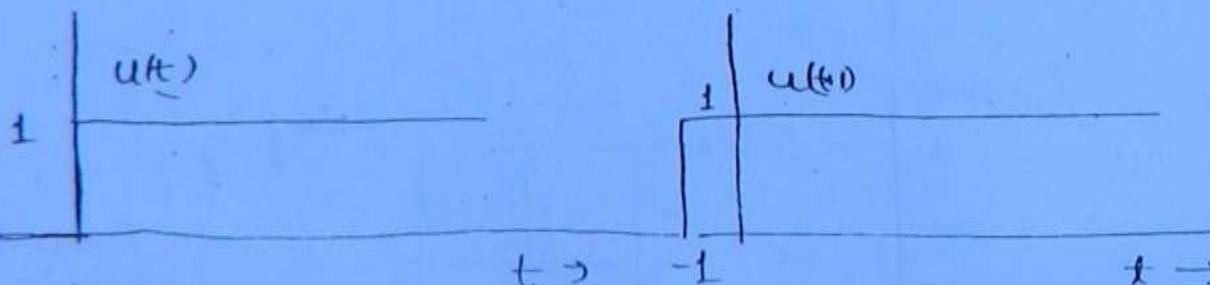
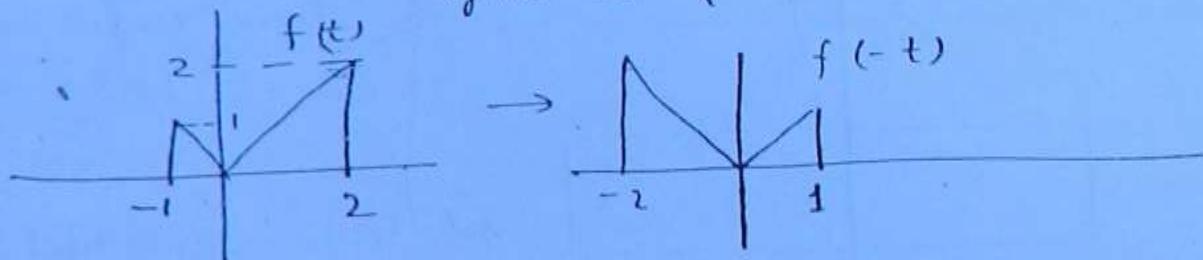


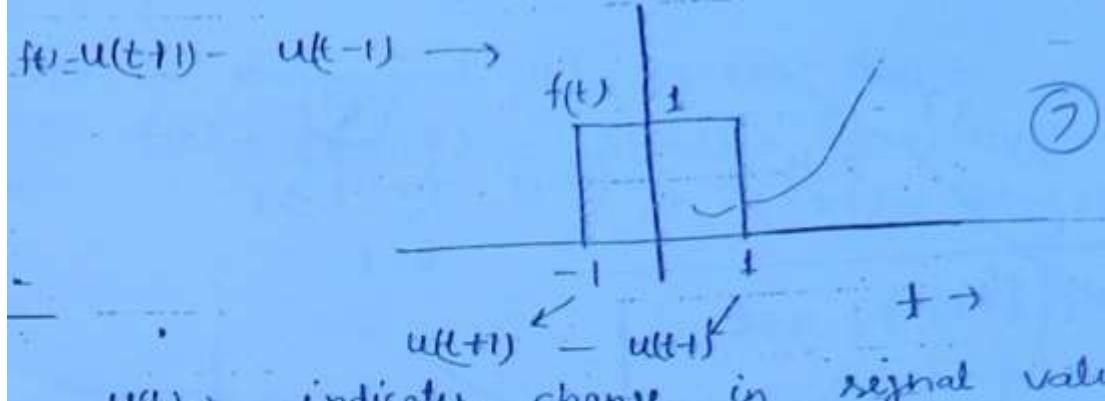
time reversal operation

$$f(t) = \begin{cases} 1 & -1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} f(t) &= -1 \leq -t \leq 2 \\ &= 1 \geq t \geq -2 \end{aligned}$$

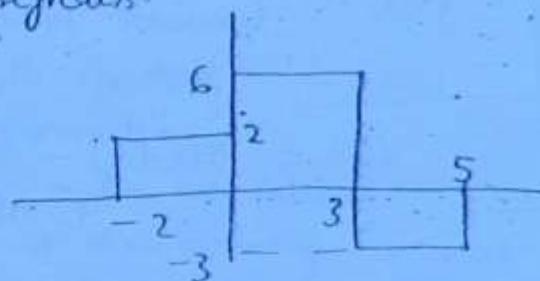
obtained by rotating signal  $\sim$  Y-axis/by  $180^\circ$   
or by taking mirror image of  
signal abt Y-axis.





$u(t) \rightarrow$  indicates change in signal value from 0 to 1 exactly at  $t=0$

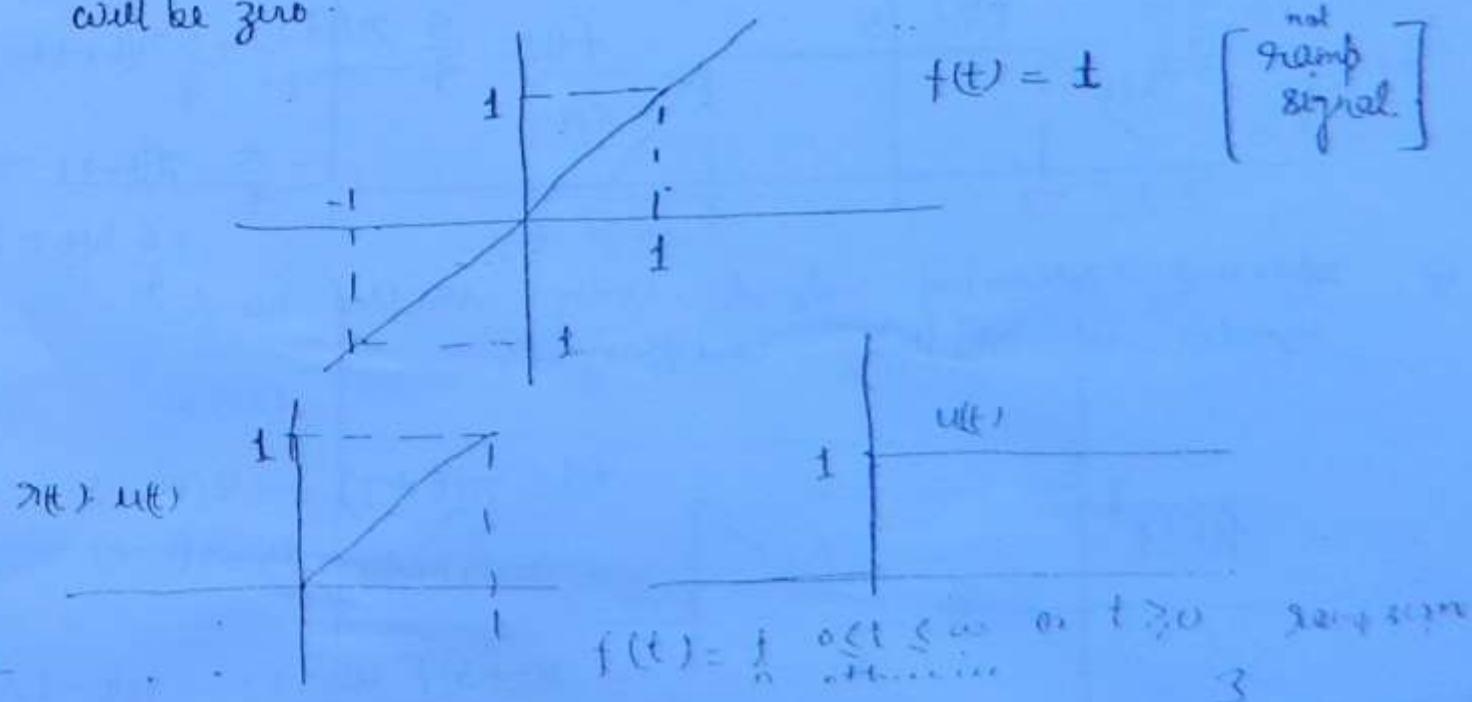
- Q. Represent the following signal using shifted unit step signals.

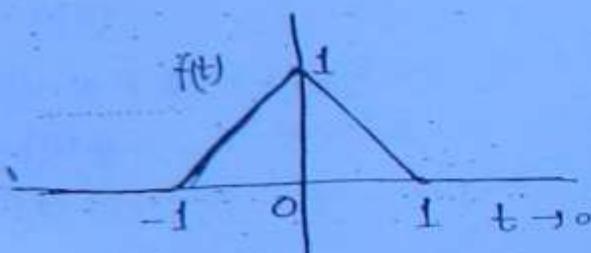
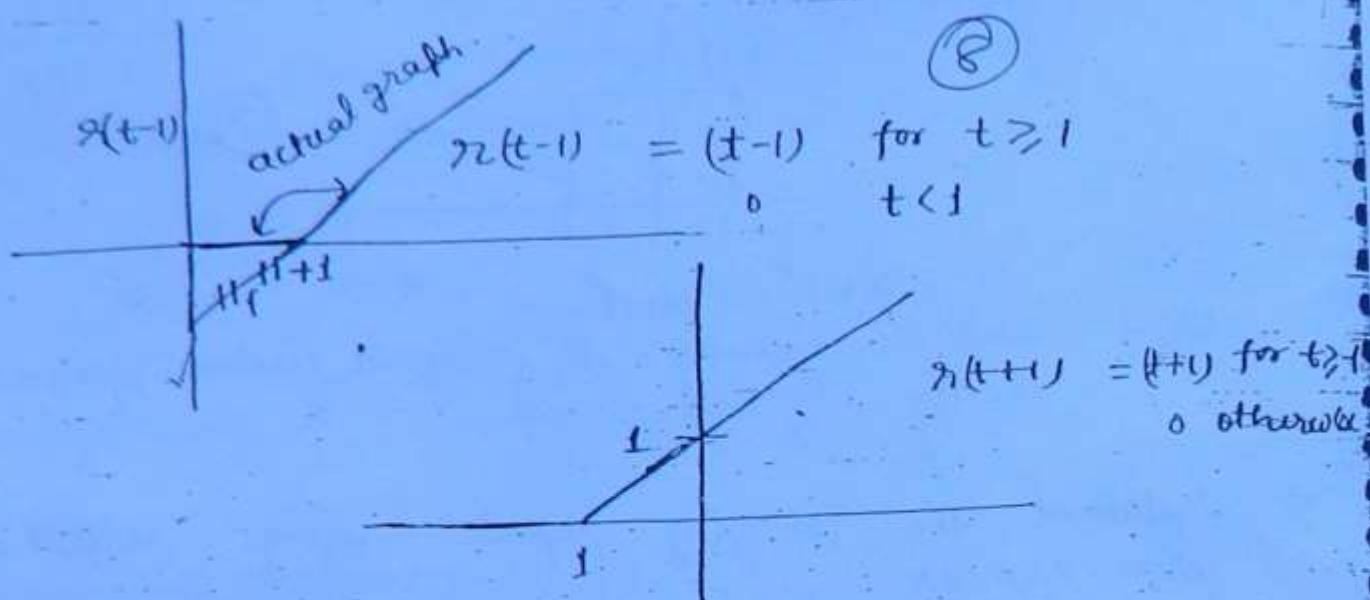


$$2u(t+2) + 4u(t) - 9u(t-3) + 3u(t-5)$$

as long as signal is nonzero only for a finite interval of time then some of coefficient will be zero as in above of  $\Rightarrow 2+4-9+3=0$

- (\*) A signal having step changes in finite interval time then some of all coefficient of shifted unit step signals will be zero.

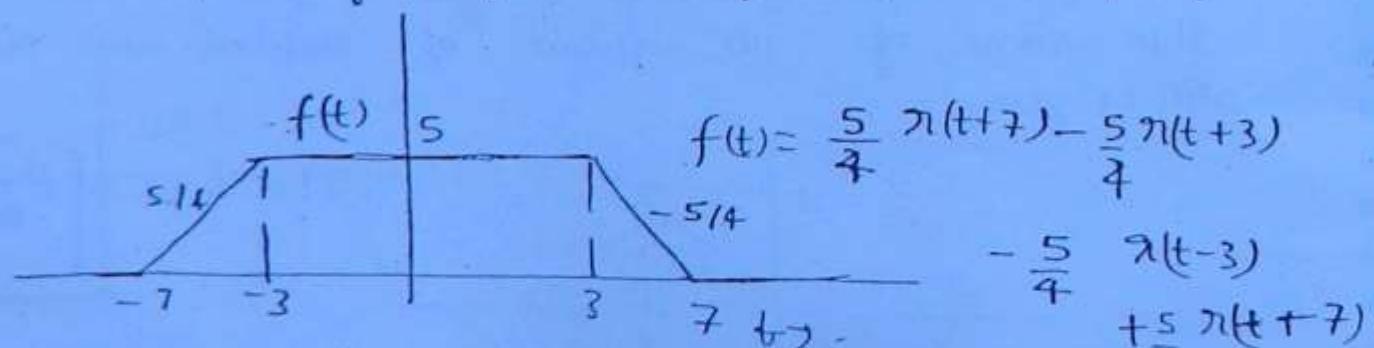




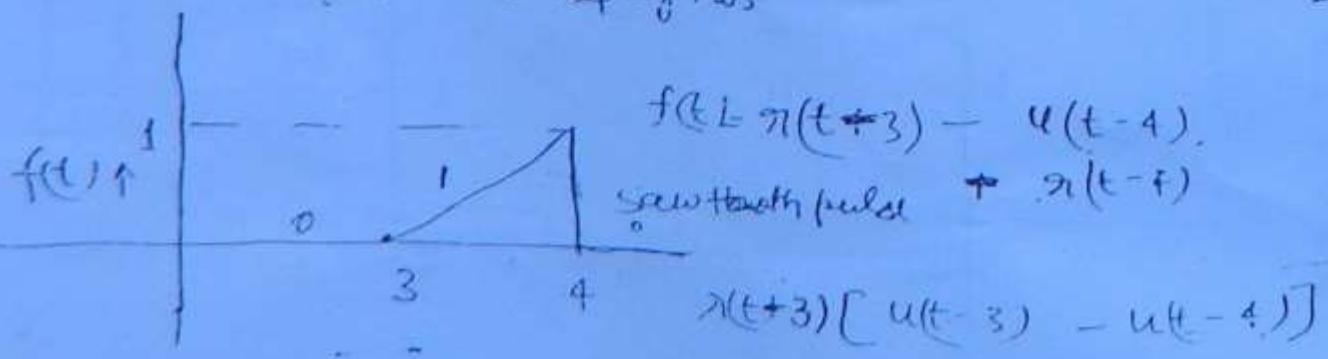
$$f(t) = \frac{n(t+1) - 2n(t)}{+ n(t-1)}$$

$\Rightarrow$  if signal is finite interval signal some of coefficients will be zero  $\rightarrow$  e.g.  $1 - 2 + 1 = 0$

- Q. Represent following signal using shifted ramp signal.



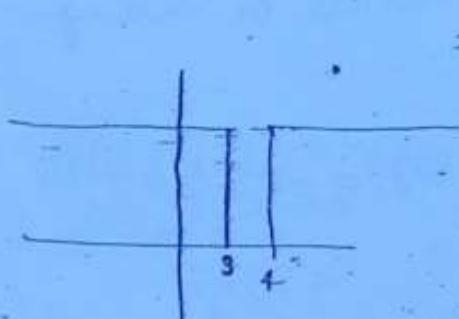
- Q. represent following signal using shifted ramp signals and shifted unit step signals.



for the above signal  $f(t) + f(-t+7)$  will be

(9)

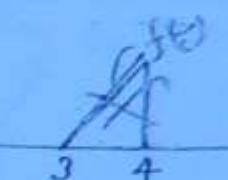
$$f(t) + f(-t+7) = \pi(t-3) - \pi(t-4) - u(t-4) \\ + \pi(-t+4) - \pi(-t+3) - u(-t+3)$$



=

$$y(t) = f(-t+7)$$

$$+ f(t)$$



$$y(t) \downarrow$$

$$y(t) \downarrow$$

$$f(-t+7)$$

$$f(t)$$

$$3 \quad 4$$

$$y(t) \downarrow$$

$$3 \quad 4$$

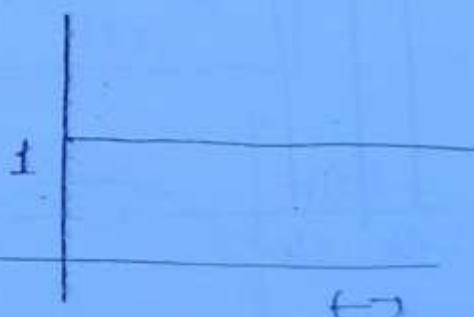
$$u(t-3) - u(t-4)$$

$$+ (\oplus)(t-7)$$

$$f(at^b)$$

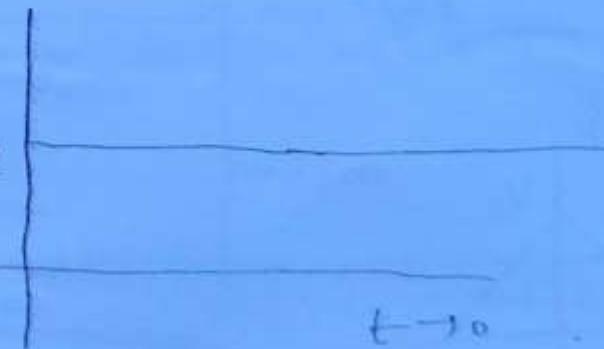
$$f(a(t+b)^b)$$

$$u(t)$$



$$u(at)$$

$$u(at)$$



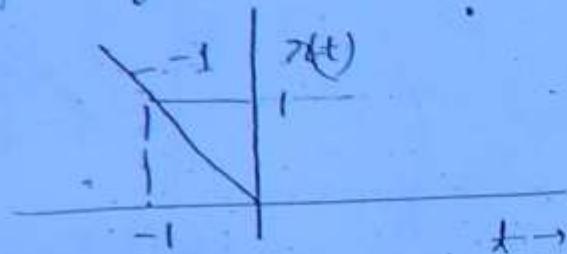
$$t \rightarrow 0$$

4

$$\text{sign} = \begin{cases} 1 & \text{at } t > 0 \\ 0 & \text{at } t = 0 \\ -1 & \text{at } t < 0 \end{cases} \rightarrow \left\{ \begin{array}{l} a > 0 \\ 0 \\ a < 0 \end{array} \right\}$$

(10)

- \*  $r(t) = a \pi(t)$
- \* in comp signal scaling of time results in scaling of magnitude (amplitude).



$$r(t) = \begin{cases} -t & t \leq 0 \\ 0 & t > 0 \end{cases}$$

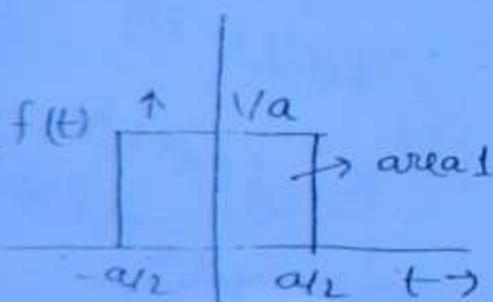
definition of delta function

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad \text{also called dirac delta function.}$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

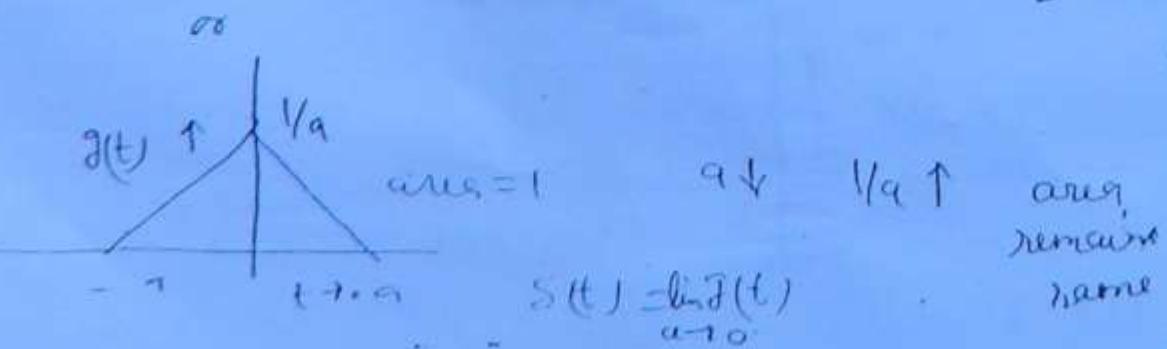
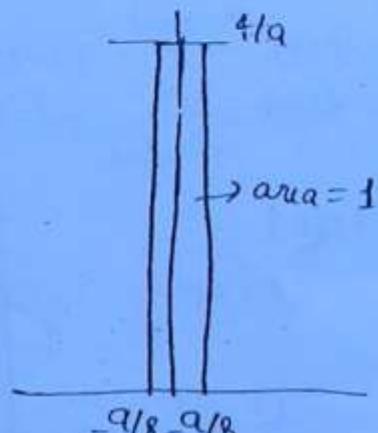
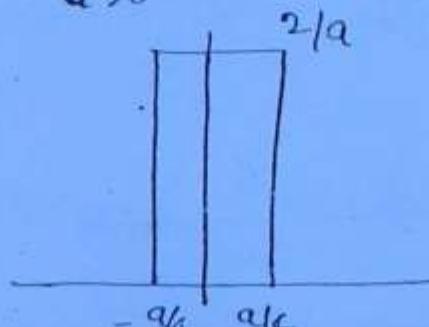
unit impulse means

$$\text{mean area } \int_{-\infty}^{\infty} \delta(t) dt = 1 \quad \text{if } f(t) = \lim_{a \rightarrow 0} \frac{f(t+a)}{a}$$



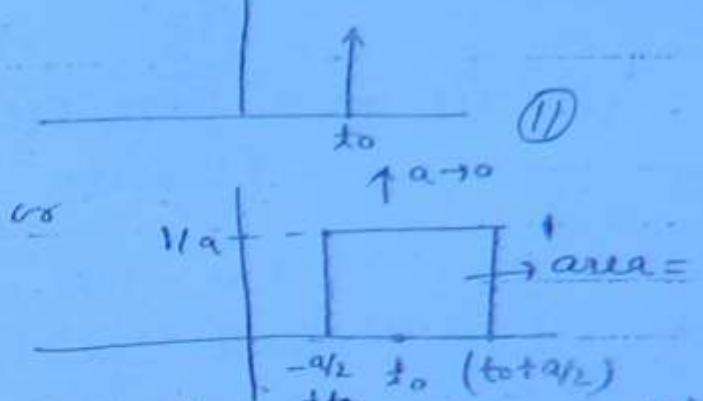
$$a \rightarrow 0$$

$$\text{height} \rightarrow \infty$$



$a \downarrow$        $1/a \uparrow$       area remains same

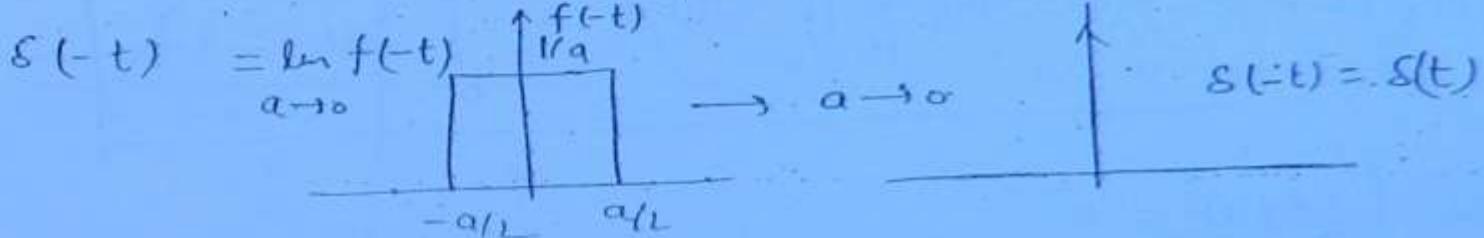
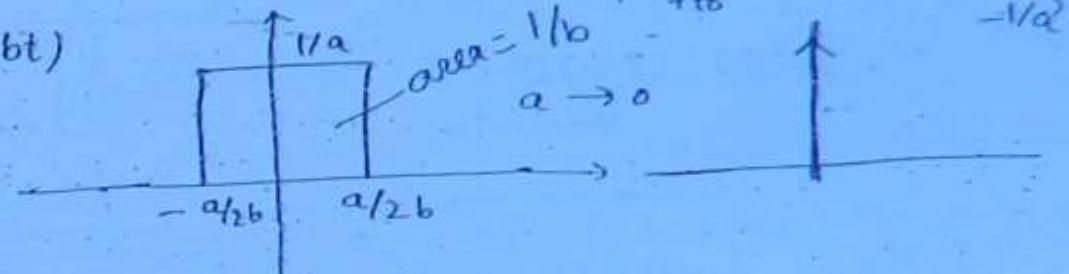
$$\delta(t-t_0) = \lim_{a \rightarrow 0} f(t-t_0) =$$



$$\delta(bt) = \lim_{a \rightarrow 0} f(bt)$$

$$= \frac{1}{b} \delta(t)$$

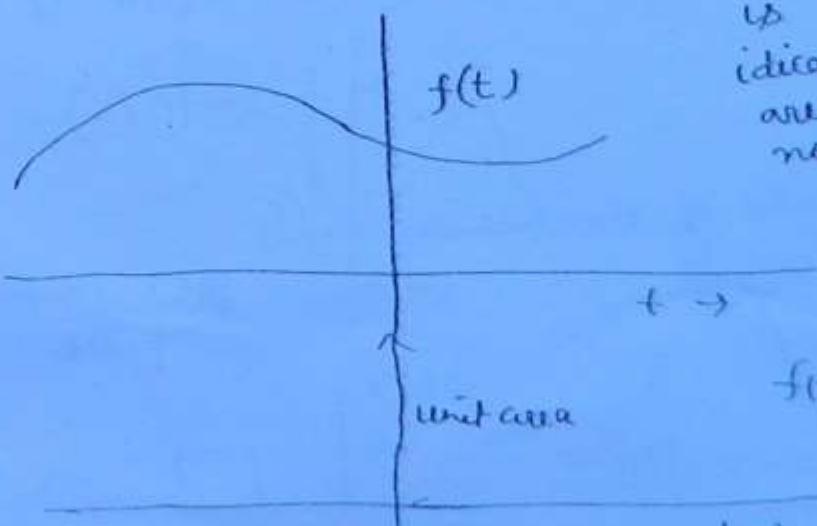
$$= 1/b \delta(t)$$



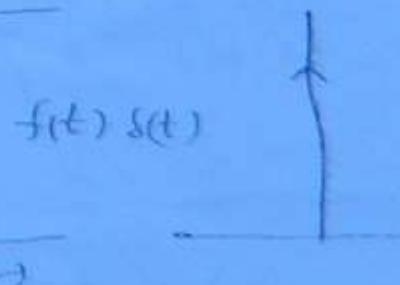
$$\delta(-2t) = \frac{1}{2} \delta(t)$$

$$\delta(-2t) = \delta(2t) = 1/2 \delta(t)$$

$$\boxed{\delta(bt) = \frac{1}{|b|} \delta(t)}$$



height  
is  
indicating  
area  
not the value



$$f(t) \cdot \delta(t) = f(0) \cdot \delta(t) = \delta(t) \quad (\text{if } f(0) > 0)$$

(12) amplitude  $\propto$  but effect on area  
area will become  
 $= f(a)$ , b/w

$$f(t) \cdot \delta(t-a) = f(a) \cdot \delta(t-a) \rightarrow$$

$$= f(a) \cdot \delta(t-a)$$

sampling Property of  $\int_{-\infty}^{\infty}$  current  
impulse signal.

$$\int_{-\infty}^{\infty} f(t) \cdot \delta(t-a) dt = f(a) \int_{-\infty}^{\infty} \delta(t) dt = f(a)$$

$$\int_{-\infty}^{\infty} f(t) \cdot \delta(t-a) dt = f(a) = \int_a^{\infty} f(t) \cdot \delta(t-a) dt$$

$$= f(a) \cdot \int_{-\infty}^{\infty} \delta(t-a) dt = f(a)$$

$$\left. \int_{-\infty}^{\infty} \delta(t) dt = \int_{-1}^1 \delta(t) dt = \int_{-a}^a \delta(t) dt = \int_0^a \delta(t) dt = 1 \right\}$$

$$\int_{-\infty}^2 \delta(t) dt = 0$$

calculate following  $\rightarrow$  (i)  $\int_{-\infty}^{\infty} \delta(t - \sin(t - \pi/2)) \delta(t - \pi/2) dt$

$$= \sin(\pi/2 - \pi) = \sin(-\pi/2)$$

$$= -\sin(\pi/2) = -1$$

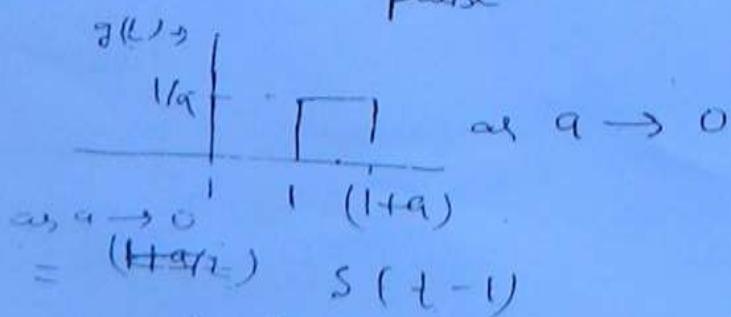
Q.  $\int_{-\infty}^{\infty} \sin(t-\pi/2) \delta(3t-\pi/2) dt = \frac{1}{3} \sin(\pi/6 - \pi)$

$$= +\frac{1}{3} \sin(-\pi/6) = -\frac{1}{3} \sin(\pi/6)$$

Q. (ii) Find the value of the following integral  $\int_{-\infty}^{\infty} \frac{\sin(\pi t - \pi/2)}{(t^2+4)} g(t) dt$  where  $g(t)$  is following pulse

(iii)

$$= \frac{\sin \pi a}{5} = 1/5$$

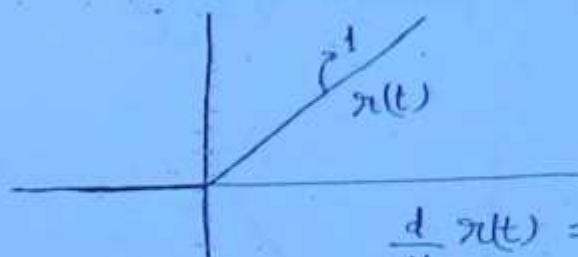


15<sup>th</sup> Oct 10

differentiation →

$$f(t), \frac{df(t)}{dt}$$

(13)



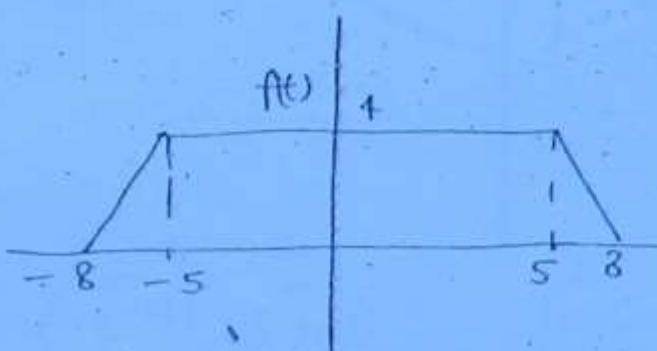
$$f(t) = mt + c$$

$$\frac{df(t)}{dt} = m$$

$$\frac{d}{dt} r(t) = 1$$

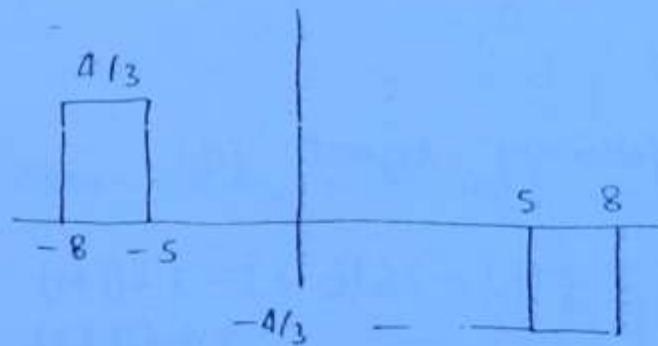
$$0 \leq t \leq 1 = u(t).$$

0 otherwise

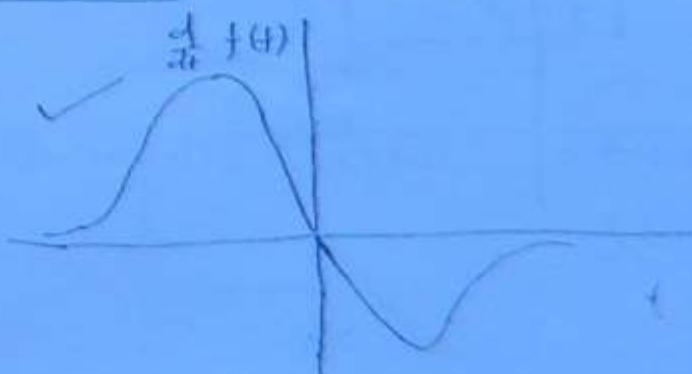
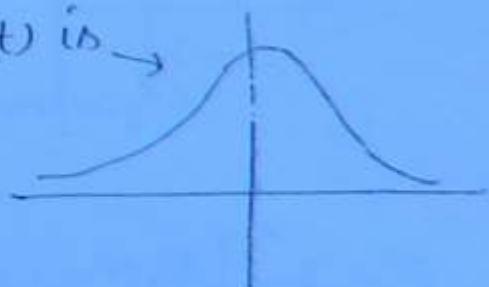
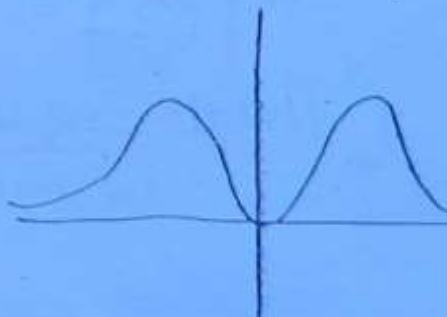


or K  $f(t) = \begin{cases} 4/3 t & t+8 \\ -4/3 t+5 & t \geq 5 \end{cases} + 4$

$$\left\{ \begin{array}{l} \frac{d}{dt} f(t) = 4/3 - 4/3 - 4/3 + 4 \\ = 0 \text{ for } t \geq 8 \end{array} \right.$$



Derivative of the following signal  $f(t)$  is →



$$\int_{-\infty}^{\infty} f(t) dt = \text{area}$$

K (scalar, may be finite or infinite)

$$\int_{-\infty}^{+\infty} f(t) dt = g(t) \quad \text{area as a function of time}$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

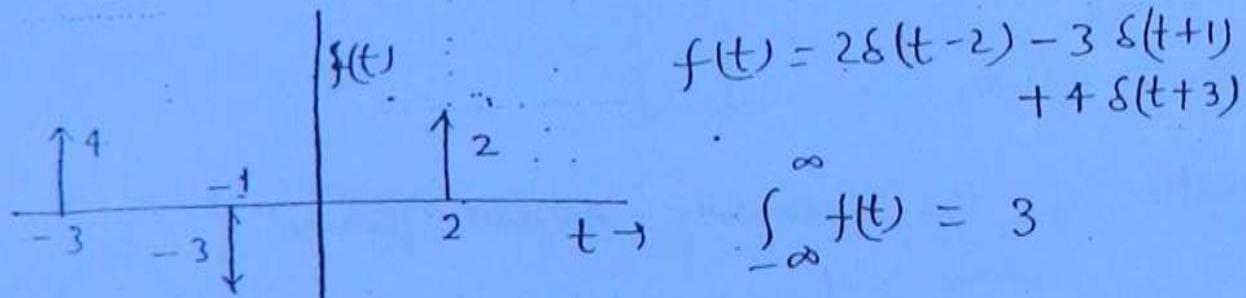
$$\int_{-\infty}^t \delta(\tau) d\tau = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

$$= u(t)$$

$$\frac{d}{dt} u(t) = \delta(t)$$

$$x(t) = \int_{-\infty}^t u(t) dt$$

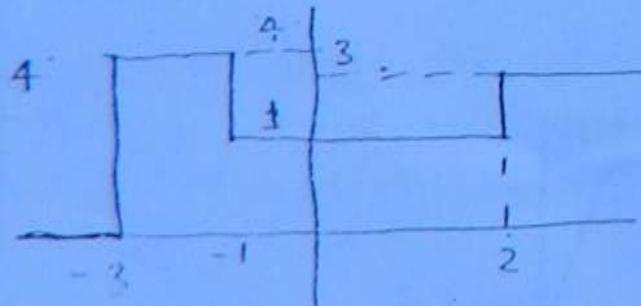
Calculate the integral of following signal  $f(t)$



$$f(t) = 2\delta(t-2) - 3\delta(t+1) + 4\delta(t+3)$$

$$\int_{-\infty}^{\infty} f(t) dt = 3$$

$$\int_{-\infty}^t f(\tau) d\tau = 2u(t-2) - 3u(t+1) + 4u(t+3)$$



$$\frac{d}{dt} e^{-at} = -a e^{-at}, \quad \frac{d}{dt} e^{at} = a e^{at}$$

$$\int_{-\infty}^t e^{-at} dt = \left[ \frac{e^{-at}}{-a} \right]_{-\infty}^t = \frac{1}{a} [e^{-at} - \infty] = \text{not defined unmanageable.}$$

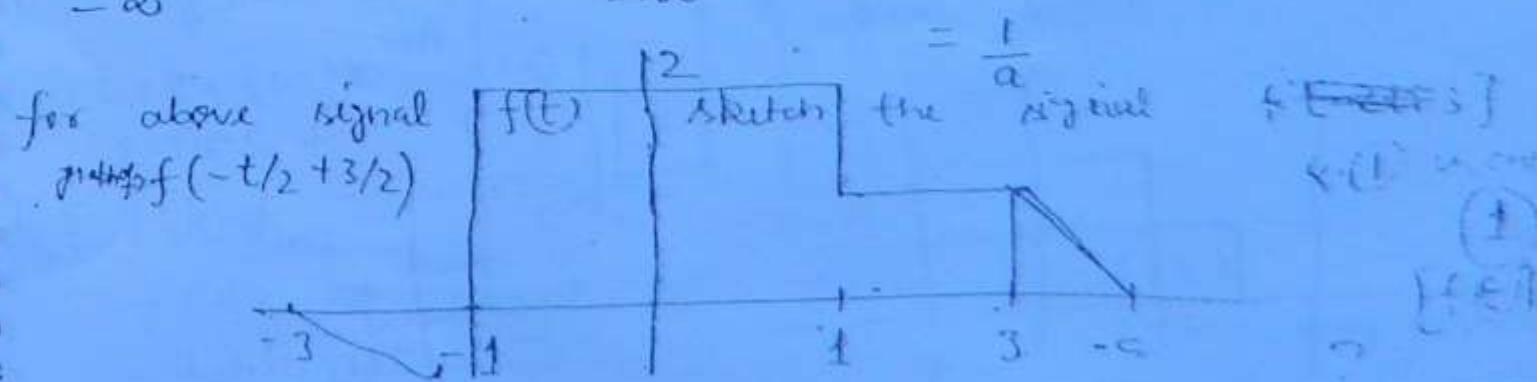


$$\int_{-\infty}^t e^{-at} u(t) dt = \int_0^t e^{-at} dt \\ = -\frac{1}{a} [e^{-at} - 1]$$

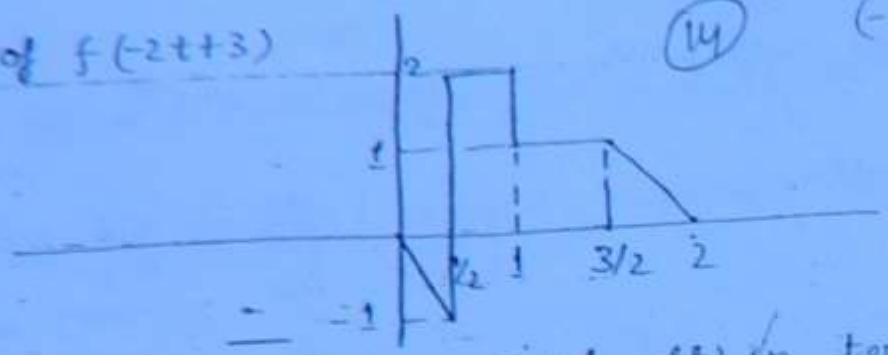
$$\text{area under } e^{-at} u(t) = \int_{-\infty}^{\infty} e^{-at} u(t) dt \\ = \int_0^{\infty} e^{-at} u(t) dt$$

$$\int_{-\infty}^t e^{at} dt = \left[ \frac{e^{at}}{a} \right]_{-\infty}^t = \frac{1}{a} [e^{at}] \text{ unmanageable}$$

$$\int_{-\infty}^t e^{at} u(-t) dt = \int_{-\infty}^0 e^{at} dt = \frac{1}{a} [e^{at}]_{-\infty}^0$$



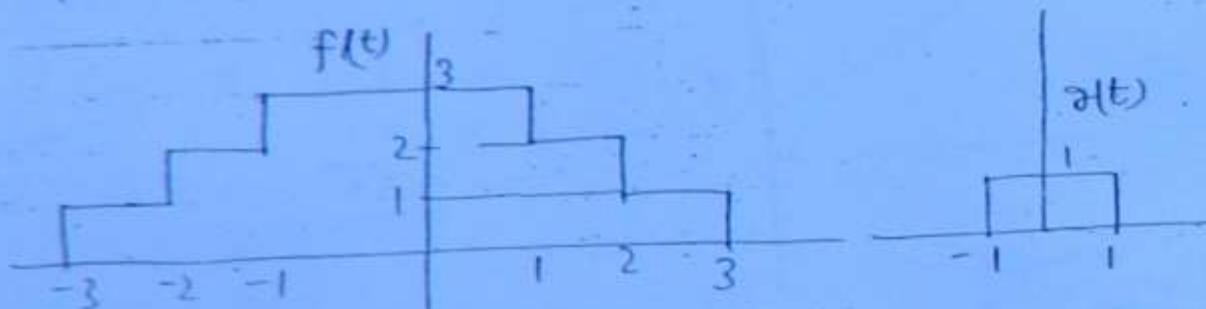
Graph of  $f(-2t+3)$



(14)

$$(-2t+3) = t_1 \\ t = -t_1/2 + 3/2$$

Q. Represent the following signal  $f(t)$  in terms of  $g(t)$

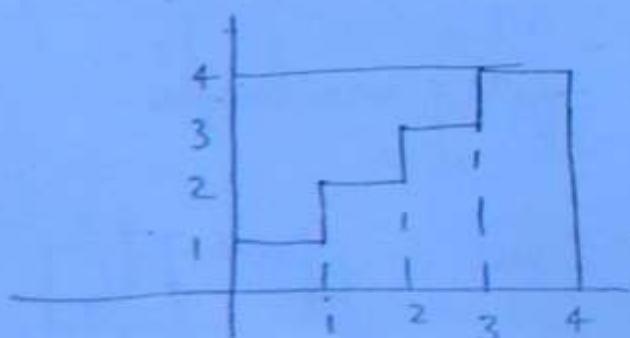


$$f(t) = \left\{ \begin{array}{l} g(t+2) + g(t+1) + g(t) + g(t-1) - g(t-3) \\ \text{with } g \\ - g(t-4) \end{array} \right\}$$

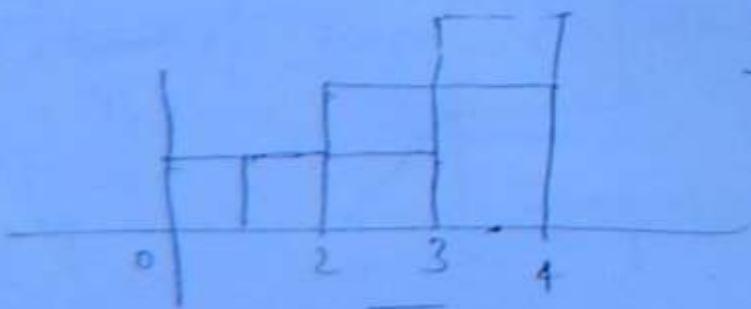
$$f(t) = g(t/3) + g(t/2) + g(t)$$

Q. Represent the following signal  $f(t)$  in terms of  $g(t)$

$g(t)$  is same  
in previous  
equation



$$g(t-1) + g(t-2) + 2g(t-3) \\ + \cancel{g(t-4)} \\ + 2g[3(t-7/2)]$$



$$\text{Ans} - \\ f(t) = g(t-1) + g(t-2) \\ + 2g(t-3) \\ + 2g[2(t-7)]$$

(B)

Types of signal  $\rightarrow$ 

can be real value signal or complex value signal

 $f(t) = \text{at real valued}$  $f(t) = a + ib \text{ complex valued}$ 

$$e^{jt} = \cos t + j \sin t \quad \text{Euler's identity}$$

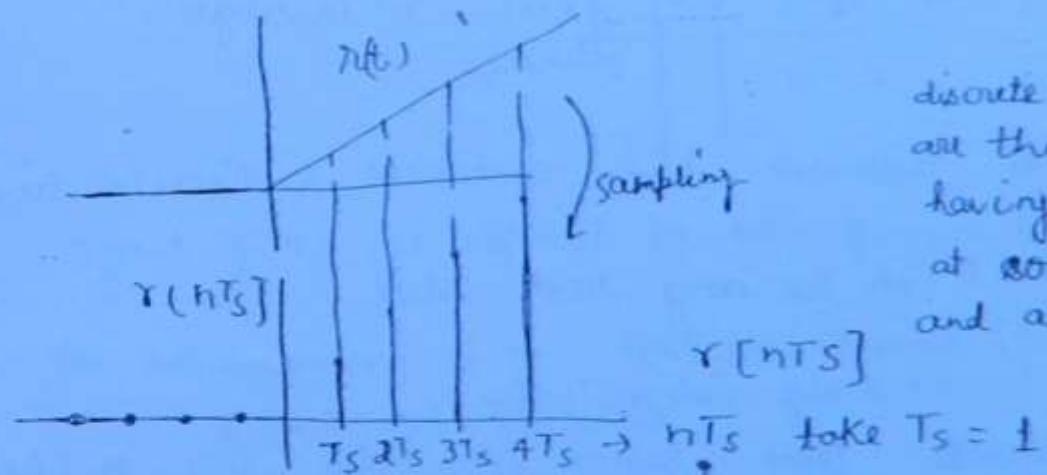
- \* Signals having only real value they are called as real valued signal  $f(t) = \cos t, \sin t$ , real valued signal
- \* Signals having imaginary value along with real value are defined as complex valued signal.

$$\text{g. } e^{jt} = \cos t + j \sin t$$

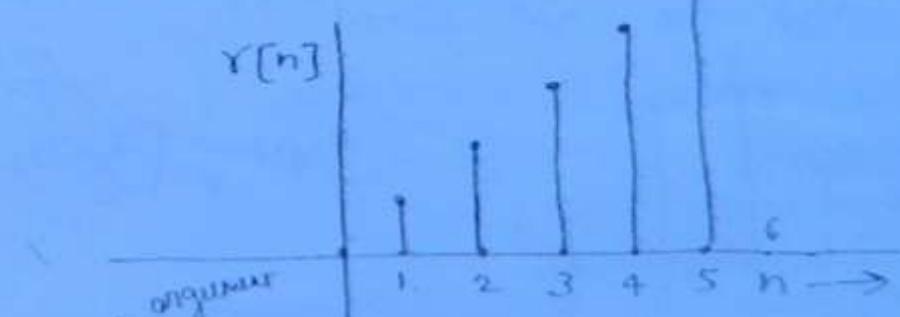
real part =  $\cos t$ , imaginary part =  $\sin t$

Continuous time & discrete time signal  $\rightarrow$ 

- (1) Continuous time signals are those signals having def value for every real value of time.



discrete-time signals  
are those signals  
having defined values  
at some fixed instants  
and at other instant  
undefined.



$f[n] \rightarrow n \text{ must be integer}$ ;  $f(t) \rightarrow \text{any real value}$   
discrete time signal

$$f[n] = \left\{ -1, \underset{n=0}{1}, 2, -2 \right\}$$

(16)

$$-1 \rightarrow n = -1$$

$$1 \quad n = 0$$

$$2 \quad n = 1$$

$$-2 \quad n = 2$$

0 otherwise other values of  $n$ .

\* Continuous time signal is that signal which is defined for all values of time.

→ A discrete time signal is a signal which is defined only for specific values of time. It is not defined for other values of time. A discrete time signal is derived from a continuous time signal by a procedure called as uniform sampling and then selecting uniform sampling interval value to be 1.

$f(t) \rightarrow f[nT_s] \rightarrow T_s = 1 \rightarrow f(n) \quad n \text{ is integer}$   
continuous

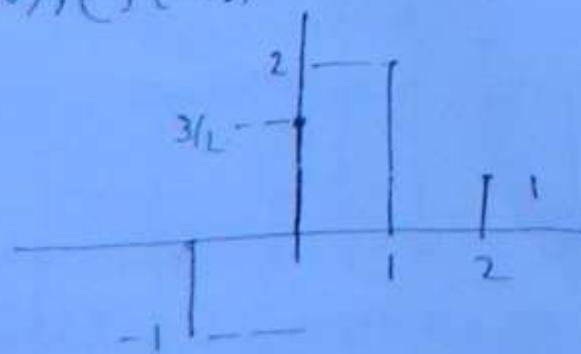
Fundamental difference b/w continuous & discrete time signal is  $f(t)$

↳  $t$  can be any real value

$f(n)$   
↳  $n$  only integer value.

