

5

258

260

-: HAND WRITTEN NOTES:-

OF

ECE

④

Electronics &
Communication Engg

-: SUBJECT:-

NETWORK THEORY

5

Network Theory

1. Basics
2. Steady State AC Circuits (Resonance).
3. Theorems (obj & conv).
4. Transients (")
5. Two port (")
6. Graph theory and magnetic circuits
7. Filters
8. Synthesis (-obj)

F (4)

Books

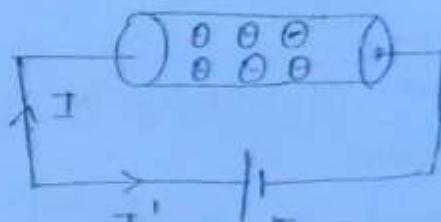
1. Fundamentals of Electric Circuits — (PB)
By Alexander Sadiku.
- * 2. Engg Circuit Analysis
By Hayt kennedy.
- * 3. Networks and Systems
By Roy Choudhary.
- * 4. Network Analysis — Van Valkenburg.

is basic quantity in the circuit is charge. The charge of the e^- is given by $-1.602 \times 10^{-19} C$.

A flow of electrons is called as current (or) the time rate of charge is also called as current.

$$I = \frac{dq}{dt} \text{ C/s (or) A}$$

$I \rightarrow$ conventional current.



(natural current).

In the network theory, while developing KVL & KCL equation conventional current is used.

To move the e^- from one point to other point in particular direction, external force is required.

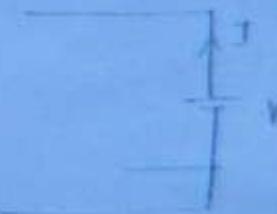
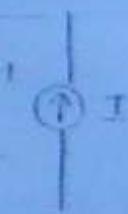
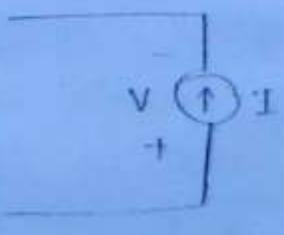
In the circuit, external force is provided by EMF and it is given by,

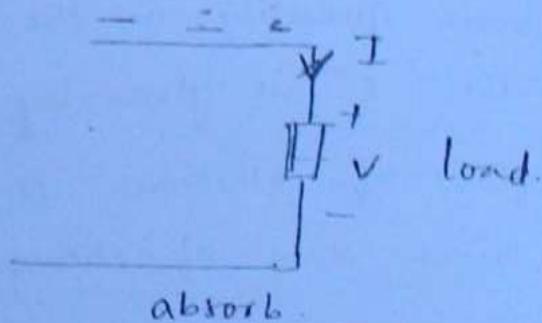
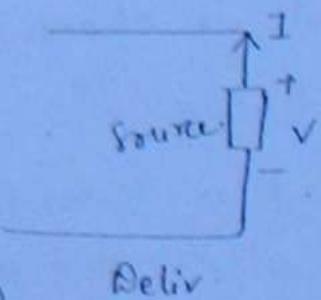
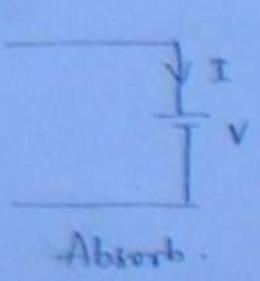
$$V = \frac{dw}{dq} \text{ J/C (or) Volt.}$$

The time rate of energy is called as power.

$$P = \frac{dw}{dt} \text{ J/s (or) Watts.}$$

$$P = \frac{dw}{dt} = \frac{dq}{dt} \cdot V \Rightarrow P = VI$$





Note :

(6)

1. When current is entering at positive terminal element is absorbing power.
2. When current is leaving from the +ve terminal, element is delivering the power.
3. Find power of each element, of the circuit shown.

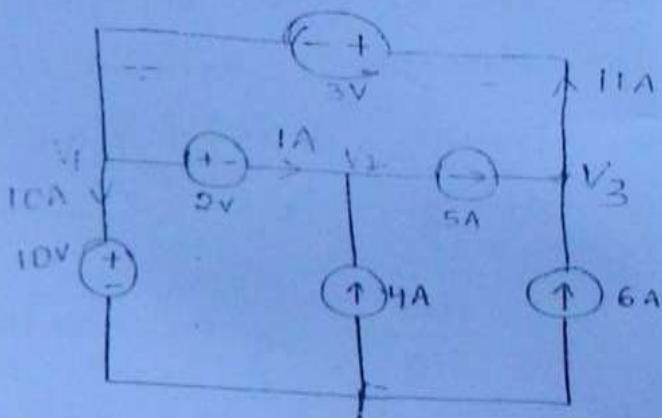
$$V_1 - V_2 = 2V$$

$$V_2 = 10 - 2 = 8V$$

$$\underline{V_2 = 8V}$$

$$V_3 - V_1 = 3$$

$$\underline{V_3 = 13V}$$



$$P_I = 8 \times 4 = 32W \quad (\text{Del})$$

$$P_L = 13 \times 6 = 78W \quad (\text{Del})$$

$$P_C = 5 \times 5 = 25W \quad (\text{Del})$$

$$P_3 = 11 \times 3 = 33W \quad (\text{abs.})$$

$$P_{\text{for}} = 10 \times 10 = 100W \quad (\text{abs.})$$

$$P_2 = 2 \times 1 = 2W \quad (\text{abs.})$$

$$(P_T)_{\text{Del}} = -(P_T)_{\text{Del}} = 135W.$$

The capacity to do the work is called as Energy.

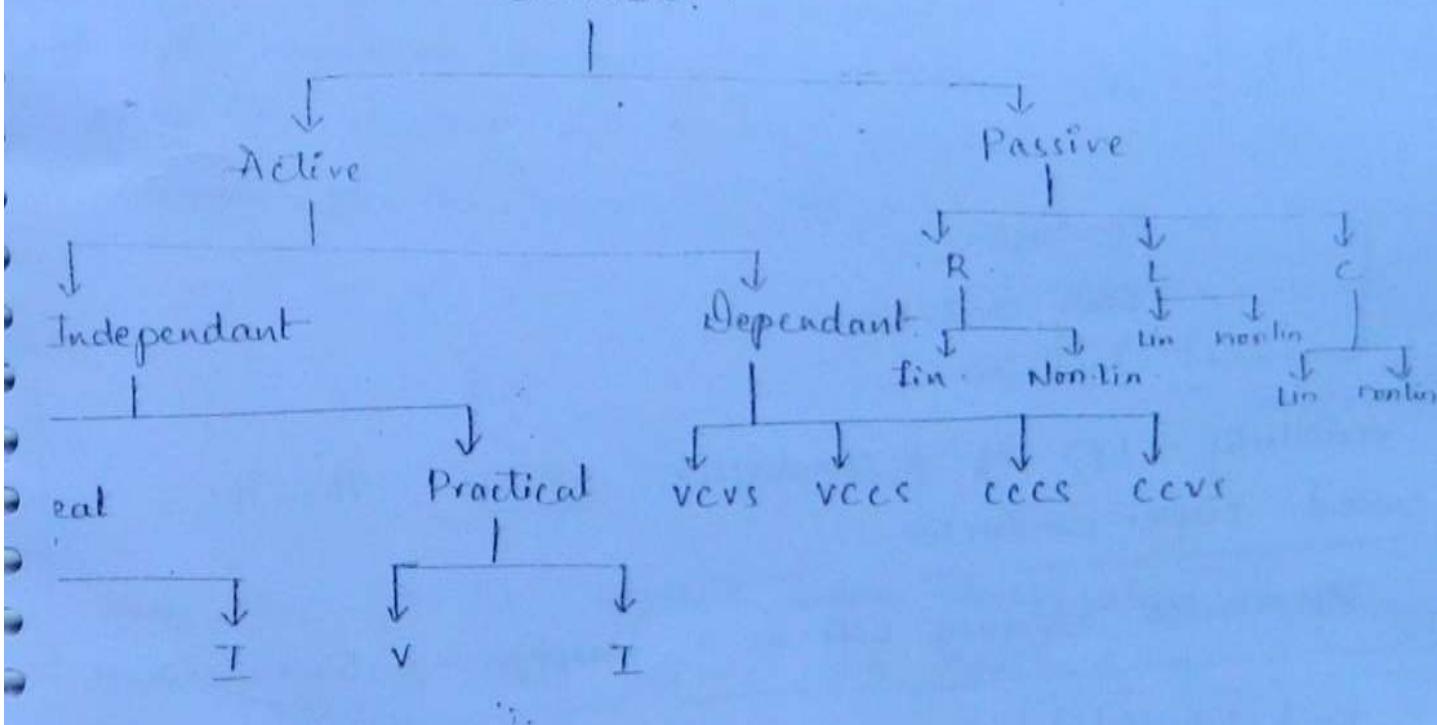
$$W = \int_0^t P dt \quad \text{Watt-sec (or) J.}$$

Classification of Elements

(7)

1. Active and Passive Elements.
2. Unidirectional & Bidirectional elements.
3. Linear and non linear elements.
4. Time variant and invariant elements.
5. Tumped and distributed elements.

Elements



Active Elements :- When element is capable of delivering Energy independantly for infinite time (or) when the element is having property of internal amplification then the element is called as active element.

Ex. \sqrt{T} ^{Independent source} Transistor ...

Passive Elements

When the element is not capable of delivering energy independently for infinite time, then the element is called as passive element. 8

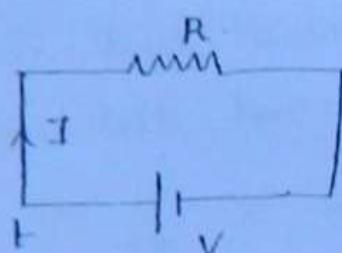
Ex. R, L, $\frac{1}{C}$, bulb, transformer.

Resistance is a property of the resistor. It opposes flow of current. By doing so, it converts electrical energy to heat energy.

$$P = I^2 R$$

$$W = I^2 R t$$

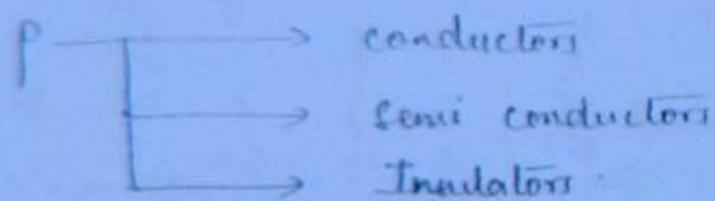
\downarrow
(heat)



$$R = \frac{\rho l}{a}$$

R

$\Omega \rightarrow \text{Dim.}$



When resistivity (ρ) of a conductor, $\rho = 0$, then it is called super conductor

Ex. At -4.15°K Mercury acts as a super conductor.

$$R_t = R_0 (1 + \alpha \Delta t)$$

where, R_0 = resistance of the material at 0°C

α = temperature coefficient per $^\circ\text{C}$.

Δt = change in temperature.

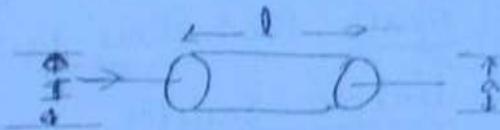
Ohm's law.

→ Ohm's law states that, at constant temperature, current density is directly proportional to electric field intensity.

$$J \propto E$$

$$J = \sigma E$$

(9)



$$\frac{I}{A} = \frac{1}{\rho} \cdot \frac{V}{L}$$

$$R = \frac{PL}{A}$$

$$\frac{V}{L} = \frac{PL}{A} = R.$$

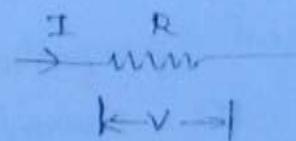
$$J = \frac{T}{A}$$

$$E = \frac{V}{L}$$

$$\sigma = \frac{1}{\rho} \text{ mho/m.}$$

→ At constant temperature, potential difference across an element is directly proportional to the current flowing across the element.

$$V \propto i$$



$$V = RI.$$

$$R = V/I = \text{constant.}$$

Ohm's law can be applied when temperature and conductivity of the material are constant.

units

1st form $J = \sigma E$

$$G_I = \frac{1}{R} \text{ mho (or si)}$$

2nd form $V = iR$

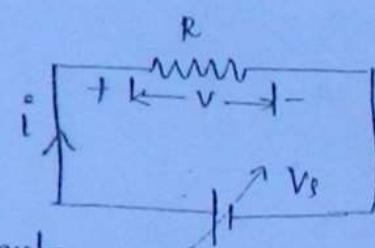
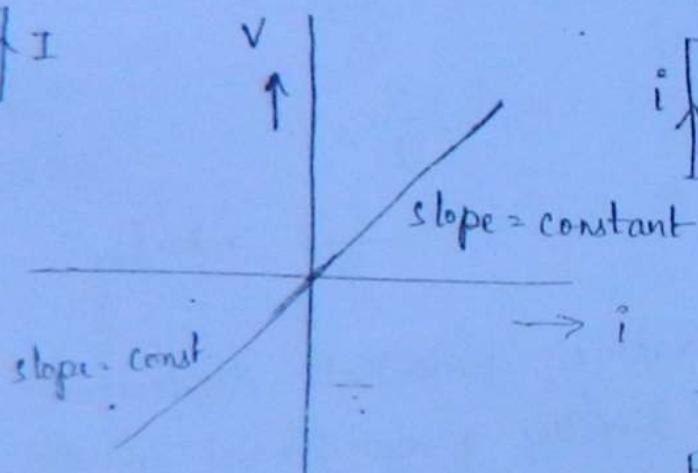
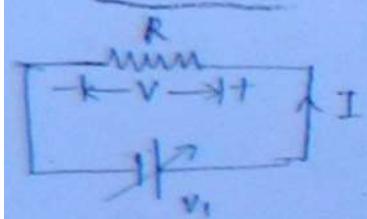
3rd form $i = G_V$

4th form $V = \frac{dq}{dt} R$

→ When element properties and characteristic are independent on the direction of the current, then the element is called as bi-directional Element. (10)

→ When element obeys the Ohm's law, then the element is called as linear resistor.

Every linear element should obey the bi-directional property. But not vice versa.



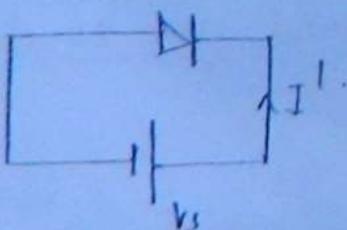
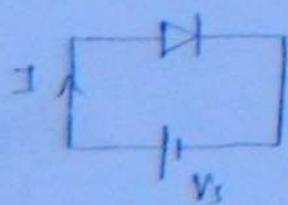
$$i \uparrow 10\% \Rightarrow v \uparrow 10\%$$

$$i \uparrow 90\% \Rightarrow v \uparrow 90\%$$

$$R = \frac{V}{I} = \text{constant}$$

When element properties and characteristic depends on direction of the current, then the element is called as unidirectional element.

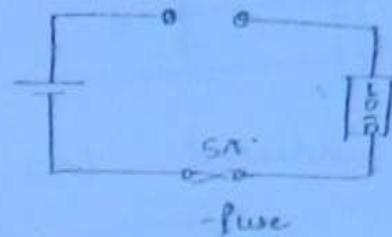
→ When element does not obey the Ohm's law, then the element is called as non-linear resistor.



$$|I| \neq |I'|$$

Open circuit O.C.

Properties of O.C. (Ideal).

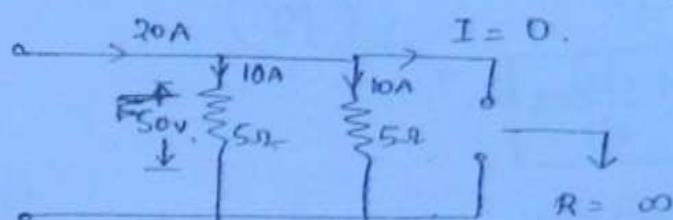


(11)

$$R = \infty$$

$$I = 0$$

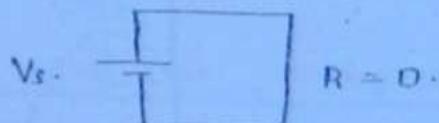
$$V = \infty \text{ (max. possible).}$$



$$V_{oc} = 50V.$$

Short circuit (S.C.)

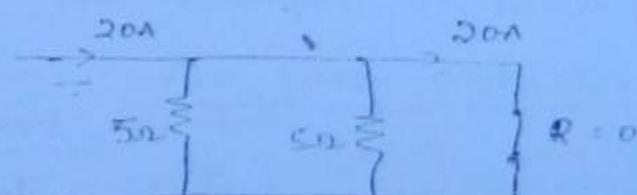
Properties of S.C. (IDEAL)



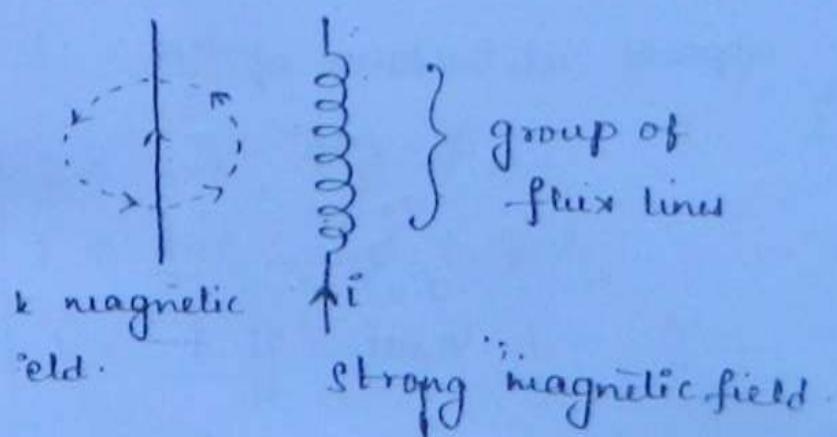
$$R = 0$$

$$I = \infty$$

$$V = 0$$



Inductor



flux direction —
right hand thumb rule.

Faraday's 1st law

When conductor cuts a magnetic lines of the force an emf is induced in the conductor.
The emf induced is proportional to



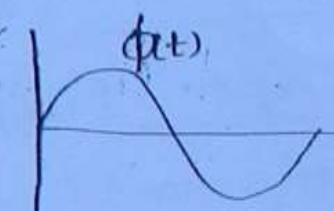
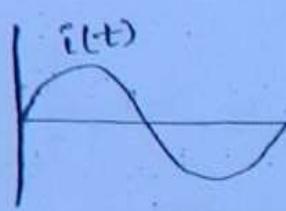
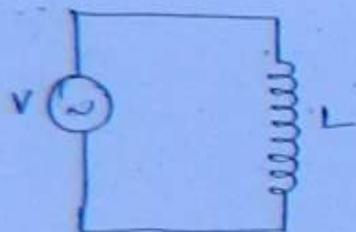
1. emf is induced (c)

$$2. e \propto \frac{d\phi}{dt}$$

(12)

$$\boxed{e = -N \frac{d\phi}{dt}}$$

This is dynamically induced emf, Ex. - generator



$$e \propto \frac{d\phi}{dt}$$

→ This is statically induced emf,
Ex. Transformer (T/F)

$$\boxed{e = -N \frac{d\phi}{dt}} \rightarrow \text{Lenz Law}$$

where -ve sign indicates, based on
Lenz law, induced voltage opposes its cause of
existence.

$$\Psi = N\phi$$

$$V = \frac{d\Psi}{dt}$$

$$\phi \propto i$$

$$\Psi \propto i^2$$

$$\Psi = N\phi$$

↳ flux linkage.

$$V = \frac{d\Psi}{dt}$$

$$\left. \begin{array}{l} \phi \propto i \\ \Psi \propto \phi \end{array} \right\}$$

$$\Psi \propto i \rightarrow \boxed{\Psi = Li}$$

$$V = L \frac{di}{dt}$$

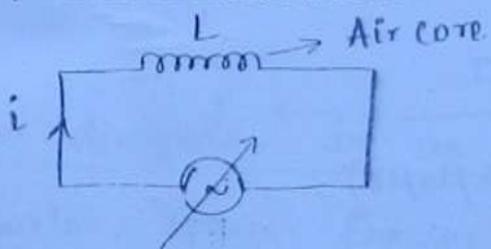
$$L = \frac{V}{(di/dt)}$$

$$\Psi = N\phi \\ \Psi = Li$$

(13)

$$L = \frac{N\phi}{I} \quad H.$$

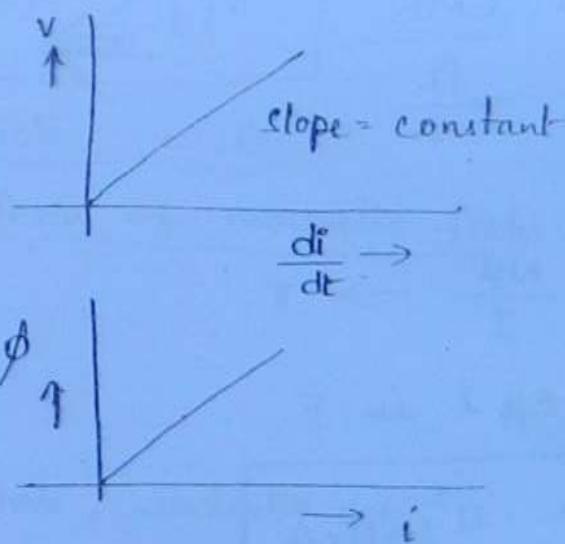
→ when inductance of the inductor independent on current magnitude then the inductor is called as linear inductor. Ex: Air core inductor.



$$L = \frac{N\phi}{I} = \text{constant}$$

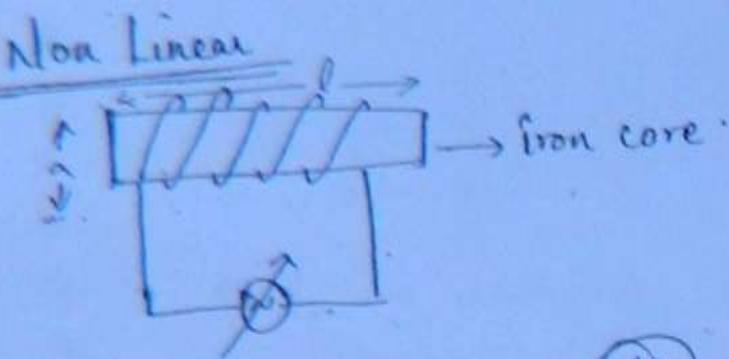
$$i \uparrow 10\%, \phi \uparrow 10\%$$

$$i \uparrow 90\%, \phi \uparrow 90\%$$



$$V = L \frac{di}{dt} \Rightarrow i = \frac{V}{(di/dt)}$$

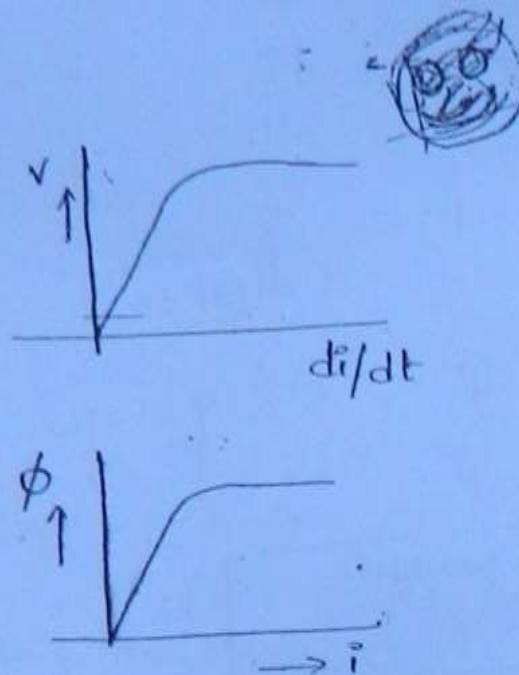
when inductance of the inductor depends on current magnitude, Then the inductor is called as non linear inductor. Ex: IRON CORE inductor



$i \uparrow 10\%$, $\phi \uparrow 10\%$. (14)

$i \uparrow 60\%$, $\phi \uparrow 60\%$.

$i \uparrow 90\%$, $\phi = \text{constant}$
(saturation)



Electric Circuit

$$1) * E = \frac{\text{EMF}}{R}$$

Magnetic circuit

$$\phi = \frac{\text{MMF}}{S} \quad (\text{Reluctance})$$

$$2) E = \frac{\text{EMF}}{\frac{PL}{a}} \Rightarrow \phi = \frac{NI}{(\frac{l}{a\mu_0\mu_r})} \rightarrow I.$$

$$L = \frac{N\phi}{I} \rightarrow I.$$

$$\text{MMF} = \underline{NI}$$

Sub eq I in II

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\mu_r = \left(\begin{array}{c} \mu_0 = 1 \\ \uparrow \\ \text{air} \end{array} \right).$$

*
$$I = \frac{N^2 \mu_0 \mu_r a}{l}$$

permeability is the property of the medium, in which magnetic field exists.

$$L = \frac{N^2}{l/a\mu_0\mu_r}$$

$$S = \frac{l}{N \cdot \mu_0 \mu_r}$$

$$L = \frac{N^2}{S} \quad *$$

(15)

$$S = \frac{MMF}{\phi} = \frac{NI}{\phi} \quad AT/Weber$$

$$L = \frac{N^2}{S} = \frac{\mu_0 \mu_r N^2}{l} \quad \text{AT/Weber}$$

law

form \rightarrow

$$V = L \frac{di}{dt}$$

u

mm \rightarrow

$$i = \frac{1}{L} \int_{-\infty}^t V dt$$

$$P = NI$$

$$P = L \frac{di}{dt} i$$

$$W = \int P dt$$

$$W = \frac{1}{2} Li^2$$

or dissipation in an ideal inductor = 0.

inductor stores Energy in the form of magnetic field (Kinetic Energy).

Inclusion:

For dc supply ~~at~~ steady state, inductor acts as a short circuit.

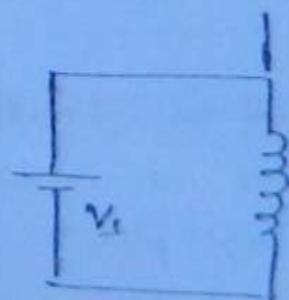
$$V = L \frac{di}{dt}$$

Steady state $\rightarrow di/dt = 0$

$$V = 0$$



S.C.



- Inductor does not allow sudden change of current & since to allow sudden change of current infinite voltage is required and time constant of the circuit should be equal to zero.

(16)

i) $V = L \frac{di}{dt} = \infty \quad \text{dt} \rightarrow 0$

ii) Time constant = L/R .

$r = 0$ mm

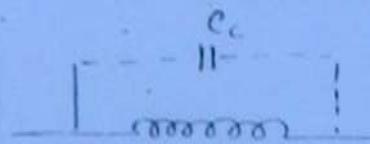
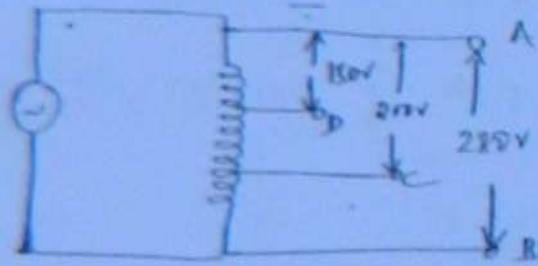
ideal
inductor

mm mm

$r \approx m\Omega$

practical

r : internal resistance.



C_c : inter turn capacitance
(or)

Self capacitance

$$X_c = \frac{1}{2\pi f C}$$

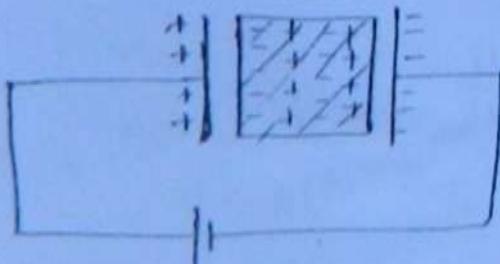
Interturn capacitance is present when inductor is operated either at high frequency or high voltage.

Capacitor

$$Q \propto V$$

$$Q = CV$$

$$C = \frac{Q}{V} \quad \text{or} \quad F$$



$$Q = CV$$

$$\frac{dq}{dt} = C \frac{dv}{dt}$$

$$i = C \frac{dv}{dt}$$

$$C = \frac{V}{(dv/dt)}$$

(17)

$$i = C \frac{dv}{dt}$$

Ohm's law
7th form.

$$P = Vi$$

$$P = C \frac{dv}{dt} V$$

$$W = \int P dt$$

$$W = \frac{1}{2} CV^2$$

$$V = \frac{1}{C} \int_{-\infty}^t i dt$$

8th form.

Energy stored in the
form of electric field.

Power dissipation in ideal capacitor = 0.

Capacitor stores energy in the form of electric field.
(potential energy).

$$C = \frac{\epsilon A}{d}$$



Permittivity is the property of
the medium in which electric
field exists.

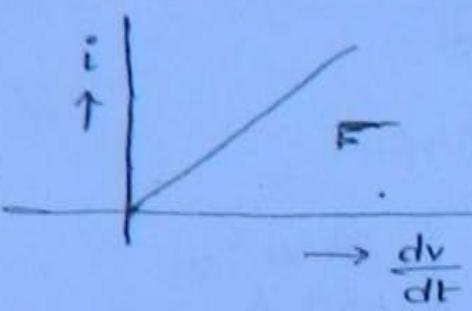
A \rightarrow area of cross section of each conducting plate

d = distance b/w 2 conductors

→ When capacitance of the capacitor independent on the voltage magnitude, then the capacitor is called as linear capacitor.

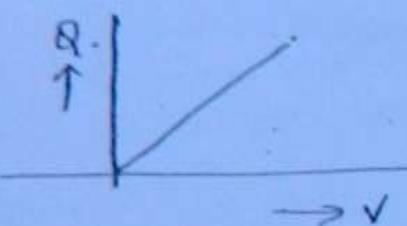
$$C = \frac{Q}{V} = \text{constant.}$$

(15)



$V \uparrow 10\%$, $Q \uparrow 10\%$.

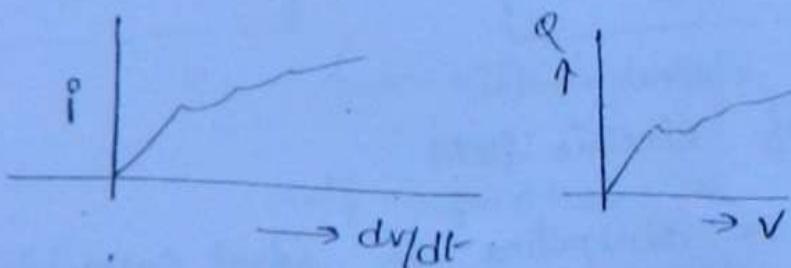
$V \uparrow 90\%$, $Q \uparrow 90\%$.



$$\text{i. } C \frac{dV}{dt} \Rightarrow C > \frac{i}{(dv/dt)}$$

→ When capacitance of the capacitor depends on voltage magnitude, then the capacitor is called as non-linear capacitor. Ex. Varactor diode.

$$C = \frac{Q}{V} = \text{variable.}$$



Conclusion:

→ 1. For dc supply, at steady state, capacitor acts as D.C.

$$\text{i. } C \frac{dV}{dt} \quad \left| \begin{array}{l} \text{Steady state } \frac{dV}{dt} = 0 \Rightarrow \\ i = 0 \end{array} \right. \therefore \text{D.C.} \quad \boxed{\text{vs}}$$

$$\text{ii. } C \frac{dV}{dt}$$

Capacitor does not allow sudden change of voltages.

∴ for "sudden change of voltages" no current is required.

But practically it is not possible.

" " "



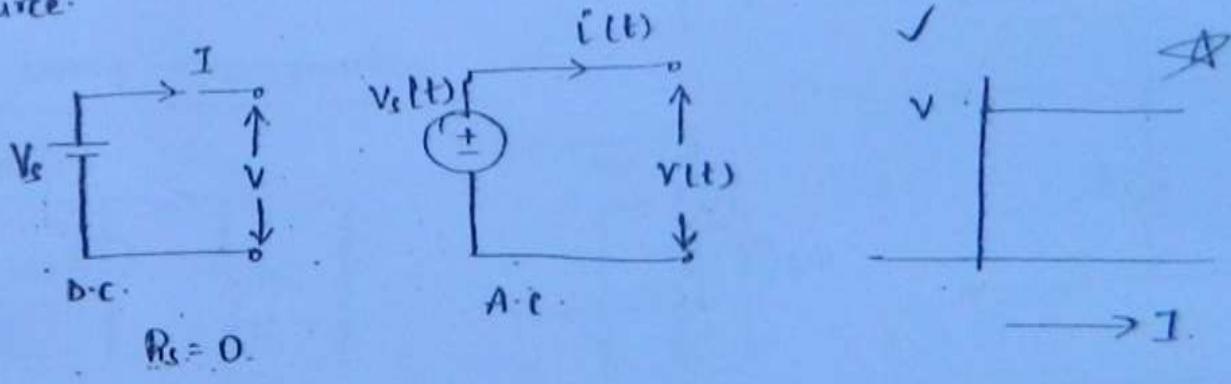
$\tau \approx 1\text{ M}\Omega$ in inductor $\tau \approx 1\text{ m}\Omega$.

\rightarrow leakage path
 $\tau = \text{leakage path resistance}$.

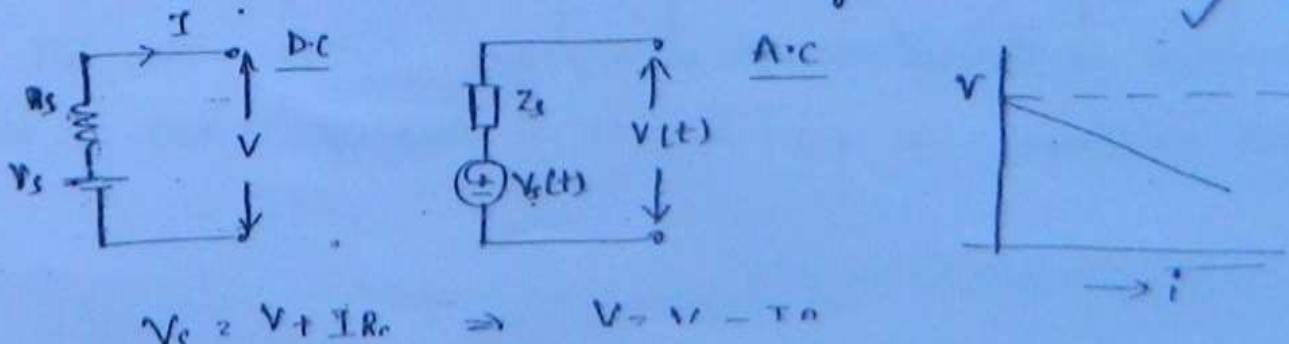
Active Elements

Voltage Source

ideal voltage source delivers energy at the specified voltage (v_s), which is independent on current delivered by this source.

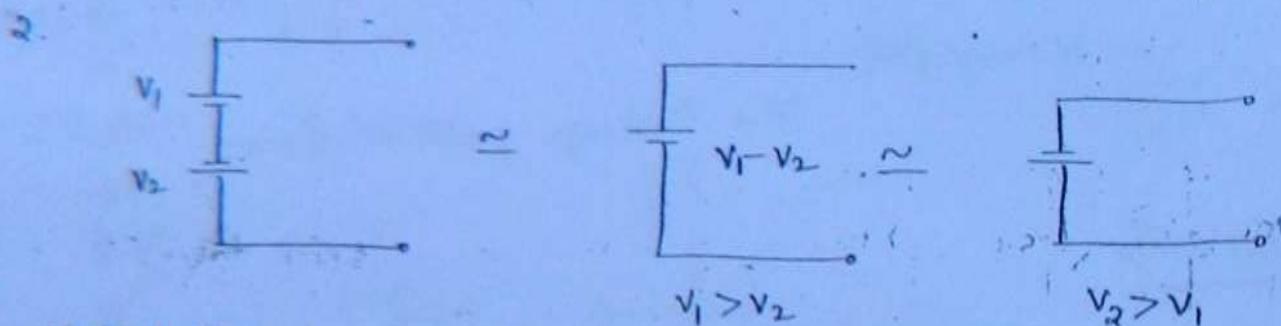
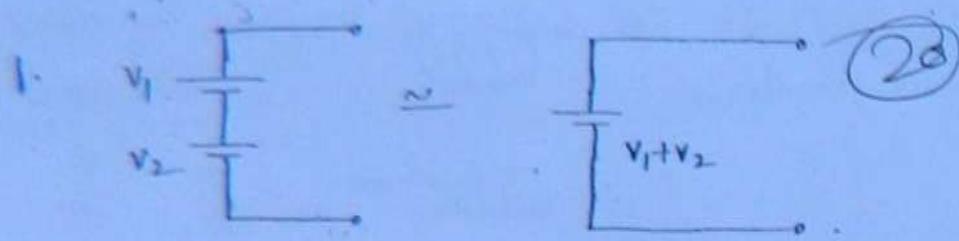


realistic voltage source delivers energy at specified voltage (v), which depends on current delivered by the source.

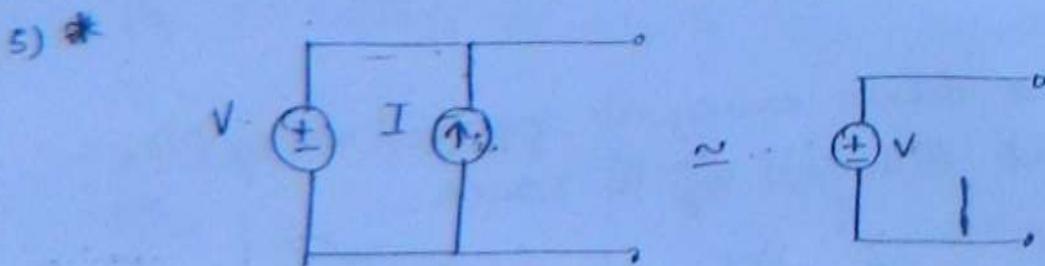
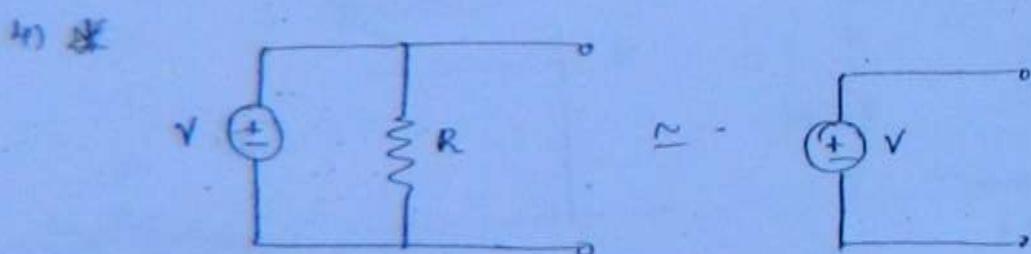
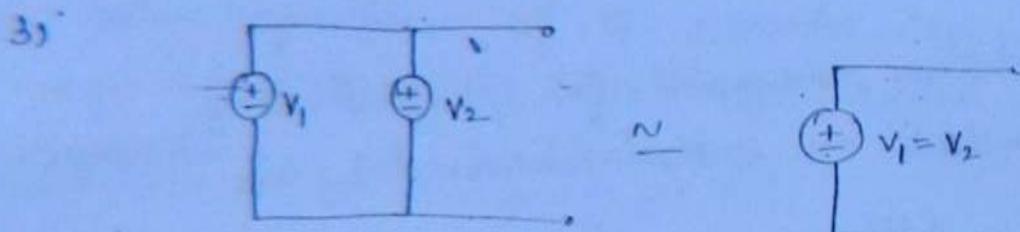


Note:

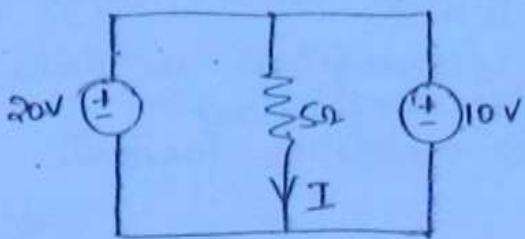
Independent voltage and current source does not obey the ohm's law since voltage and current characteristic is non-linear.



Ideal Source.



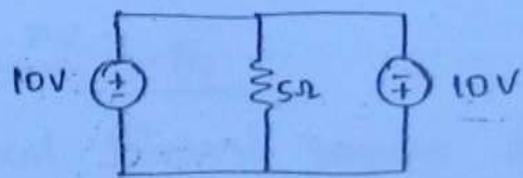
~~A/A~~ Find the value of I for this ckt. shown.



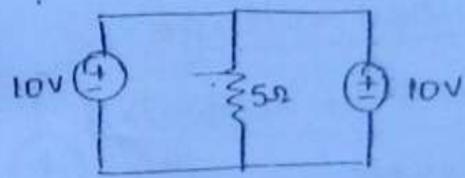
- (a) 2A (b) 4A (c) 6A
(d) none.

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wrt KVL voltage across all the // branches should be equal.



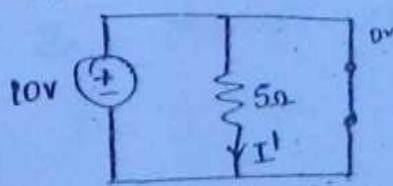
→ The given ckt doesn't satisfy KVL.



→ $I = 10/5 = 2A$

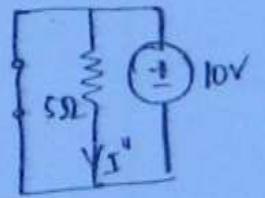
using superposition

Case (i)



$$I^1 = 0$$

Case (ii)

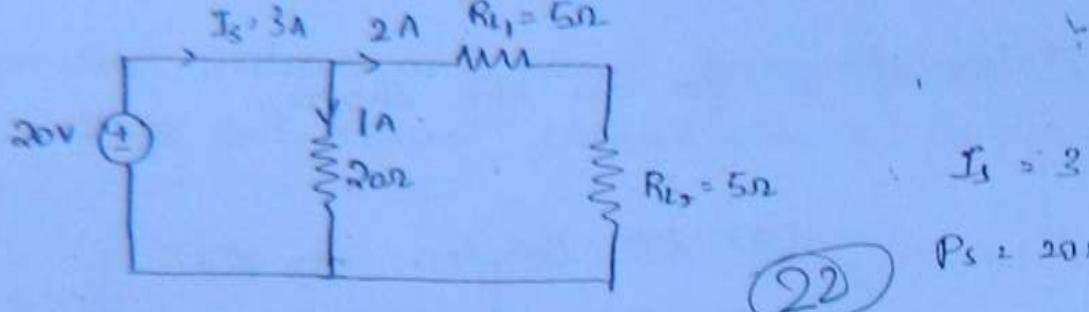


$$I'' = 0$$

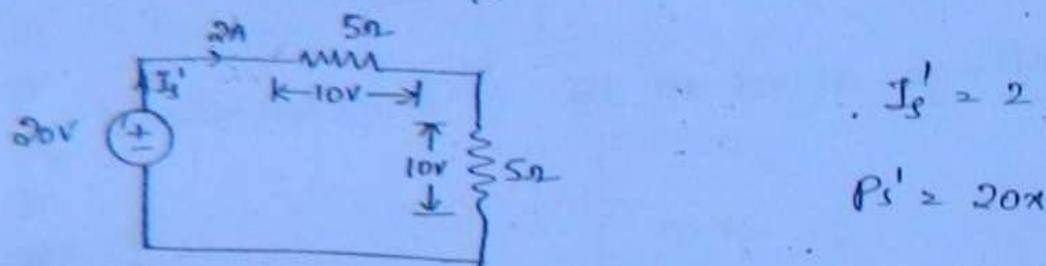
$$I_2 = I^1 + I'' = 0 \times$$

∴ 10V { 0V can't be

or the above circuit superposition theorem cannot be applied since, case 1 & case 2 circuits are not satisfying KVL.

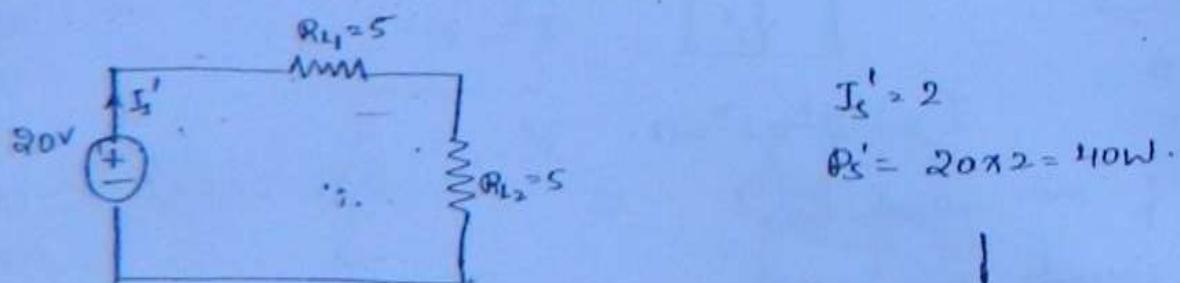
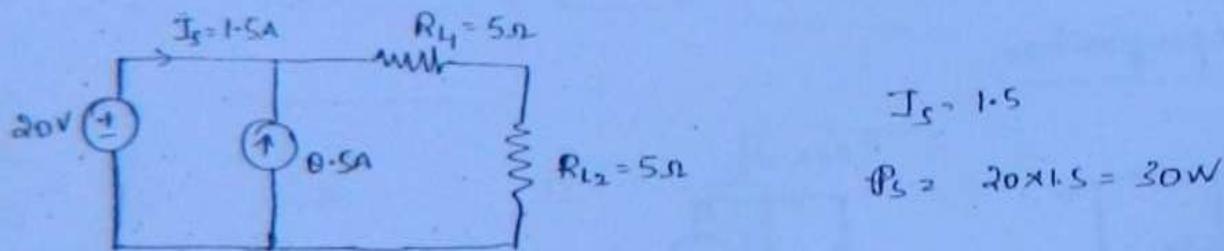


as per eq. ckt (ii) neglect $R = 20\Omega$



Note : 4

- i) In the above circuit 20Ω resistance can be neglected while calculating either load current or load voltage.
- ii) In the above circuit 20Ω resistance cannot be neglected while calculating either source current or power.

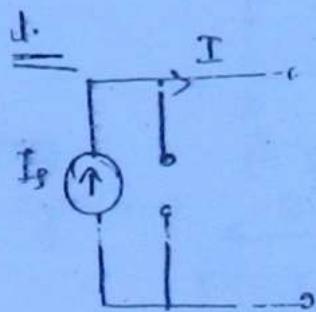


- Note : i) In the above ckt, current source can be neglected while calculating either load current or load voltage.
- ii) In the above circuit, current source cannot be neglected while calculating either voltage source current or power.

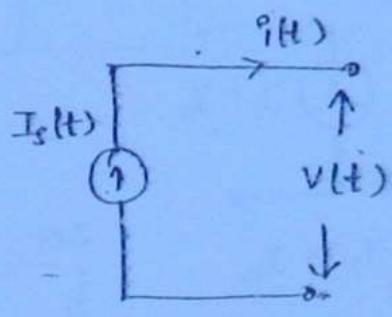
Current Sources

Ideal current source delivers energy at specified current (I) which is independent on voltage across the source.

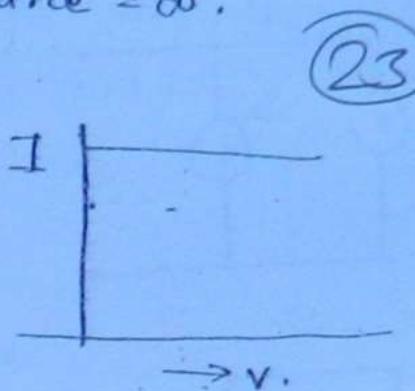
Internal resistance of ideal current source = ∞ .



$$\text{D.C.} \quad R_s = \infty$$

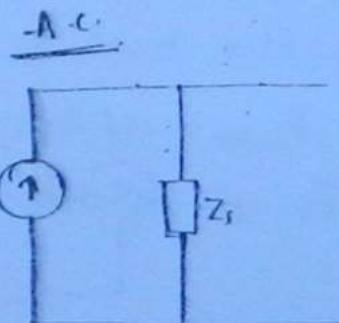
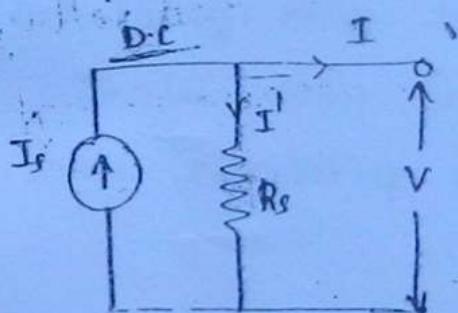


$$\text{A.C.}$$



(23)

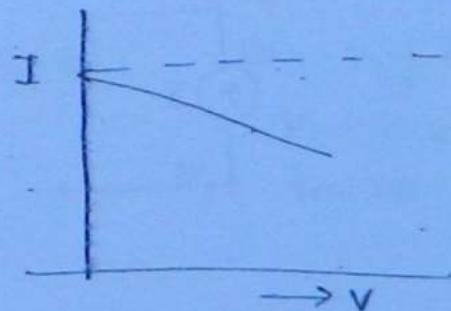
Ideal current source delivers energy at specified current (I) which depends on voltage across the source.



$$I_s = I' + I$$

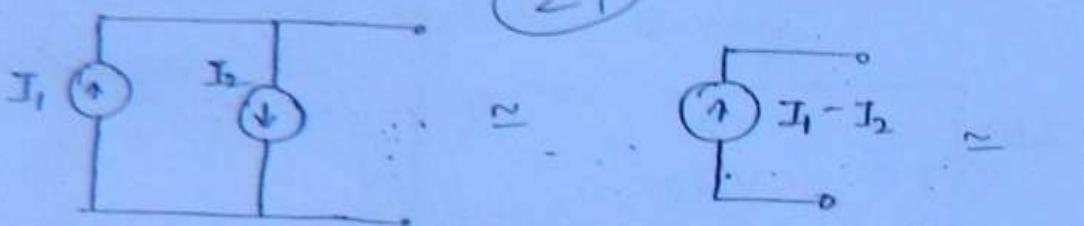
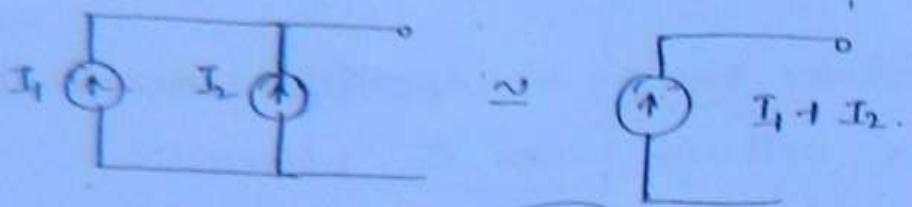
$$I = I_s - I'$$

$$I = I_s - \frac{V}{R_s}$$



In the real time system no independent current source exist.

Equivalent circuit



(24)

$$\approx \quad \text{Top terminal negative} \quad I_2 - I_1$$

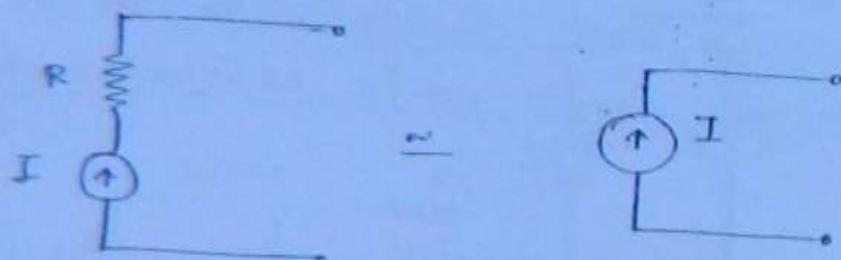
$I_1 > I_2$

$I_2 > I_1$

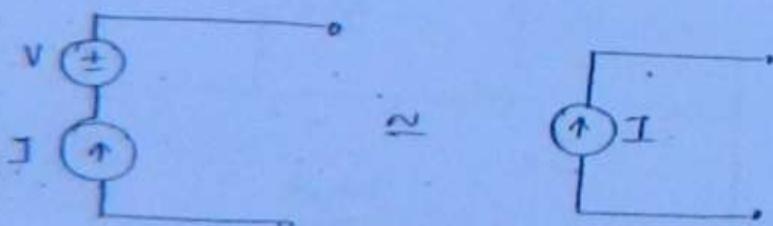
3.



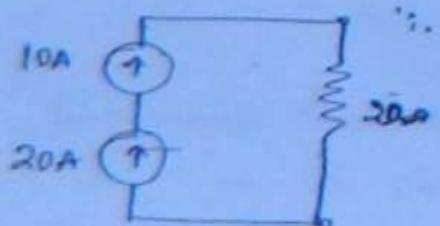
4.



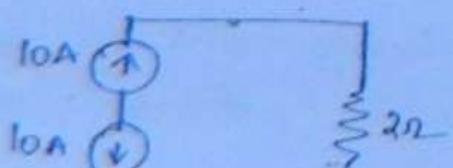
5.



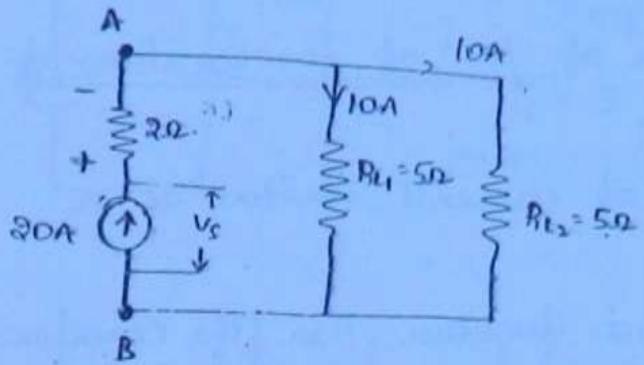
Ex 1.



(a) 10A, (b) 20A (c) 30A none
not satisfying KCL



not satisfying KCL



$$V_{AB} = V_s - 40$$

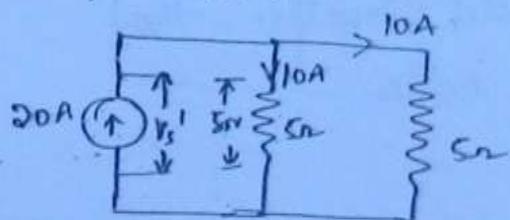
$$50 = V_s - 40$$

$$V_s = 90$$

$$P_s = 90 \times 20$$

(25)

neglecting 2Ω,



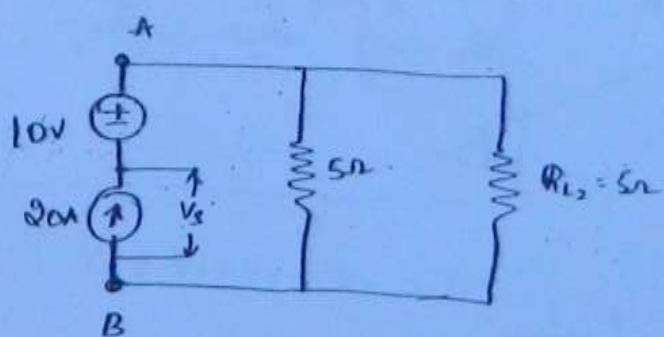
$$V_s' = 50V$$

$$P_s' = 50 \times 20$$

50

In the above circuit 2Ω resistance can be neglected while calculating either load currents or load voltage.

In the above circuit 2Ω resistance cannot be neglected while calculating either source voltage or source power.

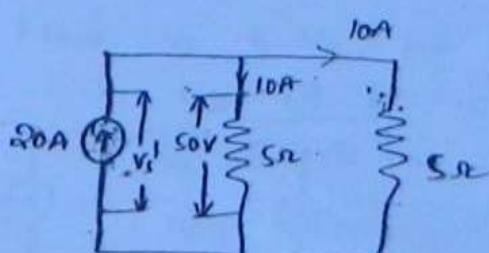


$$V_{AB} = V_s + 10$$

$$50 = V_s + 10$$

$$V_s = 40V$$

$$P_s = 40 \times 20$$



$$V_s' = 50V$$

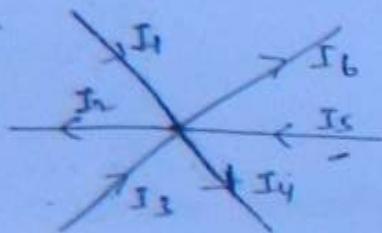
$$P_s' = 50 \times 20$$

To: 1) In the above circuit voltage source can be neglected while calculating either load current or load voltage.

In the above ckt voltage source cannot be neglected

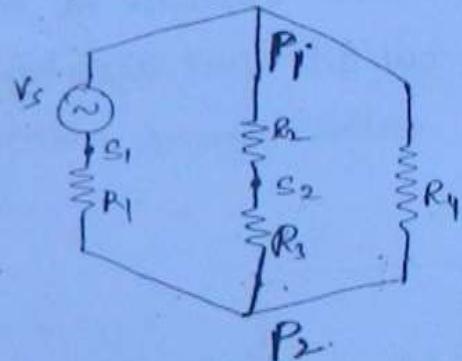
KCL

1. KCL works based on the principle of law of conservation of charge.
2. KCL states that algebraic sum of currents meeting at a point is equal to zero.
3. When two elements are connected together then the common point is called as simple node.
4. When more than 2 elements are connected together, then the common point is called as principle node.

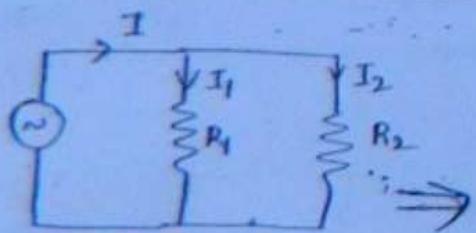


node
simple
principle

$$I_1 - I_2 + I_3 - I_4 + I_5 - I_6 = 0$$

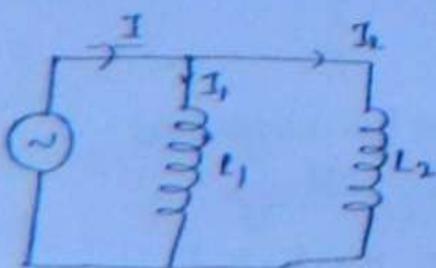


Current dividing rules



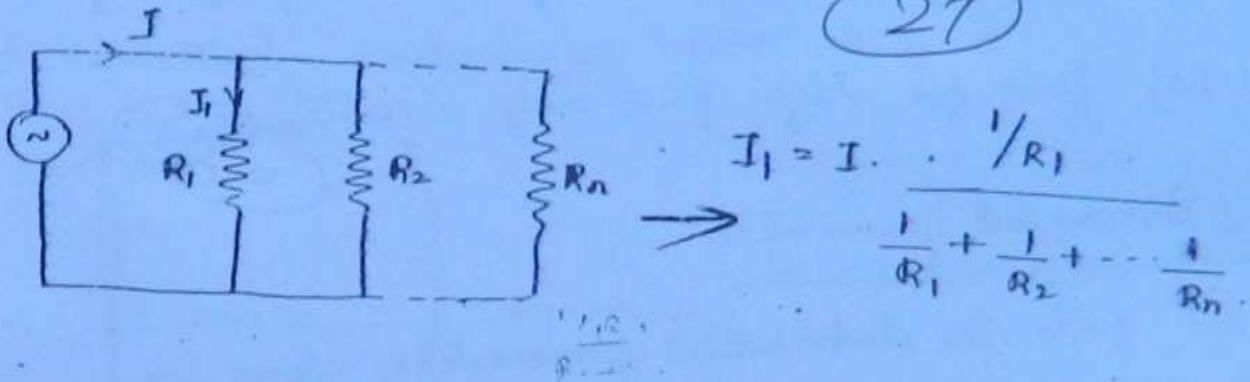
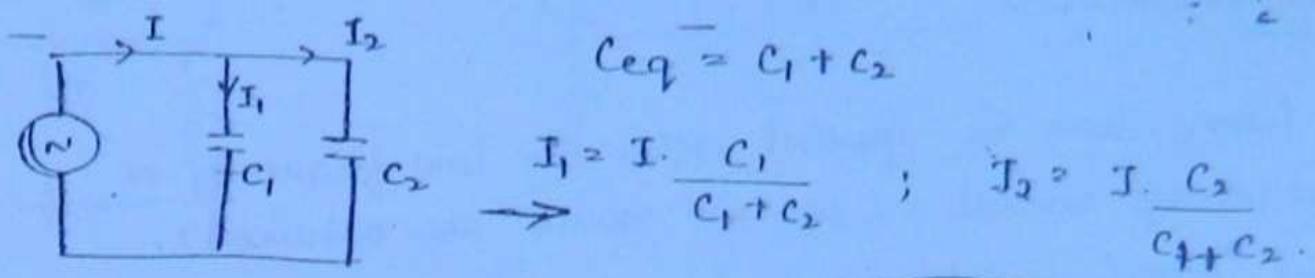
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$I_1 = \frac{I \cdot R_2}{R_1 + R_2} ; \quad I_2 = \frac{I \cdot R_1}{R_1 + R_2}$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

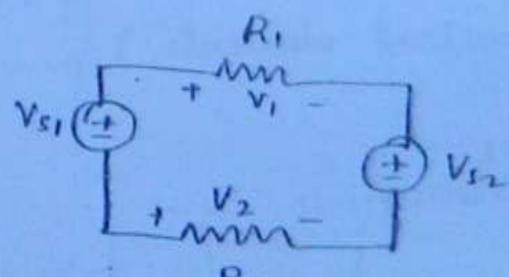
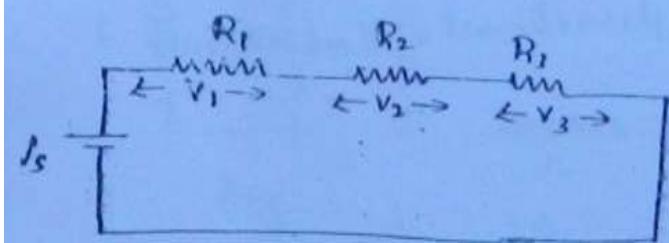
$$I_1 = \frac{I \cdot L_2}{L_1 + L_2} ; \quad I_2 = I \cdot \frac{L_1}{L_1 + L_2}$$



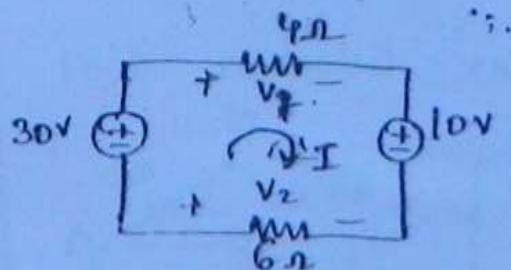
KVL

• KVL works based on the principle of law of conservation of energy.

• KVL states that the algebraic sum of the voltages in a closed loop is equal to zero.



Find V_1 & V_2 of the ckt shown.



$$30 - V_1 - 10 + V_2 = 0$$

$$V_1 - V_2 = 20 \rightarrow \textcircled{1}$$

$$V_1 = 4i, \quad V_2 = -6i$$

$$4i + 6i = 20$$

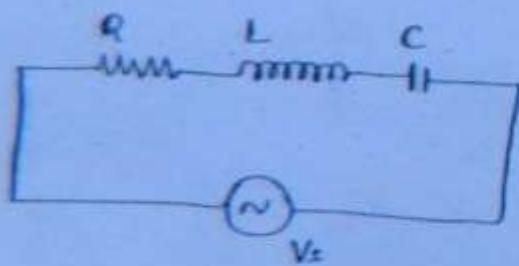
Conclusions.

1. Field theory can be applied either for low frequency or high frequency circuits. (accurate results are obtained).
2. Network theory can be applied only for low frequency circuits.
3. KVL & KCL fails for high frequency circuits.

(28)

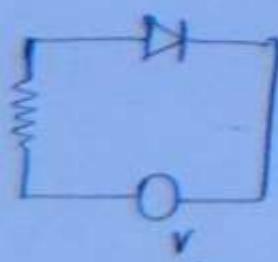
Lumped & Distributed parameters.

1. KVL & KCL fails for distributed parameters since in distributed parameters electrically, it is not possible to separate resistance, inductance and capacitance effect.
2. Ohm's law can be applied for lumped (linear) and distributed parameters.
3. KVL & KCL equations used for lumped parameters circuit. (linear, non linear, unidirectional, bidirectional, time variant, and invariant elements).



$$V_s = iR + L \frac{di}{dt} + \frac{1}{C} \int idt$$

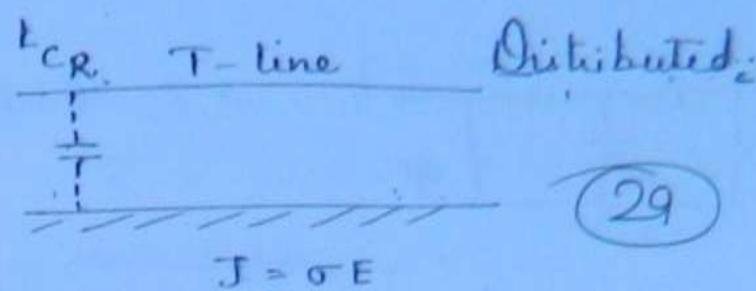
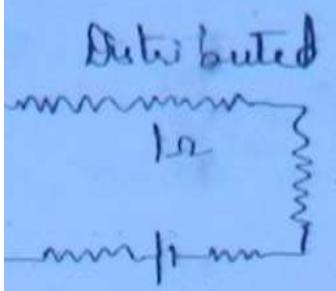
lumped (linear)



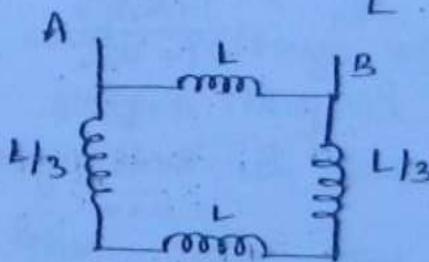
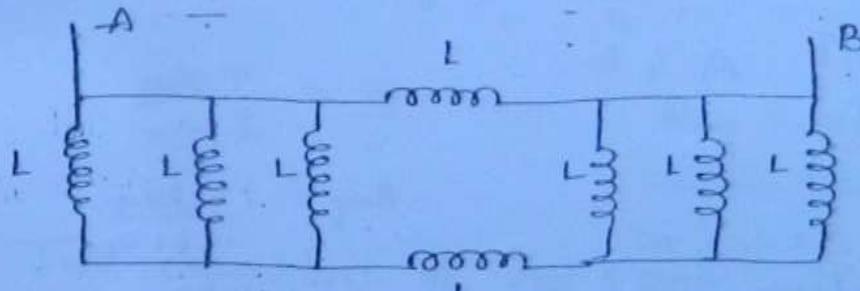
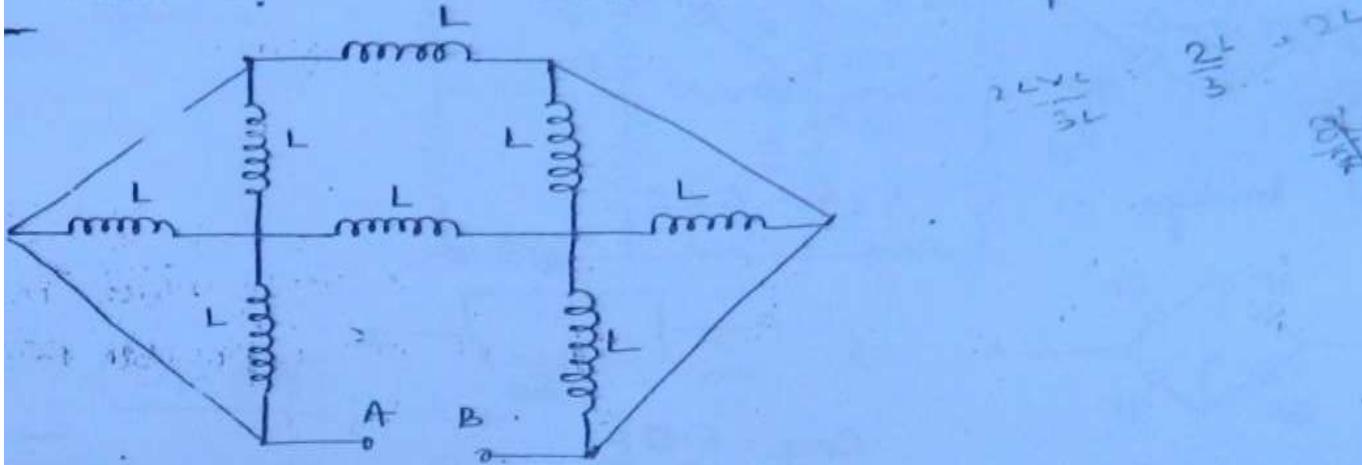
→ Lumped (non linear).



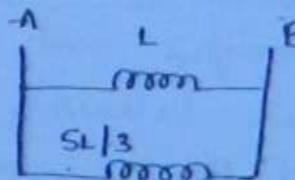
→ Lumped



Find equivalent inductance wrt A & B.

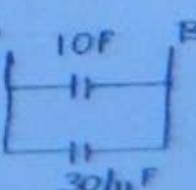
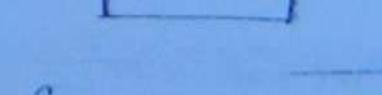
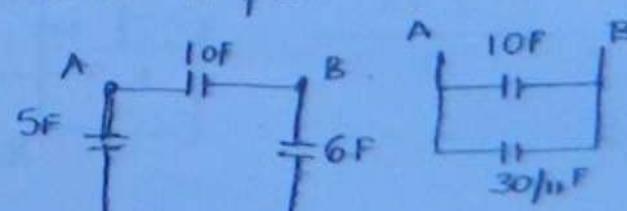
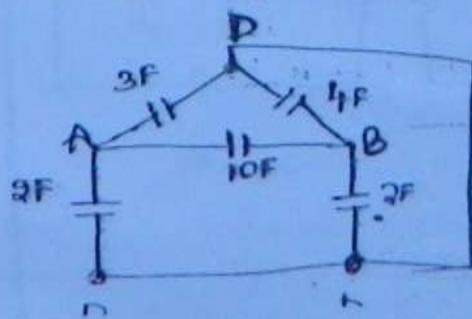


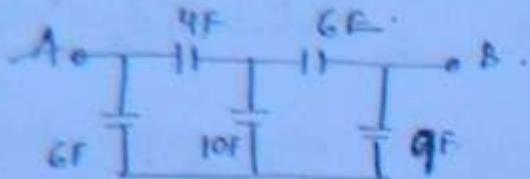
$$L_{eq} = \frac{5L}{8}$$



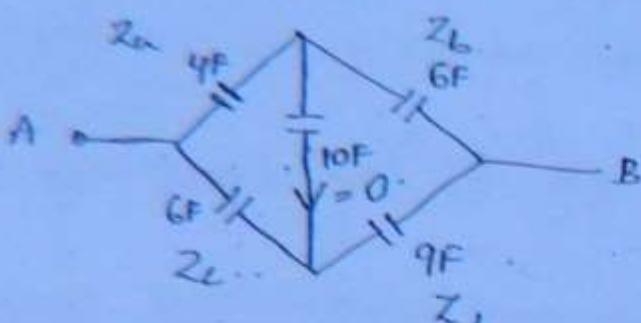
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Find equivalent capacitance wrt A & B.



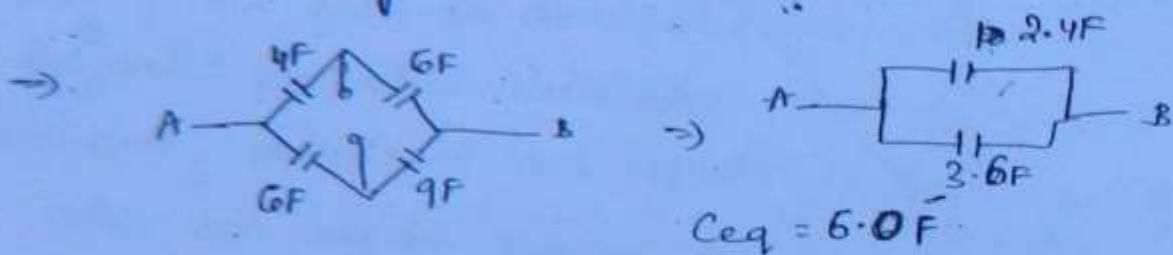


Find C_{eq} wrt A & B.

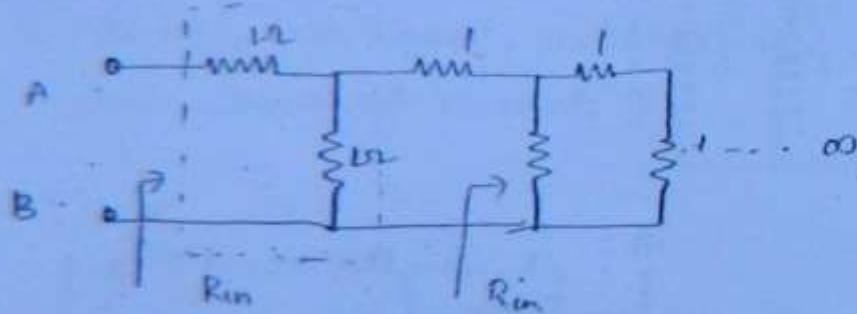


(30)

Balanced bridge $\Rightarrow 4 \times 9 = 6 \times 6$.



Q. Find eq. resistance wrt. A & B.



$$R_{eq} = 1 + \frac{R_{eq}}{1 + R_{eq}}$$

$$R_{eq} = \frac{1 + 2R_{eq}}{1 + R_{eq}}$$

$$R_{eq} + R_{eq} = 1 + 2R_{eq}$$

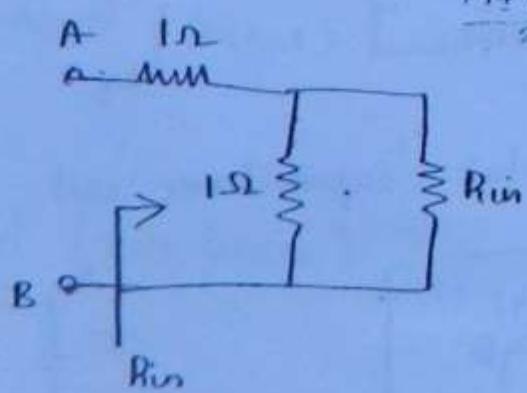
$$R^2 - R - 1 = 0$$

$$\frac{+1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}$$

$$R_{eq} = 1 + \frac{R_{eq}}{1 + R_{eq}}$$

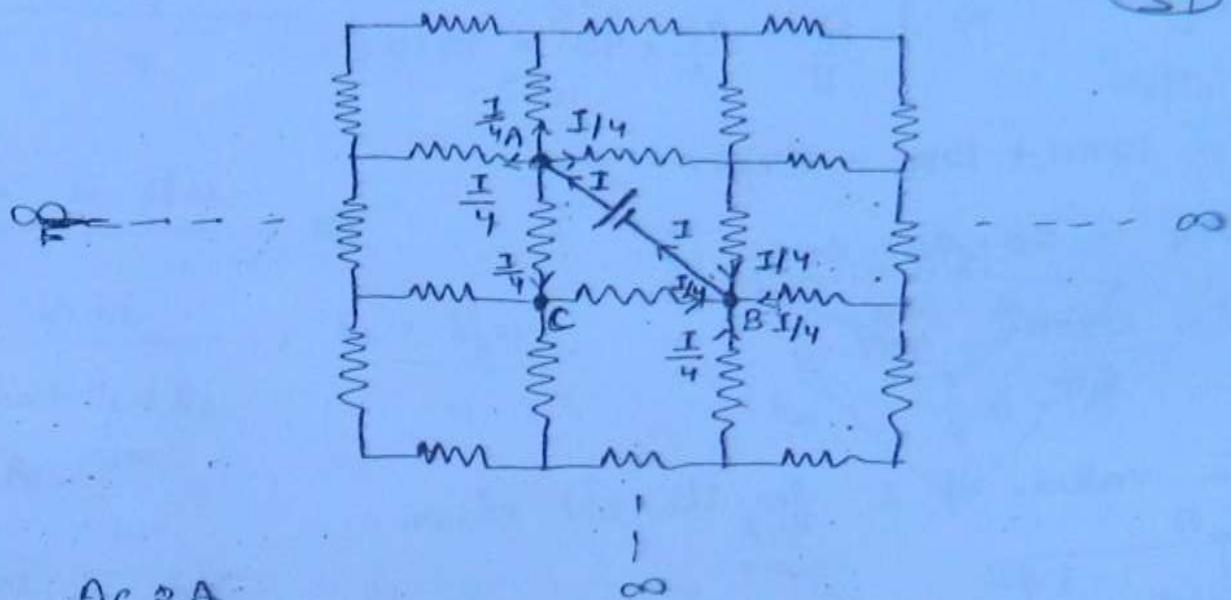
$$R_{eq} - R_{eq} - 1 = 0$$

$$R_{eq} = \frac{1 \pm \sqrt{5}}{2}$$



Find eq. resistance w.r.t. A & B. (assume each resistor is $R \Omega$)

(31)



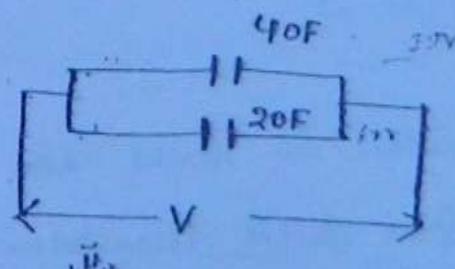
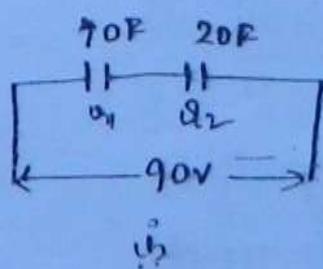
AC BA

$$V = R \cdot \frac{I}{4} + R \cdot \frac{R}{4} = \frac{R \cdot I}{2}$$

$$\Rightarrow \frac{V}{I} = \frac{R}{2} \Rightarrow \underline{\underline{R_{eq} = R/2}}$$

Two capacitors of $40F$ & $20F$ are connected in series to a source voltage of $90V$. When two capacitors are charged fully, they are connected in parallel. Find voltage across capacitors in parallel connection.

- (a) $40V$ (b) $45V$ (c) $30V$ (d) $60V$



$\frac{V}{120F}$
60

$$V_1 = \frac{90 \times 20}{60} = 30V \quad V_2 = \frac{90 \times 40}{60} = 60V$$

$$\text{i)} \quad C_{eq} = \frac{40 \times 20}{40+20} = \frac{40}{3} \text{ F}$$

(31)

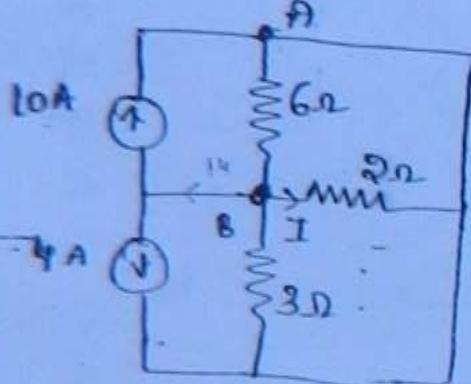
$$Q = C_{eq} \cdot V \Rightarrow Q = \frac{40}{3} \times 90 = 1200 \text{ C}$$

$$\text{ii)} \rightarrow Q_T = 1200 + 1200 = 2400 \text{ C} \quad \text{**}$$

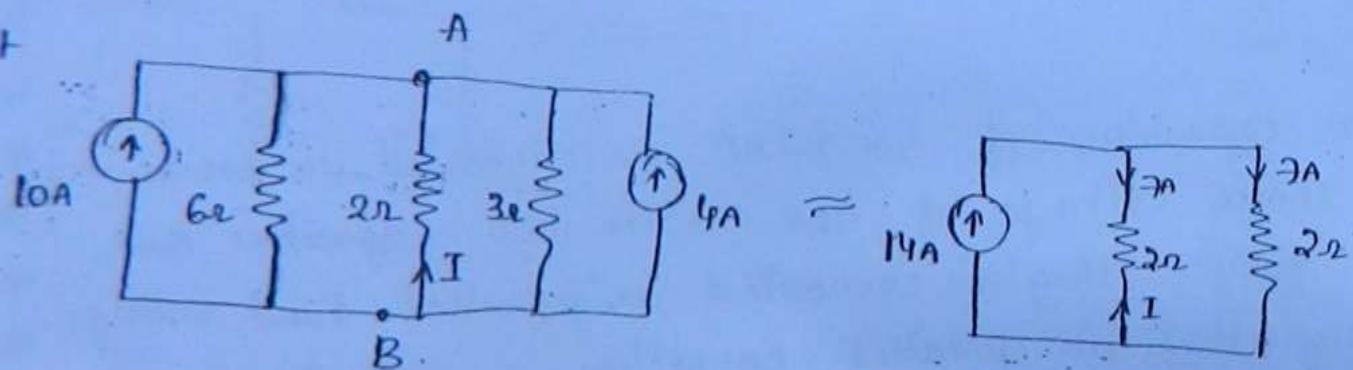
$$C_{eq} = 20 + 40 = 60 \text{ F}$$

$$V = \frac{2400}{60} = 40 \text{ V}$$

Q. Find the value of I for the ckt shown.



Eq. ckt

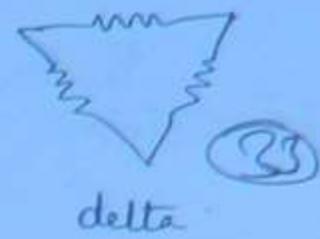
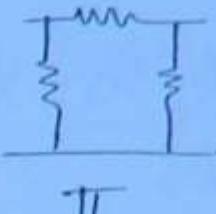
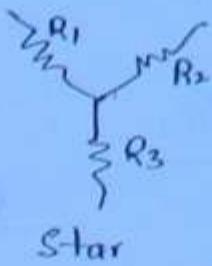
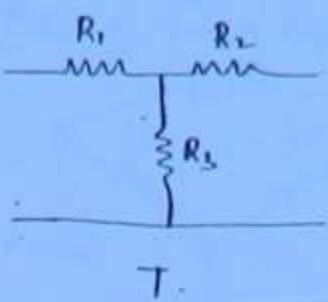


$$I = -7 \text{ A}$$

82/6/11

Note :-

When elements are connected neither in series nor in parallel, to reduce the network, Star delta transformation is used.



Delta to star.

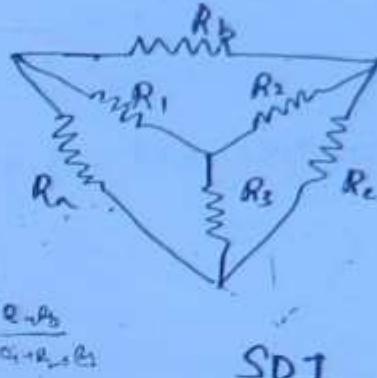
$$i = \frac{R_a R_b}{R_a + R_b + R_c}, \quad R_2 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$R_3 \dots$

Star to delta

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}, \quad R'_b = \frac{R_a R_b}{R_a + R_b + R_c}, \quad R'_c = \frac{R_a R_c}{R_a + R_b + R_c}$$

\therefore ~~for~~



SDT

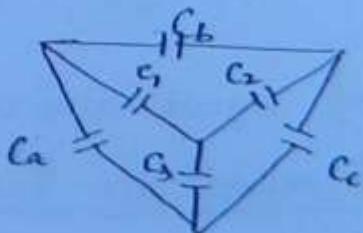
Procedure of transformation from delta to star (or) star to delta resistors, inductors and impedances is the same.

Capacitors

Delta to star.

$$i = \frac{1/C_a - 1/C_b}{1/C_a + 1/C_b + 1/C_c} \quad \therefore$$

$$i^2 = \frac{1/C_b - 1/C_c}{1/C_a + 1/C_b + 1/C_c}, \quad \frac{1}{C_3} = \frac{1/C_a - 1/C_c}{1/C_a + 1/C_b + 1/C_c}$$



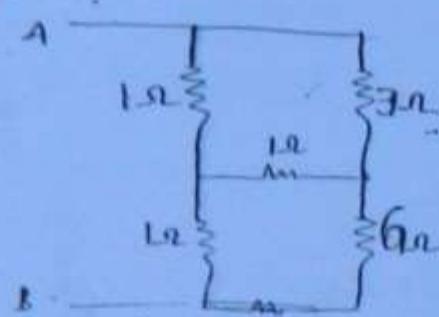
convert to delta

$$\frac{1}{C_a} = \frac{1/C_1 + 1/C_2 + 1/C_3}{1/C_2}$$

(34)

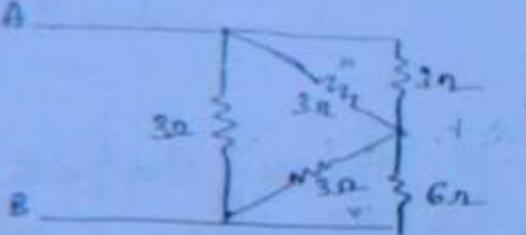
$$\frac{1}{C_b} = \frac{1}{1/C_3} \quad \frac{1}{C_c} = \frac{1}{1/C_1}$$

Q. Find eq. resistance bet A & B.



Convert R_A, R_B, R_C $\gamma \rightarrow \Delta$

Sol.



$$R_A = \frac{(1 \times 1) + (1 \times 1) + (1 \times 1)}{1}$$

$$R_A = 3\Omega$$

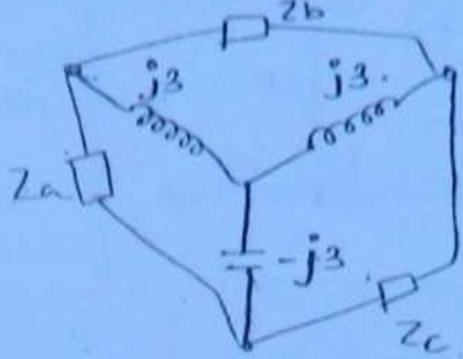
$$R_B = R_C = \frac{1.5}{3.5 \times 3} = \frac{1.5}{10.5} = 0.15\Omega$$

$$R_{AB} = 1.6\Omega$$

Note:-

1. when resistors of equal value are transformed from $\gamma \rightarrow \Delta$,
resistance is increased by 3-times SDT
2. when capacitors of equal value are transformed from $\gamma \rightarrow \Delta$
Capacitance decreases by 3-times SDT

Draw the eq. Δ network for the network shown.

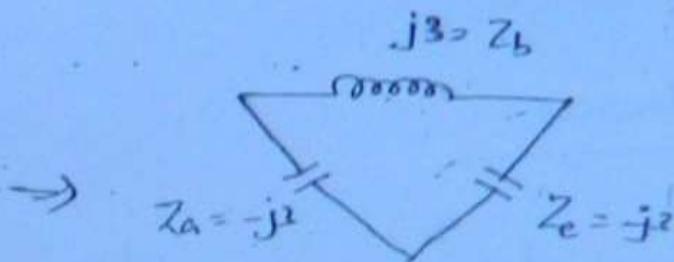


(35)

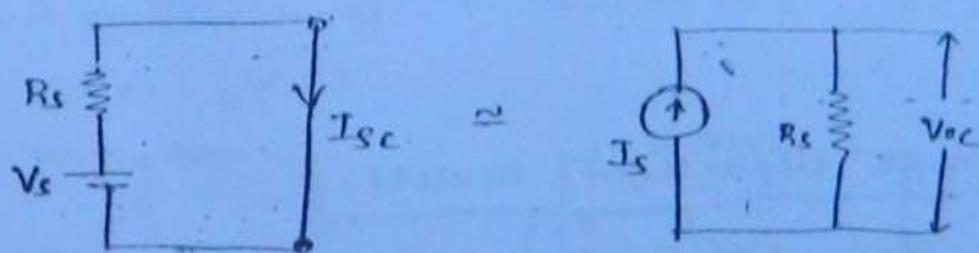
$$Z_a = \frac{(j^3)(j^3) + (j^3)(-j^3) + (j^3)(-j^3)}{(j^3)} = -j^3$$

$$Z_b = \frac{(j^3)(-j^3)}{-j^3} = j^3$$

$$Z_c = \frac{(-j^3)(-j^3)}{j^3} = -j^2$$



Source Transformation



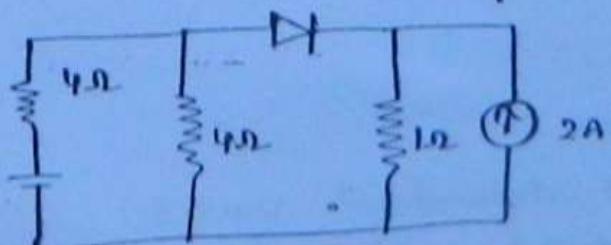
$$I_s = I_{sc} = \frac{V_s}{R_s}$$

$$R_s = R_s$$

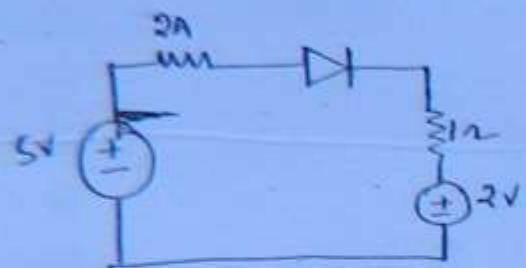
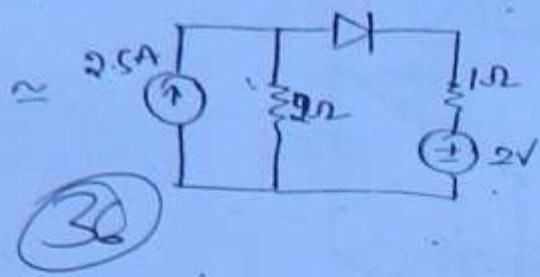
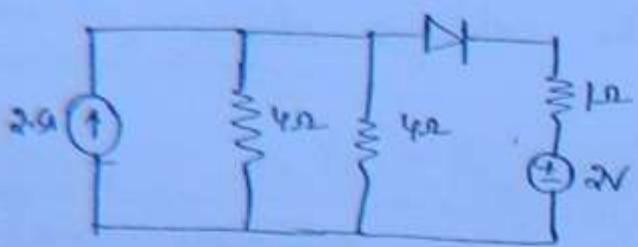
$$V_s = V_{oc} = I_s R_s$$

$$R_s = R_s$$

and current of ideal diode of the circuit shown.

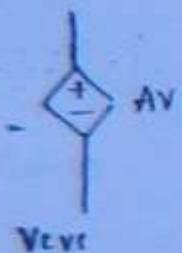


2A

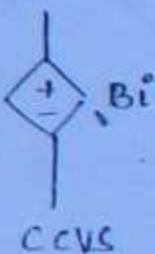


$$i = \frac{5-2}{2+1} = 1A$$

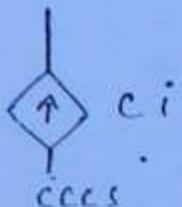
Dependant sources.



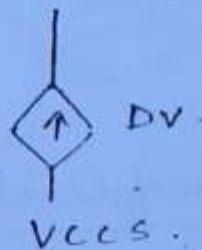
A \rightarrow no unit



B \rightarrow i
 $V = iR$



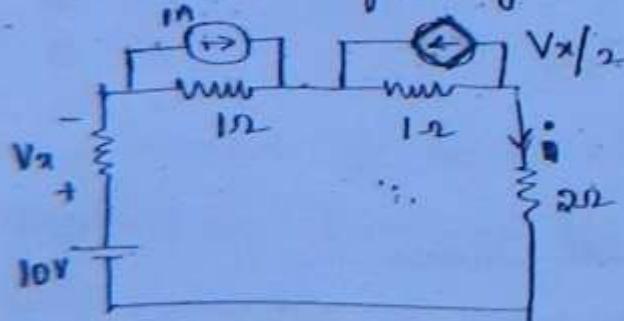
C \rightarrow no unit



D \rightarrow mho or S.
 $i = V/R = GV$

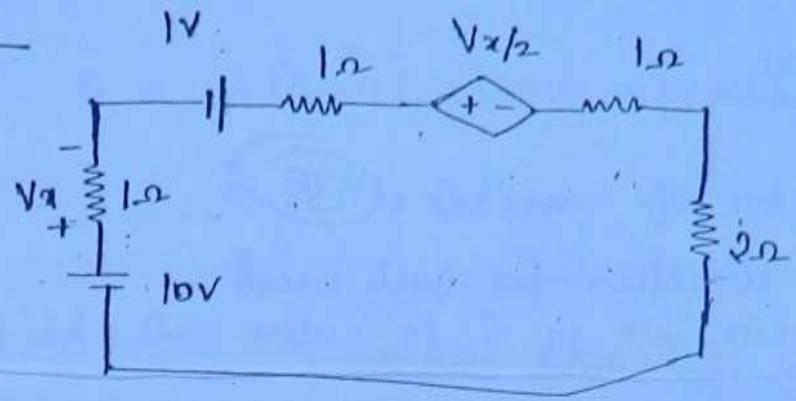
\rightarrow All the above are linear dependant sources.

Q. Find current flowing through 2Ω resistor.



Note: *

While applying source transformation for dependant source, whenever dependant source magnitude depends without distinction to about transformation.



(37)

$$\Rightarrow I = \frac{10 + 1 - V_x/2}{1 + 1 + 1 + 2}$$

$$I = 9A$$

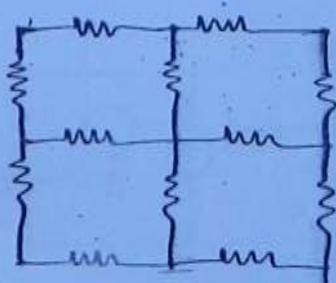
$$V_x = (\cancel{1} \times I)$$

$$V_x = I \cdot R$$

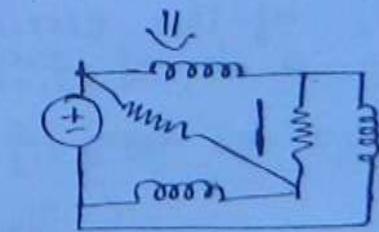
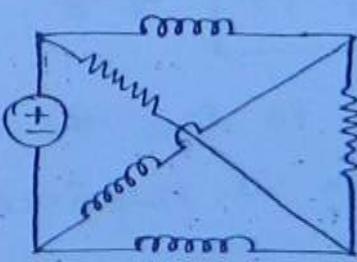
Mesh Analysis

Mesh is a loop which does not consist of any inner loop. When the network is drawn on plane without any crossover, then the network is called as planar network.

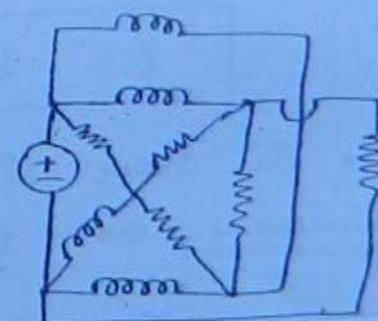
Mesh analysis can be applied only for planar networks.



planar network



non planar



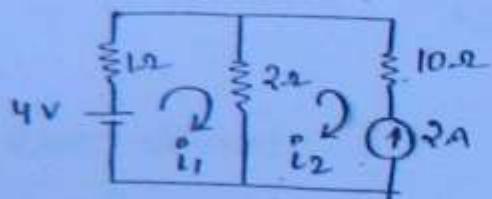
planar

W/ $\cancel{1}$
sol(3)

Procedure of Mesh Analysis

- Step 1: Identify total number of meshes. (38)
- Step 2: Assign the current direction for each mesh.
- Step 3: Develop KVL equation for each mesh.
- Step 4: By solving KVL equations, find loop currents.

Ex:



$$+4 + 3i_1 - 2i_2 = 0$$

$$i_2 = -2$$

$$i_1 = 0$$

Note:

Total no. of eqns = Total no. of meshes.

$$C = M = 2$$

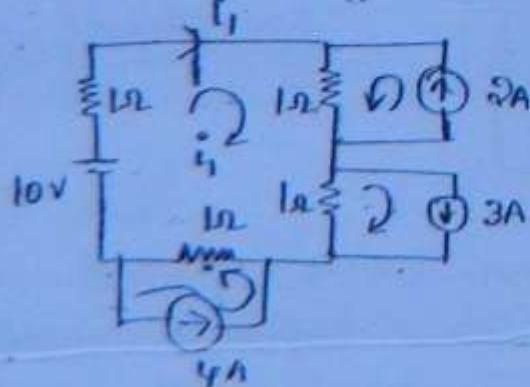
b = Total number of branches

N = Total no. of nodes.

$$C = b - (N - 1) \quad ***$$

* In the above circuit, to find the loop current, minimum one equation is required.

Q. Find the value of i_1 of this circuit shown.



$$-10 + i_1 + i_2 + 2 + i_1 + 3 + i_1 + 4 = 0$$

$$-10 + 4i_1 + 3 = 0$$

$$i_1 = 7/4$$

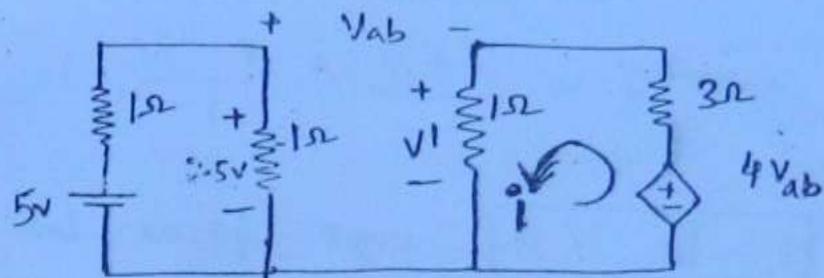
$$\frac{10 - 2 + 3 + 4}{4} = 7/4$$

$$0 = -10 + 4i_1 + (1 \times 2) - (1 \times 3) + 4 \times 1$$

$$i_1 = 7/4 \text{ A}$$

(39)

Find the value of i of the circuit shown.



$$I = \frac{4V_{ab}}{3+1} = V_{ab}$$

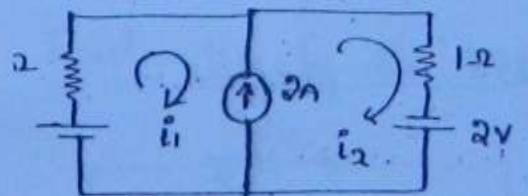
$$V^I = (1 \times I) \Rightarrow V^I = I = V_{ab}$$

$$-2.5 + V_{ab} + V^I = 0$$

$$-2.5 + V_{ab} + V_{ab} = 0 \Rightarrow V_{ab} = 1.25 \text{ V}$$

$$I = 1.25 \text{ A}$$

Find i_1 & i_2 of the circuit shown:



When current source branch is common for two nodes, it is possible to find solution by using super node technique.

$$-5 + (1 \times i_1) + (1 \times i_2) - 2 = 0 \rightarrow \text{kVL}$$

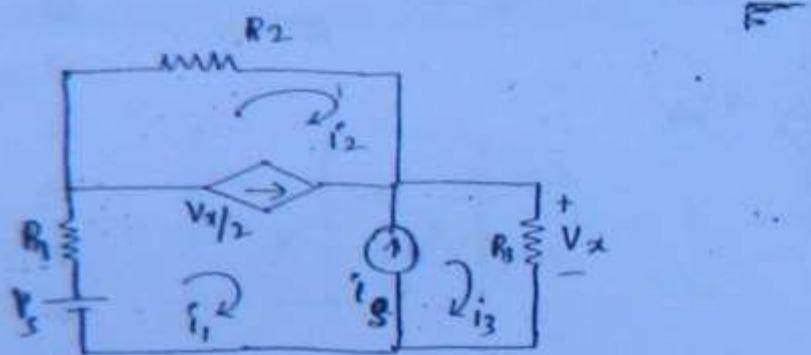
$$i_2 - i_1 = 2 \rightarrow \text{kcl}$$

Mesh \rightarrow KVL + Ohm's Law

* Super mesh \rightarrow KVL + KCL + Ohm's Law.

(46)

Q: Develop mesh equations of the circuit shown.



$$-V_s + i_1 R_1 + i_2 R_2 + i_3 R_3 = 0 \quad \rightarrow \text{KVL}$$

$$i_1 - i_2 = \frac{V_{12}}{R_1} \quad \rightarrow \text{KCL}$$

$$i_3 - i_1 = i_s \quad \rightarrow \text{KCL}$$

Nodal Analysis

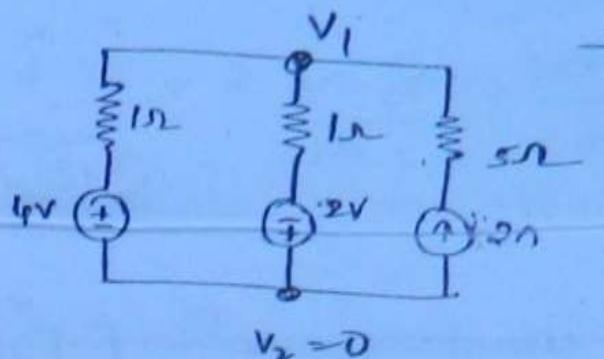
\rightarrow Nodal analysis can be applied for planar and non-planar networks.

mesh only
in plane

Procedure of nodal analysis.

1. Identify total number of nodes.
2. Assign the voltage at each node, one of the nodes is taken as a reference node and reference node potential should be equal to ground potential.
3. develop KCL equation at each non reference node.
4. By solving KCL equations find node voltages.

Ex.

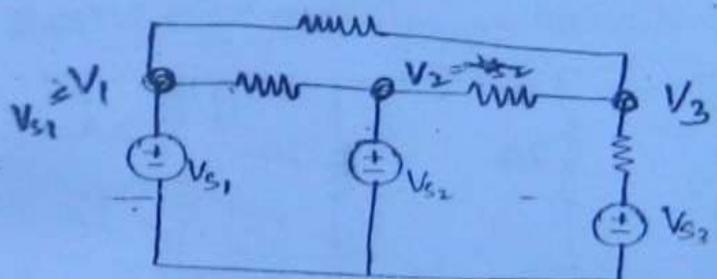


(41)

$$\frac{V_1 - 4}{1} + \frac{V_1 - 2}{1} = 2 \Rightarrow V_1 = 2V.$$

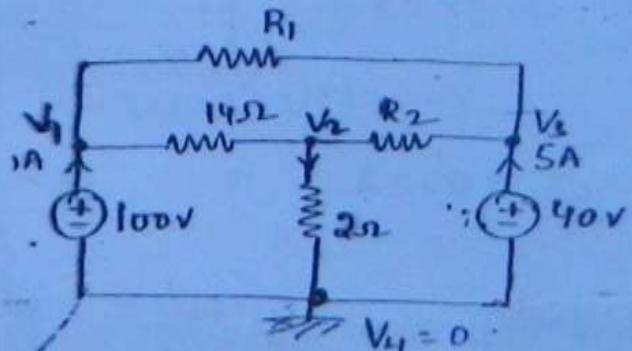
Total no. of eqns C = N - 1

N = no. of nodes.



To find node voltages in the above circuit,
minimum one equation is required.

Find R1 & R2 of the circuit shown.



$$\text{node 1} \rightarrow 10 = \frac{V_1 - V_2}{14} + \frac{V_1 - V_3}{R_1}$$

$$R_1 = 12\Omega$$

$$\text{node 4} \rightarrow I = 5 + 10 = 15A$$

$$V_2 = 15 \times 2 = 30V$$

$$V_1 = 100V$$

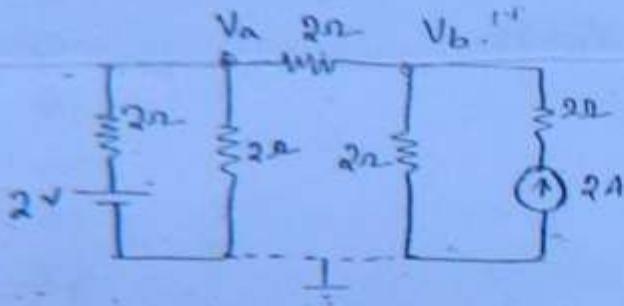
$$V_3 = 40V$$

$$\text{node } 3 \rightarrow \quad 5 = \frac{V_3 - V_2}{R_2} + \frac{V_3 - V_1}{R_1}$$

$$\frac{10}{6} = \frac{100}{60}$$

三

Q. Find V_a & V_b of the circuit shown. [DRDO]



$$\frac{V_A - 0}{2} + \frac{V_A}{2} + \frac{V_A - V_B}{2} = 0.$$

$$\frac{V_b}{2} + \frac{V_b - V_{\alpha^-}}{2} = \varrho.$$

$$V_0 = g/\zeta$$

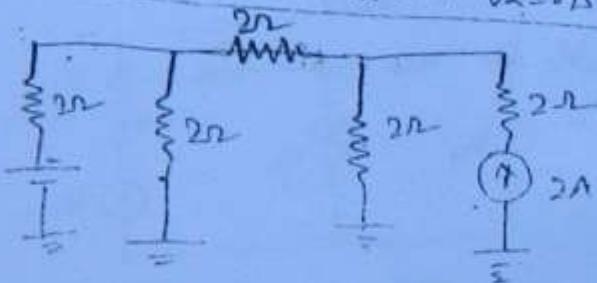
$$V_L = 14/\zeta \sqrt{}$$

$$\frac{V_A - 2}{2} + \frac{V_B}{2} + \frac{V_C - V_L}{2} = 0$$

$$2v_A - v_B = 2 \cdot \frac{v_A + v_B - v_B}{2} - 1 = 0$$

$$3V_A - \frac{4-V_A}{2} = 2 + \frac{V_B - V_A}{2} = 0$$

$$\frac{1}{2}VA + \frac{1}{2} = 6VA - 4 - VA = 4 \Rightarrow VA = 8/5$$

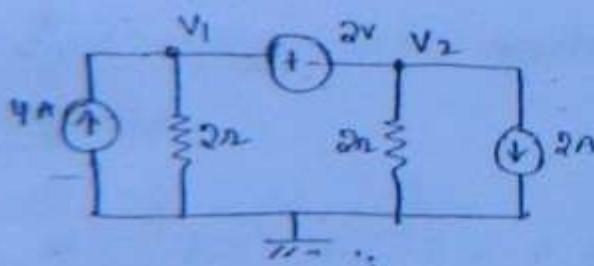


Q. - Find V_1 & V_2 of the circuit shown.

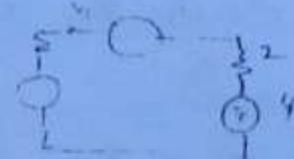
$$\frac{V_b - V_a}{2} = \eta$$

$$2V_b - V_a = 1$$

$$V_{\text{B}} = \frac{V_0 + V_{\text{A}}}{2}$$



- 54 -



Note 8-

when ideal voltage source is connected b/w two non reference nodes, it is possible to find solution by using super node technique.

$$I = \frac{V_1}{2} + \frac{V_2}{2} + 2 \rightarrow KCL$$

$$\begin{aligned} V_1 + V_2 &= 4 \\ V_1 - V_2 &= 2 \end{aligned}$$

$$V_1 - V_2 = 2 \rightarrow KVL$$

$$2V_1 = 6$$

$$V_1 = 3V, \quad V_2 = 1V$$

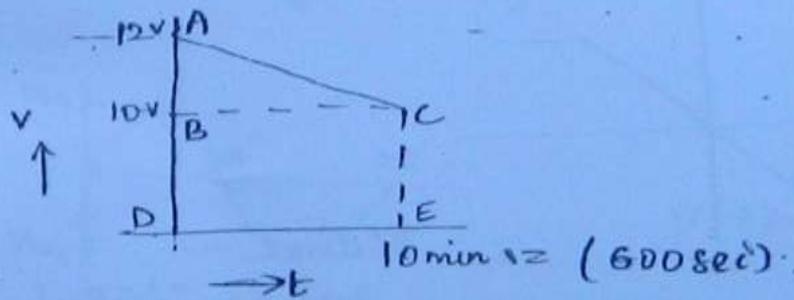
(43)

nodal \rightarrow KCL + Ohm's law

Super node \rightarrow KVL + KCL + Ohm's law.

A fully charged mobile phone is good for 10 min talk time. During talk time, battery delivers a constant current.

Q4. The voltage characteristic of battery is as shown in the figure. Find energy of the battery during talk time.



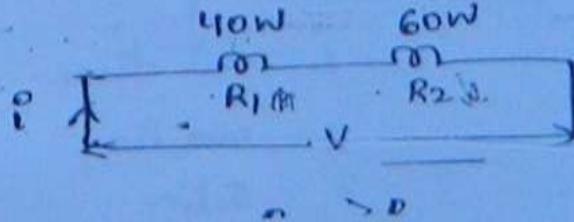
$$VT = \Delta ABC + \square BCED$$

$$\therefore \frac{1}{2} \times 2 \times 600 + 600 \times 10 = 6600$$

$$W = Vit$$

$$\underline{\text{eg}}. = 6600 \times 2A = \underline{13.2 \text{ kJ} > W}$$

which bulb gives more brightness.



210V
V, P, f \rightarrow 50Hz

$$P = \frac{V^2}{R} \Rightarrow P \propto \frac{1}{R}$$

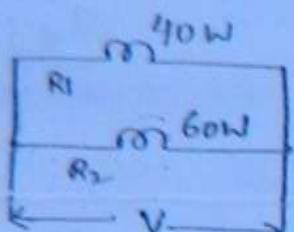
$$P_1 = i^2 R_1 \quad ; \quad P_2 = i^2 R_2 .$$

$$\therefore P = i^2 R_1 \\ P \propto R_1$$

$$P_1 > P_2 .$$

(44) BI.

vii,



$$R_1 > R_2 .$$

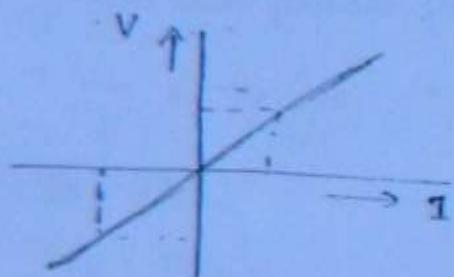
$\therefore B_2$

$$P_1 = \frac{V^2}{R_1}$$

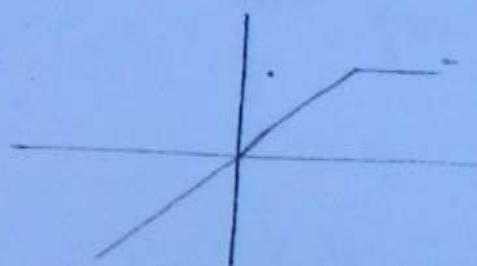
$$P_2 > \frac{V^2}{R_2} \Rightarrow P_1 < P_2 .$$

F

Steady state $\star \star \star$



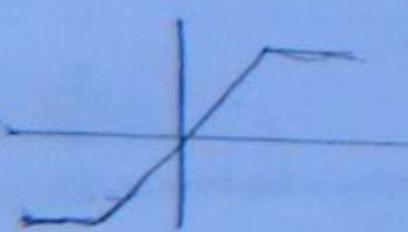
- 1) Bi-directional
- 2) Linear
- 3) Passive



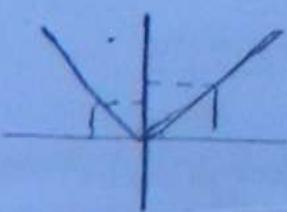
1. Non linear
2. Uni-directional
3. Passive

Bi direc —
shud be identical
in 1st & 3rd
quad. else uni-direc

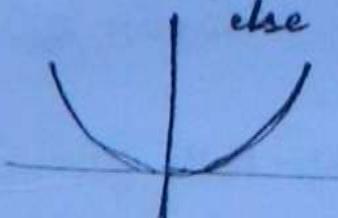
passive — $\frac{V}{I}$ shud be
+ve in 1st & 3rd quad.
else active



1. non linear
2. Bi-direc.
3. Passive



1. Uni-direc.
2. non-lin
3. active



1. Uni-direc.
2. non lin.
3. active

Steady state A.C. Circuits

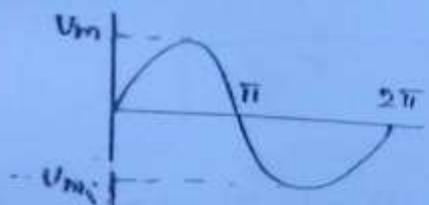
Advantages of sine wave.

(45)

- It is easy to handle mathematically. (differential and integral of the sine func. can be re-written in terms of sine function)
- The natural phenomena like the motion of the simple pendulum & response of undamped system, shows sinusoidal character.

Any periodic waveform can be expressed in terms of sine functions by using fourier analysis.

It is easy to generate in the laboratory.



$$V(t) = V_m \sin \omega t$$

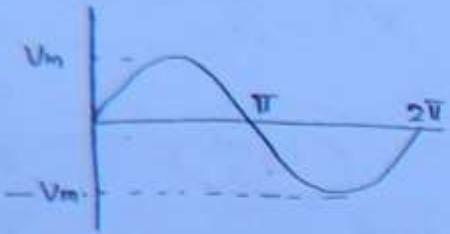
V_m = peak (or) max. value

ω = angular frequency \rightarrow rad/sec.

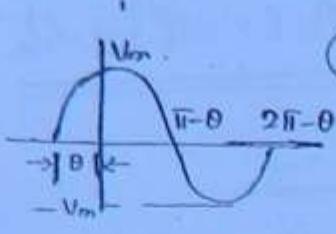
ωt = argument \rightarrow rad.

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} \text{ sec.}$$

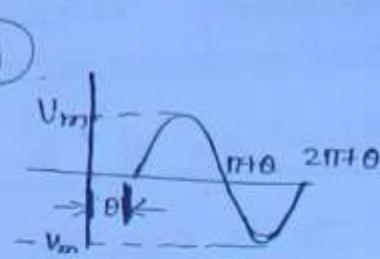
$$f = \frac{1}{T} \Rightarrow f = \frac{\omega}{2\pi} \text{ Hz (or) cycles/sec.}$$



$$(1) \quad V(t) = V_m \sin(\omega t)$$



$$V(t) = V_m \sin(\omega t + \theta)$$



$$V(t) = V_m \sin(\omega t - \theta)$$

→ wrt 1st waveform, 2nd waveform is leading by an angle θ .

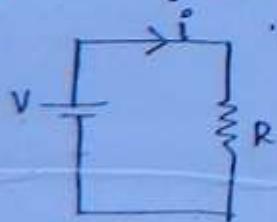
→ wrt 1st waveform, 3rd waveform is lagging by an angle θ .

→ wrt second waveform, 3rd waveform is lagging by an angle 2θ .

93/6/11

RMS Value

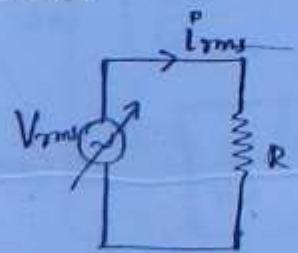
- RMS value is defined based on heating effect of the waveform.
- The voltage at which heat dissipation in AC circuit is equal to heat dissipation in DC circuit is called as V_{rms} , provided both ac & dc circuit have equal value of resistance and operated for same time



$$P = i^2 R$$

$$W = i^2 R t$$

heat D.C



$$P = i^2 R$$

$$W = i^2 R t$$

$$W_{A.C} = W_{D.C}$$

General Expressions

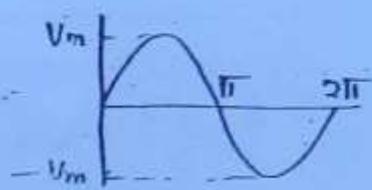
$$V_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v^2 dt}$$

(47)

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

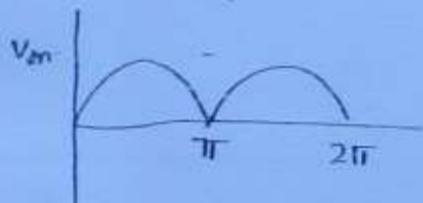
Find RMS value of the following waveforms.

1)



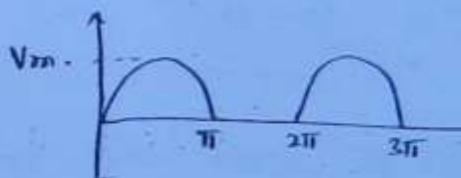
$$\begin{aligned} V_{RMS} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \sin^2 \omega t dt} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 dt} \\ &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \left(\frac{1-\cos 2\omega t}{2}\right) dt} = \boxed{\frac{V_m}{\sqrt{2}}} \end{aligned}$$

2)



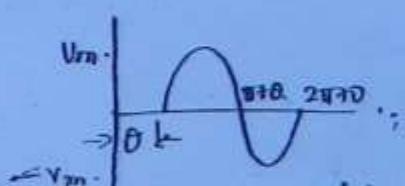
$$\text{Full wave rectified} = \frac{V_m}{\sqrt{2}}$$

3)



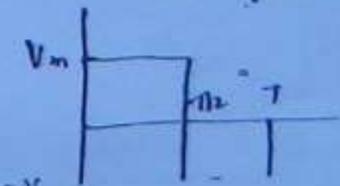
$$\text{Half wave} = \frac{V_m}{2}$$

4)

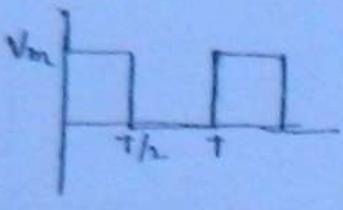


$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

Note: RMS value is independent on the position of starting of waveform. But it depends on shape of the wave-form.

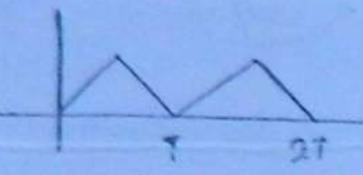


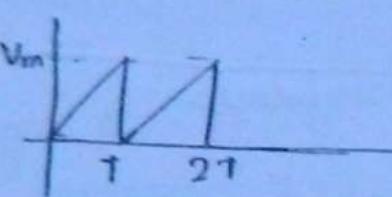
$$V_{RMS} = V_m$$

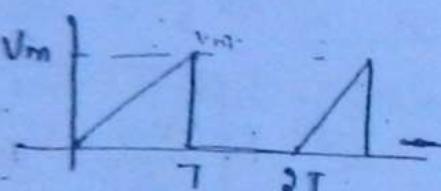
6)  = $V_{rms} = \frac{V_m}{\sqrt{2}}$

$\frac{V_m}{\sqrt{2}}$

(48)

7)  = $V_m/\sqrt{3}$

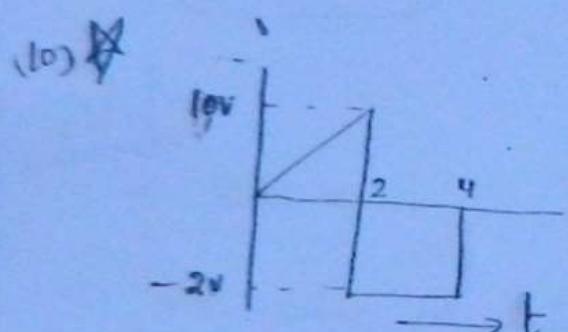
8)  = $V_m/\sqrt{3}$

9) 

$$V_{rms} = \frac{V_m}{\sqrt{6}}$$

$$\frac{V_m}{T/2} \int_0^{\frac{T}{2}} \frac{4t^2}{3} = \frac{V_m \times T^3}{2T^2 \cdot 2}$$

$$V_m = \frac{V_m}{\sqrt{6}}$$



$0 \leq t \leq 2$

$y = mx$

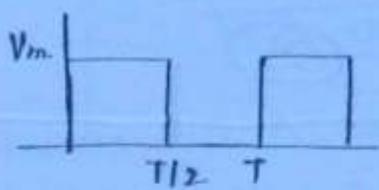
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{2 - 0} = 5.$

$v = 5t$

$$V_{rms} = \sqrt{\frac{1}{4} \left[\int_0^2 v^2 dt + \int_2^4 v^2 dt \right]}$$

$$\sqrt{\frac{1}{4} \left[\int_0^2 (5t)^2 dt + \int_2^4 (-2)^2 dt \right]} =$$

Q1 Find power dissipation in the resistor for a given waveform.



$$\text{Ans} P_{av} = P_{peak}/2$$

(49)

$$(b) P_{av} = P_F/\sqrt{2}$$

$$(c) P_{av} = P_P/\sqrt{2}$$

$$(d) P_{av} = P_{peak}$$

$$P_{av} = \frac{V_{rms}^2}{R}$$

$$P_{av} = \frac{(V_m/\sqrt{2})^2}{R} = \frac{V_m^2}{2R}$$

$$P_{av} = \frac{V_m^2}{2R}$$

$$P_{peak} = \frac{V_m^2}{R}$$

$$P_{av} = \frac{P_{peak}}{2}$$

$$\boxed{\frac{P_{D.C.}}{P_{A.C.}} = \frac{I_{av}^2 R}{I_{RMS}^2 R}}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{avg}}$$

Find RMS value of the following function:

$$V(t) = 3 + \sin t + \sin 3t + \cos t$$

$$V_{rms} = \sqrt{V_{rms,1}^2 + V_{rms,2}^2 + \dots + V_{rms,n}^2}$$

$$\sqrt{3^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2}$$

$$V_{rms} = \sqrt{2\frac{1}{2}}$$

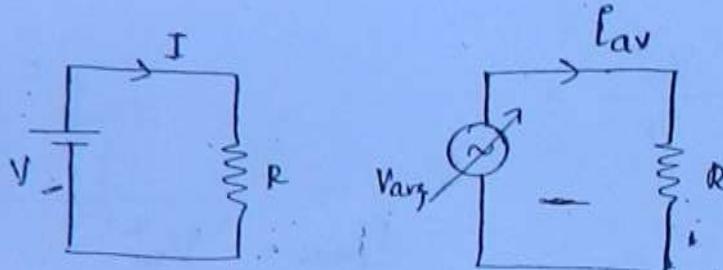
RMS value is independent on frequency of the waveform

Average Value

→ Average value is defined based on charge transfer in the circuit.

(38)

→ The voltage at which the charge transfer in A.C. circuit is equal to charge transfer in D.C. circuit is called as V_{avg} , provided both A.C. & D.C. circuits consist of equal value of resistance and operated for same time.



$$I = \frac{V}{R}$$

$$I = \frac{Q}{t}$$

$$Q = It$$

D.C.

$$I = \frac{V}{R}$$

$$I = \frac{Q}{t}$$

$$Q = It$$

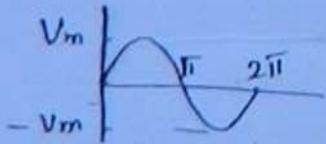
A.C.

$$\therefore Q_{A.C.} = Q_{D.C.}$$

→ Avg. value of complete cycle of symmetrical wave = 0.

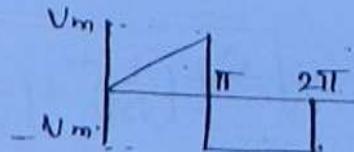
→ For analysis while finding avg. value of symmetrical wave only the $\frac{1}{2}$ cycle is considered.

→ While finding avg. value of unsymmetrical wave angle made by complete cycle is considered.



Symmetrical wave

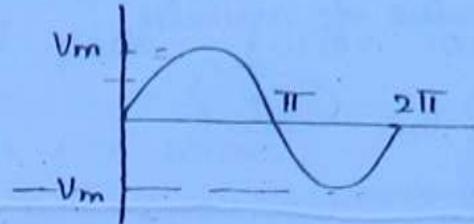
$$V_{avg} = \frac{1}{\pi} \int_0^{\pi} V dt$$



Unsymmetrical wave

$$V_{avg} = \frac{1}{2} \left(\int_0^{\pi} V dt + \int_{\pi}^{2\pi} V dt \right)$$

-Find avg. value of the following waveforms:



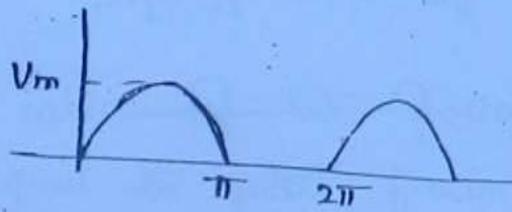
$$V_{av} = \frac{1}{T} \int_0^T V dt \quad (5)$$

$$= \frac{1}{T} \int_0^T V_m \sin \omega t dt$$

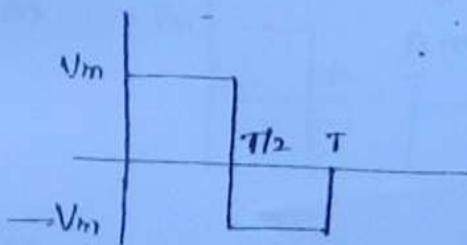
$$\boxed{V_{av} = \frac{2V_m}{\pi}}$$



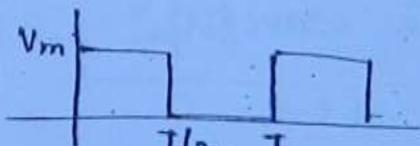
$$\boxed{V_{av} = \frac{2V_m}{\pi}}$$



$$\boxed{V_{av} = \frac{V_m}{\pi}}$$

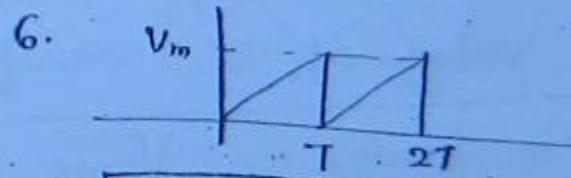


$$\boxed{V_{av} = V_{rms} = V_m}$$

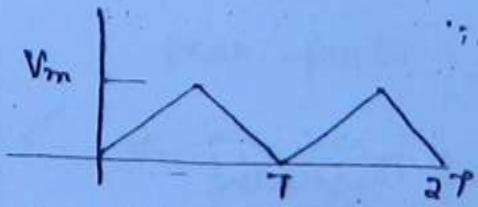


$$\boxed{V_{av} = V_m/2}$$

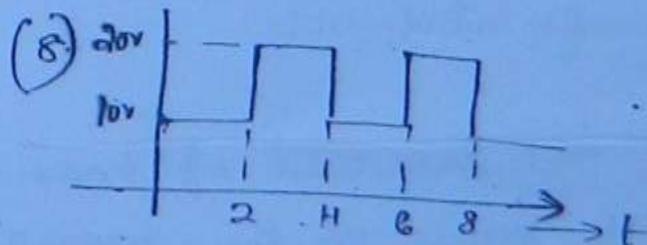
$$V_{rms} = V_m/\sqrt{2}$$



$$\boxed{V_{av} = V_m/2}$$



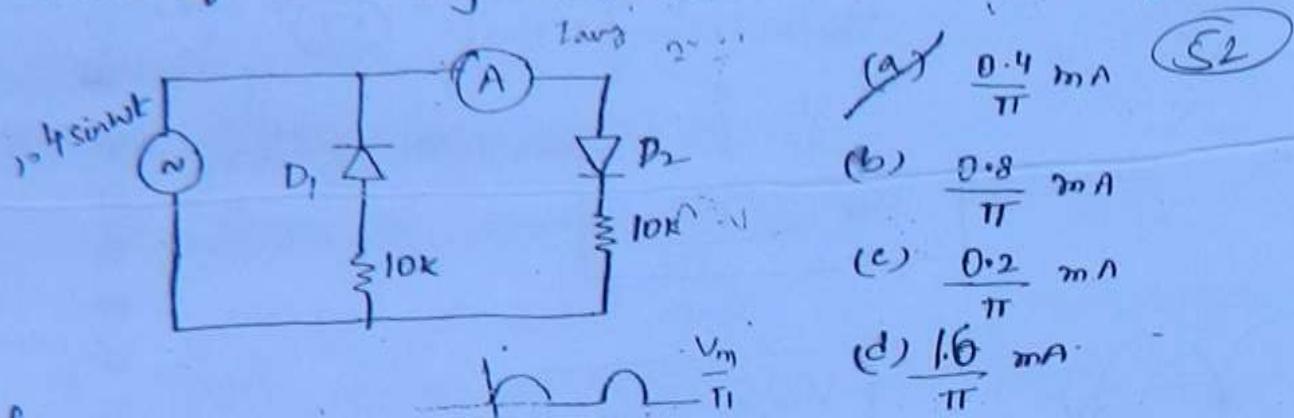
$$\boxed{V_{av} = V_m/2}$$



$$V_{avg} = \frac{1}{6} \left[\int_0^2 10dt + \int_2^4 20dt \right]$$

$$V_{avg} = 15$$

When circuit is having ideal diodes and average value of indicating ammeter . Find the value of ammeter .



current flows in D_2 branch only for +ve $\frac{1}{2}$ cycle.
 \therefore we get half wave output.

$$\text{D/P} = V_{\text{av}} = \frac{V_{m_-}}{\pi} = \frac{4}{\pi}$$

$$I_{\text{av}} = \frac{V_{\text{av}}}{R} = \frac{4/\pi}{10\text{k}} = \frac{0.4}{\pi} \text{ mA}$$

FORM FACTOR

Form factor is the ratio of RMS value of the waveform to average value of the waveform.

$$\text{form factor} = \frac{V_{\text{RMS}}}{V_{\text{avg.}}}$$

PEAK FACTOR

Peak factor is the ratio of max. value of the waveform to RMS value of the waveform.

$$\text{Peak factor} = \frac{V_m}{V_{\text{RMS}}}$$

To justify abt shape of the waveform, form factor and peak factor are introduced.

(53)

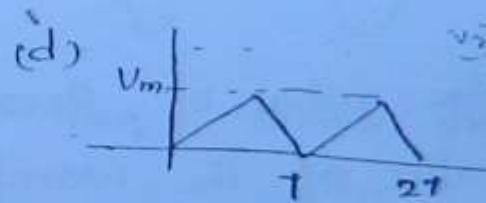
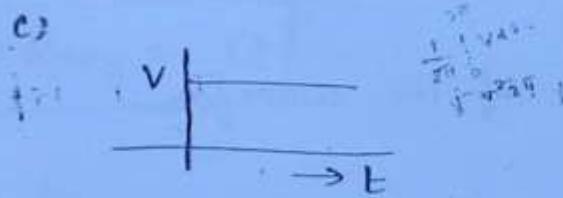
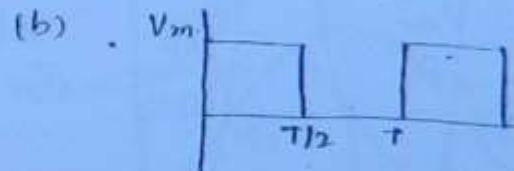
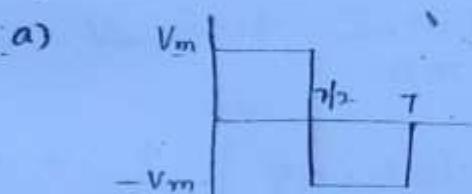
For sine wave, form factor = $\frac{V_m/\sqrt{2}}{2V_m/\pi} = 1.11$.

Power System,

= 11 kV, 33 kV, 66 kV, 132 kV, 220 kV. 3 multiples of 11.

These power systems are chosen based on the form factor = 1.11.

which of the following waveforms have form factor equal to peak factor.



$V_{rms} = V_{avg} = V_m$

form factor = 1.

peak factor = 1.

$V_{rms} = V_{avg} = V$

form factor = peak factor = 1

$V_{rms} = \frac{V_m}{\sqrt{2}}$, $V_{avg} = \frac{V_m}{2}$

$V_{rms} = V_m/\sqrt{2}$

form factor = $\frac{V_{rms}}{V_{avg}} = \sqrt{2}$

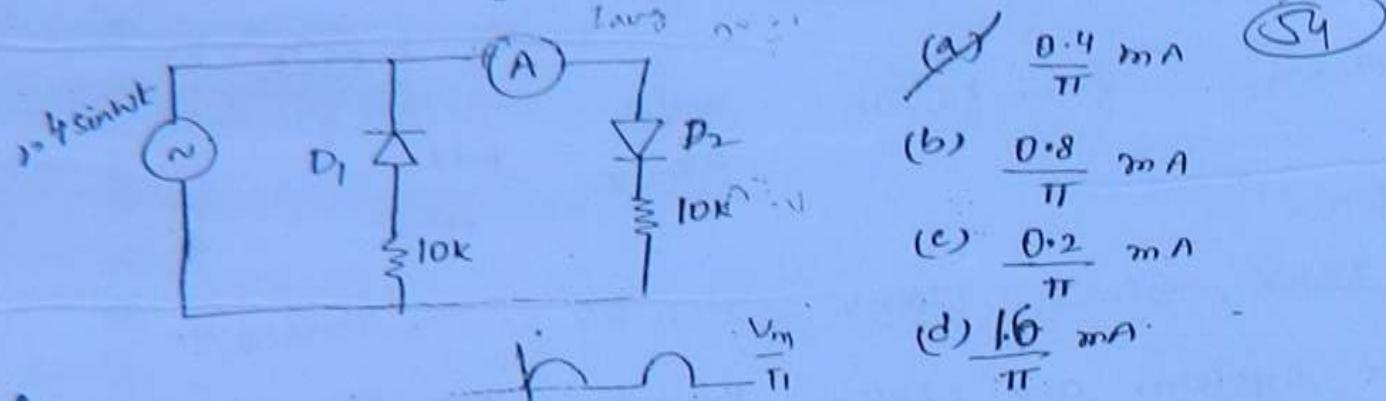
$V_{avg} = V_m/2$

form factor = $2/\sqrt{2}$

peak factor = V_{max}/V_{avg}

Peak Factor = 1.

When circuit is having ideal diodes and average value of indicating ammeter. Find the value of ammeter.



- (54)
- (a) $\frac{0.4}{\pi} \text{ mA}$
 (b) $\frac{0.8}{\pi} \text{ mA}$
 (c) $\frac{0.2}{\pi} \text{ mA}$
 (d) $\frac{1.6}{\pi} \text{ mA}$

Current flows in D_2 branch only for +ve $\frac{1}{2}$ cycle.
 \therefore we get half wave output.

$$\text{O/P} = V_{\text{av}} = \frac{V_{m_-}}{\pi} = \frac{4}{\pi}$$

$$I_{\text{av}} = \frac{V_{\text{av}}}{R} = \frac{4/\pi}{10\text{k}} = \frac{0.4}{\pi} \text{ mA}$$

FORM FACTOR

Form factor is the ratio of RMS value of the waveform to average value of the waveform.