\* For a discrete time signal  $f(n)$ , value like  $f(4/3)$ ,  $f(5/6)$ ,  $f(-4/3)$  are not defined.

an ex →

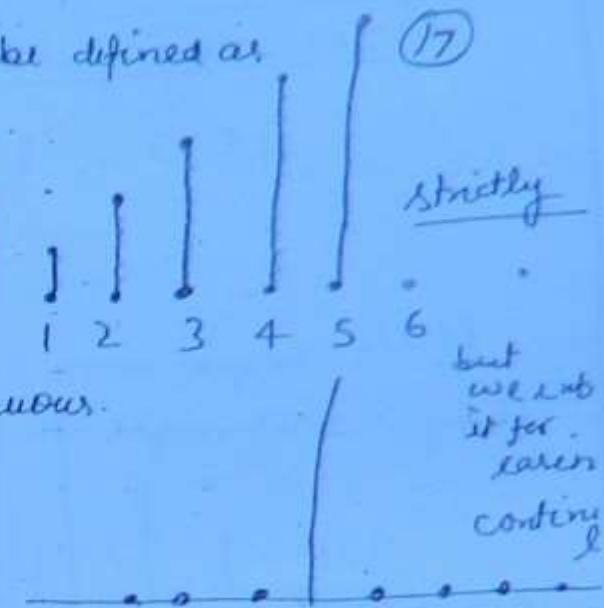


$$f(n) = \left\{ -1, \underset{n=0}{3/2}, 2, 1 \right\}$$

discrete time ramp signal can be defined as

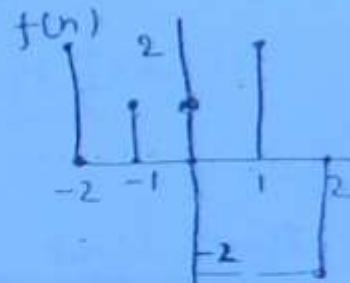
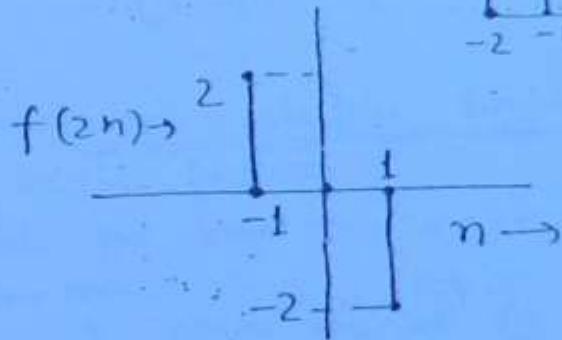
$$r(n) = \begin{cases} n & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

n-axis discrete amplitude axis is continuous.



$$f(n) \xrightarrow{n \rightarrow n-3} f(n-3)$$

$$f(2n) \rightarrow$$



$$\begin{matrix} f(n) \\ f(2n) \end{matrix} \xrightarrow{\quad} = \boxed{?}$$

Q. Signal  $f(n)$  is defined to be  $f(n) = \{ 1, 2, 3, 4, 5 \}$

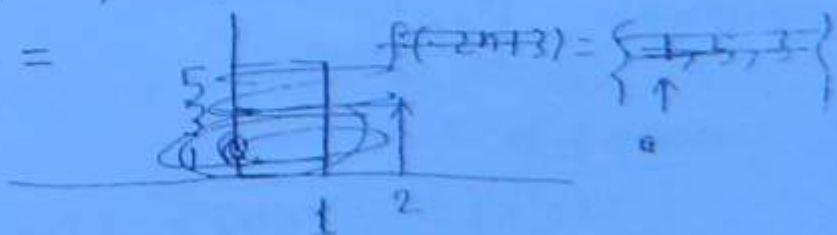
for what values of  $n$  will the signal

$$f[-2n+3] \text{ be zero}$$

$$f[-2(n-3/2)]$$

Ø at  $n = 0, -2, -4, \dots, +2, 4, \dots$

$$\varnothing f[-2n+3] =$$



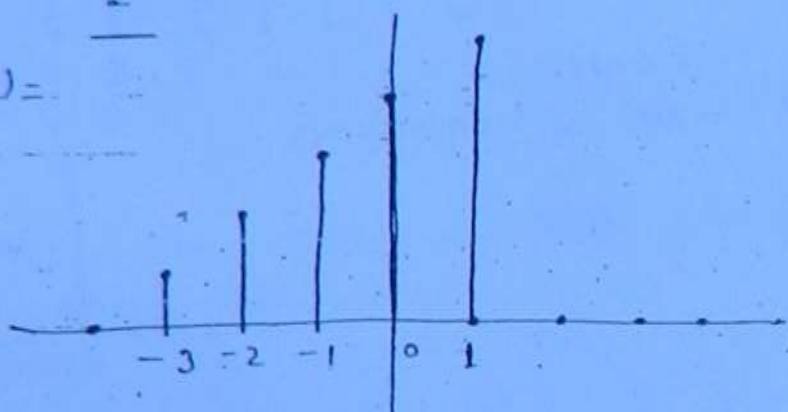
$$f(-2n+3) = \{ 1, 3, 5 \} \text{ and } 0 \text{ for } n \leq 0 \text{ and } 0 \text{ for } n > 3$$

discrete time unit step signal :-

(78)

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$f(n) =$$



$$f(n) = u[n+3] + u[n+2] + u[n+1] + u[n] + u[n-1] - 5u[n-2]$$

discrete time unit impulse function →  
area of  $\delta(t) = 1$

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases} \rightarrow \text{real by defining value 1 we are making it manageable.}$$

17<sup>th</sup> Oct 10:

$$\delta[n] = u[n] - u[n-1]$$

$$\delta(t) = \frac{du(t)}{dt}$$

$$u[n] = \sum_{k=-\infty}^n \delta[n-k]$$

$$u(t) = \int_{-\infty}^t \delta(t) dt$$

$$u[n] = \sum_{k=-\infty}^n \delta[k]$$

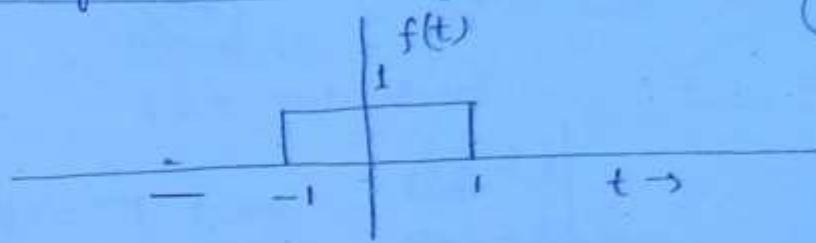
$$r[n] =$$

$$[u[n] = r[n] - r[n-1]]$$

$$r[n] = \sum_{k=-\infty}^n u[k]$$

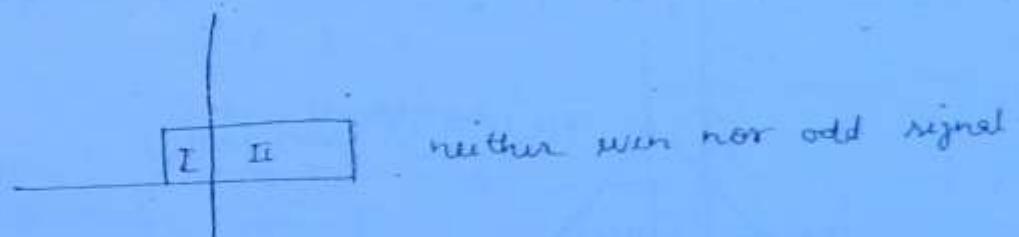
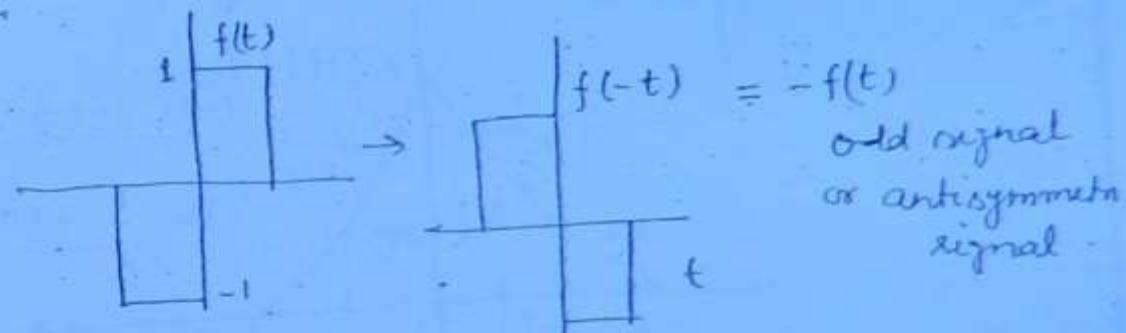
Even signal or odd signals:

(19)



$f(-t) = f(t)$  even signal.

or symmetrical signals



$f(t) + f(-t) \rightarrow$  even signal (always)

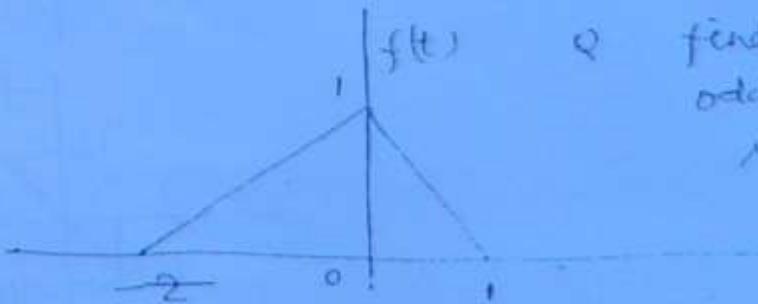
$f(t) - f(-t) \rightarrow$  odd signal

$$f(t) = \frac{1}{2} [\text{even signal} + \text{odd signal}]$$

even part of  $f(t)$       odd part of  $f(t)$

$\cos(t) = \cos(-t) \rightarrow$  even signal or symmetrical signal

$\sin(t) = -\sin(-t) \rightarrow$  odd signal or antisymmetrical signal  
(opposite symmetry)

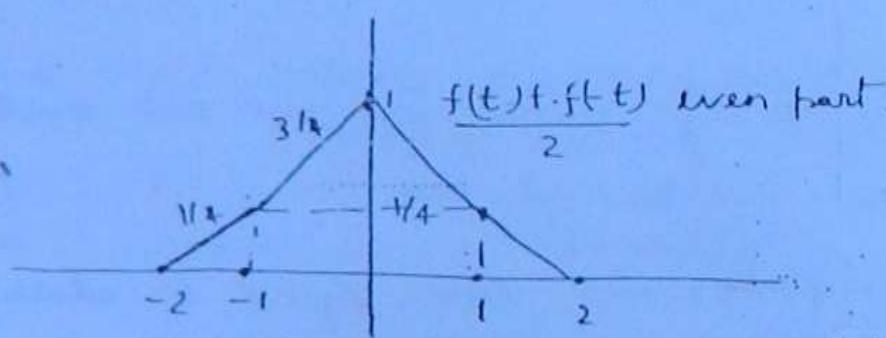
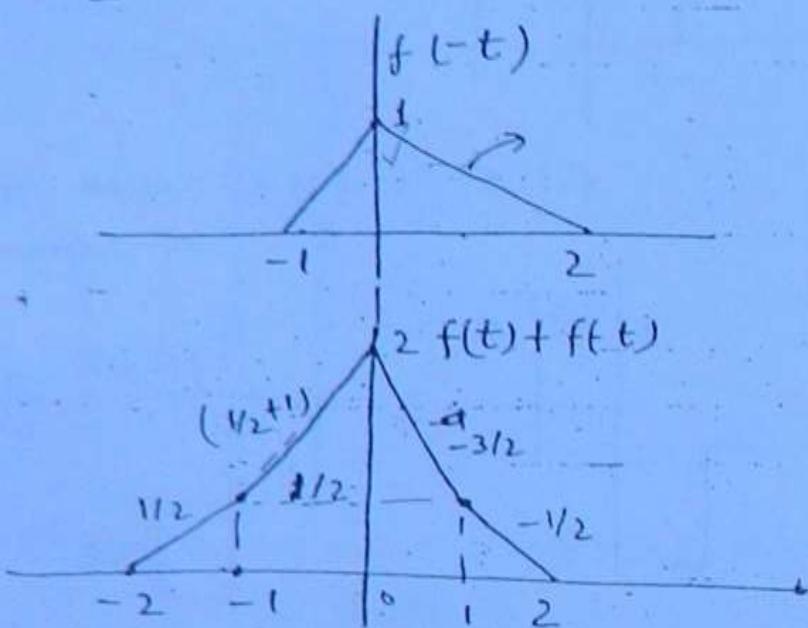


find the even and  
odd part of following  
signal  $f(t)$ .

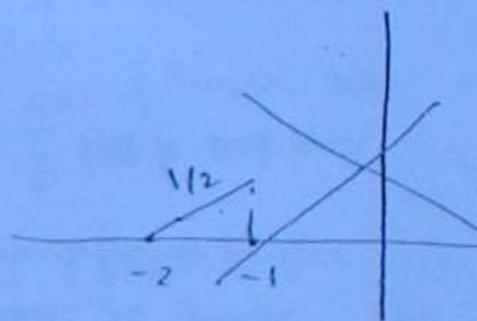
(10)

$$\text{even part} = \frac{1}{2} [f(t) + f(-t)]$$

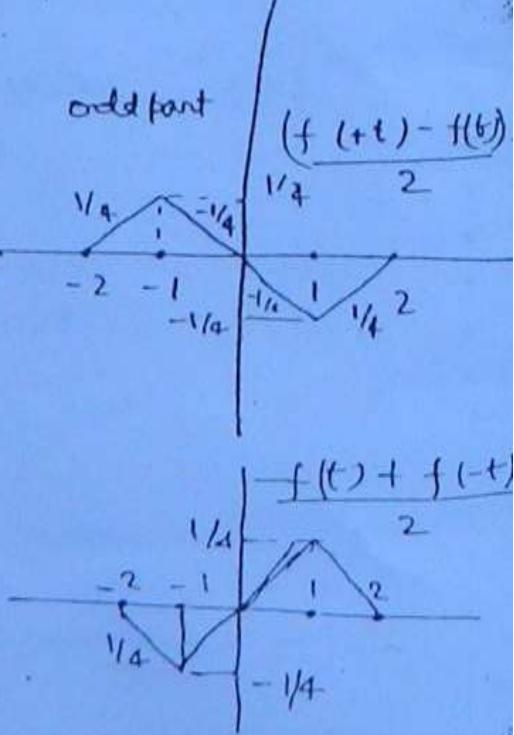
(20)



odd part



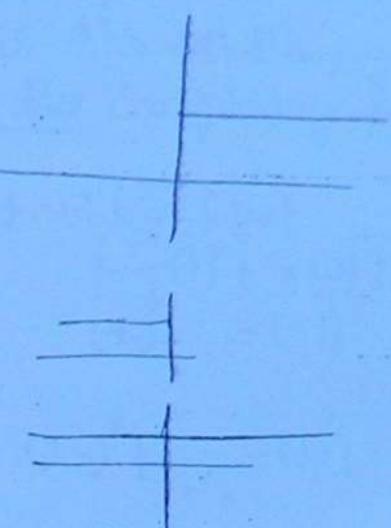
odd part



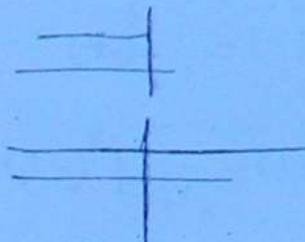
$$\frac{1}{2} [u(t) + v(t)],$$

(21)

$$\left\{ \begin{array}{ll} u(t) = & 1 \quad t > 0 \\ & 0 \quad t < 0 \\ & t = 0 \text{ not defined} \end{array} \right.$$



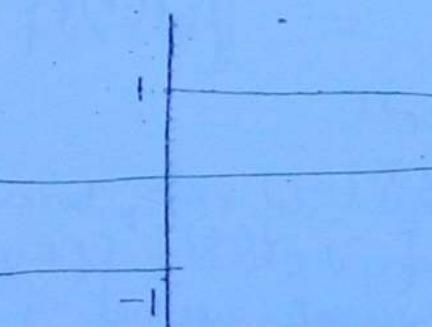
$$\left\{ \begin{array}{ll} u(t) = & -1 \quad t > 0 \\ & 0 \quad t < 0 \\ & 1/2 \quad t = 0 \end{array} \right. \quad \begin{array}{l} \text{take} \\ \text{any definition} \end{array}$$



$$u_e(t) = \frac{u(t) + v(t)}{2} = 1/2$$

$$u_o(t) = \frac{u(t) - v(t)}{2} = \frac{1}{2} \operatorname{sgn}(t)$$

$$\operatorname{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$



four definitions }  $\left\{ \begin{array}{l} t=0 \text{ undefined} \\ \text{or } 1 \\ \text{or } -1 \\ \text{or } 0 \end{array} \right.$

Q. find @ even and odd part of  $f(t) = \sin(t) u(t)$

$$f(t) = e^{jt} = \cos t + j \sin t$$

(22)

$$f_e(t) = \frac{e^{jt} + e^{-jt}}{2} = \cos t$$

$$f_o(t) = j \sin t$$

\*  $f(t), f^*(t)$

$f(t) = f^*(-t)$  even conjugate signal

conjugate symmetric signal

$f(t) = -f^*(-t)$  odd conjugate signal

or conjugate antisymmetric signal

$\cos t \rightarrow$  even conjugate signal

$$f(t) = f^*(-t) \\ = [e^{j(-t)}]^* = e^{-jt} = \cos t$$

$$j \sin t$$

$$-f^*(-t)$$

$$- [j e^{j(-t)}]^* = -[-j e^{jt}] = j \sin t$$

odd conjugate signal

$$f(t) = e^{jt} = \underbrace{\cos t}_{\text{even}} + \underbrace{j \sin t}_{\text{odd}}$$

\* for an even conjugate signal

Real part  $[f(t)] =$  even part of  $f(t) \rightarrow$  even in nature

odd part  $[f(t)] =$  odd imaginary part of  $f(t)$

$$j \sin t = -\sin t + j \cos t$$

$$\downarrow \quad \downarrow$$

odd      even

(\*) for a complex valued signal which is even conjugate in nature real part is always even and imaginary part is always odd.

④ For a complex valued signal which is odd conjugate in nature real part is always odd and imaginary part is always even.

$$f(t) = f_{\text{re}}(t) + j f_{\text{im}}(t)$$

(23)

$$f^*(t) = f_{\text{re}}^*(t) + j f_{\text{im}}^*(t)$$

$$f^*(-t) = f_{\text{re}}^*(-t) + j f_{\text{im}}^*(-t)$$

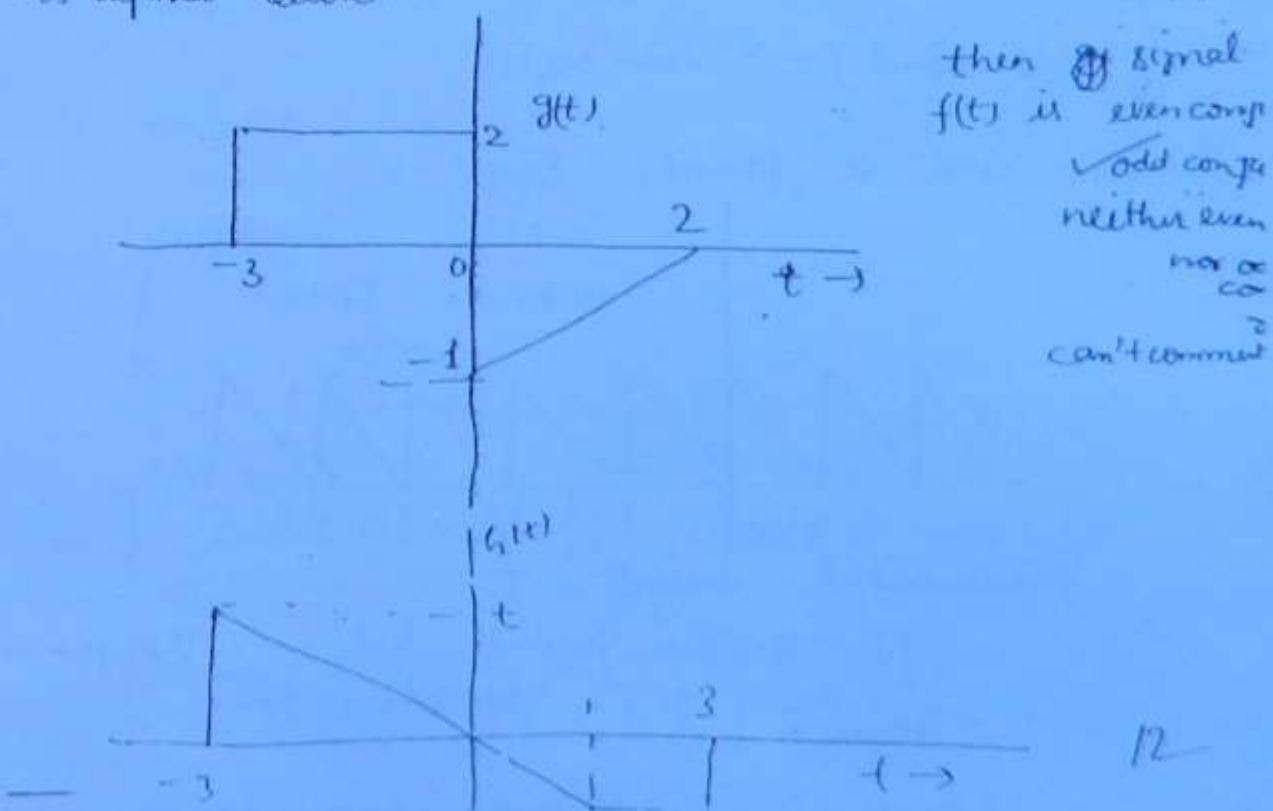
$$= f_{\text{re}}(t) - j f_{\text{im}}(t)$$

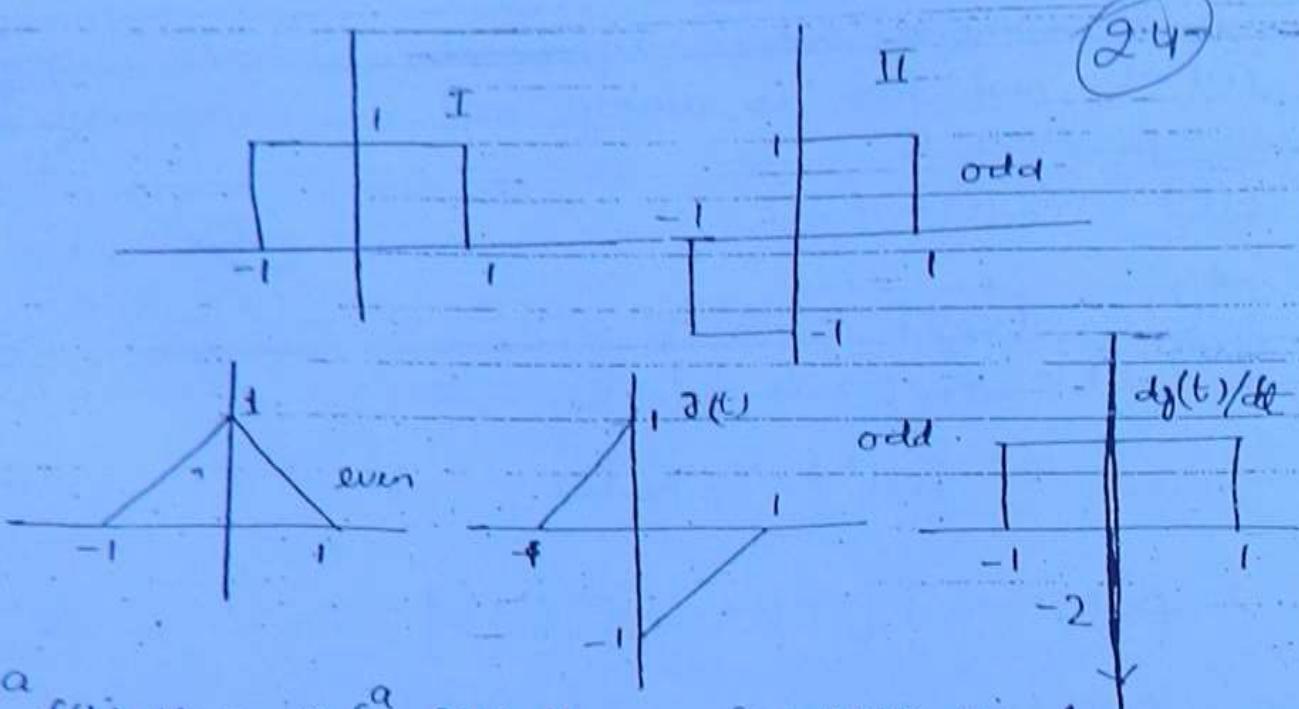
$$f_{\text{re}}(t) = \frac{1}{2} [f(t) + f^*(-t)]$$

$$f_{\text{im}}(t) = \frac{1}{2j} [f(t) - f^*(-t)]$$

$$\begin{aligned} f(t) &= \frac{1}{2} [f(t) + f^*(-t)] \\ &= \frac{1}{2} \left[ f(t) + \frac{f(-t) - f(t)}{j} \right] \\ &= \frac{1}{2} \left[ f(t) + \frac{-f(t) - j f'(t)}{j} \right] \\ &= \frac{1}{2} \left[ f(t) - \frac{j f'(t)}{2} \right] \end{aligned}$$

Q. A complex valued signal  $f(t)$  is defined with a real part  $\rightarrow [g(t) - g(-t)]$  and imaginary part which is  $[h(t) + \{h(-t)\}]$  where  $g(t)$  &  $h(t)$  are as defined below.





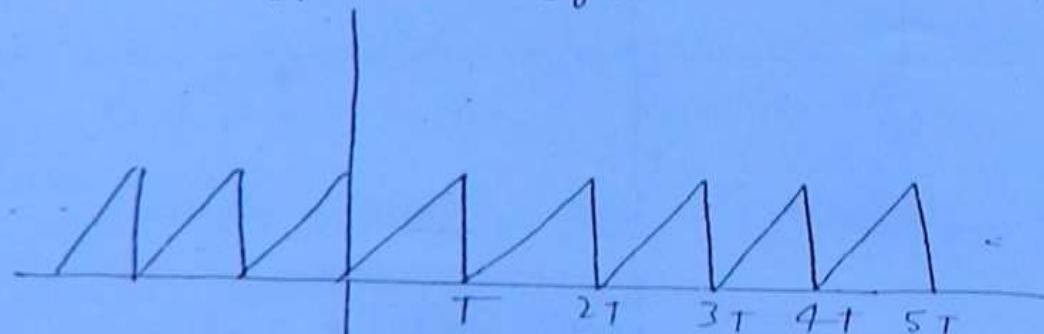
$$\int_{-a}^a f(t) dt = 2 \int_0^a f(t) dt \quad \text{for even signal}$$

$$\int_a^a f(t) dt = 0 \quad \text{for odd signal}$$

$$\frac{d}{dt} [\text{even signal}] = \text{odd signal}$$

$$\frac{d}{dt} [\text{odd signal}] = \text{even signal}$$

periodic and a periodic signal  $\rightarrow$



fundamental period =  $T$

$kT \rightarrow$  time period but not the fundamental period.  
integer

any shift in time will not change period.

(24)

$$f(t) = f(t \pm KT) \xrightarrow{\text{fundamental period}} \text{integer}$$

(25)

$$\text{fundamental period of } \sin KT \xrightarrow[\substack{K \text{ integer} \\ \downarrow}]{} \frac{2\pi}{|KT|} \xrightarrow{\text{fundamental}} \frac{2\pi \times m}{|KT|} \xrightarrow{\substack{\text{also a per} \\ \downarrow \\ \text{iod}}} \frac{2\pi}{m}$$

Carries  $2\pi$ .

$\sin t$   
 $\sin 2t$   
 $\sin mt$   
 $\vdots$   
 $\sin KT$

$\left. \begin{array}{l} \sin t \\ \sin 2t \\ \sin mt \\ \vdots \\ \sin KT \end{array} \right\} \rightarrow \text{all has period } 2\pi \quad (\text{but this is not fundamental period for all except } \sin t)$

$$\int_0^{2\pi} \sin KT dt = 0 \quad (\text{K complete cycle of period } \frac{2\pi}{|KT|})$$

$\uparrow$   
 $\text{so area} = 0$

$K \text{ is integer.}$

$$\begin{aligned} \sin \omega_0 t &\rightarrow \frac{2\pi}{\omega_0} \text{ fundamental period} \\ \sin 2\omega_0 t &\rightarrow \frac{2\pi}{2\omega_0} \rightarrow \frac{\pi}{\omega_0} \\ \sin 3\omega_0 t &\rightarrow \frac{2\pi}{3\omega_0} \end{aligned} \quad \left. \begin{array}{l} \text{common} \\ \text{knew} \\ \text{of} \\ \frac{2\pi}{\omega_0} \end{array} \right\}$$

$$\int_0^{\frac{2\pi}{\omega_0}} \sin \omega_0 t \cdot \sin 2\omega_0 t dt$$

$$= \frac{1}{2} \int_0^{\frac{2\pi}{\omega_0}} (\cos \omega_0 t - \cos 3\omega_0 t) dt = 0$$

$$\int_0^{\frac{2\pi}{\omega_0}} \cos \omega_0 t \cos 2\omega_0 t = \frac{1}{2} \int_0^{\frac{2\pi}{\omega_0}} (\cos 3\omega_0 t + \cos \omega_0 t) dt$$

$$= 0$$

$$\int_0^{\frac{2\pi}{\omega_0}} \sin \omega_0 t \cos 2\omega_0 t dt = \frac{1}{2} \int_0^{\frac{2\pi}{\omega_0}} [\sin 3\omega_0 t - \sin \omega_0 t] dt$$

$$= 0$$

B

$$\sin \omega t \left\{ 2\pi/\omega_0 \right.$$

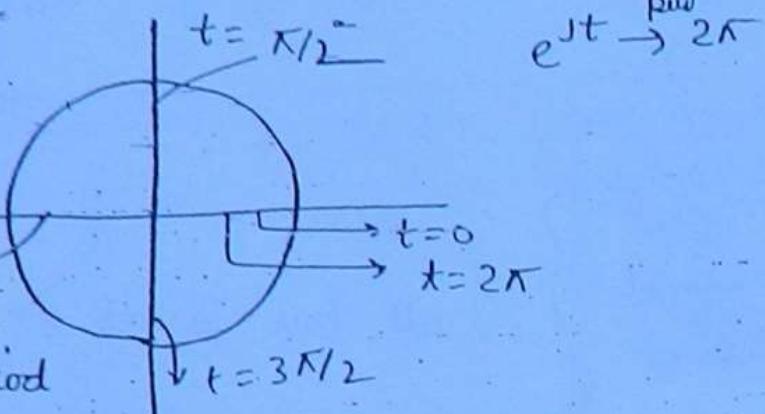
Cos  $\omega t$

$$e^{j\theta} = \cos \theta + j \sin \theta = 1 \angle \tan^{-1}(\tan \theta)$$

$$e^{jt} = 1 \angle t$$

phasor quantity

covering the circumference  
of circle of unit radius and periodic  
with fundamental period  $2\pi$ .



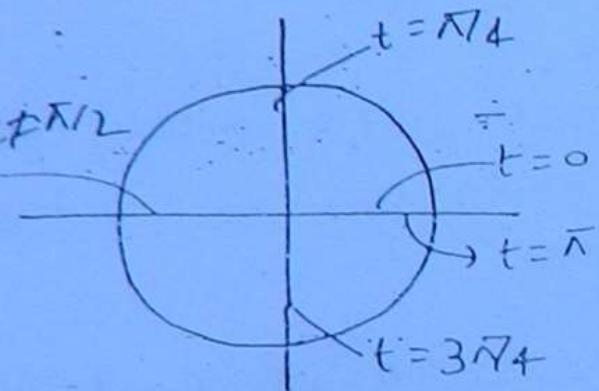
(26)

$$e^{jt} \xrightarrow{\text{fund}} 2\pi$$

$$e^{j(2t)}$$

$$= 1 \angle 2t$$

$$t \neq \pi/2$$



fundamental

period =  $\pi$

$$e^{jk\pi t} \rightarrow \text{fundamental period } \frac{2\pi}{k}$$

$$e^{j\omega_0 t} \rightarrow \frac{2\pi}{\omega_0}$$

$$e^{-j\omega_0 t} \rightarrow \frac{2\pi}{\omega_0} \text{ rotate in opposite direction.}$$

Minimum No. of samples taken to repeat itself is defined as the fundamental time period of a discrete time periodic signal.

No. of samples is always a integer and hence time period of a discrete time signal is always an integer.

\* For a discrete time complex exponential  $e^{j\omega_0 n}$  to be period the condition is ratio  $\frac{2\pi}{\omega_0}$  must be rational, if it is rational the period

$$N = m \cdot \frac{2\pi}{\omega_0}$$

where  $m$  is selected to be a minimum possible integer such that above product is an integer.

$$e^{j\omega_0 n} \rightarrow \omega_0 \rightarrow (\omega_0 + 2\pi k) \quad \downarrow \text{integer}$$

\* A discrete time complex exponential signal there is no change in signal even if  $\omega_0$  is replaced by  $(\omega_0 + 2\pi k)$  i.e. discrete time complex exponential signals the frequencies  $\pi + 2\pi, \pi + 4\pi, \pi + 6\pi \dots$  so on,  $\pi - 2\pi, \pi - 4\pi \dots$  so on, all denote the same discrete time complex exponential signal.

same as  $\cos \omega_0 n$  }  $\rightarrow \left( \frac{r(2\pi)}{\omega_0} \right)$  rational  
 $\sin \omega_0 n$  } integer

### Some of signals

$$\sin t + \sin 2t$$

$$\downarrow \quad \downarrow$$

$$2\pi \quad \pi$$

$$\underbrace{\quad}_{\text{over all period}} = 2\pi$$

$$\frac{T_1}{T_2} = 2 \rightarrow \text{rational number}$$

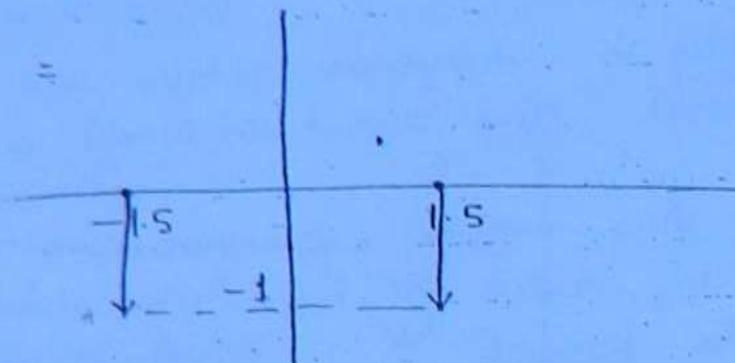
$$\frac{\sin t + \sin \pi t}{t - \pi} \quad T = \pi$$

$$\therefore T_1 = \pi \text{ not rational}$$

Repeat the above problem  $\mathfrak{I}(t)$  is defined as 28

$$\mathfrak{I}(t) = f(t) [-s(t-1.5) + s(t+1.5)]$$

then  $\mathfrak{I}(t) =$



Q if the signal

$$f(t) = 3 + 4 \sin(\pi/3t + \pi/2) + 6 \cos(\pi/4t + \pi/3)$$

Power, signal  $\rightarrow$  calculate the power.

$$= 9 + \frac{4^2}{2} + \frac{6^2}{2} = 35 \text{ W}$$

Q If signal  $f(t) = e^{j2t}$  energy signal or power signal  
power signal periodic signal

$$\text{Power} = 1$$

In calculation of power or energy we concentrate on amplitude not on phase.

In case of discrete time signals

$$E = \sum_{n=-\infty}^{\infty} |f[n]|^2$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N} \sum_{n=-N}^N |f[n]|^2$$

Find energy in the even conjugate part of the signal.

$$f[n] = \left\{ \begin{matrix} -4-j5 & 1+j2 & 4 \\ & \uparrow & \end{matrix} \right\}$$

$$x_{ec}[n] = \frac{x[n] + x^*[n]}{2}$$

(29)

$$= \left\{ -2.5j \quad 1 \quad 2.5j \right\}$$

$$\begin{aligned} E_{ec} &= 1 + (2.5)^2 + (2.5)^2 \\ &= 1 + 6.25 + 6.25 \\ &= 13.5 \text{ J} \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} f(t)^2 dt &= \int_{-\infty}^{\infty} [f_e(t) + f_o(t)]^2 dt \\ &= \int_{-\infty}^{\infty} f_e^2(t) dt + \int_{-\infty}^{\infty} f_o^2(t) dt + 2 \int_{-\infty}^{\infty} f_e(t) f_o(t) dt \\ &= E_e + E_o + 0 \end{aligned}$$

$E = E_e + E_o$

odd & even

Causal or Non causal signal

↳ the signal which not start before zero

↳ If signal start before zero then it is a non causal signal. (all periodic signals are non causal)

Deterministic & Random signal :

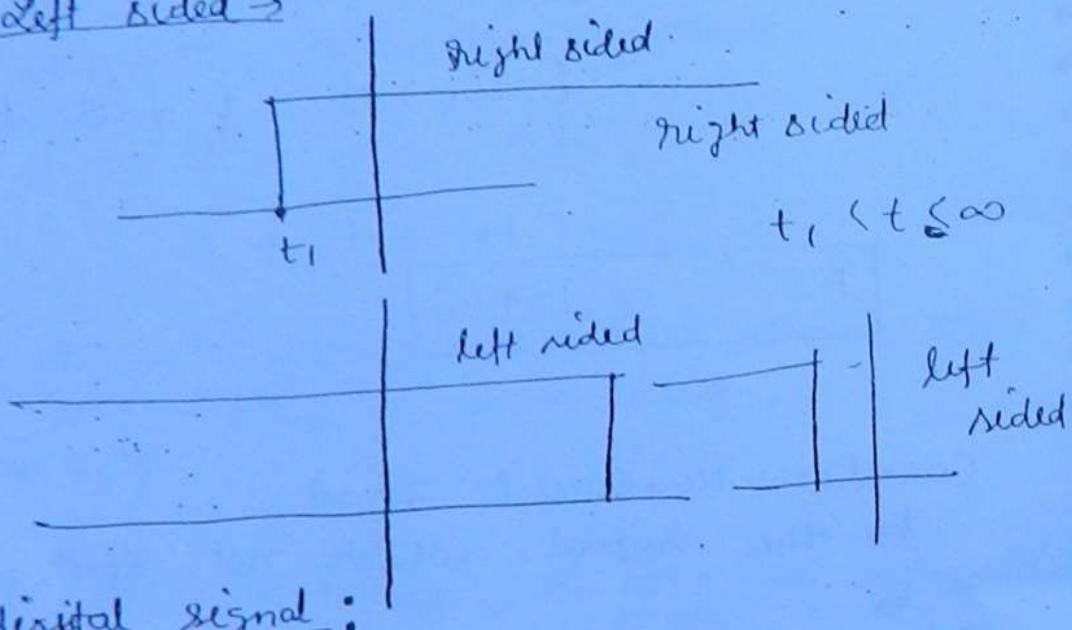
- \* A signal which can be defined by well defined mathematical expression, it is called as deterministic signal.
- \* A signal for which we can't give a well defined mathematical expression is defined as a random signal.

(36)

- Bounded or unbounded signal  $\rightarrow$
- \* If the amplitude of signal have some finite boundaries for all values of time it is called as bounded signal
- $|f(t)| < \infty$  } for all  $t$ , bounded signal  
 $|f(t)| < M \rightarrow$  finite }

If signal value become infinite for any value of time, it is called as unbounded signal.

Right sided or Left sided  $\rightarrow$



Analog or digital signal:

- \* A signal which can assume infinite no. of values for its amplitude is defined as analog signal.
- \* If a signal is allowed only to assume finite no. of amplitude then the corresponding signal is a digital signal. A digital signal is that signal which discrete in both on time axis & amplitude axis.

$$f(t) u(-t) = 0$$

$$f(t)$$

Match the following:

(B1)

List - I

expression of  $f(t)$

$$A. \quad f(t)[1 - u(4+t)] = 0$$

$$B. \quad f(t) + K \frac{df(t)}{dt} = 0 \quad ((K = +ve))$$

$$C. \quad f(t) + K \frac{d^2 f(t)}{dt^2} = 0$$

$$D. \quad f(t)[g(t) - g(0)] = 0$$

arbitrary  $g(t)$

List - II

nature of  $f(t)$

(i) increasing exponential

(ii) causal signal

(iii) decreasing exponential

(iv) sinusoidal

(v) impulse

$$A \rightarrow f(t) = f(t) u(t)$$

$$t < 0$$

$$f(t) = 0$$

$$C \rightarrow (iv)$$

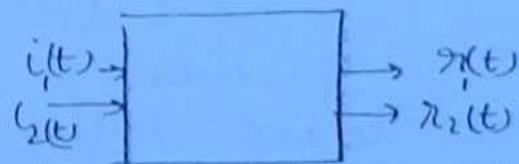
$$B \rightarrow (ii),$$

$$A \rightarrow (i)$$

$$D \rightarrow (v)$$

$$f(t) g(t) = f(t) g(0)$$

System :



★ A system is a quantity which maps a set of i/p signals to a set of o/p signals  
we can understand a system by

(i) i/p - o/p relationship.

(ii) Physical composition  $\rightarrow$  (NE, ECD, AE, DE)

(iii) differential equation or difference equations.

(iv) Unit impulse response  $h(t)$ ,  $h[n]$

(v) Transfer function  $H(\omega)$ ,  $H(S)$ ,  $H(Z)$

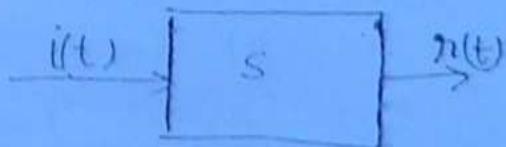
(vi) State variable

$$V = L \frac{di}{dt}$$

$$i = \frac{du}{dt}$$

our

concern



(32)

$$i(t) \xrightarrow{S} r(t)$$

Linear system or Nonlinear system.

$$i(t) \xrightarrow{S} r(t)$$

$$a i(t) \xrightarrow{S} a r(t)$$

homogeneity principle [like  $z = 2i$ ]

where  $a$  is a real

or imaginary quantity

$a$  constant

$$i_1(t) \xrightarrow{S} r_1(t)$$

$$i_2(t) \xrightarrow{S} r_2(t)$$

$$a i_1(t) + b i_2(t) \xrightarrow{S} a r_1(t) + b r_2(t)$$

Linearity principle.

$$\left\{ \begin{array}{l} i_1(t) + i_2(t) \xrightarrow{S} r_1(t) + r_2(t) \\ \text{superposition principle or additivity principle} \end{array} \right.$$

- \* A system satisfying both homogeneity & superposition principle then it is said to linear system.  
If it can not satisfy one of these principle or both, it is defined as non linear system.

$$r(t) = 2i(t) + 3 \rightarrow \text{non linear}$$

$$r(t) = \log i(t) \rightarrow \text{non linear}$$

$$r(t) = i^2(t) \rightarrow \text{non linear}$$

$$r(t) = +i(t) \rightarrow \text{linear}$$

$$r(t) = \sin t i(t) \rightarrow \text{linear}$$

$$r(t) = \int_{-\infty}^t i(\tau) d\tau \rightarrow \text{linear} /$$

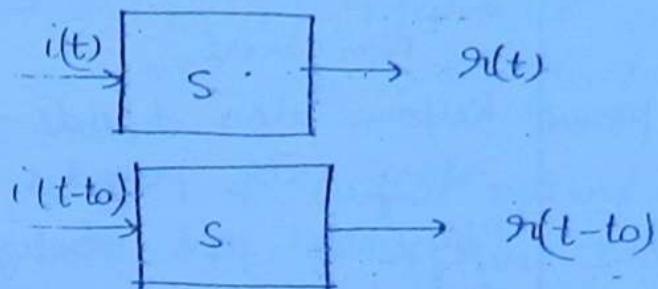
$$r(t) = \int_{-5}^5 i(\tau) d\tau \rightarrow \text{linear}$$

$$\checkmark r(t) = \text{real part of } \{i(t)\} \text{ non linear.}$$

it does not hold homogeneity for an  $a = jb$   
since if  $a i(t)$  will not be  $a r(t)$   $\therefore$  non linear  $\checkmark$  real

time variant or time invariant :-

(33)



time invariant system.

- \* The system is defined as time invariant system if the response is delayed by the same amount as delay given to the system.

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- \* If response to  $i(t-t_0)$  is not equal to  $r(t-t_0)$ , system will be called as time variant system.

$$\begin{aligned} i(t) &\rightarrow i(-t) & r(t-t_0) &\rightarrow r(-t+t_0) \\ i(t-t_0) &\rightarrow r_1[-t-t_0] & \downarrow & \\ \rightarrow i'(t) &\rightarrow r'_1(t) \rightarrow i(-t). & & \\ && i(t-t_0) & \\ && \downarrow & \\ && \neq & \\ && r(t-t_0) & \end{aligned}$$

time variant system

$$r(t) = 2i(t) + 3 \rightarrow \text{time invariant}$$

$$r(t) = \log i(t) \rightarrow \text{time invariant}$$

$$r(t) = i^2(t) \rightarrow \text{T.I.}$$

$$r(t) = t i(t) \rightarrow \text{T.V}$$

$$r(t) = \int_{-\infty}^t i(\tau) d\tau \rightarrow \text{T.V}$$

\* Causal system & Non causal system  $\rightarrow$

Cause  $\rightarrow$  Causal

(34)

Causal

- (i) response depends upon present & past I/Ps
- (ii) Physically realizable
- (iii) Nonanticipatory

Non causal

- Response also depends on future along with present & past I/Ps
- Physically not realizable
- Anticipatory

\* Non causal system also become physically realizable when the data <sup>with</sup> is being operated upon or the <sup>old</sup> I/P data is recorded data. But by default we consider data to be real time data and hence only causal systems are physically realizable systems.

$$\begin{array}{ccc} i(t) & i(t-t_0) & i(t+t_0) \\ i=0 & i(0) & i(-t_0) \\ & & i(t_0) \end{array}$$

$$r(t) = f[i(t), i(t-t_0)]$$

$$r(t) = f[i(t-t_0)]$$

$$t_0 \geq 0$$

\*  $r(t) = i(-t)$   
non causal

$$r(1) = i(-1)$$

$$r(-1) = i(1) \rightarrow \text{depend future I/P}$$

$$r(t) = i(t) + i(t-2) + i(t-4) \rightarrow \text{causal system}$$

$$r(t) = i(2t) \rightarrow \text{non causal system}$$

$$r(t_2) = i(1)$$

$$r(t) = i(1/2t) - \text{non causal}$$

(35)

$$r(-1) = i(-1/2) \rightarrow \text{future i/p}$$

\*  $r(t) = i(at)$   $\rightarrow$  always non causal

$$r(t) = i^2(t) \rightarrow \text{causal system}$$

$$r(t) = i(t) i(t^2) \rightarrow \text{non causal}$$

$$r(t) = i(\sin t) \rightarrow \text{non causal}$$

$$r(\pi/2) = i(1)$$

$$r(-\pi/2) = i(-1) \rightarrow \text{future i/p}$$

$$r(\pi) = \delta i(\sin \pi) = i(0)$$

$$r(-\pi) = i(0)$$

\* Systems can be static or dynamic.

If there is no arrangement for memory in electrical system, it is called as static system.

If there is arrangement of memory in electrical system, it is called as dynamic system.

$$r(t) = i^2(t) - \text{static system}$$

$$\text{or } \log i(t), t i(t)$$

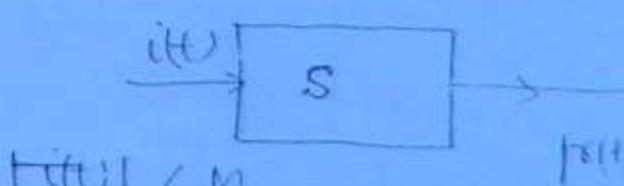
$$\sin i(t)$$

$$2 i(t) + 3$$

$A \cdot i(t)$  { - Linear, static, time invariant }

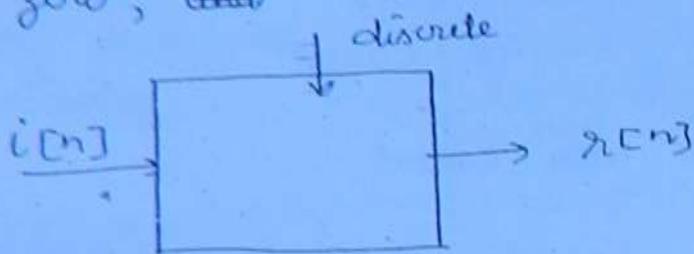
\* For a static system to be linear & time invariant only way the response can be related to i/t is  $r(t) = A i(t)$

Stable or Unstable system:-



BIBO stability

- \* A system can become noninvertible in following cases
  - if more than one I/P to system is generating the same response for the system.
  - if the response of the system to a nonzero I/P is zero; and



(36)

- \* Verify whether the following discrete time system is linear, time invariant, causal, static stable, invertible

$$r[n] = \sum_{k=-\infty}^n i[k] \rightarrow \text{accumulator}$$

$$i'[n] = i[n-n_0]$$

$$i'[k]$$

linear, time invariant,  
causal, dynamic,  
unstable,  $i[k \neq n_0]$   
invertible

$$i[k-n_0] = i[n-n_0]$$

$$\begin{aligned} r[n] &= \sum_{k=-\infty}^n i[k] \\ &= i[0] + \sum_{k=1}^n i[k] \\ &= i[0] + \sum_{k=1}^{n-n_0} i[k+n_0] \\ &= i[0] + \sum_{k=1}^{n-n_0} i'[k] \end{aligned}$$

Q. Find whether system defined as

$$r[n] = i[n] - i[n-1] ; \text{ check}$$

for L, T, C, S, I.

this system  $\rightarrow$  linear, time invariant,  
causal, stable

Invertible system because we can select a system  
having r/pn  $r[n] = \sum_{k=-\infty}^n r_k[n] = \sum_{k=0}^n \{i[k] - i[k-1]\}$

$$\left\{ \begin{array}{l} (i[n-2] + i[n-1] + i[n]) = (i[n]) \\ \text{if } i[n] \text{ is not zero, } i[n-1] \text{ and } i[n-2] \text{ will be zero} \end{array} \right.$$

Q. If P → defined: linearity

Q → defines Time invariant

R → Causality

S → Stability

(37)

A discrete time system defined by i/p o/p relationship.

$$y[n] = x[n] \quad n > 0$$

$$0 \quad n = 0$$

$$x[n+1] \quad n < 0$$

where  $x[n]$  and  $y[n]$ , i/p and o/p of the system  
system is

(i) P, Q, R, S

(ii) P, Q, S but NOT R

~~(iii)~~ P, S but NOT, Q, R

(iv) P but no Q, R, S.

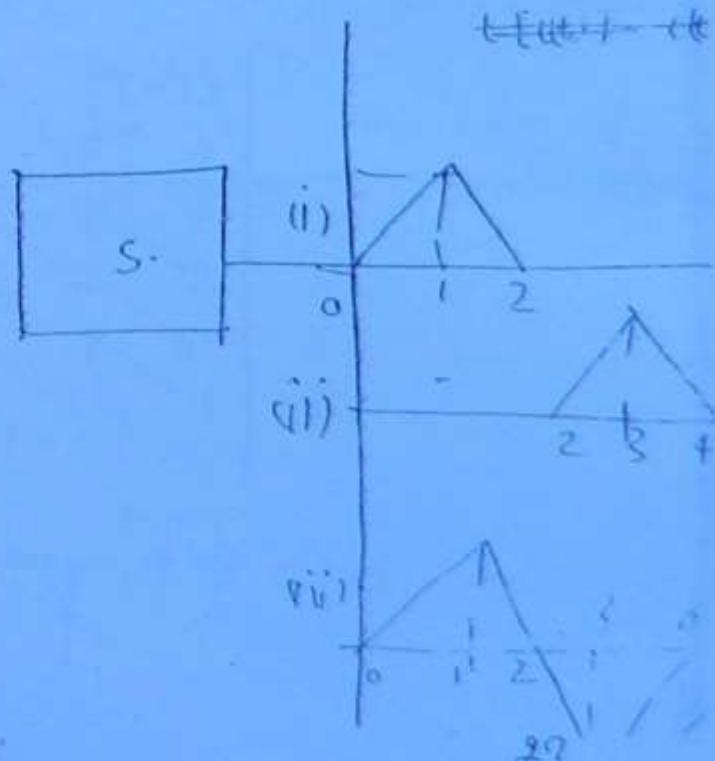
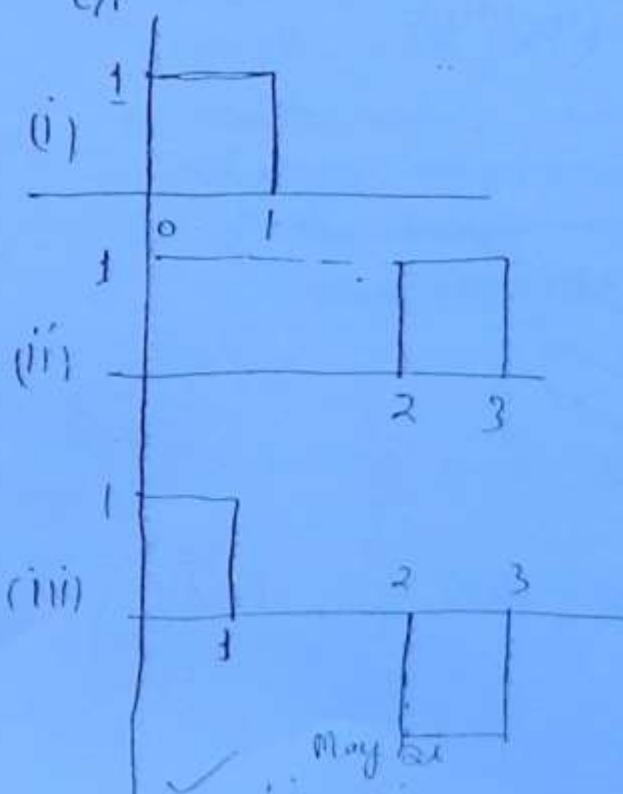
$$y[n] = u[n-1] x[n] + u[n+1] x[n+1]$$

Linear, Not causal,

Stable, time variant

Q. A system  $s$  has the following for the considered

i/p



length of resultant signal  $l$  always =  $L_1 + L_2 - 1$

$$\text{Answer end of resultant signal} = \underline{(n_1 + n_2)} \quad (B8)$$

→ Lower rate at end of  $f[n]$

lower end of h[n]

Then we ,convolve  $f[n]$  &  $h[n]$

$$\text{Summation of all samples of } f[n] = \sum_{k=-\infty}^{\infty} f[k]$$

$$A_2 = \dots \quad \dots \quad \dots \quad \text{def } h[n] = \sum_{k=-\infty}^{\infty} h[k]$$

$$A_1 \cdot A_2 = \sum_{k=-\infty}^{\infty} y[k] \quad y[n] = f[n] * h[n]$$

二

$m \rightarrow$  upper end of  $f[n]$

$$n_2 \rightarrow n \quad n \rightarrow h[n]$$

$$\text{upper end of } y[n] = f[n] \oplus h[n] \\ = (h_2 + h_3)$$

		$h[n]$	4	3	2	1
		1	4	3	2	1
		2	8	6	4	2
		3	12	9	6	3
		4	16	12	8	4

$$f(n) = \left\{ \begin{matrix} n=1 \\ 4, 11, 20, 30, 20, 11, 4 \end{matrix} \right. \left. \begin{matrix} n=5 \end{matrix} \right\}$$

- \* Whenever two discrete time signals are convolved
  - Resultant of convolution will have a length which is equal to sum of individual lengths of the signal being convolved - 1. (minus 1) (39)
  - Resultant of convolution will have extends which is equals to some of individual extends of the signal being convolved.
  - Some of the sample values in resultant of convolution value is same as the product of the some of the sample values of individuals signals being convolved

If  $[h[n] = u[n]]$  not possible to follow this procedure.