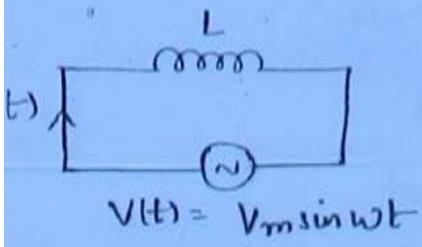
$$\boxed{\text{form factor} = \frac{V_{\text{RMS}}}{V_{\text{avg.}}}}$$

PEAK FACTOR

Peak factor is the ratio of max. value of the waveform to RMS value of the waveform.

$$\boxed{\text{Peak factor} = \frac{V_m}{V_{\text{RMS}}}}$$

AC source across inductor



$$i_L = \frac{1}{L} \int v dt$$

$$i_L = \frac{1}{L} \int V_m \sin \omega t dt$$

$$i_L = -\frac{V_m}{\omega L} \cos \omega t$$

$$\therefore \frac{V_m}{\omega L} \sin(\omega t - 90^\circ) \quad (X_L = \omega L)$$

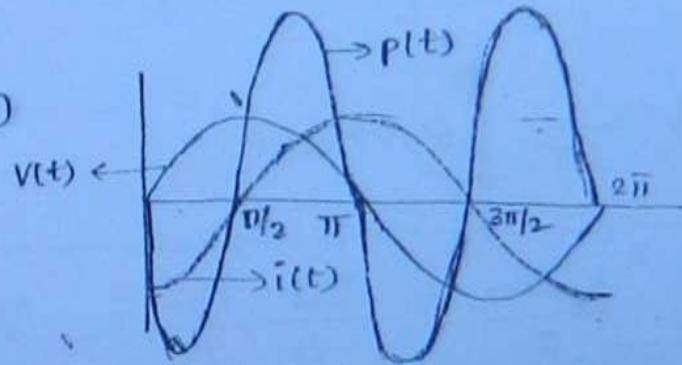
$$i_L = I_m \sin(\omega t - 90^\circ) \quad \therefore X_L = \omega L; I = \frac{V}{X_L}$$

$$P = V(t) i(t)$$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t - 90^\circ)$$

$$P = \frac{1}{2\pi} \int_0^{2\pi} P(t) d\omega t$$

$$\text{Pavg} = 0$$



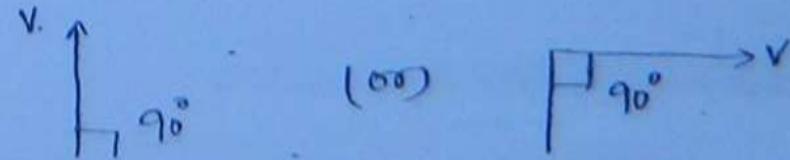
$P \rightarrow$ freq = double $V(t)$

ing the $\frac{1}{2}$ cycle of the power, inductor takes energy
out the source & in the $\frac{1}{2}$ cycle of the power,
inductor delivers energy to the source.

hereby, net power taken from the source = 0

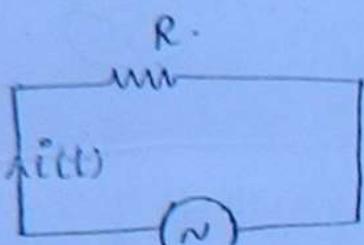
$$f = 50\text{Hz} \quad \{ \quad f_p = 100\text{Hz}$$

use diagram :-



AC source across resistance

(Q6)



$$V(t) = V_m \sin \omega t$$

$$i(t) = v(t)/R$$

$$i(t) = \frac{V_m \sin \omega t}{R}$$

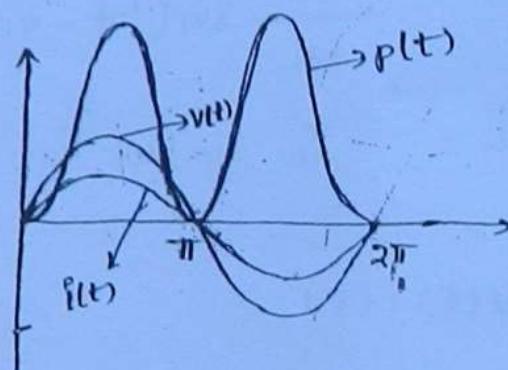
$$i(t) = I_m \sin \omega t$$

$$P(t) = V(t) i(t)$$

$$= (V_m \sin \omega t)(I_m \sin \omega t)$$

$$P(t) = \frac{V_m I_m}{2} [1 - \cos 2\omega t]$$

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P(t) dt$$



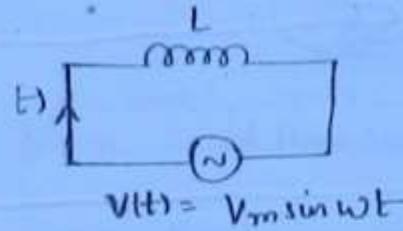
$$P_{avg} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$$

$$f = 50 \text{ Hz} \Rightarrow f_p \text{ (power frequency)} = 100 \text{ Hz}$$

\therefore When voltage completes one cycle from $0 - 2\pi$, power completes 2 cycles \therefore it has double the freq. of voltage.

AC source across inductor

(57)



$$i_L = \frac{1}{L} \int v dt$$

$$i_L = \frac{1}{L} \int V_m \sin \omega t dt$$

$$i_L = -\frac{V_m}{\omega L} \cos \omega t$$

$$\therefore \frac{V_m}{\omega L} \sin(\omega t - 90^\circ) \quad (X_L = \omega L)$$

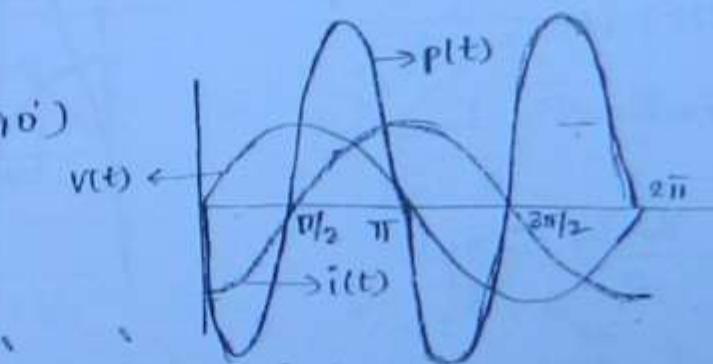
$$v_L = I_m \sin(\omega t - 90^\circ) \quad X_L = \omega L; I = \frac{V}{X_L}$$

$$P = V(t) i(t)$$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t - 90^\circ)$$

$$\omega = \frac{1}{2\pi} \int_0^{2\pi} P(t) d\omega t$$

$$\text{Pavg} = 0$$



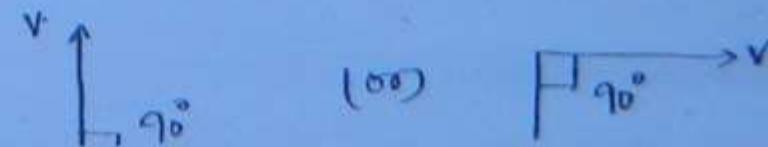
$P \rightarrow \text{freq} = \text{double } v(t)$

During the $\frac{1}{2}$ cycle of the power, inductor takes energy from the source & in next $\frac{1}{2}$ cycle of the power, inductor delivers energy to the source.

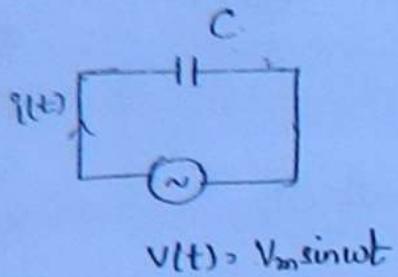
thereby, net power taken from the source = 0

$$f = 50\text{Hz} \quad \{ \quad f_p = 100\text{Hz}$$

phasor diagram



AC source across Capacitor



$$i = C \frac{dv}{dt}$$

$$= C \frac{d}{dt} (V_m \sin \omega t)$$

$$i = \omega C V_m \cos \omega t$$

$$i(t) = \omega C V_m \cos \omega t$$

$$i(t) = \frac{V_m}{\omega C} \sin(\omega t + 90^\circ) \quad (\because X_C = 1/\omega C)$$

$$i(t) = I_m \sin(\omega t + 90^\circ)$$

$$P(t) = V(t) i(t)$$

$$P(t) = V_m \sin \omega t \cdot I_m \cos \omega t$$

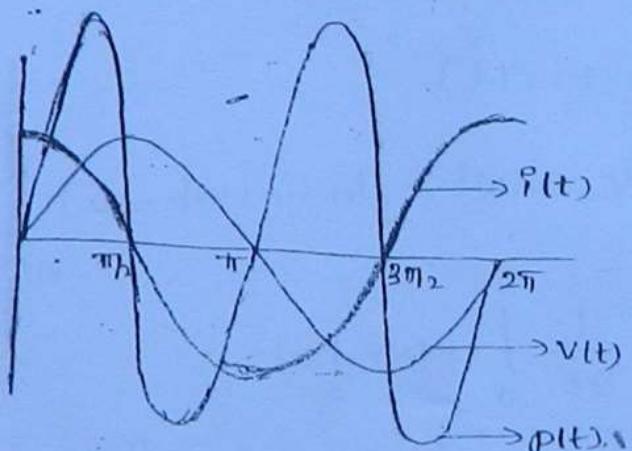
$$P(t) = \frac{V_m I_m}{2} \sin 2\omega t$$

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P(t) d\omega t$$

$$\boxed{P_{avg} = 0}$$

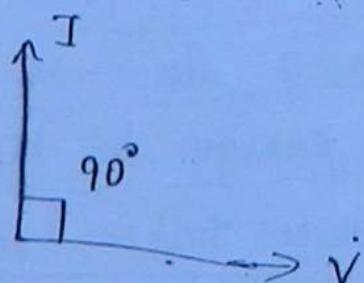
phasor diagram :

(6)



$f = 50\text{Hz}$ — voltage

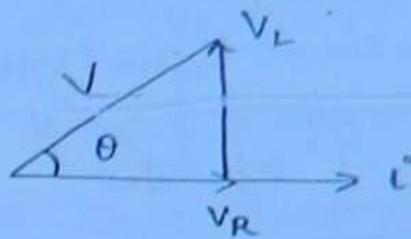
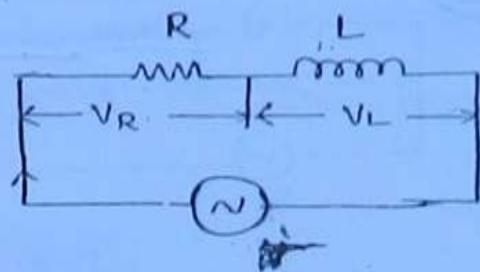
$f_P = 100\text{Hz}$ — power



R-L Series Circuit

phasor

(59)



$$\text{KVL}, \quad V = V_R[0^\circ] + V_L[90^\circ]$$

$$IZ = I_R[0^\circ] + I_{X_L}[90^\circ]$$

$$IZ = I(R + jX_L) \Rightarrow Z = R + jX_L$$

angle $\Delta\theta$

$$\begin{array}{c} \sqrt{V_R^2 + V_L^2} \\ \theta \\ \hline V_R = I_R \end{array}$$

$$V_L = I_{X_L}$$

impedance $\Delta\theta$

$$\begin{array}{c} Z \\ \theta \\ \hline R \\ jX_L \end{array}$$

power $\Delta\theta$

$$\begin{array}{c} \tilde{Z} = S \\ \theta \\ \hline I_R = P \\ jX_L = Q_L \end{array}$$

$$\therefore \sqrt{V_R^2 + V_L^2}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$S = \sqrt{P^2 + Q_L^2}$$

$$\therefore \tan \theta = \left(\frac{V_L}{V_R} \right)$$

$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\theta = \tan^{-1} \left(\frac{Q_L}{P} \right)$$

$\Rightarrow P \rightarrow$ active power (or)
(W). True " (or)
real " avg. "

effective power

$Q_L \rightarrow$ Inductive reactive power

$Q_L \rightarrow$ unit \rightarrow VAR

(volt-ampere reactive) \Leftarrow

$S \rightarrow$ Apparent power (or) Complex power

$\rightarrow S \rightarrow VA$ (volt-ampere).

Power Expressions

$$P(t) = V(t) I(t)$$

(6d)

$$P(t) = I_m \sin \omega t V_m \sin(\omega t + \theta)$$

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P(t) d\omega t$$

V_m, I_m

$$P_{avg} = \frac{V_m I_m}{2} \cos \theta$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \theta = \underline{\underline{V I \cos \theta}}$$

| V, I are
Rms values.
in any ac ckt.

$$\text{Power factor} = \cos \theta = \frac{V_R}{V_i} = \frac{R}{Z} = \frac{P}{S} \quad \text{w.r.t. i lag}$$

→ While defining power factor for any circuit, voltage phasor is taken as a reference. Since,

- In the real time systems, only independant voltage source exists. (source voltage constant).
- In the real time systems, loads are connected in parallel. (voltage is constant).

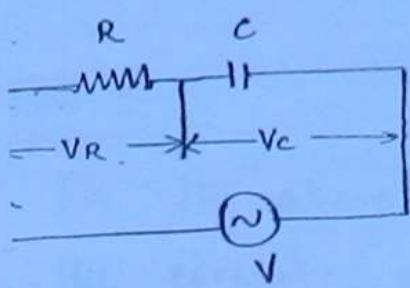
⇒ Power factor angle indicates position of current phasor w.r.t. voltage phasor.

$$\sin \omega t \sin \omega t \cos \theta + \cos \omega t \sin \theta$$

$$S_{12} \cos \theta + S_{21} \cos \theta$$

$$2 \cos \theta$$

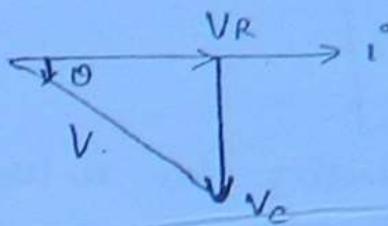
C Series Circuit



By

phaser

(6)



F

KVL,

$$2. \quad V_R \angle 0^\circ + V_C \angle -90^\circ$$

$$Z = IR \angle 0^\circ + IX_C \angle -90^\circ$$

$$IZ = I(R - jX_C) \Rightarrow Z = R - jX_C$$

Voltage Δe

$$\begin{array}{l} V_R = IR \\ \theta \\ = IZ \\ V_C = IX_C \end{array}$$

Impedance Δe

$$\begin{array}{c} R \\ j\theta \\ Z \\ X_C \end{array}$$

Power Δe

$$\begin{array}{c} IR = P \\ jIX_C = Q_C \\ IZ = S \end{array}$$

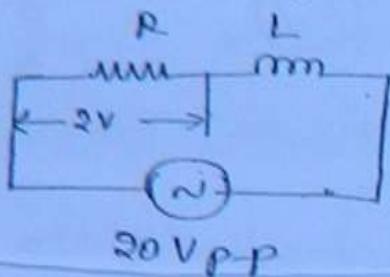
$$S = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1} \left(\frac{-Q_C}{P} \right)$$

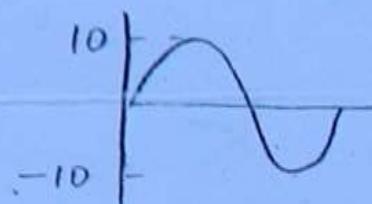
VC - 's CW

$$\text{Power factor} = \cos \theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S} \quad (\text{lead wrt } V)$$

8. Find voltage across the inductor of the ckt shown:



(62)



$$\frac{10\text{ V}_m}{\sqrt{2}} = \text{ rms}$$

$$\frac{1000}{\sqrt{2}} = 707$$

$$V_m = 10$$

$$V_{rms} = 10/\sqrt{2} = V$$

$$V = \sqrt{V_R^2 + V_L^2} \Rightarrow$$

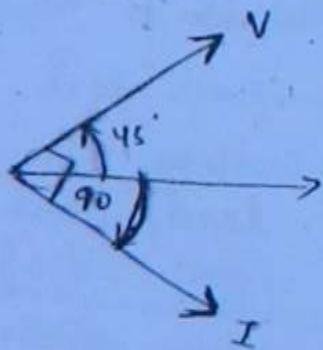
$$V_L = \sqrt{V^2 - V_R^2}$$

$$V_L = \sqrt{46}$$

- 9/6/2011

a. Find the circuit elements for a given current and voltage eqn

$$v(t) = 9\sin(t + 45^\circ) \quad i(t) = 3\sin(t - 45^\circ)$$



wrt I, V is leading by 90°

∴ it is an inductor

$$X_L = \frac{V}{I}$$

$$\Rightarrow \frac{9/\sqrt{2}}{3/\sqrt{2}} = 3$$

$$\therefore X_L = 3 \Rightarrow \omega L = 3$$

$$\omega = 1 \therefore L = 3H$$

1. Find circuit elements for a given voltage and current eqns.

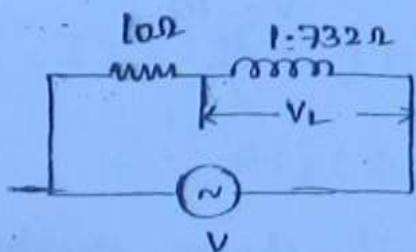
$$v(t) = 9 \sin(t + 30^\circ) \quad i(t) = 3 \sin(12t + 60^\circ)$$

(67)

Note :-

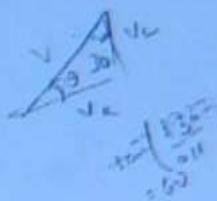
For the above equations, it is not possible to design the circuit since, frequency of voltage and current are unequal.

2. Find angle made by source voltage wrt V_L .



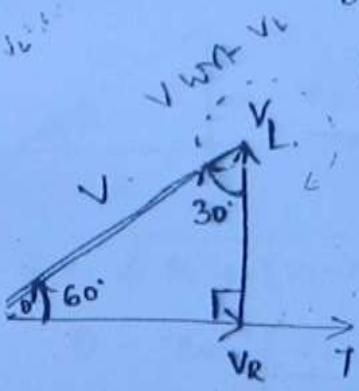
$$(a) 30^\circ \quad (b) 60^\circ$$

$$\checkmark -30^\circ \quad (d) -60^\circ$$



$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right) = \tan^{-1} \left(\frac{1.732}{10} \right) = \tan^{-1}(\sqrt{3})$$

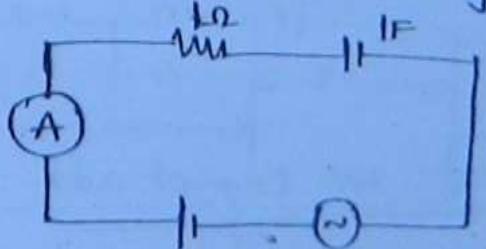
$$\theta = 60^\circ$$



wrt. V_L angle made by source voltage is 30° . The angle vector is rotating in clockwise direction. \therefore it is -ve

Thus, the angle made by source voltage wrt to V_L is -30°

Find ammeter reading of the circuit shown.



$$350 \quad 5 \angle -45^\circ$$

$$v(t) = 10\sqrt{2} \sin t$$

Apply Superposition theorem.

D.C.

$$C \rightarrow 0 \cdot C \quad \therefore i_{DC} = 0$$

(64)

A.C.

$$X_C > \frac{1}{\omega C} = 1$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{1+1} = \sqrt{2}$$

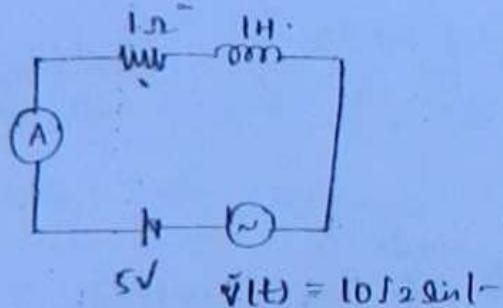
$$V = V_{rms} = \frac{10\sqrt{2}}{\sqrt{2}}$$

$$I_{AC} = \frac{V}{Z} = \frac{10}{\sqrt{2}}$$

$$V = 10$$

ammeter reading: $\frac{10}{\sqrt{2}} + 0 = \frac{10}{\sqrt{2}} A$

Q. Find the ammeter reading of the circuit shown.



$$\check{V}(t) = 10\sqrt{2}\sin t$$

Sol:

$$I_{DC} = \frac{5}{1} = 5A \quad L \rightarrow \infty$$

Superposition

$$X_L = \omega L = 1\Omega$$

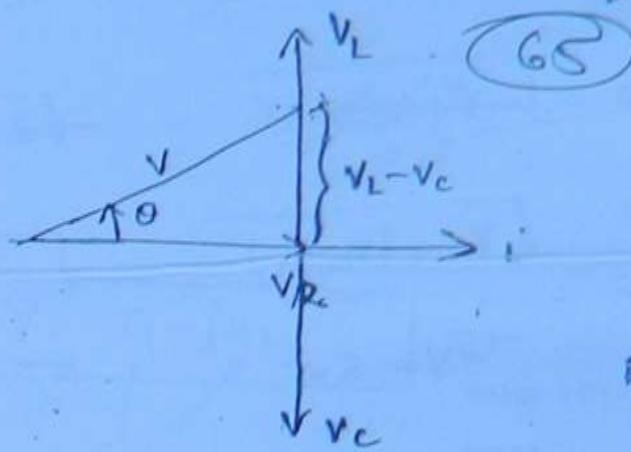
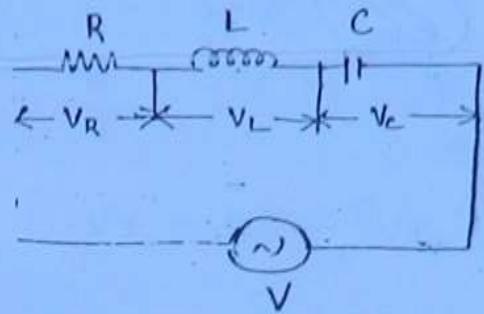
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{1+1} = \sqrt{2}$$

$$I_{AC} = \frac{V}{Z} = \frac{10}{\sqrt{2}}$$

Two different frequencies are present we cannot add i_{DC} & i_{AC} for total current.

$$\therefore I = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = \sqrt{5} \text{ A}$$

RLC Series Circuit



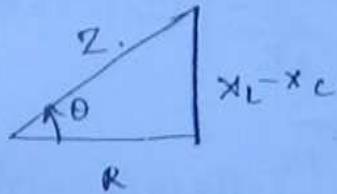
By KVL,

$$V = V_R[0^\circ] + V_L[90^\circ] + V_C[-90^\circ]$$

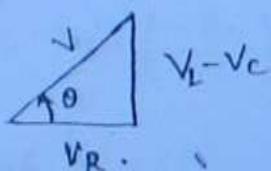
$$IZ = I[R + j(x_L - x_C)]$$

$$Z = R + j(x_L - x_C)$$

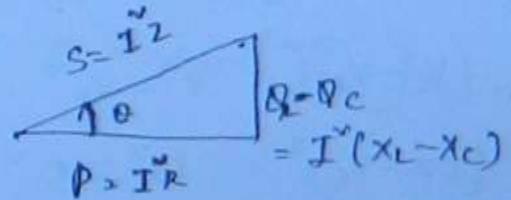
Impedance Δ



Voltage Δ



Power Δ



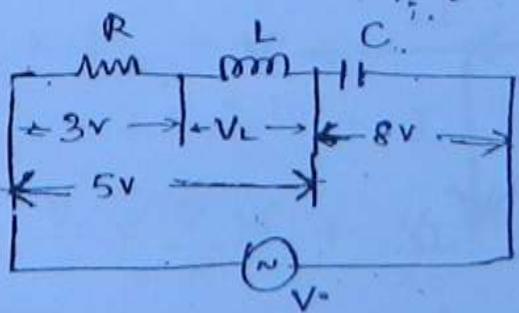
$$Z = \sqrt{R^2 + (x_L - x_C)^2} \rightarrow V = \sqrt{V_R^2 + (V_L - V_C)^2} \rightarrow S = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\theta = \tan^{-1}\left(\frac{x_L - x_C}{R}\right)$$

$$\theta = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$\theta = \tan^{-1}\left(\frac{Q_L - Q_C}{P}\right)$$

Find V_L & V of the circuit shown.



$$5 = \sqrt{V_L^2 + 3^2} \Rightarrow V_L = 4.$$

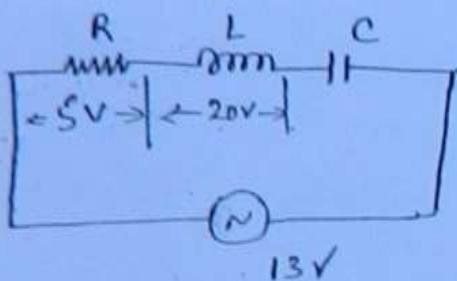
~~$$V = \sqrt{3^2 + V_L^2 + 5^2} = \sqrt{32}$$~~

(66)

$$\rightarrow V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{3^2 + (4-8)^2}$$

$$V = 5V$$

Q. Find voltage across capacitor.



- (a) 8V (b) 32V
 ✓ (c) 8 or 32V (d) 10V

$$13 = \sqrt{5^2 + (20 - V_C)^2}$$

$$13^2 = 5^2 + (20 - V_C)^2$$

$$V^2 = V_R^2 + (V_L - V_C)^2$$

for

$$20 - V_C = 12$$

$$(V_L - V_C)^2 = V^2 - V_R^2$$

$$V_C = 8V$$

$$(20 - V_C)^2 = 13^2 - 5^2$$

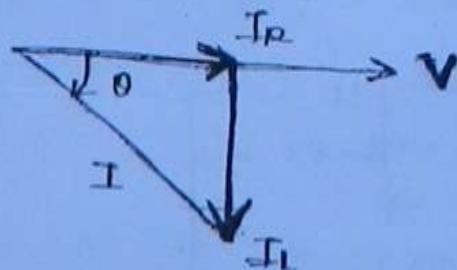
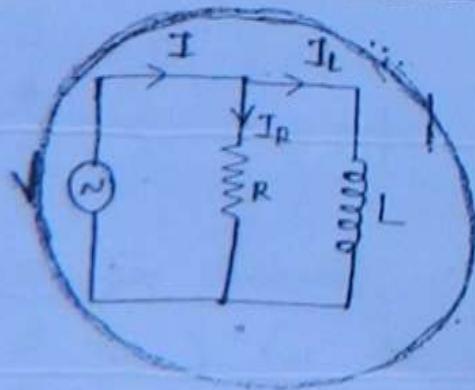
$$20 - V_C = -12$$

$$20 - V_C = \pm 12$$

$$32 = V_C$$

$$V_C = 8 \text{ or } 32V$$

→ RL parallel circuit



By KCL

$$I = I_R \angle 0^\circ + I_L \angle 90^\circ$$

$$\frac{V}{Z} = \frac{V}{R} \angle 0^\circ + \frac{V}{X_L} \angle 90^\circ$$

$$VY = VG_I - jVB_L$$

\Rightarrow

$$Y = G_I - jB_L$$

V
mhos (or) S.

mhos (or) S

Current A.C.

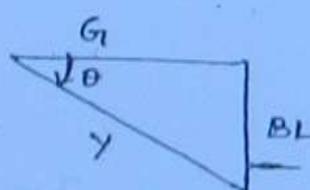
$$I_R = VG_I$$



$$I = \sqrt{I_R^2 + I_L^2}$$

$$\theta = \tan^{-1} \left(\frac{-I_L}{I_R} \right)$$

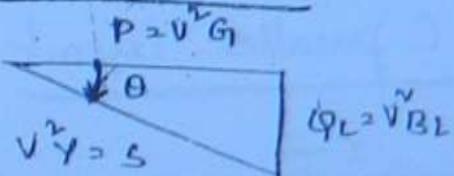
admittance A.C.



$$Y = \sqrt{G_I^2 + B_L^2}$$

$$\theta = \tan^{-1} \left(\frac{-B_L}{G_I} \right)$$

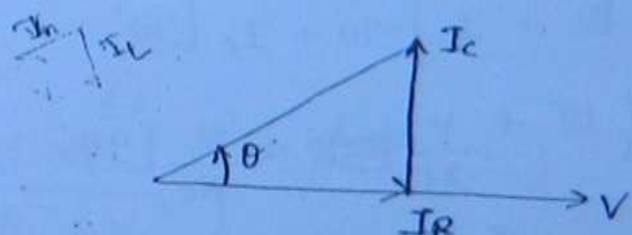
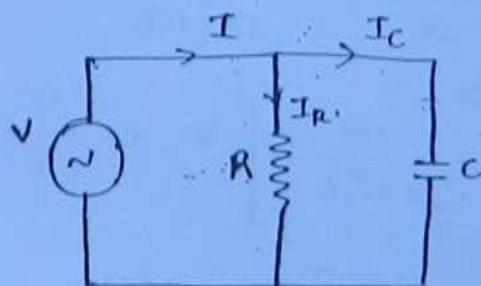
power A.C.



$$S = \sqrt{P^2 + Q_L^2}$$

$$\theta = \tan^{-1} \left(\frac{-Q_L}{P} \right)$$

P.C. - parallel circuit



By KCL,

$$I = I_R \angle 0^\circ + I_C \angle 90^\circ$$

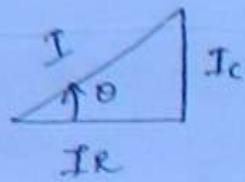
$$\frac{V}{Z} = \frac{V}{R} \angle 0^\circ + \frac{V}{X_C} \angle 90^\circ$$

$$VY = VG_I + jVB_L$$

current A.R.

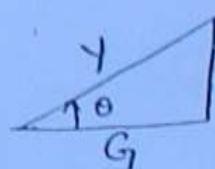
admittance A.R.

Power A.R.



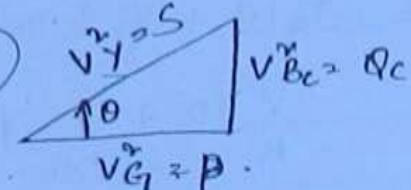
$$I = \sqrt{I_R^2 + I_C^2}$$

$$\theta = \tan^{-1} \left(\frac{I_C}{I_R} \right)$$



$$Y = \sqrt{G_Y^2 + B_C^2}$$

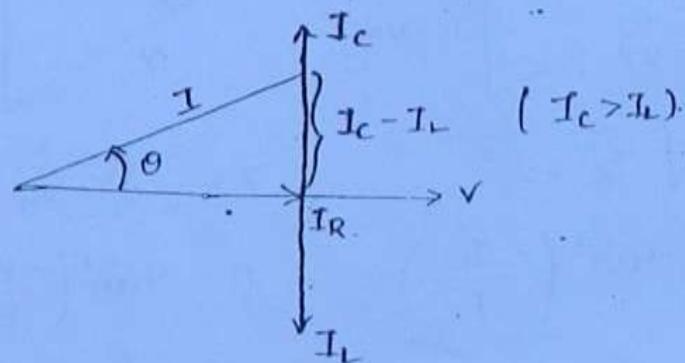
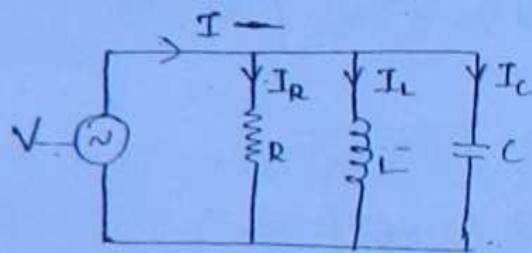
$$\theta = \tan^{-1} \left(\frac{B_C}{G_Y} \right)$$



$$S = \sqrt{P^2 + Q_C^2}$$

$$\theta = \tan^{-1} \left(\frac{Q_C}{P} \right)$$

R L C parallel Circuit



By KCL

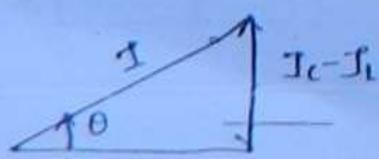
$$I = I_R \angle 0^\circ + I_L \angle -90^\circ + I_C \angle 90^\circ$$

$$\frac{V}{Z} = \frac{V}{R} \angle 0^\circ + \frac{V}{X_L} \angle -90^\circ + \frac{V}{X_C} \angle 90^\circ$$

$$\nabla Y = -V G_Y + j V B_L + j V B_C$$

$$Y = G_Y + j (B_C - B_L) \Rightarrow$$

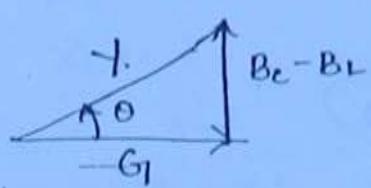
Current A.R.



$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

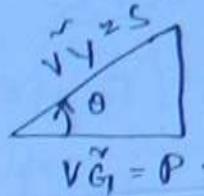
$$\theta = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right)$$

admittance triangle



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Power triangle



$$\sqrt{G_1^2 + (B_c - B_L)^2} = Q > Q_c - Q_L$$

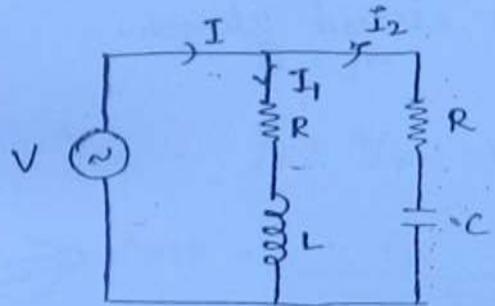
$$Y = \sqrt{G_1 + (B_c - B_L)^2}$$

$$S = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1} \left(\frac{B_c - B_L}{G_1} \right).$$

$$\theta = \tan^{-1} \left(\frac{Q}{P} \right)$$

RC parallel



$$I_1 = \frac{V}{R + jX_L} \times \frac{R - jX_L}{R_0 - jX_L}$$

$$\frac{V}{Z_1} = V \left[\frac{R_1}{R_1^2 + X_L^2} - \frac{jX_L}{R_1^2 + X_L^2} \right] \quad \Leftarrow$$

$$VY_1 = V [G_1 - jB_L]$$

$$Y_1 = G_1 - jB_L$$

$$I_2 = \frac{V}{R_2 - jX_C} \times \frac{R_2 + jX_C}{R_2 + jX_C}$$

$$\frac{V}{R + jX_L} = \frac{V}{R_1^2 + X_L^2} + j\frac{X_L}{R_1^2 + X_L^2}$$

$$\frac{V}{Z_2} = V \left[\frac{R_2}{R_2 + X_C} + j \frac{X_C}{R_2 + X_C} \right]$$

$$V Y_2 = V (G_2 + j B_C)$$

$$Y_2 = G_2 + j B_C$$

$$I = I_1 + I_2$$

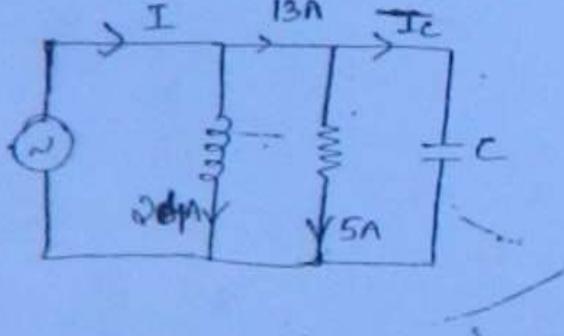
$$V Y_{eq} = V Y_1 + V Y_2$$

$$Y_2 = I - I_1 + I_2$$

$$Y_{eq} = Y_1 + Y_2$$

$$Y_{eq} = Y_1 + Y_2$$

Q. Find the value of I_c & I of the circuit shown.



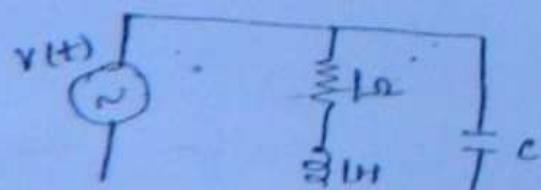
$$13 = \sqrt{I_R^2 + I_c^2} = \sqrt{5^2 + I_c^2}$$

$$I_c = 12A$$

$$\Rightarrow I = \sqrt{I_R^2 + (I_c - I_t)^2} = \sqrt{5^2 + (24 - 12)^2}$$

$$I = 13A$$

Find the value of C when power factor of the ckt is 0.8 lag.



i.e., $\cos \phi = 0.8$

$\Rightarrow \phi = 36.87^\circ$

$$X_L = \omega L$$

$$X_L = 1$$

$$G_{II} = \frac{R_1}{R_1^2 + X_L^2} = \frac{1}{1^2 + 1^2} = 1/2$$

$$B_L = \frac{X_L}{R_1^2 + X_L^2} = \frac{1}{1^2 + 1^2} = 1/2$$

$$Y_1 = G_{II} - j B_L$$

$$Y_1 = 1/2 - j 1/2$$

$$Y_2 = j B_C = j \omega C = j C$$

$$Y_2 = j C$$

$$Y_{eq} = Y_1 + Y_2 \Rightarrow Y_{eq} = \frac{1}{2} + j(c - 1/2)$$

$$\Rightarrow \cos \theta = \frac{G}{\sqrt{G^2 + (B_C - B_L)^2}}$$

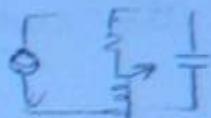
$$0.8 = \frac{1/2}{\sqrt{\left(\frac{1}{2}\right)^2 + (c - 1/2)^2}} \Rightarrow c = \frac{7}{8}, \frac{1}{8}$$

But power factor is 0.8 LAG. which means the ext is inductive in nature. $\therefore \frac{B_L > B_C}{B_L > B_C}$

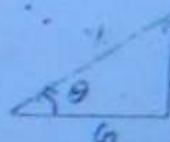
$$B_L = 1/2 \therefore \text{for } C = 1/8 \text{ satisfies } B_L > B_C$$

if 0.8 lead $\Rightarrow C = 7/8$

if neither lead nor lag is mentioned $C = \frac{7}{8}/1/8$



$$\begin{aligned} I &= I_1 + j I_2 \\ Y_{eq} &= V Y_1 + Y_2 \end{aligned}$$

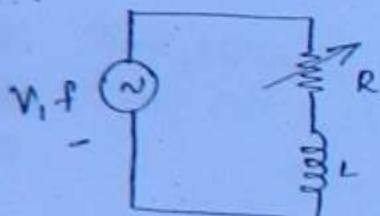


Locus Diagram

(72)

- Locus diagrams are useful for analysis and designing of the circuits. Ex. filters.
- The path traced by terminals of the current vectors by varying anyone of the circuit elements (or) by varying frequency is called as current locus.

Q. Draw the current locus of the circuit shown.



$$R = 0$$

$$Z = X_L$$

$$I = \frac{V}{X_L}$$

$$\theta = 90^\circ$$

$$R \uparrow$$

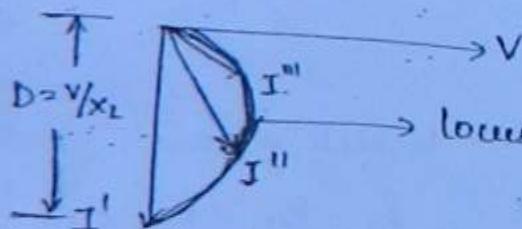
$$Z \uparrow$$

$$I \downarrow$$

$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right) \downarrow$$

$$R = \infty$$

$$I = 0$$

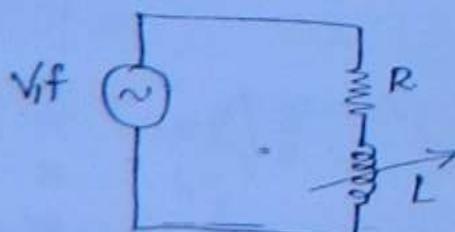


locus of I

$D \rightarrow$ diameter of locus.

$$\text{radius } r = \frac{V}{2X_L}$$

Q. Draw the current locus of the ckt shown.



$$X_L = 2\pi f L$$

$$X_L = 0$$

$$Z = R$$

$$I = \frac{V}{R}$$

$$\theta = 0$$

$$X_L \uparrow$$

(73)

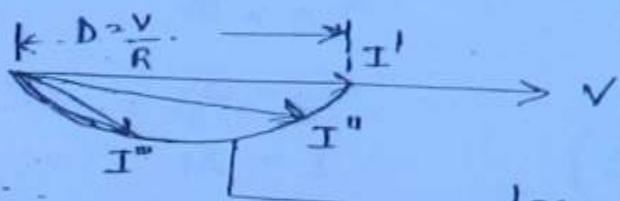
$$X_L = \infty$$

$$I = 0$$

$$Z \uparrow$$

$$I \downarrow$$

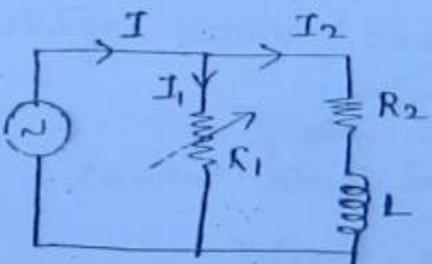
$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right) \uparrow$$



Locus of I.

For the above ckt if $R \neq L$ are kept constant & freq. f is varied, then also the locus is the same.

Draw the current locus of i_1 and i of the ckt shown.



$$I_2 = \frac{V}{R_2 + jX_L} = \text{const.}$$

Let

$$R = 0.1 \Omega$$

$$I_1 = \frac{V}{R_1}$$

$$\theta_1 = 0$$

$$I = I_1 + I_2$$

$$R_1 \uparrow$$

$$I_1 \downarrow$$

$$\theta_1 = 0$$

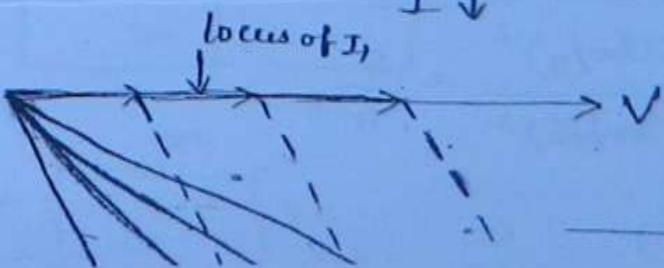
$$R_1 = \infty$$

$$I_1 = 0$$

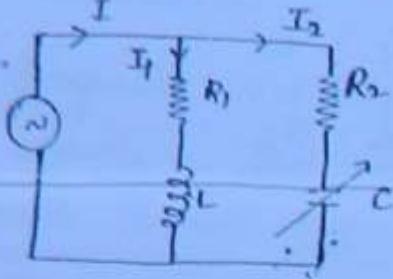
$$I = I_2$$

$$I_2 = \text{const.}$$

$$I \downarrow$$



Q. Draw the current wave of I_2 & i of the ckt shown.



$$X_C = \frac{1}{2\pi f C}$$

$$X_C \approx 0$$

$$I_2 = \frac{V}{R_2}$$

$$\theta_2 = 0$$

$$I = I_1 + I_2$$

$$X_C \uparrow$$

$$I_2 \downarrow$$

$$\theta = \tan^{-1}\left(\frac{X_C}{R_2}\right)$$

$$I_1 = \text{constant}$$

$$I \downarrow$$

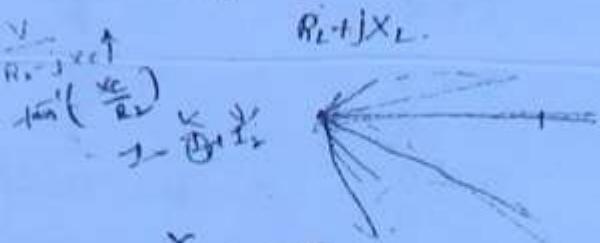
$$X_C = \infty$$

$$I_2 = 0$$

$$I = I_1$$

74

$$I_1 = \frac{V}{R_L + jX_L}$$

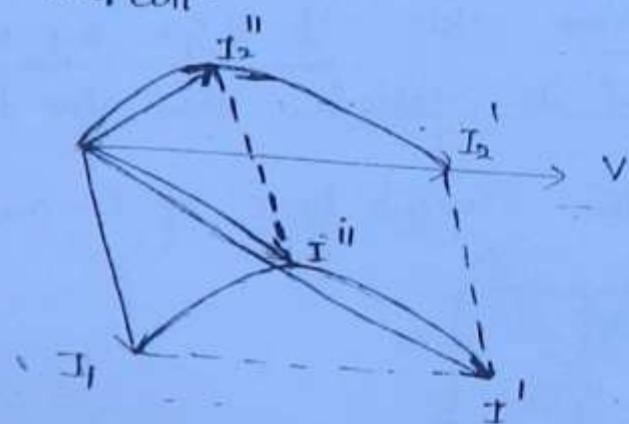


Work book

$$1. \quad V_{av} = -\frac{1}{2\pi} \left\{ \int_0^{\frac{\pi}{2}} V \sin(\omega t + \phi) d\omega t + \int_{\frac{\pi}{2}}^{\pi} 0 \right\} V$$

$$= \frac{V}{\pi} \cos \phi$$

$$2. \quad \frac{P_{D.C.}}{P_{A.C.}} = \frac{I_{av}^2 R}{I_{avg}^2 R} = \frac{\left(\frac{2E_0}{\pi}\right)^2}{\left(\frac{E_0}{\pi}\right)^2} = \frac{8}{\pi^2}$$



$$5. \quad X_L > 20$$

$$X_C > 20$$

$$R = 10, \quad V = 200.$$

$$Z = R + j(X_L - X_C) = 10.$$

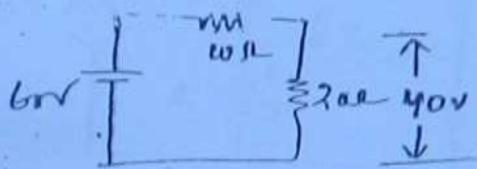
(75)

$$I = \frac{V}{Z} = \frac{200}{10} = 20 \text{ A}$$

$$V_C = -j I X_C = (20)(20) L - 90^\circ = 400 L - 90^\circ \text{ V}$$

$$I = \sqrt{I_R^2 + I_L^2}$$

$$V_1 = 40 \text{ V}$$



So, the current source has no effect on 2Ω across 60V .

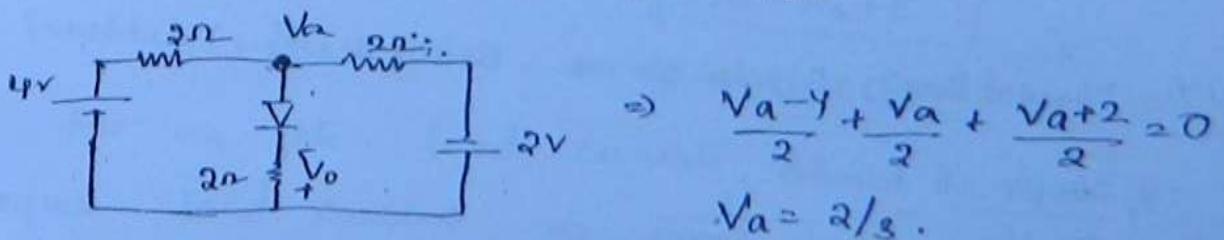
Voltage across $2\Omega = 40\text{V}$. $\therefore E_R = 0 \Rightarrow V_1 = V_2 = 40\text{V}$

Current mag. is const.

Voltage doubles for double resistor

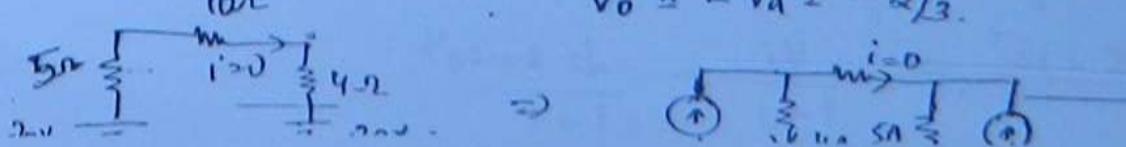
$$I_L = e^{at} + e^{bt} \Rightarrow V = L \frac{di}{dt} \Rightarrow V = a e^{at} + b e^{bt}$$

$$10 + 5 + E + 1 + 0 = 0 \Rightarrow E = -16 \text{ V}$$

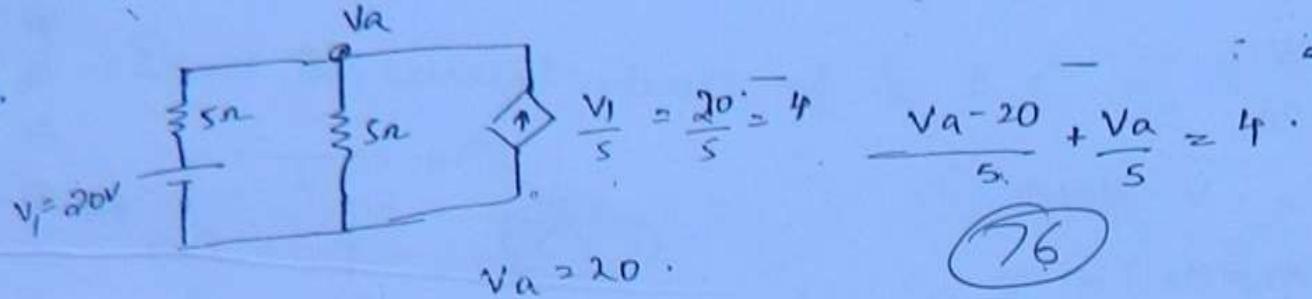


$$V_a = 2/3.$$

$$V_o = -V_a = -2/3.$$

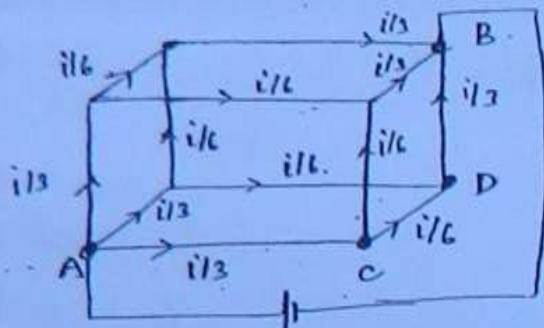


22.



Current source $\Rightarrow 90 \times 4 = 80W$ deliver.

24.



Consider equal resistors of value $= R$.

loop ACDB.

$$V = \frac{I}{3}R + \frac{I}{6}R + \frac{I}{3}R$$

$$\frac{V}{I} = \frac{5}{6}R = R_{AB}$$

For Resistors, $R_{AB} = \frac{5}{6}R$.

inductors $L_{AB} = \frac{5}{6}L$.

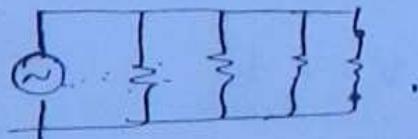
capacitors $C_{AB} = \frac{6}{5}C$.

25.

variable angle \rightarrow semicircle
cont. w \rightarrow st. line.

28.

Consider,



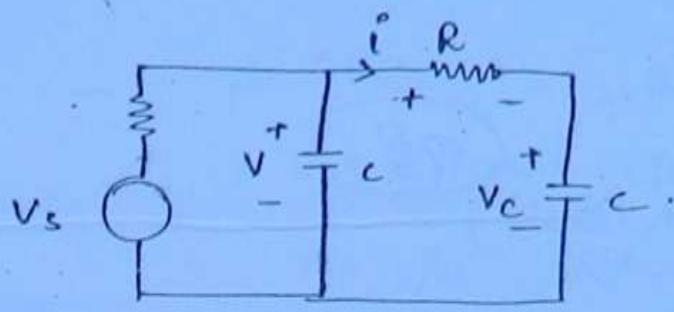
29.

$$I = VY = (3 - 2j + 6j) \sin 2t = 5 \sin(2t + 53.1^\circ)$$

30.

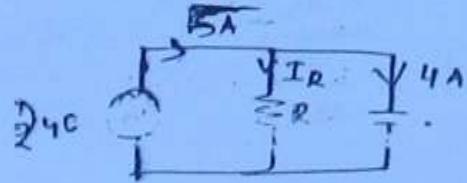
$$V_o = V_i \frac{1/j\omega C}{R + 1/j\omega C} = \frac{V_i}{1 + j\omega RC}$$

$$\omega = 10^3, R_C = 10^3 \Rightarrow \frac{V_i}{1 + j} = \frac{\sqrt{2} \sin 10^3 t}{1 + j}$$



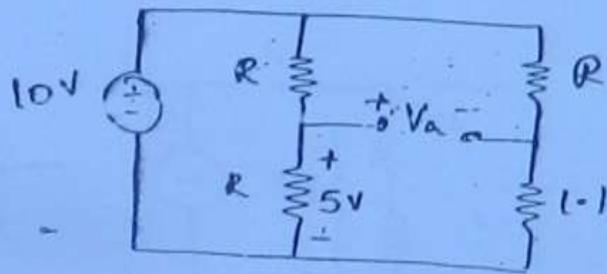
(77)

$$V = iR + V_C \Rightarrow V = \frac{cdV_C}{dt} + V_C$$



$$V_R = \sqrt{5^2 - 4^2} = 3V$$

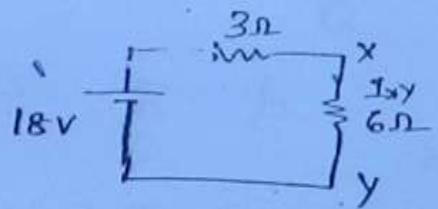
$$R = \frac{240V}{3A} = 80\Omega$$



$$V_I = \frac{10 \times 1.1R}{1.1R + R}$$

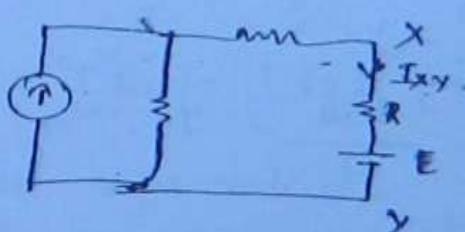
$$V_I = 5.238V$$

$$V_A = 5 - 5.238 = -0.238V$$



$$I_{xy} = \frac{18}{9} = 2A$$

$$V_{xy} = 2 \times 6 = 12V$$



$$V_{xy} = RI_{xy} + E$$

$$12 = R_2 + E$$

Substitute the given set of values and check.

For eq. ckt, load currents should be equal.
equal load current \Rightarrow equal load voltage.

38

$$V_{ta} = \frac{e_1 + e_2(t)}{3}$$

$$= \frac{1}{3} (\sqrt{3} \cos(\omega t + 30^\circ) + j\sqrt{3} \sin(\omega t + 60^\circ))$$

$$= \frac{1}{3} \cdot \sqrt{3} [\cos(\omega t + 30^\circ) + \cos(\omega t + 60 - 90^\circ)]$$

Convert it to either cos or sine.

$$= \frac{1}{\sqrt{3}} [\cos(\omega t + 30^\circ) + \cos(\omega t - 30^\circ)]$$

$$= \frac{1}{\sqrt{3}} [1[\underline{30^\circ}] + 1[\underline{-30^\circ}]] = 1V \quad \text{or} \quad 1\text{D}$$

$$= 1 \cos(\omega t + 0^\circ) = \underline{\cos \omega t}$$

$$43. V = \sqrt{V_R^2 + V_L^2}$$

$$250 = \sqrt{V_R^2 + 150^2} \Rightarrow V_R = 200V$$

$$i = \frac{V_R}{R} = \frac{200}{100} = 2A \Rightarrow i_{rms} = 2A$$

$$X_L = \frac{V_L}{I} = \frac{150}{2} = 75 \Rightarrow \omega L = 75$$

$$L = \frac{75}{300} = 0.25H$$

Let current thru 60V = i^1

$$i + i^1 = 12 \Rightarrow i^1 = 12 - i^1 = 12A$$

Ans: 10A

$$\boxed{R = \frac{V^2}{P}}$$

$$R = 192 \Omega$$

$$\boxed{R_{eq} = \frac{V_{ea}^2}{P_{ea}}} \quad n = \frac{V_{ea}}{V_{open}^2}$$

$$R_{eq} = nR$$

$$n = 2$$

$n = \text{no. of bulbs}$

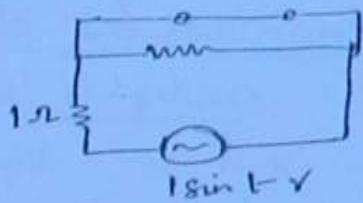
The bridge is unbalanced becoz, the angles θ_L & θ_C are different for inductor & capacitor.

\therefore Go for $\nabla \rightarrow \Delta$ transformation (79)

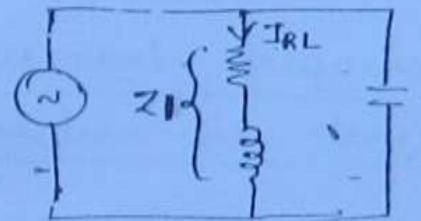
$$\omega = 1$$

$$X_L = \omega L = 1$$

$$X_C = \frac{1}{\omega C} = 1 \Rightarrow j(X_L - X_C) = 0 \rightarrow S.C.$$



$$\left(\frac{1}{R^2 + X_L^2} \right)^{1/2}$$



$$Z_1 = \frac{V_s}{I_{RL}}$$

$$Z_1 = \frac{110}{\sqrt{2} \angle -45^\circ} = \frac{1}{\sqrt{2}} \angle 45^\circ$$

$$Z_1 = \frac{\cos 45^\circ + j \sin 45^\circ}{\sqrt{2}} = \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} = R + j X_L$$

$$R = \frac{1}{\sqrt{2}} \Rightarrow P = \frac{V_s I_{RL}}{\sqrt{2}} = (\sqrt{2}) \times (\frac{1}{\sqrt{2}}) = 1 W$$

phasor sum is done with rms values.

$$I_s = I_{RL} + I_C \quad \xrightarrow{\text{phasor sum}} \text{rms value}$$

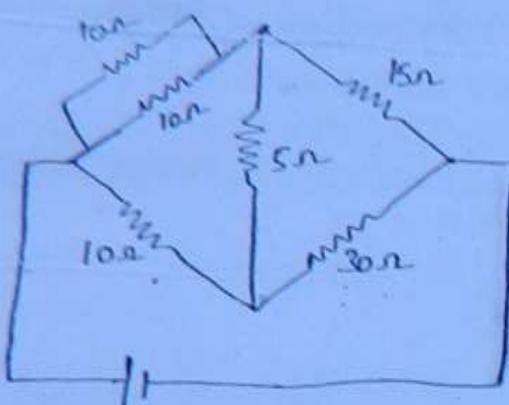
$i_{rms} \cdot \sqrt{2}$ by default

Note : In real time systems, the voltage or the current values are given in rms values.

\therefore if nothing is specified, by default we take it as rms value.

Conventional ques.

1.



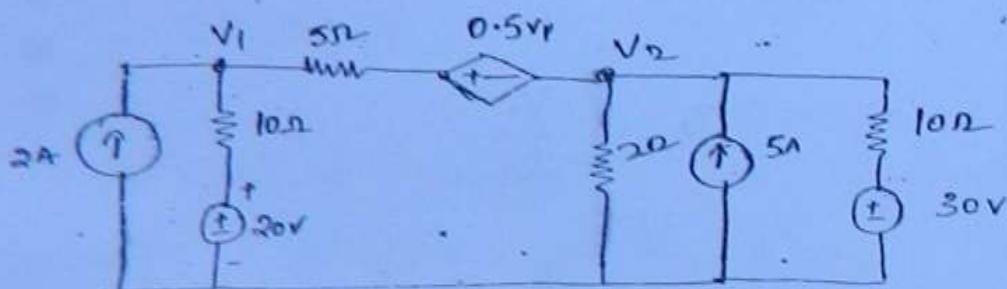
(80)

Balanced bridge

$$20 \times 30 = 10 \times 15$$

$$150 = 150 \text{ V}$$

$$\therefore i_{5\Omega} = 0$$



IES 2009

By KCL, at node 1

$$2 = \frac{v_1 - 20}{10} + \frac{v_1 - 0.5v_1 - v_2}{5} \rightarrow ①$$

at node 2.

$$5 = \frac{v_2}{2} + \frac{v_2 - 30}{10} + \frac{v_2 + 0.5v_1 - v_1}{5} \rightarrow ②$$

$$2 = 0.1v_1 - 2 + 0.1v_1 - 0.2v_2$$

$$4 = 0.2v_1 - 0.2v_2 \Rightarrow v_1 - v_2 = 20$$

$$5 = 0.5v_2 + 0.1v_2 - 3 + 0.2v_2 - 0.1v_1$$

$$8 = -0.8v_2 - 0.1v_1 \Rightarrow 8v_2 - v_1 = 80$$

$$8v_2 - 20 + v_2 = 80$$

Resonance

(81)

For occurrence of resonance in any system, two energies are required.

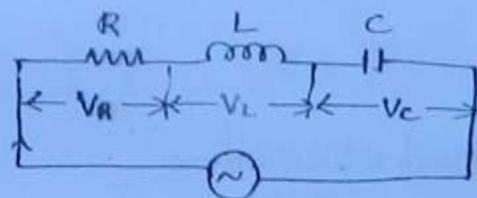
In RLC circuit, inductor consists of magnetic field energy, and capacitor consists of electric field energy.

The circuit is said to be resonant, when source voltage and source current are in phase.

The frequency at which $X_L = X_C$ is called as resonant frequency.

The resonant frequency indicates the rate at which energy transformation is done between inductor and capacitor.

Series resonance



$$V = V_R \angle 0^\circ + V_L \angle 90^\circ + V_C \angle -90^\circ$$

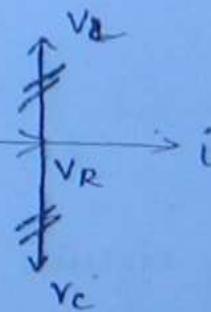
for resonance,

$$V_L = V_C$$

$$I X_L = I X_C$$

$$\omega L = \frac{1}{\omega C} \Rightarrow$$

$$\text{resonant freq, } f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$



$$\boxed{\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \text{ rad/sec} \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \end{aligned}}$$

$$1. \quad Z = R + j(X_L - X_C)$$

$$\boxed{Z_{\min} = R}$$

(82)

$$2. \quad I_{\max} = \frac{V}{Z_{\min}} = \frac{V}{R}$$

$$3. \quad \cos \theta = 1 \quad (\text{power factor} > 1 \Rightarrow \text{inphase})$$

$$4. \quad V_R = V$$

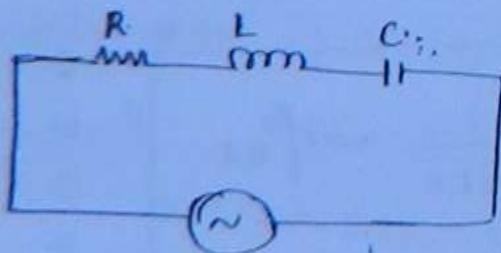
(active component of the voltage) $\rightarrow V_R = V$ $\rightarrow i$

$$5. \quad \text{Net reactive voltage} = 0$$

6. voltage across inductor and voltage across capacitor greater than source voltage. This phenomena is called as voltage magnification.

Applications

1. Oscillation
2. Filters (Band pass & Band elimination filter)
3. Tuning circuits.
4. Induction heating.



$$V_C = I X_C$$

$$X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi f C}$$

* The graph & derivation for $L_1 \approx L_2$ are in next page

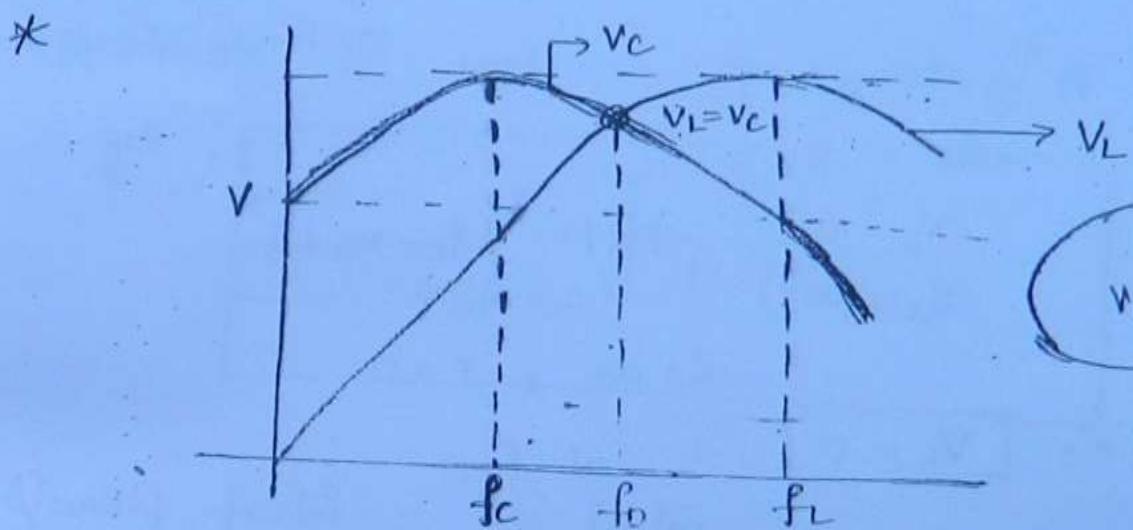
Let $f \geq 0$ & f is varied (increased).

$$x_L = 0 \quad |x_L - x_C| = \infty \Rightarrow |x|.$$

$$x_C = \infty \Rightarrow Z = \infty, I = 0,$$

$$\therefore \boxed{V_C = V} \text{ at } f \geq 0.$$

(83)



$$f \uparrow, x_L \uparrow, x_C \downarrow \quad |x| = |x_L - x_C| \downarrow \downarrow$$

(10 Hz) (1000 Hz) 900

$Z \downarrow \downarrow$ \rightarrow $\uparrow I = \frac{V}{Z \downarrow \downarrow} \Rightarrow V_C \uparrow$

higher frequencies,

$$f \uparrow \uparrow \quad x_L \uparrow \uparrow \quad x_C \downarrow \downarrow$$

(very low)

$$V_C = I x_C.$$

$$V_C = \frac{V x_C}{\sqrt{R^2 + (x_L - x_C)^2}} \rightarrow$$

$$V_C = \frac{V \cdot 1/\omega_C}{\sqrt{R^2 + \left(\omega_L - \frac{1}{\omega_C}\right)^2}} \rightarrow \textcircled{1}$$

differentiate - eq ① wrt ω . & equate it to zero.

we obtain,

(84)

series resonance

$$f_c = \frac{1}{2\pi\sqrt{LC}} \quad \boxed{\sqrt{1 - \left(\frac{R^2}{2L}\right)}}$$

for V_L :

$$\frac{1}{2\pi f LC} \quad \boxed{1 - \frac{R^2}{2L}}$$

$$V_L = IX_L$$

for $f = 0$;

$$X_L = 2\pi f L$$

$$X_L = 0 \quad |x| = |X_L - X_C|$$

$$X_C > \frac{1}{2\pi f C}$$

$$X_C = \infty \quad = \infty$$

$$Z = \infty, I = 0$$

$$\rightarrow \therefore \boxed{V_L = 0} \quad \text{at } f = 0$$

for low freq.

$$f \uparrow, X_L \uparrow, X_C \downarrow \quad |x| = |X_L - X_C| \downarrow \downarrow$$

(low) (high) $q_{0\Omega}$

$$Z \downarrow \downarrow \rightarrow \uparrow \uparrow I = \frac{V}{Z \downarrow \downarrow} \Rightarrow V_L \uparrow$$

for high freq.

$$f \uparrow \uparrow, X_L \uparrow \uparrow, X_C \downarrow \downarrow, Z = R + j(X_L - X_C) \uparrow \uparrow \uparrow$$

(very low)

$$\therefore I \downarrow \downarrow \downarrow \quad \& \quad V_L \downarrow$$

$$V_L = I \times L$$

$$V_L = \frac{V \times X_L}{\sqrt{R^2 + (X_L - X_C)^2}} \quad (85)$$

$$V_L = \frac{V \omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \rightarrow (2) .$$

differentiate (2) wrt ω & equating it to zero, we

get,

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{1 - (\omega^2 C^2 / 4L)}}}$$

Quality factor - Q-factor

$Q = \frac{2\pi}{\text{power dissipation per cycle}} \frac{\text{Max. energy stored in the ckt}}{\text{power dissipation per cycle}}$

$$i(t) = I_m \sin \omega t$$

$$V_C = \frac{1}{C} \int i(t) dt = \frac{1}{C} \int I_m \sin \omega t dt$$

$$\boxed{V_C(t) = \frac{I_m}{\omega C} \cos \omega t}$$

$$\omega^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega^2 C}$$

$$\frac{I_m}{\omega C}$$

$$W_g = \frac{1}{2} L I^2 + \frac{1}{2} C V_c^2$$

(86)

$$W_g = \frac{1}{2} L (I_m \sin \omega t)^2 + \frac{1}{2} C \left(-\frac{1}{\omega C} I_m \cos \omega t \right)^2$$

$$W_g = \frac{1}{2} L I_{max}^2 \sin^2 \omega t + \frac{1}{2} C \cdot \frac{1}{\omega^2 C} I_{max}^2 \cos^2 \omega t$$

$$W_g = \frac{1}{2} L I_{max}^2 \sin^2 \omega t + \frac{1}{2} L I_{max}^2 \cos^2 \omega t \quad \because \frac{1}{\omega^2 C} = L$$

$$W_g = \frac{1}{2} L I_{max}^2$$

$$W_g = \frac{1}{2} C V_{cmax}^2$$

where

$$V_{cmax} = \frac{I_{max}}{\omega C}$$

$$Q = \frac{2\pi}{R} \frac{\frac{1}{2} L I_{max}^2}{\left(\frac{I_{max}}{\sqrt{LC}}\right)^2 R \cdot 1/f}$$

$$Q = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

$$Q = \frac{\omega L}{R} : \quad \left(\omega = \frac{1}{\sqrt{LC}} \right)$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = \frac{x_L}{R} = x_C$$

$$Q = \frac{x_L}{R} = \frac{I x_L}{I_R} = \frac{V_L}{V_R} = \frac{V_L}{V} \quad \therefore x_L = \omega L$$

$$\text{But } V_L = V_C \quad \underline{=} \quad \underline{=}$$

$$Q = \frac{V_C}{V} = \frac{I x_C}{V_R} = \frac{I x_C}{V_0} = \frac{x_C}{V_0} = \underline{\underline{1}}$$

$$Q = \frac{I^2 X_L}{I^2 R} = \frac{Q_L}{P}$$

(87)

6/11

Band width

f_0

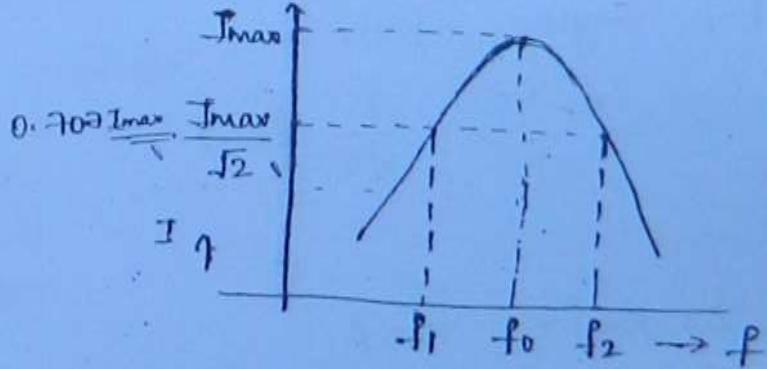
P

When the curve is drawn between current and frequency, then the curve is called as resonance curve.

Band-width is a range of frequencies on either side of the resonant frequency where the current falls from maximum value to $\pm 0.7\%$ of the max. value. It is given by,

$$\text{BW} = f_2 - f_1 \rightarrow \text{lower cut off}$$

↓
upper cut off



$$f_0 \rightarrow I_{\max}$$

$$f_0 \rightarrow Z = R$$

$$f_1, f_2 \rightarrow \frac{I_{\max}}{\sqrt{2}}$$

$$f_1, f_2 \rightarrow Z = \sqrt{2}R$$

$$f_0 \rightarrow \cos \theta = 1$$

$$f_1, f_2 \rightarrow Z = R \pm jX$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{2}R$$

$$\therefore X = R$$

from the curve,

$$X > X_L - X_C$$

$$X_L = 2\pi f L, \quad X_C = \frac{1}{2\pi f C}$$

(88)

$$f_1 \rightarrow X_C > X_L \Rightarrow X \rightarrow -ve.$$

$$f_2 \rightarrow X_L > X_C \Rightarrow X \rightarrow +ve.$$

For f_1 :

$$f_1 \rightarrow Z = R - jX, \quad X = R.$$

$$\text{Impedance angle} = \tan^{-1}\left(\frac{-X}{R}\right), -45^\circ$$

$$I = \frac{\sqrt{10}}{Z[-45^\circ]} = \frac{\sqrt{10}}{Z} [+45^\circ]$$

$$\text{P.f. angle} = +45^\circ$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \text{ lead}$$

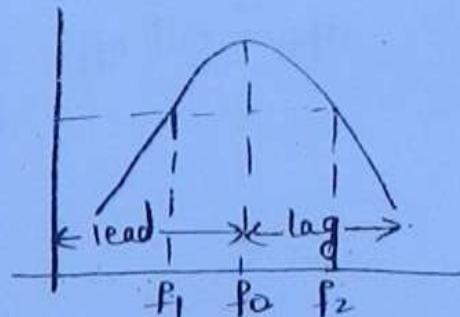
for f_2 :

$$f_2 \rightarrow Z = R + jX, \quad X = R.$$

$$\text{Impedance angle}, \quad \tan^{-1}\left(\frac{X}{R}\right) = 45^\circ$$

$$\text{P.f. angle} = -45^\circ$$

$$\cos(-45^\circ) = \frac{1}{\sqrt{2}} \text{ lag}$$



Note:

Impedance angle & P.f. angle always have same magnitude but have opposite sign.

ippsku

$$f_0 \rightarrow I_{max}, \quad P = I_{max}^2 R$$

$$f_1, f_2 \rightarrow \frac{I_{max}}{\sqrt{2}}, \quad P' = \left(\frac{I_{max}}{\sqrt{2}} \right)^2 R. \quad (89)$$

$$\therefore P' = \frac{I_{max}^2 R}{2} = \frac{P}{2}.$$

$$\boxed{P' = \frac{P}{2}}$$

Power at $f_1, f_2 = \frac{1}{2}$ power of f_0 .

$\therefore f_1, f_2 \rightarrow \underline{\text{half power frequencies}}$

$$f_1: X_c > X_L, \quad X = R.$$

$$\frac{1}{\omega_1 C} - \omega_1 L = R \quad \rightarrow (1)$$

$$f_2 \rightarrow X_L > X_c, \quad X = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \rightarrow (2)$$

$$(1) = (2) \quad \omega_1 \omega_2 = \frac{1}{LC} \quad \rightarrow (3)$$

$$\omega_0^2 = \frac{1}{LC} \rightarrow (4)$$

$$(4) = (3)$$

$$\omega_0^2 = \omega_1 \omega_2$$

$$\boxed{\omega_0 = \sqrt{\omega_1 \omega_2}} \quad **$$

$$\boxed{f_0 = \sqrt{f_1 f_2}} \quad **$$

dd eqn (1) & (2)

$$\frac{1}{C} \left[\frac{1}{\omega_1} - \frac{1}{\omega_2} \right] + L [\omega_2 - \omega_1] = 2R$$

$$\frac{1}{C} \left[\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right] + L [\omega_2 - \omega_1] = 2R$$

$$L (\omega_2 - \omega_1) + L (\omega_2 - \omega_1) = 2R$$

(90)

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$L = \frac{1}{\omega_1 \omega_2 C}$$

* *

$$B \cdot \omega = \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/s}$$

$$B \cdot \omega = f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz}$$

R/L

$f_2 - f_1$

$$5.) Q = \frac{\omega_0 L}{R}$$

$$\frac{\omega_0}{(R/L)} \Rightarrow$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

$$Q = \frac{f_0}{f_2 - f_1}$$

* *

Objectives

1. $R \uparrow$ $B \cdot \omega \uparrow$ $f_1 \downarrow$ $f_2 \uparrow$, $f_0 = \text{const.}$

$$2. Q = \frac{\omega L}{R} = \frac{X_L}{R} = \frac{V_L}{V}, \quad Q > 1$$

$$X_L > R$$

$$X_C > R$$

* In series resonance, always $V_L > V \Rightarrow X_L > R \text{ & } X_C > R.$

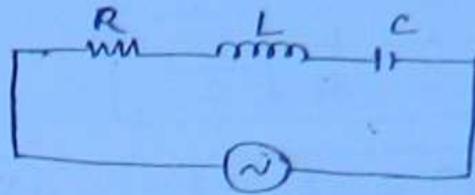
$$f_1 = \frac{1}{2\pi f_{LC}} \sqrt{1 - \frac{R^2}{4L^2}}$$

$$f_2 = \frac{1}{2\pi f_{LC}} \sqrt{\frac{1}{1 - \frac{R^2}{4L^2}}}$$

$\because X_L = X_C \text{ at } f_0$

By KVL,

$$V = RI + L \frac{di}{dt} + \frac{1}{C} \int i dt \quad (91)$$



diff. wrt t.

$$V' = R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{i}{C}$$

$$\frac{1}{L} \frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = \frac{V'}{L}$$

$\downarrow \omega_2 - \omega_1$ $\downarrow \omega_0^2 = 1/LC$.

If a given diff. eqn is of the above form, then we can obtain $B\omega$ & ω_0 .

coeff of $\frac{di}{dt} = \omega_2 - \omega_1 = \frac{R}{L} = B\omega$.

coeff of $i^2 = \frac{1}{LC} = \omega_0^2$.

$$\Rightarrow Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC} \right) i = 0 \quad \left. \begin{array}{l} \\ s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \end{array} \right\} \quad \frac{d}{dt} = D$$

$$\Rightarrow \omega_n^2 = \frac{1}{LC} \Rightarrow \omega_n = \frac{1}{\sqrt{LC}}$$
$$2\zeta\omega_n = \frac{R}{L}$$

Damping ratio $\zeta = \frac{R}{2} \sqrt{\frac{C}{L}}$

$$D = \frac{1}{2} \sqrt{\frac{C}{L}}$$

But

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

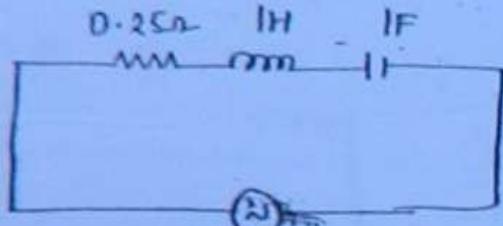
\Rightarrow

$$\mathcal{E}_I = \frac{1}{2Q}$$

(92)

$$\frac{R}{2} \sqrt{\frac{C}{L}}$$

Q.



$$V(t) = 10\sin\omega t$$

Find

$$(i) f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1 \times 1}} = \frac{1}{2\pi}$$

$$(ii) Q = \frac{1}{R} \sqrt{\frac{C}{L}} = \frac{1}{0.25} \sqrt{\frac{1}{1}} = 4$$

$$(iii) \frac{B\omega}{(f_2 - f_1)} = \frac{-f_0}{Q} = \frac{1}{8\pi}$$

$$(iv) \mathcal{E}_I = \frac{1}{2Q} = \frac{1}{8}$$

$$(v) f_2 - f_1 = 1/8\pi \quad \text{and} \quad f_0^2 = f_1 f_2$$

$$\Rightarrow f_1 f_2 = 1/4\pi^2$$

$$(f_1 + f_2)^2 - (f_2 - f_1)^2 = 4 f_1 f_2$$

$$(f_1 + f_2)^2 = \frac{1}{\pi^2} + \frac{1}{64\pi^2} =$$

$$f_1^2$$

6.

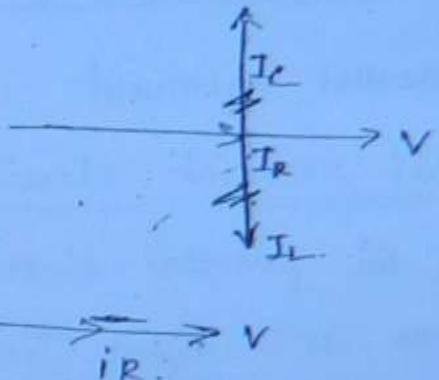
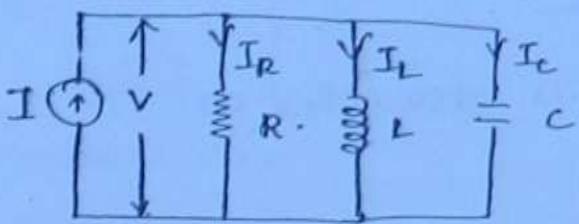
$$I = \frac{V}{Z} = \frac{V}{R}$$

$$I = \frac{10/\sqrt{2}}{0.25} = 20\sqrt{2}$$

(Q3)

Parallel Resonance

Ans 1.



$$I = I_R \perp^0 + I_L \perp 90^\circ + I_C \perp 90^\circ$$

$$i_R$$

$$I_L = I_C$$

$$\frac{X_L}{X_C} = \frac{L}{C} \Rightarrow$$

~~**~~

$$B_L = B_C$$

$$\frac{1}{\omega L} = \omega C \Rightarrow \boxed{\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \text{ rad/sec} \\ \omega_0 &= \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \end{aligned}}$$

$$Y = G + j(B_C - B_L)$$

4πn

$$Y_{\min} = G$$

$$Z_{\max} = \frac{1}{Y_{\min}}$$

$$I_{\min} = \frac{V}{Z_{\max}}$$

5. $I_a = I$

(active component of the current).

(94)

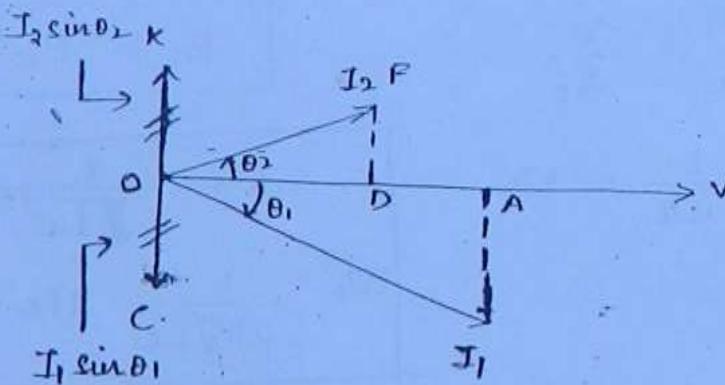
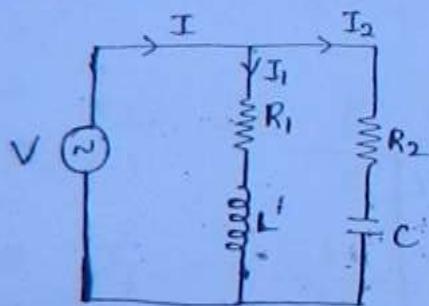
6. Net reactive current = 0.

7. Current flowing through inductor and capacitor greater than total current. This phenomena is called as current magnification.

8. Parallel resonant circuit is also called as Anti-resonant circuit.

9. In the parallel circuit at resonance if $G_f = 0$, circuit behaves as Open circuit.

Case 2 :



$$I = I_1 \cos \theta_1 + I_2 \cos \theta_2$$

$$OA = I_1 \cos \theta_1$$

$$OC = AB = I_1 \sin \theta_1$$

$$OD = I_2 \cos \theta_2$$

$$OK = DF = I_2 \sin \theta_2$$

$$\rightarrow B_L = B_C -$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{X_C}{R^2 + X_C^2}$$

(95)

$$\frac{\omega L}{R^2 + (\omega L)^2} = \frac{1/X_C}{R_2^2 + (1/X_C)^2} \rightarrow ①$$

*
$$i_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}$$

$$\frac{1}{2\pi f C} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}$$

Resonance Condition for all frequencies

from eqn ① above.

$$\frac{\omega L}{R^2 + (\omega L)^2} = \frac{1/X_C}{R_2^2 + (1/X_C)^2}$$

$$\frac{1}{\frac{R_1^2}{\omega L} + \omega L} = \frac{1}{R_2 C \omega + \frac{1}{\omega C}}$$

Comparing the coefficients, (\because for resonance condition in the above eqn $\forall f \Rightarrow$ it should be independent of ω)

$$\omega \rightarrow L = R_2^2 C \Rightarrow R_2 = \sqrt{\frac{L}{C}}$$

$$\frac{1}{\omega} \rightarrow \frac{R_1}{L} = \frac{1}{C} \Rightarrow R_1 = \sqrt{\frac{L}{C}}$$

**
$$R_1 = R_2 = \sqrt{\frac{L}{C}}$$

$$Y = (G_{11} + G_{12}) + j(G_{21} - G_{22})$$

$$Y = \frac{R_1}{R_1^2 + X_L^2} + j \frac{R_2}{R_2^2 + X_C^2}$$

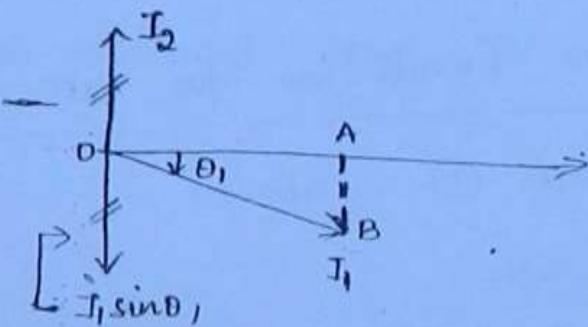
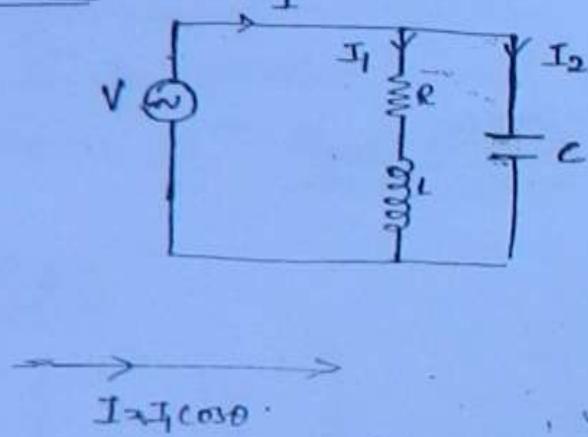
$$I = VY$$

$$I = V \left[\frac{R_1^2}{R_1^2 + X_L^2} + j \frac{R_2^2}{R_2^2 + X_C^2} \right]$$

96

//

Case 3: TANK Ckt



$$I = I_1 \cos \theta$$

$$OA = I_1 \cos \theta$$

$$OC = AB = I_1 \sin \theta$$

$$\begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \quad Z_1 = \frac{X_L}{\cos \theta} \quad \theta = \tan^{-1} \frac{X_L}{Z_1}$$

$$\sin \theta = \frac{X_L}{Z_1}$$

$$\cos \theta = \frac{R}{Z_1}$$

$$I_2 = I_1 \sin \theta$$

$$\frac{V}{X_C} = \frac{V}{Z_1} \frac{X_L}{Z_1}$$

$$Z_1^2 = X_L X_C = \frac{\omega L}{\omega C} = L/C$$

$$* * \sqrt{Z_1} = \sqrt{\frac{L}{C}}$$

$$I = I_0 \cos \theta$$

$$I = \frac{V}{Z_L} \cdot \frac{R}{Z_L}$$

$$I = \frac{VR}{L/C}$$

(97)

$$I = \frac{V}{(L/R_C)}$$

$$Z_{DY} = \frac{L}{RC} \quad \Omega$$

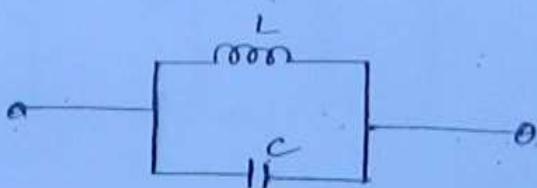
Dynamic impedance.

$$Z_L^2 = R^2 + X_L^2$$

$$\frac{L}{C} = R^2 + (2\pi f_L)^2$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$



$$Z_{DY} = \frac{L}{RC} = \infty$$

Ideal Tank ckt i.e. $R=0$

Q-factor

$$Q = \frac{V_L \text{ (or) } V_C}{V} \quad \text{for series.}$$

parallel resonant

$$Q = \frac{I_L \text{ (or) } I_C}{I}$$

$$Q = \frac{I_L}{I} = \frac{I_L}{I_R} = \frac{\text{Reactance component of current}}{\text{Active component of current.}}$$

$$Q = \frac{I_L}{R} = \frac{V/X_L}{V/R} = \frac{R}{X_L} = \frac{R}{\omega L}$$

(98) for Case ii // resonance

$$\omega = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{R}{\omega} \sqrt{\frac{C}{L}}$$

$$Q = \frac{V/X_L}{V/R} = \frac{B_L}{G_I}$$

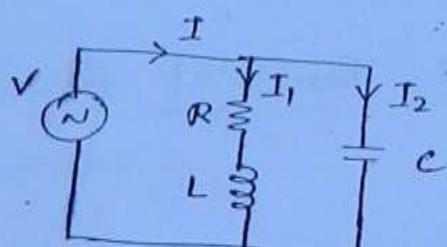
$$\rightarrow Q^2 = \frac{I_C}{I} = \frac{I_C}{I_R}$$

$$\rightarrow Q = \frac{V/X_C}{V/R} = \frac{R}{X_C} = \underline{R_{WC}}$$

For tank ckt:

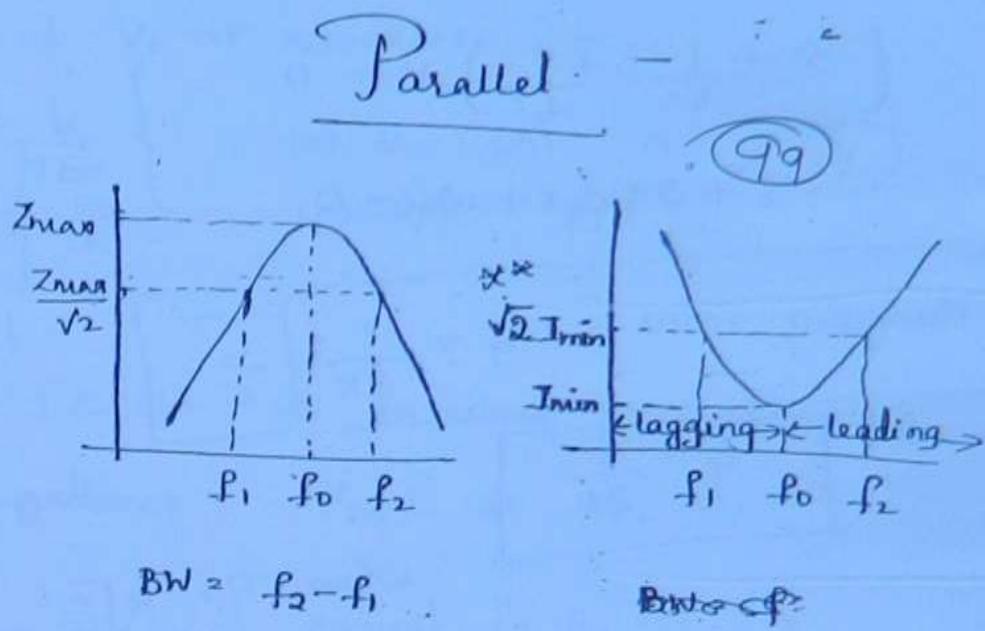
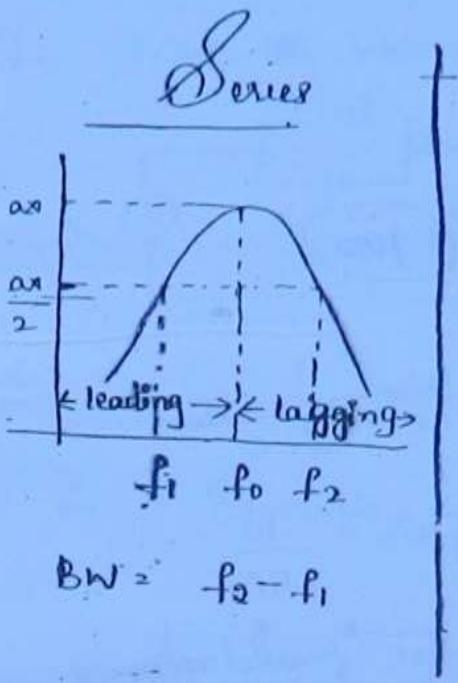
$$Q = \frac{I_2}{I}$$

$$Q^2 = \frac{V/X_C}{V/(L/R_C)} \therefore = \frac{1/(1/w_C)}{1/(L/R_C)}$$

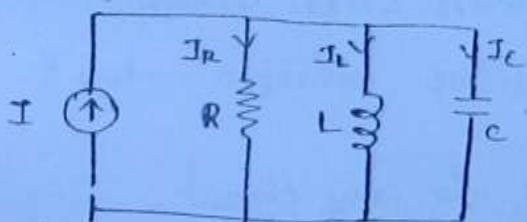


$$Q = \frac{wL}{R}$$

* * for tank circuit \Rightarrow series



Relation between damping factor and Q-factor.



$$I = \frac{V}{R} + C \frac{dv}{dt} + \frac{1}{L} \int v dt$$

differentiate - wrt - t .

$$I' = \frac{1}{R} \frac{dv}{dt} + C \frac{d^2v}{dt^2} + \frac{V}{L}$$

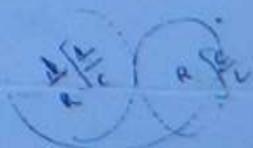
$\therefore C$

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{V}{LC} = \frac{I'}{C}$$

$$\omega_2 - \omega_1 = \frac{1}{RC}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$



$$\left(D^2 + \frac{1}{RC} D + \frac{1}{LC} \right) V = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} \frac{d}{dt} = D \quad (QD)$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

damping ratio = $\xi = \frac{1}{2R} \sqrt{\frac{L}{C}}$

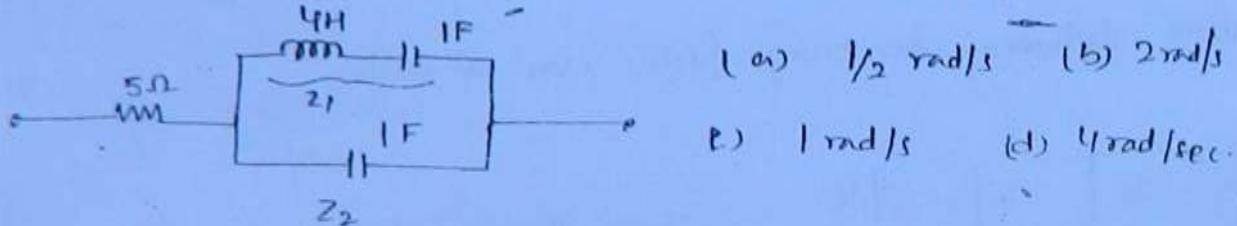
$$\omega_n = \frac{1}{\sqrt{LC}}$$

****** $\xi = \frac{1}{2R}$

$$2\xi\omega_n = \frac{1}{RC}$$

where $\xi = R\sqrt{\frac{C}{L}}$. for parallel resonance.

* * * Q. Find resonant frequency of the circuit shown.



(a) $1/2 \text{ rad/s}$ (b) 2 rad/s

(c) 1 rad/s (d) 4 rad/sec.

Note :-

To find resonant frequency for any circuit,

→ i) find equivalent impedance

→ ii) Equate imaginary part of the impedance to zero.

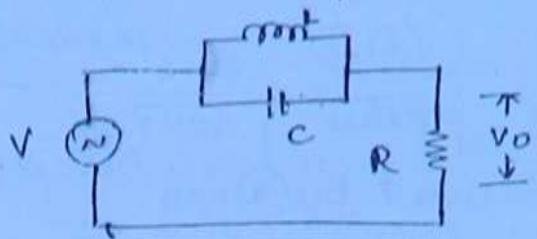
$$Z_1 = j(\omega_L - \omega_C) = j\left(\omega L - \frac{1}{\omega C}\right) = j\left(4\omega - \frac{1}{\omega}\right).$$

$$Z_2 = \frac{-j}{\omega}$$

$$Z_1 // Z_2 \Rightarrow Z_{eq} = \frac{j\left(4\omega - \frac{1}{\omega}\right)\left(-\frac{j}{\omega}\right)}{j\left(4\omega - \frac{1}{\omega}\right) - \frac{j}{\omega}} = \frac{\frac{1}{\omega}\left(\frac{1}{\omega} - 4\omega\right)}{\frac{1}{\omega}(4\omega^2 - 1) - 1} = 0.$$

$$\left(4\omega - \frac{1}{\omega}\right) \frac{1}{\omega} = 0 \Rightarrow \omega = 0.5 \text{ rad/sec.}$$

Q. Find the value of V_o at resonance.

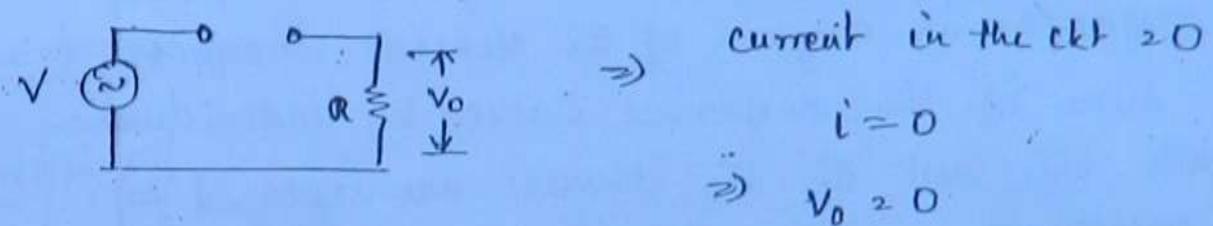


- (a) $V \sqrt{b^2 - 1}$ (b) 0 (c) $V/2$ (d) ∞ . 101

Sol.

The given LC ckt is an ideal tank ckt.

∴ Dynamic impedance $Z_{dy} = \infty$.



A parallel RLC circuit have.

$$BW = 1\text{ kHz} \quad \& \quad C = 0.1\ \mu\text{F}$$

Find effective resistance of the circuit.

$$BW = \omega_2 - \omega_1 = \frac{1}{RC} \text{ rad/s}$$

$$\Rightarrow f_2 - f_1 = \frac{1}{2\pi RC} \quad \begin{matrix} BW \text{ in Hz} \\ \text{convert to } f \end{matrix}$$

$$R = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 10^3} = \frac{10^4}{2\pi}$$

* For conventional, Thenevin's & NPT are very important -

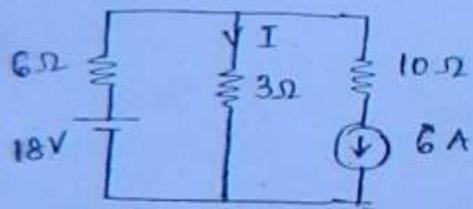
(102)

- when network is having several nodes, meshes & sources, the response in any element can be obtained by using theorems

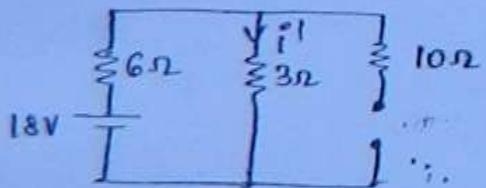
Superposition theorem

In any linear bidirectional circuit having more number of sources the response in anyone of the elements is equal to algebraic sum of the responses caused by individual sources, while the rest of the sources are replaced by its internal resistance.

- Find the value of i using superposition theorem



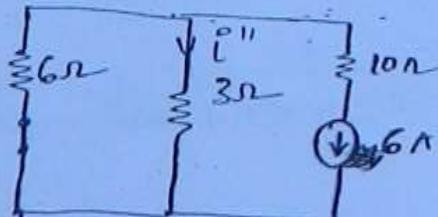
case i) (18V)



$$i^1 = \frac{18}{9} = 2A$$

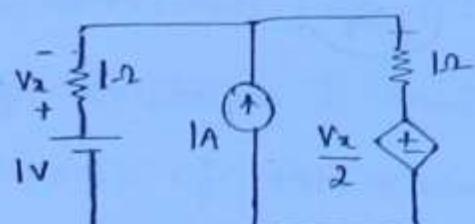
$$i^2 = i^1 + i'' = 2 - 4 = -2A$$

case ii) (6A)



$$i'' = \frac{-6 \times 6}{9} = -4A$$

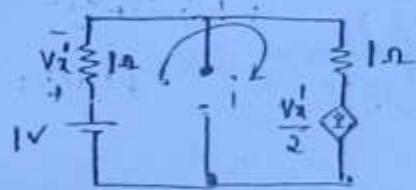
Q. Find the value of V_a by using superposition theorem.



(103)

Note: while applying superposition theorem, dependant source remains same as the original circuit.

Case i^b: 1V.



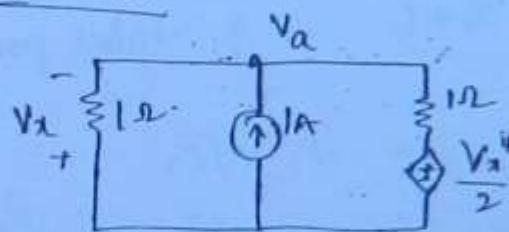
$$I = \frac{1 - V_x'/2}{1 + 1}$$

$$2i = 1 - \frac{V_x'}{2}$$

$$V_x' = 1 \times I = 1.$$

$$V_x = 2/5 V$$

Case ii^b



$$V_a + V_a - \frac{V_x''}{2} = 1.$$

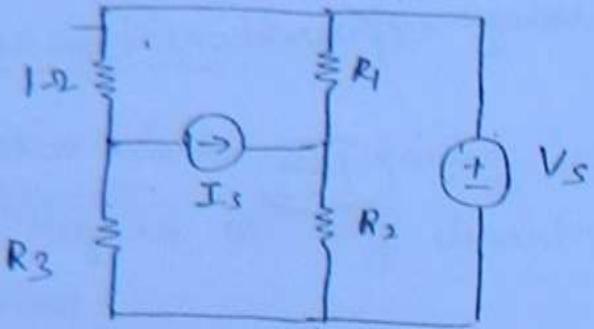
$$V_a = -V_x''.$$

$$V_x'' = -2/5 V$$

$$V_x = V_x' + V_x''$$

$$V_x = 0 V$$

In the circuit shown, power dissipation in the 1Ω resistor is 546W when voltage source is acting alone & power dissipation in 1Ω resistor is 1W when current source is acting alone. Find total power dissipation in 1Ω resistor.



104

$$P_1 = I_1^2 R \quad , \quad P_2 = I_2^2 R$$

$$I_1 = \sqrt{\frac{P_1}{R}} \quad I_2 = \sqrt{\frac{P_2}{R}}$$

total $I = I_1 + I_2$; total power dissipation = IR .

$$R = 1\Omega$$

$$I = \sqrt{P_1} + \sqrt{P_2}$$

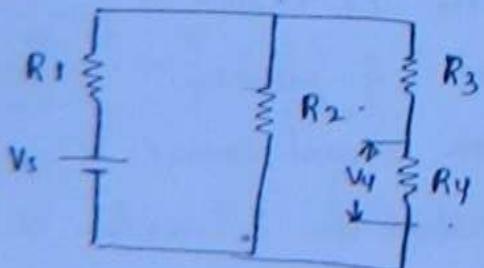
$$P = (\sqrt{P_1} + \sqrt{P_2})^2 \cdot 1\Omega = (\sqrt{576} + \sqrt{1})^2$$

$$P = 625W$$

General Expression for $R = 1\Omega$ total power dissipation

$$P = (\pm \sqrt{P_1} \pm \sqrt{P_2})^2$$

In the circuit shown, if source voltage is increased by 10%. Find change in power of R_4 resistor.



(a) 10% (b) 20%

(c) 21% (d) 30%

$$\frac{V \cdot R_4}{R_3 + R_4} \cdot \frac{(1+0.1)}{1.1V_1} = 1.1V_1 \cdot 1.21$$

Note: When circuit is having linear and bi directional elements, based on homogeneity principle, if excitation is multiplied with constant k , the response of each element is also multiplied with const. k .

$$P_4 = \frac{V_4^2}{R_4}$$

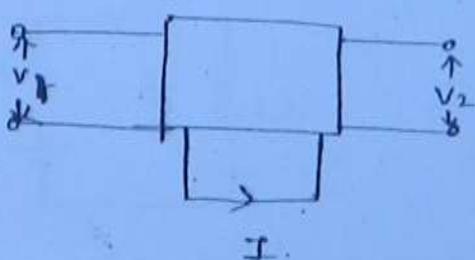
$$P_4 = \frac{(1.1 V_4)^2}{R_4} \quad \therefore \uparrow 10\% = 1 + 0.1 = 1.1$$

$$= 1.21 \frac{V_4^2}{R_4}$$

increase = 21 %.

Find the value of I when $V_1 = 10V$

$$V_2 = -4V$$



V_1	V_2	I
2	0	3
0	4	-2

$$V_1 = 2V \quad I = 3A$$

$$V_1 = 10V \quad I = 3 \times 5 = 15A$$

$$V_2 = 4 \quad I = -2$$

$$V_2 = -4 \quad I = 2A$$

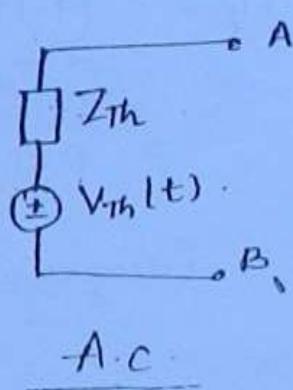
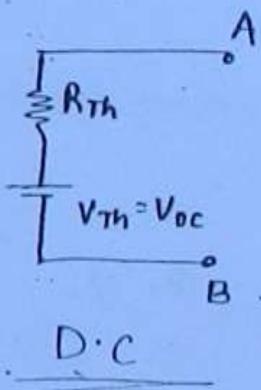
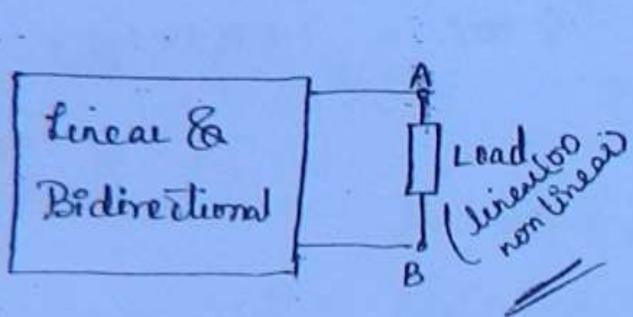
$$I = 15 + 2 = 17A$$

$\frac{15}{12}$

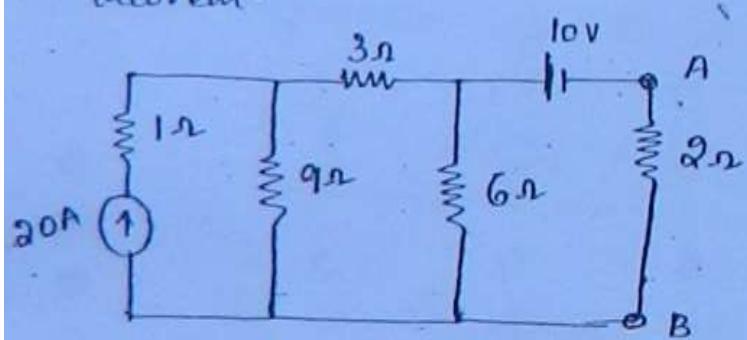
Thevenin's Theorem

106

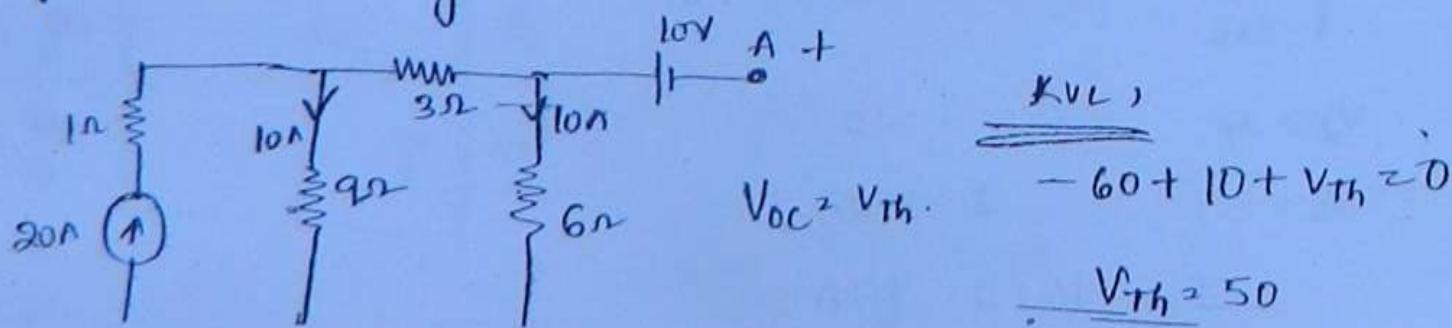
- In any linear bidirectional circuit having more number of elements it can be replaced by single equivalent circuit consisting of equivalent voltage (V_{Th}) in series with eq. resistance (R_{Th}).
- ⇒ By using Thevenin's theorem load current can be calculated either in linear or non-linear load.



Find current flowing through 2Ω resistor using Thevenin's Theorem.



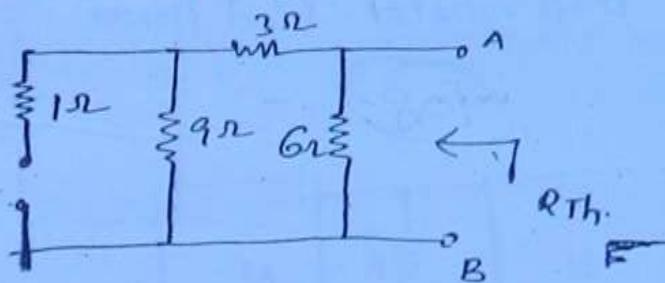
Case i, (V_{Th}) \therefore disconnect the load resistor and find open circuit voltage.



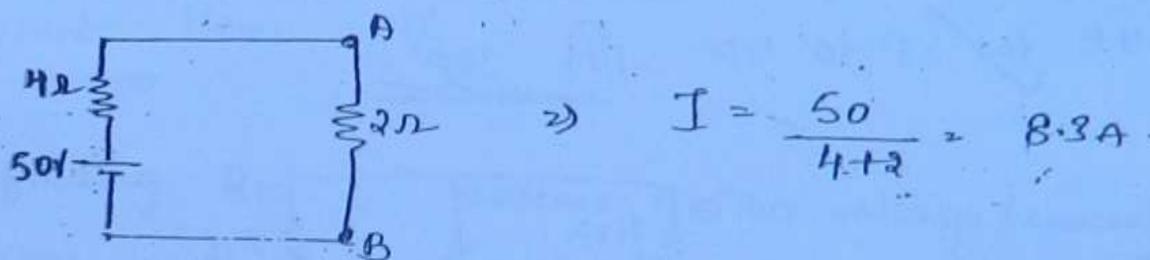
Case (2) (R_{Th}):

Deactivate all independent sources and find equivalent resistance
at load terminals.

(107)

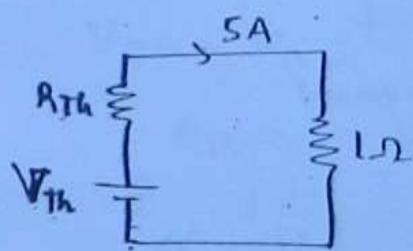


$$R_{Th} = \frac{12 \times 6}{12 + 6} = 4\Omega$$

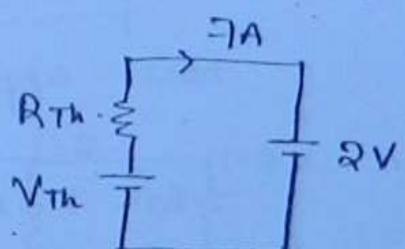


$$I = \frac{50}{4+2} = 8.3A$$

A battery charger drives a current of 5A when it is connected to load resistance of 1Ω. When the same battery charger is used for charging of ideal 2V battery at 7A rate find V_{Th} & R_{Th} .



$$V_{Th} = 5(R_{Th} + 1) \rightarrow ①$$



$$7 = \frac{V_{Th} - 2}{R_{Th}}$$

$$V_{Th} = 5(1.5 + 1)$$

$$7R_{Th} = V_{Th} - 2$$

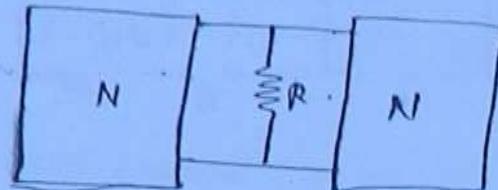
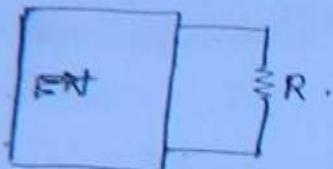
$$V_{Th} = 12.5V$$

$$7R_{Th} = 5R_{Th} + 5 - 2$$

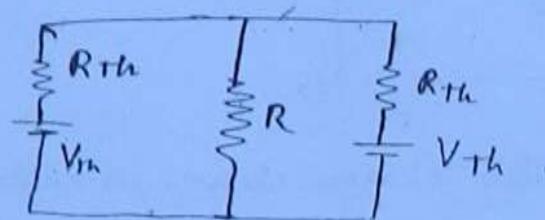
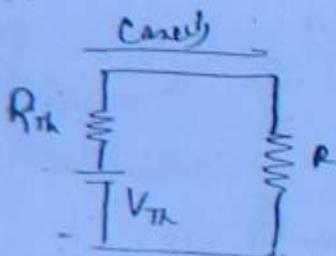
$$R_{Th} = 1.5\Omega$$

(Q) A complex network of N is connected to load resistor of R . Power dissipation in the resistor is $P_{N \text{ with } R}$. When two identical complex networks are connected to load resistor. Find power dissipation in the load.

(108)



- (a) $2P$ (b) $4P$ (c) P to $4P$ (d) $3P$



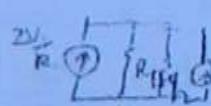
$$I = \frac{V_{th}}{R_{th} + R}$$

$$\Phi = IR$$

$$P = \left(\frac{V_{th}}{R_{th} + R} \right)^2 R \rightarrow \textcircled{1}$$

now apply Thvenin

$$\frac{1}{R_{th}} \parallel \frac{1}{R_{th}} \Rightarrow V_{th} = V_{th}$$



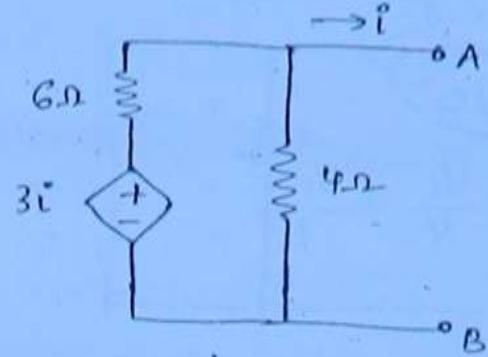
$$\frac{1}{R_{th}} \parallel \frac{1}{R_{th}} \Rightarrow R_{th} = R_{th}/2$$

$$R_{th} \parallel R_{th} \Rightarrow R_{th} = \frac{R_{th}}{2}$$

$$P = I^2 R = \left(\frac{2V_{th}}{R_{th} + 2R} \right)^2 R \rightarrow \textcircled{2}$$

$$I^2 = \frac{V_{th}}{R_{th} + R}$$

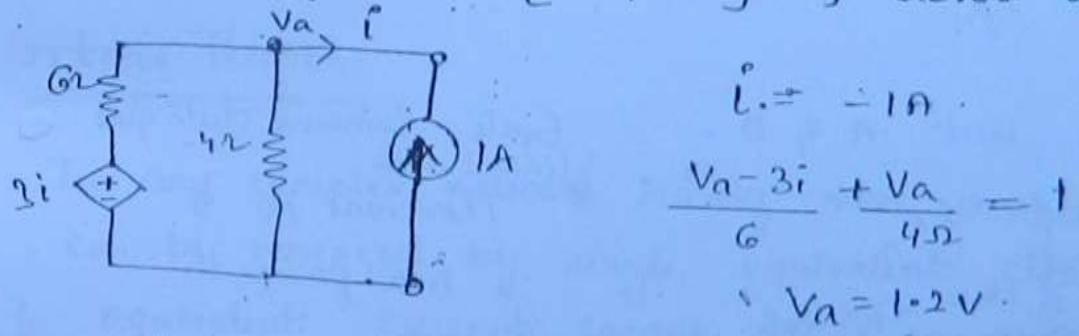
Find V_{Th} & R_{Th} wrt A & B.



(109)

In the above circuit no independent source is present. Hence $V_{Th} = 0$

or finding R_{Th} , assume either voltage/current source to any magnitude [preferably 1] across load terminals.



$$i_s = -1A$$

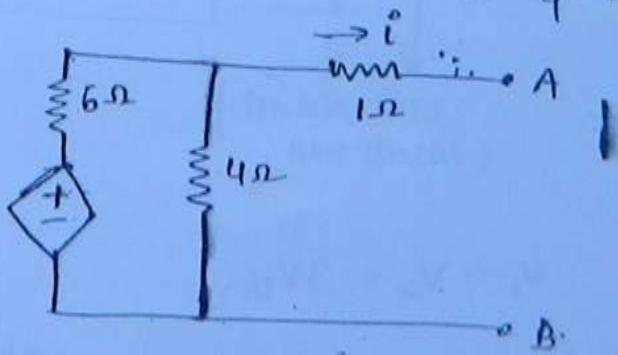
$$\frac{V_A - 3i}{6} + \frac{V_A}{4\Omega} = 1$$

$$\therefore V_A = 1.2V$$

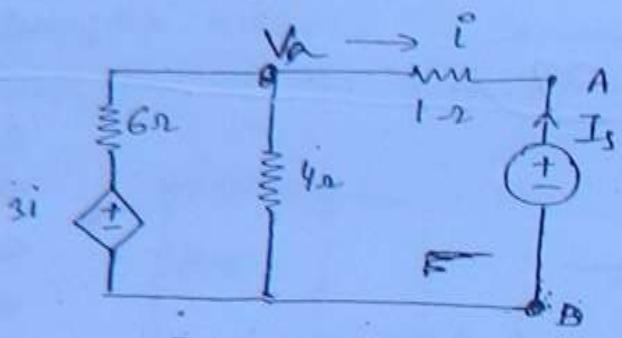
$$V_{AB} = V_A = 1.2V$$

$$R_{Th} = \frac{V_{ab}}{I_s} = \frac{1.2}{1} = 1.2 \Omega$$

Find R_{Th} wrt A & B.



Here it is easy to solve if we connect a voltage source across terminal.



(110)

$$0 = \frac{4-3i}{6} + \frac{V_1}{4} + \frac{V_1-1}{1}$$

$$i = V_1 - 1$$

$$\frac{V_1-3V_1+3}{6} + \frac{V_1}{4} + \frac{V_1-1}{1} = 0$$

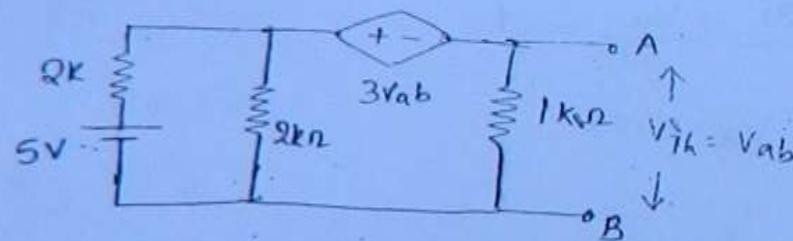
$$V_1 = 6/11 \Rightarrow i = -\frac{5}{11}$$

$$I_s = -i = -\left(-\frac{5}{11}\right) = \frac{5}{11}$$

$$R_{Th} = \frac{V_{ab}}{I_s} = \frac{1}{5/11} = 2.2 \Omega$$

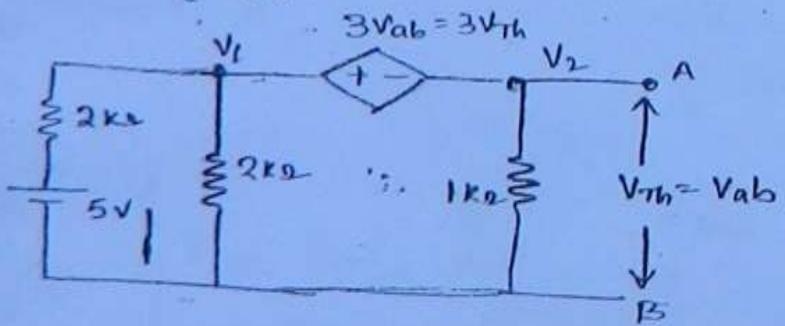
Q. Find V_{Th} & R_{Th} wrt A & B.

Gate Common data
linked data
* Thevenin 80



max power th.
& imp.

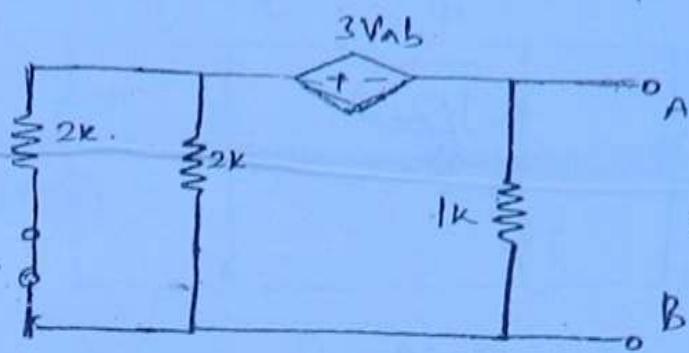
Case i) (V_{Th})



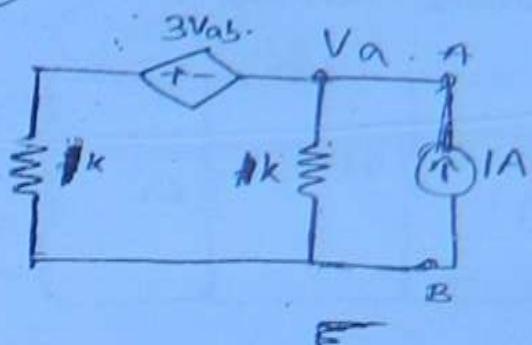
$$\frac{V_1-5}{2k} + \frac{V_1}{2k} + \frac{V_2}{1k} = 0 \quad \text{&} \quad V_1 - V_2 = 3V_{Th}$$

$$\frac{V_{Th}-5}{2k} + \frac{V_{Th}}{2k} + V_{Th} = 0 \quad \rightarrow \quad V_2 = V_{Th}$$

use (2) (R_{Th})



III



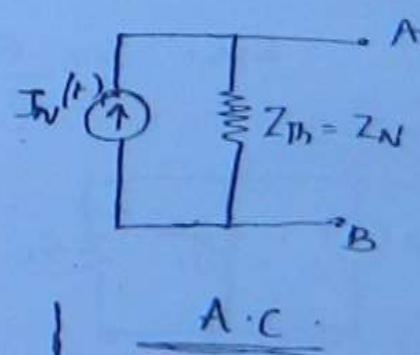
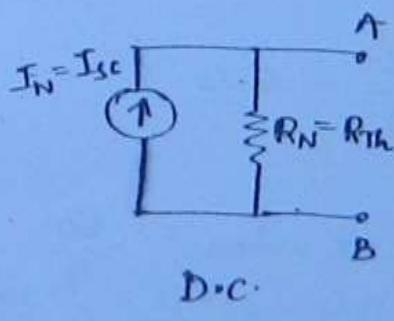
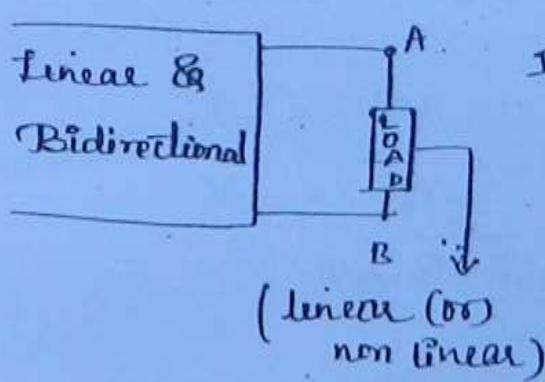
$$\frac{V_a - 3V_{ab}}{1k} + \frac{V_a}{1k} = 1 \quad V_a = V_{ab}$$

$$V_{ab} = 200V$$

$$R_{Th} = \frac{V_{ab}}{I_s}, \quad \frac{200}{1} = 200\Omega$$

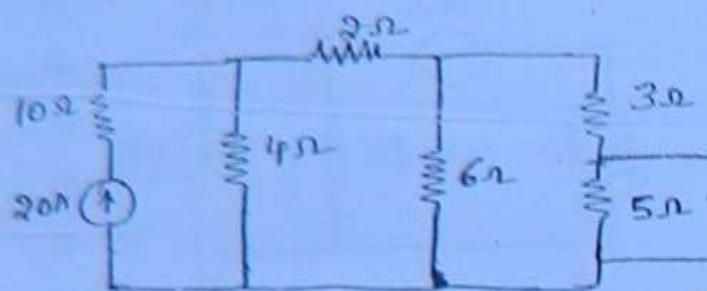
Orrton's Theorem

In any complex network having more number of elements, can be replaced by single equivalent circuit consisting of equivalent current source (I_N) in parallel with resistance (R_N).



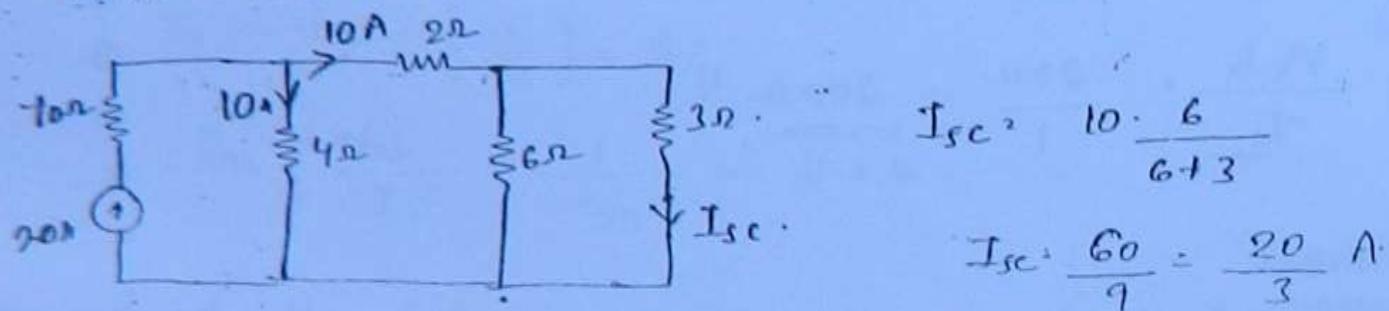
Q. Find current flowing through 5Ω resistor using Norton theorem.

1/2

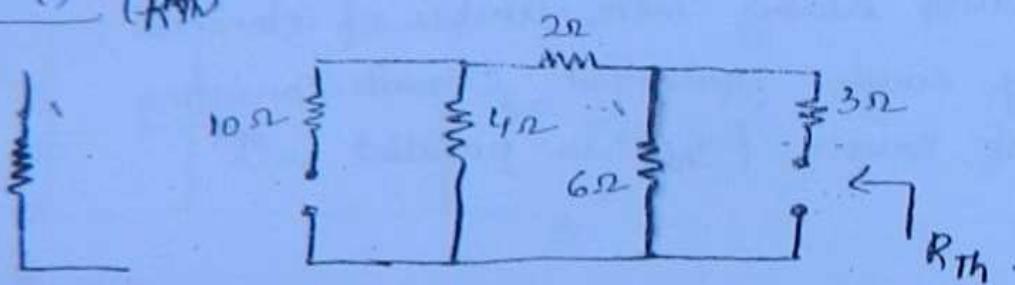


Case i) (I_{sc})

Replace the load resistor by short circuit and find short circuit current.

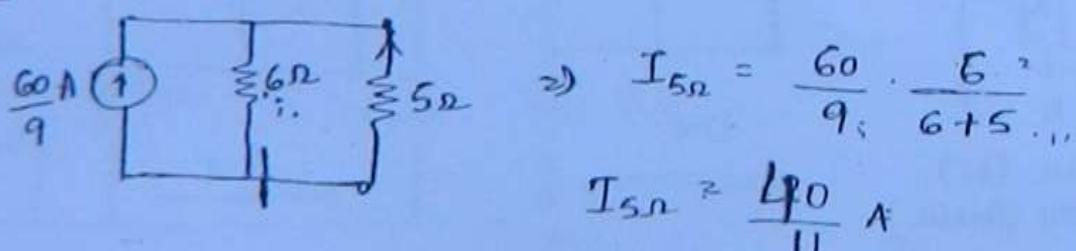


Case ii) (R_{Th})

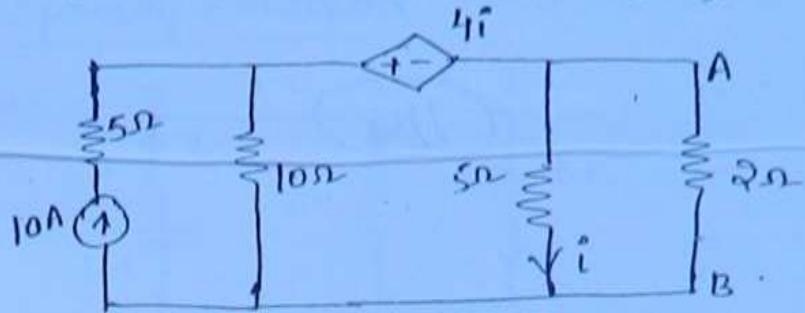


$$R_{Th} = 3 + \frac{6 \times 6}{6+6} = 6\Omega$$

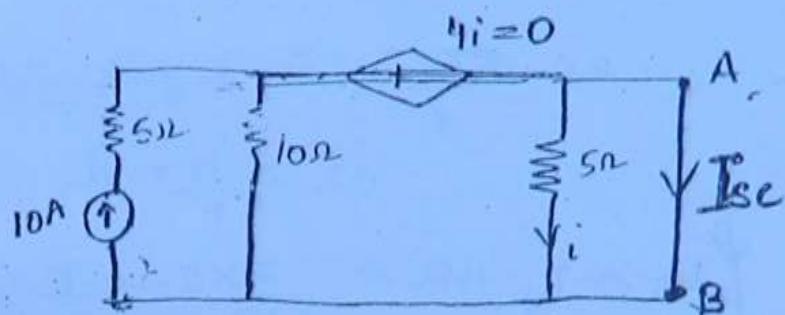
Eq. ckt



Q. Find short ckt current (I_{sc}) wrt A & B.



(113)

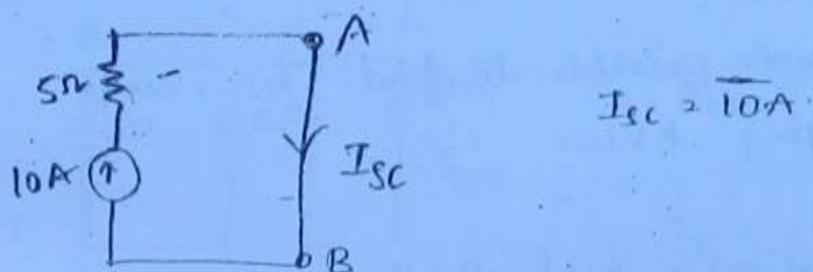


$\therefore AB$ is short ckted,

$$i = 0$$

$$\therefore i_i = 0$$

\Rightarrow voltage source is I.S.C.



$$I_{sc} = 10A$$

Find current flowing through 4Ω resistor by using the following data

$$V_0 = 0 \quad 60$$

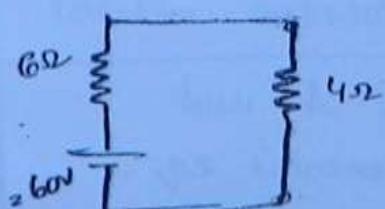
$$I_{sc} = 10A$$

$$I_0 = 10 \quad 0$$

$$I_{sc} = 10A$$

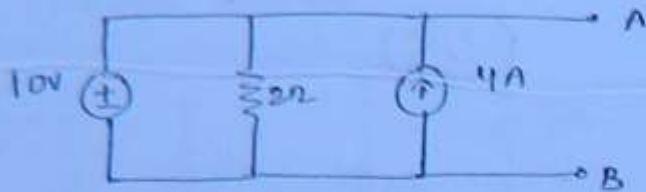
$$V_{oc} = 60V$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{60}{10} = 6\Omega$$



$$i_{4\Omega} = \frac{60}{6+4} = 6A$$

Q. Obtain Thvenin's Norton eq. ckt wrt. A & B.



(114)

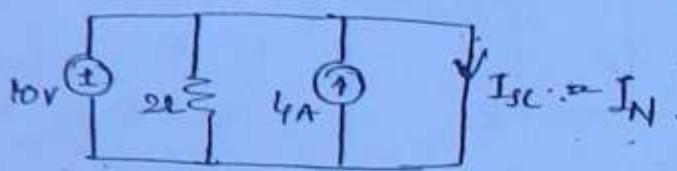
Soln:-

$$R_{Th} = 2\Omega.$$

$$V_{Th} = 10V.$$

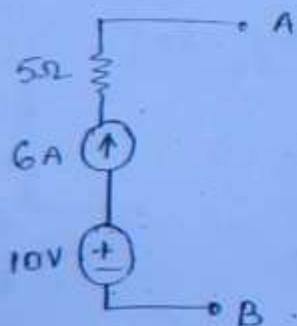
For I_{sc} .

Note:

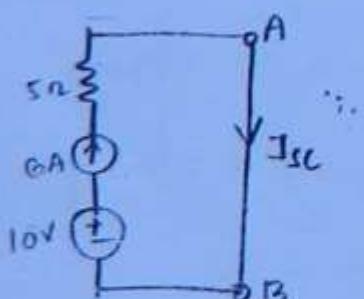


In the above ckt, it is not possible to find I_N , since above circuit is not satisfying KVL.

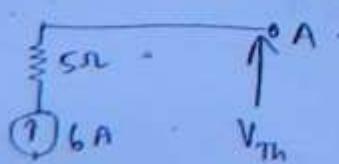
Q. Develop Thvenin's Norton eq. wrt. A & B.



Soln:-

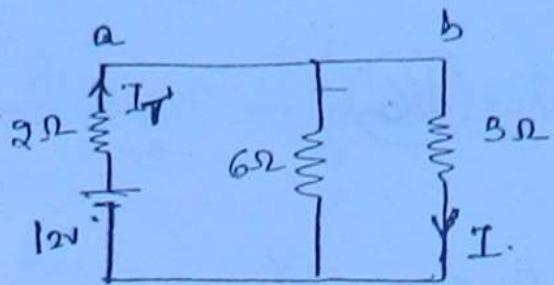


$$I_{sc} = 6A$$



Note: For the 2nd ckt, it isn't possible to develop Thvenin's eq ckt since the ckt is not satisfying

Reciprocity theorem



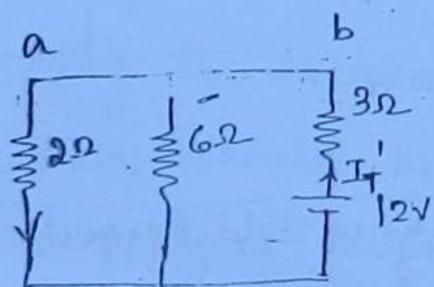
(115)

$$R_{eq} = \frac{2 + 6 \times 3}{9} = 4 \Omega$$

$$I_F = \frac{12}{4} = 3A$$

$$I = \frac{3 \times 6}{6+3} = 2A$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{2}{12}$$



$$R_{eq}' = -3 + \frac{6 \times 2}{6+2}$$

$$I_F' = \frac{12}{R_{eq}'}$$

$$I = I_F \frac{6}{6+2}$$

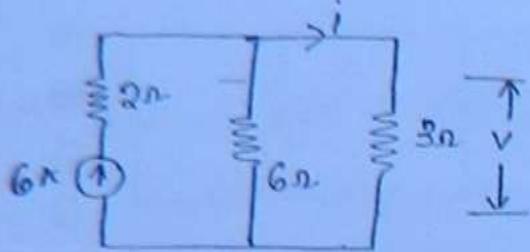
$$I = 2A$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{2}{12}$$

In the above ckt after interchanging position of response and excitation, the ratio of response to excitation is const. Hence, above network satisfies reciprocity.

Using Reciprocity theorem, it is possible to conclude whether network is linear or non linear.

Q. - Verify reciprocity th. for the circuit shown.



(1/6)

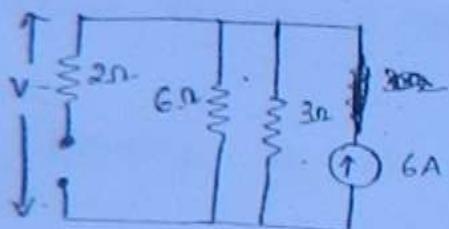
$$\frac{6 \times 6}{6+3}$$

Soln.

$$i = \frac{6 \times 6}{6+3} = 4A$$

$$V = 4 \times 3 = 12V$$

$$\frac{\text{Res}}{\text{Exci}} = \frac{12}{6} = 2$$



Current source when interchanged,
should be connected in parallel.

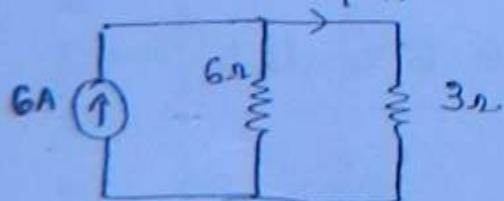
$$i_{6\Omega} = \frac{6 \times 3}{9} = 2A$$

$$V_{6\Omega} = 2 \times 6 = 12V$$

$$V = V_{6\Omega} = 12V$$

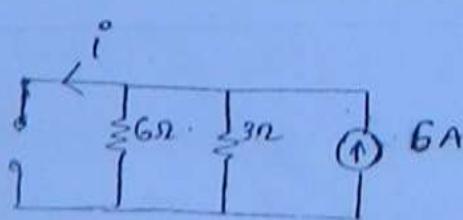
$$\frac{\text{Res}}{\text{Exci}} = \frac{12}{6} = 2$$

Q. Verify reciprocity theorem of the circuit shown.



$$\frac{6 \times 6}{9} \times 6\Omega$$

$$i = \frac{6 \times 6}{6+3}$$



In the above 2 problems.

$$\frac{\text{Res.}}{\text{Exc.}} = \frac{i}{V_s} \text{ mho}$$

(117)

$$\frac{\text{Res.}}{\text{Exc.}} = \frac{V}{I_s} \rightarrow \Omega$$

$$\frac{\text{Res.}}{\text{Exc.}} = \frac{I}{I_s}$$

$$\frac{\text{Res.}}{\text{Exc.}} = \frac{V}{V_s}$$

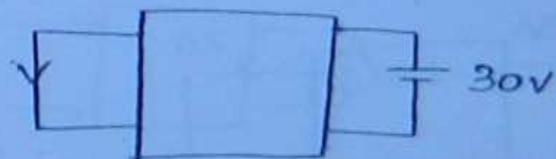
}

→ No unit

To apply the reciprocity theorem, unit of response excitation should be either mho (Ω) Ω .