- The above method is suitable when the no. of samples in the I/P signal and the impulse response are finite in no.

- Q. Two discrete time signals  $x[n]$  &  $h[n]$  each of lengths 3 & 5 are convolved. The maximum possible sample value of  $x[n]$  is L, max<sup>m</sup> possible value sample value is K for  $h[n]$ . What is maximum value of the sum of all the sample value in resultant convolue signal

~~Ans. (L+K) (40)~~

$$x[n] = \{L, L, L\}$$

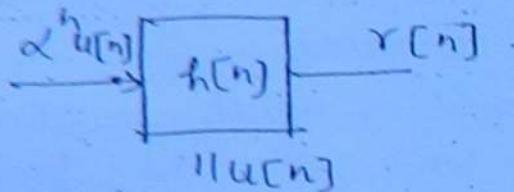
$$h[n] = \{K, K, K, K, K\}$$

$$\sum_{n=0}^{\infty} x[n]h[n] = 3L \times 5K = 15LK$$

$$\sum_{K=-\infty}^{\infty} r[K] = \sum_{K=-\infty}^{\infty} h[K] * \sum_{K=-\infty}^{\infty} x[K]$$

(48)

$$\left[ \sum_{K=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x[m] h[K-m] = \sum_{K=-\infty}^{\infty} h[K] \sum_{K=-\infty}^{\infty} x[K] \right]$$



$$r[n] = \alpha^n [u[n]] \otimes u[n]$$

$$r[n] = \sum_{K=-\infty}^{\infty} \alpha^K u[K] u[n-K]$$

$$= \left[ \sum_{K=0}^{\infty} \alpha^K u[K] u[n-K] \right] \quad (n \geq 0)$$

$$r[n] = \left[ \alpha u[n] + \alpha u[n-1] + \alpha^2 u[n-2] + \dots + \alpha^{n-r} u[n-r] \right]$$

$$h=0 \quad r[n] = \begin{cases} 0 & n < 0 \\ \frac{\alpha^{n+1}-1}{\alpha-1} & n \geq 0 \end{cases}$$

$n=1 (1+\alpha)$

$$h=2 1+\alpha+\alpha^2$$

$$h=r \rightarrow 1+\alpha+\alpha^2+\dots+\alpha^r$$

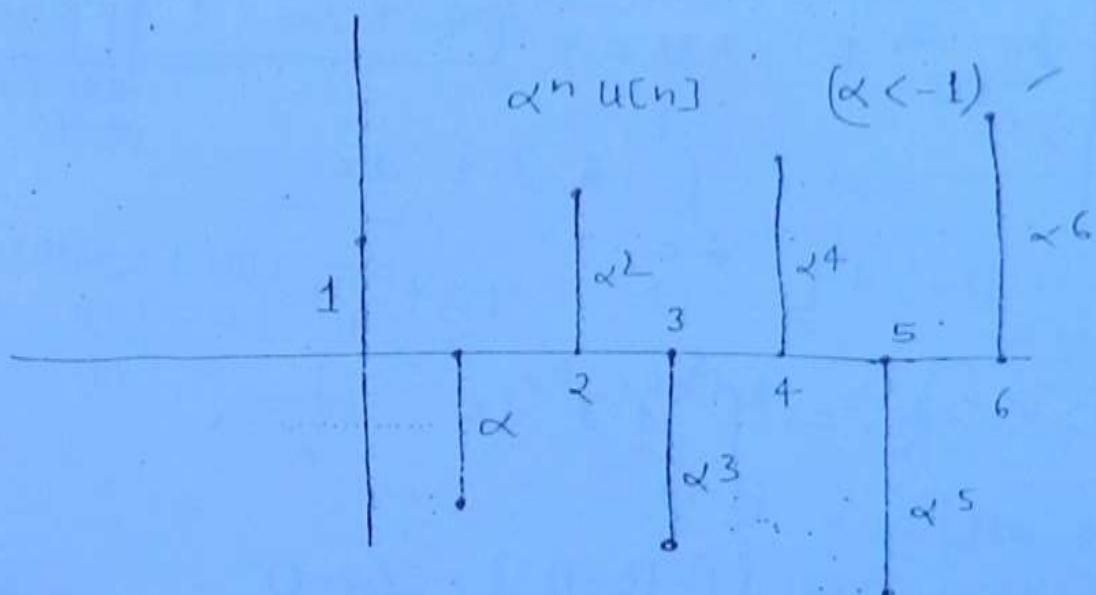
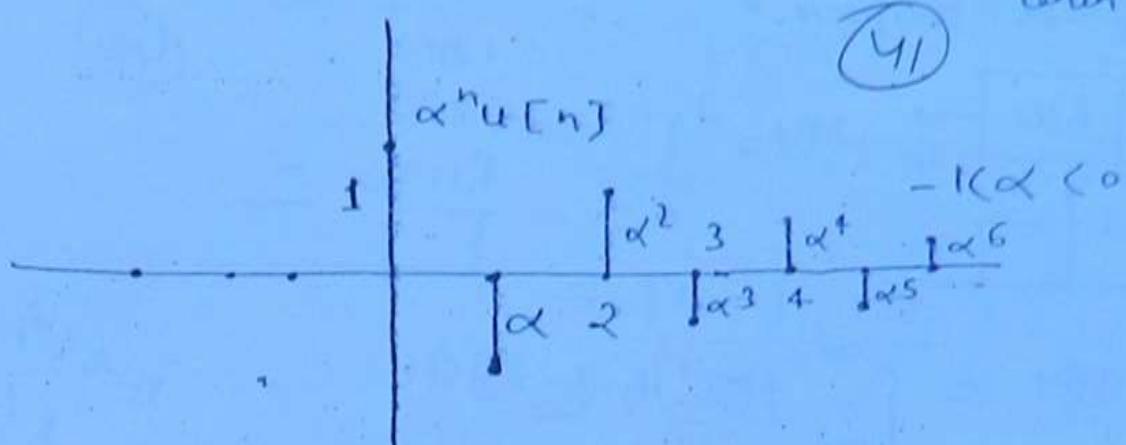
$$= \frac{1[\alpha^{r+1}-1]}{\alpha-1}$$

$$r[n] = \left( \frac{\alpha^{n+1}-1}{\alpha-1} \right) u[n]$$

Sketch the graphs of  $\alpha^n u[n]$  when  $0 < \alpha \leq 1$

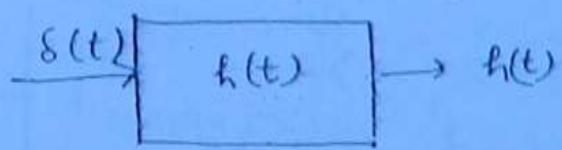
when  $-1 < \alpha < 0$   
 $\alpha \leq -1$

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$$f(t) = \int_{-\infty}^{\infty} f(\tau) \delta(t-\tau) d\tau$$



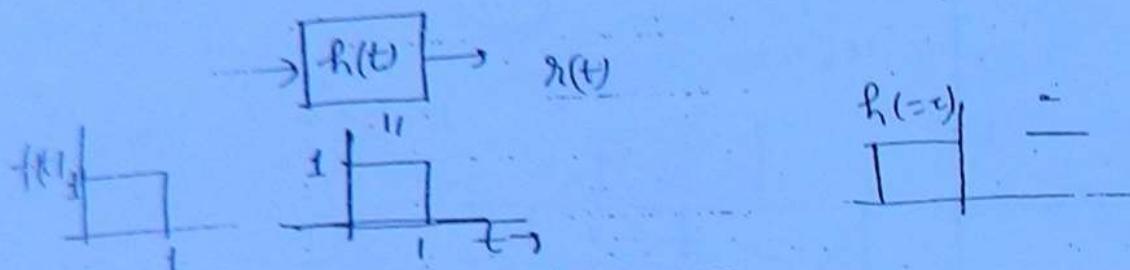
for any i/p  $f(t)$  response  $r(t)$

$$r(t) = \int_{-\infty}^{\infty} f(\tau) h(t-\tau) d\tau \rightarrow \text{convolution, discrete}$$

$$f(t) \otimes h(t)$$

Q. Find the response of a continuous time LTI system with impulse response.

(42)



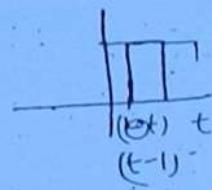
$$r(t) = \int_{-\infty}^{\infty} f(\tau) h(t-\tau) d\tau$$

$$h(-t)^0 \\ h(-t)$$

$$\text{for } t < 0 \Rightarrow r(t) = 0$$

$$\text{for } t \geq 0,$$

$$= \int_{0}^{t} 1 \cdot 1 \cdot 1 d\tau$$



$$= t \quad t \geq 0$$

$$\text{for } 1 \geq t > 0 = \int_{(t-1)}^1 1 \cdot 1 \cdot 1 d\tau$$

$$= [1 - (t-1)] = (2-t)$$

$$\text{for } t \geq 2$$

$$r(t) = 0$$

$$r(t) = \begin{cases} t, & 1 \geq t > 0 \text{ or } t > 2 \\ 0, & t \leq 1 \text{ or } t < 2 \text{ (input)} \\ 0, & t < 0 \end{cases}$$

both correct

Repeat the above problem if the  $f(t) = u(t)$  and the  $h(t) = u(t)$

$$u(t) = 0 \quad t < 0$$

$$u(t) = t \quad t \geq 0 \quad \text{Ans. ✓}$$

Q. Repeat the problem if  $f(t) = u(t) \cdot e^{-t}$ ,  $h(t) = u(t)$

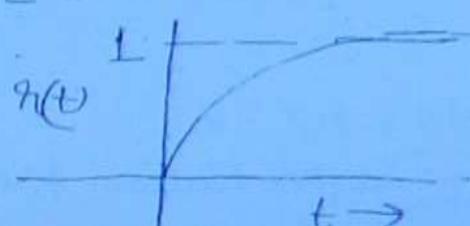
$$r(t) = 0 \quad t < 0$$

$$r(t) = \int_0^t e^{-t} \cdot 1 dt \quad t > 0$$

$$= -[e^{-t}]_0^t$$

$$r(t) = (1 - e^{-t}) \quad t > 0$$

$r(t) \rightarrow 1 \text{ as } t \rightarrow \infty$



Q.  $e^{-t} u(t) \oplus t u(t) \rightarrow$

$t < 0 \quad r(t) = 0$

$$\begin{aligned} t > 0 \quad r(t) &= \int_0^t e^{-\tau} - \cancel{\int_{-\infty}^{\tau} (-\tau+t) d\tau} = -\int_0^t \tau e^{-\tau} d\tau + \int_0^t t e^{-\tau} d\tau \\ &= -[(1-e^{-t}) - t e^{-t}] \end{aligned}$$

$$r(t) = t e^{-t} + (t-1)e^{-t} \quad t > 0$$

Q.  $e^{-t} u(t) \oplus e^{-t} u(t)$

$t < 0 \quad r(t) = 0$

$$\begin{aligned} t > 0 \quad r(t) &= \int_0^t e^{-\tau} e^{\tau-t} d\tau \\ r(t) &= t e^{-t} \quad t > 0 \end{aligned}$$

$$\begin{matrix} e^{\tau} u(\tau) \\ e^{t-\tau} \end{matrix}$$

Q.  $e^{at} u(t) \oplus e^{-bt} u(t) =$

$t < 0 \quad r(t) = 0$

$$\begin{aligned} \text{for } t > 0 \quad r(t) &= \int_0^t e^{a\tau} e^{b(a-b)\tau} d\tau = e^{ab} \int_0^t e^{(a+b)\tau} d\tau \\ -t > 0 \quad r(t) &= e^{at} - e^{-bt} \end{aligned}$$

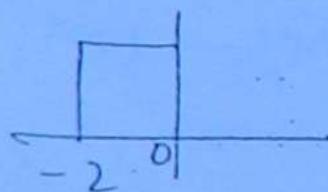
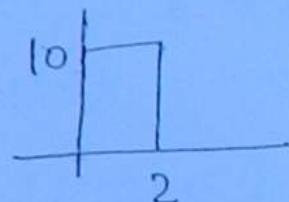
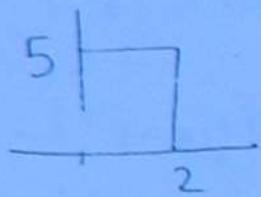
$$r(t) = e^{ab} [e^{(a+b)t} - 1]$$

$$\textcircled{+} \quad r(t) = f(t) \otimes h(t) = \int_{-\infty}^{\infty} f(\tau) h(t-\tau) d\tau \quad (44)$$

$$\begin{aligned} \frac{d}{dt} r &= \frac{d}{dt} [f(t) \otimes h(t)] = \int_{-\infty}^{\infty} f(\tau) \frac{d}{dt} h(t-\tau) d\tau \\ &= \int_{-\infty}^{\infty} f(\tau) h'(t-\tau) d\tau \\ &= f(t) \otimes h'(t) \end{aligned}$$

differentiation property  $\boxed{\frac{dr}{dt} = f(t) \otimes \frac{dh}{dt}}$

⑧ Calculate the convolution of following two pulses



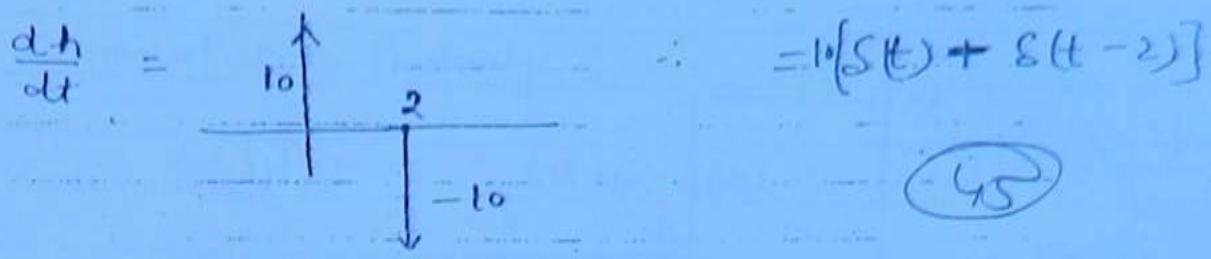
$$r(t) = 0 \quad t < 0$$

$$2 \geq t > 0$$

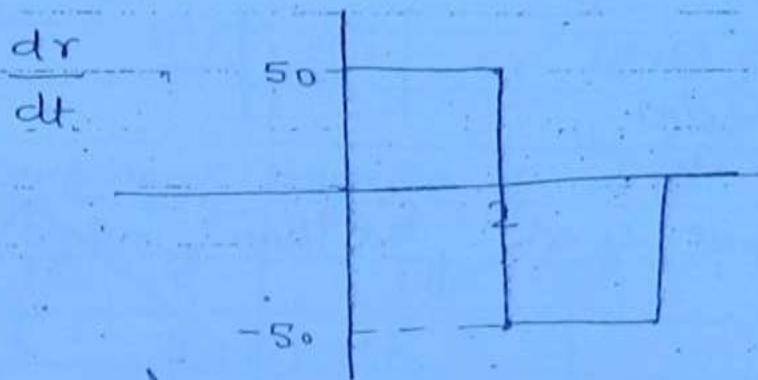
$$r(t) = \int_0^t 5 \times 10 dt = 50t$$

$$t = 2 \quad r(t) = 100$$

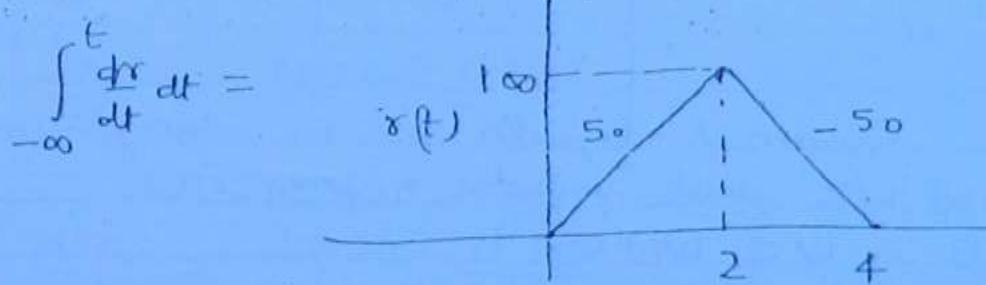
$$\begin{cases} 0 & t > 2 \\ \int_{t-2}^2 5 \times 10 dt & 0 \leq t \leq 2 \\ 50(t-2) & 2 < t < 4 \\ 0 & t \geq 4 \end{cases}$$



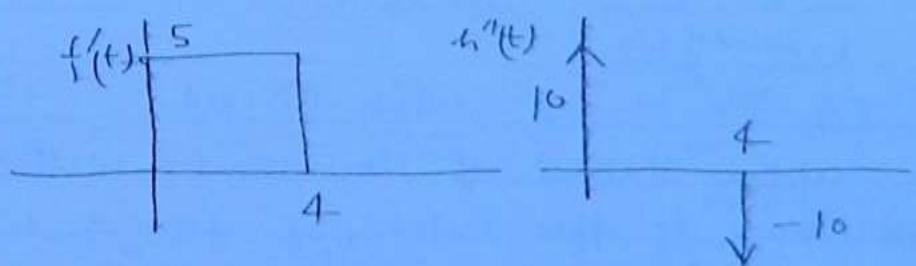
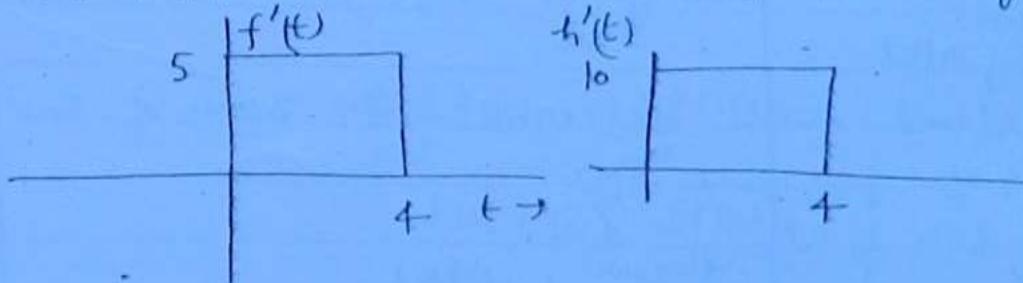
$$f(t) \otimes \frac{dh}{dt} = 10 f(t) - 10 f(t-2)$$



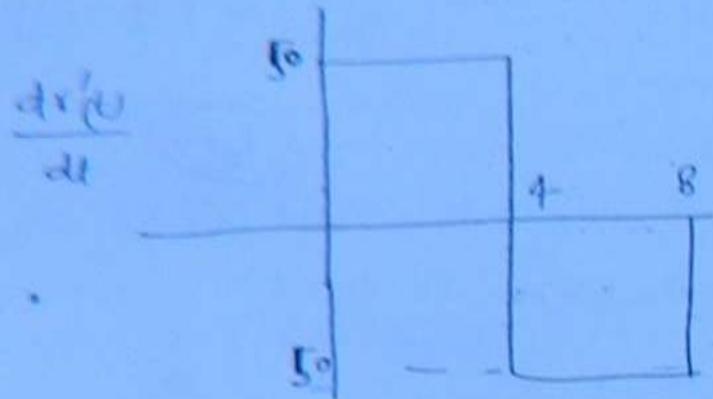
$$r = 50t + (-100) \underline{-50t}$$



Q. Calculate the convolution of following signals

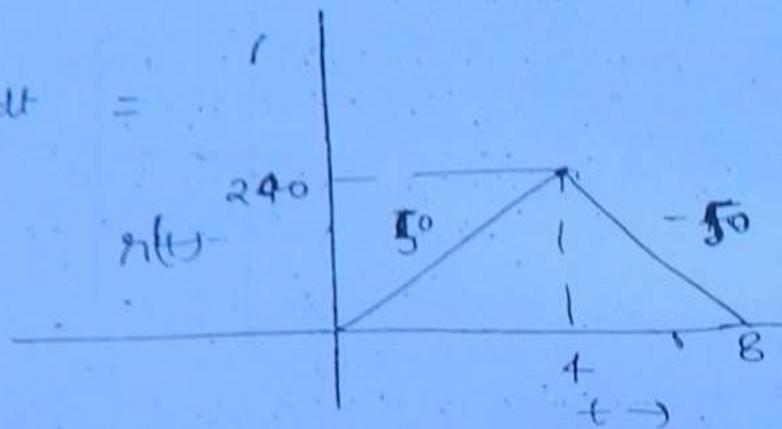


$$f'(t) \otimes h''(t) = 10 f'(t) - 10 f'(t-2)$$



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$$r'(t) = \int_{-\infty}^t \frac{df'(t)}{dt} dt =$$



\* Width of resultant signal will be always sum of widths of I/P and impulse response  $h(t)$

i.e.  $w = w_1 + w_2$

$\downarrow$  width of  $r(t)$        $\downarrow$  width of  $f(t)$        $\rightarrow$  width of  $h(t)$

④ Lower extent will be equal to sum of lower extents of  $f(t)$  and  $h(t)$

$\downarrow$   $l = l_1 + l_2$       lower end of  $h(t)$   
lower end of  $f(t)$       lower end of  $h(t)$

Similarly upper extent of resultant will be equal to sum of upper extents of  $f(t)$  &  $h(t)$

$U = U_1 + U_2$

## Commutative Property :-

$$f(t) \otimes h(t) = h(t) \otimes f(t) \quad (7)$$

$$\left| \begin{array}{c} f(t) \\ h(t) \end{array} \right| \rightarrow f(t) \otimes h(t) = h(t)$$

" Same "

$$\left| \begin{array}{c} h(t) \\ f(t) \end{array} \right| \rightarrow h(t) \otimes f(t) = f(t)$$

⊗ Interchanging positions of input and impulse response for an LTI system does not change response of the system

$$\left| \begin{array}{l} \frac{d\gamma}{dt} = f(t) \otimes \frac{dh}{dt} \\ \frac{d\gamma}{dt} = h(t) \otimes \frac{df}{dt} \end{array} \right|$$

$$\left| \begin{array}{l} \frac{d^2\gamma}{dt^2} = h(t) \otimes \frac{d^2f}{dt^2} \\ = f(t) \otimes \frac{d^2h}{dt^2} \\ = \frac{df}{dt} \otimes \frac{dh}{dt} \\ = \frac{dh}{dt} \otimes \frac{df}{dt} \end{array} \right|$$

$$\frac{d^m \gamma}{dt^m} = h(t) \otimes \frac{d^m f}{dt^m} = \frac{d^m h(t)}{dt^m} \otimes f(t)$$

$$= \frac{d^{m-1} h(t)}{dt^{m-1}} \otimes \frac{df}{dt}$$

$$= \frac{d^n f}{dt^n} \otimes \frac{d^{k-m} h}{dt^k}$$

$$= \frac{d^{m-2} h(t)}{dt^{m-2}} \otimes \frac{d^2 f}{dt^2}$$

where  $(n+k) = m$

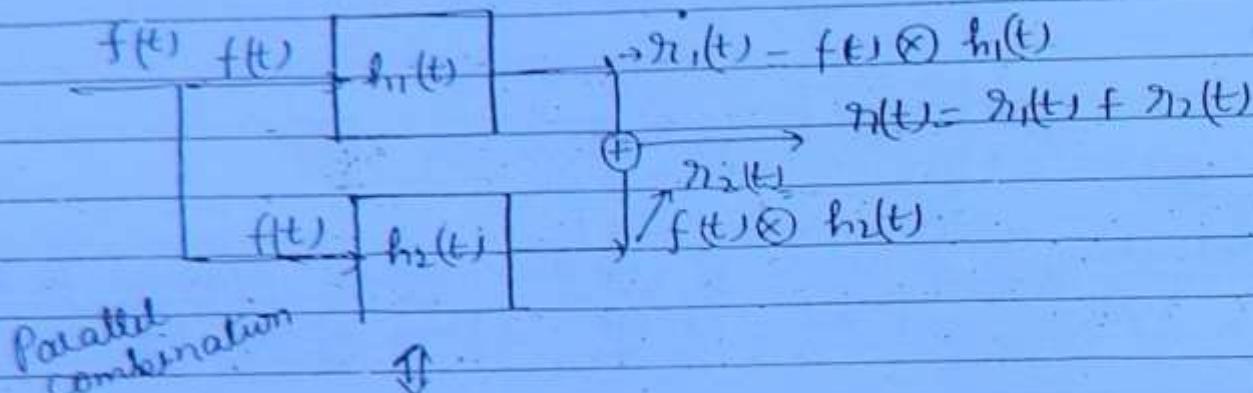
$$= d^{m-k} h(t) \otimes \frac{d^k f}{dt^k}$$

$$ax[b+c] = axb + axc$$

distributive property  $\rightarrow$

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$$h \otimes [f_1(t) + f_2(t)] = h(t) \otimes f_1(t) + h(t) \otimes f_2(t)$$



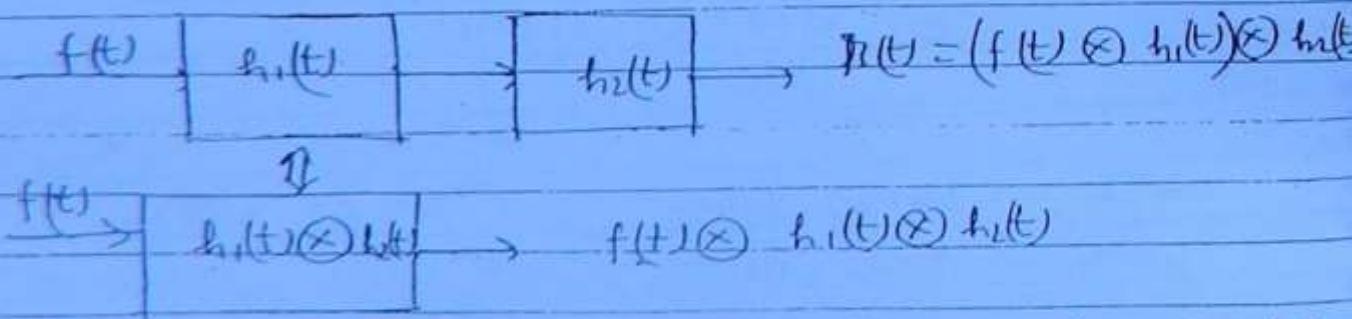
$$f(t) \rightarrow \boxed{h_1(t) + h_2(t)} \rightarrow f(t) \otimes h_1(t) + f(t) \otimes h_2(t)$$

- ⊗ Any no. of systems connected in parallel can be combined into a single system whose impulse response is sum of all the individual <sup>impulse</sup> responses.

Associative Property  $\rightarrow$

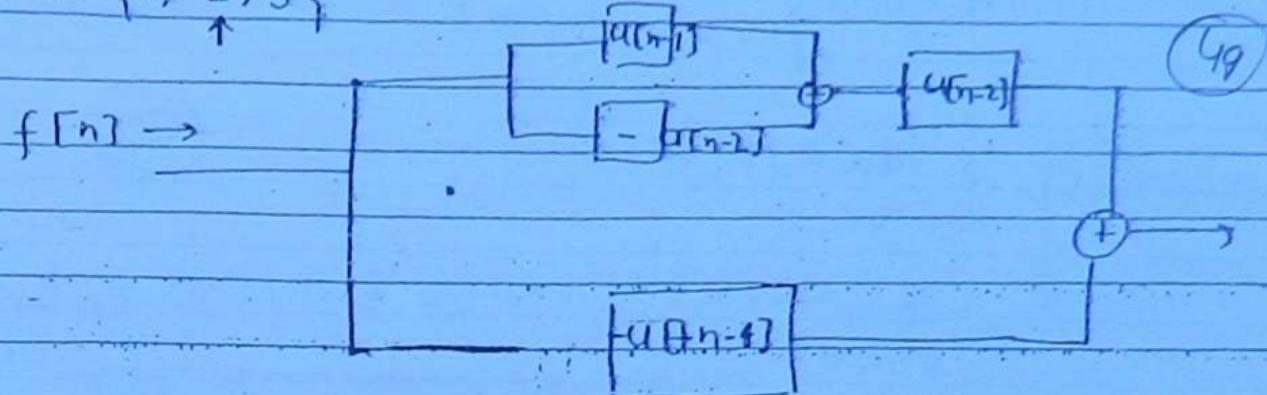
$$f(t), h_1(t), h_2(t)$$

$$f(t) \otimes h_1(t) \otimes h_2(t) = [f(t) \otimes h_1(t)] \otimes h_2(t)$$



- ⊗ Any no. of systems connected in cascade can be combined into a single system whose impulse response is convolution of all the individual impulse responses.

\* Q. what is the response of following system to an i/p  
 $f[n] = \{1, 2, 3\}$



$$r[n] = f[n] \otimes \left[ -u[n-4] + \{ u[n-1] + u[n-2] \} \otimes u[n-2] \right]$$

$$= f[n] \otimes \left[ -u[n-4] + \delta[n-1] \otimes u[n-2] \right]$$

$$= f[n] \otimes \left[ \cancel{-u[n-4]} + \cancel{\delta[n-1]} u[n-2] \right]$$

$$= f[n] \otimes [s[n-2]]$$

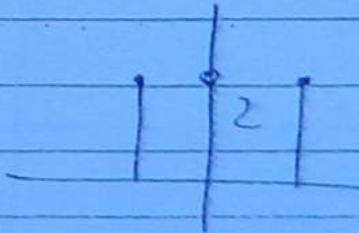
$$= f[n-2]$$

-4

2

$$\left\{ r[n], f[n] \otimes u[n-2] \right\}$$

~~49~~



$$f[n] \xrightarrow{AEM} \sum_{k=-\infty}^{\infty} f[k] h[n-k] = \sum_{k=-\infty}^{\infty} h[k] f[n-k]$$

(50)

$$g[n] = \sum_{k=-\infty}^{\infty} h[k] f[n-k]$$

$$= \sum_{k=-\infty}^{-1} h[k] f[n-k] + h[0] f[n] + \sum_{k=1}^{\infty} h[k] f[n-k]$$

for system to be causal  $[h[n] = 0 \quad n < 0]$

$$h(t) = 0 \quad t < 0$$

$$f(t) \xrightarrow{h(t)} \int_{-\infty}^{\infty} h(\tau) f(t-\tau) d\tau$$

$t - \tau = m$   
 $\tau = (t-m)$

↓  
causal system

$$\int_0^{\infty} h(\tau) f(t-\tau) d\tau = \int_{-\infty}^{\infty} h(t-m) f(m) (-dm)$$

$$r(t) = \int_{-\infty}^t h(t-\tau) f(\tau) d\tau$$

both are causal  $f(t)$  & system is causal

↓  
causal signal

$$n(t) = \int_0^t f(\tau) h(t-\tau) d\tau$$

$$g(t) = \int_0^t f(\tau) h(t-\tau) d\tau$$

$$f(t) \rightarrow 0$$

+ 1. 2

$$n(t) = \int_{-\infty}^{(t+2)} f(\tau) h(t-\tau) d\tau$$

$$= (t+2) \int_{-\infty}^{t+2} f(t+\tau) h(t-\tau) d\tau$$

$$\left. \begin{array}{l} (t+\tau - \tau) = t \\ \tau - \tau = 0 \\ f(\tau) \end{array} \right\} \quad \left. \begin{array}{l} (t+\tau - M) = t - M \\ t - \tau = -M \\ f(t-M) \end{array} \right\}$$

✳ h(t) is not equal to a impulse function if it is representing a dynamic system.

$$f(t) \rightarrow \boxed{\quad} \quad \Rightarrow n(t) = \int_{-\infty}^{\infty} f(\tau) h(t-\tau) d\tau$$

$$|f(t)| \leq m$$

$$= \int_{-\infty}^{\infty} f(t-\tau) h(\tau) d\tau$$

$$= \left| \int_{-\infty}^{\infty} f(t-\tau) h(\tau) d\tau \right| < \infty$$

$$\left| \int_{-\infty}^{\infty} f(\tau) h(t-\tau) d\tau \right| < \infty$$

$$\int_{-\infty}^{\infty} |h(\tau)| |f(t-\tau)| d\tau < \infty$$

$$\int_{-\infty}^{\infty} |h(\tau)| \cdot m d\tau < \infty$$

$$\text{① } \left| \int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty \right| \rightarrow \text{system to be stable}$$

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

$$\boxed{\sum_{n=-\infty}^{\infty} |h(n)| < \infty}$$

$\{f(t) = \sin t \rightarrow \text{unstable system oscillatory}$

(52)

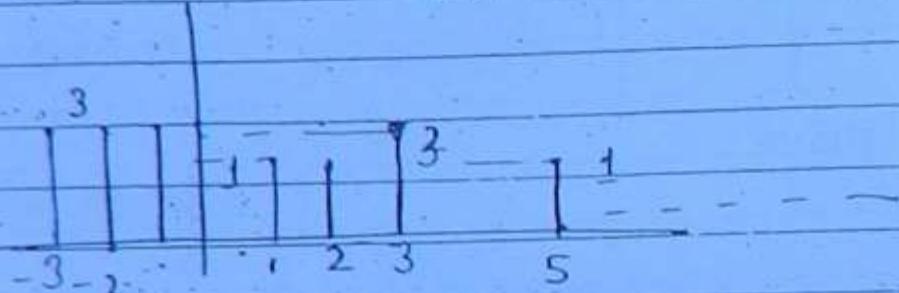
$h(t) = e^{-at} u(t) \rightarrow 1/a \text{ area stable system}$

$h[n] = u[n] \rightarrow \text{unstable}$

$\{h[n] = (-1)^n u[n] \rightarrow \text{unstable oscillatory}$

Q. A discrete time system is defined by the impulse response  $h[n] = 3u[n+3] - 2u[n-1] + 2u[n-3] - 2u[n-5]$

$h[1] = 3 \rightarrow \text{not causal}$



unstable, not causal

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Invertible or non invertible  $\rightarrow$

$$\begin{array}{c|c|c|c|c} f(t) & \xrightarrow{S} & r(t) & \xrightarrow{S'} & [f(t) \otimes h(t)] \otimes h'(t) \\ & \downarrow & & \downarrow & -f(t) \end{array}$$

$$\begin{aligned} f(t) \otimes h(t) &= f(t) \otimes [h(t) \otimes h'(t)] \\ &- f(t) \otimes S(t) \end{aligned}$$

for condition of invertibility  $= f(t)$

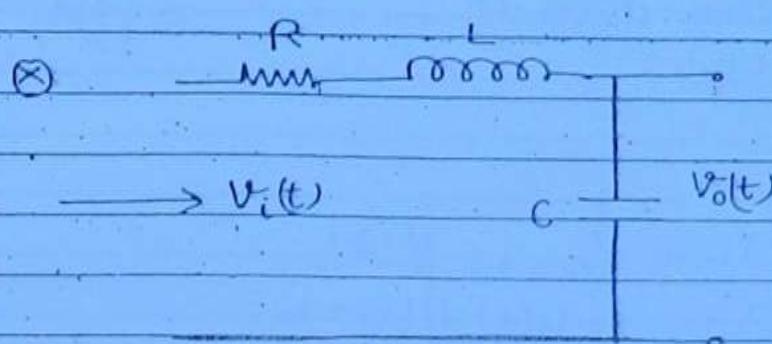
$$f(t) \otimes h(t) \otimes h'(t) = \delta(t)$$

Q. A discrete time system's impulse response is defined as  $h[n] = \alpha^n u[n] + \beta^n u[-n-1]$  the system is stable only if

$$0 < |\alpha| < 1$$

$$|\beta| > 1$$

(S3)



$$LC \frac{d^2 V_o}{dt^2} + RC \frac{dV_o}{dt} + V_o(t) = V_i(t)$$

④ A system is general can be represented by a differential eq<sup>n</sup> given below

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b_1 \frac{dx}{dt} + b_0 x(t)$$

In above differential eq<sup>n</sup> there are no product terms of the i/p  $x(t)$  or o/p  $y(t)$  or their derivatives (i.e. there are no terms like  $x^2(t)$ ,  $x^3(t)$ ,  $(\frac{dy}{dt})^2$ ,  $(\frac{dx}{dt})^2$ ,  $(\frac{d^2y}{dt^2})^2$ ,  $(\frac{d^2x}{dt^2})^2$ ,  $(y(t))^2$ ,  $(x(t))^2$ ,  $(\frac{dx}{dt} \frac{d^2y}{dt^2})$ )

such a differential equation is called a linear differential and it directly corresponds to a linear system. In the (4) In the above differential eq<sup>n</sup> if all the coefficients  $a_2, a_1, a_0, b_1, b_0$  are ~~const~~ independent of time (i.e. constants) then the above differential eq<sup>n</sup> can be written as a constant coefficient differential eq<sup>n</sup> and it directly corresponds to time invariant system

Q. A system represented by following differential eq<sup>n</sup>

$$2 \frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 4 + y(t) = 2x(t)$$

(54)

linear but time variant

$$2 \frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 2y^2(t) = 2x(t)$$

Non linear, but time invariant

$$2 \frac{d^2y}{dt^2} + \sin t \cdot \frac{dy}{dt} + y(t) = 2x(t)$$

Linear, time variant

$$\begin{array}{c|c|c} u(t) & h(t) & \\ \hline & & \end{array} \rightarrow y(t) = \int_{-\infty}^{\infty} u(\tau) h(t-\tau) d\tau$$
$$= \int_{-\infty}^{\infty} h(\tau) u(t-\tau) d\tau$$
$$y(t) = \int_{-\infty}^t h(\tau) d\tau$$

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

∴ system is linear

$$\therefore \text{response to } u(t) \text{ is } \int_{-\infty}^t h(\tau) d\tau$$

$$h(t) = \frac{d y(t)}{dt} = \frac{d S(t)}{dt}$$

$S(t)$  is step response

Q. The following four signal define step responses & impulse responses of continuous time LTI systems

(i)  $(5 - 4e^{-2t}) u(t)$

of this which two signals corresponds to step response & impulse

(ii)  $(4 - 3e^{-4t}) u(t)$

of the same LTI system

iii)  $5\delta(t) + 8e^{2t} u(t)$

(iv)  $2\delta(t) + 6e^{-2t} u(t)$

Ans

(i) & (iii)

(55)

(i)

(i) step response

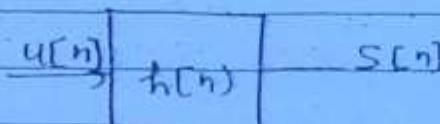
(iii) is impulse response:  $h(t) = 5\delta(t) + 8e^{-2t}u(t)$

$$s(t) = \int h(\tau) d\tau = S(t) + B \int_0^t 8e^{-2\tau} u(\tau) d\tau$$

$$S(t) = (5 - 4e^{-2t}) u(t)$$

$$= S(t) + B \int_0^t e^{-2\tau} d\tau \\ = S(t) + B(t - e^{-2t} - 1)$$

$$h(t) = \frac{ds(t)}{dt} = 5s(t) - 4s(t) + 8e^{-2t}u(t) \\ = \delta(t) + 8e^{-2t}u(t)$$



$$h[n] = s[n] - s[n-1]$$

↑  
Impulse response

s[n] Step response

$$s(n) = \sum_{k=-\infty}^n b_k h[n]$$

Q. For a discrete time system, the response to

a step i/p is known to be  $\{1, 2, 1\}$

find h[n] response of the system for f[n]

$$s[n-1] = \left\{ \begin{array}{l} 1, 2, 1 \\ \uparrow \end{array} \right\}$$

$$= \left\{ \begin{array}{l} 1, 2 \\ \uparrow \end{array} \right\}$$

$$h[n] = s[n] - s[n-1] = \left\{ \begin{array}{l} 1, 1, -1, -1 \\ \uparrow \end{array} \right\}$$

↑ impulse response

$$f[n] = \left\{ \begin{array}{l} 1, 2, 3 \\ \uparrow \end{array} \right\}$$

$$r[n] =$$

	1	1	-1	-1	=	$\left\{ \begin{array}{l} 1, 3, 0, -5 \\ \uparrow \end{array} \right\}$
1	1	1	-1	-1		
2	2	2	-2	-2		
3	3	3	-3	-3		

## Eigen Function :

5b

$$f(t) \xrightarrow{A\delta(t)} A f(t)$$

\* If the response of a given LTI system is same as the i/p function  $f(t)$  except for a scalar multiple, the function is defined as eigen function of the system. So, for a system with unit impulse response  $A\delta(t)$ , any general signal  $f(t)$  forms an eigen function.

$$\frac{f(t)}{f(t-t_0)} \xrightarrow{A\delta(t-t_0)} \frac{A f(t-t_0)}{A f(t)}$$

$$f(t) \otimes A\delta(t-t_0)$$

$$A f(t-t_0)$$

$$f(t-t_0) = f(t)$$

↙ ✓  $f(t)$  should be periodic with period  $t_0$ .

⊗ For a system with impulse response  $A\delta(t-t_0)$ , any signal which is periodic with a period  $t_0$  forms an eigen function.

$$\cos(\omega_0 t) \xrightarrow{h(t)} h(t) \otimes \cos\omega_0 t$$

$$= \int_{-\infty}^{\infty} h(\tau) \cos[\omega_0(t-\tau)] d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) [(\cos\omega_0 t \cos\omega_0 \tau) +$$

$$\int_{-\infty}^{\infty} h(\tau) \sin\omega_0 t \sin\omega_0 \tau d\tau]$$

$$H(t) = \cos\omega_0 t \int_{-\infty}^t h(\tau) \cos\omega_0 \tau d\tau - \sin\omega_0 t \int_{-\infty}^t h(\tau) \sin\omega_0 \tau d\tau$$

even signal

$$\left. \begin{aligned} \int_{-\infty}^{\infty} h(t) \cos \omega_0 t dt &= A \\ \int_{-\infty}^{\infty} h(t) \sin \omega_0 t dt &= 0 \end{aligned} \right\} \begin{array}{l} \text{scalar} \\ \text{(constant)} \end{array}$$

(S7)

condition for  $\cos \omega_0 t$  to become eigenfunction of system.

↓  
should be even signal

- (\*) For a system with general impulse response  $h(t)$ , if  $\cos \omega_0 t$  is to form eigen function then impulse response  $h(t)$  must be an even function of time.

LTI

$$e^{j\omega_0 t} \xrightarrow{h(t)} e^{j\omega_0 t} \otimes h(t)$$

$= \int_{-\infty}^{\infty} h(\tau) e^{j\omega_0(t-\tau)} d\tau$

*always form an eigen function for system having impulse response  $h(t)$ .*

$$= e^{j\omega_0 t} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega_0 \tau} d\tau$$

~~$\int_{-\infty}^{\infty} h(\tau) e^{j\omega_0 \tau} d\tau$~~

$$= e^{j\omega_0 t} \left[ \int_{-\infty}^{\infty} h(\tau) \cos \omega_0 \tau d\tau - j \int_{-\infty}^{\infty} h(\tau) \sin \omega_0 \tau d\tau \right]$$

$h(\tau) \rightarrow$  should be even

$h(t) \rightarrow$  should be an even function of time to then  $e^{j\omega_0 t}$  can form eigen function for system LTI having impulse-response  $h(t)$  if  $h(t)$  is an even function of time or  $h(t)$  should be even conjugate function

$$e^{j\omega_0 t} \rightarrow H(j\omega_0) \cdot e^{j\omega_0 t}$$

$$c_1 e^{j\omega_0 t} \rightarrow c_1 \cdot H(j\omega_0) e^{j\omega_0 t}$$

$$c_2 e^{j\omega_1 t} \rightarrow c_2 \cdot H(j\omega_1) e^{j\omega_1 t}$$

$$f(t) = \sum C_n e^{jn\omega_0 t} \Rightarrow r(t) \rightarrow \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} H(jn\omega_0) \quad (58)$$

Signal  $e^{jn\omega_0 t}$  forms an eigen function for any general LTI system with impulse response, response of any LTI system when i/p is  $e^{jn\omega_0 t}$  can be written as  $e^{jn\omega_0 t} \cdot H(jn\omega_0)$  where  $H(jn\omega_0)$  is defined as

$$H(jn\omega_0) = \int_{-\infty}^{\infty} h(\tau) e^{-jn\omega_0 \tau} d\tau$$

If we can now express any signal  $f(t)$  as sum of complex exponentials, response also will be sum of no. of complex exponential signal multiplied by a suitable value

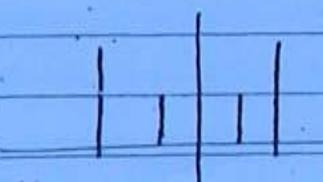
i.e.  $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \Rightarrow$  will have a response

$$r(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \cdot H(jn\omega_0)$$

$$\text{where } H(jn\omega_0) = \int_{-\infty}^{\infty} e^{jn\omega_0 \tau} h(\tau) e^{-jn\omega_0 \tau} d\tau$$

Q. A discrete time system with an impulse response

$$h[n] = \begin{cases} 2 & n=2, -2 \\ 1 & n=1, -1 \\ 0 & \text{otherwise} \end{cases}$$



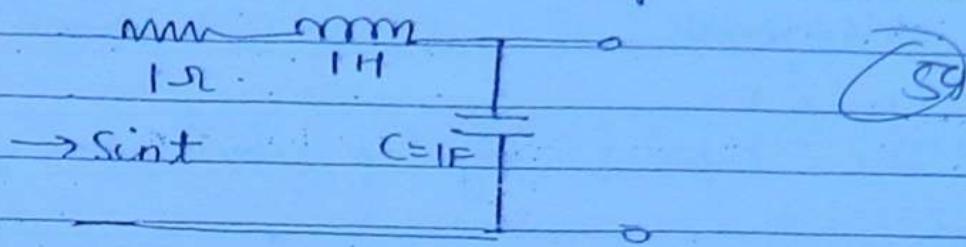
Given an i/p  $f[n] = e^{jn\pi/2}$  find the response of the system.

$$\begin{aligned} r[n] &= \sum_{k=-\infty}^{\infty} h[k] f[n-k] = \sum_{k=-\infty}^{\infty} h[k] e^{j(n-k)\pi/2} \\ &= e^{jn\pi/2} \sum_{k=-\infty}^{\infty} h[k] e^{-jk\pi/2} \\ &= e^{jn\pi/2} [ -2 \cdot e^{j2\lambda} + 1 e^{j\lambda} \\ &\quad + 1 e^{-j\lambda} + 2 e^{-j2\lambda} ] \\ &= e^{jn\pi/2} [ -4 e^{j\lambda} ] \end{aligned}$$

(a)  $\leftarrow$   
For i/p value  $e^{jn\pi/2}$  and  $e^{-jn\pi/2}$  in denominator

$$r[n] = -4 e^{jn\pi/2} \quad \text{for } n = \dots, -1, 0, 1, 2, \dots$$

Q. Find the response of following ckt to an i/p Sint



$$\boxed{H(j) = \frac{\text{Sint}}{1+j + \frac{1}{j \times 1}}} \quad \text{Sint} = e^{jt} - e^{-jt}$$

- Sint

$$1+j + \frac{1}{j \times 1}$$

$$i(t) = \text{Sint}$$

$$\therefore V(t)_1 = \frac{1}{j} \text{Sint} = -\sin(-t - \pi/2) = -\cos t$$

$$= -j \text{Sint}$$

$$V(t) = \text{Sint} \quad \boxed{3\pi/2}$$

$$-\sin(t + 3\pi/2)$$

c/p

$$\left\{ \begin{array}{l} \text{l.c. } \frac{d^2 v_i(t)}{dt^2} + RC \frac{dv_i(t)}{dt} + v_i(t) = v_o(t) \\ \text{o/p} \end{array} \right.$$

$$1 \cdot \frac{d^2 \text{Sint}}{dt^2} + 1 \cdot \frac{d \text{Sint}}{dt} + \text{Sint} = v_o(t)$$

$$\text{LTI system} \rightarrow -\cos t + \cos t + \text{Sint} = v_o(t)$$

$$v_o(t) = \cos t$$

$$v_o(t) = \cos(-\sin(\pi/2 + t)) \\ = \sin(t - \pi/2)$$

$$H(j\omega_0) = -j$$

$$H(j\beta) = -j$$

$$H(j\omega_0) = \frac{1}{(j\omega_0)^2 + j\omega_0 + 1}$$

inner product

$$\bar{x} \cdot \bar{y} = 0$$

$\{\bar{x}, \bar{y}\}$  → orthogonal

then we can express any vector in form of  $\bar{x} + \bar{y}$   
→ independent  
Vectors

Similarly  $f(t), g(t)$

$$\int_0^T f(t) g(t) dt = 0$$

in interval  $(0, T)$ ,  $f(t), g(t)$  are orthogonal

$$2\pi/\omega_0$$

$$\int_0^{2\pi/\omega_0} \sin m\omega_0 t \cdot \sin n\omega_0 t dt$$

$m, n$  integer (+ve)  $\geq 0$  or -ve integer  $\leq 0$   
and  $m \neq n$

then above integral will be zero always.

then  $\sin m\omega_0 t$  is orthogonal  $\sin n\omega_0 t$  is ~~orthogonal~~ in  
interval  $(0, 2\pi/\omega_0)$

$$\int_0^{2\pi/\omega_0} \cos m\omega_0 t \cos n\omega_0 t dt$$

if  $m \neq n$  and  $m > 0$  or  $m < 0$   
conditions are

and  $n > 0$  or  $n < 0$

$m$  and  $n$  are integers

then  $\cos m\omega_0 t$  &  $\cos n\omega_0 t$  both  
will be orthogonal in interval  $(0, 2\pi/\omega_0)$

Some will be true for

$$2\pi/\omega_0$$

$$\int_0^{2\pi/\omega_0} \sin K\omega_0 t \cos K\omega_0 t dt = 0$$

\* if  $K$  is any integer other than 0

so we can define any  $f(t)$  in terms of independent  
 fundamental oscillations & sinusoids

$$f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \dots + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \dots$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad (6)$$

$f(t)$  is periodic  
 with period  
 $= \frac{2\pi}{\omega_0}$

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$$f(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2} \cos(n\omega_0 t - \phi_n)$$

$$f(t) = \sum_{n=0}^{\infty} Y_n \cos(n\omega_0 t - \phi_n)$$

⊕ Fourier series represents the information of the signal  $f(t)$  as amplitude and angles at different angular frequencies which are integer multiples of a fundamental angular frequency  $\omega_0$  i.e.  $n\omega_0$ .

where:

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos n\omega_0 t dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin n\omega_0 t dt$$

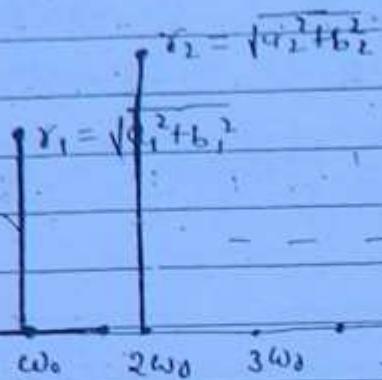
$$a[n\omega_0] = a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos n\omega_0 t dt$$

$$b[n\omega_0] = b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin n\omega_0 t dt$$

### Magnitude spectrum

(62)

$\gamma_n$

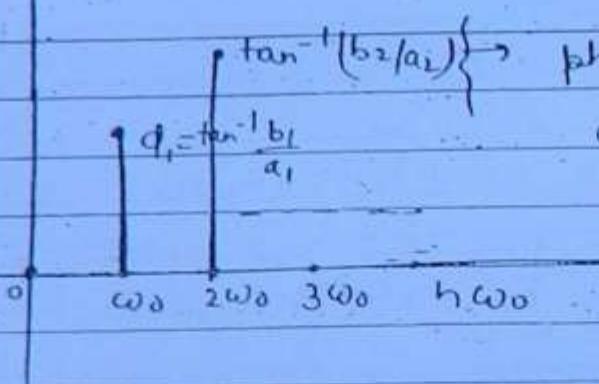


or  
amplitude  
spectrum

Any graph  
w.r.t.  $\omega$

frequency  
is called  
spectrum.

$\phi_n$



$\tan^{-1} \left( \frac{b_2}{a_2} \right) \rightarrow$  phase spectrum

or angle spectrum

Gen Cos wot

Sin wot

$\{ e^{jn\omega_0 t} \} \rightarrow n \rightarrow -\infty \rightarrow \infty$  → all have time period  
integers  $\left( \frac{2\pi}{\omega_0} \right)$

$\int_0^T f(t) g^*(t) dt = 0$  → for  $f(t)$  &  $g(t)$  → two complex  
valued signal to be orthogonal  
in interval  $(0, T)$

$2\pi/\omega_0$

$$\int_0^{2\pi/\omega_0} e^{jm\omega_0 t} (e^{jk\omega_0 t})^* dt = 0$$

$$f(t) = c_0 + c_1 e^{j\omega_0 t} + c_2 e^{j2\omega_0 t} + c_3 e^{j3\omega_0 t} + \dots + c_{-1} e^{-j\omega_0 t} + c_{-2} e^{-j2\omega_0 t} + \dots$$

$$f(t) = c_0 + \sum_{n=1}^{\infty} (c_n e^{jn\omega_0 t} + c_{-n} e^{-jn\omega_0 t})$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \rightarrow \text{Exponential Fourier Series}$$

(63)

Periodic to with  $T = 2\pi/\omega_0 \rightarrow$  fundamental period of  $f(t)$ .

$$\begin{aligned} c_n &= \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt \\ &= \frac{1}{T} \int_{-T/2}^{T/2} f(t) \cos n\omega_0 t dt - \frac{1}{T} \int_{-T/2}^{T/2} j f(t) \sin n\omega_0 t dt \\ &= \frac{1}{2} a_n - \frac{1}{2} j b_n \end{aligned}$$

$$c_n = \frac{1}{2} [a_n - j b_n] \rightarrow \text{complex in nature}$$

$\rightarrow$  valid for only  $n \neq 0$

$$c_{-n} = \frac{1}{2} [a_n + j b_n]$$

$$c_n^* = \frac{1}{2} [a_n + j b_n] = c_{-n}$$

$$[c_n^* = c_{-n}] \quad \text{providing } f(t) \text{ to be real}$$

$$\begin{aligned} c_n^* &= \frac{1}{T} \int_{-T/2}^{T/2} f^*(t) (e^{-jn\omega_0 t})^* dt \\ &= \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{+jn\omega_0 t} dt \end{aligned}$$

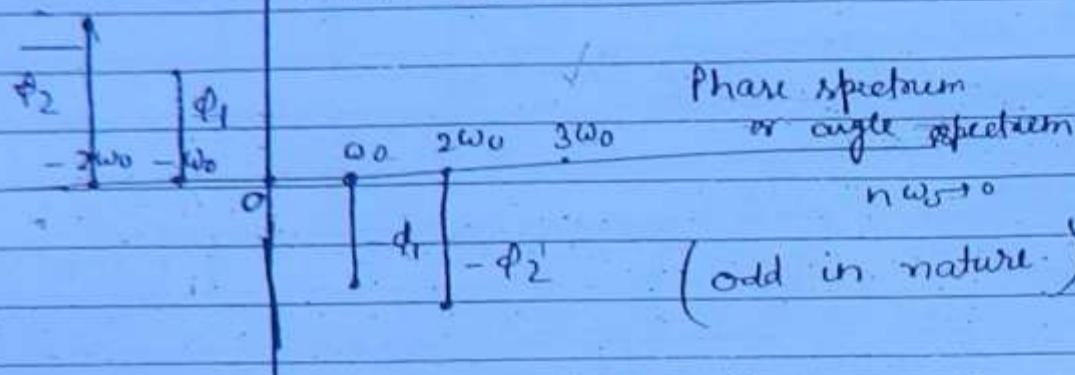
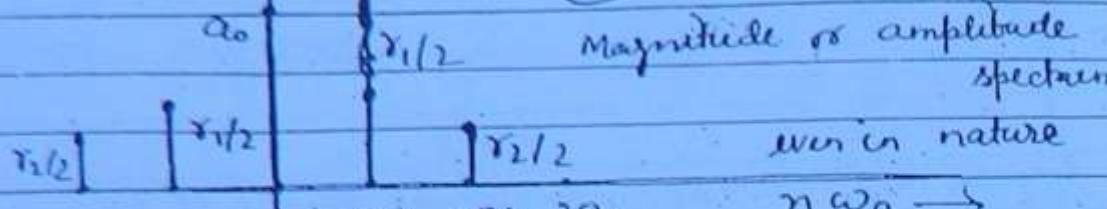
$$[c_n^* = c_{-n}]$$

$$|c_n| = \frac{1}{2} \sqrt{a_n^2 + b_n^2} \rightarrow |c_n| = -\tan^{-1}(b_n/a_n) = \theta \rightarrow \phi_n$$

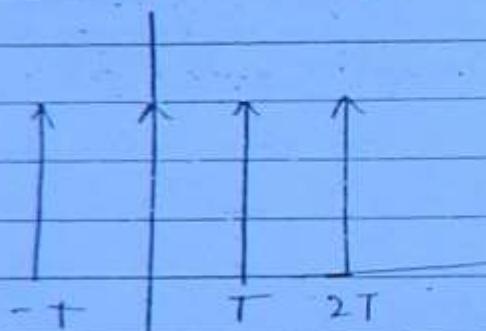
$$K_n = \frac{1}{2} r_n = |c_n^*| = |c_{-n}| = -|c_{-n}|$$

$|C_n|$

(84)



Q. Find the Fourier series of signal  $f(t)$  shown below



$$\delta \cdot f(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

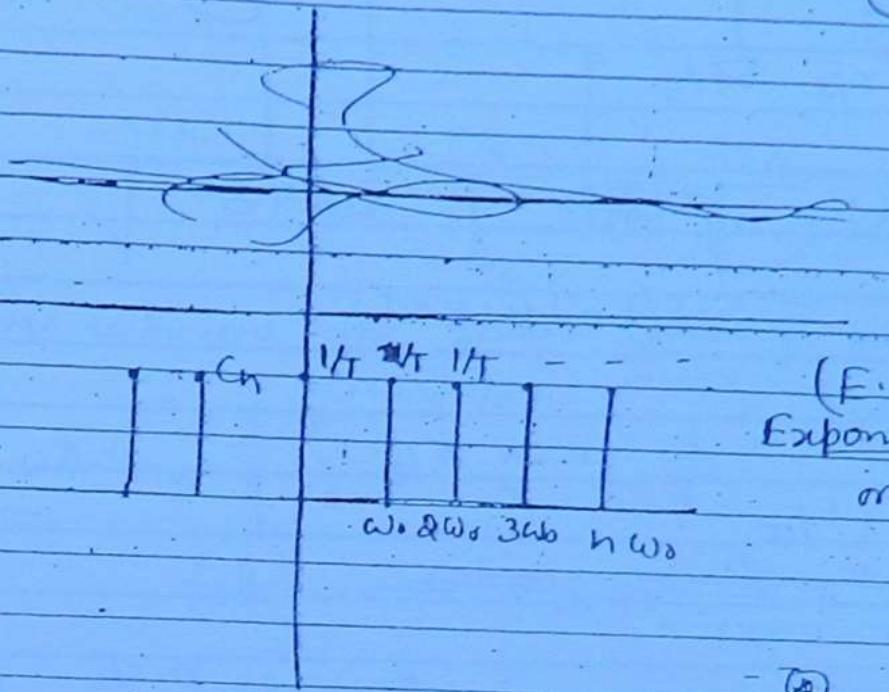
$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt = \frac{1}{T}$$

$$\begin{aligned} c_n &= \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt \\ &= \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) e^{-jn\frac{2\pi}{T} t} dt \end{aligned}$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) dt = 1/T$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_0 t}$$

(65)



(E.F.S.) or C.F.S.  
Exponential fourier spectra  
or complex fourier spectra

$$C_n = \frac{a_n}{2} - j \frac{b_n}{2}$$

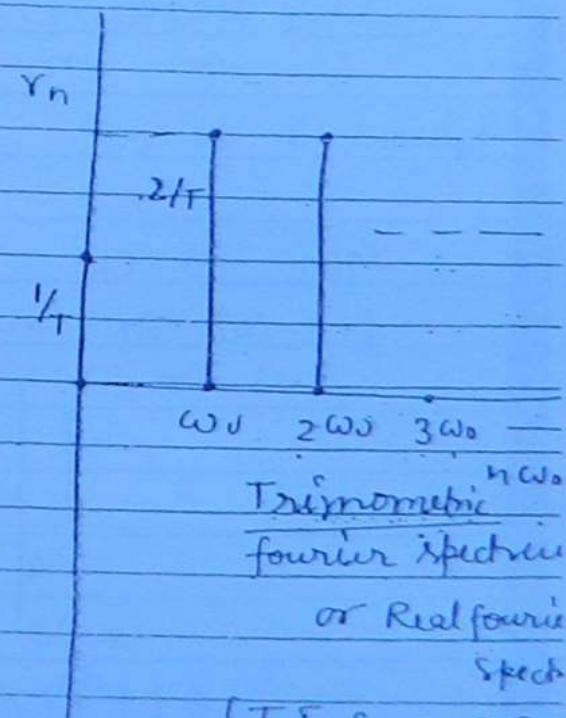
$$\therefore |C_0| = 1/T = a_0$$

$$a_n = \frac{2}{T}, \quad b_n = 0, \quad \phi_n = 0$$

$$f(t) = \sum_{n=0}^{\infty} r_n \cos(n\omega_0 t - \phi_n)$$

$$f(t) = \frac{1}{T} + \sum_{n=1}^{\infty} \frac{2}{T} \cos(n\omega_0 t)$$

$$= \frac{1}{T} \sum_{n=1}^{\infty} \frac{2}{T} \cos\left(n \frac{2\pi}{T} t\right)$$



Trigonometric  
fourier spectra  
or Real fourier  
spectra

(T.F.S. or R.F.S)

(66)

$$T/4 \quad 5T/4$$

$$\begin{aligned} f(t) &= \sum_{n=-\infty}^{\infty} \delta(t - T/4 + KT) \delta(t - \\ &= \sum_{K=-\infty}^{\infty} \delta(t - T/4 + KT) \end{aligned}$$

$$C_n = \frac{1}{T} \cdot e^{-jn\frac{2\pi}{T}T/4} = \frac{1}{T} e^{-jn\pi/2} = \frac{(-1)^{n/2}}{T}$$

$$f(t) = \frac{1}{T} + \sum_{n=1}^{\infty} \frac{2}{T} e^{jn\omega_0 t}$$

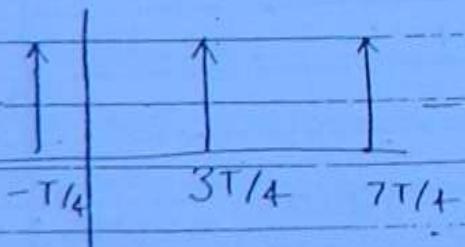
$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{-jn\pi/2} e^{jn\omega_0 t}$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} (-1)^{n/2} e^{jn\omega_0 t} \quad \text{cos } \frac{n\pi}{2} + j \sin \frac{n\pi}{2}$$

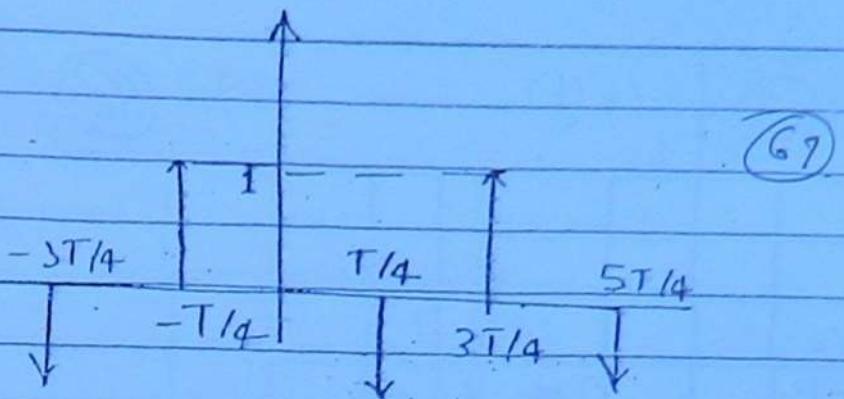
$$(-1)^{n/2}$$

$$f(t) \rightarrow c_n$$

$$f(t-t_0) \rightarrow c_n e^{-jn\omega_0 t_0}$$



$$\begin{aligned} f(t+T/4) &\rightarrow \frac{1}{T} e^{-jn\omega_0(-T/4)} \quad \text{or} \quad e^{-jn\omega_0 3T/4} \\ &\rightarrow \frac{1}{T} e^{jn\omega_0 T/4} \end{aligned}$$



fourier coefficient for given train of pulse  $\rightarrow$

$$C_n = \frac{1}{T} e^{jn\omega_0 T/4} - \frac{1}{T} e^{jn\omega_0 T/4} = \frac{1}{T} \left[ a_j \sin n\omega_0 T/4 + b_j \cos n\omega_0 T/4 \right]$$

$f_1(t) \rightarrow C_m$

$f_2(t) \rightarrow C_{2n}$

$$\text{then } a f_1(t) + b f_2(t) \rightarrow a C_m + b C_{2n}$$

$C_n$  is imaginary for odd signal and odd in nature.

Q. Find the fourier coefficients of  $\sin \omega_0 t$  &  $\cos \omega_0 t$

$$\sin \omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} =$$

$$C_1 = \frac{1}{2j} \quad \text{for } n \\ 0, \text{ otherwise}$$

$$C_{-1} = -1/2j$$

$$\cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

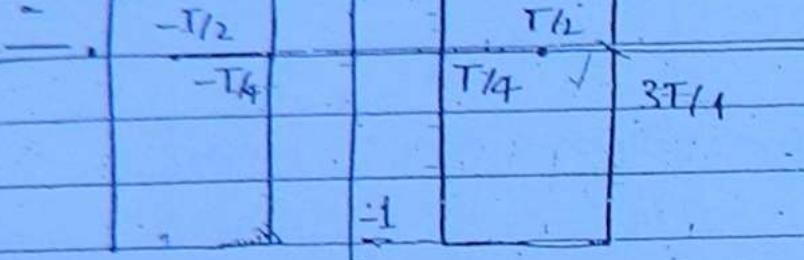
$$C_1 = 1/2 \quad \left\{ \begin{array}{l} \text{for other } n \\ 0, \text{ else} \end{array} \right.$$

$$C_{-1} = +1/2$$

Q Calculate the fourier coefficient of the following signal.

$$f_4(t)$$

(68)



$$C_n = \frac{1}{T} \int_{-T/4}^{3T/4} f(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{T} \int_{-T/4}^{T/4} 1 e^{-jn\omega_0 t} dt + \frac{1}{T} \int_{T/4}^{3T/4} e^{-jn\omega_0 t} dt$$

$$= \frac{1}{T} \left[ \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right]_{-T/4}^{T/4} + \frac{1}{T} \left[ \frac{e^{-jn\omega_0 t}}{jn\omega_0} \right]_{T/4}^{3T/4}$$

$$= \frac{1}{T} \left[ \frac{1}{jn\omega_0} [e^{jn\omega_0 T/4} - e^{-jn\omega_0 T/4}] \right]$$

~~$$+ \frac{1}{T j n \omega_0} \left[ e^{-j n \omega_0 3T/4} - e^{-j n \omega_0 T/4} \right]$$~~

$$= \frac{1}{T j n \omega_0} \left[ 2e^{-j n \omega_0 3T/4} - 2e^{-j n \omega_0 T/4} \right]$$

$$= \frac{1}{T j n \omega_0} \left[ e^{j n \omega_0 T/4} - e^{-j n \omega_0 T/4} \right]$$

$$= \frac{1}{T j n \omega_0} 2 j \sin n \omega_0 T/4$$

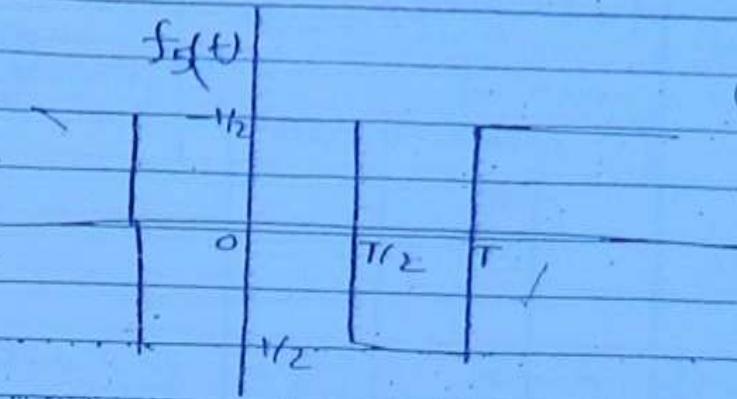
$$C_n = \frac{2}{T j n \omega_0} \sin n \pi/2 = \frac{2}{n \pi} \sin \frac{n \pi}{2}$$

$$C_0 = 0$$

= 0 for  $n \neq 0$

$f_5(t)$

(69)



$f(t)$

$$c_n \rightarrow \frac{1}{n\pi} \sin \frac{n\pi}{2} e^{-jn\omega_0 T/4}$$

$$c_0 = 0$$

$$c_n \rightarrow \left[ \frac{1}{n\pi} \sin \frac{n\pi}{2} e^{-jn\pi/2} \right] \rightarrow \frac{1}{2jnn}$$

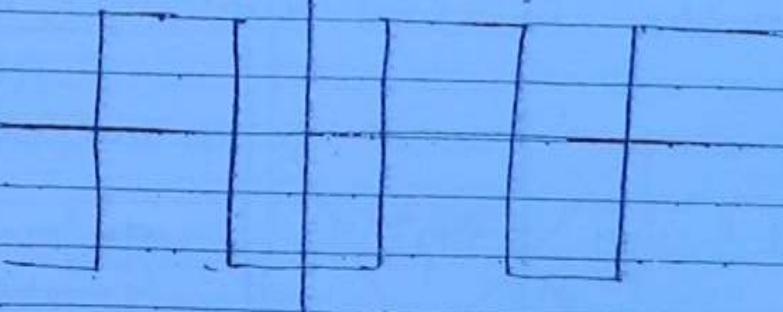
$$f(t) \rightarrow c_n$$

$$\frac{d}{dt} f(t) \rightarrow +c_n j n \omega_0$$

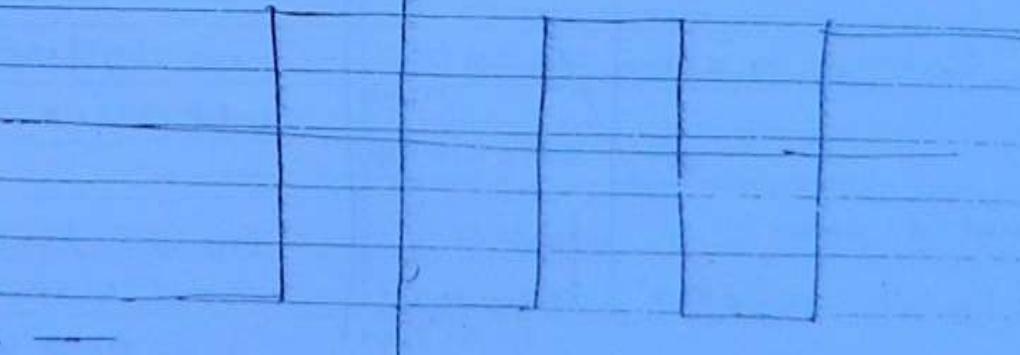
↳ imaginary  
and odd for  
 $n \neq 0$

$$c_n = 0$$

$f_5(t-T/4)$

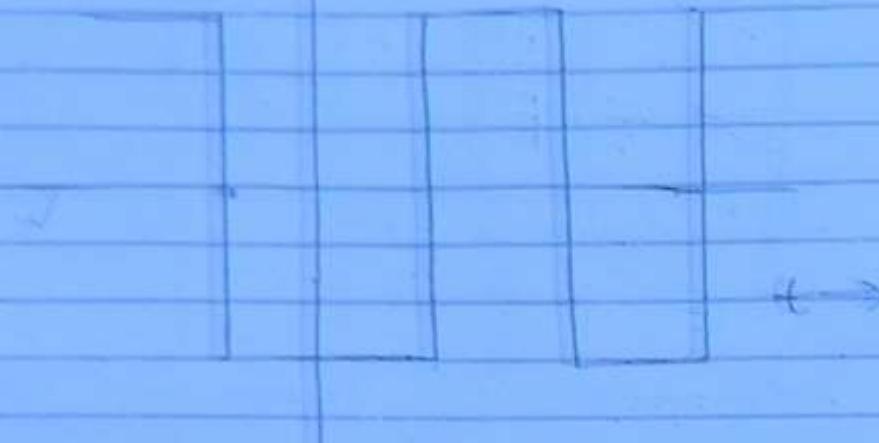


$$f_5(t+T/2) = -f_5(t)$$



$$f_4(t+T_0) = -f_4(t)$$

(70)



$$f(t \pm T_0) = -f(t) \quad c_n = 0 \text{ unless}$$

↳ hidden symmetry or called as half wave symmetry

### Symmetry table

$f(t)$	$c_n$	Trigonometric Coefficient	harmonic
real & even	$c_n = c_{-n}$	$a_0, a_n, b_n = 0$	none
real & odd			$f(t) = a_0 + \sum a_n \cos nt + b_n \sin nt$
Even	real & even	$b_n = 0$	dc term, cosine term, sine term (most important)
odd	Purity imaginary & odd	$a_n = 0, a_{-n}$	<del>dc term</del> & sine terms
half wave symmetry	$c_n = 0 \quad n = \text{even}$	$a_n = 0 \quad n = \text{even}$ $b_n = 0 \quad n = \text{even}$	no dc term odd cosine & odd sine terms
even & half wave symmetrical	real & even	$b_n = 0$ $a_n = 0 \quad n = \text{even}$	no dc terms odd harmonics & only cosine terms

odd &

half wave symmetry

$a_n = 0$   $c_n = 0$

for even

$a_n = 0$  always

$a_n = 0$  always

$b_n = 0$  for even

odd term

odd harmonics

& only sine terms

(i.e. odd sine terms)

Q

Q. After following signal which of signals will have only odd sine terms in T.F.S.  $\rightarrow$  defined in one time period.

(i)

$T/2$   $T$

Ans (i) & (iv)

(ii)

$T/4$   $T$   
 $3T/4$

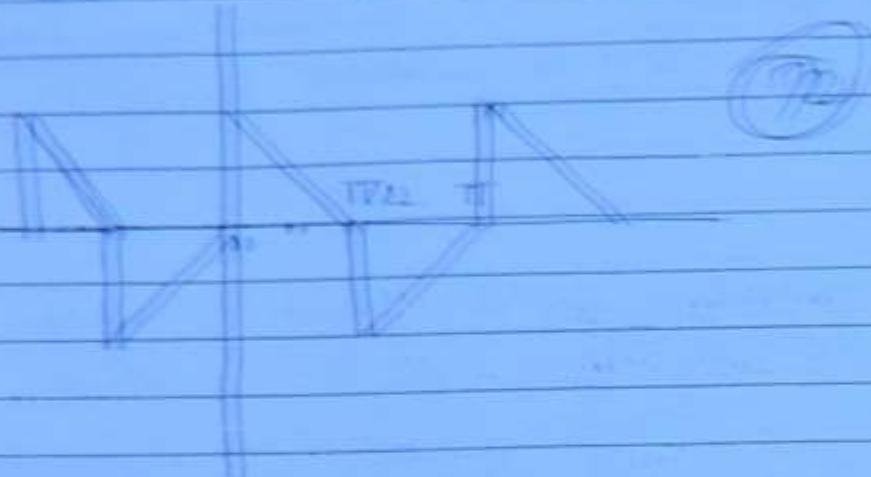
(iii)

$T/4$   $T/2$   
 $5T/4$   $T$

(iv)

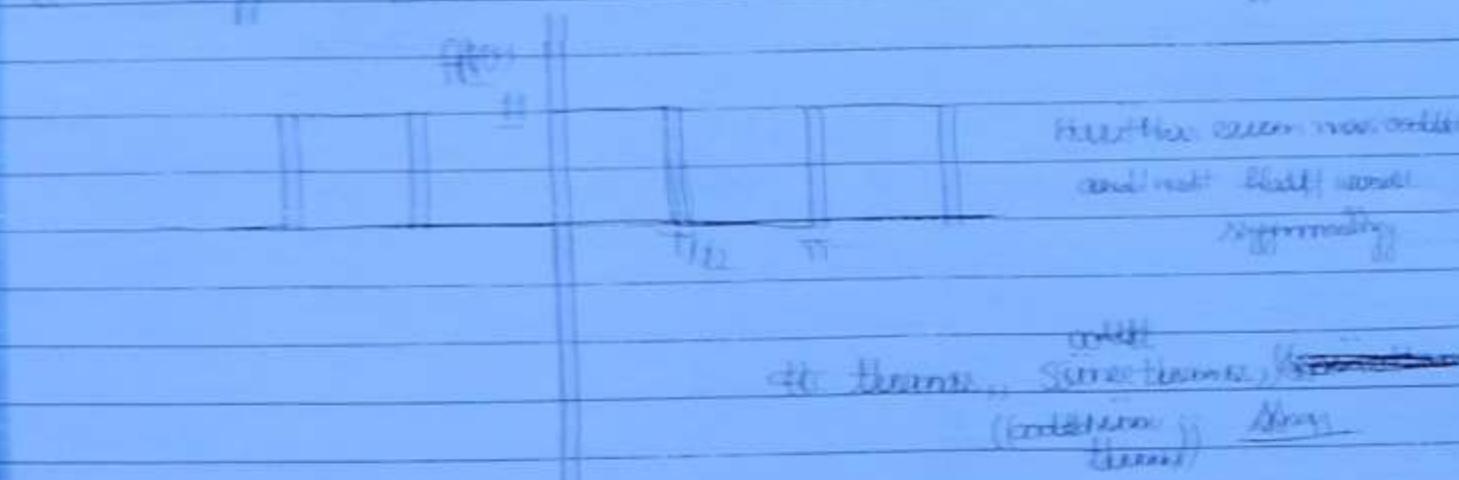
$T/2$   $T$

Q. A second field is offshoot of stream below  $\rightarrow$



✓ only credit give to count them.

Q. A second field is offshoot of stream below



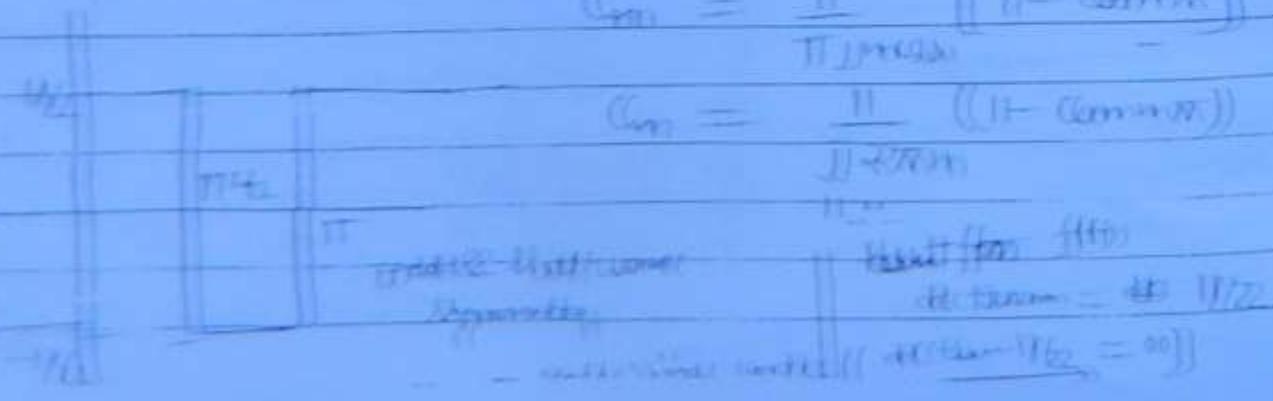
at them, sometimes ~~at them~~ (at them) along

$$C_m = \frac{H}{T} \left( \frac{H}{H - e^{-\lambda m t}} \right) = \frac{H}{T H_m} \left[ \frac{H}{H - e^{-\lambda m t}} \right]$$
$$= \frac{H}{T H_m} \left[ 1 + e^{-\lambda m t} \right]$$

product of  $H_m - H_m$

$$C_m = \frac{H}{T H_m} \left[ \frac{H}{H - C_m m t} \right]$$

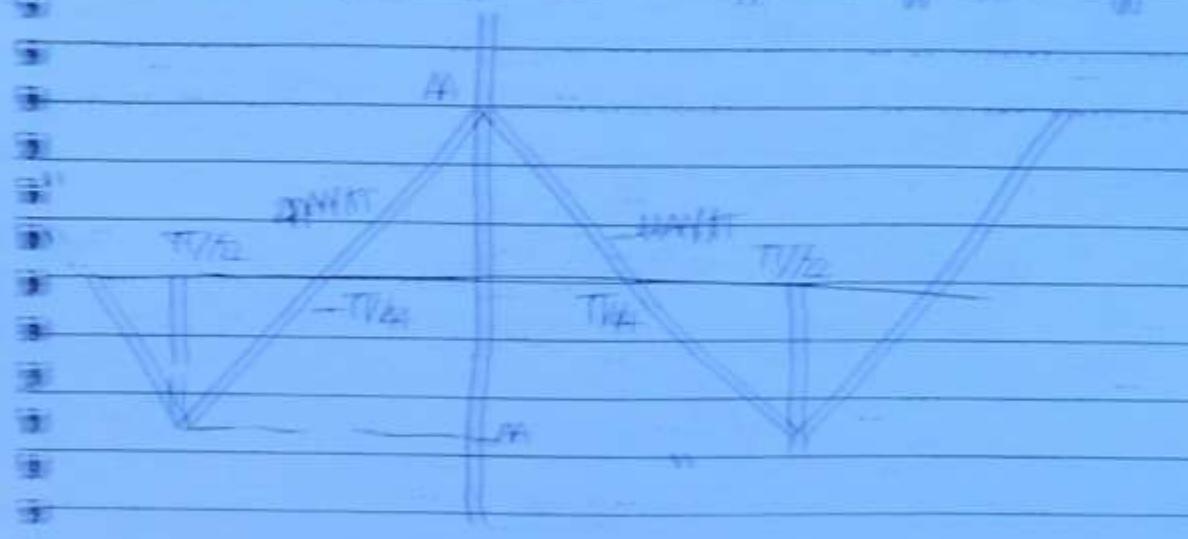
$$C_m = \frac{H}{H - C_m m t}$$



Q. If we add some constant to the coefficients then  
only the term will change and other terms  
coefficient will remain same -

(23)

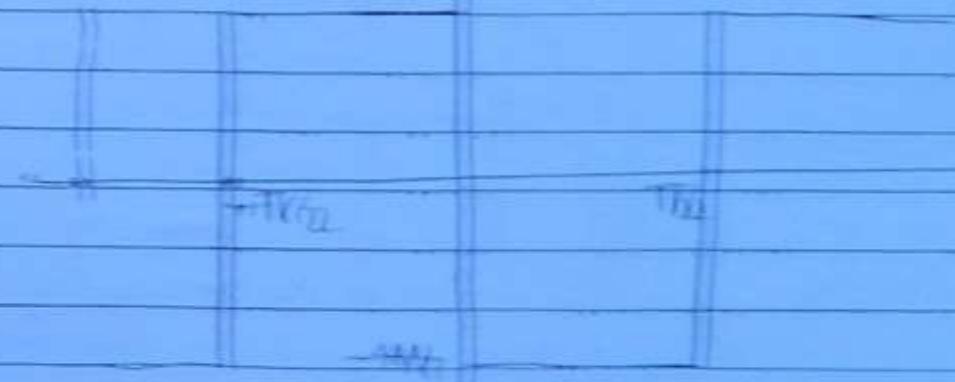
Q. Calculate the Fourier coefficients of following signals.



$$C_0 = 0$$

$$C_1 =$$

$\frac{2\pi}{T}$



$$\frac{2\pi}{T}$$

$$\frac{2\pi}{T}$$

$T/2$

$-T/2$

0

44

for this impulse

$$C_n = \frac{-8A}{T} \cdot \frac{1}{T} e^{-jn\omega_0 T/2} - \frac{8A}{T} \cdot \frac{1}{T} \quad (74)$$
$$= \frac{8A}{T^2} \left[ e^{-jn\omega_0 T/2} - 1 \right]$$
$$= \frac{8A}{T^2} \left[ e^{-j\pi n} - 1 \right].$$

$$C_n = \frac{8A}{T^2} \left[ \cos n\pi - 1 \right]$$

fourier coefficient of  $f(t)$

$$f \quad F_n = \frac{C_n}{(j\omega_0 n)^2}$$
$$= -\frac{8A}{T^2 \omega_0^2 n^2} [\cos n\pi - 1]$$
$$= \frac{8A}{T^2 \omega_0^2 n^2} [1 - \cos n\pi]$$
$$\boxed{F_n = \frac{2A}{\pi^2 n^2} [1 - \cos n\pi]} \quad \text{Ans}$$

real & even  
because signal  
is real & even

$$F_n = \frac{2A}{\pi^2 n^2} [1 - (-1)^n] \rightarrow \text{and only even terms}$$

" 0 for n even because of half wave symmetry

$$F_n \propto \frac{1}{n^2} \rightarrow F_n \propto 1/n^2$$

Triangular pulse

for rectangular pulse

$$C_n \propto \frac{1}{n}$$

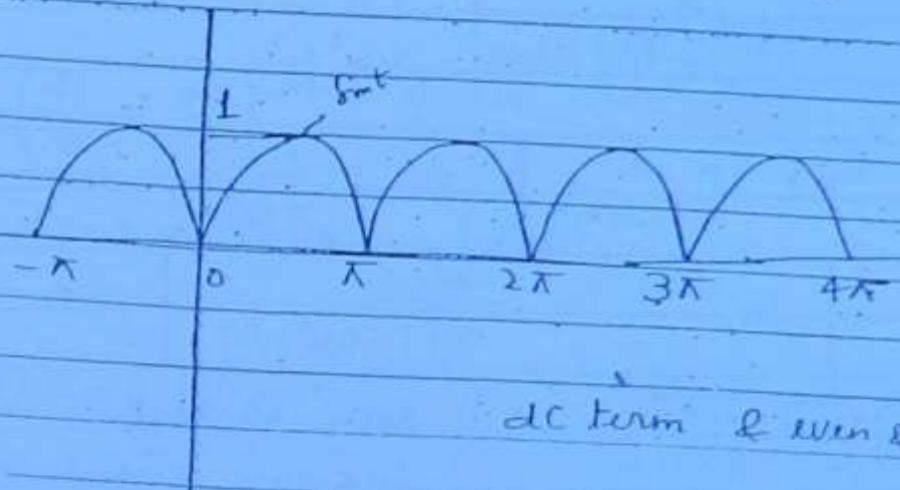
If function is varying as parabolic variation in  $\omega t$   
means as a function  $t^2$ .

then fourier series coefficient will be

(75)

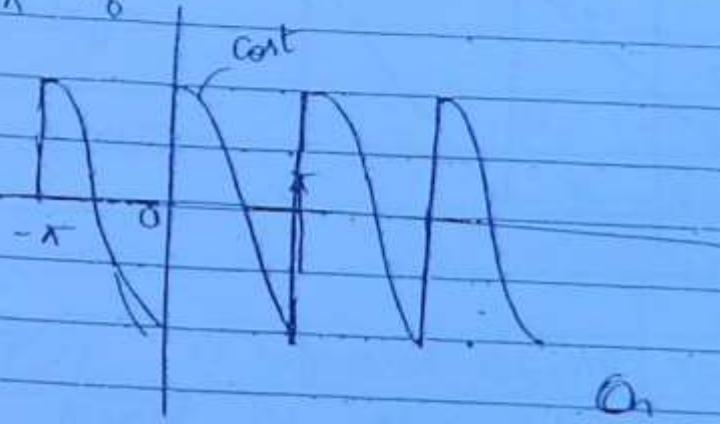
$$C_n \propto \frac{1}{n^3} \rightarrow C_n \propto 1/n^3$$

Q. Find the fourier coefficient of following signal

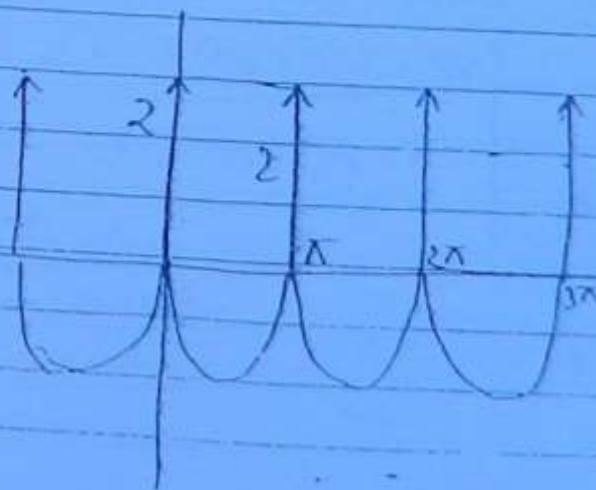


dc term & even & real

$$C_n = \frac{1}{\pi} \int_0^\pi \sin t e^{-jn\omega_0 t} dt \quad \text{we can find}$$



$C_n$



$F_m = C_m \text{ for initial}$

$-f_m$

$$2F_m = 2$$

IV

$$C_n = \left( \frac{\text{impulse}}{\text{ton}} - F_n \right)$$

$$C_n = \left[ \frac{2}{\pi} - F_n \right]$$

(76)

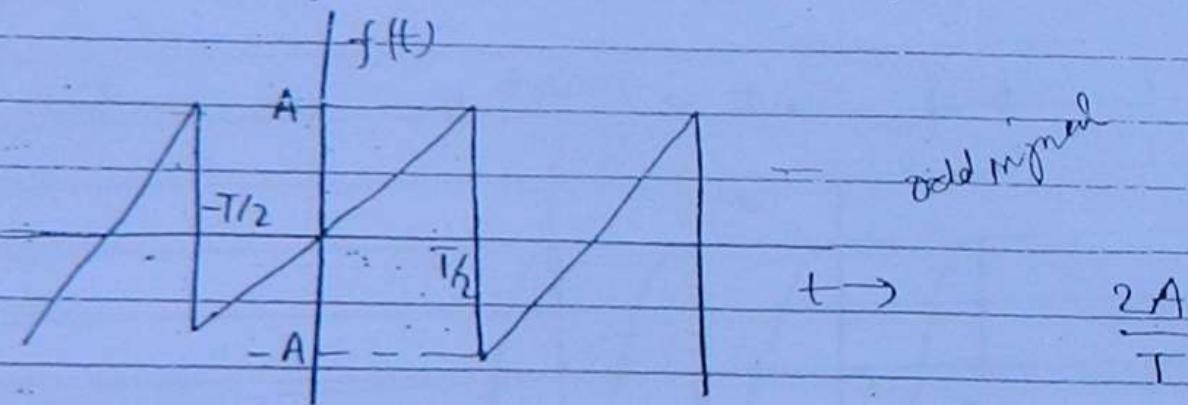
$$F_n = \frac{C_n}{(\omega_0 h)^2}$$

$$= -\frac{2}{\pi \omega_0^2 h^2} + \frac{F_n}{\omega_0^2 h^2}$$

$$F_n \left[ 1 - \frac{1}{\omega_0^2 h^2} \right] = -\frac{2}{\pi \omega_0^2 h^2}$$

$$\boxed{F_n = \frac{2}{\pi (1 - \omega_0^2 h^2)}} = \frac{2}{\pi (1 - 4h^2)}$$

even &amp; real

because signal  $f(t)$  is real  
& even

$$\frac{2A}{T}$$

$$\frac{df(t)}{dt}$$

$$\frac{2A}{T}$$

$$T/2$$

$$T/2$$

$$3T/2$$

$$T/2$$

$$-2A$$

a) for  $\frac{df(t)}{dt}$

$$f[n] = \frac{2A}{T} \frac{d}{dt} e^{j\omega_0 t} \Big|_{t=nT/2} \quad (7)$$

b)

$$C_n = -\frac{2A}{T} \frac{2A}{T} e^{-jn\omega_0 T/2}$$

$$C_n' = of \left( \frac{df}{dt} \Big|_{t=nT/2} \right) \rightarrow \text{only effect of coefficient}$$

$$\frac{2A}{T} e^{jn\omega_0 nT} = f[n] \circ jn\omega_0 + \{n \neq 0\}$$

$$f_n(f[0]) = -\left( \frac{2A}{T} e^{-jn\pi} \right) \rightarrow \frac{2A}{jn2\pi} [e^{-jn\pi}]$$

$$f_0(f[0]) = -\frac{2A}{T} + \frac{2A}{T} = 0 \quad = -\frac{A}{jn\pi} [(-1)^n]$$

$$f[n] = (-1)^{n+1} \frac{A}{jn\pi} \quad n \neq 0$$

$$f_0 = 0$$

image  
and odd

Q. A signal  $f(t)$  is expressed as a sum of sinusoidal signals as shown below

$$f(t) = 2 + 3 \cos\left(\frac{5\pi}{6}t + \pi/3\right) + 4 \sin\left(\frac{6\pi}{5}t + \pi/4\right)$$

d)

$$+ 3 \cos\left(\pi/7 t\right) = 2 + 3 \left[ \frac{e^{j6\pi/6t + \pi/3}}{2} + e^{-j6\pi/6t - \pi/3} \right] + 4 \left[ e^{j(6\pi/5)t + \pi/4} + e^{-j(6\pi/5)t - \pi/4} \right]$$

$$\frac{5\pi}{6}, \frac{6\pi}{5}, \pi/7$$

Overall

$$\omega_0 = \frac{\pi}{2T_0}$$

$$T = \frac{2\pi}{\omega_0}$$

$$4-20$$

$$+ \frac{3}{2} \left[ e^{j\pi/7 t} - e^{-j\pi/7 t} \right]$$

$$x(t) = x_0 + \sum_{n=1}^{\infty} r_n \cos(n\omega_0 t - \phi)$$

78

$$f(t) = \omega + \frac{3}{2} e^{j\pi/3} e^{j\frac{\pi}{210} \times 5 \times 210 t} + \frac{3}{2} e^{-j\pi/3} e^{-j\frac{\pi}{210} \times 5 \times 210 t}$$

$$+ \frac{3}{2} e^{j\pi/4} e^{j\frac{\pi}{210} \times 6/5 \times 210 t} + \frac{3}{2} e^{-j\pi/4} e^{-j\frac{\pi}{210} \times 6/5 \times 210 t}$$

$$+ \frac{3}{2} e^{j\frac{\pi}{210} \times 2 \frac{210}{7} t} + \frac{3}{2} e^{-j\frac{\pi}{210} \times 2 \frac{210}{7} t}$$

dc terms, +175<sup>th</sup>, +252<sup>th</sup>, +30<sup>th</sup> harmonics  
 175<sup>th</sup>, 252<sup>th</sup>, 30<sup>th</sup> harmonics.

Q. Find the following signal  $f(t)$  defined as

$$f(t) = (\cos 2\pi t + \cos 7\pi t)^2$$

$$= \cos^2 2\pi t + \cos^2 7\pi t + 2 \cos 2\pi t \cos 7\pi t$$

$$= \frac{1}{2} [ 2 \cancel{1} + \cos 4\pi t + 1 + \cos 14\pi t ]$$

$$+ 2 [\cos 5\pi t + \cos 9\pi t]$$

$$\omega \rightarrow 4\pi, 14\pi, 5\pi, 9\pi \rightarrow \text{H.C.F of all } \omega \rightarrow \pi$$

$\star$

dc terms are 4<sup>th</sup>, 14<sup>th</sup>, 5<sup>th</sup>, 9<sup>th</sup> harm.

(\*)

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$P_t = a_0^2 + \sum_{n=1}^{\infty} \left( a_n^2 + \frac{b_n^2}{2} \right)$$

$$= a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

$$P_t = a_0^2 + \sum_{n=1}^{\infty} r_n^2$$

$$|C_n|^2 = \frac{a_n^2 + b_n^2}{4} = |C_{-n}|^2$$

$$\begin{aligned} f(t) &= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \\ C_n e^{jn\omega_0 t} &= |C_n|^2 e^{jn\omega_0 t} \\ P &= \sum_{n=-\infty}^{\infty} |C_n|^2 \end{aligned}$$

$$|C_n|^2 + |C_{-n}|^2 = 2 \frac{a_n^2 + b_n^2}{4}$$

$$\begin{aligned} P &= C_0^2 + \sum_{n=1}^{\infty} \{ |C_n|^2 + |C_{-n}|^2 \} \\ &= C_0^2 + 2 \sum_{n=1}^{\infty} |C_n|^2 \end{aligned}$$

$$= C_0^2 + \sum_{n=-\infty}^{\infty} |C_n|^2$$

$n \neq 0$

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2$$

Parseval's power relation for periodic power signal

Q. A periodic saw tooth waveform as one considered in previous problem has a period  $T=2$ , find the no. of fourier coefficient to be considered such that considered fourier coefficients 90% of the total power.

$$\sum_{n=-\infty}^{\infty} |C_n|^2 = P_{\text{total}}$$

$$\begin{aligned} P_{\text{total}} &= \sum_{n=-\infty}^{\infty} \left( \frac{A}{n\pi} \right)^2 = \frac{2A^2}{\pi^2} \cdot \sum_{n \neq 0}^{\infty} \left( 1 + \frac{1}{n} \right)^2 \dots \\ &= \frac{2A^2}{\pi^2} \cdot \sum_{n=1}^{\infty} \left( \frac{1}{n} \right)^2 \end{aligned}$$

$$P_{\text{total}} = \frac{2A^2}{\pi^2}$$

$$\text{Total} = \frac{1}{2} \int_{-1}^1 (At)^2 dt$$

$$\begin{aligned} &= \frac{A^2}{2} \left[ \frac{t^3}{3} \right]_{-1}^1 = \frac{A^2}{2} \times \frac{A^2}{2 \times 3} [2] \\ &= A^2 / 12 \end{aligned}$$

Power in n terms

(80)

$$\frac{\alpha A^2}{\pi^2} \sum_{m=1}^n \left(\frac{1}{m}\right)^2 = 0.9 \times \frac{A^2}{3}$$

$$\sum_{m=1}^n \left(\frac{1}{m}\right)^2 = \frac{0.3}{6^2}$$

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} = \frac{3}{20} \pi^2$$

1.25

9.89 x 3

1.47

29.67

6 terms > 29.07

(QK)

$$\frac{1+1}{1+9} \frac{20}{9+1} \frac{20 \cdot 30}{30} \frac{130}{30} \frac{0.04}{6.2}$$

$$1.38 + 09(0.04) +$$

Q. A periodic signal  $f(t) = \sum_{n=0}^{\infty} C_n n \pi e^{jn\pi/100t}$

In this signal is complex valued signal or real signal

$$C_n = C_n^*$$

$$= \sum_{n=0}^{\infty} C_n n \pi e^{jn\pi/100t}$$

$$= \sum_{n=-\infty}^0 C_{-n} (-n) \pi e^{j(-n)\pi/100t}$$

$$C_n = C_{-n}$$

$$\left. \begin{aligned} C_n &= \sum_{n=-\infty}^0 C_{-n} (-n) \pi \\ C_{-n} &= \sum_{n=-\infty}^0 C_n n \pi \\ C_{-n} &= \sum_{n=0}^{\infty} C_n n \pi \\ C_n &= \sum_{n=0}^{\infty} C_n n \pi \end{aligned} \right\}$$

for  $n \leq 0$

0 otherwise

$$C_n = C_{-n}$$

$n \geq 0$

complex valued signal

A discrete time system defined by (81)

$$y[n] - y[n-1] = x[n] \rightarrow \text{Recursive system response}$$

find its impulse response  

$$h[n] - h[n-1] = \delta[n]$$

$\frac{dy[n]}{dx[n]} = \frac{1}{z-1}$  is depending on previous resp.

$$n=0 \quad h[0] = 1 + h[-1]$$

~~for~~

$$Y[z] - z^{-1} Y[z] = X[z]$$

$$\rightarrow H[z] = \frac{1}{1-z^{-1}} = u[n] = h[n]$$

IIR system

$$\left\{ \begin{array}{l} y[n] = x[n] + x[n-1] \quad \text{non recursive form} \\ y[n] - y[n-1] = x[n] - x[n-2] \quad \text{recursive form} \end{array} \right.$$

FIR system

Name system

find the value of the integral

(82)

$$\int_{-\infty}^{\infty} [\cos 3t \cdot s(3t-3) + e^{-3t} s'(2t-2)] dt$$

$$= \frac{1}{3} [-1] + \int_{-\infty}^{\infty} e^{-3t} s'[-1] dt$$

$$= -\frac{1}{3} + \frac{1}{4} \left[ -\frac{d}{dt} e^{-3t} \right]_{t=1}$$

$$= -\frac{1}{3} + \frac{3}{4} e^{-3}$$

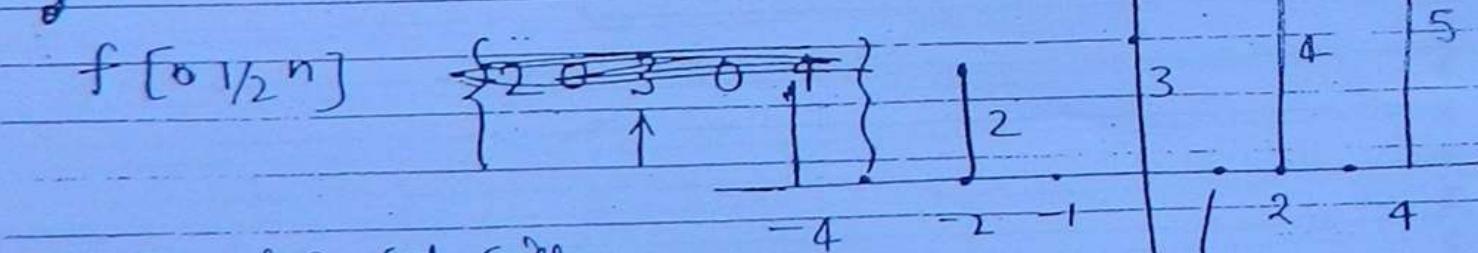
$$= -\frac{1}{3} + \frac{3}{4} e^{-3} \quad \underline{\text{Ans}}$$

$f[n] \rightarrow$

$$\{ 1, 2, 3, 4, 5 \}$$

$$f[2n] \rightarrow \begin{cases} 1 & 3 & 5 \\ \downarrow & \uparrow & \end{cases}$$

derivation



we can also feed fill.

these samples by 1 called  
unit interpolation.