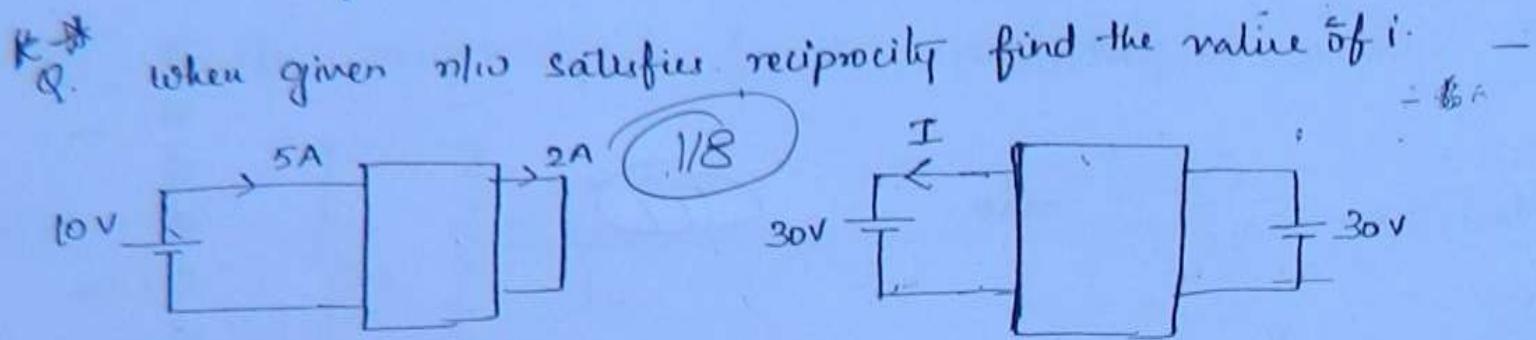
While applying reciprocity theorem circuit should consist of only one independant source.

When given network satisfies reciprocity find the value of I.

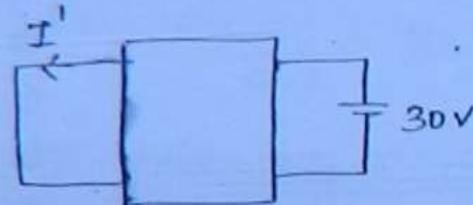


$$\text{const.} = \frac{\text{Res.}}{\text{Exc.}} = \frac{2}{10} = \frac{I}{30}$$

$$I = 6A$$



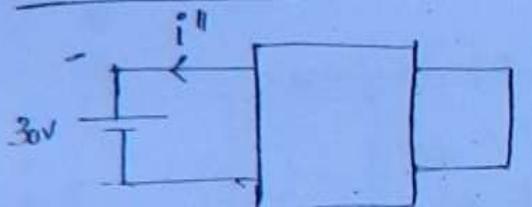
Case 1) (30V)



$$\frac{R_{eq}}{\text{exc}} = \frac{2}{10} = \frac{I'}{30}$$

$$I' = 6A$$

Case 2 (18V)

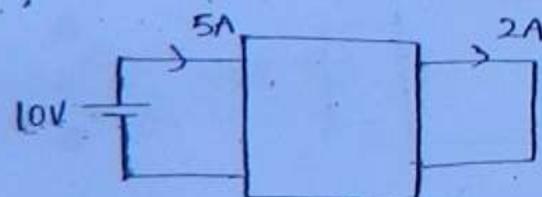


$$\frac{5}{10} = \frac{-i''}{30}$$

$$i'' = -15A$$

$$I = I' + I'' = 6 - 15 = -9A$$

Q. When given network satisfies reciprocity find the value of i .



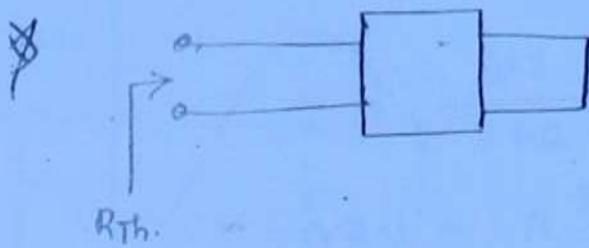
Sol Here we use norton eq. coz 2A branch is short circuited. Hence to find i we go for norton eq.

Case 1) (I_{sc})



$$\frac{2}{10} = \frac{I_{sc}}{30}$$

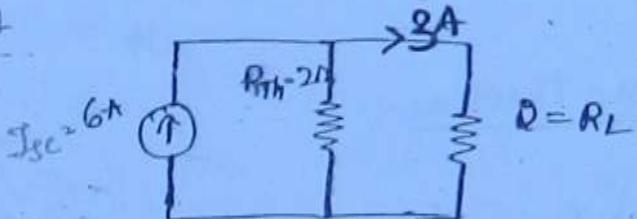
Case(2) (R_{Th})



(119)
The physical connections of
this circuit are similar to the
first one.

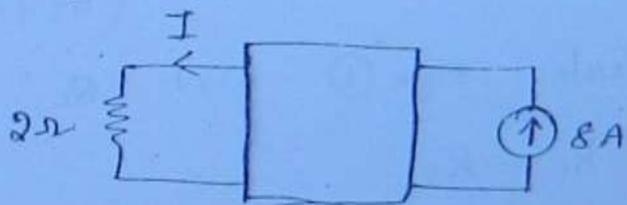
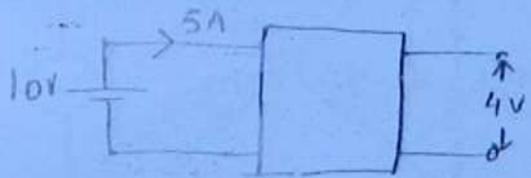
$$R_{Th} = \frac{10V}{5A} = 2\Omega$$

Eq ckt



$$I_{2\Omega} = 3A$$

when given network satisfies the reciprocity, find the value of i .



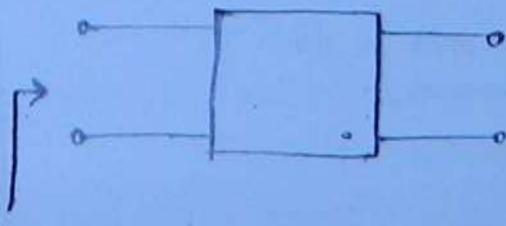
Case(3) (V_{Th})

$$\frac{5}{4} = \frac{8}{V} \quad \text{Res} = \frac{V_{Th}}{8} = \frac{4}{5}$$



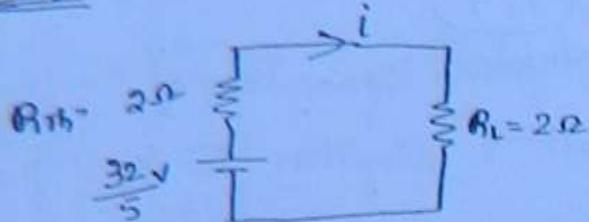
$$V_{Th} = \frac{32}{5}V$$

Case(4) (R_{Th})



$$R_{Th} = \frac{10}{5} = 2\Omega$$

Eq.

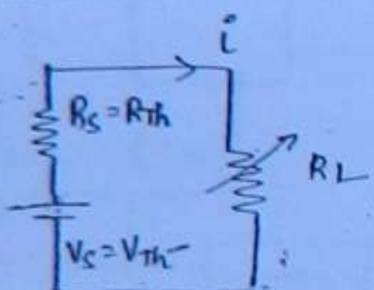


(120)

$$i = \frac{32/5}{2+2}$$

$$i = \frac{8}{5} A = 1.6 A$$

Maximum Power Transfer theorem.



$$i = \frac{V_s}{R_s + R_L}$$

$$P_L = I^2 R_L$$

$$= \left(\frac{V_s}{R_s + R_L} \right)^2 \cdot R_L \rightarrow (1)$$

differentiate eqn (1) wrt R_L & equate it to zero.

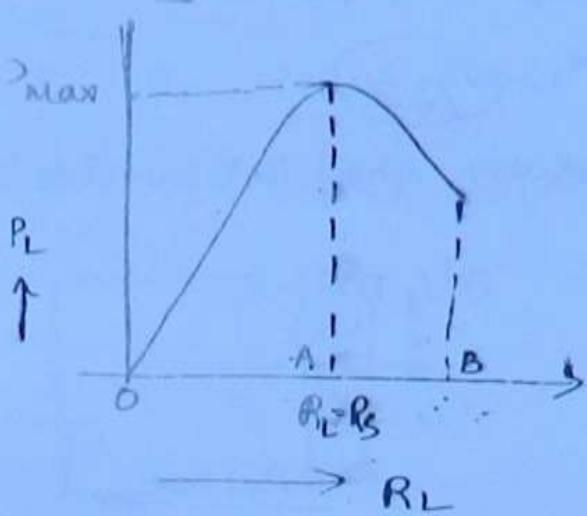
$$\Rightarrow R_L = R_s$$

$$P_{\max} = \frac{V_s^2}{(R_s + R_L)^2}, R_L$$

** $P_{\max} = \frac{V_s^2}{4R_s}$

$$\eta = \frac{O/P}{E/P} \times 100 \Rightarrow \frac{I^2 R_L}{I^2 (R_L + R_s)} \times 100$$

** $\eta = \frac{R_L}{R_L + R_s} \times 100 = 50\%$



(i) $OA \rightarrow R_s > R_L$

$$\underline{\eta < 50\%}$$

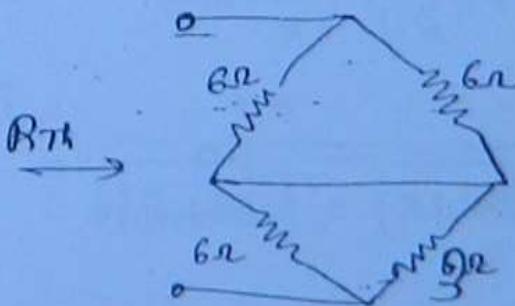
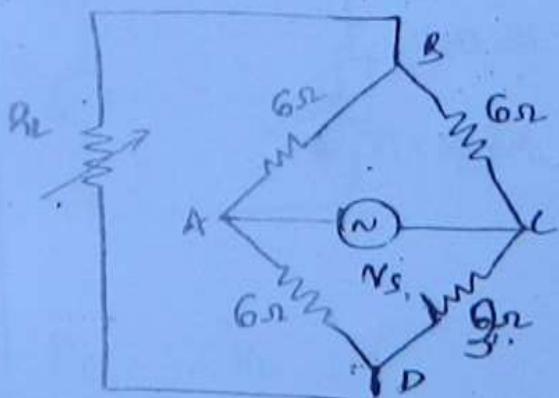
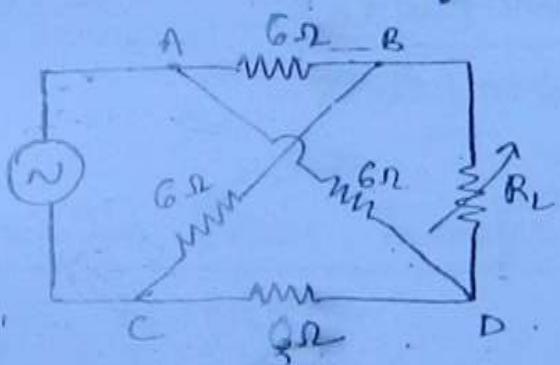
2) $A \rightarrow R_L = R_s$

$$\underline{\eta = 50\%}$$

3) $AB \rightarrow R_L > R_s$

$$\underline{\eta > 50\%}$$

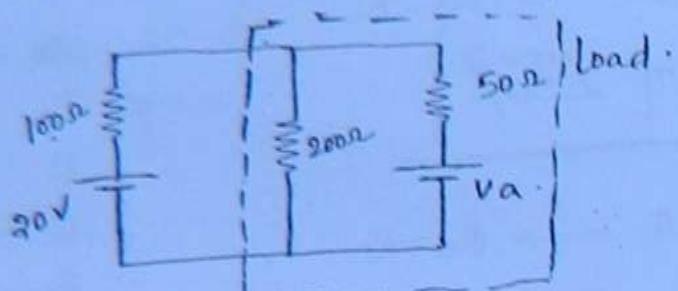
In the circuit shown, at what ~~had~~ value of R_L power delivered from source to load is maximum?



$$R_{Th} = 3+2 = 5\Omega$$

$$\underline{R_L = R_{Th} = 5\Omega}$$

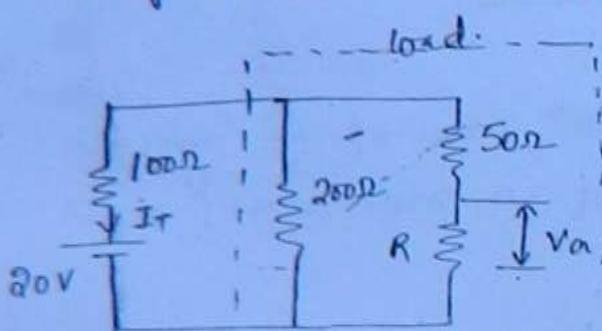
Q. In the circuit shown, at what value of V_a , power delivered from source to load is maximum. (12)



- (a) 7.5V (b) 10V
(c) 15V (d) 0V

Soln.

Here V_a is nothing but voltage drop across a resistor.



$$(R_{eq})_L = \frac{200(50+R)}{200+50+R}$$

$(R_{eq})_L$ as per MPT $R_{eq} = 100\Omega$

$$R_{eq} = R_s = 100$$

$$100 = \frac{200(50+R)}{250+R} \Rightarrow R = 150\Omega$$

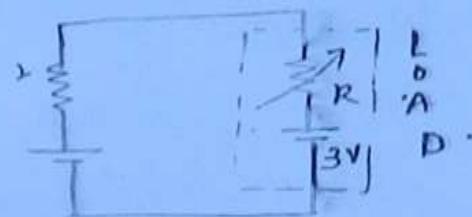
$$I_T = \frac{20}{R_s + (R_{eq})_L} = \frac{20}{100 + 100} = \frac{1}{10}$$

$$V_a = \frac{I_T \cdot R}{2} = 7.5V$$

$$\begin{aligned} & \frac{(R+50) 200}{R+250} \\ & 2 \frac{R}{R+250} = \frac{200}{250} \\ & \frac{2R}{250} = \frac{200}{250} \\ & R = 100 \end{aligned}$$

V_a as $R: 2A$
 $3\Omega \xrightarrow{\frac{2A}{12V}} 6\Omega \approx \frac{3\Omega}{1} \xrightarrow{\frac{2A}{12V}}$ $\therefore V_a$ can be represented as drop

In the circuit shown at what value of R , the power delivered from source to load is maximum. (12.3)



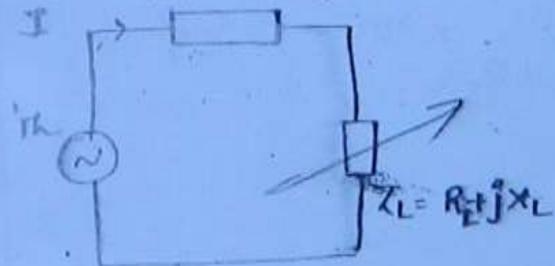
$$(R_{eq})_L = R_L = 2 \Omega$$

$$I = \frac{10}{R_L + (R_{eq})_L} = \frac{10}{2+2} = 2.5 A$$

$$I = \frac{10 - 3}{2 + R} = 0.5 \Rightarrow R = 10.8 \Omega$$

Maximum Power Transfer Theorem (A.C.)

$$Z_{th} = R_{th} + jX_{th}$$



$$I = \frac{V_{th}}{(R_{th} + R_L) + j(X_{th} + X_L)}$$

$$I = \frac{V_{th}}{\sqrt{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}}$$

$$P_L = I^2 R_L$$

$$P_L = \frac{V_{th}^2 R_L}{(R_L + R_{th})^2 + (X_L + X_{th})^2} \rightarrow \textcircled{1}$$

~~Case 1)~~ Both R_L & X_L are variable.

(12y)

- Differentiate eqn ① wrt R_L and equate it to zero.
→ Differentiate eqn ① wrt X_L and equate it to zero.

$$R_L + jX_L = R_{th} - jX_{th}$$
$$Z_L = Z_{th}^*$$

$$P_{max} = \frac{V_{th}^2}{4R_L}$$

$$\eta = 50\%$$

Case 2: Only R_L is variable. ($X_L = \text{constant}$)

- Differentiate eqn ① wrt R_L & equate it to zero.

$$R_L = \sqrt{R_{th}^2 + (X_L + X_{th})^2}$$

$$\eta > 50\%$$

Ex: $R_L > R_{th}$.
i.e. 3

$$\eta = \frac{R_L}{R_L + R_{th}} \times 100 > > 50\%$$

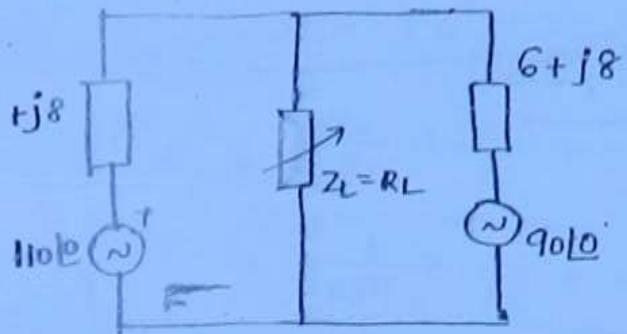
~~Case 3:~~ R_L is variable ($-X_L = 0$)

$$P_L = \frac{V_{th}^2 R_L}{(R_L + R_{th})^2 + X_{th}^2} \rightarrow ②$$

Differentiate eqn ② wrt R_L and equate it to zero.

$$R_L = \sqrt{R_{th}^2 + X_{th}^2}$$
$$R_L = |Z_{th}|$$
$$\eta > 50\%$$

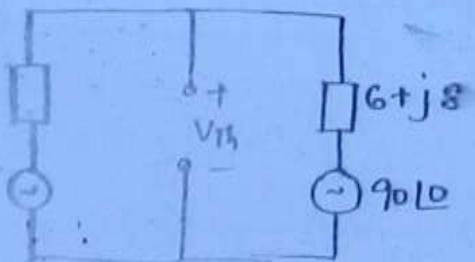
Find max power dissipation in the load impedance.



(125)

First find Thvenin eq.

[Case 3, Prob.]



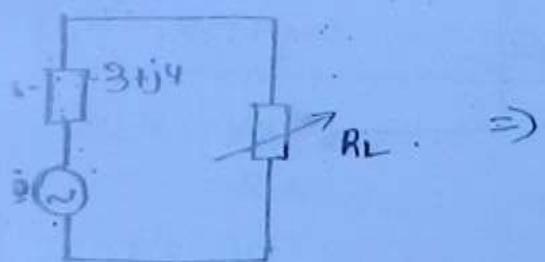
$$\frac{V_{Th} - 110\angle 0^\circ}{6+j8} + \frac{V_{Th} - 90\angle 0^\circ}{6+j8} = 0$$

$$2V_{Th} = 200\angle 0^\circ$$

$$V_{Th} = 100\angle 0^\circ$$

$$R_{Th} = \frac{(6+j8)(6-j8)}{2(12+j16)} = \frac{36-64+j96}{-12+j16}$$

$$Z_{Th} = 3+j4$$



$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2}$$

$$R_L = \sqrt{3^2 + 4^2} = 5\Omega$$

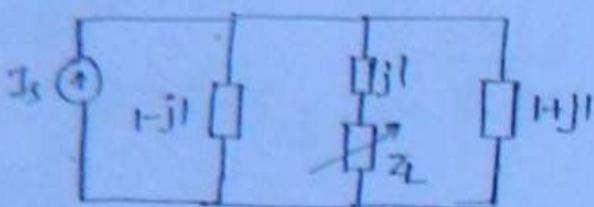
$$R_L = 5\Omega$$

$$I = \frac{100\angle 0^\circ}{(3+j4)} = \frac{100\angle 0^\circ}{8+j4} = \frac{100\angle 0^\circ}{\sqrt{8^2+4^2}}$$

$$P_2 = I^2 R_L = \frac{100^2}{(8^2+4^2)} \times 5$$

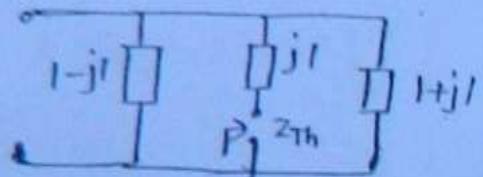
$$P_2 = 625W$$

Q. At what value of Z_L , power delivered from source load is max.



(126)

Z_{Th}



$$Z_{Th} = \frac{(1+jI)(1-jI)}{1+jI+1-jI} + jI$$

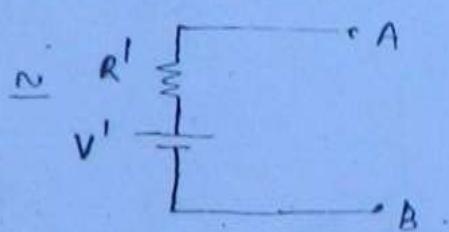
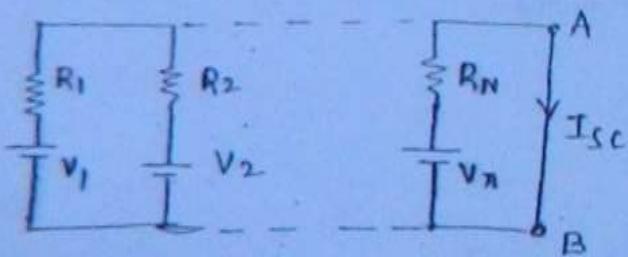
$$Z_{Th} = 1+jI$$

Case (i) form.

$$\therefore Z_L = Z_{Th}^* = 1-jI$$

$$\therefore Z_L = 1-jI$$

Millman's Theorem



$$R' = R_{Th}$$

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

* $R' = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right)} = \frac{1}{G_1 + G_2 + \dots + G_n}$

$$V_{oc} = I_{sc} R_{th}$$

$$V^I = I^I R^I$$

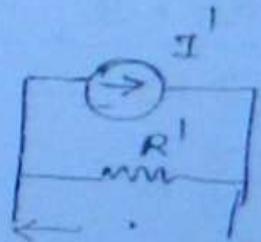
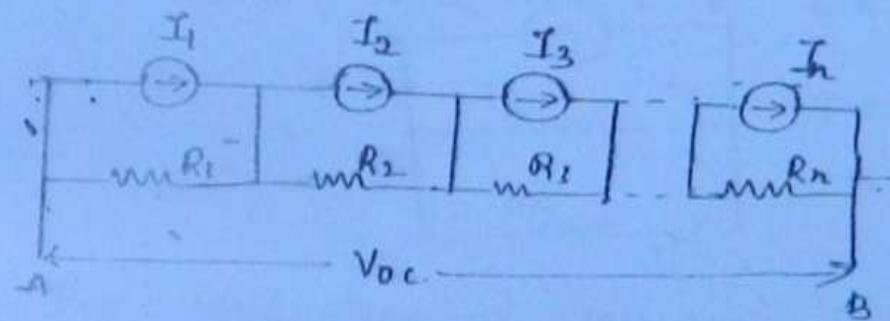
(127)

$$V^I = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n}$$

$$\frac{1}{R^I} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

$$V^I = \frac{V_1 G_{11} + V_2 G_{12} + \dots + V_n G_{1n}}{G_{11} + G_{12} + \dots + G_{nn}}$$

$$V^I = V_{Th}$$



$$R^I = R_{th}$$

$$R^I = R_1 + R_2 + \dots + R_n$$

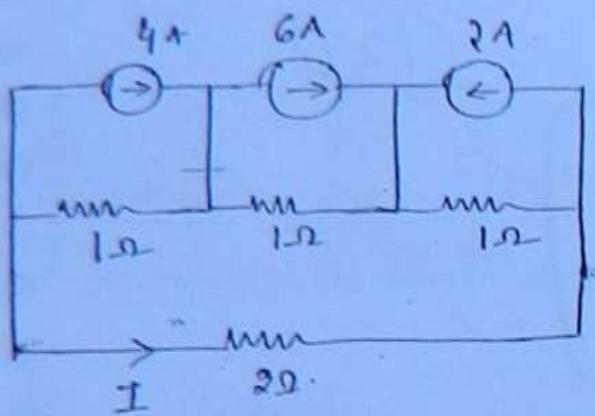
$$I_{sc} = \frac{V_{oc}}{R_{th}}$$

$$I^I = \frac{V^I}{R^I}$$

$$I^I = \frac{I_1 R_1 + I_2 R_2 + \dots + I_n R_n}{R_1 + R_2 + \dots + R_n}$$

$$I^I = \frac{V^I}{R^I}$$

Find the value of I in the ckt shown.



(128)

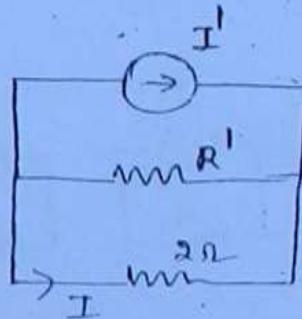
oh.

$$I^1 = \frac{4 \times 1 + 6 \times 1 - 2 \times 1}{1+1+1} = 8/3$$

$$R^1 = 1+1+1 = 3\Omega$$

$$I = \frac{8/3 \cdot 3}{3+2}$$

$$I = -\frac{8}{5} A$$

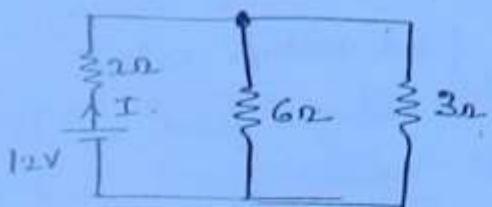


Tellegen's Theorem

→ Tellegen's theorem states that algebraic sum of the powers in any circuit (linear, non-linear, unidirectional, bi-directional, time variant and invariant elements) at any instant = 0.

And it is given by,

$$\sum_{k=1}^n V_k i_k = 0$$



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$$R_{eq} = \frac{2 + 6 \times 2}{9} = 4\Omega$$

$$I_T = \frac{12}{4} = 3A$$

$$V_{6\Omega} = \frac{2 \times 2}{9} = 1V \Rightarrow I_{3\Omega} = 2A$$

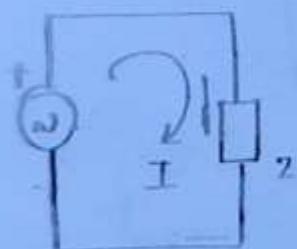
$$V_2 = 2 \times 3 = 6V, \quad V_3 = V_G = 3 \times 2 = 6V$$

$$\begin{aligned} V_2 I_1 + V_L I_L + V_3 I_3 - V_S I_T &= 6 \times 3 + 6 \times 1 + 6 \times 2 - 12 \times 3 \\ &= 18 + 6 + 12 - 36 \\ &= 0 \end{aligned}$$

For verification of Tellegen's Theorem KVL & KCL equations are used.

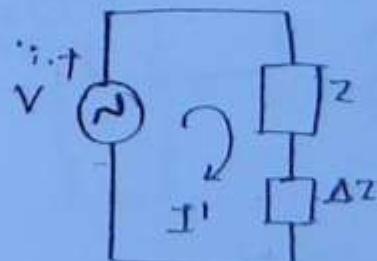
Tellegen's theorem works based on the principle of Law of Conservation of Energy.

Compensation Theorem



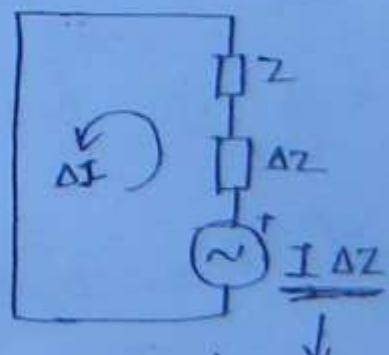
$$I = V/Z$$

1st law, 1st rule



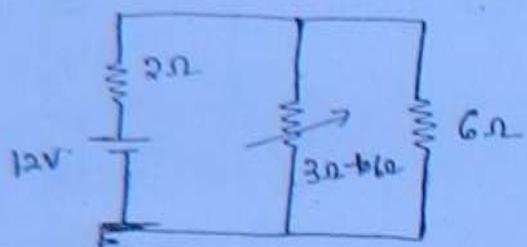
$$I^1 = \frac{V}{Z + \Delta Z}$$

modified circuit



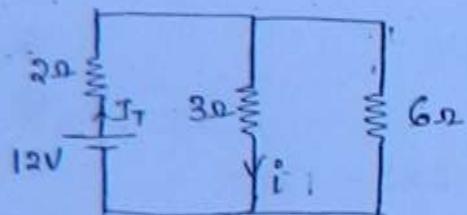
compensation emf (m)

Q. Find change in current in 2Ω & 6Ω resistor when resistance in the variable branch is changed from 3Ω to 6Ω .



(136)

Step 1: Find original current circulating in the variable branch.



$$Req = 2 + \frac{3 \times 6}{9} = 4\Omega$$

$$I_T = 12/4 = 3A$$

$$i_{3\Omega} = 2A$$

Step 2: Find compensation emf.

$$= I \Delta Z = 2(6-3) = 6V$$

Step 3: Develop modified circuit.

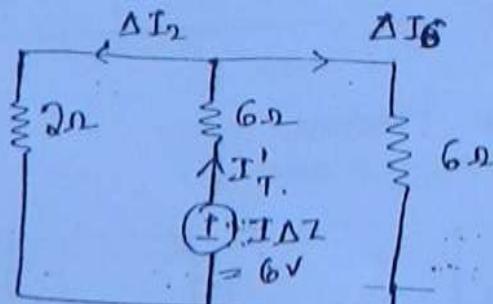
while developing modified circuit, deactivate all the independant sources and connect the compensation emf in series to variable branch.

$$Req = 6 + \frac{2 \times 6}{2+6} = 22/4$$

$$I_T' = \frac{6}{Req} = \frac{8}{22} A = 0.8A$$

$$\Delta I_2 = I_T' \cdot \frac{6}{6+2} = 0.6A$$

$$\Delta I_6 = I_T' - \Delta I_2 = 0.2A$$

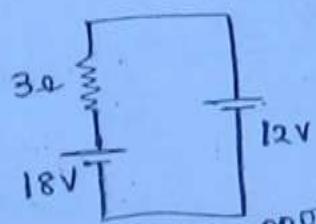
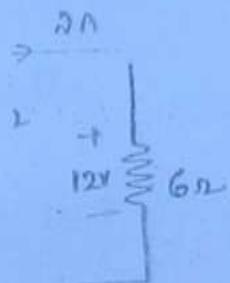


$$\begin{aligned} &\frac{8}{22} \cdot \frac{6}{8} \\ &= 0.6 \end{aligned}$$

In the Bridge circuit, to obtain null deflection in the galvanometer, compensation theorem is used.

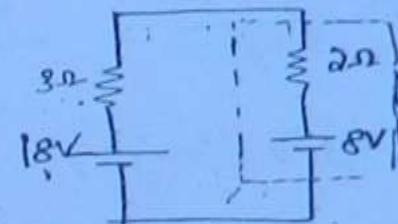
(13)

STITUTION THEOREM

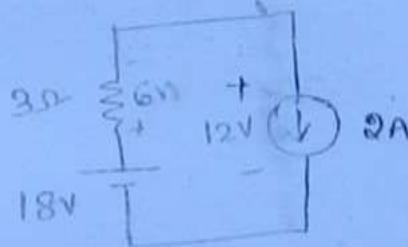


$$I = \frac{18 - 12}{3} = 2A$$

Opposing end.



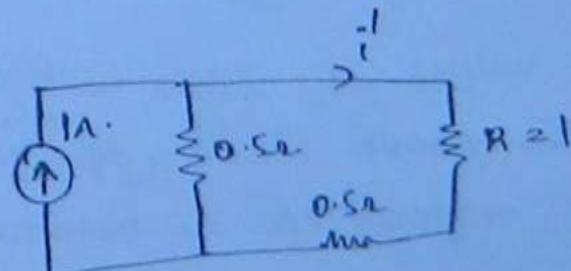
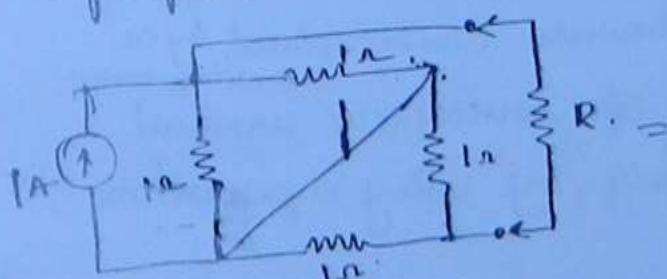
$$I_2 = \frac{18 - 8}{3 + 2} = 2A$$



Result

Req. is equal in both cases. $\therefore \frac{V}{I} = \text{Req.}$

Superposition:



$$i' = 1 \cdot \frac{0.5}{0.5 + 0.5 + 1} = 0.25A$$

as 1A is O.C. \Rightarrow balanced bridge

$$(R_1 + j\omega L_1)(R_4 - j/\omega C_4)$$

$$(R_1 + j\omega L_1)(R_4 - j/\omega C_4) = R_2 R_3.$$

$$\omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0 \Rightarrow \omega^2 L_1 R_4 = \frac{R_1}{C_4}. \quad (132)$$

$$\frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

F

$$4. \quad V_{Th} = \frac{100[0^\circ + j4]}{3+j4} - \frac{100(3-j4)j4}{25}$$

$$= j16(3-j4).$$

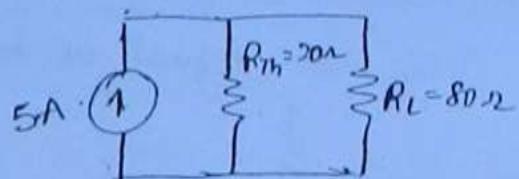
$$5. \quad P = \left(\pm \sqrt{P^1} \pm \sqrt{P^2} \pm \sqrt{P^3} \right)^2.$$

$$P_{max} = \left(\sqrt{18} + \sqrt{50} + \sqrt{18} \right)^2$$

$$P_{min} = \left(\sqrt{18} - \sqrt{50} - \sqrt{18} \right)^2$$

$$6. \quad R_{Th} = \frac{100}{5.00} = \frac{100}{5} = 20\Omega. \quad 20\Omega.$$

$$I_L = I_{20\Omega} = \frac{5 \times 20}{80+20} = 1A.$$



7. ω values are same, voltage branches can be replaced by eq. voltage source.

In the above circuit, if freq. of sources are unequal
Current response can be obtained only by using superposition theorem.

$$V_{OC} = 2 \cdot \frac{2}{2+3} = 4/5 \text{ V}$$

$$R_{Th} = \frac{3 \times 2}{5} + \frac{4}{5} = 2\Omega$$

$$I_{SC} = \frac{V_{OC}}{R_{Th}} = \frac{4}{5 \times 2} = \frac{2}{5} \text{ A}$$

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2} = 14.14 \Omega$$

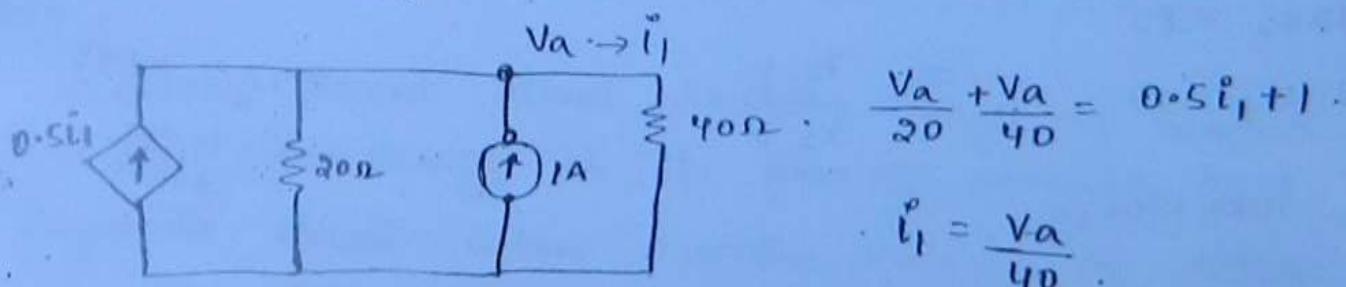
$$P_{max} = \frac{V_s^2}{4R_L} = \frac{12^2}{4 \times 2} = 18W$$

17. millman's theorem.

$$V^I = \frac{10/6 + 5/4}{1/6 + 1/4} = 7V$$

$$R^I = \frac{1}{\frac{1}{6} + \frac{1}{4}} = 2.4 \Omega$$

$$R_L = R_{Th}$$



$$\frac{V_\alpha}{20} + \frac{V_\alpha}{40} = \frac{V_\alpha}{80} + 1$$

$$R_{Th} = \frac{V_\alpha}{I_S} = \frac{16}{1} = 16\Omega$$

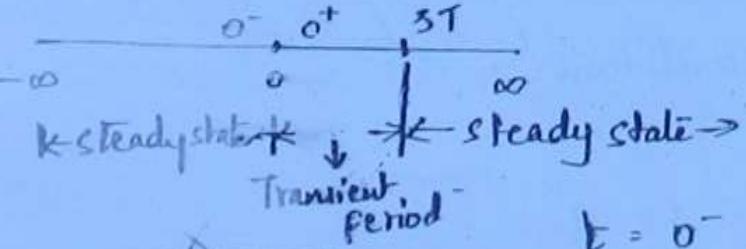
$$V_\alpha = 16$$

$$I_{y,0} = \frac{3}{4} \text{ A}$$

$$I_{y,V} = \frac{3}{4} + 0.25 = 1A$$

TRANSIENTS

(135)

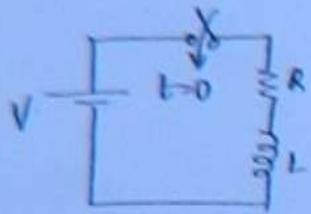


$t = 0^-$	$i = 0$
$t = 0^+$	$i = 0$ (o.c.)
$t = \infty$	$V_L = L \frac{di}{dt} = 0$ (s.c.)

Transients are present in the circuit when the circuit subjected to any changes either by changing source magnitude or while changing any circuit elements, provided circuit consists of any energy storage elements.

Inductor doesn't allow sudden change of current and it stores energy in the form of magnetic field.
Capacitor doesn't allow sudden change of voltage and it stores energy in the form of electric field.

When circuit is having only resistive elements, no transients are present in the circuit. Since resistor allows sudden change of current and voltage and it doesn't store any energy.



$t = 0^-$ indicates immediately before operating the switch.

$t = 0^+$, indicates immediately after operating switch.

$t = \infty$ indicates steady state condition.

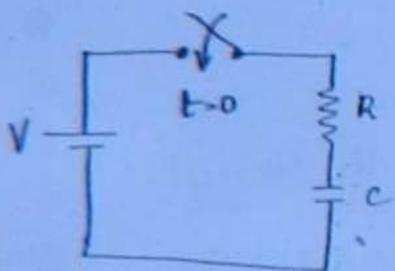
136

F

$$t = 0^- \quad i = I_0$$

$$t = 0^+ \quad i = I_0 \text{ (current source)}$$

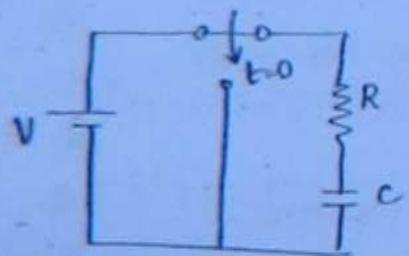
$$t = \infty \quad i = 0$$



$$t = 0^- \quad V_C = 0$$

$$t = 0^+ \quad V_C = 0 \text{ (S.C.)}$$

$$t = \infty \quad V_C = V \quad i = 0 \text{ (O.C.)}$$

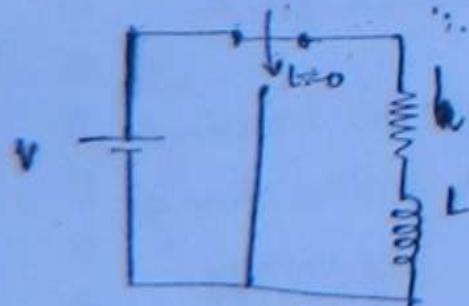


$$t = 0^- \quad V_C = V_0 \quad (V_0 = V)$$

$$t = 0^+ \quad V_C = V_0 \text{ (voltage source)}$$

$$t = \infty \quad V_C = 0$$

Source Free RL circuit



$$t = 0^- \quad i = I_0$$

$$t = 0^+ \quad i = I_0$$

RL circuit with source

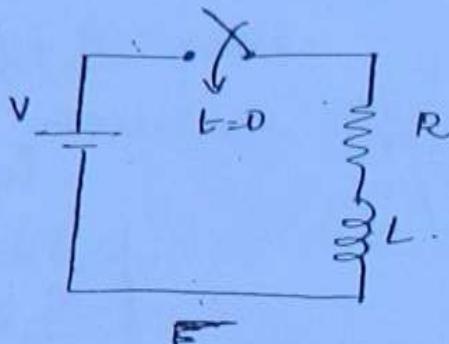
By KVL,

$$V = iR + L \frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

$$i(t) = C.F + P.I$$

(138)



CF \Rightarrow complementary func.

P.I - particular integral.

C.F \rightarrow Transient response (or) source free response.

$$\frac{di}{dt} + \frac{R}{L} i = 0 \quad \Rightarrow \quad i(t) = Ae^{-Rt/L}$$

P.I \rightarrow Steady state response / Final value. \rightarrow S.c.
 $i = V/R$.

$$i(t) = C.F + P.I$$

$$i(t) = Ae^{-Rt/L} + V/R$$

$$t = 0 \quad i = 0$$

$$t = 0^+ \quad i = 0$$

$$0 = A + \frac{V}{R} \Rightarrow A = 0 - \frac{V}{R}$$

$$A = i(0^+) - i(\infty)$$

Note:

* This formula is only applicable

$$i(t) = [i(0^+) - i(\infty)] e^{-Rt/L} + i(\infty)$$

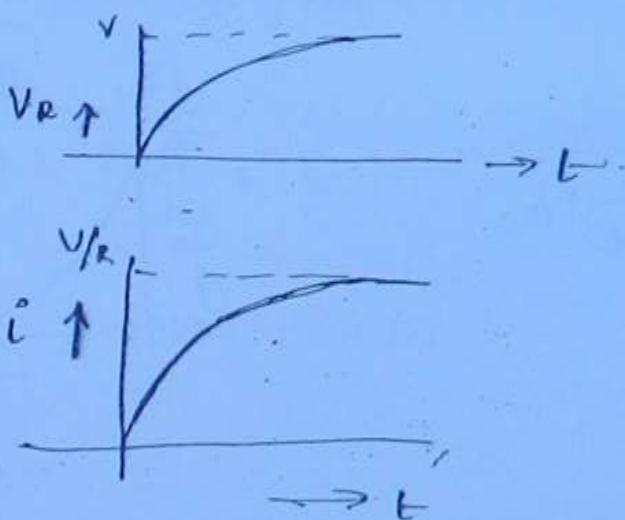
$$i(t) = \left[(i(0^+) - i(\infty)) e^{-\frac{R}{L}t} + i(\infty) \right] \downarrow I.V. \quad \downarrow F.V. \quad \downarrow F.V.$$

(B9)

$$v = iR$$

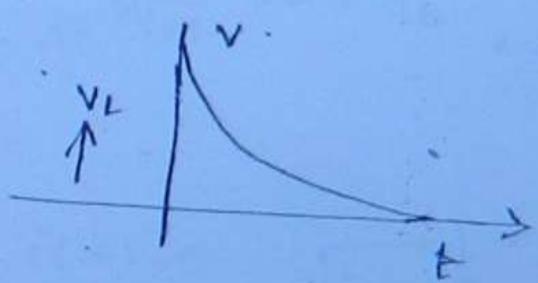
$$v = V \left(1 - e^{-\frac{Rt}{L}} \right)$$

$$i(t) = \frac{V}{R} \left(1 - e^{-\frac{Rt}{L}} \right)$$

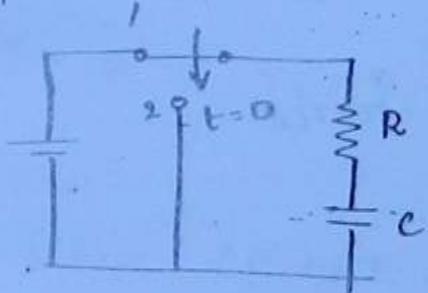


$$L \frac{di}{dt} = E - \frac{V}{R} \left(1 - e^{-\frac{Rt}{L}} \right)$$

$$v_L = V e^{-\frac{Rt}{L}}$$



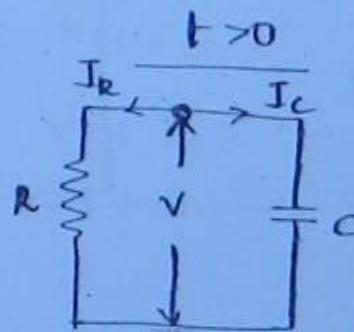
source Free RC ckt.



$$t=0^-, \quad v_c = v_0$$

$$t=0^+, \quad v_c = v_0$$

$$I_R + I_C = 0$$



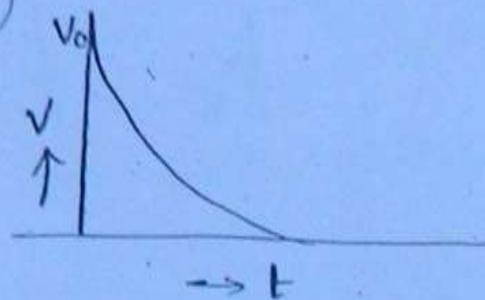
Assume a virtual voltage source v

$$\frac{V}{R} + C \frac{dv}{dt} = 0 \Rightarrow \frac{V}{R} = -C \frac{dv}{dt}$$

$$- \frac{1}{C} \frac{dV}{dt} = \frac{dv}{dt}$$

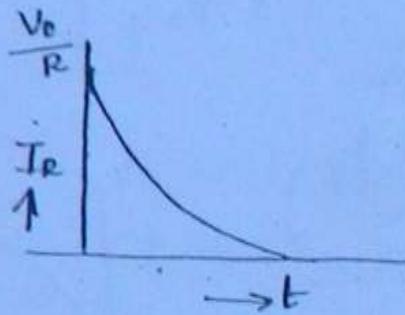
$$\int_0^t \frac{1}{RC} dt \rightarrow \int \frac{dv}{v} \quad \text{140}$$

$$V(t) = V_0 e^{-t/RC}$$



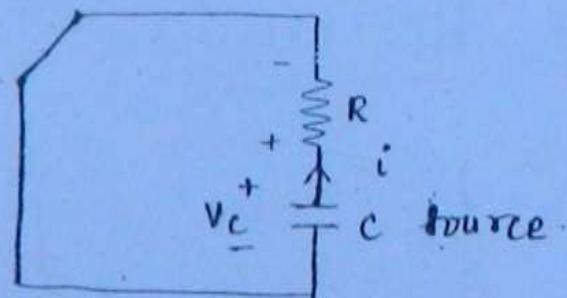
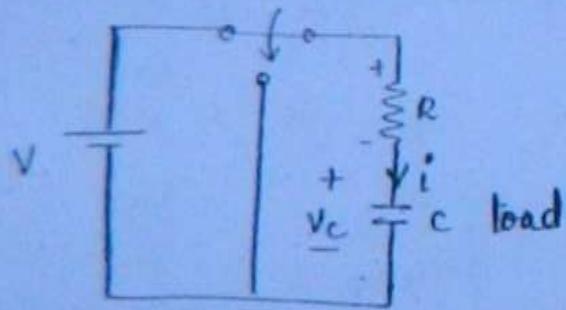
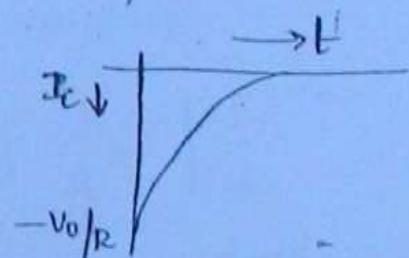
$$I_R = \frac{V}{R}$$

$$I_R = \frac{V_0}{R} e^{-t/RC}$$



$$I_C = C \frac{dv}{dt}$$

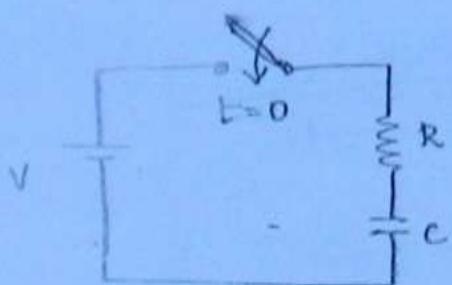
$$I_C = C \frac{d}{dt} (V_0 e^{-t/RC}) = -\frac{V_0}{RC} e^{-t/RC}$$



Note

In the discharging capacitor voltage across capacitor polarities do not change. But current direction of the capacitor is reversed.

RC circuit with source.



By KVL,

(141)

$$V = iR + \frac{1}{C} \int i dt$$

diff. wrt

$$0 = R \frac{di}{dt} + \frac{i}{C}$$

$$\frac{di}{dt} + \frac{i}{RC} = 0$$

$$i(t) = CF + PI.$$

CF = Transient response.

$$\frac{di}{dt} + \frac{i}{RC} = 0 \Rightarrow i(t) = Ae^{-t/\tau_{RC}}$$

PI \rightarrow Steady state :

$$C \rightarrow 0 \cdot C \Rightarrow i = 0 \Rightarrow i(\infty) = 0$$

$$t = 0^-, V_C = 0$$

By KVL,

$$t = 0^+, V_C = 0$$

$$V = V_R + V_C$$

$$t = 0^+, V = IR + 0 \Rightarrow V(0^+) = V/R$$

$$i(t) = CF + PI$$

$$i(t) = Ae^{-t/\tau_{RC}} + 0$$

$$i(t) = \frac{V}{R} e^{-t/\tau_{RC}}$$

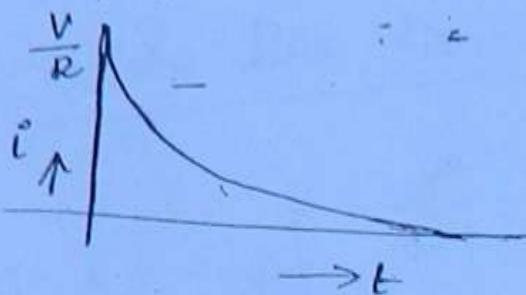
$$A = i(0^+) - i(\infty)$$

$$A = V/R - 0$$

$$A = V/R$$

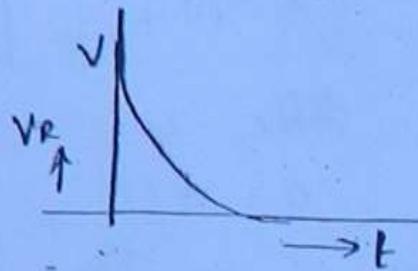
$$i(t) = \frac{V}{R} e^{-t/RC}$$

(142)



$$V_R = i R$$

$$V_R = V e^{-t/RC}$$

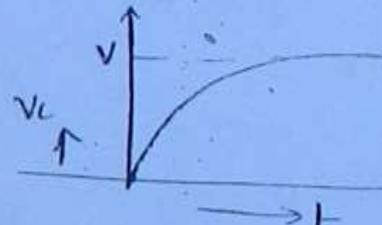


$$V_C = \frac{1}{C} \int_0^t i dt = \frac{1}{C} \int \frac{V}{R} e^{-t/RC} dt$$

$$V_C(t) = -V e^{-t/RC} + V$$

* $V_C(t) = [V_C(0^+) - V_C(\infty)] e^{-t/RC} + V_C(\infty)$

$$V_C(t) = V (1 - e^{-t/RC})$$



Time Constant

→ Time constant is the time taken for response to rise 63.2% of the max. value & is given by,

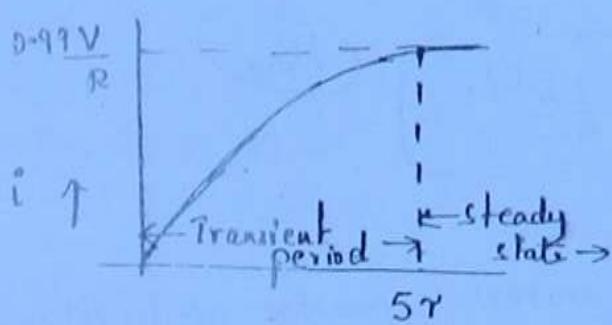
$$\begin{aligned} \tau &= L/R & - RL \\ \tau &= RC & - \alpha C \end{aligned} \quad \left\{ \text{Unit - sec.} \right.$$

$$t = \tau$$

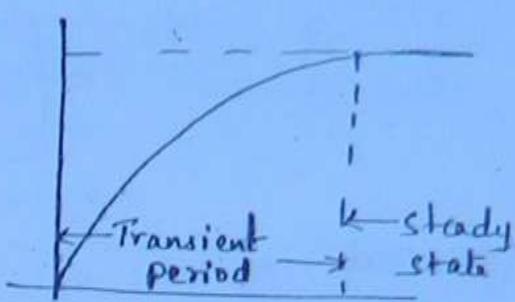
$$i(t) = \frac{V}{R} (1 - e^{-t/\tau}) = 0.63 V / R$$

R-L with source

(143)



RC with source

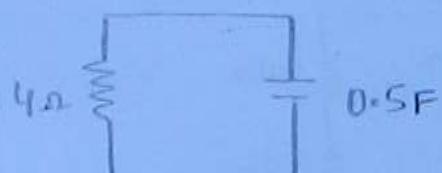
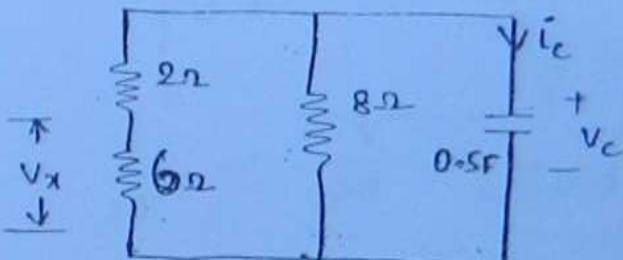


$$\Rightarrow i(t) = \frac{V}{R} \left(1 - e^{-\frac{Rt}{L}} \right) \Rightarrow V_c = V \left(1 - e^{-\frac{t}{RC}} \right)$$

$$i(t) = \frac{V}{R} \left(1 - e^{-\frac{t}{T_f}} \right)$$

Find response of V_c , i_c , and v_x when initial value of the capacitor is 3V. i.e. $V_0 = 3V$

First find out the equivalent resistance of capacitance.



$$V_c = V_0 e^{-t/RC}$$

$$V_c = 3 e^{-t/2} \quad \text{source free}$$

$$\therefore V_x = V_c \cdot \frac{6}{6+2} = V_c \cdot \frac{6}{8}$$

$$V_x = 3 e^{-t/2} \cdot \frac{6}{8} \Rightarrow$$

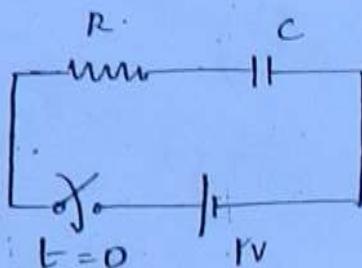
$$V_x = \frac{9}{4} e^{-t/2}$$

$$V_C = C \cdot \frac{dv_c}{dt} \Rightarrow i_C = C \cdot \frac{d}{dt} (3e^{-t/2})$$

$$i_C = \frac{1}{2} \cdot \frac{-3}{2} \cdot e^{-t/2} \Rightarrow \boxed{i_C = -\frac{3}{4} e^{-t/2}}$$
144

Q. Find the rate of rise of voltage across capacitor at $t=0^+$.

- (a) RC (b) $1/RC$
 (c) 0 (d) V



Soln:-

$$V_C = -Ve^{-t/RC} + V$$

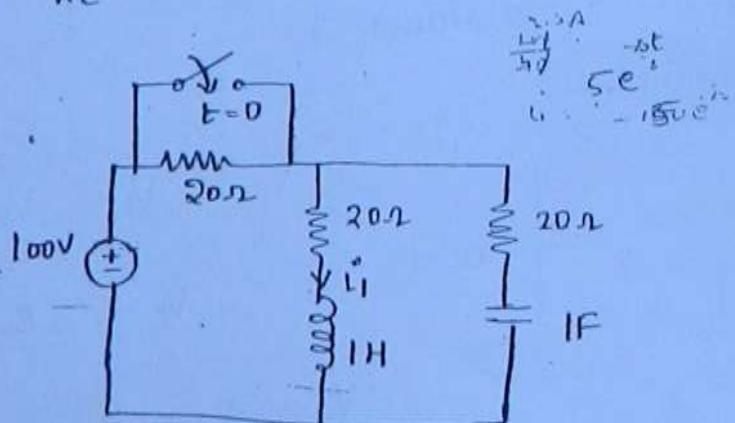
$$\frac{dv_c}{dt} = +V \cdot \frac{e^{-t/RC}}{RC} + 0$$

$$\left. \frac{dv_c}{dt} \right|_{t=0^+} = \frac{V}{RC} = \frac{1}{RC} \quad \therefore V = 1V$$

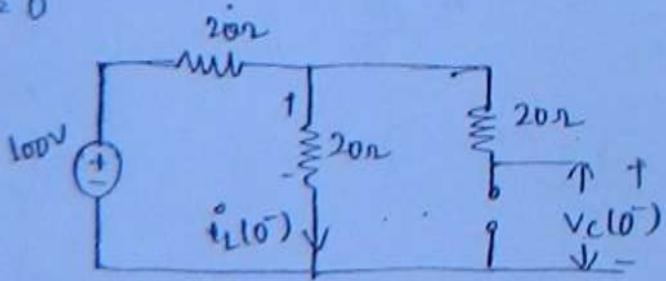
Q.

Find $\frac{di_i}{dt}$ at $t=0^+$

Soln
first find ext at $t=0^-$
to calculate initial values

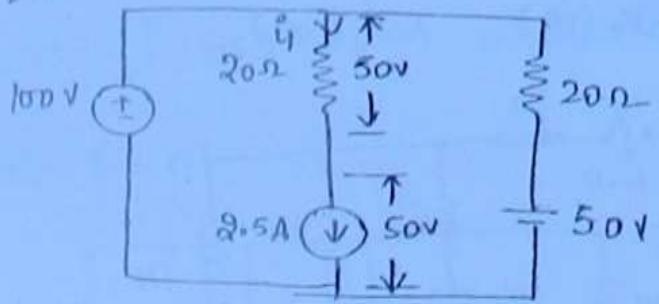


$t=0^-$



$$i_L(0-) = \frac{100}{20+20} = 2.5A$$

$$\therefore V_C(0-) = 2.5 \times 20 = 50V$$



$$145 \quad i_L(0^+) = i_L(0^-) = 2.5A$$

$$V_C(0^-), \quad V_C(0^+) = 50V$$

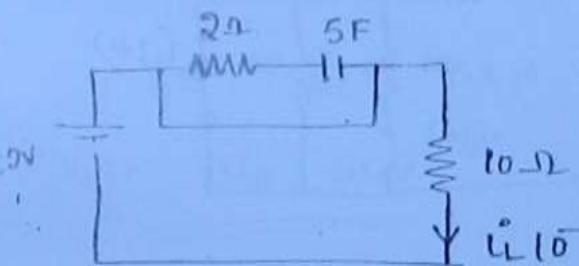
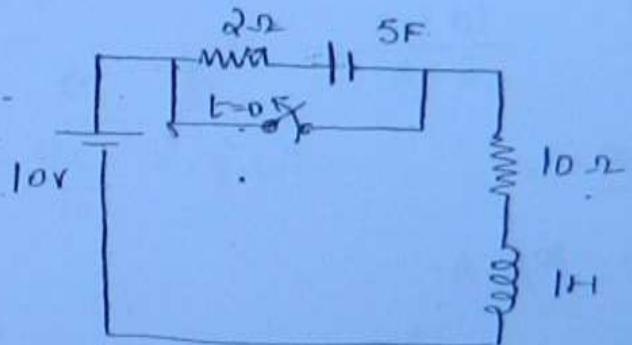
$$V_L = L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_L}{L}$$

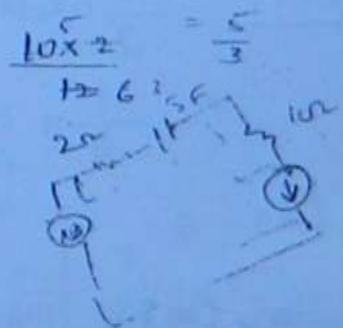
$$\left. \frac{di_L}{dt} \right|_{t=0^+} = 50 \text{ A/s}$$

Find voltage across the switch at $t=0^+$

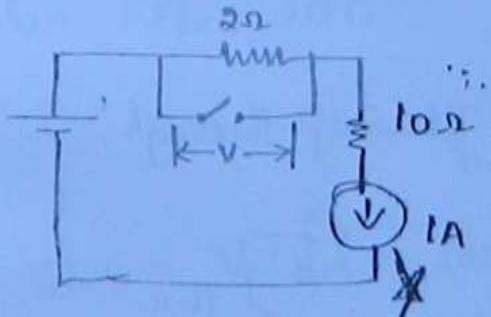
- (a) 5V (b) 10V
 (c) 0 (d) none



$$i_L(0^-) = \frac{10}{10} = 1A, \quad V_C(0^-) = 0$$



$$t=0^+, \quad V_L(0^+) = 0 \quad i_L(0^+) = 1A$$



$$V = 1 \times 2\Omega$$

$$V = 2V$$

Q. Find $i_c(0^+)$, $v_L(0^+)$, $\omega_c(\infty)$, $w_L(\infty)$

at $t=0^-$,

$$v_c(0^-) = 0, \quad i_L(0^-) = 0.$$

$t=0^+$

$$v_c(0^+) = 0, \quad i_L(0^+) = 0.$$

$$i_c(0^+) = \frac{10}{3+2} = \frac{2A}{2} = 1A$$

$$\underline{i_c(0^+) = 1A}$$

$$v_L(0^+) = \frac{10 \times 2}{2+3} = 4V. \quad \Rightarrow \quad v_L(0^+) = 4V$$

at $t=\infty$

$$i_L(\infty) = \frac{10}{3} A$$

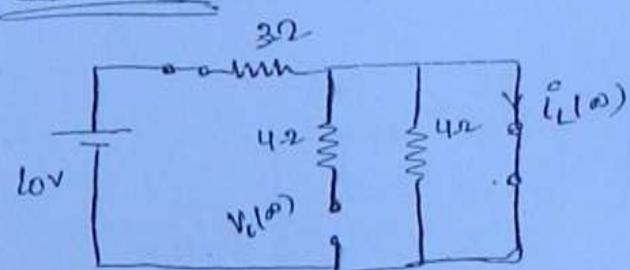
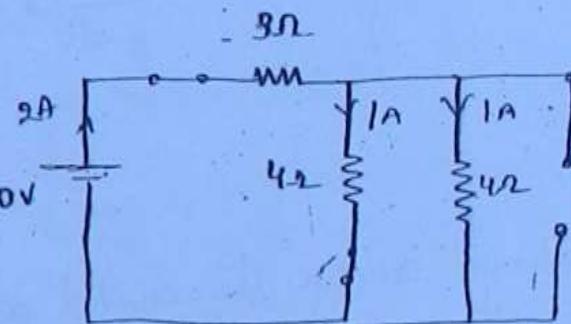
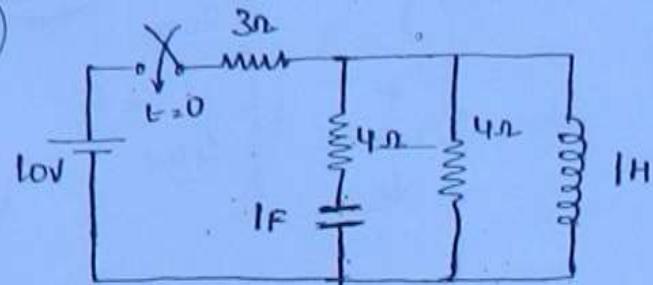
$$\underline{\omega_L(\infty) = \frac{1}{2} L i_L(\infty)}$$

$$= \frac{1}{2} \times 1 \times \left(\frac{10}{3}\right)^2 = \frac{50}{3}$$

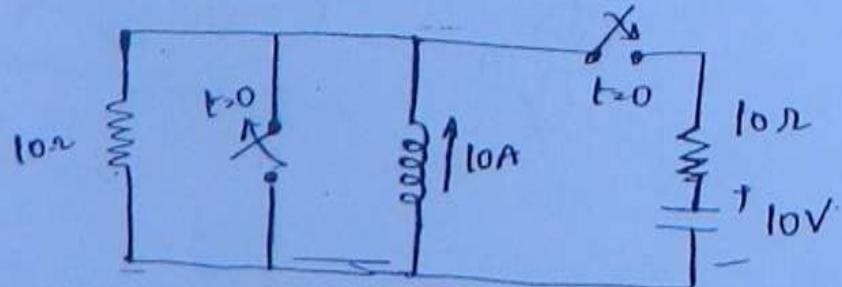
$$v_c(\infty) = 0.$$

$$\omega_c(\infty) = \frac{1}{C} v_c(\infty) = 0.$$

(146)



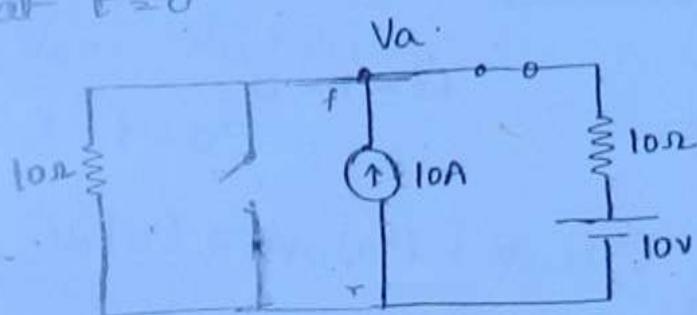
Q. Find voltage across inductor and current flowing through capacitor at $t=0^+$



$$i_L(0^+) = i_L(0^-) = 10 \text{ A} , V_c(0^-) = 10 \text{ V}$$

(147)

at $t=0^+$



g' kcl

$$\frac{V_a}{10} + \frac{V_a - 10}{10} = 10$$

$$\frac{V_a}{5} = 11 \Rightarrow V_a = 55 \text{ V}$$

$$i_c = \frac{V_a - 10}{10} = \frac{55 - 10}{10} = \frac{45}{10} = 4.5 \text{ A}$$

$$i_c = 4.5 \text{ A}$$

$$V_L = V_a = 55 \text{ V}$$

Find $i_c(0^+), i_c(1s)$

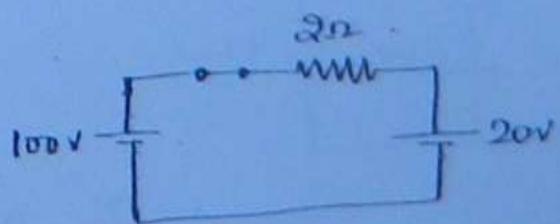
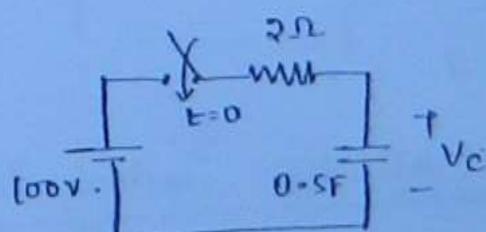
$V_c(0^+), \omega_c(\infty)$

then $Q_0 = 10 \text{ C}$

$$V_0 = \frac{Q_0}{C} = \frac{10}{0.5} = 20 \text{ V}$$

$$V_c(0^+) = V_c(0^-) = 20 \text{ V}$$

$$i_c(0^+) = \frac{100 - 20}{2} = 40 \text{ A}$$



$$L = \infty$$

$$V_c(\infty) = 100V$$

$$\omega_c(\infty) = \frac{1}{2} C \cdot V_c(\infty)$$

$$= \frac{1}{2} \cdot 0.5 \times 100^2 = 2500 \text{ rad/s}$$

$$i(t) = [i(0^+) - i(\infty)] e^{-t/\tau_{RC}} + i(\infty)$$

$$i(t) = 40 e^{-t}$$

$$\tau_{RC} = 1$$

$$i(1\text{sec}) = 40 e^{-1} = 14.7 \text{ A}$$

$$1 - e^{-1} = 0.63$$

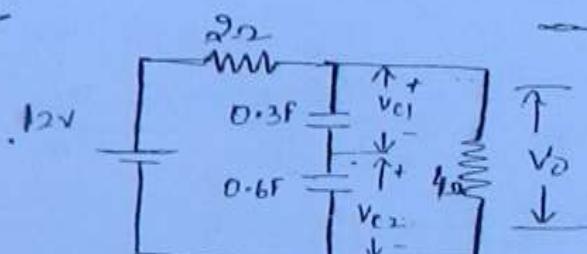
Q. Find response of V_o for the circuit shown.