& fill with zero-  
called  
zero  
interpolation

$$\delta'(-t) \rightarrow -1 \cdot \delta'(t) \rightarrow \text{add function. } (83)$$

\* all even derivative of  $f(t)$  are even functions of time and all odd derivative of  $s(t)$  are odd function of time.

$$\frac{d}{dt} [f(t) \cdot \delta(t-a)] = \frac{d}{dt} [f(a) \cdot \delta(t-a)]$$

$$f'(t) \delta(t-a) + f(t) \cdot \delta'(t-a) = f(a) \cdot \delta'(t-a)$$

$$\boxed{f(t) \delta'(t-a) = f(a) \delta'(t-a) - f'(t) \delta(t-a)}$$

$$t \delta'(t) = 0 \cdot -1 \delta(t-0) \\ \therefore = -\delta(t)$$

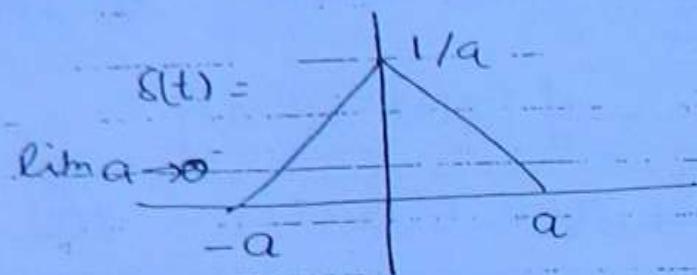
$$\int_{-\infty}^{\infty} f(t) \delta(t-a) dt = f(a)$$

$$\begin{aligned} \int_{-\infty}^{\infty} f(t) \delta'(t-a) dt &= \int_{-\infty}^{\infty} f(a) \delta'(t-a) dt - \int_{-\infty}^{\infty} f'(t) \delta(t-a) dt \\ &= \int_{-\infty}^{\infty} f(a) \delta'(t-a) dt - f'(a) \\ &= f(a) \int_{-\infty}^{\infty} \delta'(t-a) dt - f'(a) \\ &= f(a) [\delta(t-a)]_{-\infty}^{\infty} - f'(a) \\ &= f(a) \times 0 - f'(a) \\ &= -f'(a) \end{aligned}$$

derivative of unit impulse signal

(Q4)

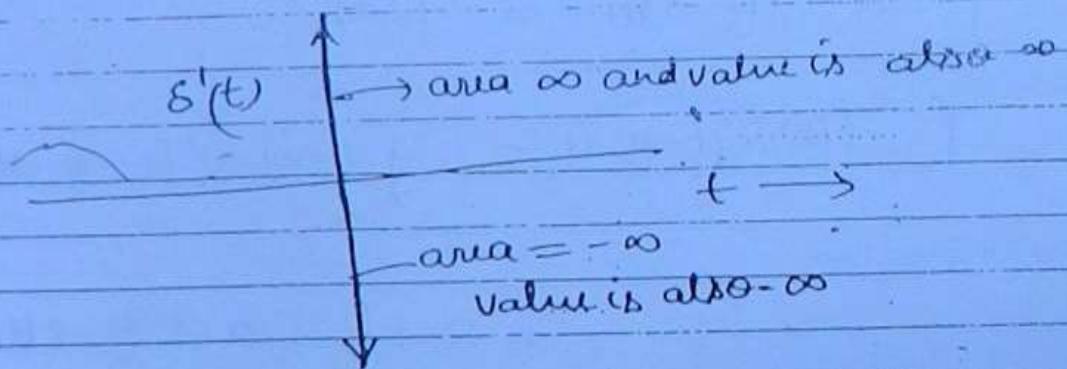
$$\delta'(t) = \frac{d}{dt} \delta(t) =$$



$$\lim_{a \rightarrow 0} \frac{d}{dt} \delta(t) =$$

lim  
 $a \rightarrow 0$

-1/a<sup>2</sup>      1/a<sup>2</sup>



$$\delta(at) = \frac{1}{|a|} \delta(t)$$

$$\frac{d}{dt} \delta(at) = \frac{1}{|a|} \cdot \delta'(t)$$

$$a \delta'(at) = \frac{1}{|a|} \delta'(t)$$

$$\boxed{\delta'(at) = \frac{1}{a|a|} \delta'(t)}$$

Cooley - Tukey

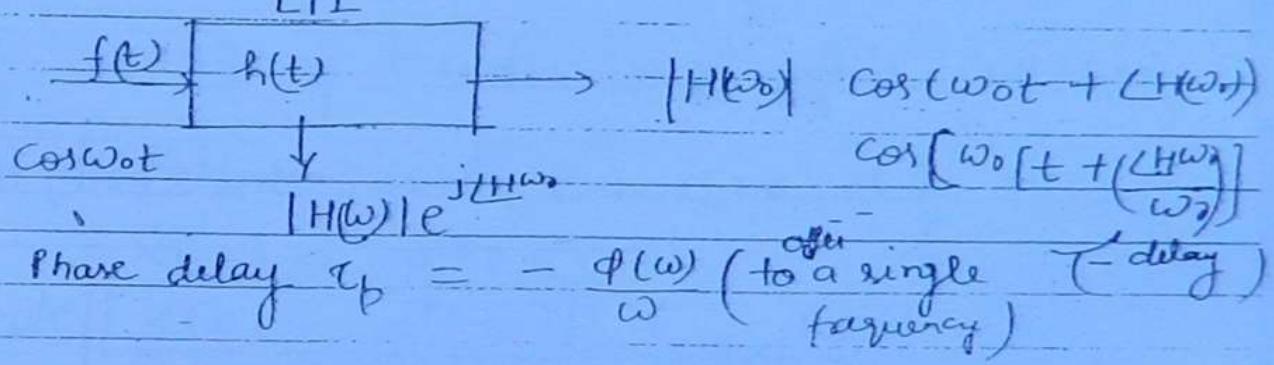
(85)

FFT (algorithm).

$\frac{N}{2} \log_2 N$  — multiplications.

$N \log_2 N$  → additions.

Group delay and phase delay →



$$\phi(\omega) = \angle H(\omega)$$

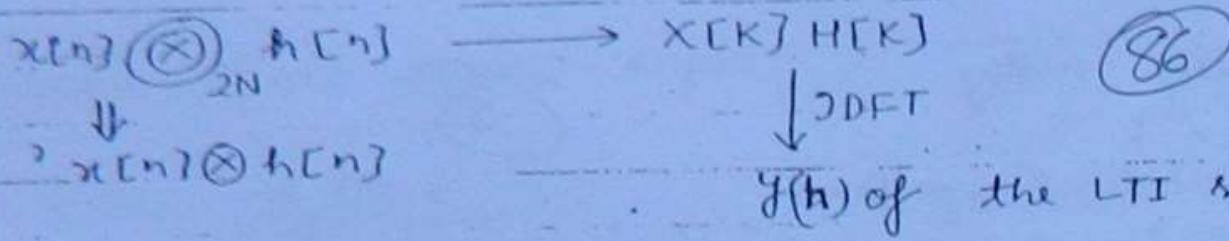
$$\tau_g = -\frac{d\phi(\omega)}{d\omega} \quad (\text{offer to group of frequencies})$$

Q. Calculate the Group delay and the phase delay of a distortionless transmission system and compare them.

$$|H(\omega)| = \omega_0$$

$$\text{Phase delay} = \omega_0$$

$$\text{group delay} = \omega_0$$



$$\sum |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2 \quad \text{Parseval's theorem}$$

Q. DFT of a real valued signal  $x[n]$  is  
 $\{10, 2+3j, 2A, 3-2j, -4, B, 1+j, C\}$

find the energy of the signal

$$= \{10, 2+3j, 1-j, 3-2j, -4, 3+2j, 1+j, 2-3j\}$$

$$E = \frac{1}{8} \sum_{k=0}^{N-1} |X[k]|^2$$

$$= \frac{1}{8} [100 + 13 + 2 + 13 + 16 + 13 + 2 + 13]$$

$$= \frac{1}{8} [215]$$

$$[E = 21.5 \text{ Joule}]$$

$N^2$  multiplication  
 $N(N-1)$  addition

$\rightarrow$  actually  $4N^2$  multiplication  
 $\rightarrow$  actually  $2N(N-1)$  addition.

$$x[n] \xleftrightarrow{\text{DFT}} X[k]$$

$$\begin{matrix} x[-n]_N & \xleftrightarrow{\text{DFT}} & X[-k]_N \rightarrow X[N-k] \\ \downarrow & & \text{circular} \\ x[N-n] & & \text{reversal} \end{matrix}$$

circular reversal

(87)

Q.  $x[n] \longleftrightarrow \{ 10, 4+2j, -4, 4-2j \}$

find DFT of  $x[-n]_N$

$$X[k] = \{ 10, 4-2j, -4, 4+2j \}$$

$$X[-k]_N = \underline{\text{Ans.}}$$

$$\begin{array}{|l|l|} \hline X[-k] & = X^*[k] \\ \hline X[k] & = X^*[-k] \\ \hline \end{array}$$

even conjugate in nature if  $x[n]$  is

a real valued signal as in case of other Fourier transforms.

all the properties of Fourier transform of CT continuous time signal for complex signals are also valid for DFT.

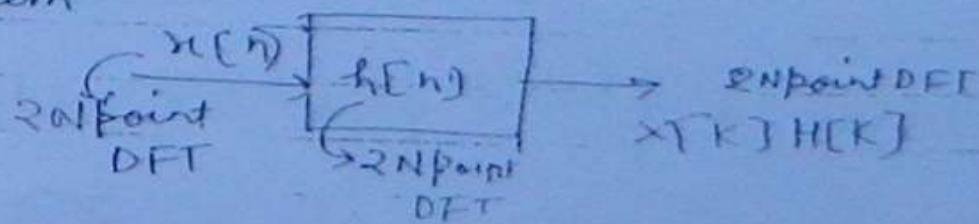
$$x[n] \xrightarrow{\text{N-point DFT}} X[k]$$

$$h[n] \xrightarrow{\text{N-point DFT}} H[k]$$

$$x[n] \underset{N}{\circledast} h[n] \longleftrightarrow X[k] H[k]$$

$$\begin{array}{ccc} \text{N zero} & & \\ x[n] & \xrightarrow{2N \text{ DFT}} & X[k] \\ h[n] & \xrightarrow{2N \text{ DFT}} & \Phi H[k] \end{array}$$

for LTI system



Q. Given two signals  $f[n]$  and  $h[n]$  each consisting of  $N$ -sample values if we zero pad each of the signal with  $N$ -zeros and carry out circular convolution of order  $2N$  this will be equal to linear convolution of two signals  $f[n]$  &  $h[n]$ .

(88)

Time shifting Property of DFT →

The DFT of a signal  $F[n] \leftrightarrow \{10, -4+2j, -4, -4-2j\}$

\* To find the DFT of  $f[n-2]_4$

$$\frac{x[k]e^{j2\pi k}}{Y[k]} = F[k] e^{-jk\omega_0 \cdot 2}$$

$$Y[k] = F[k] e^{-jk\frac{2\pi}{N} \cdot 2}$$

$$Y[k] = F[k] (-1)^k$$

$$Y[k] = \{10, -4+2j, -4, -4-2j\}$$

$$x[n] \rightarrow X[k]$$

$$x[n] e^{j2\pi k \cdot n} \rightarrow x[k-k_0]_N$$

always circular shift

Q. DFT of a signal  $x[n] = \{1, 2, 3, 4\} \leftrightarrow X[k]$   
find the inverse DFT of  $\leftrightarrow X[k-1]$ .

$$e^{j2\pi/N \cdot 1 \cdot n}$$

$$e^{j2\pi/4 \cdot n}$$

$$e^{j\pi/2 \cdot n}$$

$$(j)^n$$

$$Y[n] = \{1, 2j, -3, -4j\}$$

$f[n]$	$h[n]$	4	3	2	1
1		4	3	2	1
2		8	6	4	2
3		12	9	6	3
4		16	12	8	4

(89)

linear convolution result of two signal

$$y[n] = \{ 4, 11, 20, 30, 20, 11, 4, 2 \}$$

↓ add      ↓ add      ↓ add      ↓ add  
 24    22    24    30

$$= \{ 24, 22, 24, 30 \}$$

$$= \frac{(1 - e^{-j\omega})}{e^{-j\omega/2} [e^{j\omega/2} - e^{-j\omega}]}$$

divide the result of linear convolution into two parts such that first part having  $\frac{N}{2}$  sample and then add corresponding parts in order and write as

No of sample in each signal.

a circular convolution result

Q. circularly convolve the following two signal

$$f[n] = \{ 1, 2 \}$$

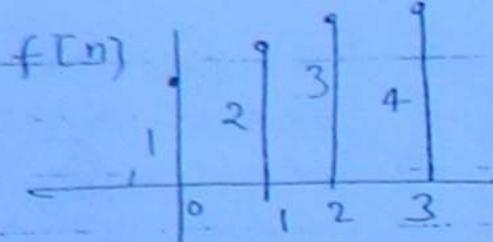
$$h[n] = \{ 2, 1 \}$$

by zero padding each of signal with two zeros

	2	1	0	0
1	2	1	0	0
2	4	2	0	0
0	0	0	0	0
0	0	0	0	0

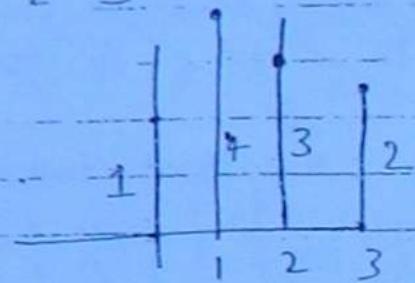
$$\text{linear conv}^n [ 2 \ 5 \ 2 \ 0 | 0 \ 0 \ 0 ] \leftarrow f[n] \otimes h[n]$$

circular conv →  $[ 2 \ 5 \ 2 \ 0 ]$  from linear



(90)

$$f[-n] \rightarrow$$



Keep the value at  $n=0$  as it is write other samples in reverse order.  
as  $2, 3, 4$  (we will write as  $4, 3, 2$ )  
 $\cdot n = (1, 2, 3)$  at  $n=0$

Q.

Evaluate the circular convolution of 2 signals

$$f[n] = \{1, 2, 3, 4\}$$

$$h[n] = \{4, 3, 2, 1\}$$

$$h[-n] = \{4, 1, 2, 3\}$$

$$f[n] \otimes h[n]$$

$$y[n] = \sum_{k=0}^{N-1} f[k]$$

symbol  
for circular convolution

$$n=0 \quad h[n-k]$$

$$\therefore y[0] = 4+2+6+12 = 24$$

$$h[-n+1] = \{3, 4, 1, 2\}$$

25

$$\therefore y[1] = 3+8+3+8$$

$$= 22$$

$$h[-n+2] = \{2, 3, 4, 1\}$$

$$y[2] = 2+6+12+4 = 24$$

$$h[-n+3] = \{1, 2, 3, 4\}$$

$$\therefore y[3] = 1+4+9+16 = 30$$

$$y[n] = \{24, 22, 24, 30\}$$

$F[0]$  is always  $f[0] + \dots + f[N-1]$   
 $F[N/2]$   $f[0] - f[1] + f[2] - \dots$  (91)  
 If  $N$  is even  
 $F[K] = F^*[N-K]$  → symmetry satisfied

These symmetries are valid till  $f[n]$  is a real valued signal.

DFT of a real valued signal is given as

$$\left\{ \begin{array}{ccccccc} 10, A, 2+3j, B, -4, 3-2j, C+1j \\ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \end{array} \right\}$$

Find  $A, B, C$ .

$$\left. \begin{array}{l} A = 1-j \\ B = 3+2j \\ C = 2-3j \end{array} \right\} \text{ using } F[K] = F^*[N-K]$$

Q. Find the DFT of the following 8-point signal

$$f[n] = \{ 1, \dots, 1 \}$$

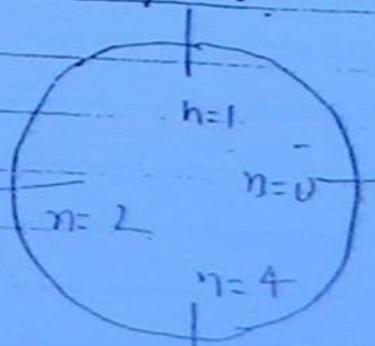
$$F[0] = 8$$

$$F[1] = [1 + e^{-j\omega_0} + e^{-2j\omega_0}] \dots = e^{-7j\omega_0}$$

$$F[4] = 0 \quad F[1] = 1 \left[ \sum_{k=0}^{7} [1 - e^{-8j\omega_0}] \right]$$

$$\begin{aligned} F[7] &= \frac{e^{-4j\omega_0}}{e^{-j\omega_0/2}} \sum_{k=0}^{7} \frac{\sin 4\omega_0}{\sin \omega_0/2} \\ &= e^{-7j\omega_0/2} \frac{\sin 4\omega_0}{\sin \omega_0/2} \end{aligned}$$

Circular shift :-



$$f[n-1]_4$$

$$f[N-n]$$

Shifting by 1 unit but not linear  
because it is circular.

$$f[-n]_4$$

Ans

If  $N$ -sample values are picked up uniformly from DFT of a discrete time signal in its unique period of  $2\pi$ , we get the discrete fourier transform.

(92)

Evaluate DFT of  $f[n] = \{1, 2, 3, 4\}$

$$F[k] = \sum_{n=0}^3 f[n] e^{-j k \omega_0 n} \quad \omega_0 = e^{-j \frac{\pi}{2}}$$

$$F[0] = \sum_{n=0}^3 f[n] e^{-j k \omega_0 n}$$

$$\begin{aligned} F[1] &= \sum_{n=0}^3 f[n] e^{-j k \omega_0 n} \\ &= [1 + 2e^{-jk\omega_0} + 3e^{-2jk\omega_0} + 4e^{-3jk\omega_0}] \\ &= [1 + 2e^{-jk/2} + 3e^{-jk} + 4e^{-3jk/2}] \\ &= [-2 - 2j + 4j] \end{aligned}$$

$$F[1] = -2 + 2j$$

$$F[2] = \sum_{n=0}^3 f[n] e^{-2j \omega_0 n}$$

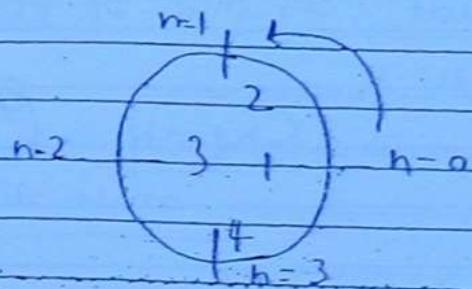
$$\begin{aligned} &= [1 + 2e^{-2j \omega_0} + 3e^{-4j \omega_0} + 4e^{-6j \omega_0}] \\ &= [1 + (-2) + 3 - 4] \end{aligned}$$

$$F[2] = -2$$

$$\begin{aligned} F[3] &= [1 + 2e^{-3j \omega_0} + 3e^{-6j \omega_0} + 4e^{-9j \omega_0}] \\ &= [1 + 2j - 3 - 4j] \\ &= -2 - 2j \end{aligned}$$

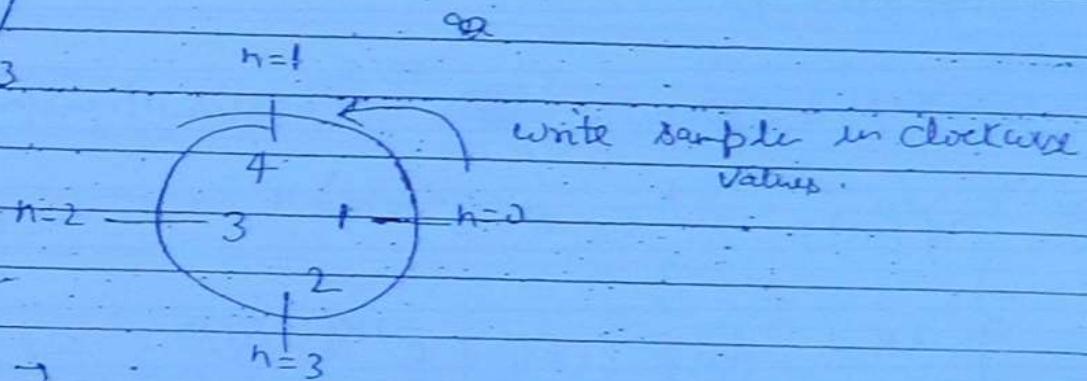
$$f(n) = \left\{ \begin{matrix} 1, 2, 3, 4 \\ \uparrow \end{matrix} \right\}$$

(93)



will be done  
representation  $\rightarrow$  shifting on this

to find  $f[-h]$ ,



write sample in clockwise values.

circular convolution  $\rightarrow$

$$f[n] \rightarrow (0 \rightarrow 3) \quad h[n] \rightarrow (0 \rightarrow 3)$$

$$y[n] = h[n] \oplus f[n] \rightarrow 0 \rightarrow 6 \text{ - linear convolution}$$

$$y[n] = f[n] \otimes_4 h[n] = \left\{ \begin{matrix} n=0 & n=3 \end{matrix} \right\}$$

circular convolution of order 4

\*  $C_K$  is defined as discrete time fourier series coefficient of signal  $f[n]$ , and it is also discrete and periodic with the same period as that of  $f[n]$ .

$$(e^{-j\frac{2\pi}{N}kn}) C_K = C_{K+N} \quad (94)$$

$m$  is an integer

D.F.T.: Discrete Fourier transform  $\rightarrow$

$$\left\{ \begin{array}{l} f[n] = \sum_{k=0}^{N-1} C_k e^{jk(2\pi/N)n} \quad -\infty < n < \infty \\ C_k = \frac{1}{N} \sum_{n=0}^{N-1} f[n] e^{-jk(2\pi/N)n} \quad k \rightarrow -\infty \text{ to } \infty \end{array} \right.$$

↓  
discrete time F.T. pairs

$$D.F.T. \quad F[k] = \sum_{n=0}^{N-1} f[n] e^{-jk(2\pi/N)n} \quad \text{for } k = 0 \text{ to } N-1$$

$$F[k] = N C_k$$

$$f[n] = \sum_{k=0}^{N-1} \frac{1}{N} F[k] e^{jk(2\pi/N)n}$$

$$f[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{jk(2\pi/N)n}$$

$$0 \leq n \leq N-1$$

↓  
discrete F.T.  
pairs

- shifting is circular and

$$f[[n-2]]_4 \rightarrow 0001$$

✓  
no. of samples  $0 \cdot 10^3$  /  
taken i.e.  $10^3$  m

$$\frac{df(t)}{dt} \leftrightarrow j\omega F(\omega)$$

$$f[n] \leftrightarrow F(\omega)$$

$$[f[n] - f[n-1]] \leftrightarrow [(1 - e^{-j\omega})] F(\omega) \quad (95)$$

$$u[n] \leftrightarrow \frac{1}{(1 - e^{-j\omega})} + \pi \delta[\omega]$$

$\cancel{\omega \leq \pi}$

$$\leftrightarrow \infty \cdot \frac{1}{e^{j\omega}} + \sum_{k=0}^{\infty} \pi \delta(\omega - 2\pi k) \quad \omega < \omega < \infty$$

$$\int_{-\infty}^t f(t) dt \leftrightarrow \frac{F(\omega)}{j\omega} + \pi \delta(\omega) \cdot F(0)$$

$$\sum_{k=-\infty}^n f[k] \leftrightarrow \frac{F(\omega)}{(1 - e^{-j\omega})} + \pi \delta(\omega) F(0)$$

### \* Duality Property of F.T. :

- D.T.F.T. and continuous time fourier series forms dual of each other.
- Discreteness in one domain leads to periodicity in the second domain and vice versa.

fourier series for discrete signals  $\xrightarrow{\text{time}}$

$$f[n] = \sum_{k=0}^{N-1} c_k e^{j k \omega_0 n} = \sum_k c_k e^{j k \frac{2\pi}{N} n}$$

periodic N  
↓  
over period N

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} f[n] e^{-j k \omega_0 n}$$

↓  
over period N

we can use the properties of F.T. (continuous or discrete) only if dirichlet condition must be satisfied.

$$F(\omega) =$$

$$E = \frac{1}{2\pi} \int_{-\pi}^{\pi} |F(\omega)|^2 d\omega$$

$$E = \frac{1}{2\pi} \int_{-\pi}^{\pi} |1|^2 d\omega$$

$$|F| = \frac{1}{2\pi} = \frac{2}{\pi}$$

(96)

### ④ Convolution property of D.T.F.T.

$$f(n) * h[n] \longleftrightarrow F(\omega) H(\omega)$$

Find the D.T.F.T. of  $y(n) = \sum_{k=-\infty}^{\infty} a^k u[n-k]$

$$\begin{aligned} f[n] &= \sum_{K=-\infty}^{\infty} f[K] \delta[n-K] \\ &= \sum_{K=-\infty}^{\infty} a^K u[K] a^{n-K} u[n-K] \\ &= \sum_{K=0}^{\infty} a^K a^{n-K} u[n-K] \\ &= a^n \sum_{K=0}^{\infty} a^{-K} u[n-K] \quad n-K \geq 0 \\ &= a^n \sum_{K=0}^n a^{-K} u[n-K] \quad \text{for } n \geq 0 \quad K \leq n \\ &= a^n \sum_{K=0}^n a^{-K} u[n-K] \quad n < 0 \end{aligned}$$

$$f(n) = (n+1) a^n u[n]$$

$$f[n] = (n+1) a^n u[n] \text{ Ans.}$$

$$f(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) d\omega$$

$$= \frac{1}{2\pi} \cdot 2\omega_0$$

(91)

$$f(0) = \frac{\omega_0}{\pi}$$

$$f[n] = \frac{\sin \omega_0 n}{\pi n} \quad \text{for } n \neq 0$$

$$\frac{\omega_0}{\pi} \quad \text{for } n=0$$

Q. Two signal  $f[n]$  &  $h[n]$  are convolved to get the signal  $\delta$   $y[n]$ , the no. of nonzero samples in  $f[n]$  is 5 and no. of nonzero samples in  $h[n]$  is 3. the maximum possible sample values of  $f[n]$  &  $h[n]$  is  $L$ , &  $K$  respectively if D.T.F.T of  $y[n]$  is  $Y(\omega)$  what is maximum possible value of  $Y(0)$

~~sol.~~

$$[Y(0) = 5L \cdot 3K]$$

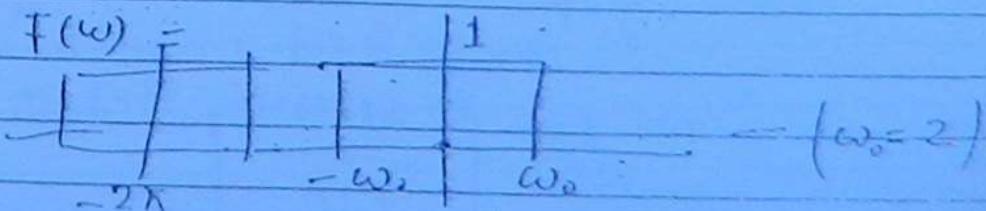
$$Y(\omega) = \sum_{n=-\infty}^{\infty} y[n] e^{-jn\omega}$$

$$[Y(0) = 15LK]$$

$$y(0) = \sum_{n=-\infty}^{\infty} y[n]$$

Find the energy of the signal  $f[n] = \frac{\sin \omega_0 n}{\pi n}, n \neq 0$

$$\frac{2}{\pi} \quad n=0$$



$$E = \sum_{n=-\infty}^{\infty} |f[n]|^2 = \frac{1}{\pi^2} + \sum_{n=-\infty, n \neq 0}^{\infty} \frac{\sin^2 \omega_0 n}{n^2 \pi^2}$$

Q. d DFT. if a signal  $f[n] \leftrightarrow 2 - 4e^{-j\omega}$   
Find the D.T.F.T.  $nf[n-2]$

(98)

$$f[n-2] \rightarrow e^{-j2\omega} [2 - 4e^{-j\omega}]$$

$$nf[n-2] \leftrightarrow j[-4je^{-j2\omega} + 4je^{+j\omega}]$$

$$\leftrightarrow j[-4je^{-2j\omega} + 12je^{-3j\omega}]$$

$$\leftrightarrow [4e^{-2j\omega} - 12e^{-3j\omega}]$$

$$\leftrightarrow 4e^{-2j\omega}[1 - 3e^{-j\omega}]$$

$\leftrightarrow$

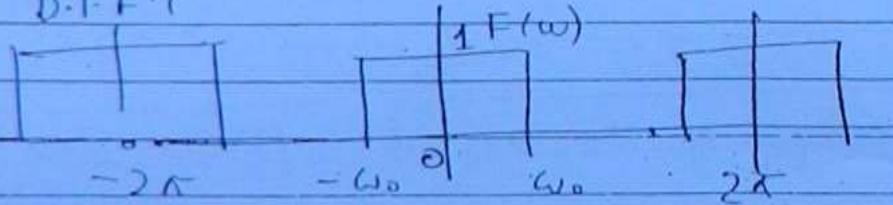
$$f[n] \leftrightarrow F(\omega)$$

$$\boxed{\sum_{n=-\infty}^{\infty} f[n] = F(0)}$$

$$\boxed{\int_{-\pi}^{\pi} F(\omega) d\omega = 2\pi f[0]}$$

Find the inverse D.T.F.T. of for following

D.T.F.T



$$f[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} 1 - e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[ \frac{e^{j\omega n}}{j\omega} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi j\omega} [2j \sin \omega n]$$

$$f[n] = \frac{1}{\pi} \sin \omega n = \frac{\omega_0}{\pi} \text{Sa}[\omega_0 n]$$

$$\text{Sin wave} \longleftrightarrow \frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$

(9)

$$\text{or } \frac{\pi}{j} \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)$$

$\omega < \infty$

$$\delta f[n] \longleftrightarrow F(\omega)$$

$$f(n - n_0) \longleftrightarrow e^{-jn_0\omega} F(\omega)$$

$$e^{j\omega_0 n} f[n] \longleftrightarrow F(\omega - \omega_0)$$

$$\text{Find DTFT of } f[n] = u[n] - u[n-9]$$

Ans

$$\begin{aligned} &= \sum_{n=0}^{\infty} f(n) e^{-j\omega n} \\ &= \frac{1 + e^{-j\omega 9}}{1 - e^{-j\omega}} = \frac{e^{-j\omega 9/2} 2j \sin \omega 9/2}{e^{-j\omega/2} 2j \sin \omega/2} \\ &= e^{-j\omega 9/2} \frac{\sin 9\omega/2}{\sin \omega/2} \end{aligned}$$

$$f(\omega) \Rightarrow e^{-j\omega 9/2} \frac{\sin 9\omega/2}{\sin \omega/2}$$

If D.T.F.T of  $f[n] \leftrightarrow F(\omega)$ , find I.D.T.F.T. of  $F(\omega - \bar{\omega})$

Ans

$$(-1)^n f[n] \leftrightarrow F(\omega - \bar{\omega})$$

$$(-1)^n e^{\bar{\omega}n} f[n] \leftrightarrow F(\omega - \bar{\omega}).$$

$$n f[n] \longleftrightarrow j \frac{dF(\omega)}{d\omega}$$

find DTFT of

$$n a^n u[n] \longleftrightarrow j \frac{d}{d\omega} \frac{1}{1 - ae^{-j\omega}}$$

$$f[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{j\omega n} d\omega$$

(100)

$$F(\omega) = \delta(\omega) \xrightarrow{-\pi < \omega < \pi} \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

$$f[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \delta(\omega) e^{j\omega n} d\omega$$

$$f[n] = \frac{1}{2\pi} \left[ \int_{-\pi}^{-\pi} + \int_{-\pi}^{-\pi+2\pi} + \int_{-\pi+2\pi}^{-\pi+4\pi} + \dots \right] = \dots$$

$$\frac{1}{2\pi} \longleftrightarrow \delta[\omega] \quad -\pi < \omega < \pi$$

$$1 \longleftrightarrow 2\pi \delta[\omega]$$

$$1 \longleftrightarrow 2\pi \sum_{k=-\infty}^{\infty} \delta[\omega - 2\pi k] \quad -\infty < \omega < \infty$$

D.D.T.F.T. of  $F(\omega) = 2\pi \delta(\omega - \omega_0)$

$$= f[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} 2\pi \delta(\omega - \omega_0) e^{j\omega n} d\omega \quad \omega - \omega_0 = w$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \delta(\omega) e^{j(\omega_0 + w)n} d\omega$$

$$f[n] = e^{j\omega_0 n} \cdot 1$$

$$e^{j\omega_0 n} \longleftrightarrow 2\pi \delta(\omega - \omega_0) \quad -\pi < \omega < \pi$$

$$e^{j\omega_0 n} \longleftrightarrow 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k) \quad -\infty < \omega < \infty$$

$$\cos(\omega_0 n) \longleftrightarrow \frac{1}{2} [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

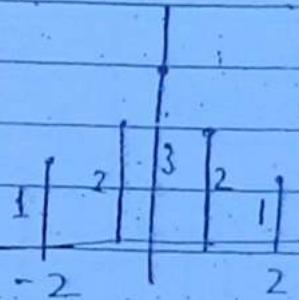
$$\text{or } \omega \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)$$

$$F(w) = e^{jwN} \cdot e^{-jw(2N+1)/2} \left[ e^{jw(2N+1)/2} - e^{-jw(2N+1)} \right]$$

(10)

$$= e^{-j\pi N} \cdot \frac{\sin(\omega(2N+1))}{2} + \frac{j \sin(\omega)}{2}$$

$$\boxed{F(w) = \frac{\sin(\omega(N+1/2))}{\sin(\omega/2)}}$$

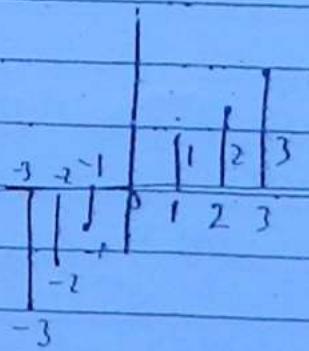


$$F(w) = \sum_{n=-2}^2 f(n) e^{-j\omega n}$$

$$= 3 + 2e^{-jw} + 2e^{jw} + 1e^{-j\omega \times 2} + 1e^{j\omega \times 2}$$

$$= 3 + 2\cos w + 2\cos 2w$$

$$\boxed{F(w) = 3 + 4\cos w + 2\cos 2w}$$



$$F(w) = 2j \sin w + 4j \sin 2w + 6j \sin 3w$$

Ans

D.T.F.T. is a continuous function of  $\omega$  repeating with a period of  $2\pi$ . i.e.

We generally consider D.T.F.T. in an interval of width of  $2\pi$  extending from  $-\pi$  to  $\pi$  for all analytical (mathematical) purposes.

$$f[n] = a^n u[n]$$

$$a^n u[n] \Leftrightarrow \left[ \frac{1}{1 - ae^{-j\omega}} \right]$$

191<1

102

$$\delta[n] \Leftrightarrow 1$$

$$a^{-n} u[-n-1] \Leftrightarrow \frac{1}{1 - ae^{-j\omega}} \quad |a| > 1$$

$$a^{|n|} = a^n u[n] + a^{-n} u[-n-1]$$

$$d^m \Leftrightarrow \frac{1}{1 - ae^{-j\omega}} + \frac{1}{(1 - \frac{1}{a} e^{-j\omega})} \quad 191 \quad |a| < 1$$

$$= \frac{a - e^{-j\omega} - (1 + ae^{-j\omega})a}{1 - a^2} = \frac{|1|}{|a|} > 1$$

$$= - \frac{(1 - ae^{-j\omega})(a - e^{-j\omega})}{e^{j\omega}(1 - a^2)} \quad (191 < 1),$$

$$= \frac{(e^{j\omega} - a)(a - e^{-j\omega})}{1 - a^2} = \frac{(1 - a^2)}{a^2 - 2a \cos \omega + 1} \quad m$$

191 < 1

$$F(w) = \sum_{n=-N}^N z^n e^{-jn\omega}$$

$$F(w) = \sum_{n=-N}^{2N} e^{-jn\omega(m-n)}$$

$$m=0$$

$\delta(nw)$   
 $\delta(mw)$

$\rightarrow Z^{-1} Z^m$

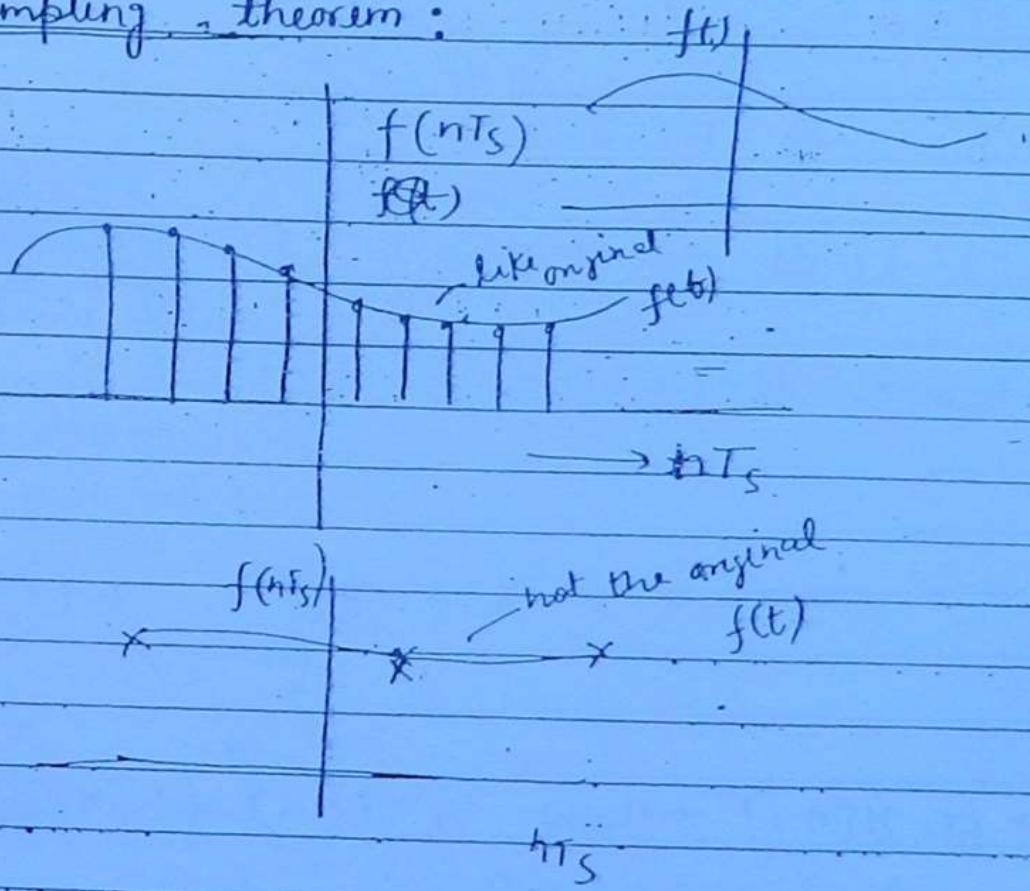
$$= e^{j\omega N} \cdot \left[ \frac{1 - e^{-j\omega(2N+1)}}{1 - e^{-j\omega}} \right]$$

$$F(w) = - \frac{e^{j\omega N} - e^{-j\omega N}}{1 - e^{-j\omega}}$$

Initial value theorem can be applied any  $F(z)$  there is no restriction.

- \* For application of Final value theorem all the poles of  $(z-1)F(z)$  or  $(1-z^{-1})F(z)$  must lie inside the unit circle (strictly i.e  $|z| \neq 1$ )  
~~Final value theorem~~ (103)

Sampling theorem:



11<sup>th</sup> NOV 10:

$$\underbrace{F(j\omega), F(e^{j\omega})}_{\text{discrete time F.T.}} \quad F(\omega) = \sum_{n=-\infty}^{\infty} f[n] e^{-j\omega n}$$

$F(j\omega)$   
 $F(e^{j\omega})$   
allowed omega corresponds  
to discrete angular frequency  
D.T.F.T.

$$Y(z) = \frac{1}{(1-2z^{-1})(1-3z^{-1})} = \frac{-\frac{1}{2}z^1 + \frac{3}{2}}{(1-2z^{-1})(1-3z^{-1})}$$

12123

$$\therefore y[n] = -2 \cdot 2^n u[n] + 3 \cdot 3^n u[n]$$

(104)

$$y[n] = 3^{n+1} u[n] - 2^{n+1} u[n]$$

Ans

Q. Property

$$f[n] \rightarrow F(z)$$

$$f[n] - f[n-1] \rightarrow (1-z^{-1}) F(z) \quad \text{ROC remains same except only exclude } z=0 \text{ from ROC.}$$

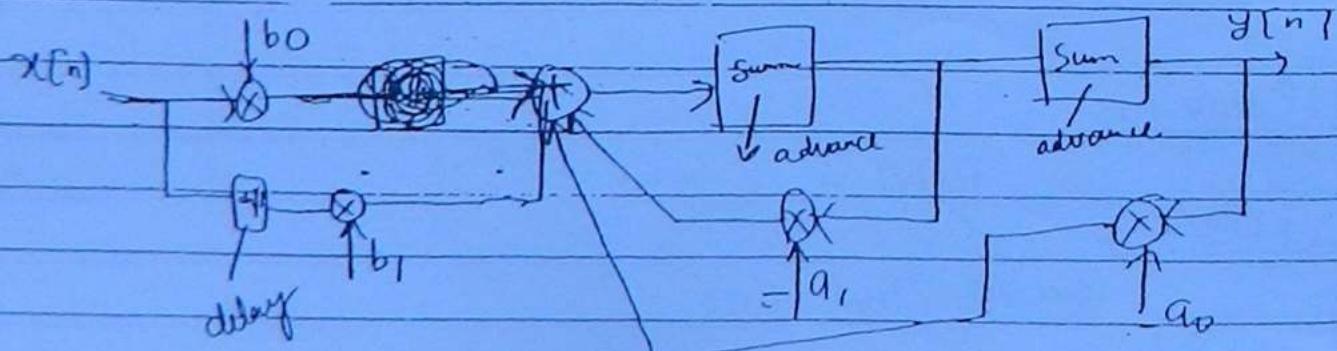
$$\sum_{k=-\infty}^n f[k] \leftrightarrow f[n] * u[n] \leftrightarrow F(z) \cdot U(z)$$

ROC intersection

of the two

$$\leftrightarrow \frac{1}{(1-z^{-1})} F(z)$$

$$a_2 y[n-2] + a_1 y[n-1] + a_0 y[n] = b_1 x[n-1] + b_0 x[n]$$



$$\left\{ \begin{array}{l} a^n u[n] \rightarrow \frac{1}{1-a z^{-1}} \\ -a^n u[n-1] \rightarrow \frac{1}{1-a z^{-1}} \end{array} \right\},$$

$$\frac{z}{(z-3)^2} \rightarrow \frac{1}{3} \cdot \frac{z/3}{(\frac{z}{3}+1)^2} \rightarrow \frac{-1}{3} \cdot \frac{z/3}{[(\frac{z}{3})-1]^2} \xrightarrow{\textcircled{P5}} \frac{-1}{3} \left(\frac{z}{3}\right)^2 n u[n]$$

$$\left\{ \begin{array}{l} n u[n] \rightarrow \frac{z}{(z-1)^2} \quad \text{ROC } |z| > 1 \\ a^n n u[n] \rightarrow \frac{z/a}{(z/a-1)^2} \quad \text{ROC } |z| > a \end{array} \right\}$$

$$h^2 u[n] \rightarrow \frac{z(z+1)}{(z-1)^3}$$

Q. Z-transform of a signal  $f[n]$  is defined as

$$F(z) = 2 - 4z^{-1} + 8z^{-2}$$

$$f[n] = \left\{ \begin{array}{l} 2, -4, 8 \\ \uparrow \end{array} \right\}$$

$$h f[n] = \left\{ \begin{array}{l} 0, -4, 16 \\ \uparrow \end{array} \right\}$$

now  $h f[n]$

$$F(z) = H(z) = -4z^{-1} + 16z^{-2} \text{ in}$$

$$-\frac{dF(z)}{dz}$$

$$-(8z^{-2})^{-2}$$

Convolution Property of z-transforms  $\rightarrow$

$$f[n] \rightarrow F(z)$$

$$h[n] \rightarrow H(z)$$

$$f[n] \otimes h[n] \rightarrow F(z) \cdot H(z)$$

Q. Find the response of a discrete time system with impulse response  $h[n] = 2^n u[n]$

$$f[n] = 3^n u[n]$$

Shifting

$$f[n] \rightarrow F(z)$$

$$f[n-h_0] \rightarrow z^{h_0} F(z)$$

(106)

ROC remains same the only  
if  $z=0$  or  $\infty$  can be  
excluded.

$$\text{Coswon } u[n] \rightarrow \frac{2z^2 - 2z \cos\omega_0}{z^2 - 2z \cos\omega_0} \quad |z| > 1$$

$$\text{Sinwon } u[n] \rightarrow \frac{z \sin\omega_0}{z^2 - 2z \cos\omega_0} \quad |z| > 1$$

$$-\text{Coswon } u[-n-1] \rightarrow \frac{2z^2 - 2z \cos\omega_0}{z^2 - 2z \cos\omega_0} \quad |z| < 1$$

$$-\text{Sinwon } u[-n-1] \rightarrow \frac{z \sin\omega_0}{z^2 - 2z \cos\omega_0} \quad |z| < 1$$

$$/a^n \text{ Coswon } u[n] \rightarrow \frac{2(z/a)^2 - 2z/a \cos\omega_0}{(z/a)^2 - 2z/a \cos\omega_0} \quad |z| > 1$$

Multiplication by  $(n)$ :

$$a^n n u[n] \rightarrow \frac{z^q}{(z-1)^2}$$

$$f[n] \rightarrow F(z)$$

$$\sqrt{n} f[n] \rightarrow -z \frac{dF(z)}{dz} \quad \text{ROC remains same}$$

$$\text{Find inverse } Z-T: F(z) = \frac{1}{(z-2)(z+3)^2} \quad \frac{1}{z-2} - \frac{1}{(z+3)^2}$$

$$F(z) = \frac{1}{2} \frac{1}{z-2} - \frac{1}{25} \frac{1}{z+3} + \frac{1}{5} \frac{1}{(z+3)^2}$$

$$\frac{3}{(2+3)} \rightarrow -\frac{1}{2} \left[ \frac{3}{(2+3)^2} - \frac{1}{2+3} \right] \quad F(z) = \frac{1}{25} \frac{z}{z-2} - \frac{1}{25} \frac{z}{z+3} - \frac{1}{5} \frac{z}{(z+3)^2}$$

$$\frac{1}{2+3} = \frac{1}{5+3} \quad = \frac{1}{25} 2^n u[n] - \frac{1}{25} (-3)^n u[n] - \frac{1}{5} n \frac{1}{4+3}$$

$$\frac{2}{(2+3)^2} \left( \frac{z}{z+3} \right)$$

$$|z| > 3 \quad \text{Ans} \frac{15}{4+3} \frac{(-3)^n}{u[n]}$$

ROC not given consider right

$$f(n) = \frac{1}{3} [2]^n u[n] - \frac{1}{3} (-1)^n u[n]$$

(07)

Right-sided  
 not stable  
 causal, DTFT  
 can't be defined

$$\text{or } -\frac{1}{3} [2]^n u[-n-1] + \frac{1}{3} (-1)^n u[-n-1]$$

|z| < 1

left-sided  
 not stable  
 not causal

$$\text{or } -\frac{1}{3} [2]^n u[-n-1] - \frac{1}{3} (-1)^n u[n]$$

DTFT not  
 def

$$1 < |z| < 2$$

two-sided, DTFT not def  
 not stable  
 not causal

Find the G.Z.T. of above signal if z-transform F(z)

Obtained for  $z = 3+j5$

$$|\Theta| = |z| = \sqrt{9+25} = \sqrt{34}$$

$$|z| > 2$$

$$f(n) = \frac{1}{3} 2^n u[n] - \frac{1}{3} (-1)^n u[n]$$

|z| > 2

Linearity:

$$a_1 f_1(n) + a_2 f_2(n) \rightarrow a_1 F_1(z) + a_2 F_2(z)$$

ROC is intersection  
of individual ROC's

$$@ a^n f_1(n) \rightarrow F(z/a) \text{ ROC } |\frac{z}{a}|$$

$$e^{j\omega_0 n} f(n) \rightarrow F(z/e^{j\omega_0}) \text{ ROC } |z| > 0$$

remains same  
because  $\left| \frac{z}{e^{j\omega_0}} \right| = |z|$

$$\frac{|z|}{|e^{j\omega_0}|} = |z|$$

if the DTFT for the previous question is defined  
for  $|b| < 1$

if it is defined then it will be stable.

(P8)

Inverse Z-transform :

$$f[n] = \frac{1}{2\pi j} \oint F(z) z^{n-1} dz$$

Q:

$$F(z) = \frac{z}{(z-2)(z+1)}$$

$$\text{Res}(z-2) \text{Res}(z+1)$$

$$= \frac{2}{z-2} + \frac{1}{z+1}$$

$$\frac{2}{z}$$

$$F(z) = \frac{z^{-1}}{1 - z^{-1} - 2z^{-2}} = \frac{z^{-1}}{2z^{-2} + z^{-1} - 1}$$

$$z^{-1}$$

$$\text{Res}(2z^{-2} + 2z^{-1} - z^{-1} - 1)$$

$$-z^{-1}$$

$$(2z^{-1} - 1)(z^{-1} + 1)$$

$$= \frac{z^{-1}}{(1 - 2z^{-1})(1 + z^{-1})} = \frac{1}{3(1 - 2z^{-1})^3(1 + z^{-1})}$$

$$f[n] = \begin{cases} 1 \\ 0 \end{cases} = 1 \quad \text{ROC entire } z\text{-plane}$$

(10g)

$$f[n] = \begin{cases} 1, 2, 3 \\ 0 \end{cases}, F(z) = 1 + 2z^{-1} + 3z^{-2} \quad \text{entire } z\text{-plane except } |z|=0$$

$$f[n] = \begin{cases} 1, 2, 3 \\ 0 \end{cases}, F(z) = 3 + 2z + 1z^2 \quad \text{entire } z\text{-plane except } |z|=\infty$$

$$f[n] = \begin{cases} 1, 2, 3 \\ 0 \end{cases} = z + 2 + 3z^{-1} \quad \begin{matrix} \text{ROC entire } z\text{-plane} \\ \text{except } |z|=0, \infty \end{matrix}$$

\* A discrete time signal - having finite no. of nonzero sample will have an ROC which is the entire  $z$ -plane except possibly  $\bullet r=0$ , or  $r=\infty$ ,  $\bullet$  both  $\circ r=0$  and  $\circ r=\infty$ .

\* If the ROC is to be outward directed, ROC must be extending till  $\bullet r=\infty$  & including  $\sigma=\infty$  circle.

$$Q. f[n] = b^n u[n]$$

$$= b^n u[n] + b^{-n} u[-n-1]$$

$$= \frac{z}{z-b} - \frac{z}{z-1/b}$$

$b > 1$

$0 < b < 1$

$$|z| > |b| \quad |z| < 1/|b| \quad \text{if } \rightarrow |b| < 1$$

[ROC  $|b| < |z| < 1/|b|$ ]

if  $|b| \geq 1$  then  $z$ -transform can't be defined.

\* Combination of left sided signal will have always a defined z-transform with an ROC which is inward directed bounded by the circle whose radius is the magnitude of the least pole.

(110)

Q.  $f[n] = (3/4)^n u[n] + (5/6)^n u[n] + \left(\frac{7}{8}\right)^n [-n-1] + \left(\frac{9}{10}\right)^n u[-n-1]$

$$F(z) = \frac{z}{z-3/4} + \frac{z}{z-5/6} - \frac{z}{z-7/8} - \frac{z}{z-9/10}$$

$$(5/6 < |z| < 7/8)$$

~~113~~ 5 <sup>112</sup>

~~6 <sup>113</sup> 5 <sup>112</sup>~~

$$\left(6^2\right)^{1/6} \left(5^3\right)^{1/6}$$

$$(3.6)^{1/6} (125)^{1/6}$$

\* Combination of right sided and left sided signal will have an ROC which is a finite circular strip bounded on the lower side by a circle whose radius is the magnitude of the greatest pole of all the right sided signals and on upper side by a circle whose radius is the magnitude of the least pole of all the left sided signal.

Find the z-transform of signal

$$f[n] = 2^n u[n] + 5^n u[n] - 2^n u[-n-1] - 5^n u[-n-1]$$

$$1/2 < |z| < 2 \quad \left\{ \begin{array}{l} \text{simultaneous} \\ \text{not possible} \end{array} \right\} \quad \left\{ \begin{array}{l} \frac{z}{z-2} + \frac{z}{z-5} + \frac{z}{z+2} + \frac{z}{z+5} \\ \text{not defined} \end{array} \right\}$$

of the impulse response of such a stable & causal system must lie inside the unit circle.

\* For a discrete time signal  $f[n]$ , fourier transform can be defined as  $F(\omega) = \sum_{n=-\infty}^{\infty} f[n] e^{-j\omega n}$ , this is defined as

discrete time Fourier transform or DTFT in short. III

④ DTFT of a discrete time signal  $F[n]$  is same as z-transform, with  $r=1$ .

⑤ So in the z-transform expression if we substitute  $r=e^{j\omega}$  then  $r=1$  it will result in discrete time Fourier transform. i.e. If we substitute  $r=e^{j\omega}$  in z-transform expression we get the DTFT provided ④ z-transform is defined for  $r=1$ i.e. ROC of the z-transform is including the unit circle.

⑥ DTFT is z-transform evaluated along the unit circle

$$\left(\frac{3}{2}\right)^n u[n] \rightarrow z \quad |z| > \frac{3}{2}$$

$$\left(\frac{3}{2}\right)^n u(n) + \left(\frac{5}{2}\right)^n u(n) \rightarrow \frac{z}{z - \frac{3}{2}} + \frac{1}{z - \frac{5}{2}} \quad \text{ROC } |z| > \frac{5}{2}$$

\* Combination of right sided non discrete time signal will always have a defined z-transform with an ROC which is outward directed bounded by a circle whose radius is the magnitude of the highest pole.

$$-2^n u[-n-1] \rightarrow \frac{z}{z-2} \quad |z| < 2$$

$$-5^n u[-n-1] - 2^n u[-n-1] \rightarrow \frac{z}{z-5} + \frac{2}{z-2}$$

S plane will be map to all the circles having radius greater than  $\omega$ , and jw axis of S plane will be map to a circle of  $\infty$  unit radius.

$$F(w) = \sum_{n=-\infty}^{\infty} f[n] e^{-jwn}$$

DFT

Discrete time fourier transform

(112)

- \* ROC of z-transform is a set of concentric circles, generally a circular strip, i.e. it is bounded by two circles.
- \* ROC of the z-transform can't include any poles but is bounded by the circle whose radius is the magnitude of pole.
- \* ROC of right sided signal is outward directed i.e. outside some circle.
- \* ROC of left sided signal is inward directed i.e. inside some circle.
- \* If a system with impulse response  $h[n]$  is to be stable condition:  $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$  and for this to be satisfied the region of convergence of the z-transform of the impulse response must be including unit circle.
- \* A system with impulse response  $h[n]$  is causal if  $h[n]=0$  for  $n<0$ . And in this case the region of convergence of z-transform of impulse response must be outward directed.
- \* If a system with impulse response  $h[n]$  is to both stable & causal ROC must be including the unit circle and must be outward directed and ROC can't include any poles.
- So for a system with impulse response  $h[n]$  to be stable and causal all the poles of its z-transform

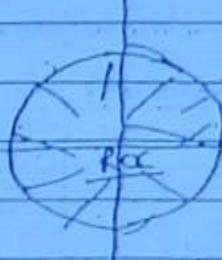
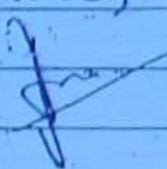
Find z-transform of  $f[n] = u[-n-1]$

$$F(z) = z + z^2 + \dots$$

1/3

$$F(z) = \frac{z}{(1-z)} \quad |z| < 1$$

$$F(z) = \frac{-1}{(1-z^{-1})} \quad |z| < 1 \text{ ROC, } r < 1$$



$r < 1$  ROC

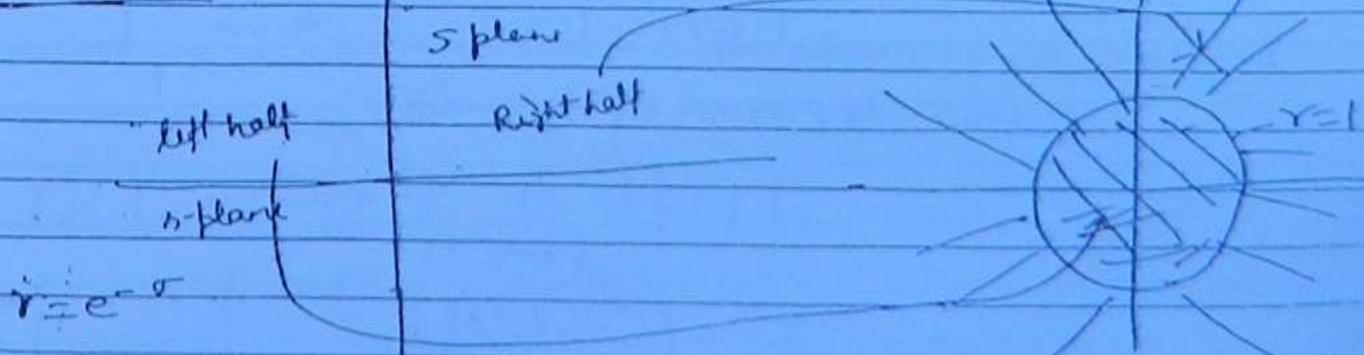
$$a^n u(n) = \frac{1}{(1-az^{-1})} \quad |az^{-1}| < 1 \quad \left|\frac{z}{a}\right| > 1$$
$$\quad \quad \quad |z| > |a| \quad |z| > |a|$$

$$-a^n u[-n-1] = \frac{1}{(1-az^{-1})} \quad |z| < |a| \quad \left|\frac{z}{a}\right| < 1$$
$$\quad \quad \quad |z| > |a|$$

$f[n] \rightarrow F(z)$  according to

$a^n f[n] \rightarrow F(z/a)$  ROC  $\rightarrow \left|\frac{z}{a}\right|$

2<sup>nd</sup> Norm to



left half of  $n$ -plane will be mapped to all circles having

radius  $0 \dots R \rightarrow \text{min max } y_0 \text{ is a tube } \pm R$

For a discrete time signal  $f[n]$ , z-transform is defined as  $F[z] = \sum_{n=-\infty}^{\infty} f[n] z^{-n}$ . (114)

where  $z$ -is complex variable defined at  $z=re^{j\omega}$ . This can be understood as Laplace transform a discrete time signal  $f[n]$ , the complex variable  $s=\sigma+j\omega$  from "rectangular form" is replaced by a complex variable  $z=re^{j\omega}$  in polar form. If the z-transform of the signal  $f[n]$  is to be defined, the condition is

$$\left| \sum_{n=-\infty}^{\infty} f[n] z^{-n} \right| < \infty$$

$$\left( \sum_{n=-\infty}^{\infty} |f[n]| r^{-n} \right) < \infty$$

$$\sum_{n=-\infty}^{\infty} |f[n]| r^{-n} < \infty$$

Based on the nature of given discrete time signal  $f[n]$  there will be a set of  $r$  values for which above condition is satisfied  $\Rightarrow$  this region of  $r$  values for which the z-transform of a signal  $f[n]$  is defined is called as region of convergence of z-transform and it is specified in terms  $r$  or  $|z|$  (ROC).

Find z-transform of

$$f[n] = \delta[n]$$

$$F(z) = 1$$

$$f[n] = u(n)$$

$$F(z) = \frac{1}{(1-z^{-1})} \quad \text{ROC } |z^{-1}| < 1$$

ROC  $\Rightarrow \{|z| > 1\}$

Calculate the final value of  $F(s) = \frac{s+2}{(s^2 + 3s + 2)}$

(115)

$$\lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} \frac{s(s+2)}{(s^2 + 3s + 2)}$$

$$= \cancel{\infty}$$

Z-transform:

$$F(z) = \sum_{n=-\infty}^{\infty} f[n] z^{-n}$$

$$z = e^{j\omega}$$

$$F(z) = \sum_{n=-\infty}^{\infty} f[n] e^{-jn\omega} e^{-j\omega n} \rightarrow S = (\sigma + j\omega)$$

$$= \sum_{n=-\infty}^{\infty} f[n] r^{-n} (e^{j\omega})^{-n}$$

$$= \sum_{n=-\infty}^{\infty} f[n] r^{-n} z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} f[n] (re^{j\omega})^{-n}$$

$$re^{j\omega} = z$$

$$F(z) = \sum_{n=-\infty}^{\infty} f[n] z^{-n}$$

for Z-transform to be defined

summation must be converging

$$\left| \sum_{n=-\infty}^{\infty} f[n] z^{-n} \right| < \infty$$

$$\sum_{n=-\infty}^{\infty} |f[n] z^{-n}| < \infty$$

$$\sum_{n=-\infty}^{\infty} |f[n] r^{-n}| < \infty \quad \text{for all } r > 0$$

when ever result comes as  $\rightarrow$  not defined  
 $\infty - \infty$  because  $\infty$  is a undefined quantity

Initial value theorem  $\rightarrow$

$$\text{Q16} \quad \lim_{t \rightarrow 0} f(t) = \lim_{S \rightarrow \infty} S F(S)$$

Initial value theorem	$f(0^+) = \lim_{S \rightarrow \infty} S F(S)$
--------------------------	---

Final value theorem	$\lim_{t \rightarrow \infty} f(t) = \lim_{S \rightarrow 0} S F(S)$
------------------------	--

$$F(S) = \frac{C\omega_0}{S + \omega_0^2}$$

$$| < f(t) < 1 |$$

④ Initial value theorem can be applied only if degree of numerator of L.T. is less than the degree of denominator.

$$F(s) = \frac{s+2}{s^2+3s+2} = \frac{A}{s+1} + \frac{B}{s+2}$$

I.L.T.  $\uparrow$   
 exists

$$F(s) = \frac{s^2+2s+3}{s^2+3s+2} = \frac{s^2}{s+1} + \frac{1}{s+2} + \frac{1-s}{s^2+3s+2}$$

Q17

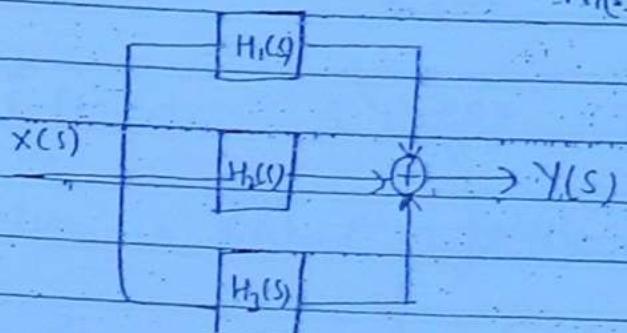
⑤ If the degree of numerator is greater than or equal to degree of denominator, we will have derivatives of impulses or impulses themselves in S.L.T. whose value is not known at  $t=0$  hence initial value theorem can't be applicable.

⑥ Final value theorem of L.T. can be applied only if all the poles of S.F(S) lie in left half of the S-plane (strictly left half i.e. excluding jωax)

## Parallel Representation.

$$H(s) = \frac{B \cdot A}{(s+a_1)} + \frac{B}{(s+B)} + \frac{C}{s^2+B^2}$$

(117)



$$H_1(s) + H_2(s) + H_3(s)$$

\*  $F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$   
 two sided L.T.  
 Bilateral L.T.

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$
 one sided L.T.  
 also called as unilateral L.T.

When analysis is started from  $t=0$  — (it may start before  $t=0$ )  
 In unilateral transform ROC does not play important role while it plays an important role in bilateral transform.

\* Unilateral transform for a signal  $f(t)$  is defined as

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt \rightarrow \text{also called as one sided L.T.}$$

L.T. and unilateral transform for any signal  $f(t)$ , is unique and hence region of convergence need not be specified to denote the uniqueness of L.T.

$$f(t) \leftrightarrow F(s) \quad [F(s) \text{ is unilateral transform of } f(t)]$$

$$f(t-t_0) u(t-t_0) \leftrightarrow F(s) e^{-s t_0}$$

$$\frac{df(t)}{dt} \rightarrow sF(s) - f(0^-)$$

start at  $t=0^-$

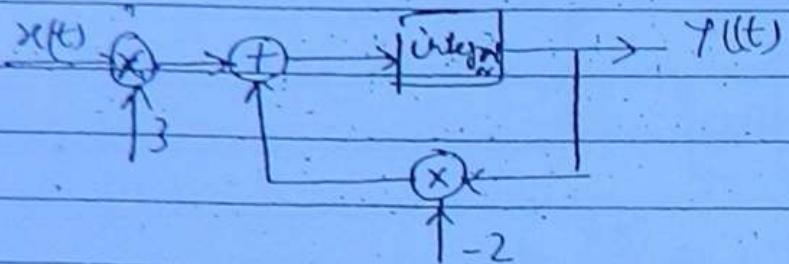
a analysis starts after  $t=0^-$  in unilateral transform

$$\frac{dy}{dt} + 2y(t) = 3x(t)$$

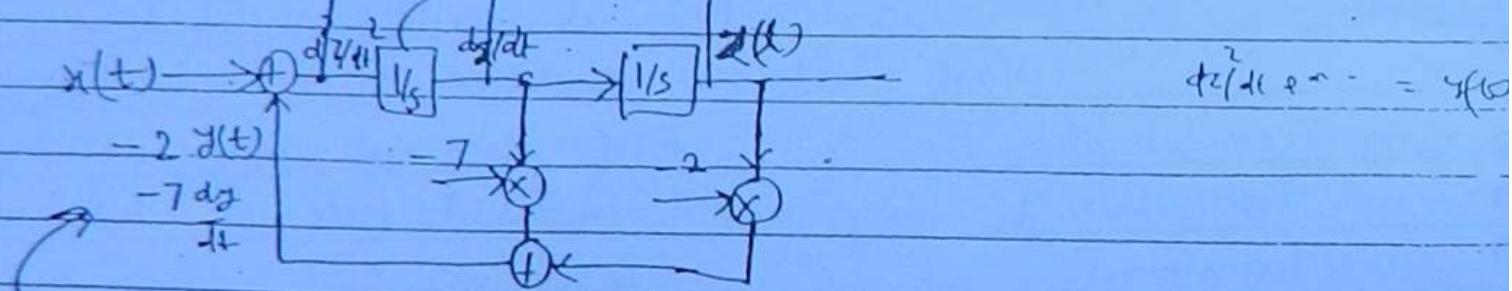
$$H(s) = \frac{3}{s+2}$$

$$\frac{dy}{dt} = -2y(t) + 3u(t).$$

$$h(t) = 3e^{-2t} u(t)$$



$$H(s) = \frac{s^2 + 3s + 2}{s^2 + 7s + 2} = \frac{1}{s^2 + 7s + 2} \cdot (s^2 + 3s + 2)$$



cascade Representation of LTI systems:

$$f(t) \leftrightarrow F(s)$$

$$\frac{d f(t)}{dt} \rightarrow sF(s) - f(0^-)$$

$$\frac{d^2 f(t)}{dt^2} \rightarrow s^2 F(s) - sf(0^-) - f'(0^-)$$

$$\int_{-\infty}^t f(\tau) d\tau \rightarrow \frac{1}{s} F(s)$$

~~assuming  $f(t) = 0$  for  $t < 0$~~

Q. A system is defined by the following differential eqn

$$\frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} + 10 y(t) = 2 \frac{dx}{dt} + 3x(t)$$

Is this linear T.I. system. Yes this is LTI system.

$$H(s) = \frac{2s+3}{(s-2)^2 + 9}$$

for  $h(t)$   
for it to be stable

$$H(s) = \frac{2[s-1] + 5}{(s-1)^2 + 9}$$

$$H(s) = \frac{2(s-1)}{(s-1)^2 + 9} + \frac{5}{(s-1)^2 + 9} \quad \sigma < 1$$

$$h(t) = \begin{cases} -2e^t \cos 3t u(-t) + \frac{5}{3} e^t \sin 3t u(-t) & t < 0 \\ 0 & t \geq 0 \end{cases}$$

Find impulse response of above system is to be causal

$$= 2e^t \cos 3t u(t) + \frac{5}{3} e^t \sin 3t u(t) \text{ Ans}$$

$\sigma > 1$

$$Q. h(t) = e^{-t} u(t), f(t) = \underline{e^{-3t} u(t)} + e^{-2t} u(-t)$$

$$H(s) = \frac{1}{s+1}, \quad F(s) = \left\{ \begin{array}{l} \frac{1}{s+3} + \frac{1}{s+2} \\ \end{array} \right\} \Rightarrow \textcircled{118}$$

$$\sigma > -1 \quad F(s) = \left[ \begin{array}{cc} \frac{1}{s+3} & - \frac{1}{s+2} \\ \end{array} \right] \quad -3 < \sigma < -2$$

$$Y(s) = H(s) F(s)$$

not defined for any common

$$\text{so } [Y(t) = 0 \text{ for all } t]$$

Convolution Property of L.T.

x

x

$$f(t) \longleftrightarrow F(s)$$

(1/9)

$$h(t) \longleftrightarrow H(s)$$

$$f(t) * h(t) \longleftrightarrow F(s) \cdot H(s)$$

Q: Find the response of an L.T.I system with impulse response,  $h(t) = e^{2t} u(t)$

for O/P  $f(t) = e^{3t} u(t)$

$$Y(s) = H(s) \cdot F(s) = \frac{1}{(s-2)(s-3)}$$

$$Y(s) = \frac{1}{s-3} - \frac{1}{s-2}$$

$$y(t) = e^{3t} u(t) - e^{2t} u(t) \rightarrow 3$$

$$y(t) = (e^{3t} - e^{2t}) u(t) \text{ Ans}$$

or

$$-e^{3t} u(-t) + e^{2t} u(t)$$

Q: Find the response of a system with impulse response  $h(t) = e^{-t} u(t)$ , for O/P  $f(t) = e^{-2|t|}$

$$Y(s) = \frac{1 - \frac{1}{s+1}}{(s+1)(s^2 - 4)} \quad -1 < s < 2$$

$$\frac{e^{-2t}}{s+2} \frac{1 - \frac{1}{s+2}}{(s+2)(s-2)}$$

$$Y(s) = -\frac{4}{(s+1)(s-2)(s+2)} \quad [-1 < s < 2]$$

$$\frac{e^{-2t} u(t)}{s+2} + \frac{e^{-2t} u(-t)}{s-2}$$

$$= \frac{4}{3(s+1)} - \frac{1}{3(s-2)} - \frac{1}{(s+2)} \quad -2 < s < 2$$

$$y(t) = \frac{4}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t) - e^{-2t} u(t), \text{ Ans}$$

$$\text{Find I.L.T. of } F(s) = \frac{\log s^2 + a^2}{(s^2 + b^2)} \quad (120)$$

$$= \log \frac{s^2 + a^2}{s^2 + b^2} = \log \frac{s^2 + b^2 + 2ab}{s^2 + b^2}$$

$$g(s) = \frac{dF(s)}{ds} = \frac{12s}{s^2 + a^2} = \frac{2as}{s^2 + b^2}$$

$$g(t) = [2 \cos at u(t) - 2 \sin bt u(t)]$$

$$f(t) = -\cancel{t} f(\cancel{t}) g(t)/t$$

$$f(t) = \cancel{t} \frac{2[\cos bt - \cos at]}{t} u(t) \quad \text{Ans}$$

$$t f(t) \rightarrow -\frac{d}{ds} F(s)$$

$$t f(t) \rightarrow -g(t)$$

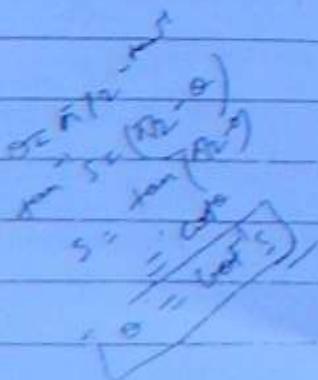
division by  $t$  property  $\cancel{t}$

$$\frac{f(t)}{t} \leftrightarrow \int_s^\infty F(s) ds$$

$$\text{Find L.T. } \frac{\sin t \cdot u(t)}{t} = S_2[t]$$

$$= \int_s^\infty \frac{1}{(s^2+1)} ds = [\tan^{-1}s]_s^\infty$$

$$L\left[\frac{\sin t}{t}\right] = \pi/2 - \tan^{-1}s \\ = \cot^{-1}s$$



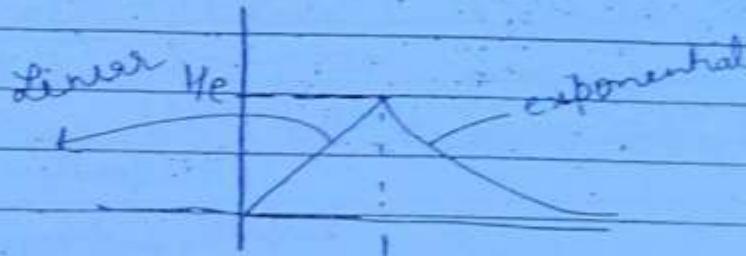
$$\frac{1}{(s-2)(s+3)^2} = \frac{1}{25(s-2)} + \frac{1}{25(s+3)} - \frac{1}{5(s+3)^2}$$

(72)

$$= \frac{1}{25} e^{2t} u(t) - \frac{1}{25} e^{-3t} u(t) - \frac{1}{5} e^{-3t} t u(t)$$

Ans /  
ROC ( $\sigma > 2$ )

Q. Find L.T. of the following signal



$$f(t) = \frac{1}{e} [1 + [u(t) - u(t-1)] + e^{-t} [u(t) - u(t-1)]$$

$$= \frac{1}{e} \left[ \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} \right] + e^{-1} \cdot \frac{1}{(s+1)}$$

~~Ans~~

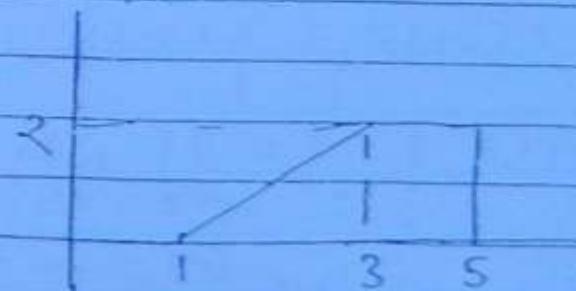
$\sigma > 0$  Ans

$$= \frac{1}{e} \left[ \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} \right] + \frac{e^{-(s+1)}}{(s+1)}$$

$$= \frac{1}{es^2} \left[ 1 - e^{-s} - se^{-s} \right] + \frac{e^{-(s+1)}}{s+1} \quad \sigma > 0$$

find L.T. of signal  $f(t)$  given below

Ans



$$= \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2} + \frac{2e^{-5s}}{s^2} - \frac{2e^{-5s}}{s} - \frac{2e^{-5s}}{s} - \frac{2(e^{-3s} - e^{-5s})}{s}$$

$$= (t-1) [u(t-1) - u(t-3)] - 2(u(t-3) - u(t-5))$$

Q. Find the I.L.T. of  $F(s) = \frac{e^{-(s+1)}}{s^2 - 2s + 5} = \frac{e^{-(s+1)}}{(s-1)^2 + 4}$

(122)

$$= \frac{e^{-(s+1)}}{(s-1)^2 + 4}$$

$$e^{-t} \times \frac{e^{-1} \cdot e^t \sin 2(t-1)}{2} : u(t-1) \text{ Ans}$$

$$\frac{1}{2} = \frac{1}{2} e^{(t-2)} \sin 2(t-1) u(t-1) \text{ Ans} \rightarrow 1, \text{ causal system}$$

$$= \frac{e^{-2} e^{t-2} \sin 2(t-1)}{2} u(-t+1) \text{ Ans, impulse response}$$

$$= -\frac{1}{2} e^{(t-2)} \sin 2(t-1) u(-t+1) \rightarrow \sigma < 1 (\text{excluding } \infty) \text{ corresponds to stable system's impulse response}$$

Multiplication by  $t$  property

$$f(t) \leftrightarrow F(s)$$

$$t^n f(t) \leftrightarrow (-1)^n \frac{d^n}{ds^n} F(s) \quad \text{ROC remains same}$$

$$t^n u(t) \leftrightarrow \frac{L_n}{s^{n+1}} = \frac{[n+1]}{s^{n+1}}$$

Q. Find I.L.T. of  $F(s) = \frac{1}{(s-2)(s+3)^2}$

$$\frac{1}{(s-2)} \frac{d^{n-1}}{ds^{n-1}} \frac{1}{s^2}$$

$$= \frac{A}{s-2} + \frac{B}{(s+3)} + \frac{C}{(s+3)^2} \quad -B = -1/25$$

$$1 = A(s+3)^2 + B(s-2)(s+3) + C(s-2)$$

$$A = 1/25, \quad 1 = \frac{9}{25} + \frac{B_0}{25} - 6B$$

$$C = -1/5$$

$$-6B = -\frac{6}{25} = -1/5$$

Linearity:

Time shifting property :-

(T23)

$$f(t) \leftrightarrow F(s)$$

$$f(t-t_0) \rightarrow e^{-s t_0} F(s)$$

$$f(t+t_0) \rightarrow e^{+s t_0} F(s)$$

$$\delta(t) \rightarrow 1$$

$$\delta(t-t_0) \rightarrow e^{-s t_0} \text{ entire } S\text{-plane excluding } s = 0$$

$$\delta(t+t_0) \rightarrow e^{+s t_0} \text{ entire } S\text{-plane excluding } s > 0$$

$$e^{-at} u(t) \rightarrow \frac{1}{s+a}, \sigma > -a$$

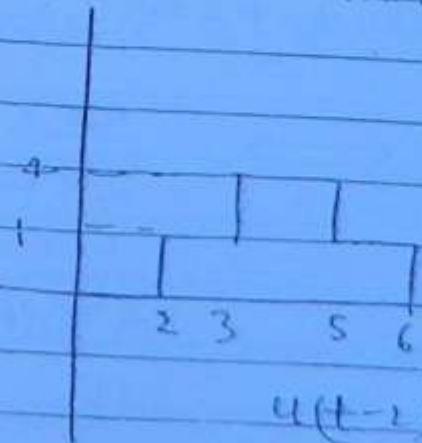
$$e^{-a(t-1)} u(t-1) \rightarrow \frac{1}{s+a} e^{-s}, \sigma > -a$$

∴ Shifting right  
will cancel the  $\sigma \rightarrow -\infty$  from  
ROC.

but  $-\infty$  is already excluded  
from ROC remains same

- \* If a signal  $f(t)$  to represent impulse response of a causal stable system ROC of its Laplace transform must be right sided extending till  $\infty$ , including  $\sigma = \infty$ .

- Q) Find L.T. of  $f(t)$  shown below.



$$u(t-2) + u(t-3) - 3 u(t-5)$$

$$\xrightarrow{\text{Laplace}} \frac{1}{s} [e^{-2s} + e^{-3s} - 3e^{-5s}]$$

For a given Laplace transform  $F(s)$ , we can define  
as many inverse L.T. as there are of ROC's.

(P24)

$$\text{No. of ILT's} = \text{No. of possible ROC}$$

Q How many inverse L.T. exist

$$\text{for } F(s) = \frac{5}{(s+1)^2 (s-2)(s+3)^3}$$

4 g.L.T. will exist

for a given  $F(s)$  if there are  $n$  poles  
then there will be  $(n-1)$  two sided g.L.T.  
&  $\frac{1}{2} \rightarrow$  one is right sided & one is left  
sided so total  $(n+1)$  g.L.T. will exist.

Properties

$$f_1(t) \rightarrow F_1(s)$$

$$f_2(t) \rightarrow F_2(s)$$

$$af_1(t) + bf_2(t) \rightarrow aF_1(s) + bF_2(s)$$

ROC  $\Phi$  ROC<sub>1</sub>  $\cap$  ROC<sub>2</sub>

(2)  $e^{j\omega_0 t} f(t) \rightarrow F(s + j\omega_0)$

ROC will remain same

(3)  $e^{-j\omega_0 t} f(t) \rightarrow F(s - j\omega_0)$  ROC will remain same

$$\frac{1}{2} e^{j\omega_0 t} u(t) + \frac{1}{2} e^{-j\omega_0 t} u(t)$$

$$\cos\omega_0 t u(t) \leftrightarrow \frac{s}{s^2 + \omega_0^2} \quad \sigma > 0$$

$$\sin\omega_0 t u(t) \leftrightarrow \frac{\omega_0}{s^2 + \omega_0^2} \quad \sigma > 0$$

$$f(t) = \frac{1}{2\pi j} \int_{-\sigma-j\infty}^{\sigma+j\infty} F(s) e^{st} ds$$

Find inverse Laplace transform of  $F(s) = \frac{1}{(s^2 + 5s + 6)}$

(125)

$$= \frac{1}{(s+3)(s+2)}$$

$$= \frac{1}{s+2} - \frac{1}{s+3}$$

$$= e^{-2t} u(t) - e^{-3t} u(t)$$

Ans  $\sigma > -2$

$$- e^{-2t} u(-t) + e^{-3t} u(-t)$$

$\sigma < -3$

Ans

$$\text{or } - e^{-2t} u(t) - e^{-3t} u(t)$$

ROC

$$-3 < \sigma < -2$$

Q. Find I.L.T. of the above  $F(s)$  if  $s = -\frac{1}{2} + j\frac{3}{2}$   
 is in ROC. i.e L.T. is defined

$$f(t) = e^{-2t} u(t) - e^{-3t} u(t) \text{ from}$$

$(\sigma > -2)$

\* at  $s = -\frac{1}{2} + j\frac{3}{2}$

(\*) at  $s = -\frac{1}{2} + j\frac{3}{2}$  for L.T. to be defined  
 this must be present in  
 ROC.

Q.  $f(t) \rightarrow F(s)$

126

a signal  $f(t)$  has Laplace transform  $F(s)$  with

exactly two poles at  $s = -2$  &  $s = +1$  another signal  $g(t)$  is defined as  $g(t) = e^{+3t} \cdot f(t)$ , if  $g(t)$  can represent impulse response of a stable system  $f(t)$  is

$\downarrow$  (unstable)

$\downarrow e^{3t} f(t) \rightarrow$  left sided Am

Newpole 1      Newpole -5

$-2 \rightarrow e^{-3t} f(t)$

Right sided if  $(e^{-3t} f(t))$  is stable

Q. Repeat the above question if  $g(t)$  is defined as  
 $g(t) = e^{-t} f(t)$

(a)  $\downarrow$  Newpole -3      0  
 $g(t)$  can't define impulse response of a stable system

B

$$F(\sigma + j\omega) = F(s) = F.T. [f(t) e^{-\sigma t}]$$

$$f(t) e^{-\sigma t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\sigma + j\omega) e^{j\omega t} d\omega$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{st} ds$$

$$f(t) = \frac{-j}{2\pi} \int_{-\infty}^{\infty} F(s) e^{st} ds$$

$$\boxed{f(t) = -j \int_{-\infty}^{\infty} F(s) e^{st} ds}$$

(\*) Combination of right sided signal & left sided signal which can be called as two sided signal & will have finite strip in ROC bounded on the lower side by the greatest poles of all right sided signal and on the higher side by least poles all the left sided signals.

(127)

$$e^{2t} u(t) + e^{5t} u(t) + e^{-2t} u(-t) + e^{-5t} u(-t)$$

$$\frac{1}{(s-2)} + \frac{1}{(s-5)} + -\frac{1}{(s+2)} + -\frac{1}{s+5}$$

$\sigma > 2$        $\sigma > 5$        $\sigma < -2$        $\sigma < -5$

- does not exist because no common values of  $\sigma$  for which all Laplace transform exist.

Laplace transform of a dc signal  $f(t)=1$

$$= \int_{-\infty}^{\infty} |e^{-\sigma t}| dt \quad (\infty \rightarrow \text{not possible})$$

this condition will not be satisfied for any  $\sigma$ . therefore Laplace transform does not exist

$\text{sgn}(t) \rightarrow$  not defined

$$f(t) \cdot e^{-bt} = e^{-bt} u(t) + e^{bt} u(t)$$

$$\text{L.T.} = \frac{1}{s+b} - \frac{1}{(s-b)} = \frac{-2b}{(s+b)(s-b)}$$

$\sigma > -b$        $\sigma < b$        $= \frac{2b}{(b^2 - s^2)}$

$$\text{ROC} \rightarrow -b < \sigma < b$$

$b > 0$  thus it is possible.

$$e^{at} [u(t) - u(t-1)] = \frac{e^{-as}}{(s-a)} - \frac{e^{-as}}{(s-a)} \rightarrow$$

any pole, so all the poles of laplace transform of impulse response of such a causal & stable system must be lying on the left of s-plane.

$$e^{2t} u(t) = \frac{1}{(s-2)}$$

(K28)

$$e^{2t} u(t) + e^{5t} u(t) \rightarrow \frac{1}{(s-2)} + \frac{1}{(s-5)} \quad \text{ROC } \sigma > 5$$

\* Combination of any no. of right sided signal will always have defined laplace transform, which is sum of all the individual laplace transform with an ROC which is right sided bounded by greatest among all the poles.

$$-e^{2t} u(-t) = \frac{1}{(s-2)} \quad \boxed{\text{Re}(s) < 2} \quad \text{ROC } \sigma < 2 \quad \text{ROC}$$

\* Any combination of left sided signals will have a defined laplace transform, which is sum of all individual laplace transforms with an ROC which left sided bounded by least among all the poles.

$$\textcircled{3} \quad f(t) = e^{-2t} u(t) + e^{-5t} u(t) + e^{2t} u(t) + e^{5t} u(t)$$

$$\frac{1}{s+2} + \frac{1}{s+5} - \frac{1}{(s-2)} - \frac{1}{(s-5)} \quad \text{Atm} \\ \text{Re}(s) > -2 \quad \text{Re}(s) > -5 \quad \text{Re}(s) < 2 \quad \text{Re}(s) < 5$$

$$s > -2, \quad s < 2$$

$$-2 < \text{Re}(s) < 2$$

$$-2 < \sigma < 2$$

if it is causal  $h(t) = 0 \quad t < 0$

right sided signal

(729)

and laplace transform for right sided signal is  
right has ROC which should be right sided.

(\*) If system is both stable & causal then ROC must be right sided & include  $\sigma=0$  line and it can't include any poles so for system to be causal & stable all the poles must lie on left half of S plane ( $\because$  ROC will be right side to poles having highest  $\sigma$ ).

(\*) ROC of left sided signal is also left sided

(\*) if F.T. of a signal is to be defined laplace transform of corresponding must be having an ROC including  $\sigma=0$  line or jw axis and corresponding F.T. can be obtained by replacing s in laplace transform expression by jw.

(\*) A system with impulse response  $h(t)$  is called as stable system if  $\int_{-\infty}^{\infty} |h(t)| dt < \infty$  which is the same

condition for the F.T. of  $h(t)$  to be defined so if a system with impulse response  $h(t)$  is to be a stable system it must be having a laplace transform with its ROC including  $\sigma=0$  line

(\*) If a system with impulse response  $h(t)$  is to be causal  $h(t) = 0 \quad t < 0$  i.e.  $h(t)$  must be generally right sided. So if a system with impulse response  $h(t)$  is to be causal its laplace transform must be having an ROC which is right sided extending till  $\sigma = \infty$ .

(\*) If a system with impulse response is to be both causal and stable ROC must be right sided including  $\sigma = 0$ . i.e.  $\sigma = 0$  is included in ROC.

Calculate Laplace T.  $\frac{-u(t)}{s}$  =  $H(s)$  ROC  $\sigma < 0$

(~~stable~~)

$A(t) \rightarrow F(s) \quad \text{Re}\{s\} > 0$

$e^{at} f(t) \rightarrow F(s+a) \quad \text{Re}\{s-a\} > 0$

(139)

$e^{-at} u(t) \leftrightarrow \frac{1}{(s+a)} \quad \text{Re}\{s+a\} > 0$   
 $\sigma > -a$

$e^{at} u(t) \rightarrow \frac{1}{(s-a)} \quad \sigma > a$   
 $\text{Re}\{s-a\} > 0$

$-u(t) \rightarrow \frac{1}{s} \quad \text{Re}\{s\} < 0$

$-e^{-at} u(t) \rightarrow \frac{1}{(s+a)} \quad \text{Re}\{s+a\} < 0 \quad \text{ROC } \sigma < -a$

$-e^{at} u(t) \leftrightarrow \frac{1}{(s-a)} \quad \text{Re}\{s-a\} < 0$   
 $a < \sigma$

- ④ For a system having impulse response  $h(t)$ , and having laplace transform  $H(s)$ , for system to be stable, ROC of  $H(s)$  must include  $\sigma = 0$  line for stability

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

this is also condition for, F.T. to exist and from laplace transform we can get F.T. only if its ROC includes  $\sigma = 0$  line.

so if it includes  $\sigma = 0$  in its ROC, then F.T. exists and system will be stable

F.T. means which can be directly find by integration

transform is defined. This region of  $\sigma$  value is called as region of convergence [ROC].

We specify ROC for the Laplace transform in terms of  $\sigma$  or real part of  $s$ .

(3)

Find Laplace T.  $f(t) = \delta(t)$

$$L[\delta(t)] =$$

$$\int_{-\infty}^{\infty} |\delta(t)e^{-\sigma t}| dt < \infty$$

$$\int_{-\infty}^{\infty} |\delta(t)e^{-\sigma t}| dt < \infty$$

$$\text{for all } \int_{-\infty}^{\infty} \delta(t) dt = 1 < \infty$$

for all values of  $\sigma$  Laplace transform will be defined.

$$L[\delta(t)] = 1 \quad \text{ROC} \rightarrow \text{all values of } \sigma$$

Laplace transform of  $u(t)$ .

$$\int_{-\infty}^{\infty} |u(t)e^{-\sigma t}| < \infty$$

$$\int_{-\infty}^{\infty} |u(t)e^{-\sigma t}| < \infty$$

$$\text{for } \sigma > 0$$

$$\int_0^{\infty} |e^{-\sigma t}| < \infty$$

for  $\sigma > 0 \rightarrow$  this integral converges

$$L[u(t)] = \int_0^{\infty} e^{-st} dt = \frac{1}{s} \text{ for } s > 0$$

but  $F(s)$  must be converging.

$\int_{-\infty}^{\infty} f(t) e^{-st} dt$ , F.T.  $\frac{1}{s-a}$   
 $\int_{-\infty}^{\infty} f(t) e^{-st} dt$  can't be defined

$$|F(s)| < \infty$$

(132)

$$\left| \int_{-\infty}^{\infty} f(t) e^{-st} dt \right| < \infty \rightarrow (\sigma + j\omega)$$

$$\int_{-\infty}^{\infty} |f(t)e^{-\sigma t}| dt < \infty$$

↓ condition to find ROC.

$\sigma$  is real part of  $s$

$\text{Re}\{s\} \rightarrow$  so ROC is defined in terms of  $\text{Re}\{s\}$ .

① For a signal  $f(t)$  Laplace transform is defined

$$a = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

where  $s$  is a complex variable and is equal  $= (\sigma + j\omega)$

so if we substitute  $\sigma = 0$  in laplace transform form

i.e., is equivalent to F.T. or laplace transform  
is more general form of F.T.

② We can also understand laplace transform as  
 $F[f(t)e^{-\sigma t}]$  where  $\sigma$  is called damping frequency factor.

③ For a laplace transform to be defined, laplace  
transform integral must be converging  
i.e.

$$\int_{-\infty}^{\infty} |f(t)e^{-st}| dt < \infty$$

$$\int_{-\infty}^{\infty} |f(t)e^{-\sigma t}| dt < \infty \quad (\because |e^{-\sigma t}| =$$

So based on the nature of the given  
signal  $f(t)$ , there will be a region of  $\sigma$ -values  
for which the integral is converging and called

Q. Repeat the previous problem for i/p PSD

133

$$PSD_{ff} = K$$

$$= \frac{1}{2\pi} K \int_{-\infty}^{\infty} \frac{1}{4\pi\omega^2} d\omega$$

(133)

$$P_y [f_s g] = K/2$$

$$\text{input power} = \frac{1}{2\pi} \int_{-\infty}^{\infty} K d\omega$$

$$P_i = \infty$$

energy

if  $f(t), g(t)$  are complex valued signal

$$R_{fg}(\tau) = \int_{-\infty}^{\infty} f(t) g^*(t - \tau) dt$$

$$R_{ff}(\tau) = \int_{-\infty}^{\infty} f(t) f^*(t + \tau) dt$$

if  $f(t) & g(t)$  are complex valued power signal

$$R_{fg}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) g^*(t - \tau) dt$$

$$R_{ff}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) f^*(t - \tau) dt$$

$$R_{ff}(T) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) f^*(t - \tau) dt$$

periodic with period T

Laplace transform :-

we can choose for a  $f(t)$  if F.T. is not defined  
by multiplying a exponential  
 $e^{-st}$ ,  $f(t)e^{-st}$  have F.T.

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-(s+is\omega)} dt = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

Q. Calculate the PSD of a signal  $f(t) = A \sin(\omega_0 t + \phi)$

$$R(\tau) = \frac{A^2}{2} \cos(\omega_0 \tau)$$

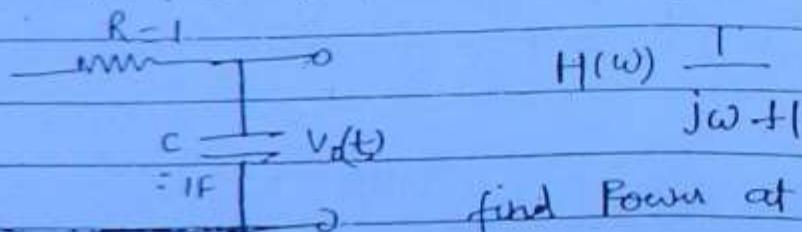
(134)

$$F[R(\tau)] = \frac{A^2}{2} [\pi \delta(\omega - \omega_0) + \pi \cdot s(\omega + \omega_0)]$$

$$\begin{aligned} \text{Total power} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} (\text{PSD}) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{A^2}{2} [\pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)] d\omega \\ &= \frac{A^2}{4} [1+1] \\ &= A^2/2 \end{aligned}$$

Q. RC system given below is given an i/p signal whose power spectral density is known to be

$$\text{PSD}_f = \pi [\delta(\omega-1) + \delta(\omega+1)]$$



find Power at o/p of system

$$P = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{PSD}_f \cdot |H(\omega)|^2 d\omega$$

$$\text{PSD}_y = \text{PSD}_f |H(\omega)|^2$$

$$P = \frac{1}{2\pi} \int_{-\infty}^{\infty} \pi [\delta(\omega-1) + \delta(\omega+1)] \frac{1}{(\omega^2 + 1)} d\omega$$

$$= \frac{1}{2} \left[ \int_{-\infty}^{\infty} \delta(\omega-1) \frac{1}{2} d\omega + \int_{-\infty}^{\infty} \delta(\omega+1) \frac{1}{2} d\omega \right]$$

$$P = \frac{1}{2} \left[ \frac{1}{2} + \frac{1}{2} \right] = 1/2 \text{ watt}$$

$$f(t) = A \cos(\omega_0 t + \phi) \rightarrow \text{Autocorrelation function.}$$

(135)

$$\begin{aligned} &= \frac{1}{(2\pi/\omega_0)} \int_{-\pi/\omega_0}^{\pi/\omega_0} A^2 \cos(\omega_0 t + \phi) \cdot \cos[\omega_0 t + \phi - \omega_0 \tau] dt \\ &= \frac{A^2 \omega_0}{2\pi} \left[ \int_{-\pi/\omega_0}^{\pi/\omega_0} [\cos^2(\omega_0 t + \phi) \cos(\omega_0 \tau) - \cos(\omega_0 t + \phi) \sin(\omega_0 \tau) : \sin(\omega_0 \tau)] dt \right] \\ &= \frac{A^2 \omega_0}{2\pi} \left[ \int_{-\pi/\omega_0}^{\pi/\omega_0} \cos^2(\omega_0 t + \phi) \cos(\omega_0 \tau) dt \right] \\ &= \frac{A^2 \cos(\omega_0 \tau)}{2\pi} \left[ \frac{t + \frac{\pi}{2} + \frac{\sin 2(\omega_0 t + \phi)}{2\omega_0}}{\omega_0} \right]_{-\pi/\omega_0}^{\pi/\omega_0} \\ &= \frac{A^2 \cos(\omega_0 \tau)}{4\pi} \cdot \frac{4\pi}{2} \times \frac{\omega_0}{\omega_0} \end{aligned}$$

$$R(\tau) = \frac{A^2 \cos(\omega_0 \tau)}{2}$$

for.  $A \sin(\omega_0 t + \phi)$

$$\therefore R_{ff}(\tau) = \frac{A^2}{2} \sin(\omega_0 \tau)$$

Properties of Autocorrelation  $\rightarrow$

$$(1) R(\tau) = R(-\tau) \text{ - even}$$

$$(2) R(0) \geq R(\tau)$$

(3)  $R(0)$  - Power of the signal

(4)  $F[R(\tau)] \rightarrow \text{PSD of } f(t)$

(5)  $R(\tau)$  is also periodic if  $f(t)$  is periodic  
and period will be same as  $f(t)$

Calculate

Property of Autocorrelation function →  
 $R(\tau)$  is even  
 $R(\tau) = R(-\tau)$ .

(136)

②  $R(0) > R(\tau)$

③  $R(0) = E_f$

④  $R(\tau) = 0$

$\tau \rightarrow \infty$

⑤  $F[R(\tau)] = ESD_f = F(\omega) F^*(\omega)$

⑥  $R_{fg}(\tau) \Rightarrow F(\omega) G^*(\omega)$

⑦  $R_{fg}(\tau) = R_{gf}(-\tau)$

⑧  $R_{gf}(\tau) = R_{fg}(-\tau)$

⑨ When  $f(t), g(t)$  are power signal

$$R_{fg}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) g(t-\tau) dt$$

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t) f(t-\tau) dt$$

for periodic signal

$$R_{ff}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) f(t-\tau) dt$$

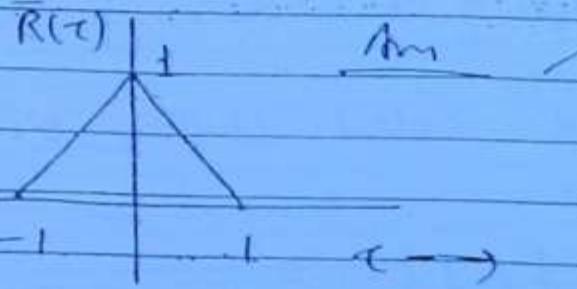
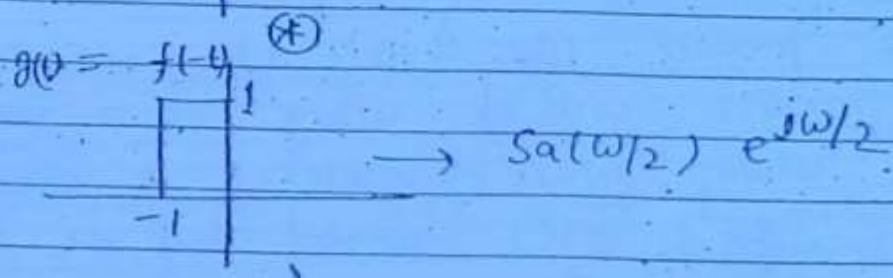
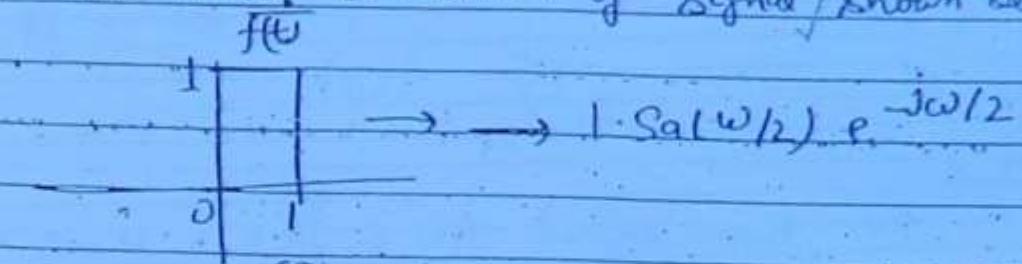
+ ip period of periodic signal

$$R(\tau) = \int_{-\infty}^{\infty} f(t) f(t-\tau) dt$$

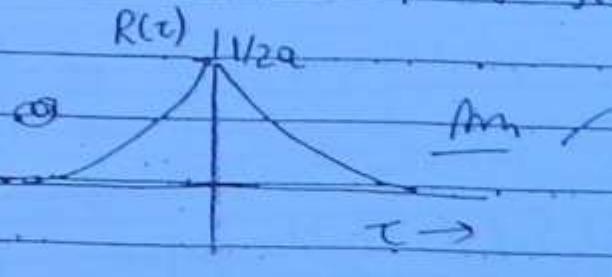
$$R(\tau) = [f(t) \otimes f(-t)]_{t=\tau}$$

(137)

Q. Find the auto-correlation of signal shown below.



Q. Find auto correlation function of  $f(t) = e^{-at} u(t)$



$$f(-t) = e^{at} u(t)$$

$$F[e^{-at} \otimes e^{at} u(t)] = \frac{1}{(a-j\omega)} \cdot \frac{1}{(a+j\omega)}$$

$$= \frac{1}{(a^2 + \omega^2)}$$

$$R(\tau) \rightarrow \frac{e^{-a|\tau|}}{2a} \rightarrow R(\tau) \Rightarrow \frac{e^{-a|\tau|}}{2a} \Delta m$$

$$-2aw_c = \ln 0.1$$

$$2aw_c = \ln 10$$

$$\boxed{w_c = \frac{\ln 10}{2a}} \quad \text{rad/sec}$$

(138)

Correlation  $\rightarrow$

for two signal  $f(t)$  &  $g(t)$ , if we take area under product  $f(t) \cdot g(t-\tau)$  will be larger or smaller will be acting as a major measure of relation b/w  $f(t)$  &  $g(t-\tau)$  & correlation of the signal  $f(t)$  &  $g(t)$  denoted by the symbol

$$R_{fg}(\tau) = \int_{-\infty}^{\infty} f(t) g(t-\tau) dt$$

$\tau \rightarrow$  can be called shifting or searching parameter

$$R_{fg}(\tau) = [f(t) \otimes g(-t)]_{(t=\tau)} \quad f(\tau) g(-\tau + \tau)$$

$$g(-\tau)$$

$$f(\tau) g(\tau)$$

$$f(t) * g(-t)$$

$$* R(\tau) = \int_{-\infty}^{\infty} f(t) f(t-\tau) dt$$

The above correlation integral

The cross correlation of  $f(t)$  &  $g(t)$  defined above is mathematically similar in it's evolution to the convolution of signal  $f(t)$  &  $g(t)$ .