(a) $22 e^{-3.75t} + 8$

(b) $22 e^{-t/3.75} + 8$

(c) $22 e^{-1.2t} + 8$

(d) $22 e^{-t/1.2} + 8$.



$$V_{C_1}(0) = 20V$$

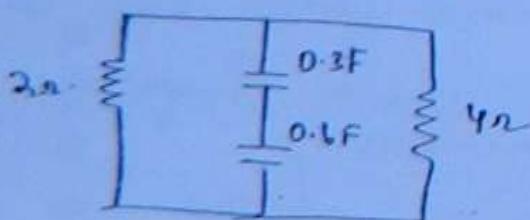
$$V_{C_2}(0) = 10V$$

$$\tau = \frac{R \cdot C}{2} = \frac{0.6 \times 0.3}{2} = 0.09 \text{ sec}$$

$$R_{eq} = \frac{0.6 \times 0.3}{0.6 + 0.3} = 0.2 \Omega$$

$$C_{eq} = \frac{0.6 \times 0.3}{0.6 + 0.3} = 0.2F$$

Ans. Deactivate all the independent sources to find the independent sources time const.



$$R_{eq} = \frac{2 \times 4}{2+4} = 1\Omega$$

$$C_{eq} = \frac{0.3 \times 0.6}{0.3 + 0.6} = 0.2F$$

$$T_2 = R_{eq} C_{eq}$$

$$= 1/0.2 = 5\text{ sec}$$

$$V_o(t) = [V_o(0^+) - V_o(\infty)] e^{-t/RC} + V_o(\infty) \rightarrow ①$$

$$V_o = V_{C1} + V_{C2}$$

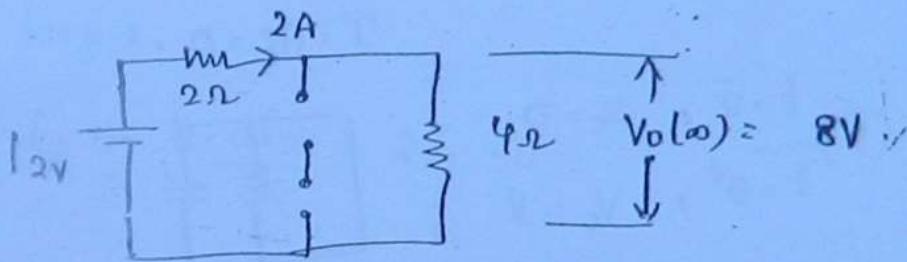
749

at $t = 0^+$

$$V_o(0^+) = V_{C1}(0^+) + V_{C2}(0^+)$$

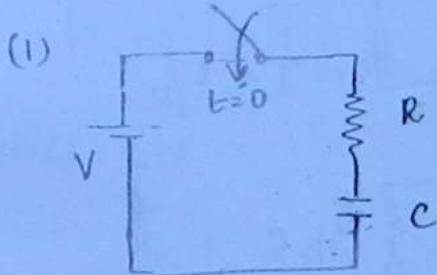
$$V_o(0^+) = 20 + 10 = 30V$$

$t = \infty$

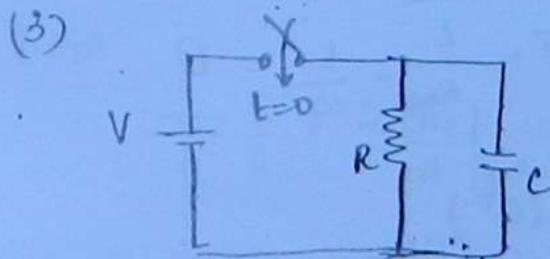
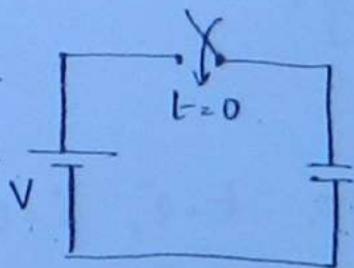


①

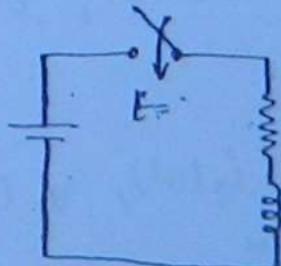
$$V_o(t) = [30 - 8] e^{-t/RC} + 8 = 22 e^{-3.75t} + 8$$



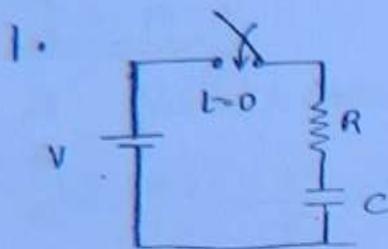
(2)



(4)

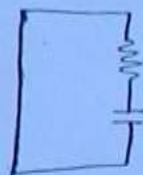


(150)

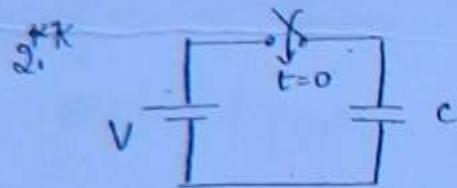


$$t = 0^-, V_C = 0$$

$$t = 0^+, V_C = 0$$

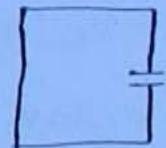


$$\tau = RC$$

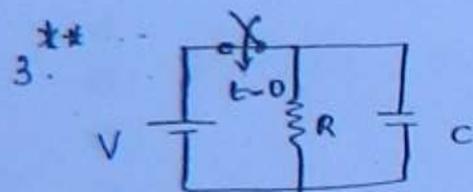


$$t = 0^-, V_C = 0$$

$$t = 0^+, V_C = V$$

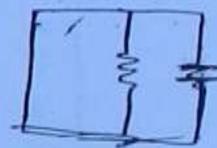


$$\tau = RC = 0$$

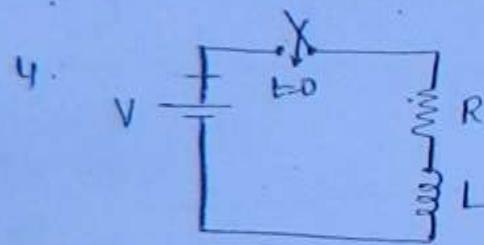


$$t = 0^-, V_C = 0$$

$$t = 0^+, V_C = V$$

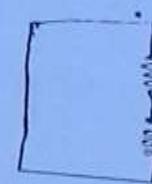


$$R_{eq} = 0 \Rightarrow \tau = RC = 0$$

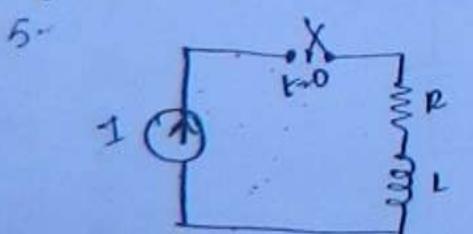


$$t = 0^-, i = 0$$

$$t = 0^+, i = 0$$

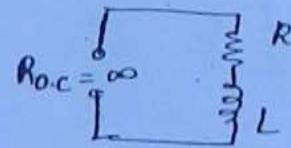


$$\tau = L/R$$



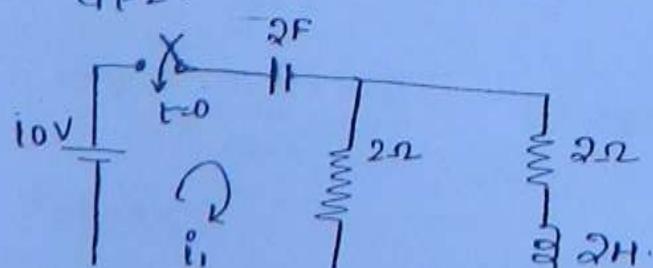
$$t = 0^-, i = 0$$

$$t = 0^+, i = I$$



$$\tau = \frac{L}{R_{eq}} = \frac{L}{\infty} = 0$$

Q. Find $i_1(0^+)$, $i_2(0^+)$, $V_C(0^+)$, $\frac{di_1}{dt}(0^+)$, $\frac{di_2}{dt}(0^+)$, $\frac{d^2i_1}{dt^2}(0^+)$, $\frac{d^2i_2}{dt^2}(0^+)$.



at $t = 0^-$,

$$i_1(0^-) = 0 .$$

$$i_2(0^-) = 0 .$$

$$V_C(0^-) = 0 .$$

at $t = 0^+$,

$$V_C(0^+) = i_2(0^+) \cancel{=} 0 .$$

$$i_1(0^+) = \frac{10}{2} = 5A .$$

KVL in loop 2:

$$4i_2 + 2 \cdot \frac{di_2}{dt} - 2i_1 = 0 . \Rightarrow ①$$

at $t = 0^+$

$$4i_2 + 2 \cdot \frac{di_2}{dt} - 2(5) = 0 \Rightarrow \frac{di_2}{dt}(0^+) = 5A/s$$

(in loop 1)

$$i_0 = \frac{1}{C} \int i_1 dt + 2(i_1 - i_2) .$$

diff wrt.

$$\frac{i_1}{2} + 2 \frac{di_1}{dt} - 2 \frac{di_2}{dt} = 0$$

at $t = 0^+$

$$\frac{5}{2} + 2 \frac{di_1}{dt} \Big|_{0^+} - 2 \times 5 = 0 .$$

$$\frac{di_1}{dt}(0^+) = 3.75 A/sec .$$

$\frac{3.75}{20 \cdot 10^{-6}}$
 3.75×10^5
 $3.75 - 20$
 22

① wrt

$$4 \frac{di_2}{dt} + 2 \frac{d^2i_2}{dt^2} - 2 \frac{di_1}{dt} = 0 .$$

$$20 \frac{di_2}{dt} + \frac{d^2i_2}{dt^2} = 0 .$$

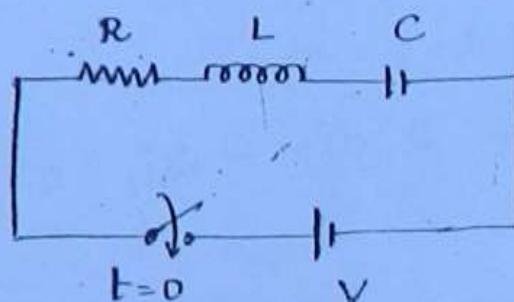
- diff eq. w.r.t.

(152)

$$\frac{1}{2} \frac{d^2 i}{dt^2} + 2 \left[\frac{d^2 V_1}{dt^2} - \frac{d^2 V_2}{dt^2} \right] = 0$$

$$\frac{d^2 V_1}{dt^2} =$$

RC series ckt with DC excitation



$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

diff wrt

$$0 = R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{i}{C}$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$

$$\left(D^2 + \frac{R}{L} D + \frac{1}{LC} \right) i = 0 \quad \frac{d}{dt} = D$$

$$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$D_1, D_2 = \frac{-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}{2}$$

Case 1:

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC} \quad \text{over damping}$$

$$V = \frac{-R^2}{2L}, \quad \beta^2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$i(t) = c_1 e^{(\alpha-\beta)t} + c_2 e^{(\alpha+\beta)t}$$

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Case 2:

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \quad \text{critical damping}$$

$$i(t) = (c_1 + c_2 t) e^{\alpha t}$$

Case 3:

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC} \quad \text{under damping}$$

$$\alpha = -\frac{R}{2L}, \quad \beta = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\Rightarrow i(t) = (c_1 \cos \beta t + c_2 \sin \beta t) e^{\alpha t}$$

$$\text{Damping coefficient} = \frac{R}{2L}$$

$$\text{Time constant} = \frac{1}{\text{damping coeff.}} = \frac{2L}{R}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\omega_d = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{4L}}$$

$$\boxed{\omega_d = \omega_0 \sqrt{1 - \xi^2}}$$

Case 4:

$$R=0$$

JSY

undamping.

$$\alpha = 0$$

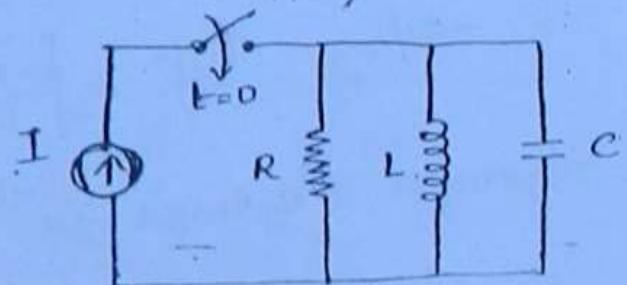
$$\beta = \frac{1}{\sqrt{LC}}$$

$$i(t) = c_1 \cos \beta t + c_2 \sin \beta t$$

RLC parallel circuit with DC excitation

$$I = \frac{V}{R} + C \frac{dV}{dt} + \frac{1}{L} \int V dt$$

diff wrt.



$$0 = \frac{1}{R} \frac{dv}{dt} + C \frac{dv}{dt^2} + \frac{V}{L}$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{V}{LC} = 0$$

$$\left(D^2 + \frac{D}{RC} + \frac{1}{LC} \right) V = 0$$

$$D_1, D_2 = -\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}$$

$$D_1, D_2 =$$

$$-\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

case 1:

$$\left(\frac{1}{2RC}\right)^2 > \frac{1}{LC} \quad \text{overdamping}$$

$$\alpha = -1/2RC$$

155

$$\beta = \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$V(t) = c_1 e^{(\alpha-\beta)t} + c_2 e^{(\alpha+\beta)t}$$

case 2:

$$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC}$$

critical damping

$$V(t) = (c_1 + c_2 t) e^{\alpha t}$$

case 3:

$$\left(\frac{1}{2RC}\right)^2 < \frac{1}{LC}$$

underdamping

$$\alpha = -\frac{1}{2RC}, \quad \beta = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$V(t) = (c_1 \cos \omega t + c_2 \sin \omega t) e^{\alpha t}$$

Damping coefficient = $\frac{1}{2RC}$

Time const. = $\frac{1}{\text{damping coeff.}} = 2RC$

$$\begin{cases} \omega_0^2 = \frac{1}{LC} \\ 2 \cdot \frac{1}{2R} \sqrt{\frac{L}{C}} \end{cases}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC - \left(\frac{1}{2RC}\right)^2}}$$

case 4:

$$G = 0$$

$$\frac{1}{R} = 0$$

$$R = \infty$$

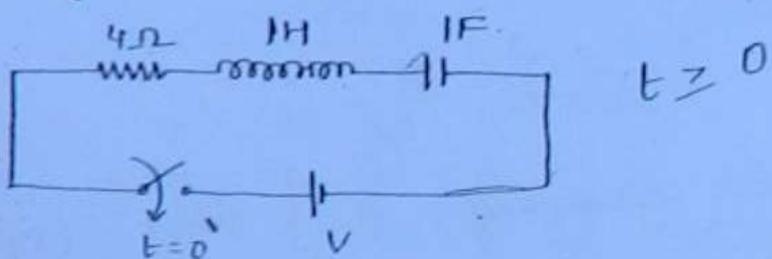
undamping

$$\beta = -\frac{1}{\sqrt{LC}}$$

(15b)

$$V(t) = E_0 \cos \beta t + c_2 \sin \beta t$$

*Q. Find current response for $t > 0$



Note:

1. Steady state current response of RLC series circuit with DC excitation = 0.

$$\xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$\xi > 1$ \Rightarrow over damped

$\xi = 1$ critically damped

$\xi < 1$ under damped

$\xi = 0$ undamped.

$$V = i e^{-\xi \omega_0 t} - \int \int i dt$$

$$\frac{dv}{dt} = i \cdot \frac{di}{dt} = \frac{d^2v}{dt^2} = \frac{d^2i}{dt^2}$$

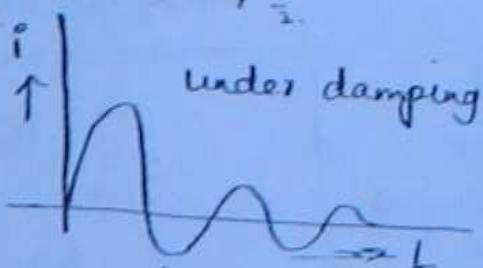
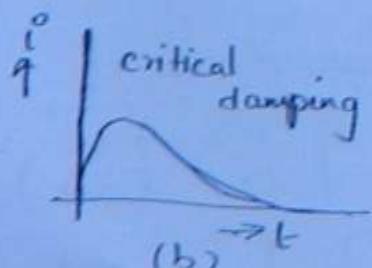
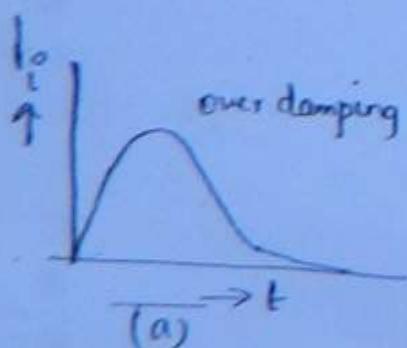
$$-\frac{1}{Lc} + \sqrt{\frac{1}{4Lc}} - 4\frac{R}{L}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{Lc} i = 0$$

$$\frac{1}{4} \left[\sqrt{\frac{R^2}{4L^2} - \frac{4}{Lc}} \right]$$

$$\frac{R}{L} \frac{dc}{dt} \xi = R/2L$$

$$4L^2 \frac{dc}{dt} \xi = R^2/4L$$



→ In over & critical damping no oscillations are present.

$$\Rightarrow \xi \geq 1 \Rightarrow \frac{R}{2} \sqrt{\frac{C}{L}} \geq 1 \quad (\text{for series ckt})$$

(157)

For under damping systems more than one oscillation is present. ($\xi < 1$) .

When response is asked,

$$i(t) = c_1 e^{(\alpha-\beta)t} + c_2 e^{(\alpha+\beta)t}$$

$$\alpha = -\frac{R}{2L}, \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

AC TRANSIENTS

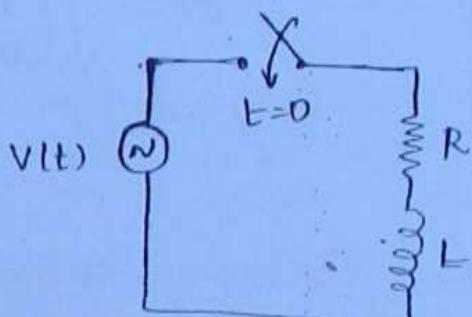
(158)

- Intensity of the DC transients are more than intensity of the AC transients.
- In the AC circuit, based on selection of circuit elements, operating frequency and switching operation it is possible to obtain TRANSIENT FREE RESPONSE. But in the DC ckt, it is not possible to obtain transient free response.

$$V = iR + L \frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

$$i(t) = CF + PI$$



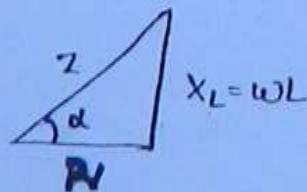
$$V(t) = V_m \sin(\omega t + \theta)$$

CF → Transient response

$$\frac{di}{dt} + \frac{R}{L} i = 0 \Rightarrow i(t) = A e^{-RT/L}$$

P.I → steady state response

$$\begin{aligned} \cancel{\Rightarrow} \quad i &= \frac{V}{Z \angle \alpha} \\ i &= \frac{V \angle -\alpha}{Z} \end{aligned}$$



$$\alpha = \tan^{-1} \left(\frac{\omega L}{R} \right)$$

$$i(t) = \frac{V_m \sin(\omega t + \theta - \alpha)}{Z}$$

$$i(t) = C \cdot F + P \cdot I$$

$$i(t) = A e^{-Rt/L} + \frac{V_m}{Z} \sin(\omega t + \theta - \alpha)$$

$$t = 0^- , i = 0$$

$$t = 0^+ , i = 0$$

$$0 = A + \frac{V_m}{Z} \sin(\theta - \alpha)$$

$$A = -\frac{V_m}{Z} \sin(\theta - \alpha)$$

(159)

$$i(t) = \underbrace{-\frac{V_m}{Z} \sin(\theta - \alpha) e^{\frac{-Rt}{L}}}_{T.R.} + \underbrace{\frac{V_m}{Z} \sin(\omega t + \theta - \alpha)}_{S.R.}$$

$$v(t) = V_m \sin(\omega t + \theta) , t = 0$$

$$\theta - \alpha = 0$$

$$\theta = \alpha$$

$$\theta = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Condition for transient free response

$$v(t) = V_m \sin(\omega t + \theta) , t = 0$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}$$

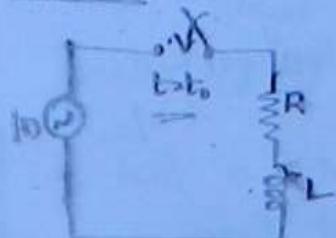
use 3:

$$i(t) = C F + P I$$

$$i(t) = A e^{-\frac{R}{L}(t-t_0)} + \frac{V_m}{Z} \sin(\omega t + \theta - \alpha)$$

$$t = t_0^- , i = 0$$

$$L \rightarrow \infty , I \rightarrow \infty$$



$$V = A + \frac{V_m}{Z} \sin(\omega t_0 + \theta - \alpha)$$

$$A = \frac{-V_m}{Z} \sin(\omega t_0 + \theta - \alpha)$$

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Case 3: $V(t) = V_m \sin(\omega t + \theta), t = t_0$

$$\omega t_0 + \theta - \alpha = 0$$

$$\omega t_0 = \alpha - \theta$$

* $\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$

Case 4:

$V(t) = V_m \cos(\omega t + \theta), t = t_0$

$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta + \frac{\pi}{2}$$

parallel
series
1. $V(t) = V_m \sin(\omega t + \theta), t = 0$
 $i(t) = I_m \sin(\omega t + \theta), t = 0$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\theta = \tan^{-1}(\omega r)$$

$$\theta = \tan^{-1}(w_{RC})$$

2. $V(t) = V_m \cos(\omega t + \theta), t = 0$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}$$

$$\theta = \tan^{-1}(\omega r) + \frac{\pi}{2}$$

$$\theta = \tan^{-1}(w_{RC}) + \frac{\pi}{2}$$

3. $V(t) = V_m \sin(\omega t + \theta), t = t_0$

$$\therefore \omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$$

$$\tan^{-1}(\omega r) - \theta$$

$$\tan^{-1}(w_{RC}) - \theta$$

4. $V(t) = V_m \cos(\omega t + \theta), t = t_0$

$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta + \frac{\pi}{2}$$

$$\tan^{-1}(\omega r) - \theta + \frac{\pi}{2}$$

$$\tan^{-1}(w_{RC}) - \theta + \frac{\pi}{2}$$

Note

→ In RLC ckt, it is not possible to obtain transient free response since ckt is having two energy storing elements.

Ex. for underdamped RLC sys.

(16)

$$i(t) = (c_1 \cos \beta t + c_2 \sin \beta t) e^{\alpha t}$$

For transient free response,

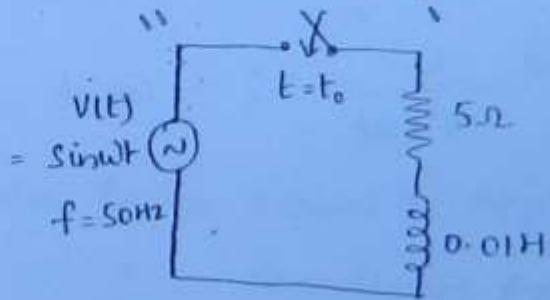
$$c_1 \cos \beta t + c_2 \sin \beta t \text{ should be } = 0$$

But cos & sin term will never be equal to zero simultaneously. Hence, there will be no transient free response.

At what value of t_0 transient free response is obtained?

$$\omega t_0 = -\tan^{-1} \left(\frac{\omega L}{R} \right) - \theta$$

$$\theta = 0^\circ$$



$$\omega t_0 = -\tan^{-1} \left(\frac{2\pi \times 50 \times 0.01}{5} \right) = -\tan^{-1} (0.2\pi)$$

$$t_0 = \frac{32.14 \times \pi}{2\pi \times 50 \times 100}$$

$-\tan^{-1}(0.2\pi)$ should be taken in radians.

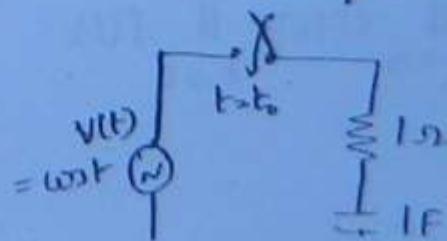
$$t_0 = 1.78 \text{ msec.}$$

$$\frac{32.14}{10000} \frac{10.67}{100} = 1.78$$

At what value of t_0 , transient free response is obtained?

$$\omega t_0 = -\tan^{-1}(\omega_{RC}) - \theta + \pi/2$$

$$\theta = 0^\circ$$



$$\omega = 1$$

$$t_0 = -\tan^{-1}(1) + \pi/2 \\ = \pi/4 + \pi/2 = \frac{3\pi}{4}$$

$$t_0 = \frac{3\pi}{4} \text{ sec.}$$

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3/8/11

Laplace Transforms

$$A \rightarrow A/s$$

$$e^{at} \rightarrow \frac{1}{s-a}$$

$$\sin \omega t \rightarrow \frac{\omega}{s^2 + \omega^2}; \quad \cos \omega t \rightarrow \frac{s}{s^2 + \omega^2}$$

$$e^{at} \sin \omega t \rightarrow \frac{\omega}{(s-a)^2 + \omega^2}; \quad U(t) \rightarrow 1/s$$
$$e^{at} \cos \omega t \rightarrow \frac{s+a}{(s+a)^2 + \omega^2}; \quad \delta(t) \rightarrow 1$$

$$\frac{df}{dt} \rightarrow SF(s) - f(0^+)$$

$\frac{d\omega}{dt}$

Initial value theorem

$$f(0^+) = \lim_{s \rightarrow \infty} SF(s) = \lim_{t \rightarrow 0} f(t)$$

f(0)

Find initial value of $F(s) = \frac{(2s+1)(s+3)}{s(s+2)(3s+4)}$.

$$F(s) = \frac{(2+1/s)(1+3/s)}{s(1+2/s)(3+4/s)}$$
(63)

$$f(0^+) = \lim_{s \rightarrow \infty} s F(s) = \lim_{s \rightarrow 0} \frac{(2+1/s)(1+3/s)}{(1+2/s)(3+4/s)}$$

$$= \frac{\frac{2 \times 1}{s}}{3} = \frac{2}{3}$$

Find initial value of the following function.

$$f(t) = 6 + 8(t).$$

Note:

In the above function initial value theorem cannot be applied.
Since, to apply the initial " " denominator power
should be $>$ numerator power.

Final value theorem.

$$f(0^+) = \lim_{s \rightarrow 0} s F(s) = \lim_{t \rightarrow \infty} f(t).$$

Find final value of the following function.

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)(s+6)}$$

$$(0^+) = \lim_{s \rightarrow 0} s F(s) = \frac{1 \times 3}{2 \times 1} = \frac{1}{4}$$

Q. Find final value of the following function

$$f(t) = 3 + e^{2t}$$

$$\frac{3}{s} + \frac{2}{s-2}$$

$$F(s) = \frac{3}{s} + \frac{1}{s-2}$$

(16y)

Note: For the above function, final value theorem cannot be applied since pole is present in right $\frac{1}{2}$ of the s -plane.
[unstable system].

Q. Find final value of the following function

$$F(s) = \frac{\omega}{s^2 + \omega^2}$$
 (a) 0 (b) 1 (c) ∞ (d) none

Soln
 $f(t) = \sin \omega t$.

Ans: dies b/w 1 & -1.

Note: For marginally stable system also, final value theorem cannot be applied.

Q. Current flowing through $4H$ inductor is given by,

$$I(s) = \frac{10}{s(s+2)}$$
 Find initial voltage of the inductor.

Soln
 $V_L = L \frac{di}{dt} \Rightarrow V_L(s) = 4 [5I(s) - i(0^+)]$

$$V(s) = 4s I(s) - 4i(0^+)$$

$$i(0^+) = \lim_{s \rightarrow \infty} s I(s) = \frac{s \cdot 10}{s + 2}$$

(105)

$$V(s) = 10s \cdot I(s) = \frac{10s \cdot 10}{s + 2}$$

$$= \frac{100}{s + 2}$$

$$I(0^+) = \lim_{s \rightarrow \infty} s V(s) = \lim_{s \rightarrow \infty} \frac{s \cdot 10}{s + 2} = 10.$$

$$V(0^+) = 10V.$$

Repetional case in Inductor

$$\hat{i}_L = \frac{1}{L} \int_{-\infty}^t v dt$$

$$\hat{i}_L(t) = \frac{1}{L} \int_{-\infty}^0 v dt + \frac{1}{L} \int_0^t v dt$$

$$\hat{i}_L(t) = \hat{i}_L(0^-) + \frac{1}{L} \int_{0^-}^t v dt$$

$$\hat{i}_L(0^+) = \hat{i}_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} v dt$$

ie $\hat{i}_L(0^+) = \hat{i}_L(0^-) + 0 \rightarrow$ Inductor doesn't allow sudden current change $\neq 0$

sudden current change $\neq 0$

$$i(0^+) = i(0^-) + \frac{1}{L} \int_{0^-}^{0^+} v dt$$

then $V \rightarrow \delta(t)$

$$\hat{i}_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} \delta(t) dt \quad \text{area} = 1.$$

$$\boxed{\hat{i}_L(0^+) = \frac{1}{L}} \rightarrow \textcircled{1} \quad i_L(0^-) = 0.$$

$$w(0^+) = \frac{1}{2} L \hat{i}_L(0^+) = \frac{1}{2} \cdot L \left(\frac{1}{L}\right)^2$$

$$\boxed{w(0^+) = \frac{1}{2L}}$$

Note :-

From above relation it is concluded that inductor doesn't allow instantaneous change for given imp.

→ from above relations it is concluded that inductor allows instantaneous changes for voltage impulse function. (2)

In case of capacitor

$$V_C = \frac{1}{C} \int_{-\infty}^t i_C dt$$

$$-V_C(t) = \frac{1}{C} \int_{-\infty}^{0^-} i_C dt + \frac{1}{C} \int_{0^-}^t i_C dt$$

$$V_C(t) = V_C(0^-) + \frac{1}{C} \int_{0^-}^t i_C dt$$

$$t=0^+, \quad V_C(0^+) = V_C(0^-) + \frac{1}{C} \int_{0^-}^{0^+} i_C dt$$

$$\underline{\text{Note :-}} \quad V_C(0^+) = V_C(0^-) + 0$$

From the above relation, it is concluded that capacitor

$$V_c(0^+) = V_c(0^-) + \frac{1}{C} \int_{0^-}^{0^+} i dt \quad (167)$$

when $i \rightarrow \delta(t)$

$$V_c(0^+) = V_c(0^-) + \left(\frac{1}{C} \int_{0^-}^{0^+} \delta(t) dt \right) \text{ area} = 1.$$

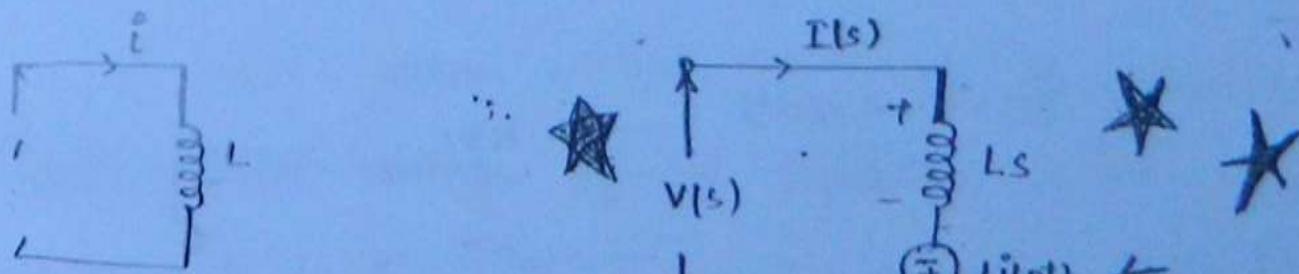
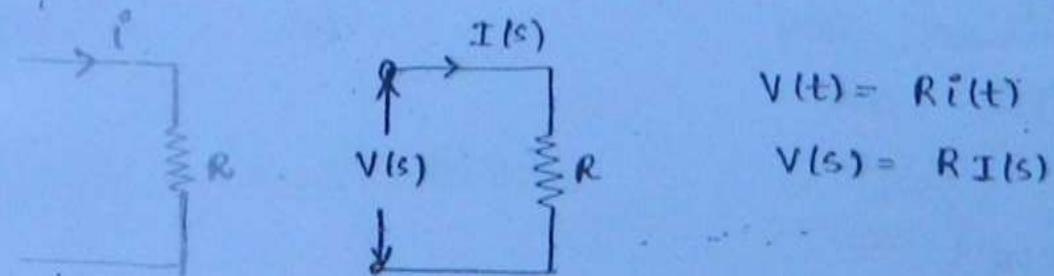
$$V_c(0^+) = 1/C$$

$$w_c(0^+) = \frac{1}{2} C V_c(0^+)^2 = \frac{1}{2} C \left[\frac{1}{C} \right]^2$$

$$w_c(0^+) = \frac{1}{2C}$$

from the above relation it is concluded that capacitors ~~do not~~ allow instantaneous voltage change for a given ~~impedance~~ current impulse function.

ablate domain

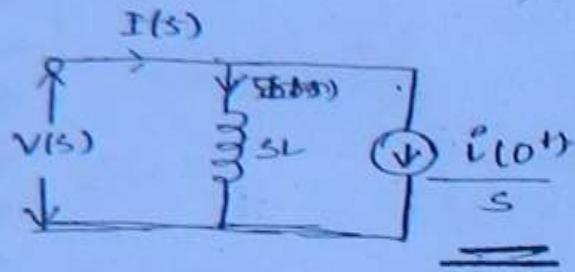


$$V(s) = L [sI(s) - i(0^+)]$$

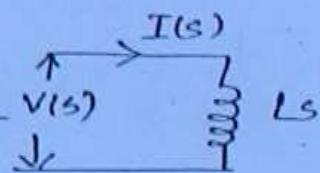
$$Z(s) = \frac{V(s)}{I(s)} = L(s) - \frac{i(0^+)}{s}$$

$$I(s) = V(s) + L i(0^+)$$

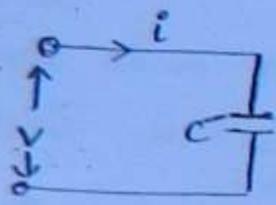
$$I(s) = \frac{V(s)}{Ls} + \frac{i(0^+)}{Ls} \Rightarrow (166)$$



$$i(0^+) = 0$$



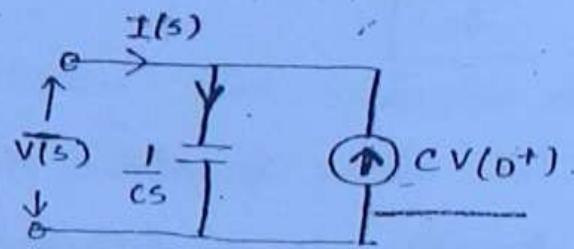
Capacitor



$$i = C \frac{dv}{dt}$$

$$I(s) = C [sV(s) - v(0^+)]$$

$$I(s) = \frac{V(s)}{1/Cs} - C v(0^+)$$

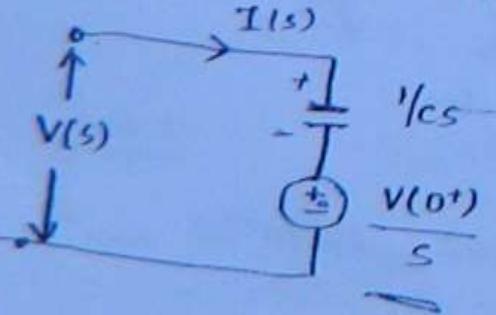


$$V(s) = \frac{I(s)}{1/Cs}$$

$$V(s) = C s - \frac{C v(0^+)}{1/Cs}$$

$$\frac{V(s)}{1/Cs} = I(s) + C v(0^+)$$

$$V(s) = \frac{1}{Cs} I(s) + \frac{C v(0^+)}{Cs} \rightarrow kV_L$$

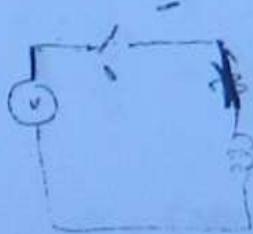
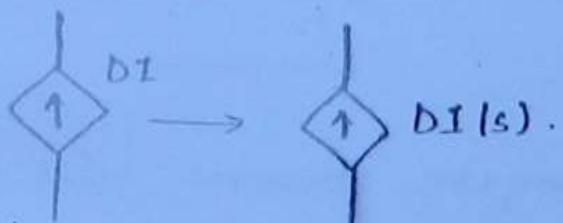
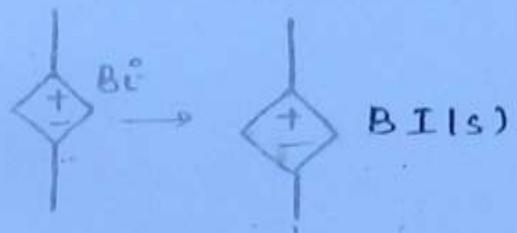
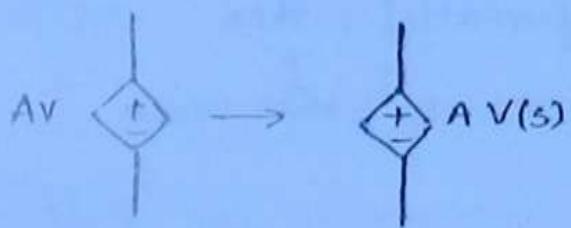


$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{Cs} + \frac{v(0^+)}{s I(s)}$$

$$V(0^+) = 0$$

Transformation of dependant sources from time domain
to s -domain

(167)



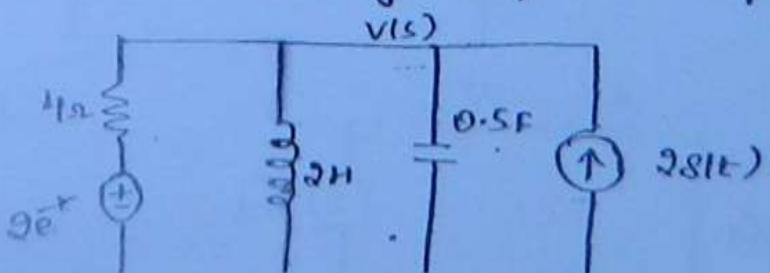
$$V(s) = R I(s) + \frac{1}{sL} I(s)$$

$$V(s) = R I(s) + s \frac{1}{L} I(s) - i(0^+)$$

$$V(s) = 3 I(s) - i(0^+)$$

$$\frac{V(s)}{3 I(s) - i(0^+)} = \frac{1}{sL} = \frac{i(0^+)}{sL}$$

Find $V(s)$ when initial current of the inductor is $2A$
and initial voltage of the capacitor is $3V$.



$V(s) = \frac{110}{sL}$
Initial value

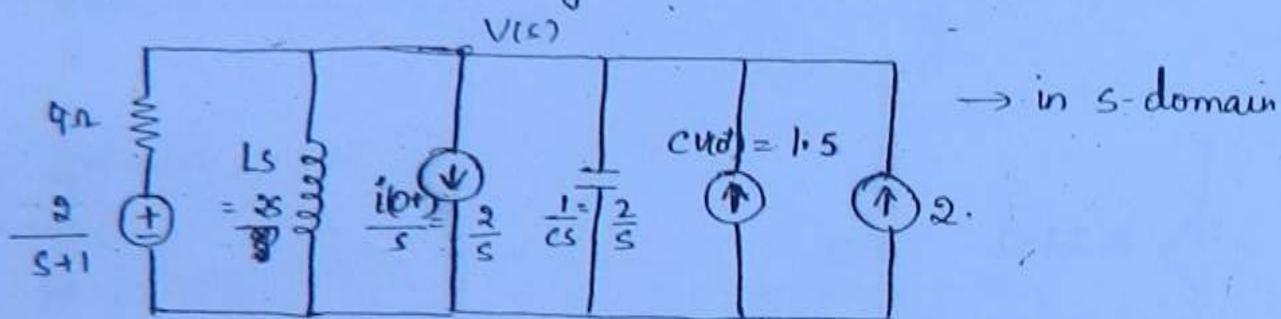
Sol

$$\frac{v}{i}(0^+) = 2A$$

$$V_0 = 3V$$

768

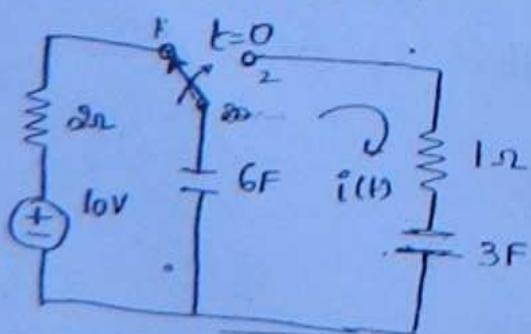
whenever, the elements in circuit are in parallel, then consider initial values as current sources & vice versa
i.e. series \rightarrow voltage sources.

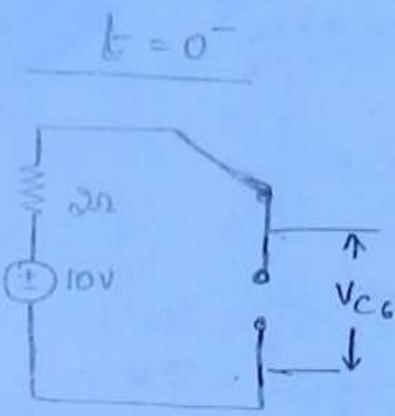


$$\frac{V(s)}{4} - \frac{2}{s+1} + \frac{V(s)}{2s} + \frac{2}{s} + \frac{V(s)}{2is} = 1.5 + 2$$

$$\frac{(s+1) V(s) - 2}{4(s+1)} + \frac{V(s)}{2s} + \frac{2}{s} + \frac{s \cdot Vs}{2} = 3.5$$

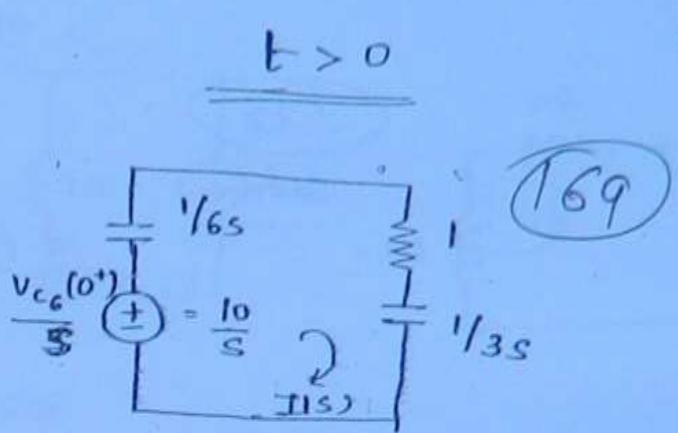
Q. Find $i(t)$ for, $t > 0$





$$V_{C_0}(0^-) = 10 \text{ V}$$

$$V_{C_0}(0^+) = 0$$

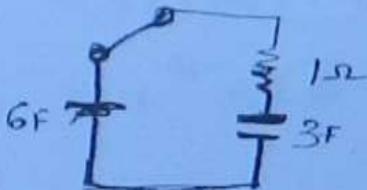


$$I(0^+) = \frac{10/2}{\frac{1}{6s} + \frac{1}{3s} + 1} = \frac{10 \times 6s}{6s + 3s + 6s} = \frac{60}{3+6s} = \frac{20}{1+2s}$$

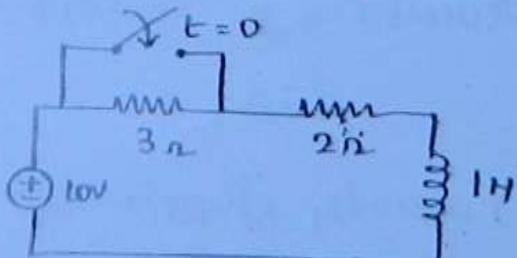
$$i(t) = 10 e^{-t/2}$$

-2 using classical method

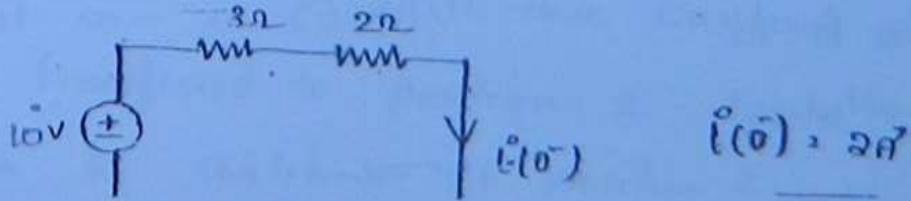
$$i(t) = \frac{V_0}{R} e^{-t/Rc}$$

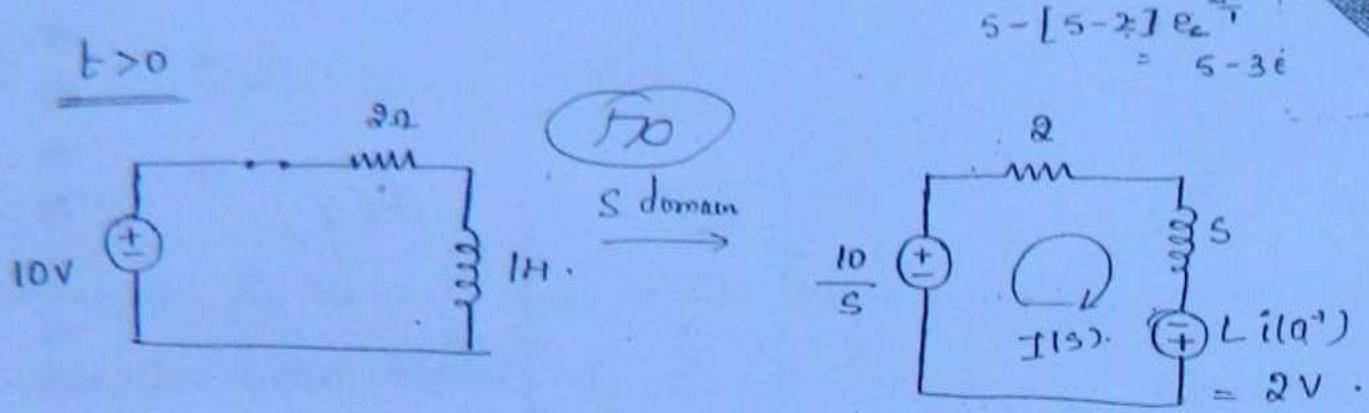


Find current response for $t > 0$.



At $t > 0^-$,



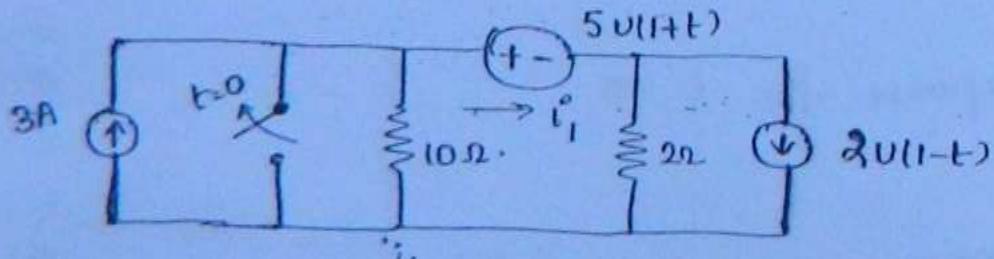


For obj

$$\hat{i}(t) = [i(0^+) - i(\infty)] e^{-Rt/L} + i(\infty)$$

$$= (2 - 5) e^{-2t} + 5 = 5 - 3e^{-2t}$$

Find the value of i_1 at $t = -2$ sec.



$$U(t) \rightarrow 0 \text{ to } \infty$$

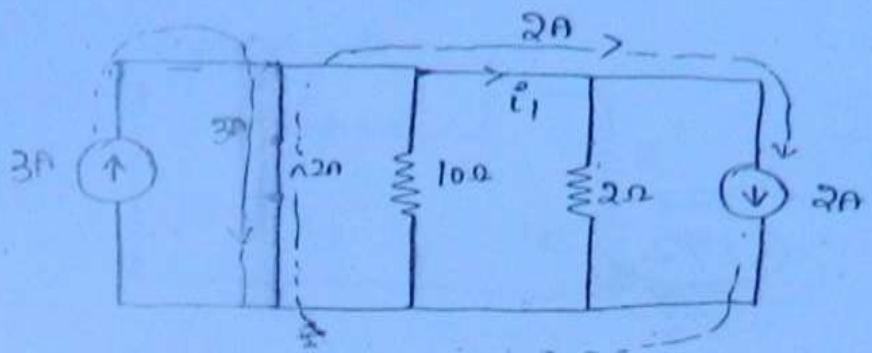
$$U(-t) \rightarrow -\infty \text{ to } 0$$

$$De \rightarrow -\infty \text{ to } \infty$$

$$U(1+t) \rightarrow -1 \text{ to } \infty$$

$$U(1-t) \rightarrow -1 \text{ to } \infty$$

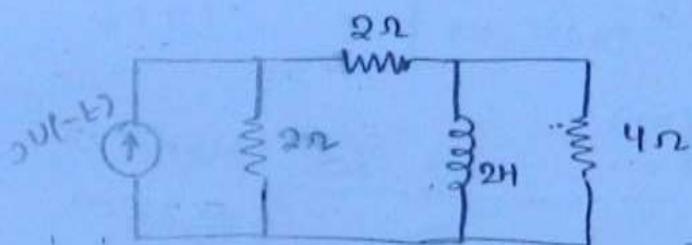
$$U(t-t) \rightarrow -\infty \text{ to } 1$$



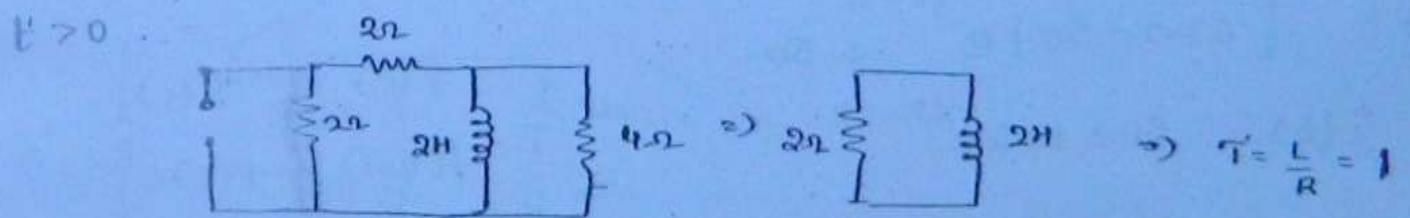
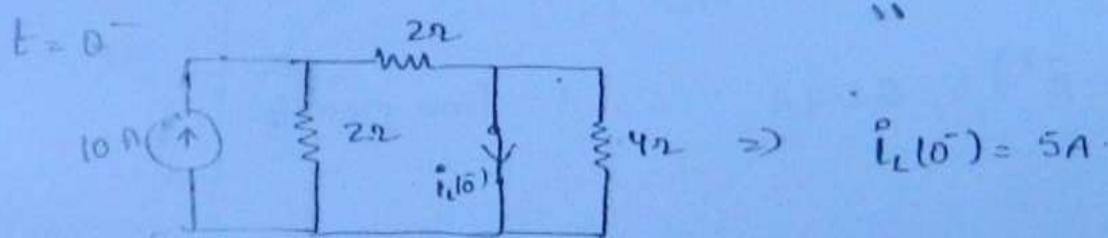
(17)

$$i_1 \Big|_{t=0} = 2A$$

Find current response in the inductor for $t > 0$

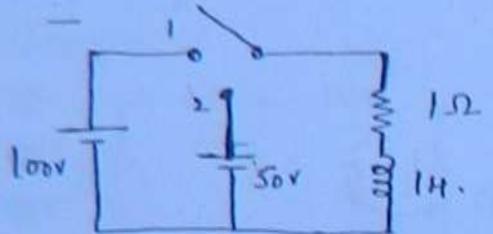


$$\frac{-2E}{R+L}$$



$$\begin{aligned} i(t) &= I_0 e^{-RT/L} \\ &= 5e^{-t} \end{aligned}$$

In the circuit shown, at $t=0$ sec, the switch is connected to 1. After 1 time constant, the switch is transferred to position 2. Find the current response when the switch is at position 2.



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Soln.

position - 1

$$i(t) = 100 \left[1 - e^{-t} \right] \quad \left[\frac{V}{R} \left[1 - e^{-Rt/L} \right] \right]$$

position 2:

$$i(t) = [i(t_0) - i(\infty)] e^{\frac{-R(t-t_0)}{L}} + i(\infty)$$

$$= 50 - [50 - 63.2] e^{-(t-1)}$$

$$= 50 + 13.2 e^{-(t-1)}$$

at $t=t_0 = 1\Omega \Rightarrow 1\gamma$ (one time const.)

$$i(t_0) = 100(1 - e^{-1}) = 63.2 A \quad (\because \text{from position 1})$$

at position 2,

$$i(t) = [63.2 - 50] e^{-(t-1)} + 50$$

$$i(t) = 50 + 13.2 e^{-(t-1)}$$

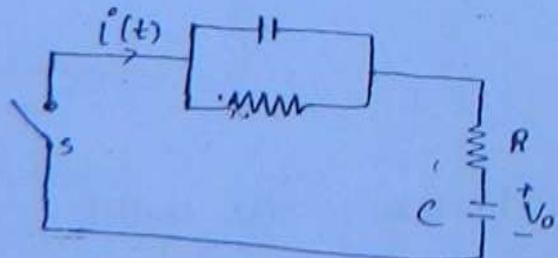
$$f(t) = f(s)$$

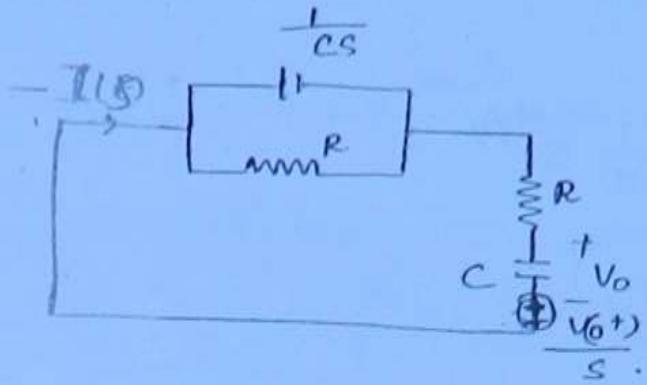
$$f(t+m) = f(s)e^m$$

Pg - 34

Conv.

1.

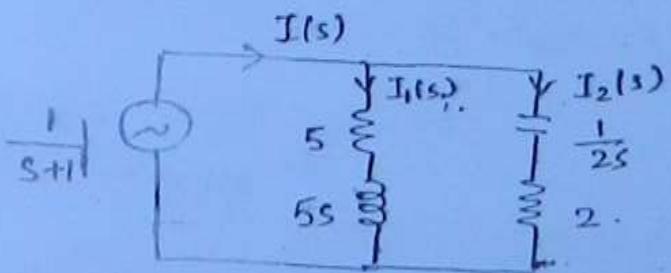
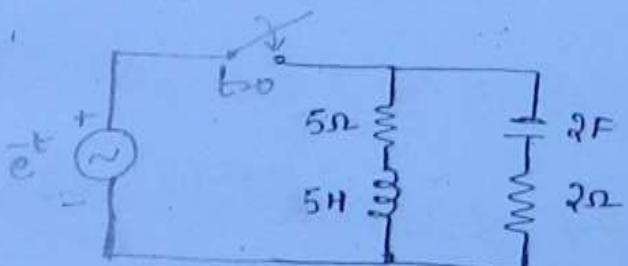
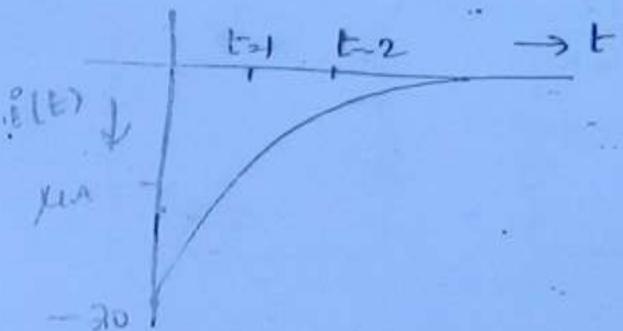




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$$I(s) = \frac{U(0^+)/s}{R + \frac{1}{Cs} + \frac{R \cdot 1/Cs}{R + 1/Cs}}$$

$$i(t) = [14.46 e^{-2.62t} + 5.54 e^{-0.38t}] \mu A$$



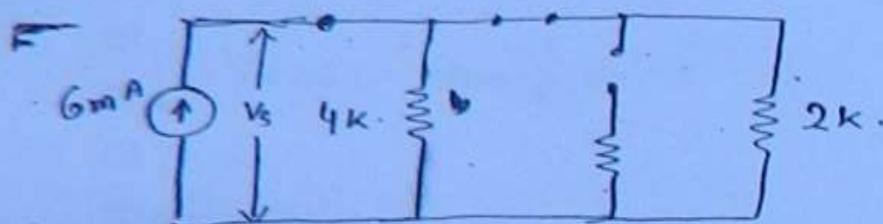
$$I(s) = I_1(s) + I_2(s) = \frac{1/(s+1)}{5+5s} + \frac{1/(s+1)}{2+1/(2s)}$$

$$i(t) = \frac{2}{3} e^{-t} + \frac{1}{5} t e^{-t} - \frac{1}{6} e^{-t/4}$$

4.

(174)

$t = 0^-$



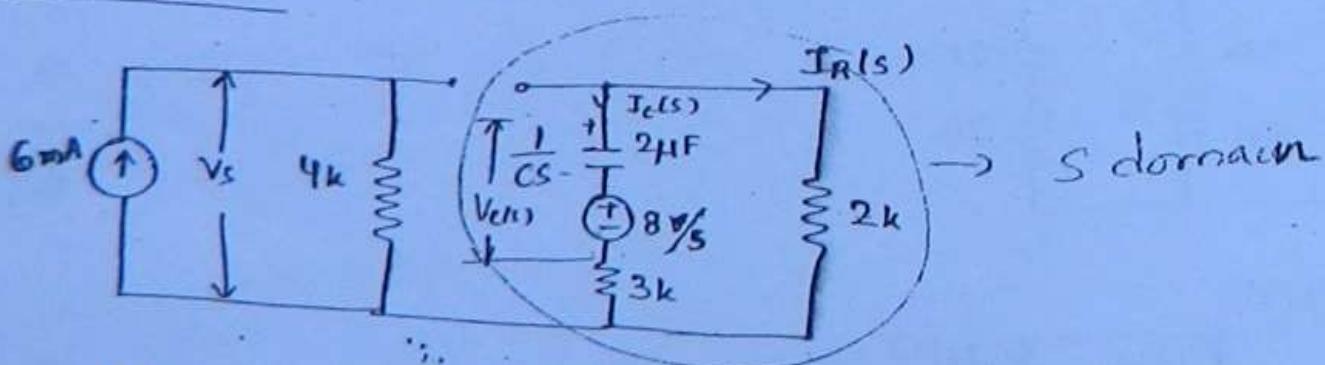
$$I_{4k} = \frac{6m \times 2k}{4k + 2k} = \frac{12}{6k}$$

$$I_{4k} = 2mA$$

$$V_{4k} = 2mA \times 4k = 8V$$

$$\Rightarrow V_C(0^-) = V_{4k} = 8V \Rightarrow V_C(0^+) = 8V$$

at $t = 0^+$



$$V_s = 6m \times 4k = 24V$$

$$T_R(s) = \frac{V(0^+)/s}{3k + 2k + 1/C_S} = \frac{8/s}{5k + \frac{500k}{s}} = \frac{8m}{5k + \frac{500k}{s}}$$

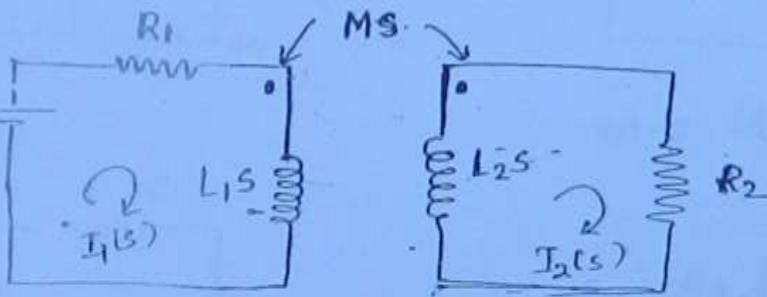
$$i_R(t) = 1.6 e^{-100t} \text{ mA}$$

$$i_C(t) = -i_R(t) = -1.6 e^{100t} \text{ mA}$$

$$V_C = \frac{1}{C} \int_{-\infty}^t i_C dt$$

$$= \frac{1}{C} \int_{-\infty}^0 i_C dt + \frac{1}{C} \int_0^t i_C dt$$

$$V_C = 8 + \frac{1}{C} \int_0^t i_C dt$$



$$-\frac{V}{s} + (R_1 + L_1 s) I_1(s) - M s I_2(s) = 0 \rightarrow ①$$

$$(R_2 + L_2 s) I_2(s) - M s I_1(s) = 0 \rightarrow ②$$

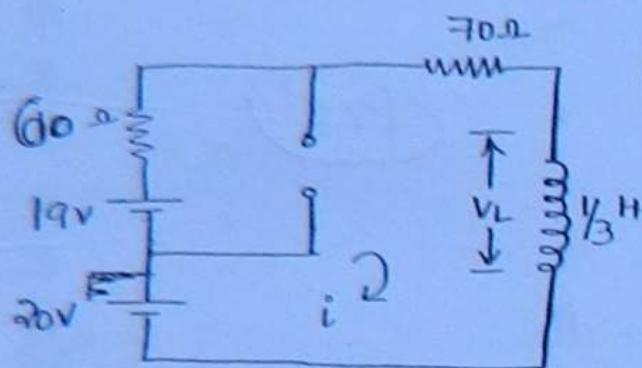
From eq. ②

$$I_2(s) = \frac{M s I_1(s)}{R_2 + L_2 s} \rightarrow ③$$

Sub eq ③ in ①

$$i_1(t) = [5 - e^{-t/s}] v(t)$$

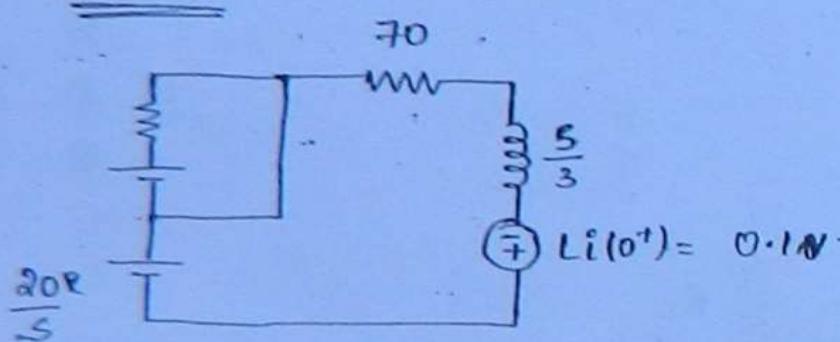
(175)

6 $t = 0^-$ 

176

$$i(0^-) = \frac{20 + 19}{60 + 70}$$

$$i(0^-) = 0.3A$$

 $t > 0$ 

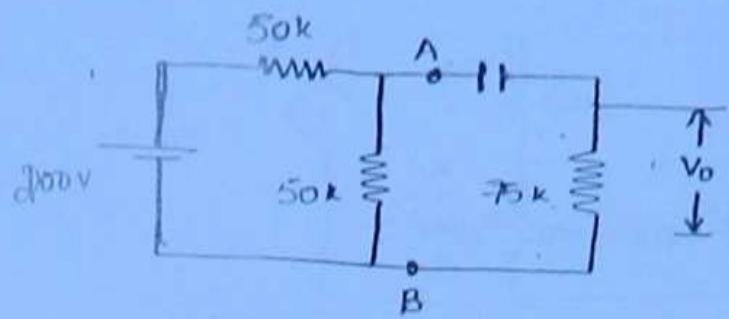
$$I(s) = \frac{\frac{20}{s} + 0.1}{70 + s/3} = \frac{3(20 + 0.1s)}{s[s + 210]}$$

$$= \frac{210}{s} + \frac{1/70}{s + 210}$$

$$i(t) = \left[\frac{2}{7} + \frac{1}{70} e^{-210t} \right] v(t)$$

$$v(t) = L \frac{di}{dt} = \frac{1}{3} \left[-\frac{210}{70} e^{-210t} \right]$$

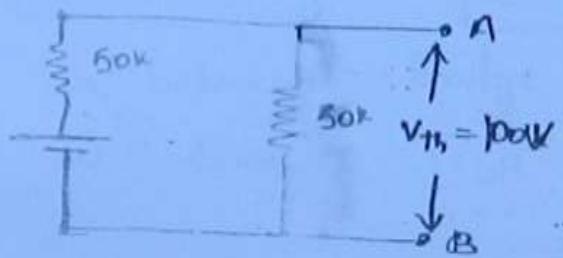
$$v(t) = -\frac{210}{70} e^{-210t}$$



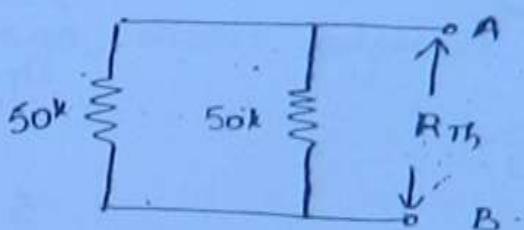
(177)

Thevenin eq. across AB.

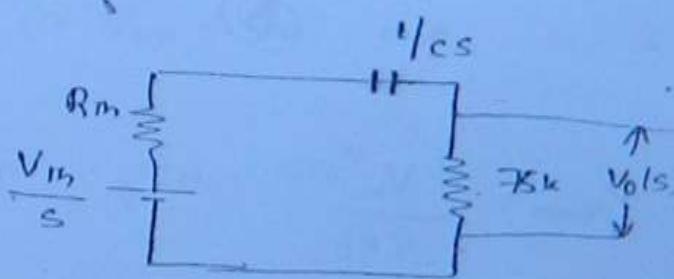
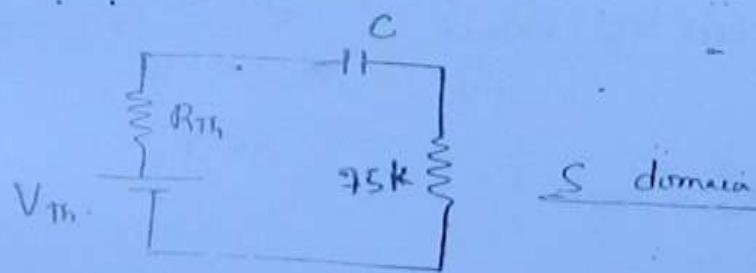
Case i) (V_{Th}).



Case ii) R_{Th} .



$$R_{Th} = 25k$$



$$V_o(s) = \frac{V_{Th}}{s} \cdot \frac{75k}{75k + \frac{1}{Cs} + R_{Th}}$$

$$V_o(t) = t V_o(s) \Rightarrow V_o(t) = 75 e^{-10t}$$

at $t = t_0$

$$V_o(t) = 75 e^{-(t-t_0)}$$

$t = t_0$

$$V_o = 75V$$

$$t = 25 \times 10^{-3} \Rightarrow V_o(t) = e^{-10(25 \times 10^{-3} - t_0)}$$

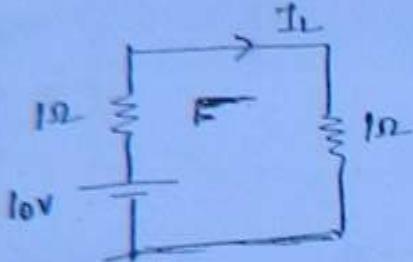
Theorems Obj.