If  $g(t)$  is happen to an even signal, functionally crosscorrelation & convolution will be same.

The above correlation if  $g(t) = f(t)$  we can defined it as autocorrelation about i.e., in general  $R(t)$

Q. An ideal L.P.F. is given an i/p  $f(t) = \frac{2a}{a^2+t^2}$  if

the response is to have 90% of the i/p Find B.W L.P.F

$$f_p = \frac{2a}{a^2}$$

$$f(t) = \frac{2a}{a^2+t^2}$$

(T39)

$$\frac{1}{(a+j\omega)}$$

8

$$e^{-at} u(t) \oplus e^{at} u(t) \rightarrow \frac{1}{(a+j\omega)^2}$$

$$e^{-at} u(t) \oplus e^{at} u(t) \rightarrow \frac{1}{(a+j\omega)} \cdot \frac{1}{(a-j\omega)}$$

$$\frac{1}{a^2+\omega^2}$$

$$e^{-at} u(t) \rightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+\omega^2} \xrightarrow{\text{F.T.}} 2\pi e^{-a|\omega|}$$

$$ESDf = |2\pi e^{-a|\omega|}|^2$$

$$= 4\pi^2 e^{-2a|\omega|}$$

$$\frac{4\pi^2}{2\pi} \int_{-\infty}^{\infty} e^{-2a|\omega|} d\omega = 2\pi \left[ \left\{ \frac{1}{2a} (1) \right\} + 1 \right]$$

$$E_f = 2\pi$$

$$\frac{2\pi}{a} \times 0.9 = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} 4\pi^2 e^{-2a|\omega|} d\omega$$

$$\frac{0.9}{a} = \frac{1}{2a} [1 - e^{-2a\omega_c}] + \frac{1 - e^{-2a\omega_c}}{2a}$$

$$1.8 = 2 - 2e^{-2a\omega_c} - 2e^{-2a\omega_c} = 0.2$$

\* When the two signal are multiplied the B.W. of resultant signal is equal to sum of B.W. of individual signal being multiplied.

(140)

$$ESD_f = |F(\omega)|^2 \quad f(t) \quad \cdot \quad h(t) \quad Y(t) = f(t) \otimes h(t)$$

$$E_f = \int_{-\infty}^{\infty} f^2(t) dt \quad ESD_y = |Y(\omega)|^2 = |F(\omega)|^2 \cdot |H(\omega)|^2$$

$$E_f = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \quad ESD_y = ESD_f \cdot |H(\omega)|^2$$

not given assume ideal

\* Q An ideal D.L.P.F. with cutoff frequency 1 rad/sec is given an i/p  $e^{-t} u(t)$ , calculate energy at response of the system

$$\left[ \text{energy at i/p} \right] = 1/2 J \quad |F(\omega)|^2 = \frac{1}{1+\omega^2} = ESD_f$$

$$ESD_y = ESD_f \cdot |H(\omega)|^2$$

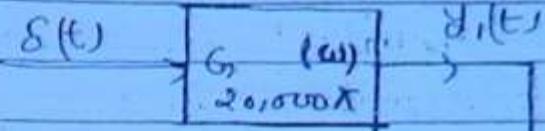
$$ESD_y = \frac{1}{1+\omega^2} \cdot 1 \quad \text{for } -1 \leq \omega \leq 1$$

$$\begin{aligned} E_y &= \frac{1}{2\pi} \int_{-\infty}^{\infty} ESD_y d\omega \\ &= \frac{1}{2\pi} \int_{-1}^{1} \frac{1}{1+\omega^2} d\omega \end{aligned}$$

$$= \frac{1}{2\pi} \left[ \tan^{-1}\omega \right]_{-1}^1$$

$$E_y = \frac{1}{2\pi} \cdot \frac{\pi/2}{4} = \frac{1}{4} J$$

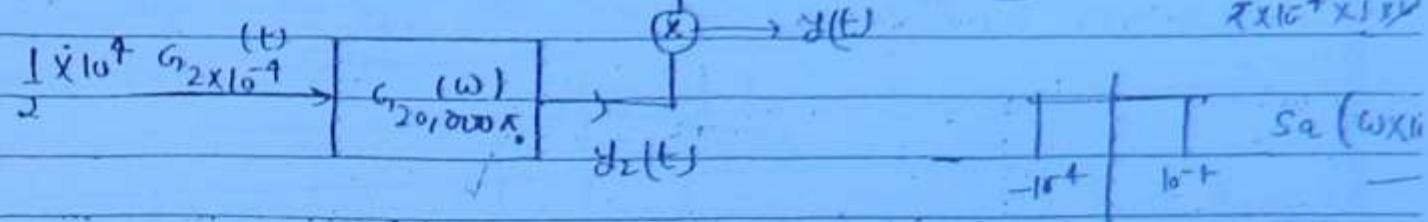
Ans



$$G_1(\omega) = 20,000\pi$$

$y_1(t)$

(1.4)



$$Y_1(\omega) = G_2(\omega) /$$

$$Y_2(\omega) = \text{Sa}(\omega \times 10^{-4}) \times G_2(\omega)$$

-  $\text{Sa}(\omega \times 10^{-4})$  from  $-10,000\pi \rightarrow 10,000\pi$

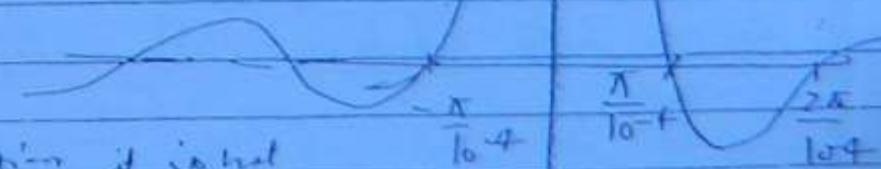
$$\text{B.W. } Y_1(t) = 10,000\pi \text{ rad/sec}$$

$$= 10,000\pi \div 5000; 5000 \text{ rad/s}$$

$$\text{B.W. } Y_2(t) = 10,000\pi \text{ rad/sec}$$

$$\left\{ \begin{array}{l} |Y_2(\omega)| \geq \frac{1}{\sqrt{2}} \\ \left| \frac{\sin \omega \times 10^{-4}}{\omega \times 10^{-4}} \right| \geq \frac{1}{\sqrt{2}} \\ \omega \times 10^{-4} \text{ can be} > \frac{1}{\sqrt{2}} \\ \text{but this is} \\ \text{not true} \end{array} \right\} \begin{array}{l} \text{sin} \geq \left( \frac{1}{\sqrt{2}} \right) \\ \frac{\sin 2}{2} \geq \left( \frac{1}{\sqrt{2}} \right) \\ \frac{1}{2} \geq \frac{1}{\sqrt{2}} \\ 2 \geq \frac{1}{\sqrt{2}} \end{array}$$

$$\text{B.W. } Y(t) = 20,000\pi \text{ rad/sec}$$



there is no question if initial

value the calculate 3rd. B.W. we are taking absolute B.W.

$$\text{Total energy} = \frac{1}{2}a$$

$$\therefore \frac{1}{4a} = \int_0^{\omega_c} \frac{1}{e^{-2at}} dt$$

(T42)

$$= \frac{1}{2a}$$

$$\frac{1}{4a} = \frac{1}{\omega_c + j\omega}$$

$$\frac{1}{4a} = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{1}{(\omega^2 + a^2)} d\omega$$

$$\frac{1}{2-4a} = \frac{1}{2\pi} \left[ \frac{1}{a} \tan^{-1} \frac{\omega}{a} \right]_{-\omega_c}^{\omega_c}$$

$$\frac{\pi}{2} = \tan^{-1} \frac{\omega_c}{a} + \tan^{-1} \frac{\omega_c}{a}$$

$$\pi/2 = 2 \tan^{-1} \frac{\omega_c}{a}$$

$$\frac{\omega_c}{a} = 1$$

$$\omega_c = a \text{ rad/sec}$$

$$\omega_c =$$

$$f_c = \frac{a}{2\pi} \text{ Hz}$$

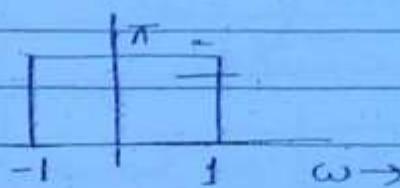
\* The above  $\omega_c$  value calculated can also be called as 50% energy B.W. of the signal likewise we can calculate  $\eta$  energy B.W. for the above signal for the specified energy required.

Q. In the following setup calculate the B.W. of signal  $y_1(t), y_2(t) \text{ & } y_3(t)$

means significant frequencies present in signal

B.W. of a signal : The width of the group of frequencies for which the magnitude spectrum of a given signal is not zero is defined as BW of signal.

$$f(t) = S_a(t) \xrightarrow{FT}$$

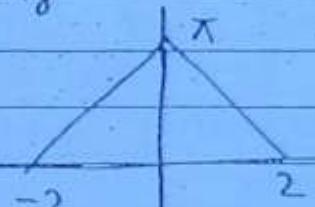


(T43)

$$B.W. = \frac{\pi}{\omega} \text{ rad/sec}$$

$$= \frac{1}{2\pi} \text{ Hz}$$

$$S_a^2(t) \xrightarrow{FT}$$



$$B.W. = 2 \text{ rad/sec}$$

$$= 1/\kappa \text{ Hz}$$

$$f(t) = e^{-at} u(t)$$

$$= \frac{1}{a+j\omega}$$

$$|F(\omega)|_1 1/a$$

B.W.

ideally  $= \infty$

$$|F(\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}$$

$$\frac{1/a}{\sqrt{1+\omega^2/a^2}}$$

$$|F(\omega)| = \frac{1}{\sqrt{2}} (1/a)$$

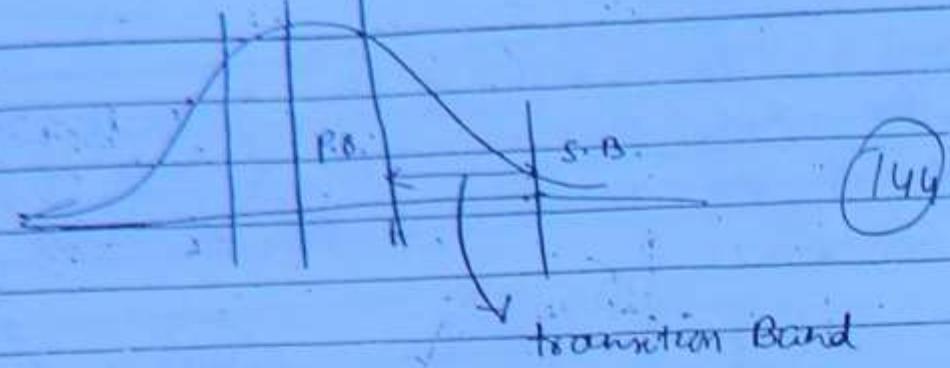
$$at \omega = a$$

$$B.W. = a \text{ rad/sec} \quad 3 \text{ d.B. B.W. of signal.}$$

$$= \frac{a}{2\pi} \text{ Hz} \cdot \text{Ans}$$

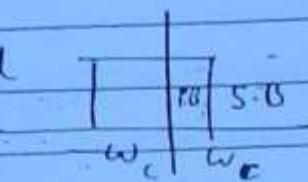
\* 3 d.B. B.W. of a signal, the width the group of frequencies for which  $|F(\omega)| \geq \frac{1}{\sqrt{2}} |F(\omega)|_{\max}$

Q. for a signal  $f(t) = e^{-at} u(t)$ , calculate the range of frequencies to be consider such that these frequencies considered contributes 50% total energy of signal



(144)

A Ideal L.P.F. has no transition band



- \* Ideal filters have sharp cutoff frequencies separating the pass band & stop band whereas practical filters have a group of frequencies separating pass band & stop band defined transition band. Moreover, the magnitude response of practically possible system can't be ~~more~~ <sup>less</sup> than absolutely flat over a range of frequency neither it can be zero for a range of frequency. Ideally speaking B.W. of the above system is  $\infty$  but it is  $|H(\omega)| > \frac{1}{\sqrt{2}} |H(\omega)|_{\max}$ , early so that this system is capable of making available only the frequency for which

\* For all practical purposes  $|H(\omega)| > \frac{1}{\sqrt{2}} |H(\omega)|_{\max}$   
i.e. Lower group of frequency of I/P  $\sqrt{2}$  at the O/P.

- \* For all practical purposes we define the width of this group of frequencies as 3dB B.W. of the system where 3dB B.W. is defined as width of this group of frequencies for which  $|H(\omega)| \geq \frac{1}{\sqrt{2}} |H(\omega)|_{\max}$

$$\text{B.W.} = \omega_c = \frac{1}{2\pi RC} H_0$$

$\frac{-1}{RC} \text{ Radians/sec}$

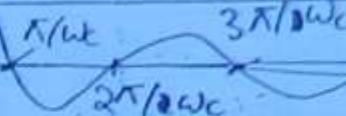
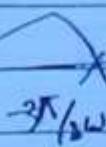
(145)

$$g(t) = \omega_c \operatorname{Sa}[\omega_c t]$$

$$\omega_c/\pi$$

↓ FF

$$G_1(\omega)$$



$$\pi/\omega_c$$

$$3\pi/\omega_c$$

$$2\pi/\omega_c$$

$$-\omega_c$$

$$\omega_c$$



ideal system

we can't realize phys  
ically

$$h(t) \neq 0 \quad t < 0$$

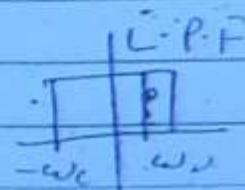
non causal

So we can't realize this system physically

$$= \frac{1}{SC} \cdot \frac{X}{R + 1/SC}$$

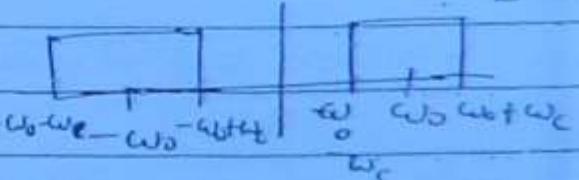
all are ideal systems

not possible

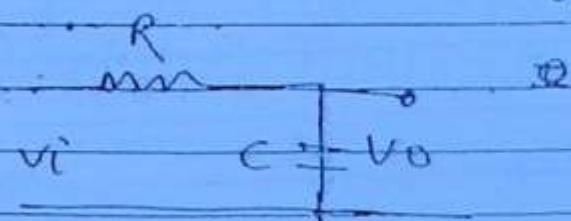


H.P.F.

B.B.F.



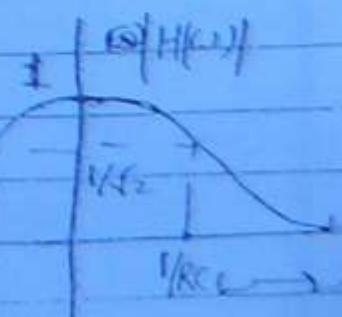
$$H(\omega) = \frac{1}{RCST + 1} = \frac{1}{j\omega RC + 1}$$



$$|H(\omega)| = \frac{1}{\sqrt{\omega^2 R^2 C^2 + 1}}$$

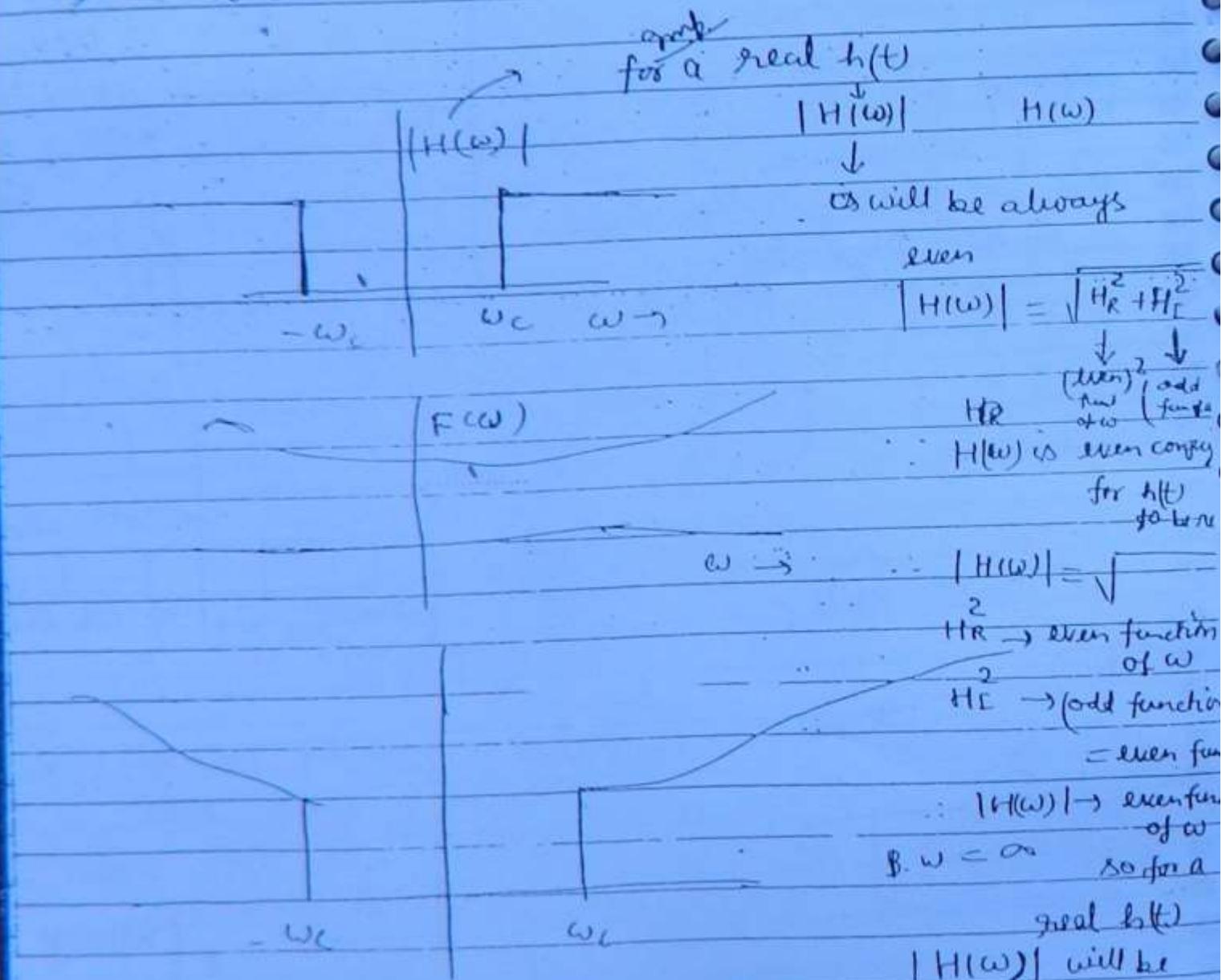
$$\omega \ll 1/RC \quad |H(\omega)| = 1$$

$$\omega \gg 1/RC \quad |H(\omega)| \rightarrow 0$$



\* Bandwidth of the system is defined as the width of group of frequencies which are made available at the O/P of a system by a system.

A system which is distortionless system has a constant magnitude response imply that it can make available all the frequencies present in the I/P at the O/P right from 0 to  $\infty$ . So width of these group of frequencies is  $\infty$ . Hence  $BW = \infty$



$|H(\omega)|$  will be always symmetric about  $y$ -axis.

$$h(t) =$$

$h(t) = A \delta(t - t_0) \rightarrow$  for a distortionless transmission system.

$$H(\omega) = A e^{-j\omega t_0} \times 2\pi$$

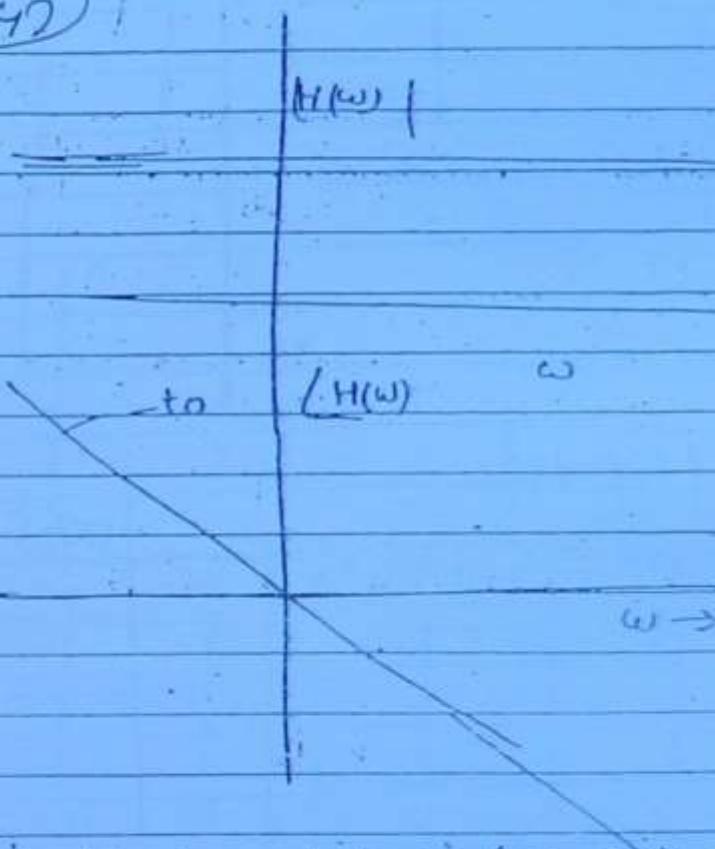
$$\{ |H(\omega)| = A$$

$$\{ \angle H(\omega) = -\omega t_0$$

(47)

$$|H(\omega)|$$

condition for distortionless transmission.



- \* A system is said to be distortionless system, if it allows only two change in the i/p. a scalar multiple  $A$  to i/p and an uniform delay to. i.e. response to an i/p  $f(t)$  should be of the form  $A f(t - t_0)$ .

Transfer func:  $H(\omega) = A e^{-j\omega t_0}$  which insures constant magnitude response & linear phase response with  $-\omega$  slope

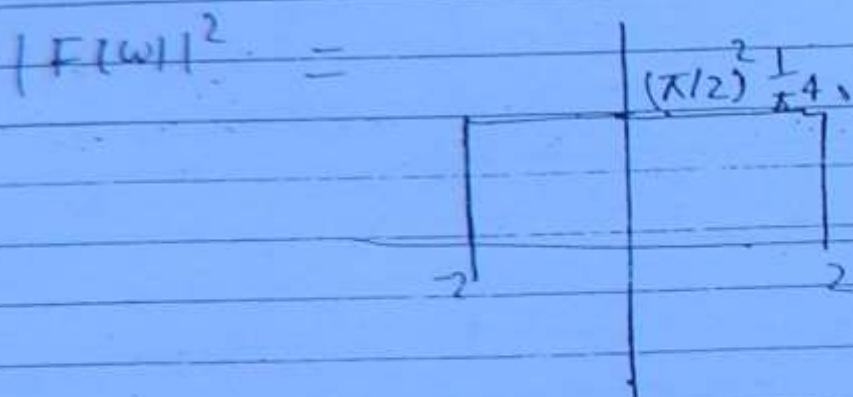
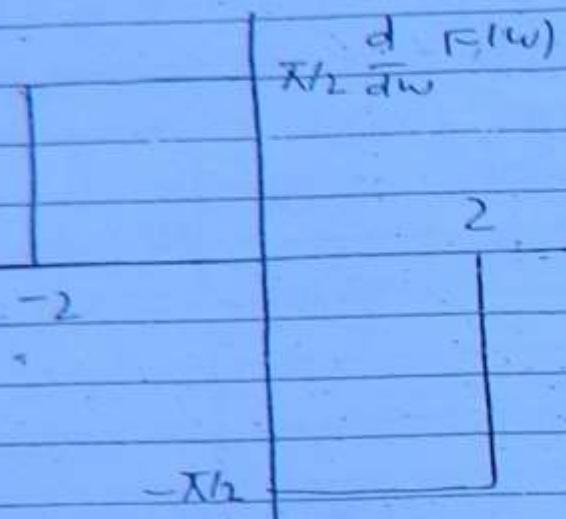
$$|H(\omega)| = A$$

$$\angle H(\omega) = -\omega t_0$$

- (\*) Constant magnitude response insures that all frequency components present at i/p will be carried to the o/p without change in shape.
- (\*) And linear phase response insures sequence of information in time domain i.e. all time components after given uniform time delay

$$P(w) = \frac{1}{\pi^2} \int_{-\infty}^{\infty} dF_1(w)$$

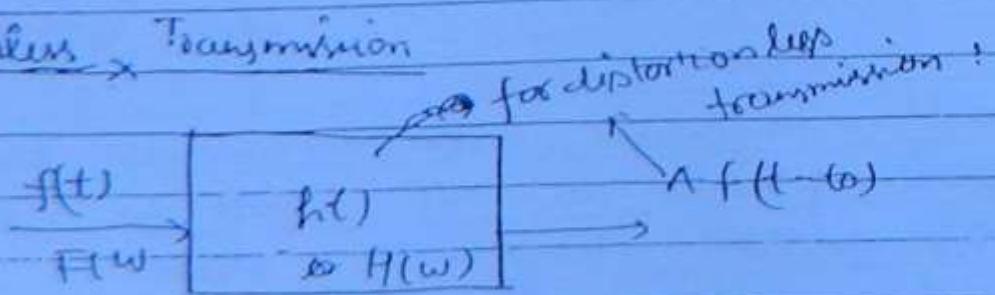
(14R)



$$E = \frac{1}{2\pi} \times 4 \times \frac{1}{2\pi^3} \cdot \frac{\pi}{2}$$

$$\begin{aligned} E &= \frac{1}{2} \frac{1}{\pi^4} \text{ Am} \\ &= \frac{1}{2\pi^3} \text{ Am} \end{aligned}$$

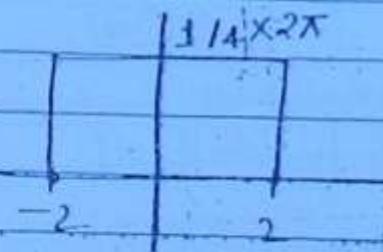
distortionless transmission



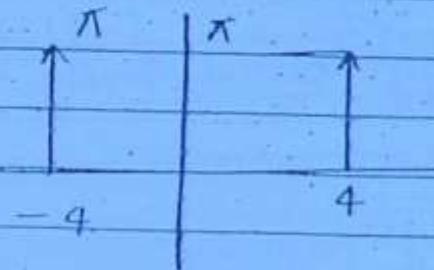
\* Calculate energy of the signal  $f(t) = \text{Sa}[\pi t] \cos t$

(149)

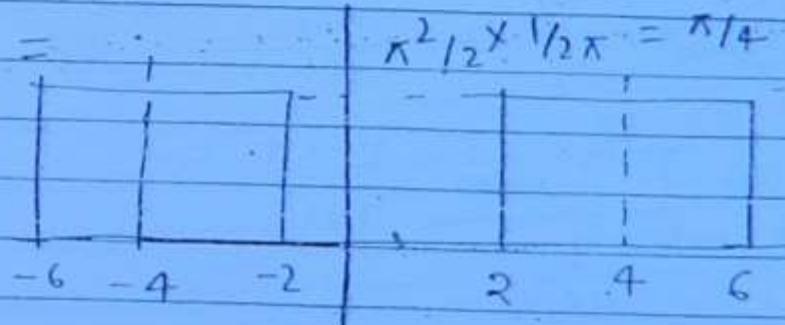
$$f_1(\omega) =$$



$$f_2(\omega) =$$



$$\frac{1}{2\pi} \int_{-\infty}^{\infty} f_1(\omega) * f_2(\omega) = \frac{\pi^2}{12} \times \frac{1}{2\pi} = \frac{\pi}{4}$$

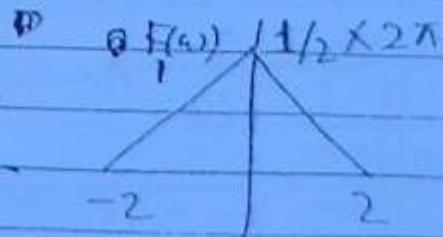


$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi} \left[ \frac{\pi^2}{12} \times 2 \times 4 \right]$$

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{\pi}{4}$$

Calculate the energy of the signal  $f(t) = t \left( \frac{\sin t}{\pi t} \right)^2$



$$= \frac{t}{\pi^2} \frac{\text{Sa}^2(t)}{f(t)}$$

$$E = \int_{-\infty}^{\infty} f(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

(150)

$$\begin{aligned} &= \int_{-\infty}^{\infty} |F(f)|^2 dt \\ &= \int_0^{\infty} |F(\omega)|^2 d\omega \end{aligned}$$

$|F(\omega)|^2 \rightarrow$  energy per unit frequency

or

energy spectral density

$$(ESD) = |F(\omega)|^2$$

↳ even function of frequency

$$f(t) = e^{-at} u(t)$$

$$F = \frac{1}{2\pi} \cdot \frac{1}{a}$$

$$F(\omega) = \frac{1}{a + j\omega}$$

$$|F(\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}$$

$$|F(\omega)|^2 = \frac{1}{a^2 + \omega^2} = ESD$$

$$\therefore E = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{a^2 + \omega^2} d\omega = \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{2\pi a^2} \frac{1}{\omega^2 + a^2} \left[ \tan^{-1} \frac{\omega}{a} \right]_{-\infty}^{\infty}$$

$$= \frac{1}{2\pi a^2} \cdot \pi$$

$$= \frac{1}{4a^2} \text{ Am}$$

Time Integration property of F.T. :

$$f(t) \rightarrow F(\omega)$$

(17)

$$\int_{-\infty}^t f(\tau) d\tau \rightarrow f(t) * u(t)$$

Q

$$F\left[\int_{-\infty}^t f(\tau) d\tau\right] \rightarrow F(\omega) \left[ \frac{1}{j\omega} + \pi \delta(\omega) \right]$$

$$\frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$$

$$\frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$$

$$= \frac{F(\omega)}{j\omega} \quad \text{when } \int_{-\infty}^{\infty} f(t) dt = 0$$

$$= F(0)$$

✳ f(t), real valued signal

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} f^2(t) dt$$

$$= \int_{-\infty}^{\infty} f(t) f(t) dt$$

$$= \int_{-\infty}^{\infty} f(t) \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \right] dt$$

$$= \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} f(t) e^{j\omega t} dt \right] d\omega F(\omega) d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(-\omega) F(\omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(\omega) F(\omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

$$f_1(\omega) * f_2(\omega) = \rightarrow$$

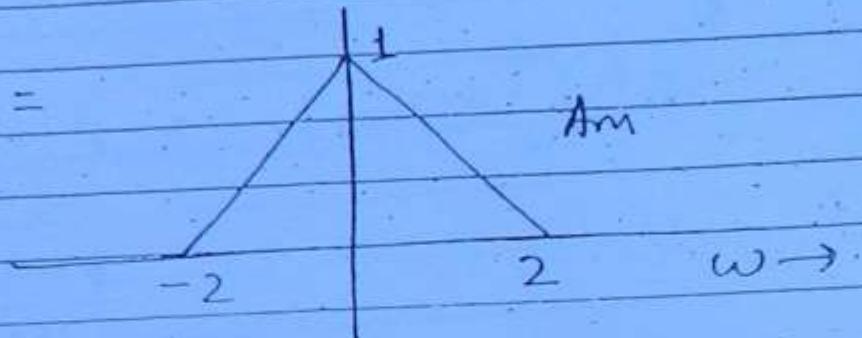
(152)

P[

-2

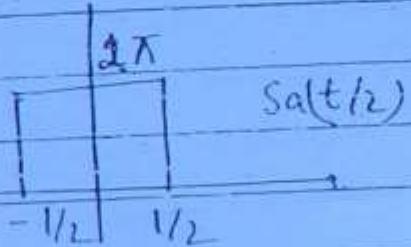
2

$$\mathcal{F}[f_1(t) \cdot f_2(t)] = \frac{1}{2\pi} [Pf_1(\omega) * f_2(\omega)]$$



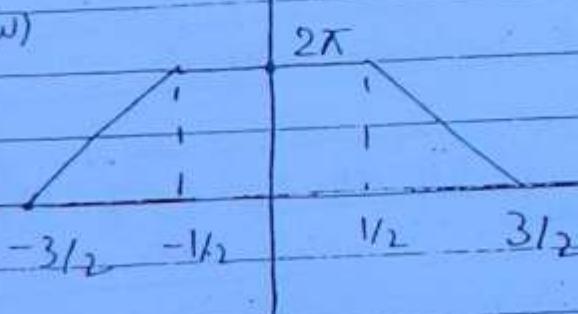
$$f_2(t) = \text{Sa}(t/2)$$

$$f_2(\omega) =$$

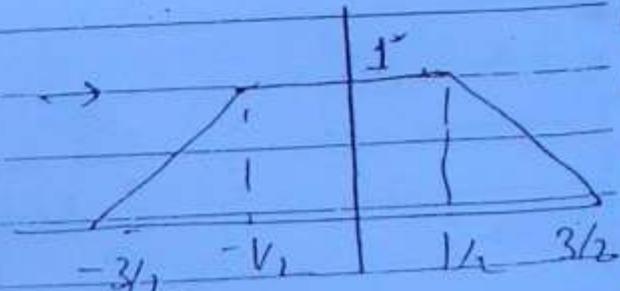


$$f_1(\omega) * f_2(\omega)$$

(check 2π)



$$\mathcal{F}(\omega) = \frac{1}{2\pi} [f_1(\omega) * f_2(\omega)]$$



$$\hat{f}(t) = \hat{f}(t) * \frac{1}{\sqrt{\pi t}}$$

(153)

$$\hat{F}(\omega) = -F(\omega) \text{j sgn}(\omega), [-\text{j sgn}(\omega)]$$

$$= -F(\omega) \text{sgn}^2(\omega) = -F(\omega)$$

$$\hat{f}(t) = -\frac{1}{\sqrt{\pi t}} \int_{-\infty}^{\infty} f(\tau) \text{j sgn}(\omega) d\omega$$

$$\hat{f}(t) = -f(t)$$

$$H[\sin \omega_0 t] = H[H(\cos \omega_0 t)]$$

$$= -\cos \omega_0 t$$

$$\left. \begin{array}{l} H\left[\frac{1}{\pi t}\right] = -\text{sgn}^2(\omega) \\ = -1 \quad \omega > 0 \\ \omega < 0 \end{array} \right\}$$

~~$$S(t) \otimes \frac{1}{\pi t} = \Theta \frac{1}{\pi t}$$~~

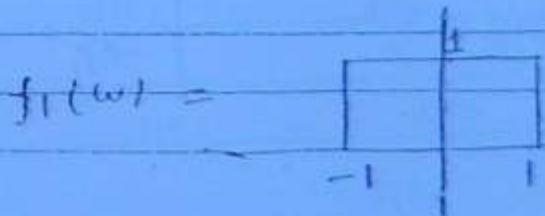
$$H[S(t)] = \Theta \frac{1}{\pi t}$$

$$H[H S(t)] = -S(t)$$

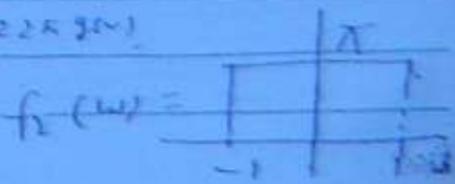
$$H\left[\frac{1}{\pi t}\right] = -\delta(t)$$

$\stackrel{\text{sa}(t)}{\parallel}$

$$f(t) = \frac{2 \sin t}{\pi t^2} \cdot \sin t/2 = \frac{1}{\pi} \frac{\text{sa}[t]}{f_1(t)} \cdot \frac{\text{sa}[t/2]}{f_2(t)}$$



$2 \text{sa}(t) \approx 22.2 \text{E-3 N}$



$$\frac{1}{\pi t} \rightarrow -j \operatorname{sgn}(w)$$

(15y)

$$f(t) \rightarrow F(w)$$

$$f(t) * \frac{1}{\pi t} \rightarrow F(w)[-j \operatorname{sgn}(w)]$$

$$F(w)e^{-j\pi/2} = -jF(w) \quad w > 0$$

$$F(w)e^{j\pi/2} = +jF(w) \quad w < 0$$

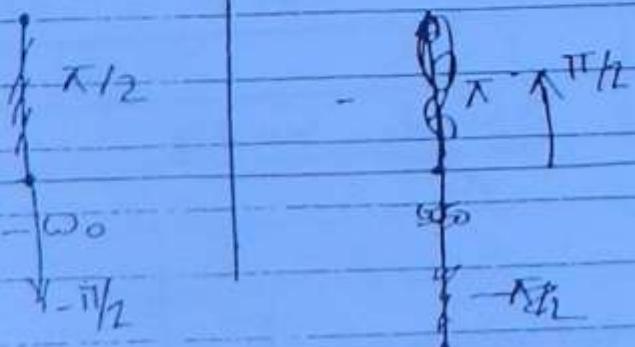
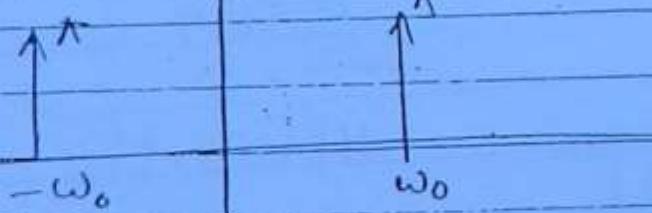
\* Hilbert transform of a signal  $f(t)$  is denoted by a symbol  $\hat{f}(t) = f(t) * \frac{1}{\pi t}$

\* The effect of taking Hilbert transform for a signal  $f(t)$ , it shifts all frequency components of  $f(t)$  uniformly by a phase of  $\pi/2$ .

\* Hilbert transform is also called as wide band phase shifter offering a shift of  $\pi/2$   
Band  $\rightarrow$  Group of frequencies

$$f(t) = \cos \omega_0 t$$

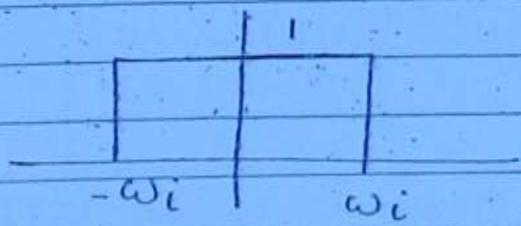
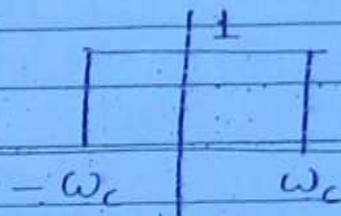
$$f(t) = \cos \omega_0 t * \frac{1}{\pi t} = \sin \omega_0 t$$



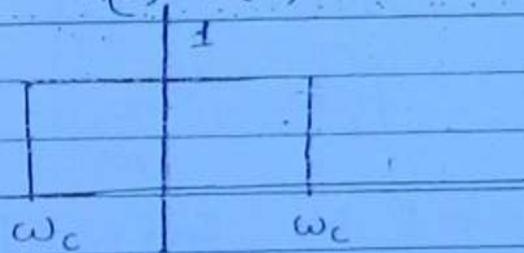
$$\text{Ansatz } Y(\omega) = H(\omega) \cdot F(\omega)$$

(15)

$$= \frac{1 + \pi^2 g''(\omega)}{4\pi^2} g'(\omega)$$



$$H(\omega) \cdot F(\omega)$$



$$Y(\omega) = \frac{1 + \pi^2 g''(\omega)}{4\pi^2} g'(\omega) \quad \text{if } \omega_i > \omega_c$$

$$= \frac{1 + \pi^2 g''(\omega)}{4\pi^2} g'(\omega) \quad \text{if } \omega_c > \omega_i$$

$$y(t) = \frac{g''(2\pi \omega_c) \sin(-\omega_c t)}{4\pi^2}$$

$$= \frac{g''(2\pi \omega_c) \sin(t\omega_c)}{2\pi} \quad \omega_i > \omega_c$$

$$y(t) = \frac{\omega_c \sin(\omega_c t)}{\pi} \quad \omega_i > \omega_c$$

$$d(t) = \begin{cases} H(t) & \omega_i > \omega_c \\ f(t) & \omega_c > \omega_i \end{cases}$$

$$Y(w) = H(w) F(w)$$

$$= \frac{1}{a+jw} + \frac{1}{b+jw}$$

15b

$$- \frac{9}{j}$$

$$= \frac{(b-a)}{(b-a)(a+jw)} + \frac{1}{(a-b)(b+jw)}$$

$$= \left(\frac{1}{b-a}\right) \left[ \frac{1}{a+jw} - \frac{1}{b+jw} \right]$$

$$y(t) = \frac{1}{(b-a)} \left[ e^{-at} u(t) - e^{-bt} u(t) \right] \text{ Ans}$$

Q. Find the response of an LTI system

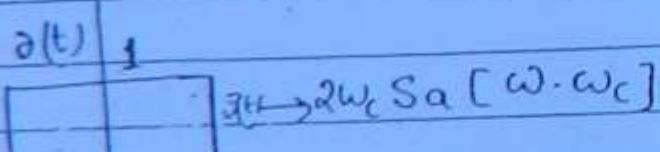
$$h(t) = \frac{\sin \omega t}{\pi t}$$

$$f(t) = \frac{\sin \omega t}{\pi t}$$



$$H(w) = F \left[ \frac{\omega_c}{\pi} \operatorname{Sa}[\omega_c t] \right]$$

$$= F \frac{\omega_c}{\pi} \left[$$



$$\xrightarrow{-\omega_c \quad \omega_c} 2\omega_c \operatorname{Sa}[\omega \cdot \omega_c] \longleftrightarrow a(-\omega) \times 2\pi / \omega$$

$$\begin{aligned} & \xrightarrow{-\omega_c \quad \omega_c} \frac{\omega_c}{\pi} \operatorname{Sa}[\omega_c t] \rightarrow \frac{1}{2\pi} g(\omega) \times 2\pi \\ & \xrightarrow{\omega_c} \frac{\omega_c}{\pi} \operatorname{Sa}[\omega_c t] \rightarrow \frac{1}{2\pi} \int g(\omega) d\omega \end{aligned}$$

$$\xrightarrow{\omega_c} \frac{\omega_c}{\pi} \operatorname{Sa}[\omega_c t] \rightarrow \frac{1}{2\pi} \int g(\omega) d\omega$$

## Convolution Property

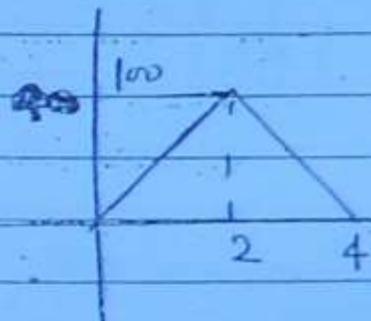
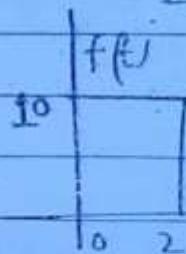
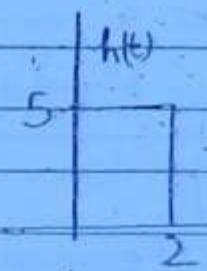
(157)

$$f(t) \rightarrow F(\omega)$$

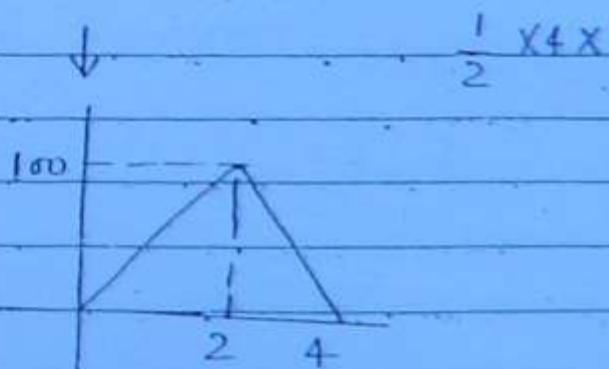
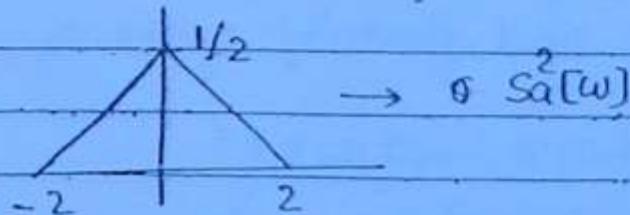
$$h(t) \rightarrow H(\omega)$$

$$f(t) \times h(t) \leftrightarrow F(\omega) \cdot H(\omega)$$

Q. Find response of a LTI system with an impulse response & I/P as shown below:



$$h(\omega) F(\omega) \rightarrow 200 \text{ Sa}^2(\omega) e^{-2j\omega}$$



Repeat the above problem for following impulse response & i/p's

$$h(t) = e^{-at} u(t)$$

$$f(t) = e^{-bt} u(t)$$

$$H(\omega) = \frac{1}{a + j\omega}$$

$$F(\omega) = \frac{1}{b + j\omega}$$

$$f^*(-t) \rightarrow F^*(\omega)$$

$c = a + jb$

(158)

$$C_r = a$$

$$j C_i = jb$$

$$f_R(t) \xrightarrow[2 \downarrow]{f^*(t) + f(t)} \frac{F(-\omega) + F(\omega)}{2}$$

real part of  $f(t)$

even conjugate of  $F(\omega)$

$$j f_I(t) \rightarrow i \left[ \frac{F(\omega) - F^*(-\omega)}{2} \right]$$

$$f(t) = f_R(t) + j f_I(t)$$

odd conjugate

$$f(t) = f_{\text{even}}(t) + f_{\text{odd}}(t)$$

$$f_{\text{even}}(t) \rightarrow \frac{F(\omega) + F^*(\omega)}{2}$$

↓  
real part of  $F(\omega)$

$$f_{\text{odd}}(t) \rightarrow \frac{F(\omega) - F^*(\omega)}{2}$$

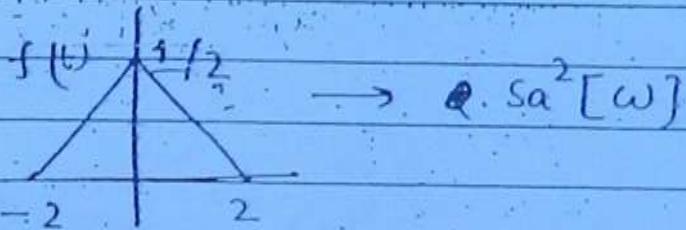
odd part j. imaginary part  
of  $F(\omega)$ .

$$\int_{-\infty}^{\infty} \text{Sa}^2(\omega) e^{-j\omega t/2} d\omega$$

(15g)

$$= f(t) - \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{+j\omega t} d\omega$$

=



$$Q = \frac{1}{4}$$

$$f(-1/2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{-j\omega t/2} d\omega$$

$$= \frac{1}{4} t + \frac{1}{2}$$

$$= -\frac{1}{8} + \frac{4}{8}$$

$$2\pi \times \frac{3}{8} = \int_{-\infty}^{\infty} F(\omega) e^{-j\omega t/2} d\omega$$

$$= \frac{3}{8}$$

Ans  $\frac{3}{8}$

Symmetry Property of Fourier transform.

$$f(t) \rightarrow F(\omega)$$

$\checkmark$   
complex valued  
signal

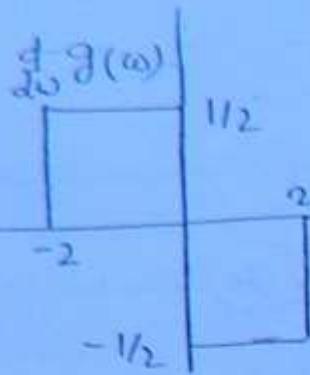
$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F^*(\omega) = \int_{-\infty}^{\infty} f^*(t) e^{j\omega t} dt$$

$$F^*(-\omega) = \int_{-\infty}^{\infty} f^*(t) e^{-j\omega t} dt$$

$$f^*(t) \rightarrow F^*(-\omega)$$

(16)



$$\rightarrow \frac{1}{2} u(\omega+2) - u(\omega) + \frac{j\omega}{2} u(\omega-2)$$

$$\frac{t [Sa(t)]^2}{\pi^2} \stackrel{\mathcal{F}^{-1}}{\longleftrightarrow} \frac{j}{2\pi} \begin{bmatrix} u(\omega+2) \\ -2u(\omega) \\ +u(\omega-2) \end{bmatrix}$$

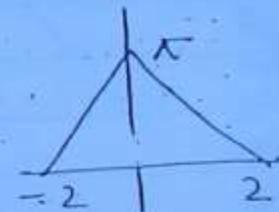
Am

$$Q, f(\omega) = \frac{4t}{(1+\omega^2)^2}$$

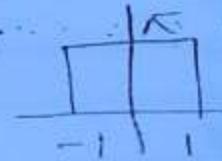
$$e^{-|t|} = \frac{2}{1+\omega^2}$$

$$t e^{-|t|} = -\frac{4\omega}{(1+\omega^2)^2} j$$

$$Sa^2(t)$$



$$Sa(t)$$



$$\left( \frac{-4t}{(1+\omega^2)^2} j \right) \longleftrightarrow 2\pi(-\omega) e^{-|\omega|}$$

$$\left( \frac{4tj}{(1+t^2)^2} \right) \longleftrightarrow +2\pi\omega e^{-|\omega|}$$

$$\left( \frac{4t}{(1+t^2)^2} \right) \longleftrightarrow -2\pi j\omega e^{-|\omega|}$$

Am

$$\int_{-\infty}^{\infty} \frac{\sin^2 t}{\pi t^2} dt = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{\pi} Sa^2(t) dt = \frac{1}{\pi} \cdot \pi = 1$$

## Duality Property:

(161)

$$f(t) \rightarrow F(\omega)$$

$$F(t) \rightarrow 2\pi f(-\omega)$$

$$f(t) = \frac{1}{jt+1}$$

$$F(\omega) \Rightarrow \frac{1}{1+j\omega} \longleftrightarrow e^{-t} u(t)$$

$$\frac{1}{1+jt} \longleftrightarrow 2\pi e^{\omega t} u(-\omega)$$

$$\int_0^\infty \frac{\sin t}{t} dt = \pi/2$$

$$\int_{-\infty}^0 \frac{\sin t}{t} dt = \pi$$

$$\frac{\sin t}{t}$$

is an even function of time

Q.  $\frac{1}{\pi t} \xrightarrow{FT} \{$

$$\frac{1}{j\omega} \rightarrow \frac{1}{2} \operatorname{sgn}(t)$$

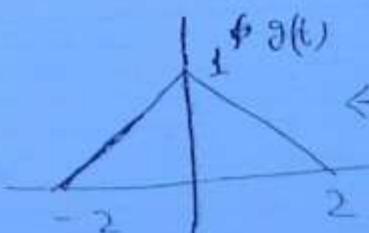
$$\frac{1}{jt} \rightarrow \frac{1}{2} 2\pi \operatorname{sgn}(-\omega)$$

$$\frac{1}{\pi t} \rightarrow \frac{1}{2} 2\pi j \operatorname{sgn}(-\omega)$$

$$\frac{1}{\pi t} \rightarrow -j \operatorname{sgn}(\omega) \text{ Ans}$$

Q.  $f(t) = t \left( \frac{\sin t}{\pi t} \right)^2$

$$= \frac{t}{\pi^2} \left( \frac{\sin t}{t} \right)^2 = \frac{t}{\pi^2} [\operatorname{Sa}(t)]^2$$



$$\leftrightarrow 2 \operatorname{Sa} t \int \operatorname{Sa}(\omega t) d\omega$$

$$\leftrightarrow 2 \int [\operatorname{Sa}(t)]^2 d\omega \rightarrow 2 \pi \operatorname{g}(-\omega) - \operatorname{g}(\omega)$$

$$\therefore 2 \int [\operatorname{Sa}(t)]^2 d\omega \rightarrow 2 \pi \int \frac{d\omega}{dt} \operatorname{g}(t) - \operatorname{g}(-t)$$

Area Property :-

(162)

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \int_a^{\infty} f(t) dt \quad \text{area under curve } f(t)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

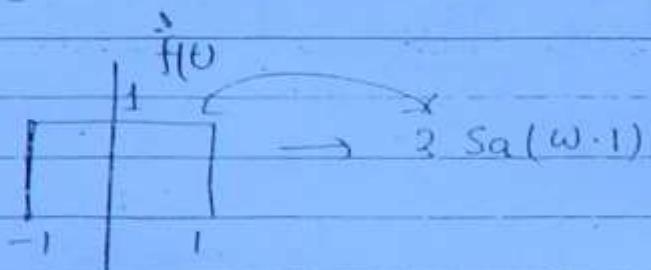
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) d\omega$$

$$\Delta f(t) = \left[ \int_{-\infty}^{\infty} F(\omega) d\omega \right]$$

$$\int_0^{\infty} \int_{-\infty}^{\infty} \text{Sa}(t) dt$$

$$= \int_{-\infty}^{\infty} \frac{\sin t}{t} dt$$

Q



$$\Delta f(t) \leftrightarrow \Delta \pi f(-\omega)$$

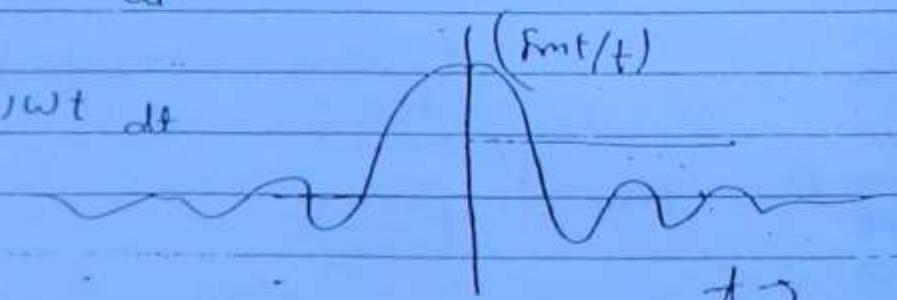
$$\Delta f(\omega)$$

$$F(\omega) = \int_{-\infty}^{\infty} \text{Sa}(t) e^{-j\omega t} dt$$

$$\pi f(\omega) = \int_{-\infty}^{\infty} \text{Sa}(t) e^{-j\omega t} dt$$

$$\pi f(t) = \int_{-\infty}^{\infty} \text{Sa}(t) dt$$

$$\pi = \int_{-\infty}^{\infty} \text{Sa}(t) dt \quad \text{Ans.}$$



$$F(\omega) = e_F e$$

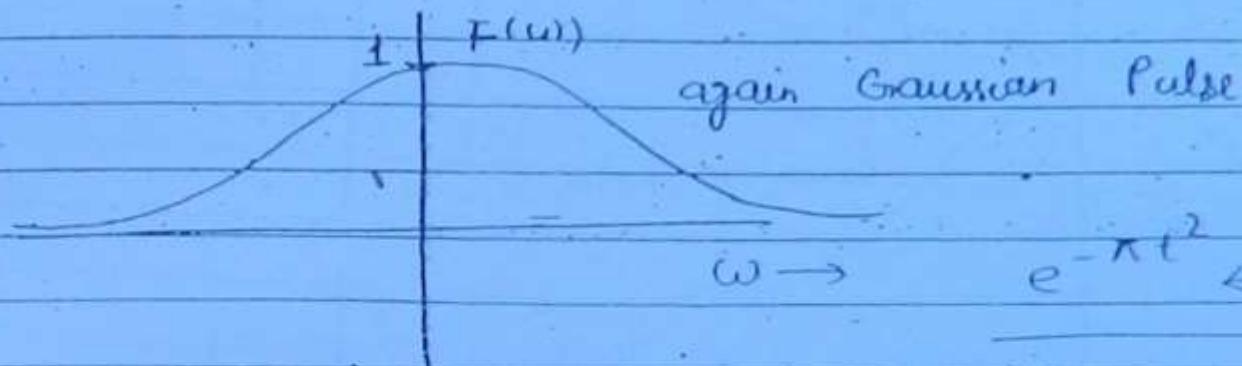
(163)

$$2\pi \frac{dF(\omega)}{d\omega} + \omega F(\omega) = 0$$

$$\frac{P}{F(\omega)} \frac{dF(\omega)}{d\omega} = -\frac{\omega}{2\pi} d\omega$$

$$\log F(\omega) = -\frac{\omega^2}{4\pi}$$

$$F(\omega) = e^{-\omega^2/4\pi} = e^{-f^2\pi}$$



$$F.T. e^{-t^2} \rightarrow$$

$$f(t) = e^{-\pi t^2} \downarrow$$
  
$$f(t/\sqrt{\pi}) \rightarrow \frac{1}{\sqrt{\pi}} e^{-\frac{t^2}{\pi}} \rightarrow \frac{1}{\sqrt{\pi}} e^{-\frac{t^2}{4}}$$
  
$$f(t/\sqrt{\pi}) \rightarrow \sqrt{\pi} e^{-\frac{t^2}{4}} \text{ Ans}$$

$$1 \rightarrow 2\pi \delta(\omega)$$

$$1 \rightarrow \delta(f)$$

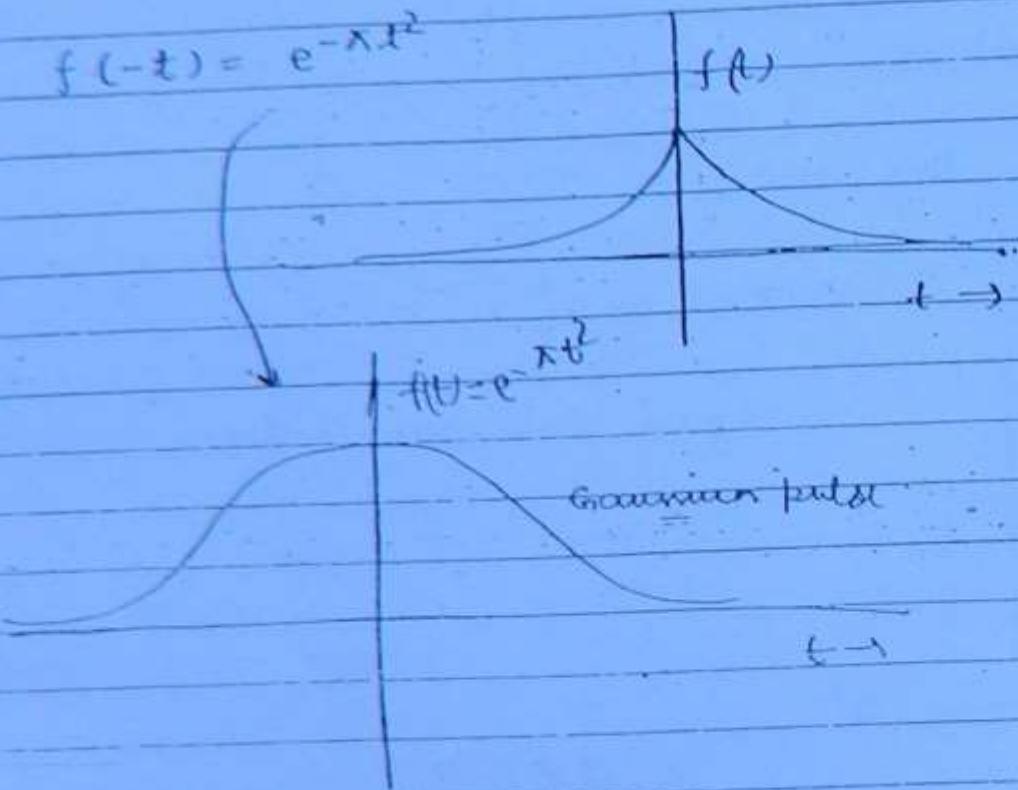
$$\text{Sgt}(t) \rightarrow f + \pi \delta(\omega)$$

$$\frac{1}{j2\pi f} + \frac{1}{2} \delta(f)$$

$$1 \quad \text{the } e^{-at} u(t) \rightarrow \frac{\frac{1}{a}}{(a+j\omega)^{n+1}} \quad (64)$$

$$f(t) = e^{-\pi t^2}$$

$$f(-t) = e^{-\pi t^2}$$



$$\frac{df}{dt} \rightarrow j\omega F(\omega)$$

$$-2\pi t e^{-\pi t^2} \xrightarrow{T.T.} j\omega F(\omega)$$

$$-2\pi j \frac{dF(\omega)}{d\omega} = j\omega F(\omega)$$

$$1 \quad R(0)) \left[ 0 \right] = 0$$

$$-2\pi \left[ -j\omega [2\pi D + \omega] F(\omega) \right] = 0$$

$$D = \frac{\omega_0}{2\pi}$$

$$Q. f(t) \rightarrow F(w)$$

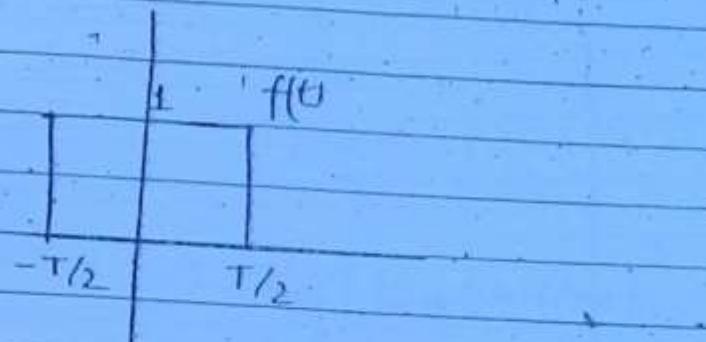
$$\text{find F.T. of } \frac{d^2}{dt^2} f(t) \quad \frac{d^2}{dt^2} [f(t-1)]$$

$$f(t-1) \rightarrow e^{-jw} F(w)$$

(165)

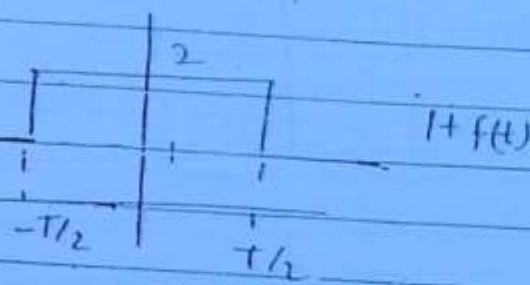
$$\frac{d^2}{dt^2} [f(t-1)] \rightarrow -\omega^2 e^{-jw} F(w)$$

Ans ✓



$$\frac{d}{dt} f(t-1)$$

$d'$



$\checkmark f(t)$  &  $1+f(t)$  are not same signal

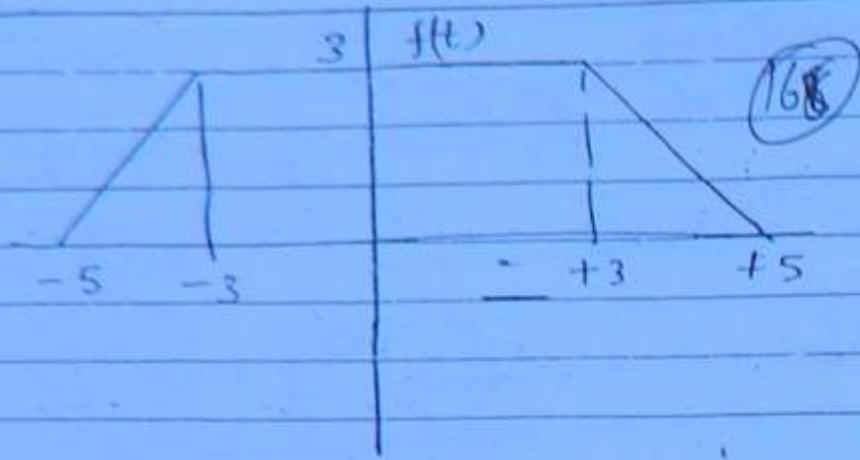
so have different F.T.

if we are going in accordance with differentiation property both have same F.T.  
which is not possible so there must be some condition for application of differentiation property.

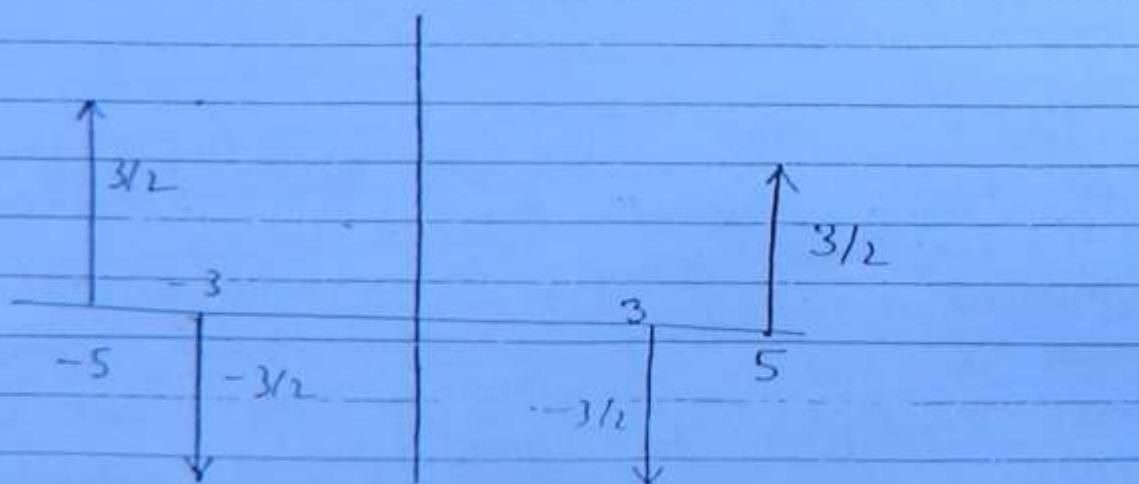
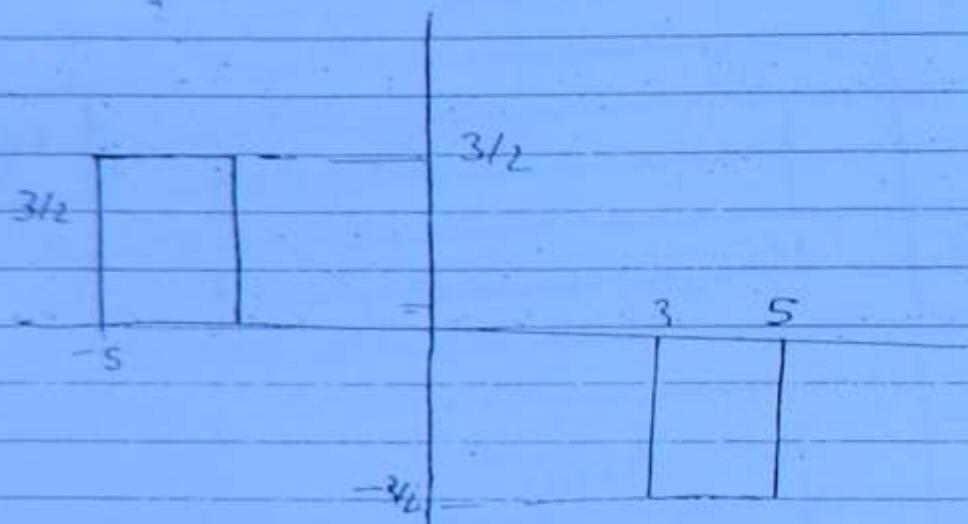
Multiplication by  $t(t) \rightarrow$

$$f(t) \rightarrow F(w)$$

$$t f(t) \rightarrow j \frac{dF(w)}{dw}$$



(168)



$$-\omega^2 F(\omega) \rightarrow \frac{3}{2} [e^{-j5\omega} + e^{+j5\omega}]$$

$$-\frac{3}{2} \left[ e^{-j3\omega} + e^{+j3\omega} \right]$$

$$-\omega^2 F(\omega) \rightarrow \frac{3}{2} \times 2 \cos 5\omega = \frac{3}{2} \cos 5\omega$$

$$F(\omega) \rightarrow \frac{3}{\omega^2} \left[ \cos 3\omega - \cos 5\omega \right] \underline{\text{Ans}}$$

$$F(\omega) = \frac{1}{\omega^2} \left[ \frac{2}{T} - \frac{2}{T} \cos(\omega T) \right]$$

(137)

$$= \frac{2}{\omega^2 T} \left[ 1 - \cos(\omega T) \right]$$

$$F(\omega) = \frac{1}{\pi \omega} \left[ 1 - \cos(\omega T) \right] \text{ fm}$$

$$= \frac{2 \sin^2 \frac{\omega T}{2}}{\pi \omega} + \frac{\frac{\omega^2 T^2}{4}}{2 \times \frac{T}{4}} \left[ \operatorname{Sa}(\omega T/2) \right]^2$$

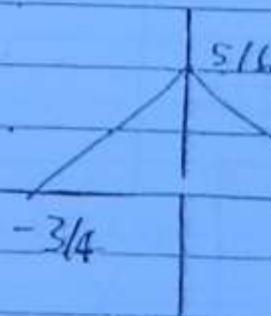
$$F(\omega) = \frac{2 \omega^2 T}{4} \left[ \operatorname{Sa}(\omega T/2) \right]^2 + \frac{2}{\pi \omega} \frac{\omega^2 T^2}{4}$$

$$F(\omega) = \frac{2 \omega^2 T}{4} \left[ \operatorname{Sa}(\omega T/2) \right]^2$$

$$\boxed{F(\omega) = T \left[ \operatorname{Sa}(\omega T/2) \right]^2}$$

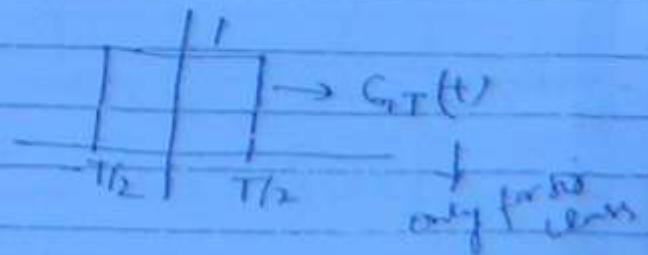
(1)

area of triangle width

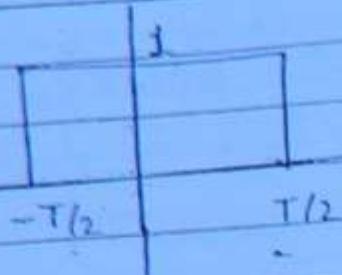


$$\frac{1}{2} \times \frac{3}{2} \times \frac{5}{16} = \frac{3}{2}$$

$$F(\omega) \rightarrow \frac{5}{8} \left[ \operatorname{Sa} \left[ \omega \cdot \frac{3}{8} \right] \right]^2$$



$T \sin \omega T/2$



168

$$\frac{df(t)}{dt} \leftrightarrow \left[ -e^{-j\omega T/2} + e^{j\omega T/2} \right]$$

$\Downarrow$

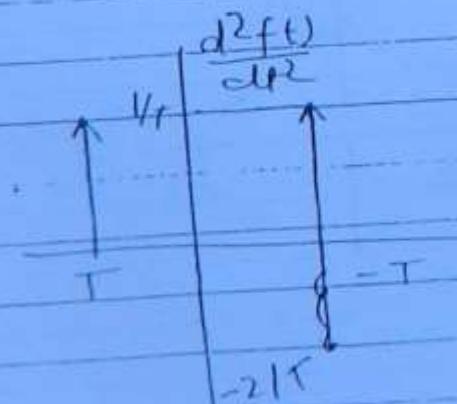
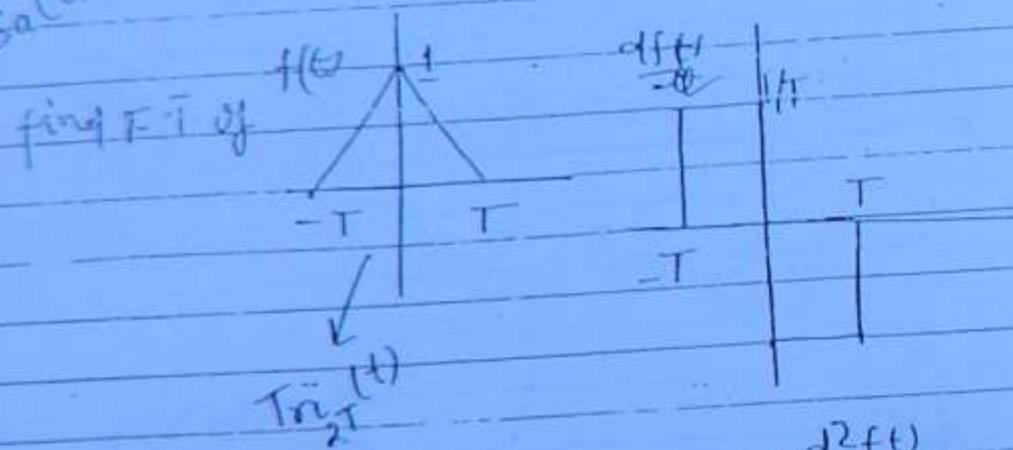
OR  $2j \sin \omega T/2$

$$F(\omega) \rightarrow i \frac{d}{dt} F\left(\frac{dft}{dt}\right)$$

$$\rightarrow \frac{2}{\omega} \sin \omega T/2$$

$$\rightarrow T \text{ sinc } \frac{\omega T}{2}$$

$\text{Sa}(t) \xrightarrow{\text{Fourier}}$



$$(j\omega)^2 F(\omega) \rightarrow -\frac{2}{T} + \frac{1}{T} (e^{j\omega T} + e^{-j\omega T})$$

$$(j\omega)^2 F(\omega) \rightarrow -\frac{2}{T} + \frac{1}{T} 2 \cos \omega T$$

# Frequency shifting Property $\rightarrow$

169

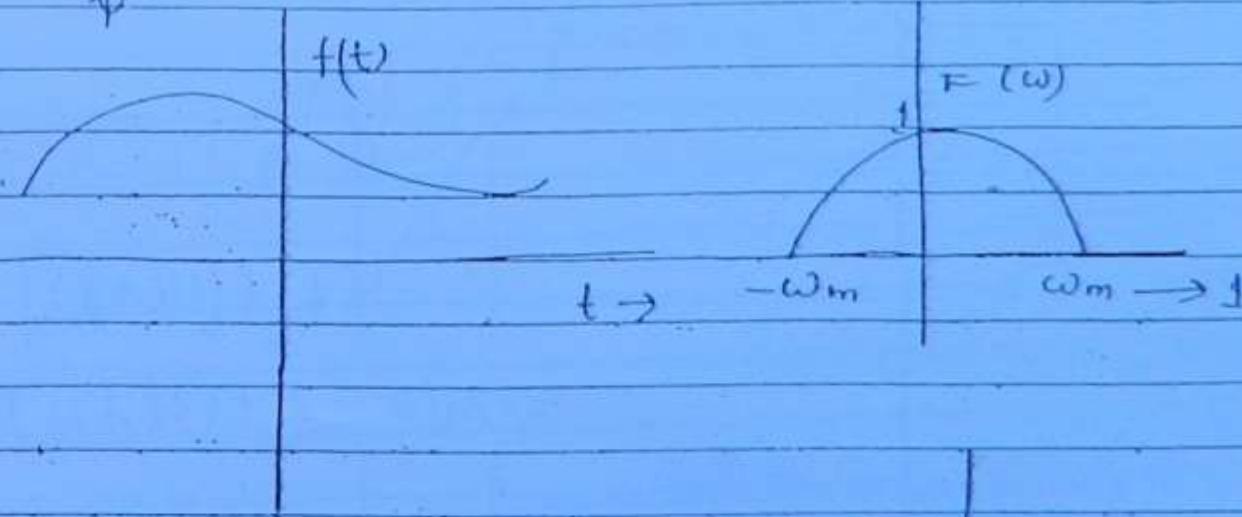
$$e^{j\omega_0 t} f(t) \rightarrow F(\omega - \omega_0)$$

$$e^{-j\omega_0 t} f(t) \rightarrow F(\omega + \omega_0)$$

$$\frac{1}{2} e^{j\omega_0 t} f(t) + \frac{1}{2} e^{-j\omega_0 t} f(t) \rightarrow \frac{1}{2} [F(\omega - \omega_0) + F(\omega + \omega_0)]$$

$$f(t) \cos \omega_0 t \rightarrow \frac{1}{2} [F(\omega - \omega_0) + F(\omega + \omega_0)]$$

modulation theorem

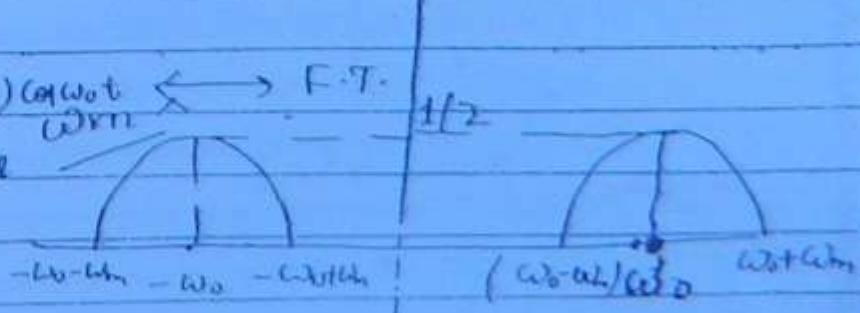


$\times \cos \omega_0 t$

$$f(t) \cos \omega_0 t \leftrightarrow F(\omega)$$

✓ very high comfort

$\omega_0 \rightarrow \omega_m$



## Time scaling

$$f(t) \rightarrow F(\omega)$$

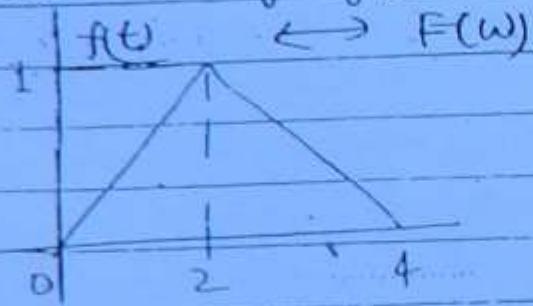
(170)

$$f(at) \rightarrow \frac{1}{|a|} F(\omega/a)$$

\* Expansion of a signal in time domain leads to compression in frequency domain by the same proportion and vice versa.

\* Always maintaining the product of time width & frequency width as a constant

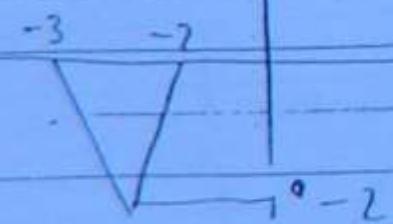
Q. F.T of  $f(t)$  shown below is known to  $F(\omega)$   
then find the F.T. of  $g(t)$



$$f(\omega/4t)$$

$$= f(\omega) f[4(t+3)]$$

$$g(t) = -2 f(4t+12)$$



$$= -2 \frac{e^{j\omega \times 12/4}}{4} F\left(\frac{\omega}{4}\right)$$

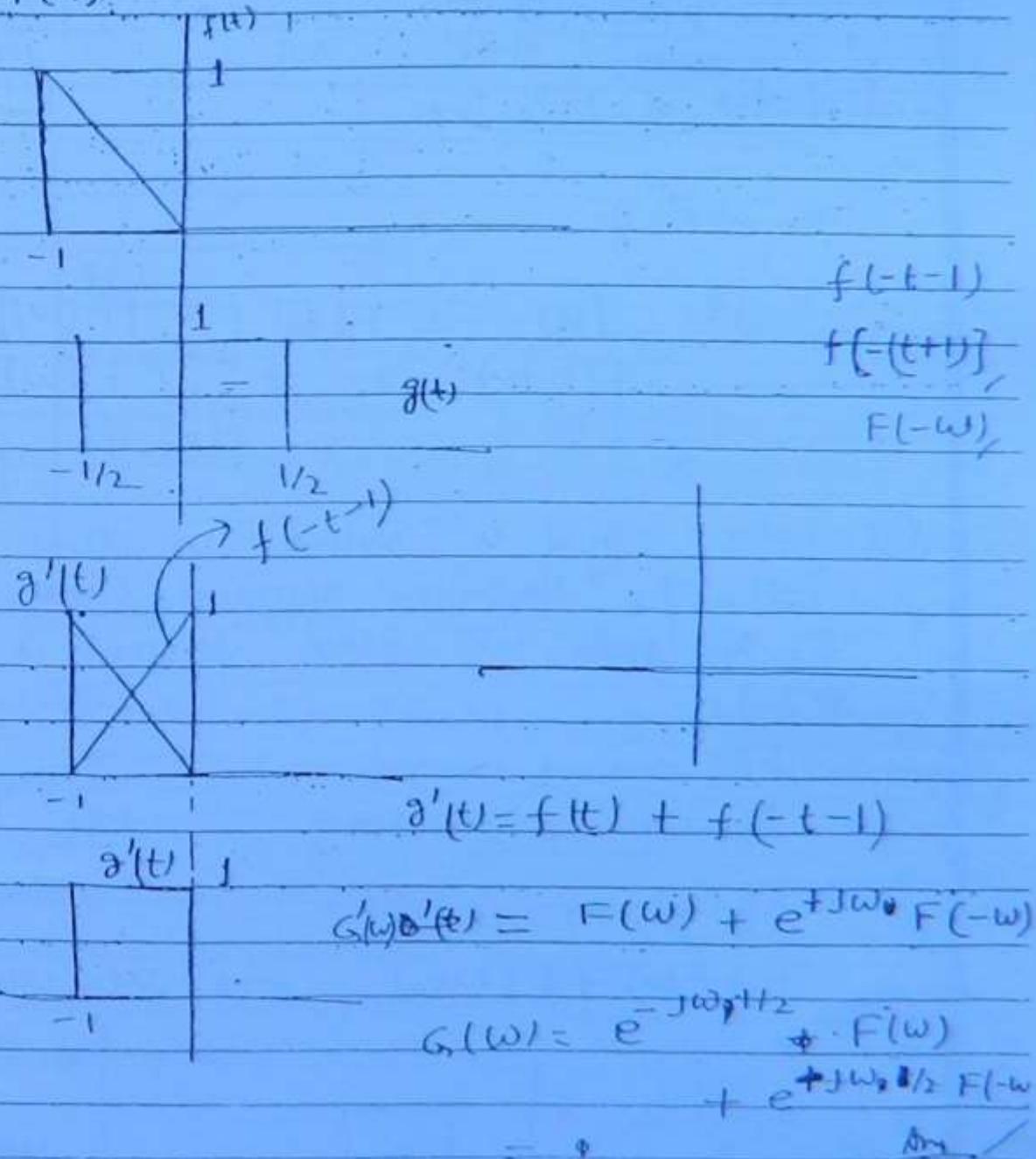
$$G(\omega) = -\frac{1}{2} e^{\frac{j\omega}{2}} F\left(\frac{\omega}{4}\right)$$

$$F(\omega) = \left\{ \frac{1}{j\omega} + \pi \delta(\omega) \right\} + 3 \left[ \frac{1 + 3e^{-j\omega \times 2}}{e^{-j\omega \times 7}} - \frac{3e^{-j\omega \times 5}}{e^{-j\omega \times 7}} \right]$$

27<sup>th</sup> Oct 10

(17)

- Q. If the Fourier transform of the signal  $f(t)$  is known to  $F(\omega)$  find F.T. of signal  $g(t)$  in terms of  $F(\omega)$ :



\* Even if F.T. is defined indirectly, for signal like  $sgn(t) \& t$  which is not satisfying the above condition ( $\int_{-\infty}^{\infty} |f(t)| dt > \infty$ ), the F.T. so derived

will not be well defined F.T. i.e. this F.T. will have an undefined value atleast for one value of  $w$ .

(72)

Properties of F.T. :-

Time shifting

$$f(t) \rightarrow F(w) = |F(w)| e^{j\theta(w)}$$

$$f(t-t_0) \rightarrow e^{-jw_0 t_0} F(w) = |F(w)| e^{j\theta(w-w_0)}$$

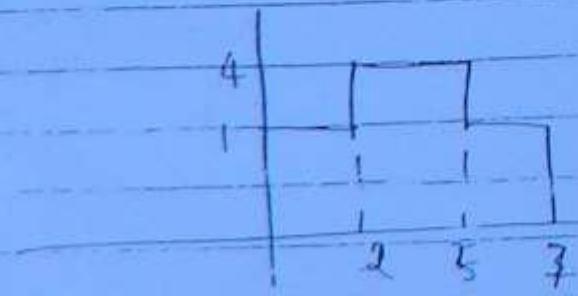
\* When a signal is shifted in time domain magnitude part of spectrum remains same, only phase spectrum is affected i.e. phase spectrum is changed by  $\omega_0 t_0$ .

$$\delta(t) \rightarrow 1$$

$$\delta(t-t_0) \rightarrow e^{-jw_0 t_0}$$

$$\delta(t+t_0) \rightarrow e^{jw_0 t_0}$$

$$f(t-t_0) + f(t+t_0) \rightarrow 2 \cos w_0 t_0 F(w)$$



$$u(t) + u(t-1) - u(t-2)$$

$$+ u(t) + 3u(t-2) + -3u(t-4) - u(t-7)$$

$$F(w) = \frac{1}{jw} \left[ e^{jwT/2} - e^{-jwT/2} \right]$$

$$= \frac{2j \sin(wT/2)}{jw}$$

(173)

$$F(w) = \frac{2 \sin(wT/2)}{\omega} - (1) \text{Sa}(wT/2)$$

$$F(w) = \frac{1}{2\pi} \text{Sinc} wT - \frac{\text{area of pulse}}{\text{half of width of } T} \cdot \frac{\sin \theta}{\theta}$$

$\sin \theta = \frac{\text{Im } A \theta}{\theta}$

$(\pi wT/2)$

4/10

$$\frac{4/10}{x} \times \frac{2 \times \sqrt{2}}{8}$$

-5/8      5/8

$$\frac{1}{2} \text{Sa}\left(\frac{5\pi}{8}\right)$$

$e^{-at} u(t)$ ,  $e^{at} u(t)$ ,  $s(t)$ ,  $\text{rect}(t) \xrightarrow{\text{FT}}$  is completely defined F.T.

Existence of Fourier Transform → not completely defined for all values of  $w$ .

$$|F(w)| < \infty$$

$$\left| \int_{-\infty}^{\infty} f(t) e^{-jw t} dt \right| < \infty$$

$$\left| \int_{-\infty}^{\infty} f(t) dt \right| < \infty$$

$$\int_{-\infty}^{\infty} |f(t)| dt < \infty \rightarrow \text{for well defined F.T.}$$

$f(t)$  must satisfy this condition.

If this condition is satisfied  
then we can use the integral formula for definition of F.T.

(174)

- t →

+

1

||

2

$$u(t) = \frac{1}{2} [\text{sgn}(t) + 1]$$

$$\therefore U(\omega) = \frac{1}{2} \left[ \frac{1}{2j\omega} + \pi\delta(\omega) \right]$$

$$U(\omega) = -\frac{j}{2} \frac{1}{j\omega} + \pi\delta(\omega)$$

$$f(t) \begin{cases} 1 & 0 \leq t < T/2 \\ 0 & T/2 \leq t \leq T \end{cases} = \int_{-T/2}^{T/2} 1 \cdot e^{-j\omega t} dt$$

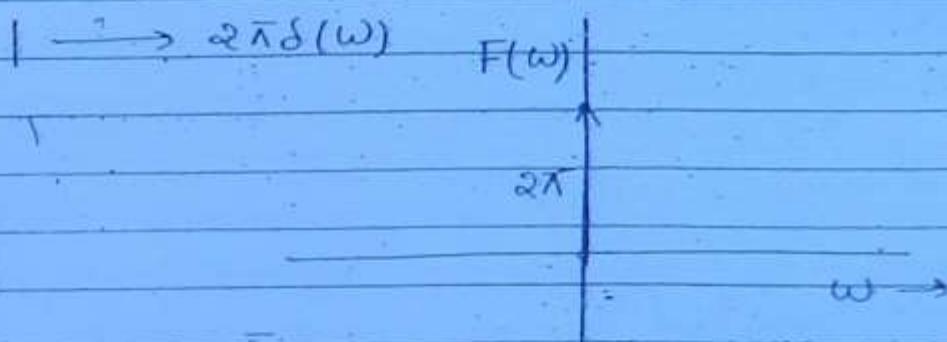
$$F(\omega) = \left[ \frac{e^{-j\omega T}}{-j\omega} \right] \frac{T/2}{T/2 - T/2}$$

$$\text{Cos}(\omega_0 t) \rightarrow \frac{1}{2} \delta(\omega + \omega_0) + \frac{1}{2} \delta(\omega - \omega_0)$$

(173)

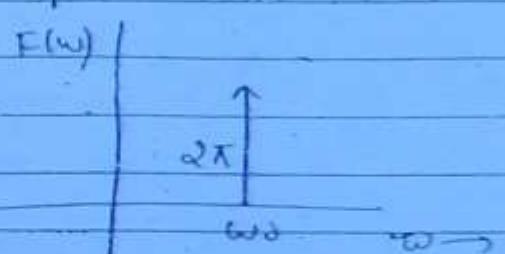
$$\frac{1}{2j} e^{j\omega_0 t} + \frac{1}{2j} e^{-j\omega_0 t} \rightarrow \frac{1}{2j} \frac{\pi}{j} \delta(\omega - \omega_0) - \frac{\pi}{j} \delta(\omega + \omega_0)$$

$$\text{Sin}(\omega_0 t) \rightarrow \frac{1}{j} [\pi \delta(\omega - \omega_0) - \pi \delta(\omega + \omega_0)]$$



$$\text{Cos}(\omega_0 t) \rightarrow 2\pi \delta(\omega - \omega_0)$$

$\downarrow$   
having  
a single  
angular  
frequency



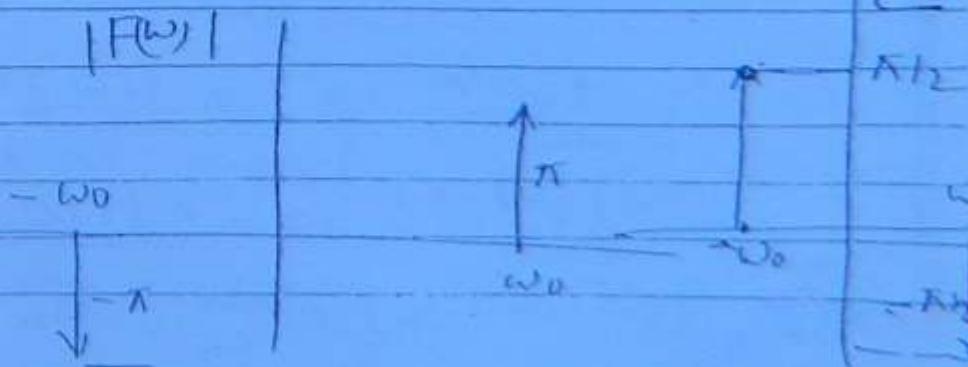
$\omega_0$   $\text{Cos}(\omega_0 t) \rightarrow |F(\omega)|$  phase spectrum

not exist  $\rightarrow$

~~because it's~~

fourier transform  
is real.

$$\text{Sin}(\omega_0 t) \rightarrow$$



$$\delta(t) \rightarrow 1$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \quad (176)$$

$$F(\omega) = \delta(\omega)$$

$$f(t) = \frac{1}{2\pi}$$

$$\frac{1}{2\pi} \rightarrow \delta(\omega)$$

$$| \xrightarrow{\text{FT}} 2\pi \delta(\omega)$$

$$F(\omega) \rightarrow \delta(\omega - \omega_0)$$

$$f(t) = \frac{1}{2\pi} e^{j\omega_0 t}$$

$$\frac{e^{j\omega_0 t}}{2\pi} \rightarrow \delta(\omega - \omega_0)$$

$$e^{j\omega_0 t} \rightarrow 2\pi \delta(\omega - \omega_0)$$

$$1 \cdot e^{j\omega_0 t} \rightarrow 2\pi \delta(\omega - \omega_0)$$

$$f(t) \rightarrow F(\omega)$$

$$f(t) e^{j\omega_0 t} \xrightarrow{\text{FT}} F(\omega - \omega_0)$$

$$f(t) e^{-j\omega_0 t} \rightarrow F(\omega + \omega_0)$$

$$\frac{1}{2} e^{j\omega_0 t} \rightarrow \pi \delta(\omega - \omega_0)$$

$$\frac{1}{2} e^{-j\omega_0 t} \rightarrow \pi \delta(\omega + \omega_0)$$

$$\frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t} \rightarrow \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$$

$f(t)$

(177)

$$\frac{1}{1+j\omega} - \frac{1}{1-j\omega}$$

$$F(\omega) = \frac{-2j\omega}{(1+\omega^2)} \text{ imaginary}$$

$f(t)$  is real & odd

$\Rightarrow F(\omega)$  is also imaginary & odd

(\*) Fourier transforms of neither even nor odd signals is generally complex in nature this complex fourier transform is conjugate symmetric.

(\*) When  $f(t)$  is an even signal Fourier transform is purely real is even function of  $\omega$ .

(\*) When  $f(t)$  is odd fourier transform is purely imaginary and this imaginary part is odd.

$$\lim g(t) = \operatorname{sgn}(t)$$

$\Leftrightarrow a \rightarrow 0$

$a$

$a$

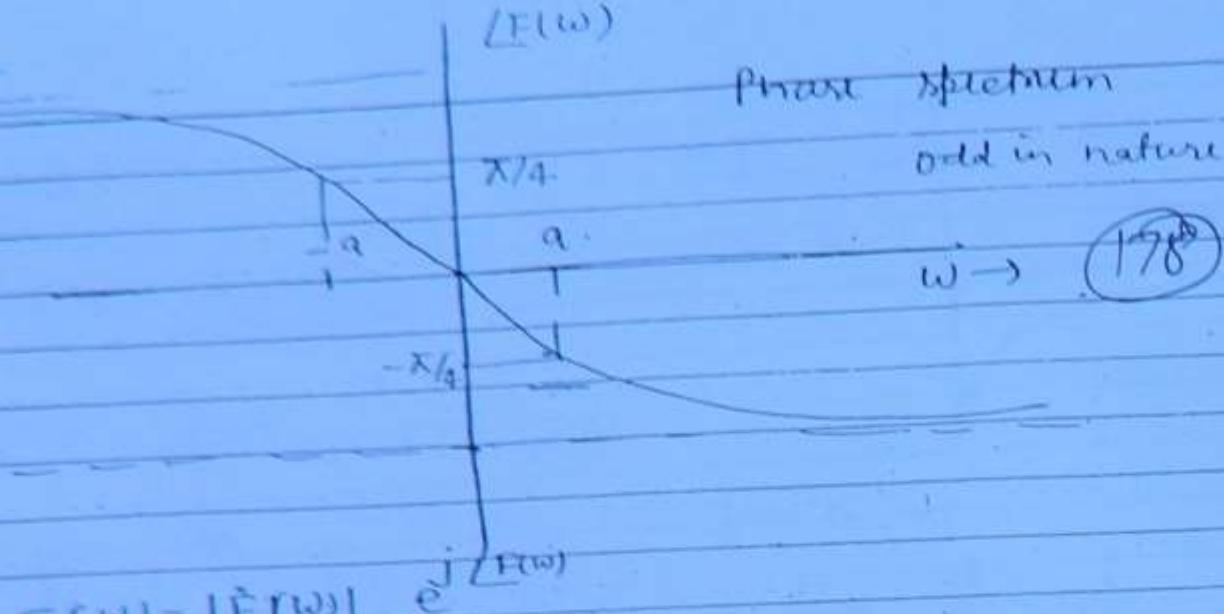
$$F[\operatorname{sgn}(t)] = \lim_{a \rightarrow 0} F[g(t)]$$

$$= \lim_{a \rightarrow 0} F[g(\omega)]$$

$$= \lim_{a \rightarrow 0} \left[ \frac{1}{a+j\omega} - \frac{1}{a-j\omega} \right]$$

$$F[\operatorname{sgn}(t)] = \frac{2}{j\omega}$$

7/2



$$e^{at} u(t)$$

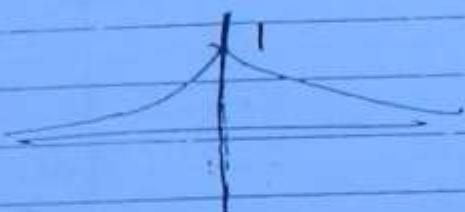
$$f(\omega) = \int_{-\infty}^{\omega} e^{at} e^{-j\omega t} dt$$

$$\left[ F(\omega) = \frac{1}{(a-j\omega)} \right] = \frac{a}{a^2+\omega^2} + j \frac{\omega}{a^2+\omega^2}$$

↓ even      ↓ odd

$$\begin{cases} f(t) \rightarrow F(\omega) \\ f(-t) \rightarrow F(-\omega) \end{cases} \quad \text{time reversal property}$$

$$f(t) = e^{at} u(t)$$



$$\frac{1}{1-j\omega} + \frac{1}{1+j\omega}$$

$$F(\omega) = \frac{2}{(4\omega^2)} \quad \text{if } f(t) \text{ is real & even}$$

$$f(t) = \frac{2}{(4\omega^2)} \rightarrow \text{real & even}$$

$$f_1(t) \rightarrow F_1(\omega)$$

$$f_2(t) \rightarrow F_2(\omega)$$

$$\alpha f_1(t) + b f_2(t) \rightarrow \alpha F_1(\omega) + b F_2(\omega)$$

Q. Fourier Transform of  $f(t) = e^{-at} u(t)$

$$= \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

(179)

$$= -\frac{1}{(j\omega + a)} [0 - 1]$$

$$\left[ F(w) = \frac{1}{a + jw} \right]$$

$$\frac{a - j\omega}{(a^2 + \omega^2)} = \frac{a}{a^2 + \omega^2} - j \frac{\omega}{a^2 + \omega^2}$$

generally  
complex  
in nature

$$F_R(w) = \frac{a}{a^2 + \omega^2}$$

$$F_R(-w) = \frac{a}{a^2 + \omega^2} = F_R(w) \rightarrow \text{even in nature}$$

$$F_I(w) = \frac{-\omega}{a^2 + \omega^2}$$

$$F_I(-w) = \frac{+\omega}{a^2 + \omega^2} = -F_I(w) \quad \text{odd in nature}$$

$F(w)$  is even conjugate or  
conjugate symmetric in nature

$$F(w) = F^*(w)$$

$$F(w) = |F(w)| \quad F(w) = \sqrt{a^2 + \omega^2} \quad (-\tan^{-1}\omega/a)$$

(179)

$$|F(w)| / a$$

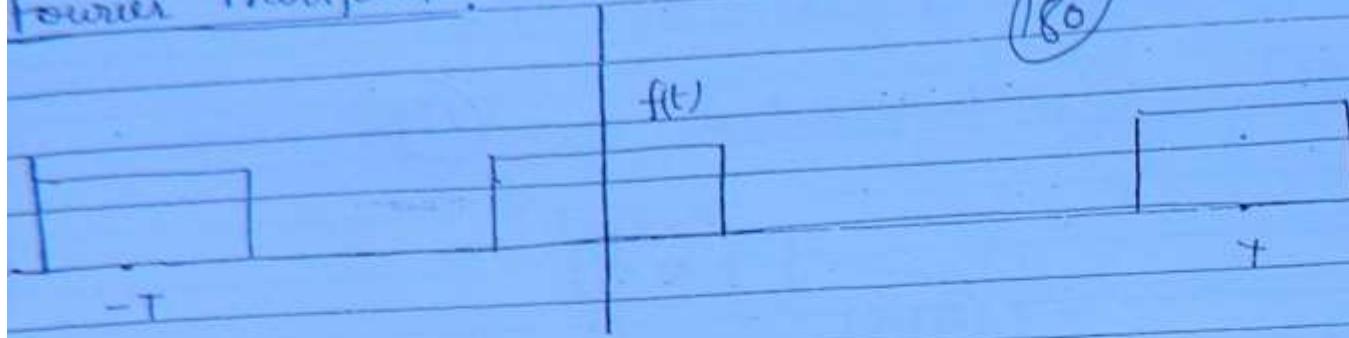
magnitude spectrum  
of

$$|F(w)|$$

$w \rightarrow$

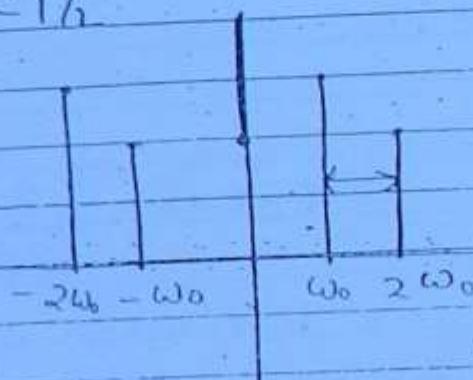
Fourier Transform :-

(180)



$$\hat{c}_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt$$

$c(n\omega_0)$



as  $T \rightarrow \infty$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$n\omega_0 \rightarrow \omega$

$$F(n\omega_0) = \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt$$

Spacing becomes zero, means

$T c_n =$

$$f(t) = \lim_{T \rightarrow \infty} \sum_{n=-\infty}^{\infty} F(n\omega_0) e^{jn\omega_0 t}$$

become continuous

$$\left[ \text{as } F(\omega) = \lim_{T \rightarrow \infty} \int_{-\omega}^{\omega} f(t) e^{-j\omega t} dt \right]$$

at  $T \rightarrow \infty$   
FT becomes a  
continuous  
function

$$f(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} F(n\omega_0) e^{jn\omega_0 t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

$$Q. f(t) = \sum_{n=-100}^{100} C_n n\pi e^{jn\pi t / 100}$$

(18)

$$C_n = \begin{cases} C_0 n\pi & n \rightarrow -100 \text{ to } 100 \\ 0 & \text{otherwise} \end{cases}$$

$$C_1 = \cos \pi, \quad C_{-1} = \cos \pi$$

$$C_n = C_{-n}^* \quad \text{real}$$

$$C_n = C_{-1} \rightarrow \text{even} \quad f(t) = \text{real & even}$$

$$f(t) = \sum_{-100}^{100} j \sin n\pi t / 2 e^{jn\pi t / 100}$$

$$C_n = j \sin n\pi t / 2 \quad n \rightarrow -100 \rightarrow 100$$

$$C_1 = j$$

$$C_{-1} = -j$$

$$C_1 = C_{-1}^* \quad \text{imaginary & odd}$$

$$\text{real & odd} \quad f(t) = \sum_{n=1}^{100} C_n e^{jn\pi t / 100}$$

Q. A signal  $f(t)$  as fourier coefficient  $C_n$   
find the fourier coefficient of:

$$f(t) + f(1-t) = C_n + f(-t+1)$$

$$f(1-t) = f(-t+1)$$

$$= C_n e^{j\pi n \omega_0}$$

$$= C_{-n} e^{-j\pi n \omega_0}$$

$$f(t) + f(1-t) = C_n + C_{-n} e^{-j\pi n \omega_0}$$