(178)

25.

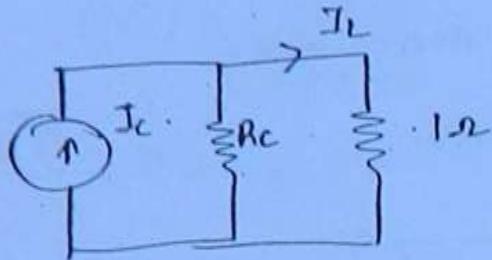
$$i = \frac{V_1 - V_2}{1 \times 10^6} \Rightarrow R_{CRO} = \frac{V_2}{i}$$

28.



$$I_L = \frac{10}{1+1}$$

$$I_L = 5A$$



$$I_L = I_c \cdot \frac{R_c}{R_c + 1}$$

$$5 = \frac{I_c R_c}{R_c + 1}$$

(b)

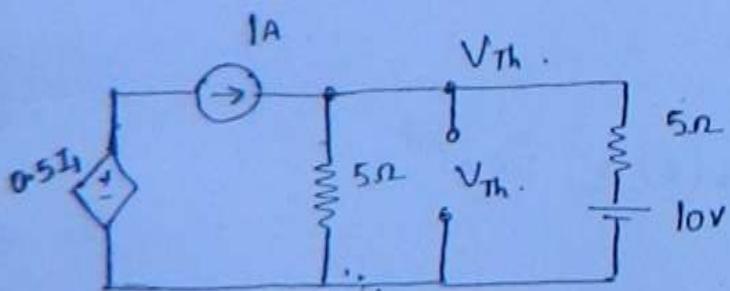
Check the values

30.

$$P_{max} = \frac{V_s^2}{4R_L} \Rightarrow R_L = R_s = 100\Omega$$

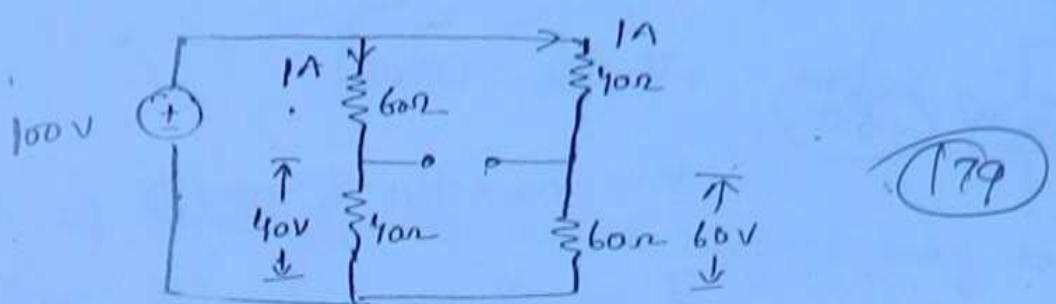
$$P_{max} = 0.25W \quad (c)$$

31.



$$I = \frac{V_{Th}}{5} + \frac{V_{Th} - 10}{5} \Rightarrow V_{Th} = 7.5V$$

$$R_{Th} = 5/2 = 2.5\Omega$$



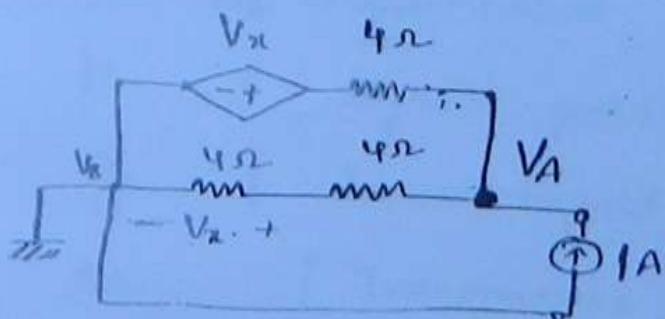
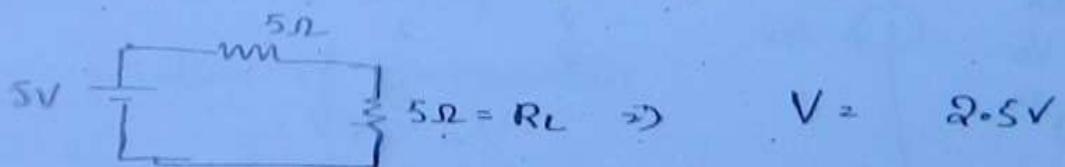
$$V_m = 60 - 40 = 20 \text{ V.}$$

Note :-
In balanced bridge voltage across adjacent branches is same.

$$E = E_1, \quad I = 0, \quad V = 5 \quad \xrightarrow[0 \cdot C]{\downarrow} V_{oc} = 5 \text{ V.}$$

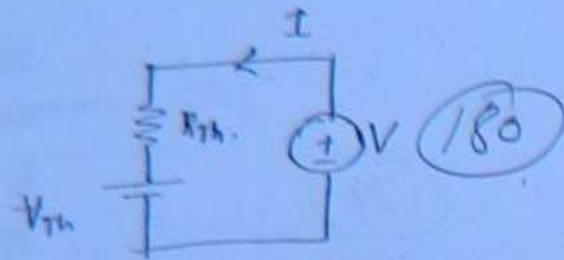
$$E = 0, \quad I = 1, \quad V = 5 \quad \Rightarrow \quad R_{th} = 5 \Omega.$$

$$E = 5 \text{ V}, \quad E = E_1, \quad I \rightarrow 5 \Omega.$$



$$I = \frac{V_A}{8} + \frac{V_A - V_B}{4} \quad , \quad V_A = V_B / 2$$

39.



$$V - V_m = I R_m$$

$$V = V_m + R_m \cdot I \rightarrow (1)$$

$$I = \frac{V - V_m}{R_m}$$

$$I = 0.2V - 2$$

$$0.2V = I + 2 \rightarrow (2)$$

$$V = 5I + 10 \rightarrow (2) \quad \text{Compare (1) \& (2)}$$

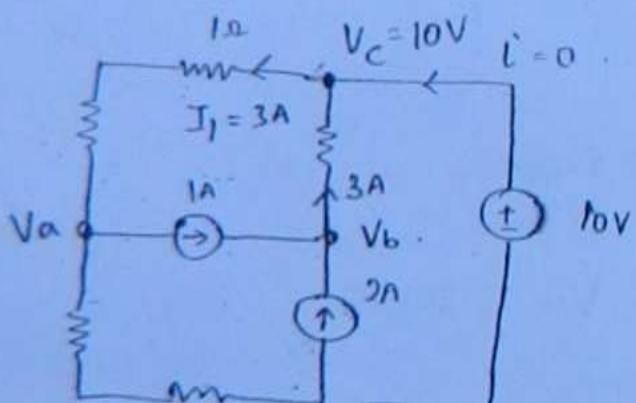
$$V_m = 10V, \quad R_m = 5\Omega$$

40.

$$Z_L = Z_S^* \Rightarrow Z_L = 3 + j4$$

$$P_{max} = \frac{V_s^2}{4R_L} = \frac{240 \times 240}{4 \times 3} = 4.8kW$$

41.



$$\frac{V_a}{2} + \frac{V_a - 10}{2} + 1 = 0$$

$$V_a = 4V$$

$$I_1 = \frac{10 - 4}{1 + 1} = 3A$$

44.

$$I_{SC} = \frac{16 \angle 0^\circ}{25 + 15 + j30}$$

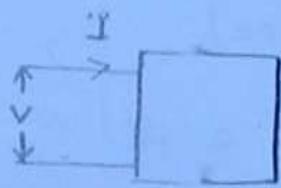
$$I_{SC} = 6.4 - j4.8$$

Two Port N/w's

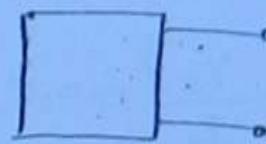
(81)

A pair of terminals at which signal may enter or leave from the network is called as port.

When n/w is having 1 pair of terminals then it is called as single port n/w.

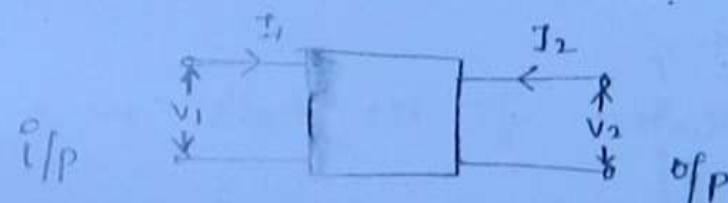


Ex. motor



Ex. Generator

When n/w is having two pair of terminals, then the b/w is called as 2 port n/w.



Ex. Transformer

Classification of parameters

- Z [Open circuit parameters].
- Y [Short circuit "].
- h [Hybrid parameters].
- β [Inverse hybrid parameters].
- ABCD [Transmission line parameters].
- abcd [Inverse " "].

Z parameters

(182)

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow \textcircled{1} \quad \text{kVL}$$

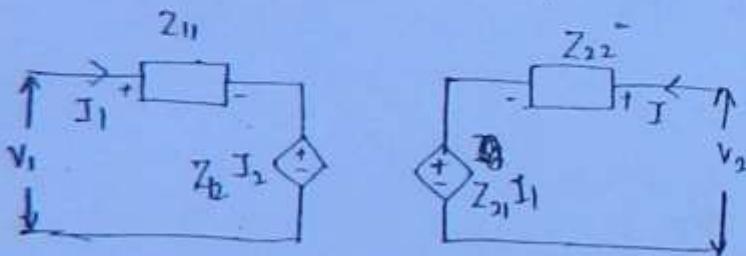
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow \textcircled{2} \quad \text{kVL}$$

$V_1, V_2 \}$ → dependant variables

$I_1, I_2 \}$ → independant variables [source]

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}, \quad Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}, \quad Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$



$Z_{11} \Rightarrow$ Open ckt i/p impedance / driving pt. i/p impedance

$Z_{21} \Rightarrow$ fwd transfer impedance.

$Z_{12} \Rightarrow$ reverse "

$Z_{22} \Rightarrow$ O.C. o/p impedance (or) driving point o/p impedance.

At 8:30

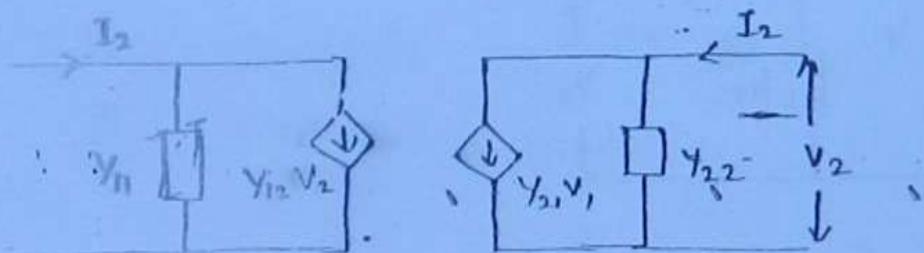
Parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow \textcircled{1} \quad \text{kcl}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow \textcircled{2} \quad \text{kcl}$$

$$h = \frac{I_1}{V_1} \Big|_{V_2=0}, \quad Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}.$$

$$z_{11} = \frac{I_2}{V_1} \Big|_{V_2=0}, \quad Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}.$$



$h_{11} \rightarrow$ short ckt i/p admittance / driving pt. i/p admittance.

$h_{12} \rightarrow$ fwd transfer admittance

$h_{21} \rightarrow$ reverse "

$h_{22} \rightarrow$ short ckt o/p admittance / driving pt. o/p admittance

parameters

$$V_1 = h_{11} I_1 + h_{21} V_2 \rightarrow \textcircled{1}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow \textcircled{2}$$

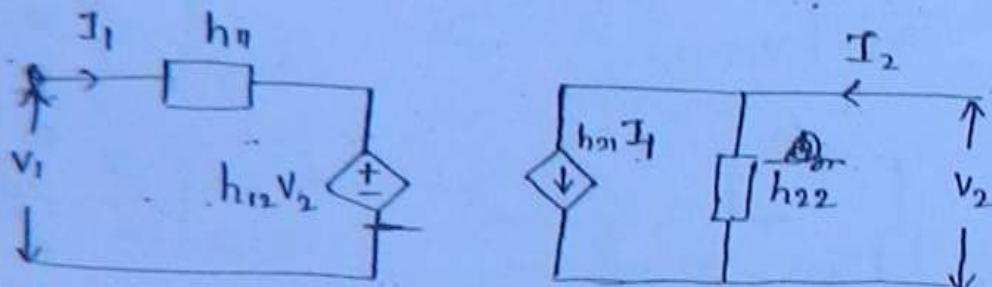
$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} \quad \left(h_{11} \neq z_{11}, \quad h_{11} = \frac{1}{Y_{11}} \right).$$

$$h_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} \quad \left\{ h_{21} = \frac{\gamma_{21}}{\gamma_{11}} = \frac{-Y_2/V_1}{-X_1/V_1} \right\}$$

[no unit] 184

$$h_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} \quad \left(h_{12} = \frac{Z_{12}}{Z_{22}} \right)$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} \quad \left(h_{22} \neq \gamma_{22}, \quad h_{22} = \frac{1}{Z_{22}} \right)$$



~~g~~ Parameters

$$I_1 = g_u V_1 + g_{12} I_2 \rightarrow ①$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \rightarrow ②$$

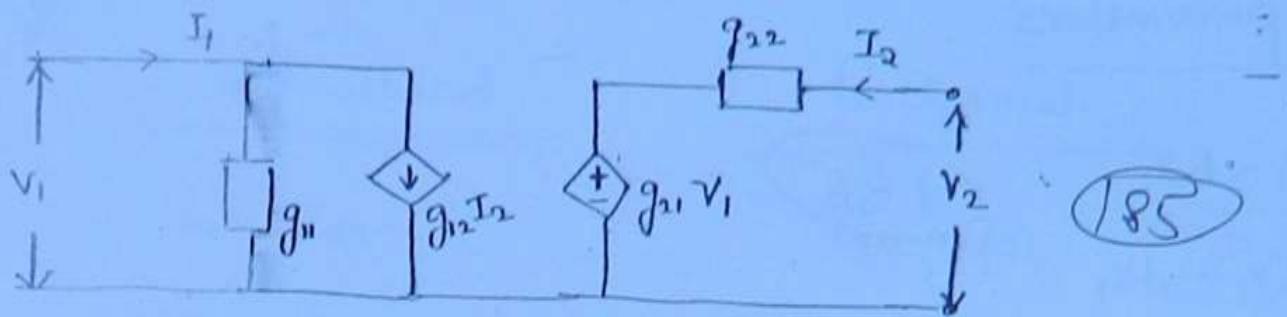
$$g_u = \frac{I_1}{V_1} \Big|_{I_2=0} \quad \left(g_u \neq \gamma_u, \quad g_u = \frac{1}{Z_{11}} \right)$$

(mho)

$$g_{12} = \frac{V_2}{I_2} \Big|_{V_1=0} \quad \left(g_{12} = \frac{\gamma_u}{\gamma_{22}} \right)$$

$$g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0} \quad \left(g_{21} = \frac{\gamma_u}{\gamma_{11}} \right)$$

$$g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0} \quad \left(g_{22} \neq \gamma_{22}, \quad g_{22} = \frac{1}{Z_{22}} \right)$$



$$\begin{aligned} V_1 &= A V_2 - B I_2 \quad \rightarrow \textcircled{1} \\ I_1 &= C V_2 - D I_2 \quad \rightarrow \textcircled{2} \end{aligned}$$

ABCD parameters

$$V_1 = A V_2 - B I_2 \quad \rightarrow \textcircled{1}$$

$$I_1 = C V_2 - D I_2 \quad \rightarrow \textcircled{2}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad \left[A = \frac{Z_{11}}{Z_{22}} \right]$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \quad \left[C = \frac{1}{Z_{21}} \right]$$

$$B = \left. -\frac{V_1}{I_2} \right|_{V_2=0} \quad \left[B = \frac{-1}{Y_{21}} \right]$$

$$D = \left. -\frac{I_1}{V_2} \right|_{V_2=0} \quad \left(D = \frac{-Y_1}{Y_{21}} \right)$$

Note: For ABCD parameters, it is not possible to develop equivalent circuit as both eqn 1 & 2 are developed wrt f/p.

abcd parameters

$$V_2 = aV_1 - bI_1$$

186

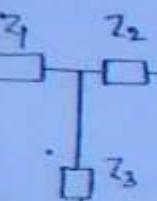
$$I_2 = cV_1 - dI_1$$

$$a = \frac{V_2}{I_1} \Big|_{I_1=0}$$

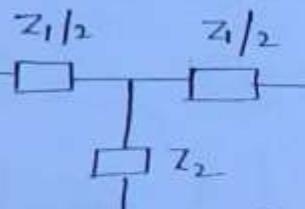
$$b = -\frac{V_2}{I_1} \Big|_{V_1=0}$$

$$c = \frac{I_2}{V_1} \Big|_{I_1=0}$$

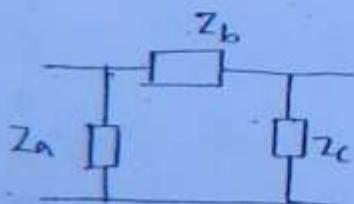
$$d = -\frac{I_2}{V_1} \Big|_{I_1=0}$$



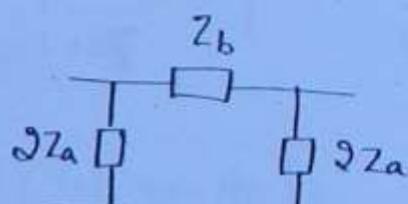
Unsymmetrical T



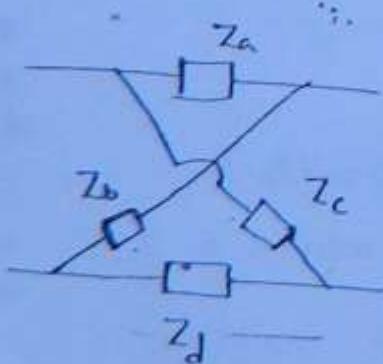
Symmetrical T



Unsymmetrical π



Symmetrical π



$\left. \begin{array}{l} z_a = z_d \\ z_b = z_c \end{array} \right\} \rightarrow \text{Symmetrical lattice}$

$\left. \begin{array}{l} z_a \neq z_d \\ z_b \neq z_c \end{array} \right\} \text{unsymmetrical lattice}$

Symmetrical

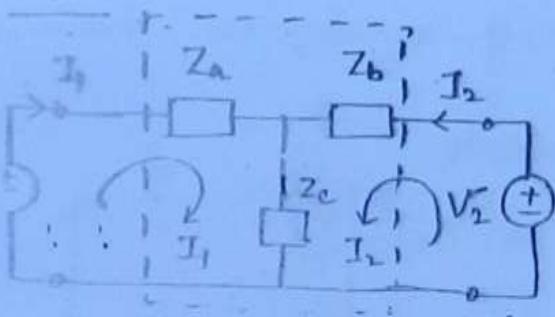
$$Z_{11} = Z_{22}$$

$$Y_{11} = Y_{22}$$

$$\Delta = h_{11}h_{22} - h_{21}h_{12} = 1$$

$$d = A = \Delta$$

η/ω



$$V_1 = (Z_A + Z_C) I_1 + Z_C I_2 \rightarrow (1)$$

$$V_2 = Z_{11} I_1 + Z_{12} I_2 \rightarrow (2)$$

$$V_1 = Z_C I_1 + (Z_B + Z_C) I_2 \rightarrow (3)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow (4)$$

$$Z_{11} = Z_A + Z_C$$

$$Z_{21} = Z_{12} = Z_C$$

$$Z_{22} = Z_B + Z_C$$

$$Z_{11} = Z_A + Z_C$$

$$Z_{12} = Z_{21}$$

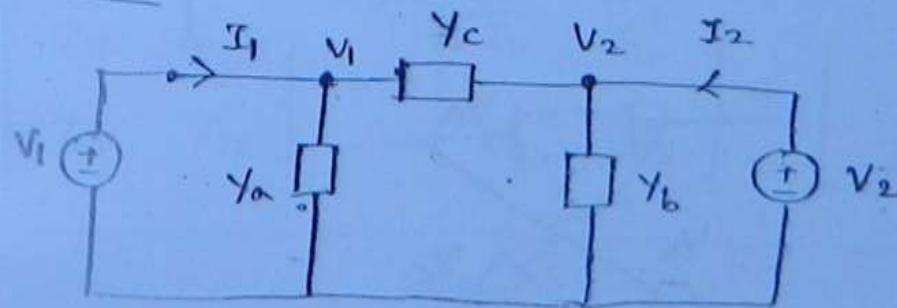
$$Z_A = Z_{11} - Z_{12}$$

$$Z_B = Z_{22} - Z_{12}$$

$$Z_C = Z_{12} = Z_{21}$$

η/ω

$A e^{j\omega t}$



$$I_1 = V_1 Y_a + (V_1 - V_2) Y_c$$

$$I_1 = (Y_a + Y_c) V_1 - Y_c V_2 \rightarrow \textcircled{1} \quad \text{188}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow \textcircled{2}$$

$$I_2 = V_2 Y_b + (V_2 - V_1) Y_c$$

$$I_2 = -Y_c V_1 + (Y_b + Y_c) V_2 \rightarrow \textcircled{3}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow \textcircled{4}$$

$$Y_{11} = Y_a + Y_c$$

$$Y_{21} = Y_{12} = -Y_c$$

$$-Y_{22} = Y_b + Y_c$$

$$Y_{11} = Y_a + Y_c$$

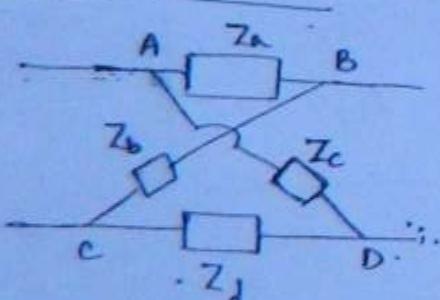
$$Y_{11} = Y_a - Y_{12}$$

$$Y_a = Y_{11} + Y_{12}$$

$$Y_a = Y_{22} + Y_{12}$$

$$Y_c = -Y_{12} = -Y_{21}$$

Lattice n/w



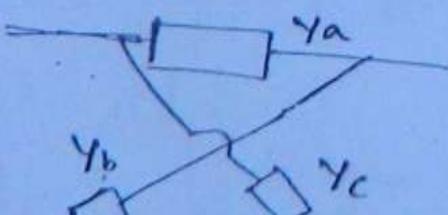
$$Y_a = Y_d, \quad Y_b = Y_c$$

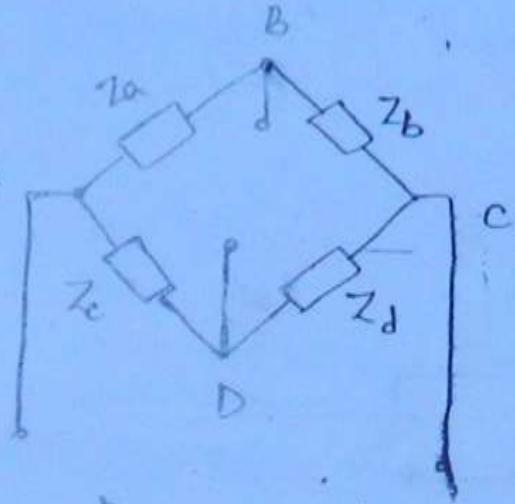
$$Y_{11} = Y_{22} = \frac{Y_b + Y_a}{2}$$

$$Z_a = Z_d, \quad Z_b = Z_c \quad \left. \right\} \text{sy-lattice}$$

$$-Z_{11} = Z_{22} = \frac{Z_b + Z_a}{2} \quad \checkmark$$

$$Z_{12} = Z_{21} = \frac{Z_b - Z_a}{2} \quad \checkmark$$

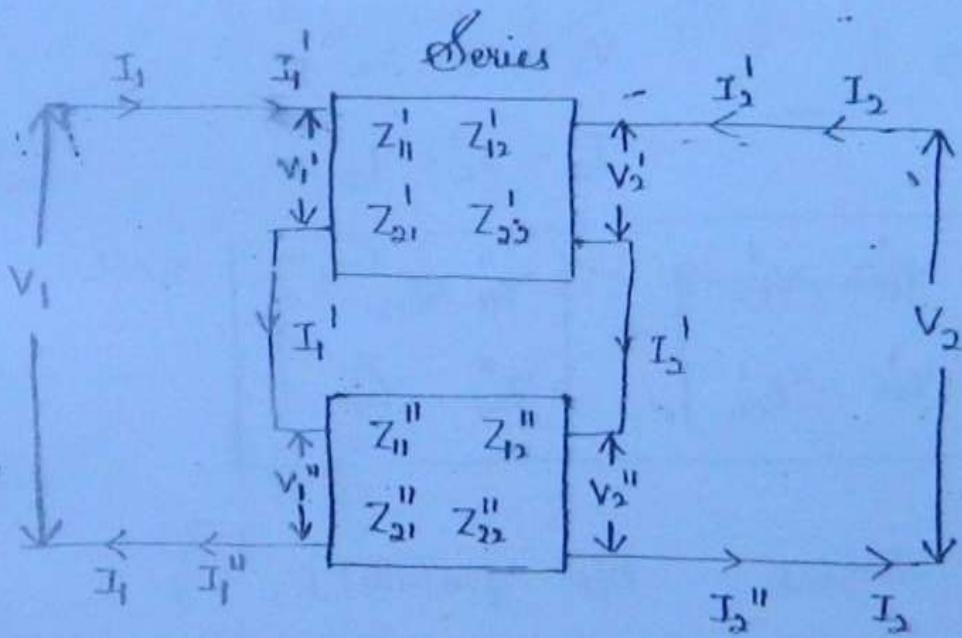




(189)

Bridge n/w

Equivalent impedance parameters



$$V_1 = V_1' + V_1''$$

$$V_2 = V_2' + V_2''$$

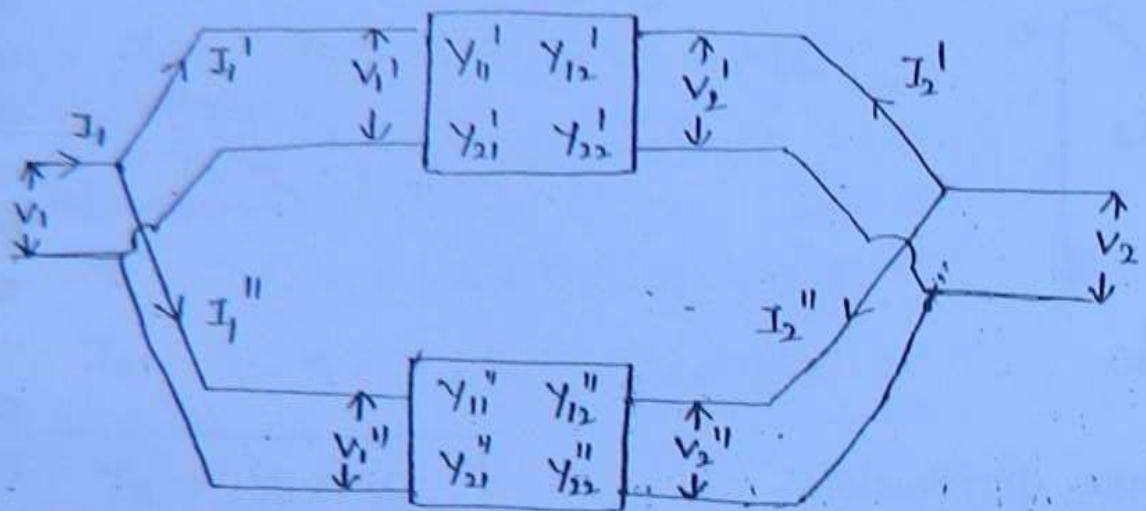
$$I_1 = I_1' = I_1''$$

$$I_2 = I_2' = I_2''$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} + \begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix}$$

Parallel

(190)



$$V_1 = V_1^1 = V_1^u$$

$$V_2 = V_2^1 = V_2^u$$

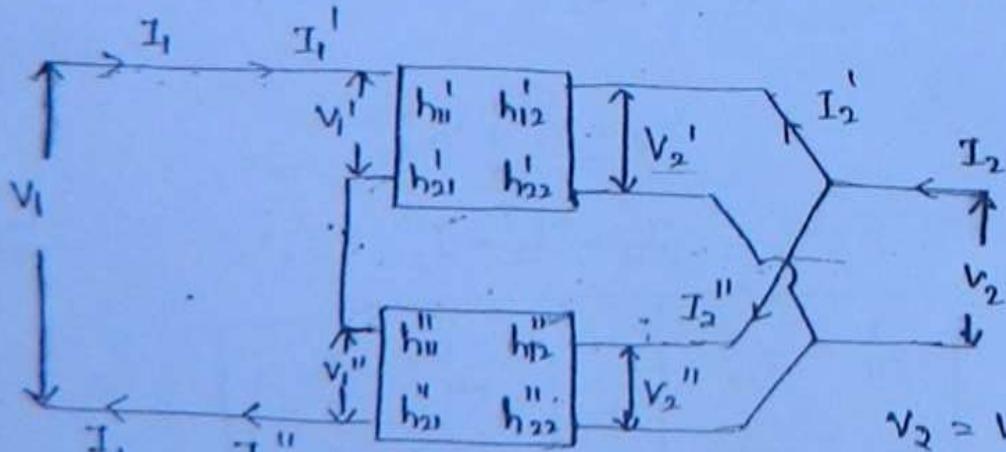
$$I_1 = I_1^1 + I_1^u$$

$$I_2 = I_2^1 = I_2^u$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}^1 & Y_{12}^1 \\ Y_{21}^1 & Y_{22}^1 \end{bmatrix} + \begin{bmatrix} Y_{11}^u & Y_{12}^u \\ Y_{21}^u & Y_{22}^u \end{bmatrix}$$

h parameters

[i/p series o/p parallel] .



$$V_1 = V_1^1 + V_1^u$$

$$V_2 = V_2^1 = V_2^u$$

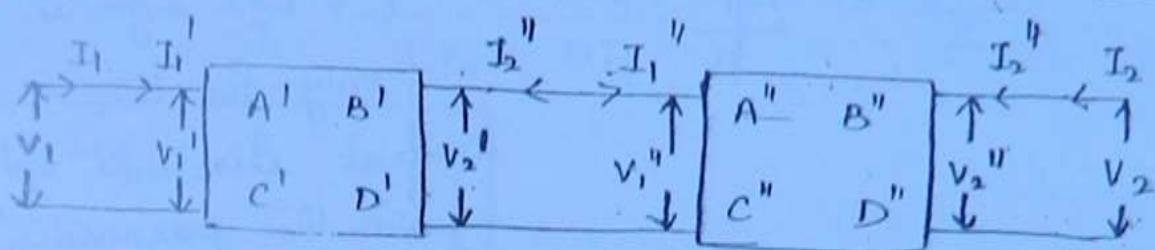
$$I_1 = I_1^1 = I_1^u$$

$$I_2 = I_2^1 + I_2^u$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11}^1 & h_{12}^1 \\ h_{21}^1 & h_{22}^1 \end{bmatrix} + \begin{bmatrix} h_{11}^u & h_{12}^u \\ h_{21}^u & h_{22}^u \end{bmatrix}$$

Cascade / Tandem n/w

(191)



$$V_1 = V_1'$$

$$V_2' = V_1''$$

$$V_2 = V_2''$$

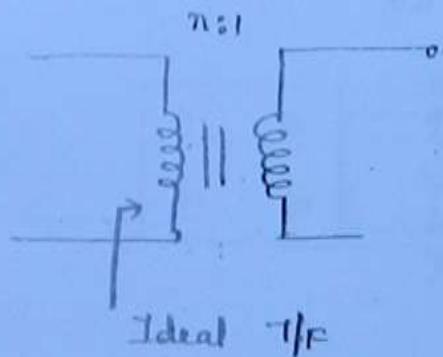
$$I_1 = I_1'$$

$$I_1'' > -I_2$$

$$I_2 = I_2''$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \cdot \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

Find Z, Y & $ABCD$ parameters of the n/w shown.



E:

In the ideal T/F it is not possible to find impedance and admittance values since self and mutual inductance of ideal T/F are ∞ .

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{n}{1}$$

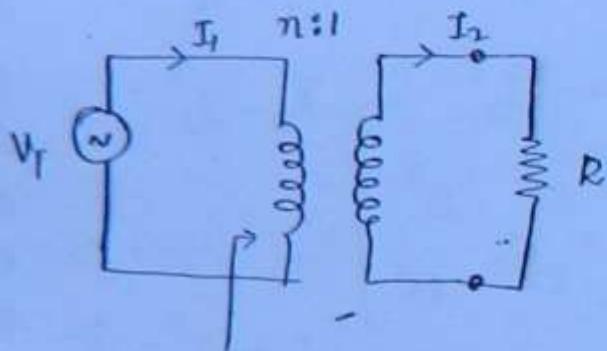
$$A = \frac{V_1}{V_2} = n$$

$$D = \frac{-I_1}{I_2} = \frac{+1}{n}$$

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(\because (-) sign is only for the current direction but not the parameter).

Equivalent resistance.



Ideal T/F

$$I_1^2 R_1 = I_2^2 R_2 \Rightarrow R_1 = \left(\frac{I_2}{I_1}\right)^2 R_2$$

$$R_1 = n^2 R$$

impedance

find the eq. if

Impedance resistance

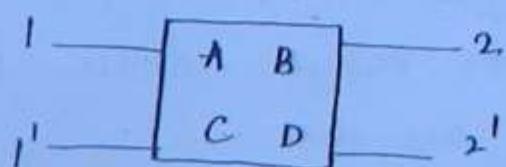
→ If inductor is given $L_1 = n^2 L$

→ " capacitor $C_1 = \frac{1}{n^2 C}$

Q. Find eq. impedance wrt

i_1 1 - 1'

\tilde{i}_1 2 - 2'



1-1'

(-193)

$$Z_{eq} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} \quad | \quad I_2 = 0$$

$$\boxed{Z_{eq} = \frac{A}{C} \omega}$$

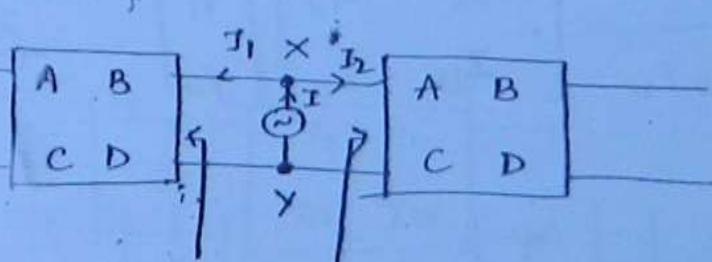
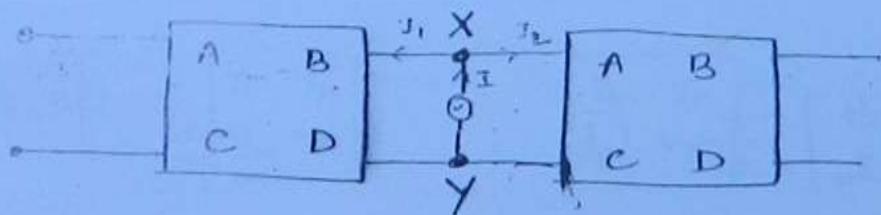
2-2'

$$Z_{eq} = \frac{V_2}{I_2} \quad | \quad I_1 = 0$$

$$I_1 = CV_1 - DI_2 \Rightarrow 0 = CV_2 - DI_2$$

$$\boxed{Z_{eq} = \frac{D}{C} \omega}$$

Find eq. impedance wrt $\times \epsilon_f Y$.



n prw prob)

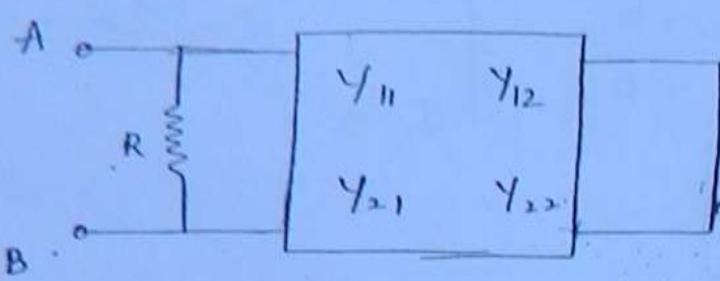
$$Z_{eq_1} = \frac{D}{C}$$

$$Z_1 \neq Z_2$$

$$Z_{eq_2} = A/C$$

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{AD/c^2}{(A+D)/c}$$

Q. Find eq. admittance w.r.t A & B.



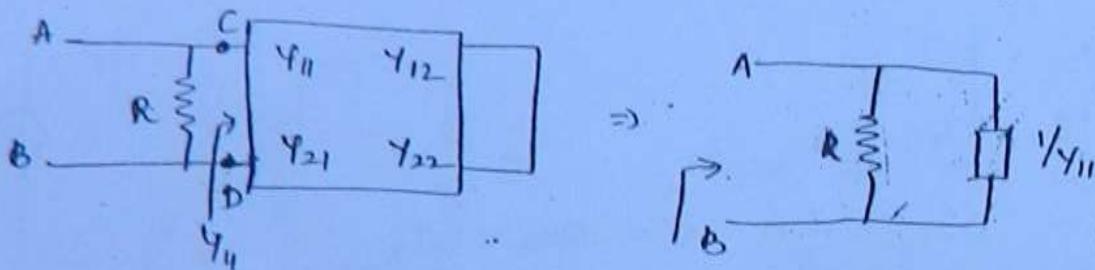
194

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad (1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad (2)$$

$$1/Y_{11}$$

Soln.



~~$$Y_{eq} = \frac{1}{R} + Y_{11}$$~~

$$Y \Leftrightarrow Z$$

$$\left[\begin{array}{c} V_1 \\ V_2 \end{array} \right] = \left[\begin{array}{cc} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{array} \right] \left[\begin{array}{c} I_1 \\ I_2 \end{array} \right] \rightarrow \textcircled{1}$$

$$\left[\begin{array}{c} I_1 \\ I_2 \end{array} \right] = \frac{1}{Z_{11}Z_{22} - Z_{12}Z_{21}} \left[\begin{array}{cc} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{array} \right] \left[\begin{array}{c} V_1 \\ V_2 \end{array} \right] \rightarrow \textcircled{2}$$

$$\left[\begin{array}{c} I_1 \\ I_2 \end{array} \right] = \left[\begin{array}{cc} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{array} \right] \left[\begin{array}{c} V_1 \\ V_2 \end{array} \right] \rightarrow \textcircled{3}$$

$$\left[\begin{array}{c} V_1 \\ V_2 \end{array} \right] = \frac{1}{Y_{11}Y_{22} - Y_{12}Y_{21}} \left[\begin{array}{cc} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{array} \right] \left[\begin{array}{c} I_1 \\ I_2 \end{array} \right] \rightarrow \textcircled{4}$$

From (1) & (2)

(195)

$$Z_{11} = \frac{\gamma_{22}}{\gamma_\Delta} ; \quad Z_{12} = \frac{-\gamma_{12}}{\gamma_\Delta} ; \quad Z_{21} = \frac{-\gamma_{21}}{\gamma_\Delta} ; \quad Z_{22} = \frac{\gamma_{11}}{\gamma_\Delta}$$

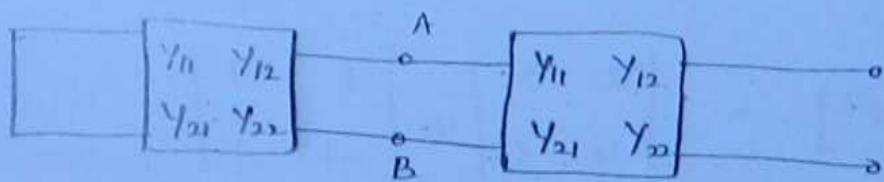
$$\therefore \gamma_\Delta = \gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}$$

From (2) & (2)

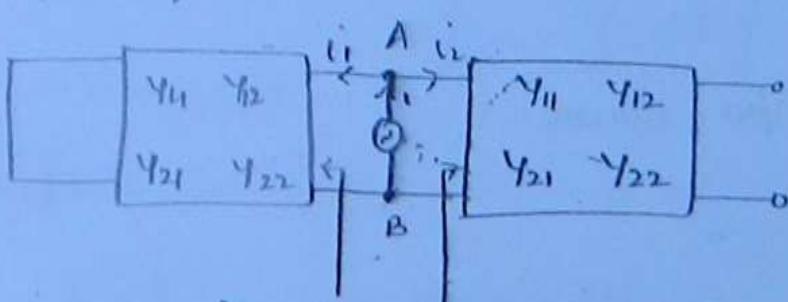
$$\gamma_{11} = \frac{Z_{22}}{Z_\Delta} ; \quad \gamma_{12} = \frac{-Z_{12}}{Z_\Delta} ; \quad \gamma_{21} = \frac{-Z_{21}}{Z_\Delta} ; \quad \gamma_{22} = \frac{Z_{11}}{Z_\Delta}$$

$$Z_\Delta = Z_{11}Z_{22} - Z_{12}Z_{21}$$

Find eq. impedance wrt A & B.



Reqd.

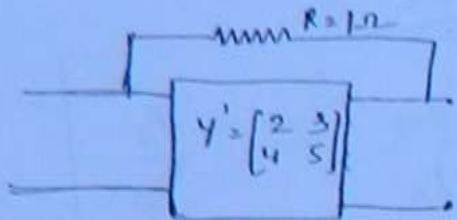


$$Z_I = \frac{1}{\gamma_{11}}$$

$$Z_{11} = \frac{\gamma_{22}}{\gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}}$$

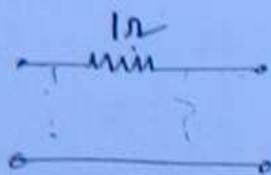
$$Z_{eq} = \frac{Z_I \cdot Z_{11}}{\gamma_{22}/\gamma_{11}}$$

Q. When two, 2-port n/w's are connected in parallel, find eq. Y parameters.

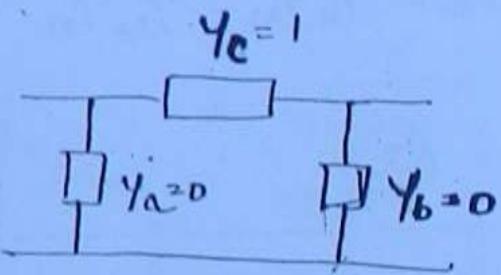


(196)

Soln.



\approx



$$Y_u^u = Y_a + Y_c = 1$$

$$Y_{22}^u = Y_b + Y_c = 1$$

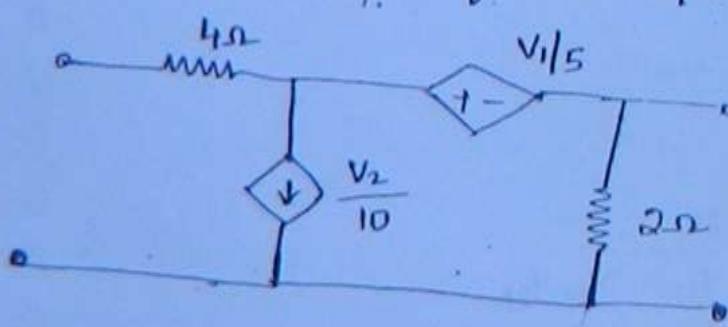
$$Y_{12}^u = Y_{21}^u = -Y_c = -1$$

$$Y^u = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

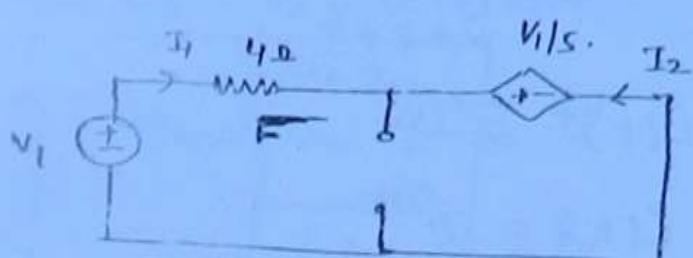
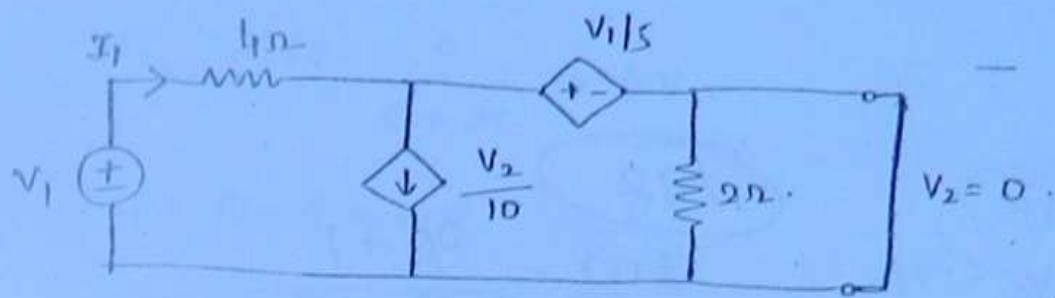
$$[Y] = \left[Y' \right] + \left[Y^u \right] = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 3 & 2 \\ 3 & 6 \end{bmatrix}$$

Q. Find B & D of the n/w shown.



$$V_1 = A V_2 - \dots$$



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$$I_1^2 - I_2 \Rightarrow$$

$$D = \frac{-I_1}{I_2} = 1$$

$$\underline{D = 1}$$

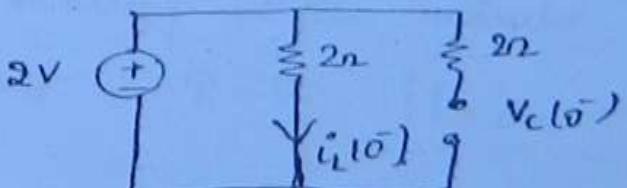
$$V_1 - I_1 - V_1/5 = 0 \Rightarrow V_1 + 4I_2 - V_1/5 = 0$$

$$\frac{4V_1}{5} = -4I_2 \Rightarrow \frac{V_1}{I_2} = -5$$

$$\underline{B = -5}$$

transients ω_B

$$t < 0^-$$



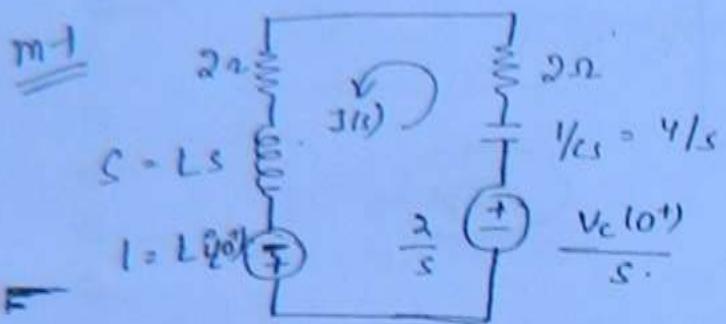
$$i_L(0^-) = \frac{2}{2} = 1A$$

$$v_c(0^-) = 2V$$

$$t > 0^+, \quad v_c(0^+) = 2V$$

$$i_L(0^+) = 1A$$

S-domain



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$$I(s) = \frac{2/s + 1}{2 + 2 + 4/s + s}$$

$$i_c(t) = e^{-t/\zeta_2}$$

$$i(t) = e^{-2t}$$

$$\gamma = 1/\zeta_2 = 0.5$$

(b)

m-2

$$\text{Time const.} = \frac{2L}{R} = \frac{2 \times 1}{4} = 1/2 \text{ sec}$$

current in inductor is decaying \therefore not oscillatory.

8. i) In the above circuit energy transformation is continuously done b/w inductor & capacitors. thereby output response is oscillatory.

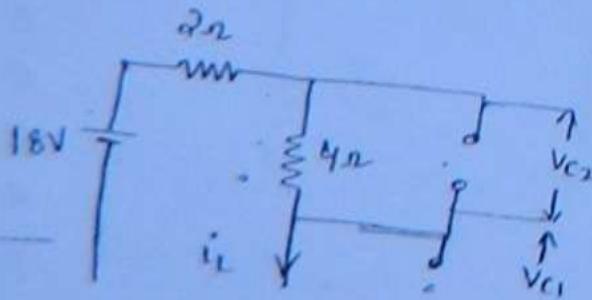
ii) In the above circuit no energy loss is present.
 \therefore resistance $= 0$.

Ans: (b)

$$I = \frac{1}{s} \cdot \frac{1/s}{1/2 + 1/s} = \frac{1}{s} \cdot \frac{1}{s+2}$$

$$V = T \cdot 1/2 = 0.1 V$$

4.

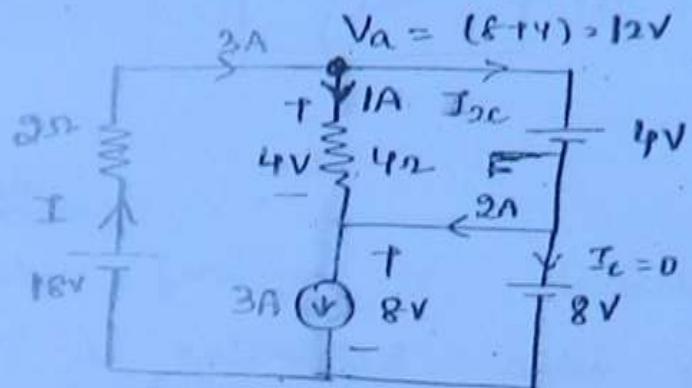


$$i_L = \frac{18}{2+4} = 3A$$

$$-V_{C_2} = 12 \cdot \frac{c}{c+2c} = 4V$$

$$V_{C_1} = 12 - 4 = 8V$$

199



$$\beta = 1 + I_{2c}$$

$$I_2 = 2\Omega$$

$$L \rightarrow \frac{18 - 12}{2} = 3A$$

$$V = IR + L \frac{di}{dt} \Rightarrow 18 = 6 \times 1 + L(6)$$

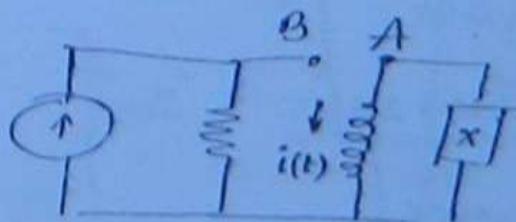
$$H(s) = \frac{V_o(s)}{V_i(s)} \Rightarrow V_o(s) = \frac{1}{(s-2)(s-3)}$$

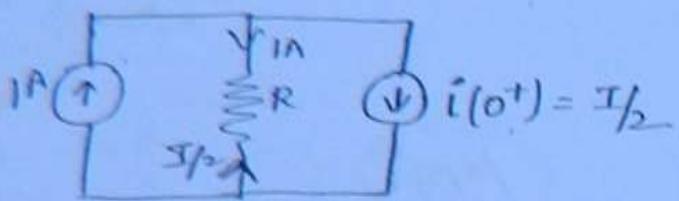
Initial & final value theorems.

at $t = \infty$, energy in the inductor is totally dissipated to current in $R=0$. $I_{t=\infty}$

$\hat{V}(t) = I \sin(\omega t + \varphi_0)$
 current leads $\xrightarrow{\text{dead angle}}$ $X = \text{Capacitor}$

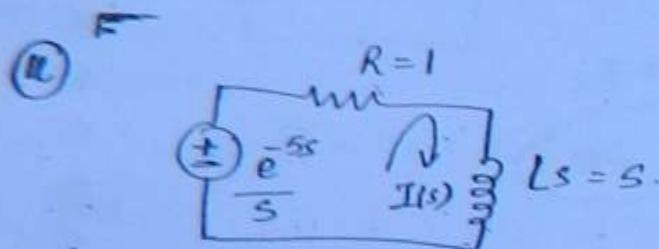
$$i(0^+) = \frac{\pi}{\omega}$$





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$$\frac{I}{2} - 1 = 1 \Rightarrow I = 4A.$$



12. $H(s) = \frac{1}{s+1}, \quad V_i = \cos t,$

$$H(j\omega) = \frac{1}{j\omega + 1} = \frac{V_o}{V_i} \Rightarrow \frac{1}{j+1} = \frac{1}{\sqrt{2}} \angle -45^\circ = \frac{V_o}{V_i}$$

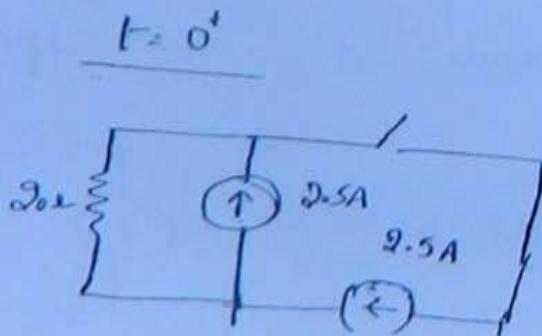
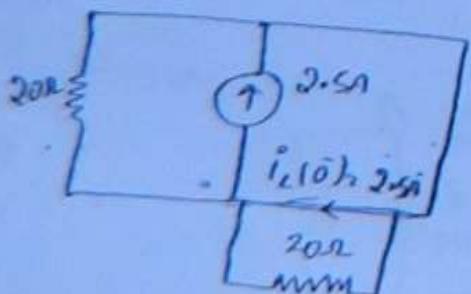
$$V_o = \frac{V_i}{\sqrt{2}} \angle -45^\circ = \frac{1}{\sqrt{2}} \omega_0 (t - 45^\circ)$$

13. $V_R = V_L = V_{f2}, \quad V_L = V_{f2}.$

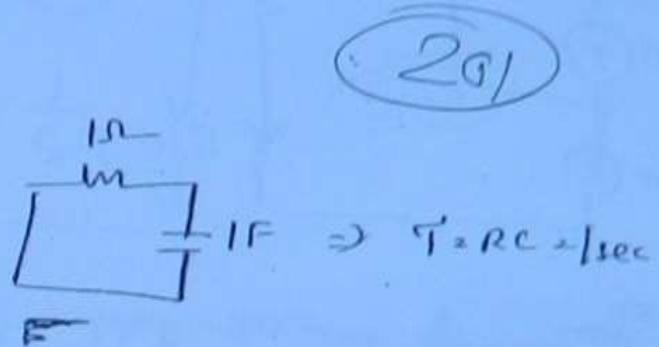
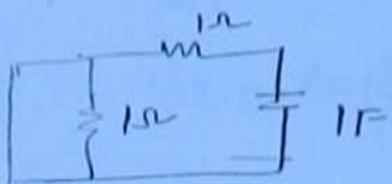
$$V e^{-Rt/L} = V_{f2} \Rightarrow e^{-t/\tau} = \frac{1}{2},$$

$$t = -\tau \ln(1/2).$$

14. $t=0^-$



$$V_{n2} = -(2.5) \times (20) = -50V$$



Note:

while finding γ deactivate all independant sources.
 γ is calculated after operating the switch.

$$t=0^+$$

$$iR + L \frac{di}{dt} + E_2 = 0$$

$$-8R + 2(3) - 4R = 0$$

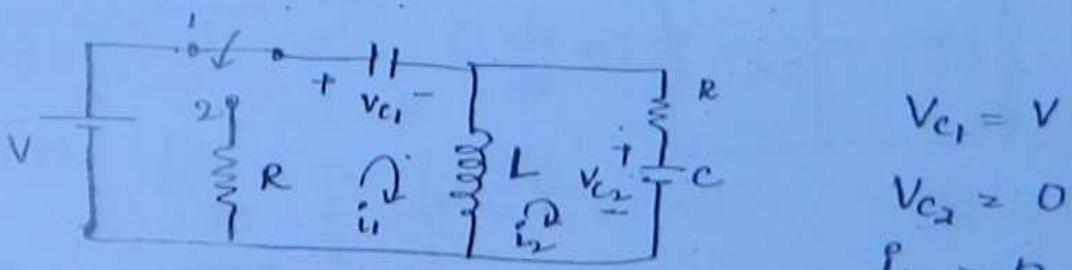
$$R = 0.5 \Omega$$

$$t=\infty, L \rightarrow \infty$$

$$E_2 = -i(\infty)R$$

$$E = -4R$$

$$V(t) = (C_1 + C_2 t) e^{-\gamma t} \Rightarrow \text{critically damped}$$



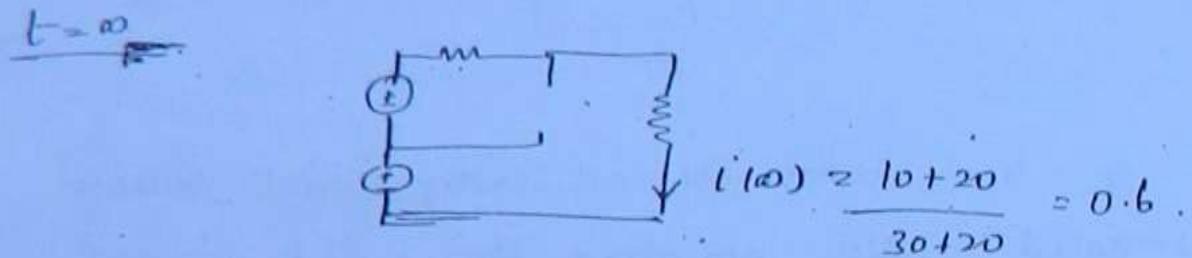
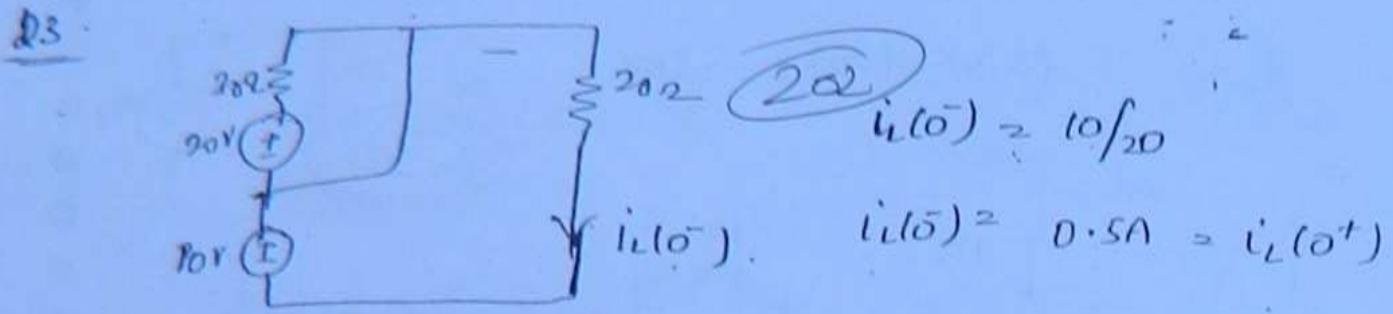
$I_C = \frac{V}{R+R}, \frac{V}{2R}$

$i_{L2} - I_C = \frac{-V}{2R}$

$i_L = i_1 - i_2$

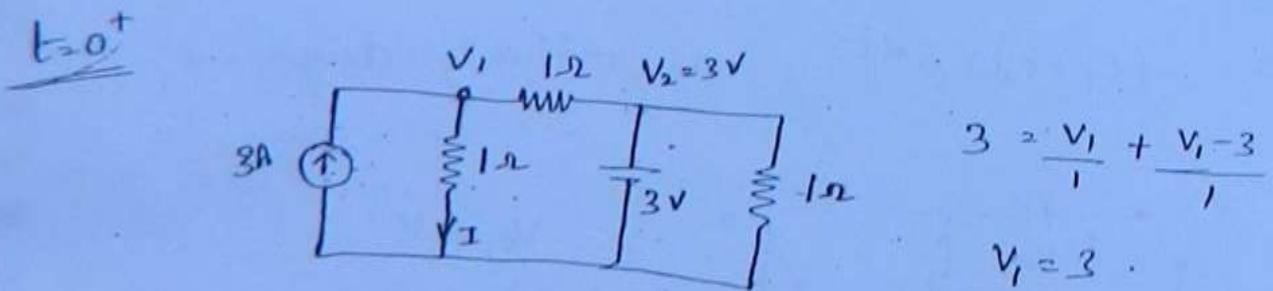
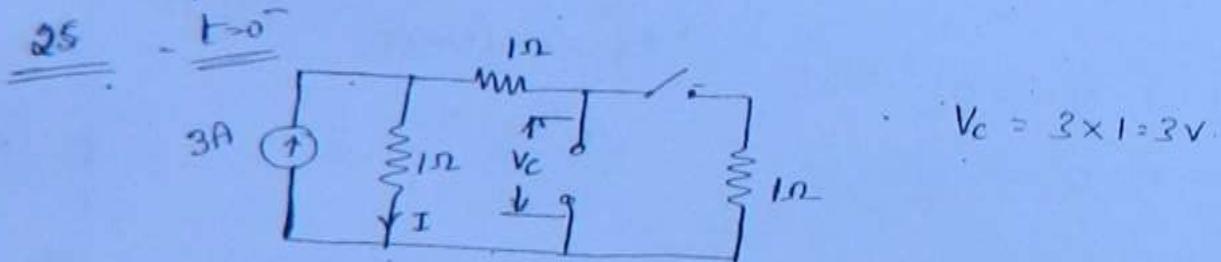
$= \frac{-V}{2R} - \left(\frac{-V}{2R} \right)$

$i_L = 0$



$$\gamma = \frac{1}{50}$$

$$i(t) = 0.6 - 0.1 e^{-50t}$$



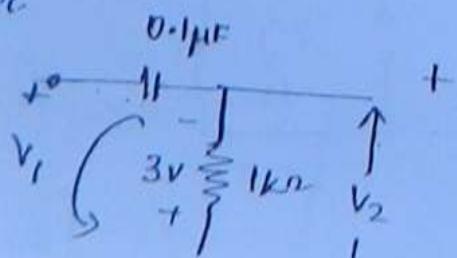
Q6.

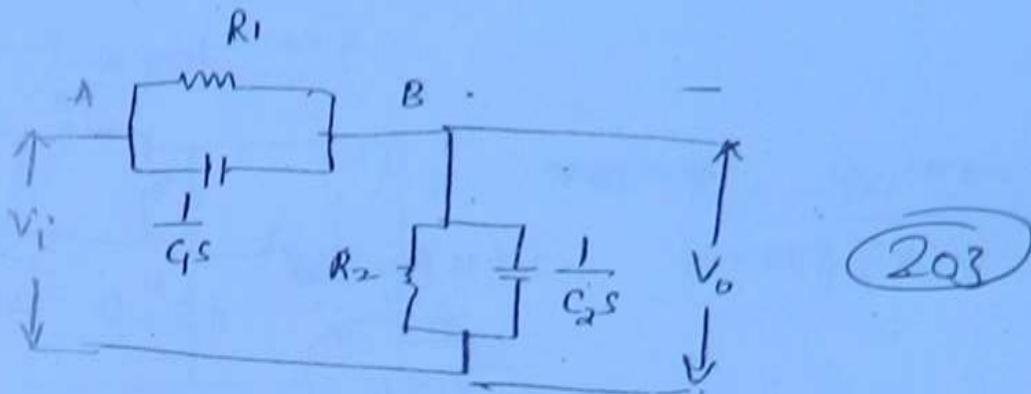
$\gamma = \frac{1}{RC}$

$I = \frac{V_1}{1} = \frac{3}{1} = 3A$

$$= 10^3 \times 10^{-9} \rightarrow \gamma = 10^{-6} \text{ sec}^{-1}$$

$$V_2 = -3V$$

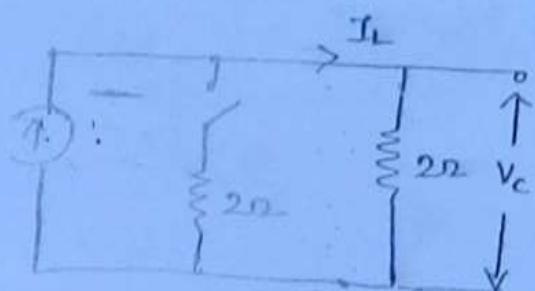




(203)

Transform in s domain & apply voltage division
ans. (c).

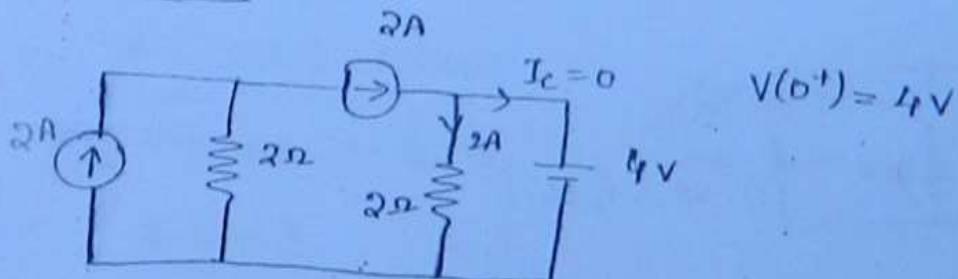
$t=0^-$



$$V_c = 2 \times 2 = 4 \text{ V}$$

$$I_L = 2 \text{ A}$$

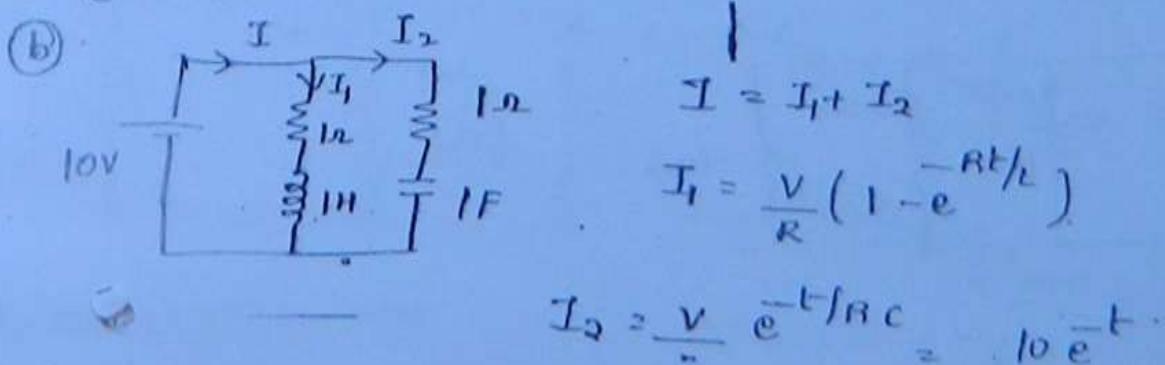
$t=0^+$



$$I_c = C \frac{dv}{dt} \quad (b)$$

$$i(t) = [i(0^+) - i(\infty)] e^{-t/\tau_{RC}} + i(\infty) \quad (b)$$

(d) 3V.

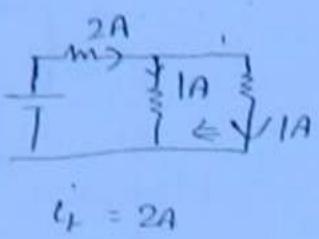


40.

(d)

$$\frac{V_1}{V_2} = \frac{10}{15} \Rightarrow \frac{V_1}{V_2} = \frac{2}{3}$$

$$V = i [10 + 5] = 2 [15] = 30V$$



42.

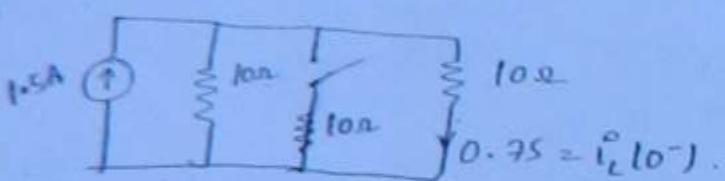
(a) $Z_{D1} = 20\Omega$

(b) $Y_{D2} = \frac{-Z_{12}}{Z_1 Z_{22} - Z_{12} Z_{21}} = 0.2$

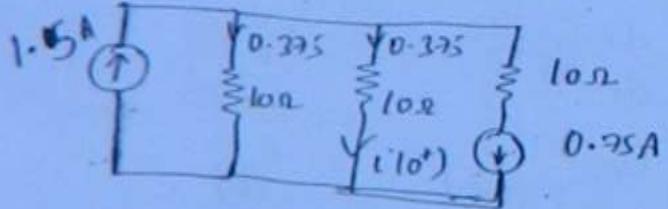
(c) $h_{12} = \left. \frac{V_1}{V_2} \right|_{I_2=0} = \frac{Z_{12}}{Z_{22}} = 20/10 = 2.0$

(d) $A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad \left(A = \frac{Z_1}{Z_{21}} \right)$

43.

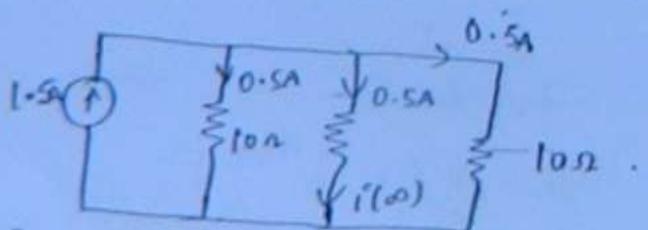


t = 0^+



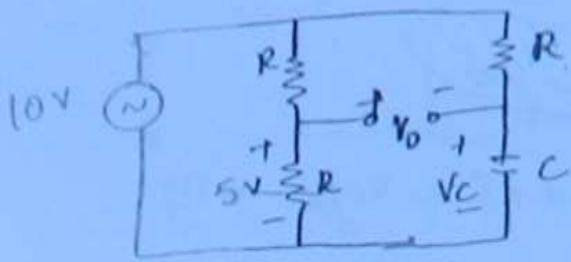
$$i_L(0^+) = 0.75A$$

t = ∞



(a)

Two port $\omega \cdot B$



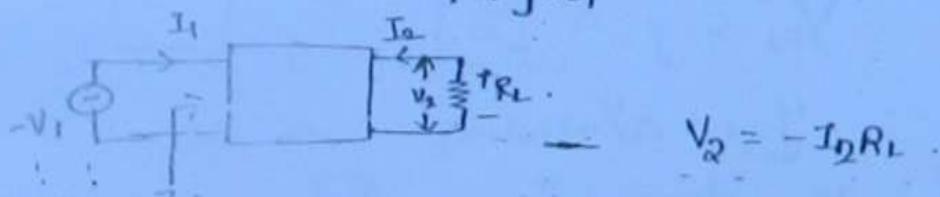
$$V_c = \frac{-jx_c}{R-jx_c}$$

(205)

$$-5 + V_o + V_c = 0$$

$$\Rightarrow 5 - V_c \Rightarrow V_o = 5 + \frac{10jx_c}{R-jx_c}$$

$$|V_o| = 5 \frac{|R+jx_c|}{|R-jx_c|} \Rightarrow |V_o| = 5V$$



$$V_2 = -I_2 R_L$$

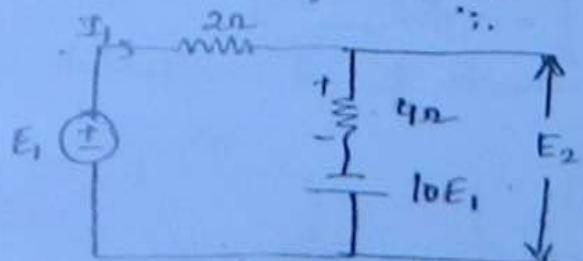
$$Z_{in} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$Z_{in} = \frac{A(-I_2 R_L) - BI_2}{C(-I_2 R_L) - DI_2} = \frac{AR_L + B}{CR_L + D}$$

$$= \frac{12j10 + 2}{12j10 + 3} = 12/B$$

$$\boxed{Z_{in} = \frac{AR_L + B}{CR_L + D}, \quad Y_{in} = \frac{1}{Z_{in}} = \frac{CR_L + D}{AR_L + B}}$$

Addition of matrices (b)



$$Z_{21} = \left. \frac{E_2}{I_1} \right|_{I_2=0}$$

$$Z_{11} = \left. \frac{E_1}{I_1} \right|_{I_2=0} = \frac{C}{I_1}$$

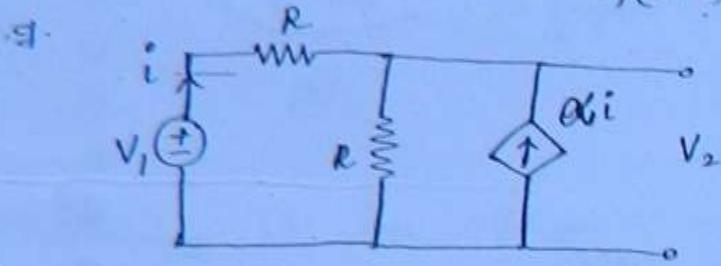
$$I_1 = \frac{E_1 + 10E_1}{2} \Rightarrow$$

$$E_1 = 6 I_1$$

5. $\Pi - T$ transform.

$$6. I(s) = \frac{V(s)}{Z(s)} = \frac{1/s}{(s+2)/(s+3)} =$$

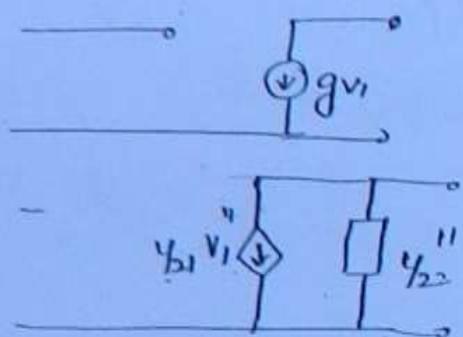
(206)



$$i + \alpha i = V_2/R \rightarrow \textcircled{1}$$

$$i = \frac{V_1 - V_2}{R} \rightarrow \textcircled{2}$$

8.



$$Y_{11} = Y_{22} = -Y_{12} = -Y_{21} = Y$$

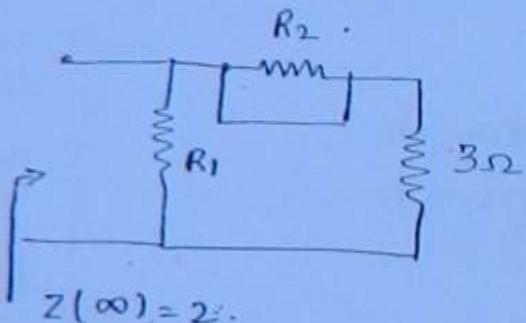
$$Y_{21}'' = g$$

$$Y_{21} = -Y$$

$$Y_{21} = Y_{21} + Y_{21}'' \quad \textcircled{c}$$

$$Y_{21} = -Y + g$$

12.



$$S \rightarrow \infty$$

$$X_C = 1/S_C = 0$$

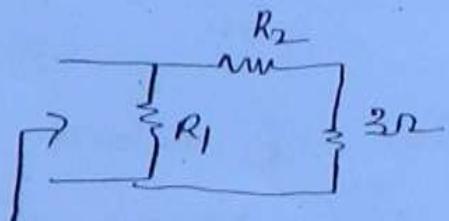
$C \rightarrow 0$. C.

$$Z = \frac{R_1 z}{3 + R_1} \Rightarrow R_1 = 6\Omega$$

$$S \rightarrow 0 \Rightarrow X_C = \infty \Rightarrow C \rightarrow 0$$

$$Z(0) = 3$$

$$Z = \frac{R_1(3 + R_2)}{R_1 + R_2 + 3} \quad \text{b} \quad R_2 = 3\Omega$$



$$\frac{V_C}{V_i} = \frac{1/100\mu s}{10k + 10m s + 1/100\mu s}$$

(d).

13.

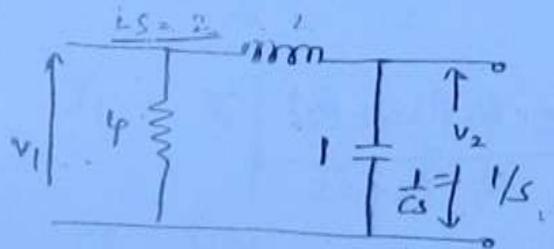
$$\begin{aligned}
 & \text{Left side: } Y_a = \frac{1}{Ls} = \frac{1}{Ls} \\
 & \text{Middle: } Y_L = 1 \\
 & \text{Right side: } \frac{1}{Cs} = \frac{1}{Cs}, \quad Y_b = 3
 \end{aligned}$$

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$$Y_{11} = Y_a + Y_c = \dots$$

$$Y_{22} = Y_s + Y_c.$$

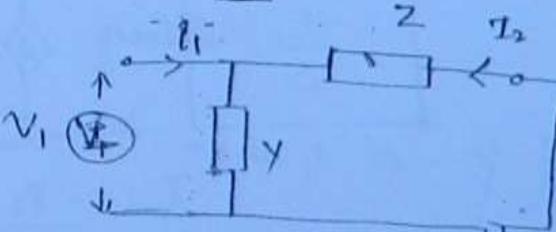
$$Y_{12} = Y_{21} = -Y_c.$$



$$\begin{aligned}
 V_2(s) &= V_1(s) \cdot \frac{\frac{1}{s}}{2s+1/s} \\
 &= \frac{1}{1+2s^2}
 \end{aligned}$$

④ \rightarrow i.e. product @.

$$D^{Y_2} = \frac{I_1}{I_2} \mid_{V_2=0}$$



$$I_2 = \frac{-I_1 / Y}{Z + Y Y} \Rightarrow$$

$$D = 1 + Y Z.$$

$\Pi \rightarrow T$.

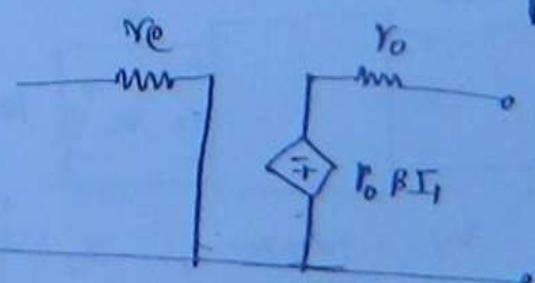
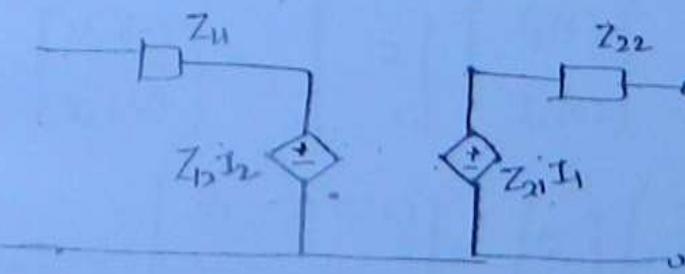
$$Y_{11} = 5, \quad Y_{12} = Y_{21} = -1, \quad Y_{22} = 1.$$

$$Y_1 = Y_0 + Y_{12}$$

$$Y_2 = Y_{22} + Y_{12}$$

$$Y_3 = -Y_{12} = -Y_{21}.$$

transform to s domain



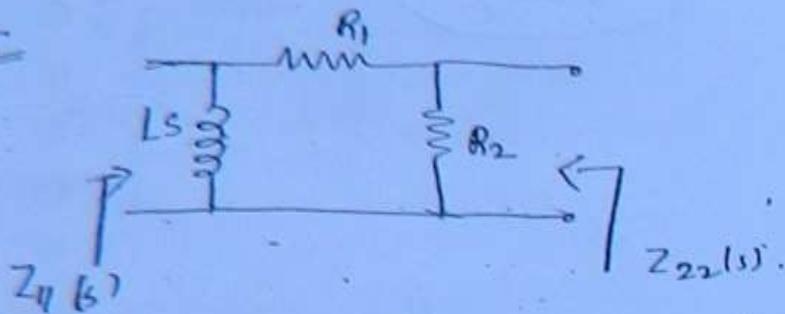
27

$$Z_1 = Z_{11} - Z_{12}$$

$$Z_2 = Z_{22} - Z_{12}$$

$$Z_3 = Z_{12} = Z_{21}$$

28

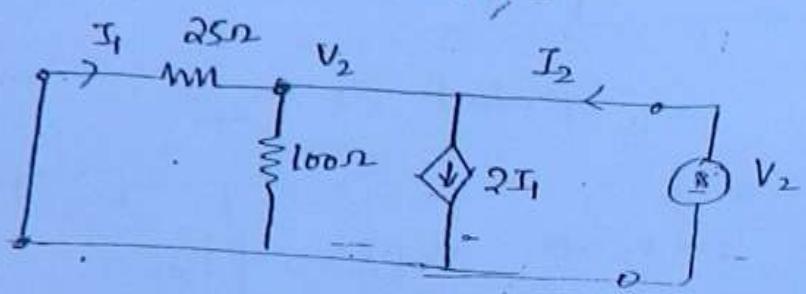


28

$$Z_{11} = \frac{Ls(R_1 + R_2)}{R_1 + R_2 + Ls}, \quad Z_{22}(s) = \frac{R_2(Ls + R_1)}{R_1 + R_2 + Ls}$$

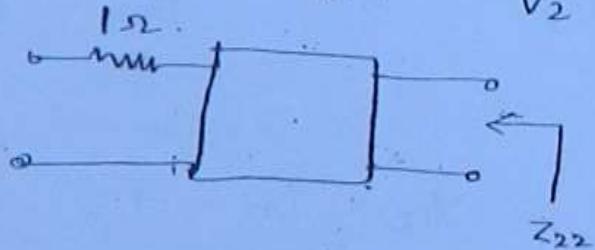
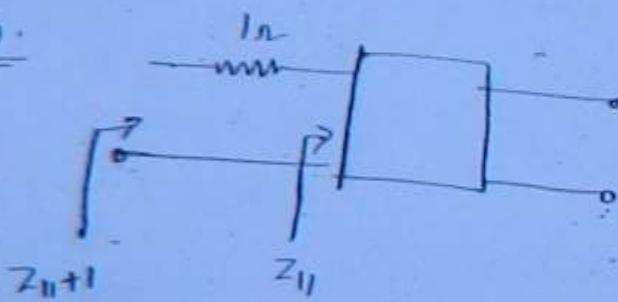
29

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$



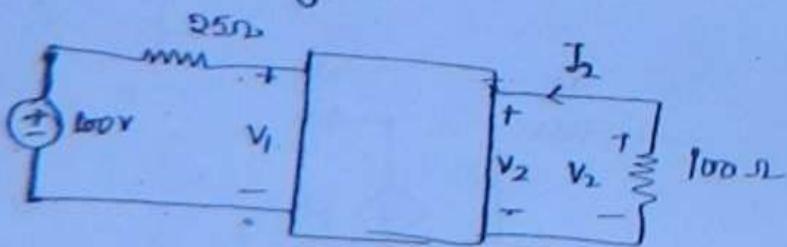
$$I_1 = -\frac{V_2}{25} \Rightarrow \frac{I_1}{V_2} = -0.04$$

30



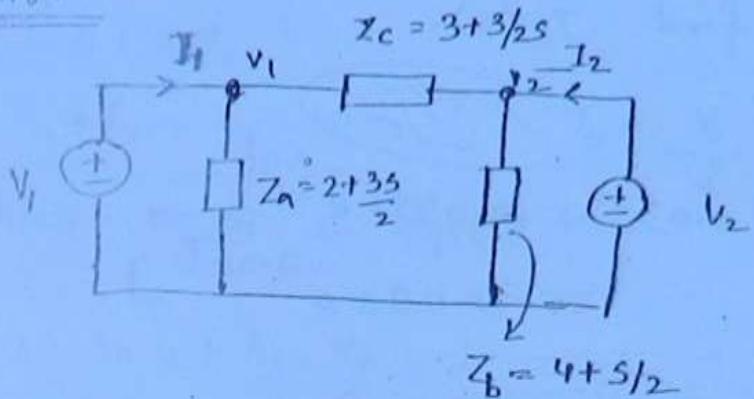
* whenever a linear element is added, there will be no variation in $Z_{12} = \frac{V_1}{I_2}$ & $Z_{21} = \frac{V_2}{I_1}$ but Z_{11} will vary.

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$$V_{22} = 100I_2, \quad I_2 = -V_2/100$$

Conv.



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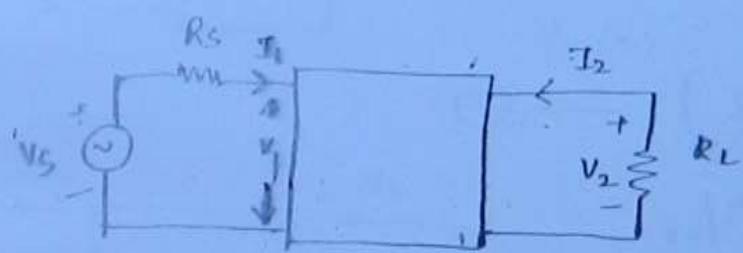
$$I_1 = \frac{V_1}{Z_A} + \frac{V_1 - V_2}{Z_C}$$

$$I_1 = V_1 \left[\frac{1}{Z_A} + \frac{1}{Z_C} \right] - \frac{V_2}{Z_C}$$

$$I_1 = V_1 Y_H + Y_{12} V_2$$

$$Y_H = \frac{1}{Z_A} + \frac{1}{Z_C}, \quad Y_{12} = \frac{1}{Z_C}$$

apply the same proc. at node 2. v_2. & find Y_{22}, Y_{21}



$$V_S = V_1 + I_1 R_S$$

$$V_D = \omega V_S e^{j\omega t} R_S$$

$$V_1 = V_S - I_1 R_S$$

$$V_2 = -I_2 R_L$$

$$V_2(s) = -I_2(s) \cdot 1$$

$$V_1(s) = \frac{1}{s} - \omega I_1(s)$$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} Z \\ Z \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$\int \left[\frac{V_S - \omega I_1(s)}{s} \right] ds = \int Z \int I_1(s) ds$$

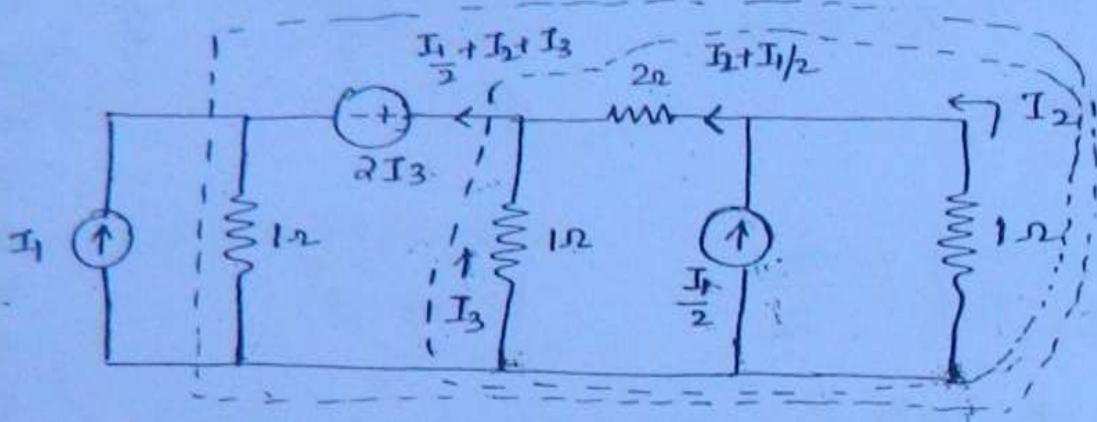
Solve the above matrix to find $I_2(s)$.

$$V_2(s) = -I_2(s)$$

(210)

-1.9t

$$V_2(t) = L^{-1}[V_2(s)] = \left[0.037 + 0.0456e^{-7.08t} - 0.083 e^{-1.9t} \right].$$



$$1 \times I_2 + 2\left(I_2 + \frac{I_1}{2}\right) + 2I_3 + 1\left(\frac{3I_1}{2} + I_2 + I_3\right) = 0 \rightarrow \textcircled{1}$$

$$(1 \times I_2) + 2\left(I_2 + \frac{I_1}{2}\right) - (I_2 \times 1) = 0 \rightarrow \textcircled{2}$$

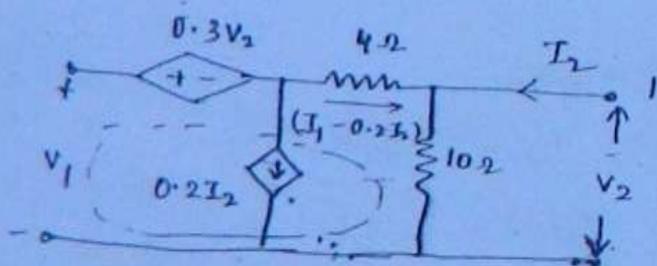
from eq. \textcircled{2}

$$I_3 = I_2 \times 1 + 2\left(I_2 + \frac{I_1}{2}\right). \rightarrow \textcircled{3}$$

Sub eq. \textcircled{3} in \textcircled{1}

$$\text{Ans} = -11/26$$

4.



$$V_1 = 0.3V_2 + 4(I_1 - 0.2I_2) + V_2 \rightarrow \textcircled{1}$$

$$V_2 = 10(I_1 + 0.8I_2) \rightarrow \textcircled{2}$$

$$V_1 = Z_{T1} I_1 + Z_{T2} I_2$$

Sub \textcircled{2} in \textcircled{1}

$$V_1 = 17.5t + 9.6 \tau.$$

$$I_2 = 10 I_1 + 8 V_2$$

$$8 I_2 = V_2 - 10 I_1 \Rightarrow I_2 = \frac{V_2}{8} - \frac{10}{8} I_1 \rightarrow \textcircled{b}$$

Substitute eq. \textcircled{b} in eqn \textcircled{a}.

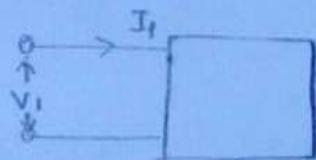
(24)

$$V_1 = h_{11} I_1 + h_{12} V_2$$

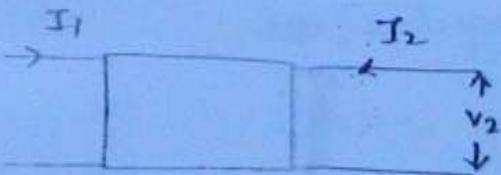
$$V_1 = 5 I_1 + 1.2 V_2 \rightarrow \textcircled{a} \Rightarrow h_{12} = 1.2$$

$$I_1 = \frac{V_1}{5} - \frac{1.2 V_2}{5} \Rightarrow Y_{12} = -\frac{1.2}{5} = -0.24$$

NETWORK FUNCTIONS



$$\begin{aligned} Z_{11}(s) &= \frac{V_1(s)}{I_1(s)} \\ Y_{11}(s) &= \frac{I_1(s)}{V_1(s)} \end{aligned} \quad \left. \begin{array}{l} \text{Immittance} \\ \text{functions} \end{array} \right\}$$



$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}, \quad Y_{11}(s) = \frac{I_1(s)}{V_1(s)} \quad \text{Driving pt. admittance function}$$

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}, \quad Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

$$Z_{12}(s) = \frac{V_1(s)}{V_2(s)} \quad \text{Impedance Transfer ratio}$$

$$Z_{21}(s) = V_1(s)$$

3. $\frac{Y_{11}(s)}{V_1(s)} = \frac{I_1(s)}{V_1(s)}$ admittance transfer ratio

(22)

4. $G_{12} = \frac{V_1(s)}{V_2(s)}$ voltage transfer ratio

$$G_{12} = \frac{V_2(s)}{V_1(s)}$$

5. $\alpha_{12} = \frac{I_1(s)}{I_2(s)}$ current transfer ratio

$$\alpha_{21} = \frac{I_2(s)}{I_1(s)}$$

→ Parameters are calculated at predefined conditions (either o.c or s.c).

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}$$

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}$$

→ To calculate network function, no predefined conditions are required.

→ To design the n/w simultaneously 4 parameters are required.
i.e. $Z_{11}, Z_{12}, Z_{21}, Z_{22}$.

→ By using only single n/w function, it is possible to design complete n/w.

Network Synthesis:

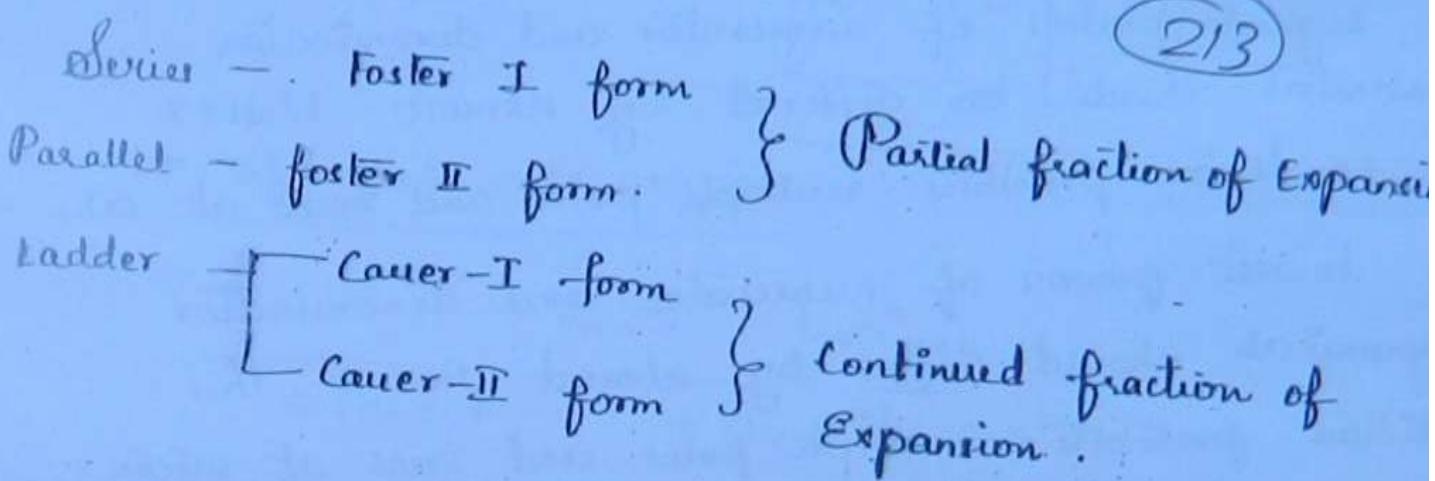
→ In the n/w synthesis for a given function, n/w is designed.

Single port → $Z(s), Y(s)$ } admittance

Two port } l. $Z_{11}(s), Z_{22}(s), Y_{11}(s), Y_{22}(s)$ - driving pt
immittance func.

In the network synthesis, for a given function, it is possible to design the following networks.

(213)



$F(s)$ should be PRF (the real func.).

$$R \geq 0, \quad L \geq 0, \quad C \geq 0.$$

If $F(s)$ is PRF, $\frac{1}{F(s)}$ is also PRF.

If $F_1(s)$ & $F_2(s)$ are PRF

$$F(s) = F_1(s) + F_2(s) \rightarrow \text{PRF}$$

$$F(s) = F_1(s) - F_2(s)$$

$$(a) \quad F_1(s) > F_2(s) \rightarrow \text{PRF}$$

$$(b) \quad F_1(s) < F_2(s) \rightarrow \text{-ve.}$$

All the poles of the function should be present in the left half of the plane.

Imaginary poles and zeroes should be conjugate pair.

In the partial fraction of Expansion, residue should be the real.

→ Numerator and denominator polynomial should satisfy Hurwitz criteria.

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→ The highest power of numerator and denominator polynomial should be different by atmost UNITY.

This condition prohibits multiple poles and zeros at ∞ .

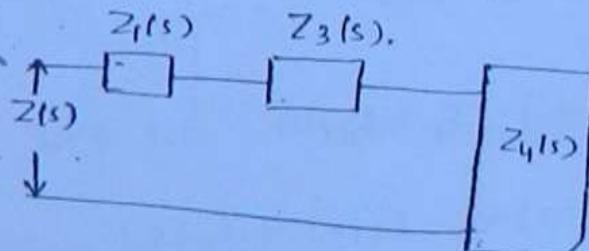
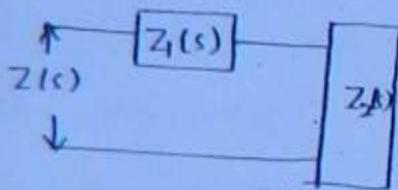
→ The lowest power of numerator and denominator polynomial should differ by atmost UNITY. This condition prohibits multiple poles and zeros at origin.

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z_3(s) + Z_4(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$

$$Z_4(s) = Z_2(s) - Z_3(s)$$



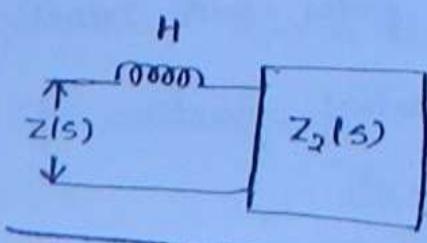
1. Removal of pole at ∞

$$Z(s) = \frac{b_{n+1} s^{n+1} + b_n s^n + \dots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_0}$$

$$Z(s) = \frac{b_{n+1} s^{n+1}}{a_n s^n} + Z_2(s) \quad \left(H = \frac{b_{n+1}}{a_n} \right)$$

$$Z(s) = H(s) + Z_2(s)$$

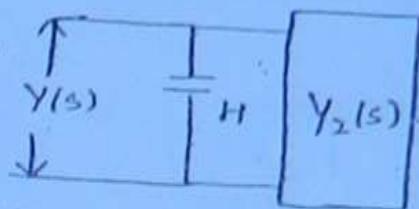
$$Z_2(s) = Z(s) - H(s)$$



$$Y(s) = Hs + Y_2(s)$$

$$B_C = SC$$

$$Y_2(s) = Y(s) - Hs$$



(215)

Removal of pole at origin

$$Y(s) = \frac{b_0 + \dots + b_{n-1}s^{n-1} + b_n s^n}{s(a_0 + \dots + a_{n-1}s^{n-1} + a_n s^n)}$$

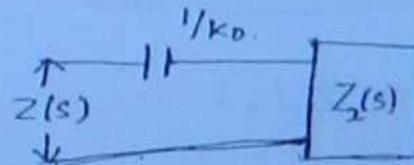
$$Z(s) = \frac{b_0}{sa_0} + Z_2(s) \quad \left(\frac{b_0}{a_0} = k_0 \right)$$

$$Z(s) = \frac{k_0}{s} + Z_2(s)$$

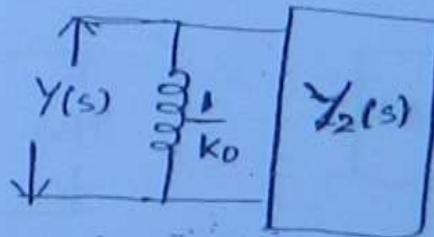
$$Z_2(s) = Z(s) - \frac{k_0}{s}$$

for admittance func.

$$B_L = 1/Ls$$



$$Y(s) = \frac{k_0}{s} + Y_2(s)$$



$$Y_2(s) = Y(s) - \frac{k_0}{s}$$

Removal of Conjugate pair of poles

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$

$$Z_1(s) \rightarrow \text{poles} \rightarrow \pm j\omega$$

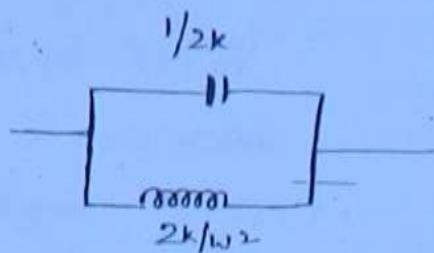
$$Z_1(s) = \frac{k_1}{s+j\omega} + \frac{k_2}{s-j\omega} \quad (k_1 = k_2 = k) \quad \text{Ans}$$

$$Z_1(s) = \frac{k}{s^2 + \omega^2} + k$$

$$Z_1(s) = \frac{2ks}{s^2 + \omega^2}$$

(2) b

$$Z_1(s) = \frac{1}{\frac{s^2}{2k} + \frac{\omega^2}{2ks}} = \frac{1}{\gamma_a + \gamma_b}$$

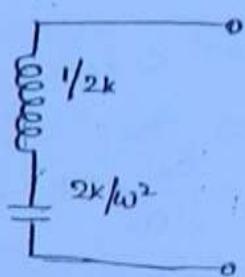


Imp with admittance function,

$$\begin{aligned} Y_1(s) &= \frac{1}{s^2 + \omega^2} \\ &= \frac{1}{2k} \end{aligned}$$

$$\begin{aligned} Y_2(s) &= \frac{1}{s^2 + \omega^2} \\ &= \frac{1}{2k} \end{aligned}$$

$$Y(s) = \frac{1}{\frac{s^2}{2k} + \frac{\omega^2}{2ks}} = \frac{1}{Z_a + Z_b}$$



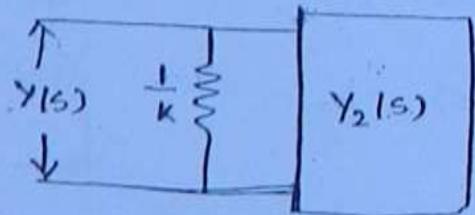
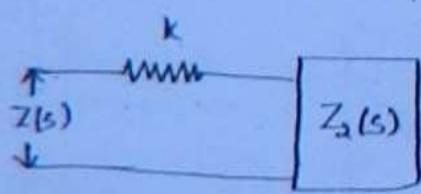
4. Removal of constant

$$Z(s) = k + Z_2(s)$$

$$Y(s) = k + Y_2(s)$$

$$Z_2(s) = Z(s) - k$$

$$Y_2(s) = Y(s) - k$$

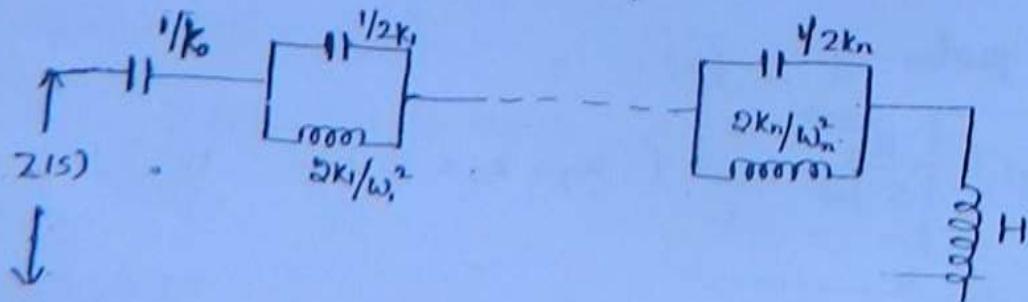


L C n/ω - Foster I form (Series)

$$X_C = 1/\omega C$$

$$Z(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{2k_i s}{s^2 + \omega_i^2} + H$$

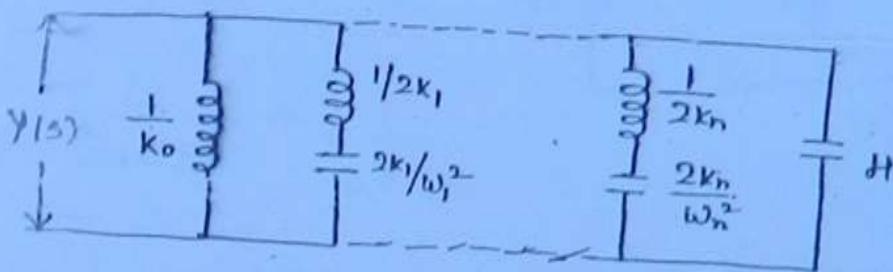
$$X_L = \omega L$$



Lc n/o Foster-II form [Parallel]

$$Y(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{2k_i s}{s^2 + \omega_i^2} + H(s)$$

(217)



Obtain Z(s) from Foster I & II form of Y(s)

$$Z(s) = \frac{(s^2+2)(s^2+4)}{s(s^2+3)}$$

$\rho \rightarrow \infty$

$\rho \rightarrow \text{origin}$

$\rho \rightarrow \text{conjugate pair}$

$$Z(s) = \frac{k_0}{s} + \frac{2ks}{s^2 + \omega^2} + Hs$$

$$\frac{(s^2+2)(s^2+4)}{s(s^2+3)} = \frac{k_0}{s} + \frac{2ks}{s^2 + \omega^2} + H(s)$$

$$k_0 = 8/3$$

$$2\omega = 4/3$$

$$H = 1$$

$$\frac{2s^2+8}{8(s^2+3)} + b$$

$$\frac{1}{s} - \frac{1}{s^2+3}$$

$$\frac{s^4 + 6s^2 + 8}{s^2 + 3s}$$

$$s^3 + 3s^2 - 8s - 6s^2 - 8s^2 - 8s$$

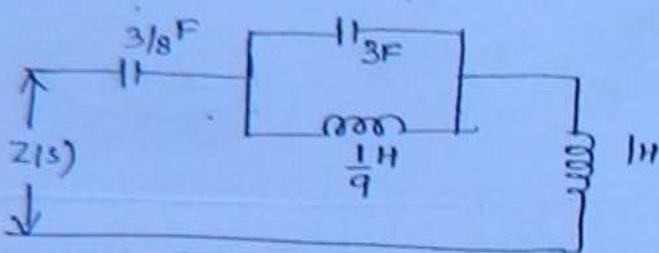
$$\frac{s^4 + 3s^2}{s^2 + 8s}$$

$$Z(s) = \frac{8/3}{s} + \frac{4/3}{s^2+3} + s$$

$$Z(s) = \frac{1}{s} + 1$$

$$\begin{aligned} k_0 &= 8 \\ \omega &= 2\sqrt{3} \\ 2\omega &= 4\sqrt{3} \\ 2\omega &\approx 11.3 \end{aligned}$$

$$B_c = sc - \left(\frac{1}{Y_A + Y_B} \right) .$$



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Foster-II form.

$$Z(s) = \frac{(s^2 + 3)(s^2 + 4)}{s(s^2 + 3)} \Rightarrow Y(s) = \frac{s(s^2 + 3)}{(s^2 + 2)(s^2 + 4)}$$

Note: (1) If $F(s)$ is given, same function is utilized to obtain Foster I & IInd form.

(2) If $Z(s)$ is given, to obtain Foster IInd form, $\frac{1}{Z(s)}$ function is used.

(3) If $Y(s)$ is given, to obtain Foster Ist form, $\frac{1}{Y(s)}$ function is used.

$$Y(s) = \frac{s(s^2 + 3)}{(s^2 + 2)(s^2 + 4)}$$

no pole at ∞
" " " " origin
conjugate pair of poles = 2.

$$Y(s) = \frac{2k_1 s}{s^2 + \omega_1^2} + \frac{2k_2 s}{s^2 + \omega_2^2}$$

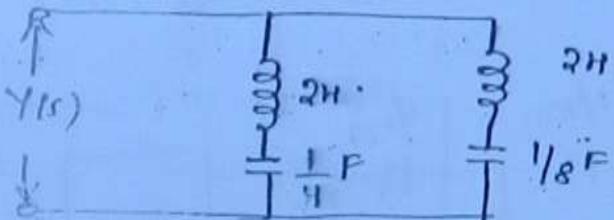
$$\frac{s(s^2 + 3)}{(s^2 + 2)(s^2 + 4)} = \frac{2k_1 s}{s^2 + 2} + \frac{2k_2 s}{s^2 + 4}$$

$$2k_1 \rightarrow 1/2, 2k_2 \rightarrow 1/2$$

$$Y(s) = \frac{s/2}{s^2+2} + \frac{s/2}{s^2+4}$$

$$Y(s) = \frac{1}{\frac{s^2}{s/2} + \frac{2}{s/2}} + \frac{1}{\frac{s^2}{s/2} + \frac{4}{s/2}} \Rightarrow \frac{1}{2s+4} + \frac{1}{2s+8}$$

$$H(s) = \frac{1}{Z_a + Z_b} + \frac{1}{Z_c + Z_d} \quad X_L = Ls \quad X_C = 1/cs$$



C n/w Cauer-T form. (ladder)
(n > d)

Removal of pole at ∞ .

$$Z_1(s) = Z(s) - H_1 G(s)$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - H_2 s$$

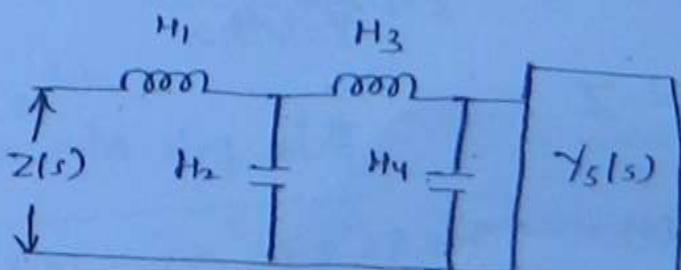
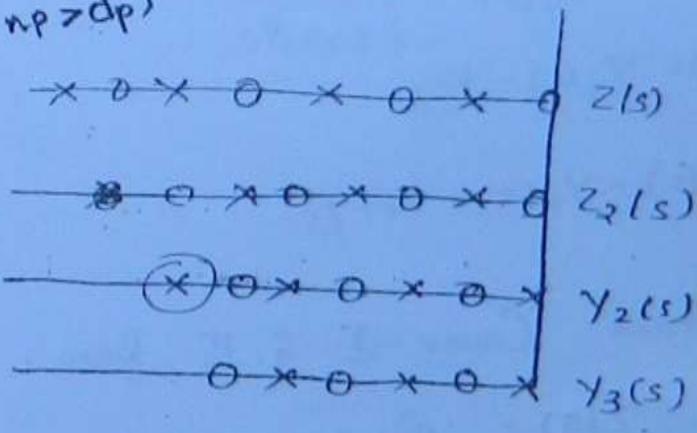
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - H_3 s$$

$$Y_4 = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - H_4 s$$

$$Y_6(s) = H_5(s) + Y_5(s) \dots$$



LC n/w, Cauer-II form.

Removal of pole at origin.

$$Z_2(s) = Z(s) - \frac{k_{01}}{s}$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - \frac{k_{02}}{s}$$

$$Z_3(s) = \frac{1}{Y_3(s)}$$

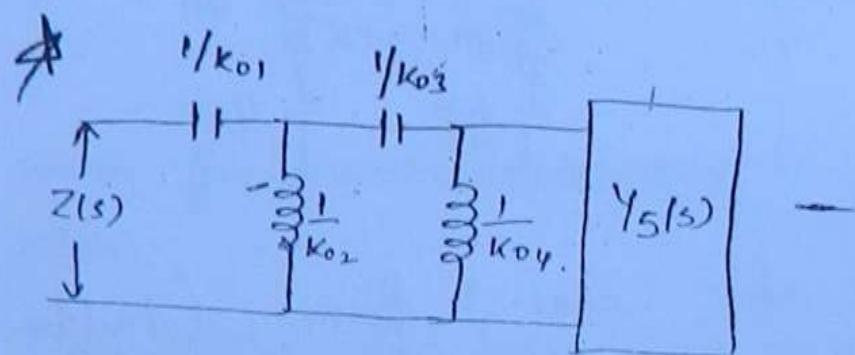
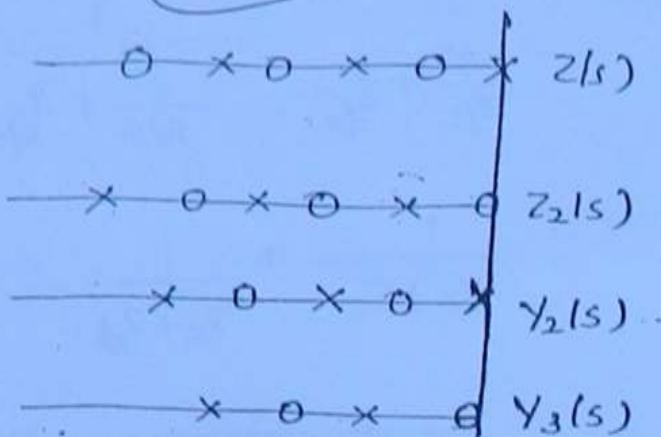
$$Z_4(s) = Z_3(s) - \frac{k_{03}}{s}$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - \frac{k_{04}}{s}$$

$$Y_4(s) = Y_5(s) + \frac{k_{04}}{s}$$

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Q. obtain Cauer-I & II form.

$$Z(s) = \frac{s^3 + 2s}{s^4 + 5s^2 + 4}$$

s.d.

Cauer-I

No pole at ∞ , \therefore consider $Y(s)$

$$Y(s) = \frac{s^4 + 5s^2 + 4}{s^3 + 2s}$$

$$s^5 + 8s^4 \left(s \right) \xrightarrow{C_1} B_C = SC$$

$$\begin{array}{c} s^4 + 2s^3 \\ \hline 3s^4 + 4 \end{array} \left(s^3 + 2s \right) \xrightarrow{L_1} Z \quad \text{221}$$

$$\begin{array}{c} s^3 + 4s/3 \\ \hline 2s/3 \end{array} \left(3s^2 + 4 \right) \left(s/6 \right) \xrightarrow{C_2} Y$$

$$\begin{array}{c} 4 \\ \hline 2s/3 \end{array} \xrightarrow{L_2} Z$$

$$\begin{bmatrix} B_H(z) & \frac{1}{6} H(z) \\ \text{from} & \text{from} \\ z(s) & \left[\begin{array}{c} s \\ z \end{array} \right] \frac{9}{2} F(y) \end{bmatrix}$$

$$Z(s) = \frac{1}{s + \frac{1}{\frac{s}{3} + \frac{1}{\frac{9s}{2} + \frac{1}{s/6}}}} \Rightarrow \frac{s^3 + 2s}{s^4 + 5s^3 + 4}$$

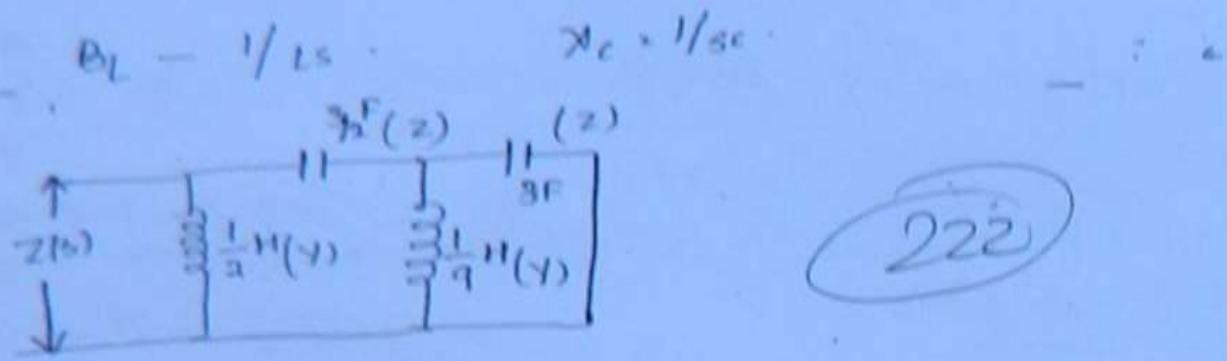
inner - di form $\rightarrow P \rightarrow \text{origin}$

$$Z(s) = \frac{s^3 + 2s}{s^4 + 5s^3 + 4}$$

$$Y(s) = \frac{s^4 + 5s^3 + 4}{s^3 + 2s} ; \quad Y(s) = \frac{s^4 + 5s^3 + 4}{s(s^2 + 2)}$$

$$\begin{array}{c} (2s + s^3) \left(4 + 5s^2 + s^4 \right) \left(\frac{a}{s} \right) \xrightarrow{L_1} Y \\ 4 + 2s^2 \\ \hline 3s^2 + s^4 \end{array} \left(2s + s^3 \right) \left(\frac{1}{3s} \right) \xrightarrow{C_2} Z$$

$$\begin{array}{c} 2s^3 + 2s^2 + 1 \\ \hline 2s^3 + 2s^2 + 1 \end{array} \xrightarrow{L_2} Z$$

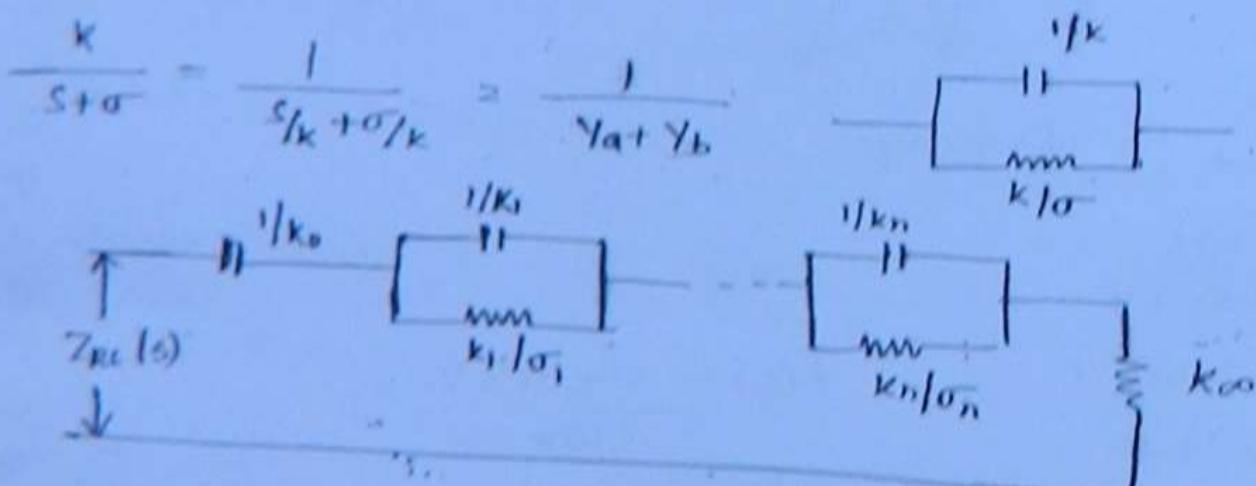


$$H_B(s) = \frac{1}{\frac{2}{s} + \frac{1}{\frac{2}{2s} + \frac{1}{\frac{2}{2s} + \frac{1}{\frac{2}{s} + \frac{1}{2s}}}}}$$

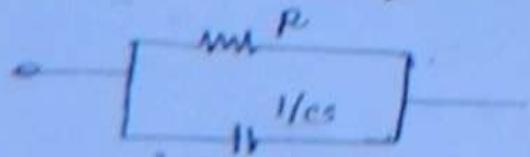
RC n/w Foster-I form (ω_{max})

$$Z_{RL}(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{2k_i s}{s + \omega_i} + R_L$$

$$Z_{RL}(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{k_i}{s + \sigma_i} + k_\infty$$



RC n/w Foster-II form



$$Z_{RL}(s) = \frac{R \cdot \sigma_{cs}}{\sigma} = R$$

$$Z_{Re} = \frac{R}{R + \frac{1}{Cs}} = \frac{1}{s + \frac{1}{Rs}}$$

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\rightarrow $Z_{Re}(s) = \frac{1}{s + \frac{1}{Rs}}$

$$Y_{Re}(s) = \frac{Cs}{s + \frac{1}{Rs}}$$

$$\Rightarrow Y_{Re}(s) = \frac{1}{s + \frac{1}{Rs}}$$

Pole zero pattern of $Z_{Re}(s)$ & $\frac{Y_{Re}(s)}{s}$ are identical.

$$\frac{1(s)}{G_i} = \frac{k_0}{s} + \sum_{i=1}^n \frac{k_i}{s + \sigma_i} + K_{\infty}$$

$$Y_{Re}(s) = k_0 + \sum_{i=1}^n \frac{k_i s}{s + \sigma_i} + s K_{\infty}$$

$$\frac{k_0}{s + \sigma_i} = \frac{1}{\frac{s}{k_i} + \frac{\sigma_i}{k_i}} = \frac{1}{Z_A + Z_B} \Rightarrow \frac{1}{\frac{s}{k_i} + \frac{\sigma_i}{k_i}} = \frac{1}{k_i / \sigma_i}$$

$$\frac{1}{k_0} \left\{ \sum_{i=1}^n \frac{1}{s + \sigma_i} \right\} = \left\{ \frac{1}{k_i} \right\}_{i=1}^n = \left\{ \frac{1}{k_i} \right\}_{i=1}^n - \left\{ \frac{\sigma_i}{k_i} \right\}_{i=1}^n = K_{\infty}$$

Properties of LC network

(224)

1. Either pole or zero should be present at origin.
 2. Either pole or zero should be present at ∞ .
 3. Poles and zeros are arranged alternately on $j\omega$ axis.
- In the partial fraction of expansion, residue should be -ve real.

5 ✓ $\frac{dx}{d\omega} > 0 \Rightarrow$ Slope is +ve

$$Z(s) = \frac{k_0}{s} + \sum_{i=1}^n \frac{2k_i s}{s^2 + \omega_i^2} + H_s$$

$$\hat{j}x = \frac{k_0}{j\omega} + \sum_{i=1}^n \frac{2k_i j\omega}{-\omega^2 + \omega_i^2} + j\omega H$$

$$\begin{aligned} Z &= \int (x_L - x_C) \\ Z &= \hat{j}x \end{aligned}$$

$$x = \frac{-k_0}{\omega} + \sum_{i=1}^n \frac{2k_i \omega}{-\omega^2 + \omega_i^2} + H\omega$$

$$\frac{dx}{d\omega} = \frac{k_0}{\omega^2} + \dots \Rightarrow \frac{dx}{d\omega} > 0$$

Properties of $Z_{RL}(s), Y_{RL}(s)$

1. Lowest critical frequency (1st critical frequency) is due to pole. It may be present either at origin or near origin.
- Poles and zeros are arranged alternately on -ve real axis.
- Highest critical freq. is due to zero, it may present either at ∞ (or) nearer to ∞ .

~~o x o x o *~~

$$1. \quad \frac{dZ_{RC}}{ds} < 0 \quad \text{slope is ve} \quad \frac{dY_{RL}}{ds} < 0$$

(225)

$$Z_{RC}(0) > Z_{RC}(\infty)$$

$$X_C = 1/s_C$$

$$Y_{RL}(0) > Y_{RL}(\infty)$$

$$S=0$$

$$S=\infty$$

$$X_C(0)=\infty \quad X_C(\infty)=0$$

Properties of $Y_{RC}(s)$ & $Z_{RL}(s)$.

$$Y_{RC} = K_0 + \sum_{i=1}^n \frac{k_i s}{s + \sigma_i} + K_\infty s.$$

Highest critical frequency is due to pole. It may be present at ∞ or near to ∞ .

Poles and zeros are arranged alternately on the real axis. ~~$*-o-x-o-x*$~~

Lowest critical frequency is due to zero, it may be present either at origin or near to origin.

~~$*-o-x-o-x*$~~ Y_{RC} ~~$-o-x-o-x-o*$~~ Z_{RL} .

$$\frac{dY_{RC}}{ds} > 0 \quad \text{slope is the} \quad \frac{dZ_{RL}}{ds} > 0$$

$$Z_{RL}(0) < |Z_{RL}(\infty)|$$

$$Y_{RC}(0) < Y_{RC}(\infty)$$

$S=0$	$S=\infty$
$X_L = Ls$	
$X_L = 0$	$X_L = \infty$

origin origin

- RC n/w Cauer-II form. (ladder)

$$LC \rightarrow Z_2(s) = Z(s) - H_1(s)$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

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$$Y_3(s) = Y_2(s) - H_2 s \quad (B_C = s_C).$$

$$Z_3(s) = 1/Y_3(s)$$

$$Z_4(s) = Z_3(s) - k_2$$

$$Y_4(s) = 1/Z_4(s) \Rightarrow Y_5(s) = Y_4(s) - H_2 s$$

$$Y_4(s) = Y_5(s) + H_2 s$$

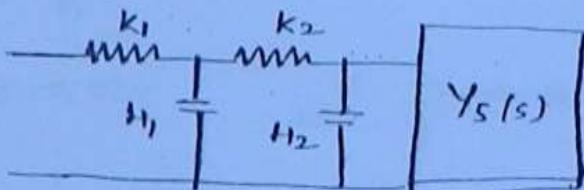
RC \rightarrow step 1: Removal of constant from $Z(s)$

$$Z_2(s) = Z(s) - k_1$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

Step 2: Removal of pole at ∞ from $Y(s)$.

$$Y_3(s) = Y_2(s) - H_1 s$$



Step 1 & Step 2 are alternately repeated until the total function is realized.

RC n/w Cauer-III form.

$$LC \rightarrow Z_2(s) = Z(s) - \frac{k_{01}}{s}$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - \frac{k_{02}}{s} \quad \left(B_L = \frac{1}{s} \right)$$

$$- Z_3(s) = \frac{1}{Y_3(s)}$$

Note :- Step 1 & step 2 are alternately repeated until the total function is realized.

$$Z_4(s) = Z_3(s) - \frac{k_{02}}{s}$$

$$Y_4(s) = \frac{1}{Z_4(s)}.$$

$$Y_5(s) = Y_4(s) - k_2.$$

$$Y_6(s) = Y_5(s) + k_2.$$

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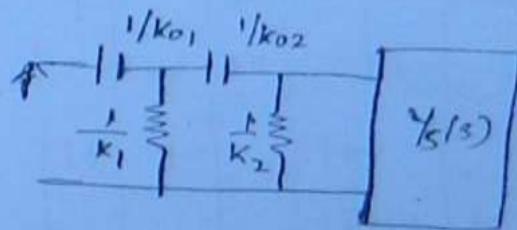
Rc \rightarrow Step 1: Removal of pole at origin from $Z(s)$

$$Z_2(s) = Z(s) - \frac{k_{01}}{s}$$

$$Y_2(s) = \frac{1}{Z_2(s)}.$$

Step 2: Removal of constant from $Y(s)$

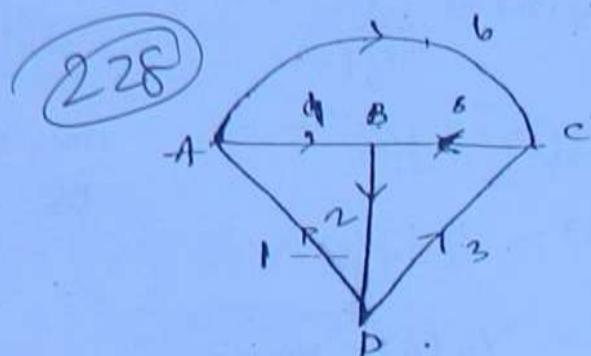
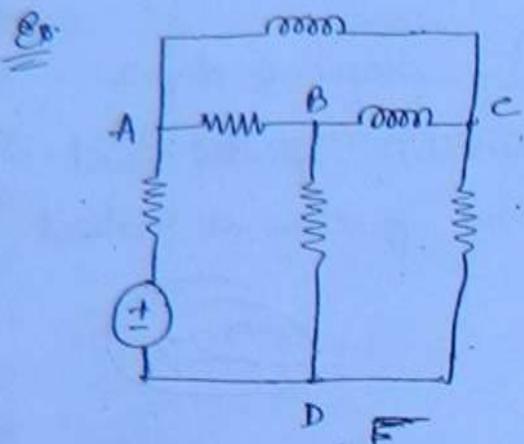
$$Y_3(s) = Y_2(s) - k_1$$



Graph Theory

Network Topology is a study of the n/w properties by investigating interconnections b/w branches and nodes, it mainly concentrates on the geometry of the network.

In the network topology, any network is replaced by graph. To develop the graph each element is replaced by either st. line or arc of the semi-circle, voltage source is replaced by short circuit & current source is replaced by o.c. and graph retains all the nodes of the original n/w.



$$C = N - 1$$

$$C = M$$

No. of branches of n/w \geq no. of branches of graph.

in+ve Augmented Incident matrix

	1	2	3	4	5	6
A	+1			-1		-1
B		-1	+1	+1		
C			+1		-1	+1
D	-1	+1	-1			

Reduced ^{Incident} matrix

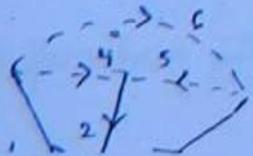
	1	2	3	4	5	6
A	+1			-1		+1
B		-1		+1	+1	
C			+1		-1	+1
D	-1	+1	-1			

$D \rightarrow \text{ref}$

→ All the information regarding the graph can be represented mathematically in concised form is called as, incidence matrix.

→ For a given graph augmented incidence matrix is unique

→ Tree is a connected sub graph, it connects all the nodes of the n/w but it does not consist of any closed path.

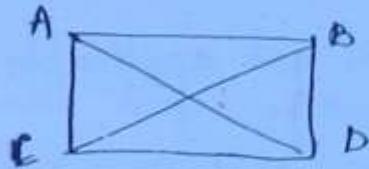


Tree {1, 2, 3}

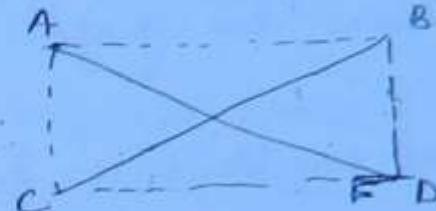
- Total tree branches = $N - 1$. . . $4 - 1 = 3$ $N = \text{no. of nodes}$

(red.) links = $b - (N - 1)$. . . $6 - (4 - 1) = 3$. . . $b = \text{branches}$

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& tree



The tree is invalid, because there is no interconnection between the nodes.

The set of branches which are disconnected, to form a tree is called as co-tree (complementary tree).

A branch which form a tree is called ar-tree branch (twig). Generally it is indicated by solid line (or thick line).

Total no. of tree branches = $N - 1$

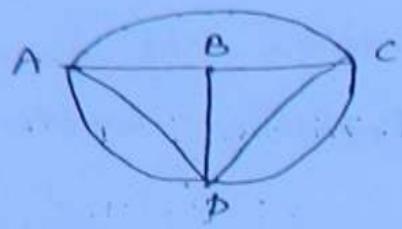
A branch which is disconnected to form a tree is called link. also known as chord. Generally it is indicated by dotted lines.

Total no. of links = $l = b - (N - 1)$.

For a given graph tree is not unique.

Total no. of possible trees = N^{N-2}

Note :- The above formula can be applied when connections should be present b/w all the nodes.



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For the above two graphs the formula is not applicable.

→ Total no. of possible trees for ~~any~~ graph \Rightarrow

$$= \det |AAT|, \text{ where } A = \text{reduced incidence matrix.}$$

Node Pair voltages

$$\text{Total no. of node pair voltages} = \frac{N(N-1)}{2} = Nc_2$$

$$= \frac{4(4-1)}{2} = 6$$

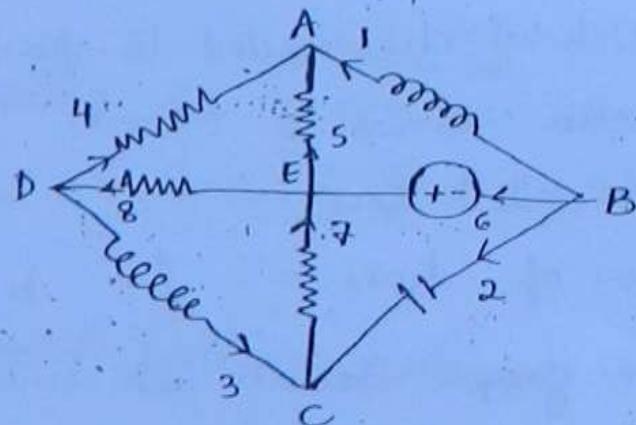
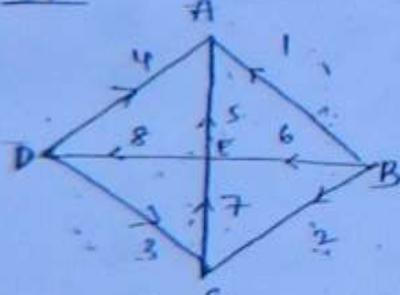
$$\rightarrow \text{Total no. of edges} = \frac{N(N-1)}{2}$$

\Rightarrow edge = branch

Q/1

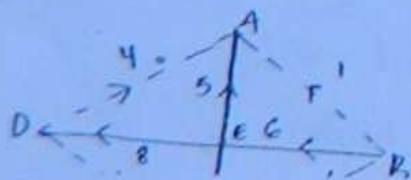
Develop ~~a graph~~ ^{a graph} for the network shown.

Step 1



Step 2

Develop a tree for the graph.



Step 3:

(23)

Identify total no. of basic loops / fundamental loops. (ex)
independant loops / -P-loops.

Basic loop should consist of only one link.

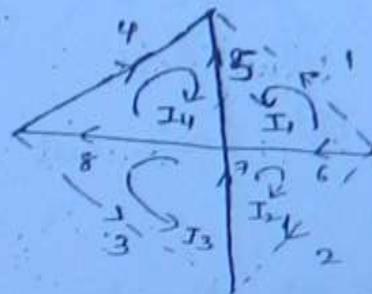
Total no. of basic loops = total no. of links.

$$l = b - (N-1)$$

Basic loop direction is same as the link current direction.

		[C]		U		C _b			
		v ₁	v ₂	v ₃	v ₄	v ₅	v ₆	v ₇	v ₈
J ₁	+1		-		-1	-1			
J ₂	+1				-1	+1			
J ₃		+1			+1	+1			
J ₄			+1	-1			+1		

$$[C] = [U; C_b] \text{ voltages}$$



kVL.

$$v_1 - v_5 - v_6 = 0$$

$$v_2 - v_6 + v_7 = 0$$

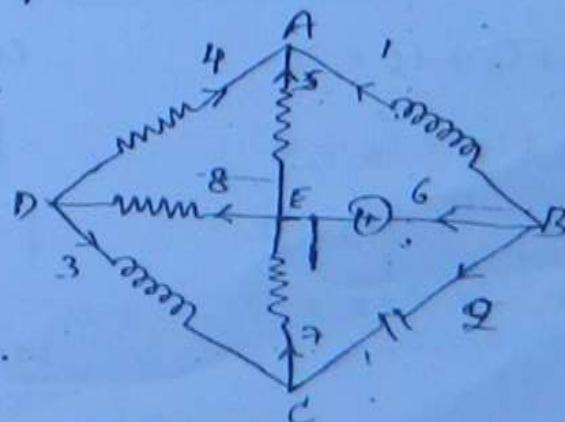
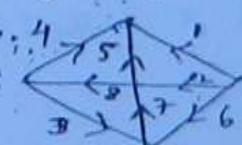
$$v_3 + v_7 + v_8 = 0$$

$$v_4 - v_5 + v_8 = 0$$

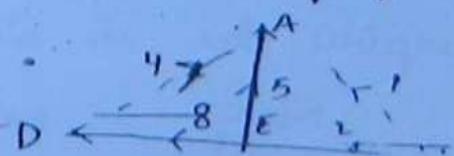
Cut-set matrix.

Develop cut-set matrix for the n/w shown.

Step 1: Develop a graph for the given n/w.



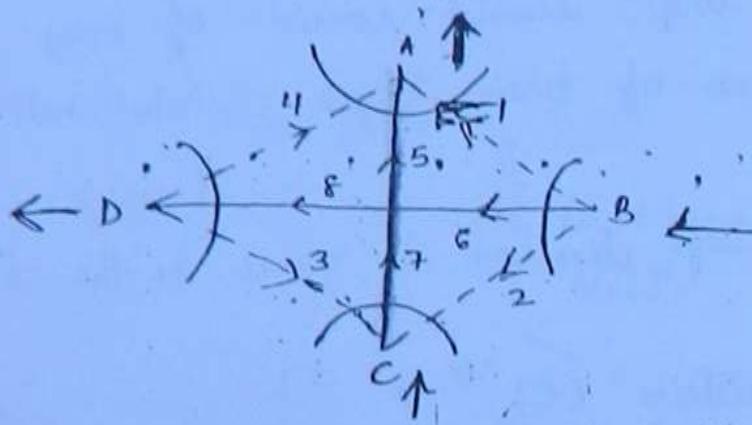
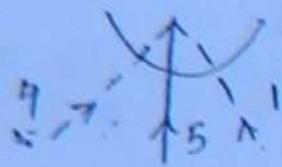
Step 2: Develop a tree for a graph.



3. i) Identify total no. of basic cut sets / fundamental cut sets / f-cut sets

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ii) Basic cut set should consist of only one tree branch.



iii) Total no. of basic cut sets = total no. of tree branches

$$= N-1$$

iv) Basic cut set direction is same as the tree branch current direction.

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
cut-set matrix : [B] :	-A	+1			+1	+1		
	B	+1	+1				+1	
	C		-1	-1				+1
	D			-1	-1			+1

key

$$i_1 + i_4 + i_5 = 0$$

currents

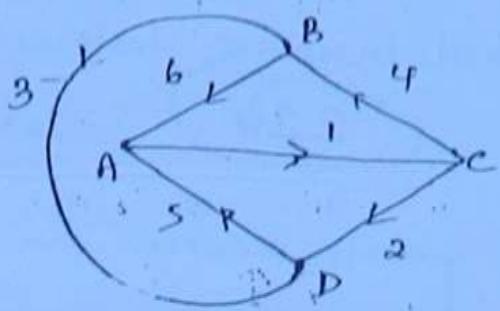
$$i_1 + i_2 + i_6 = 0$$

$$[B] \rightarrow [B_L : v]$$

$$-i_2 - i_3 + i_7 = 0$$

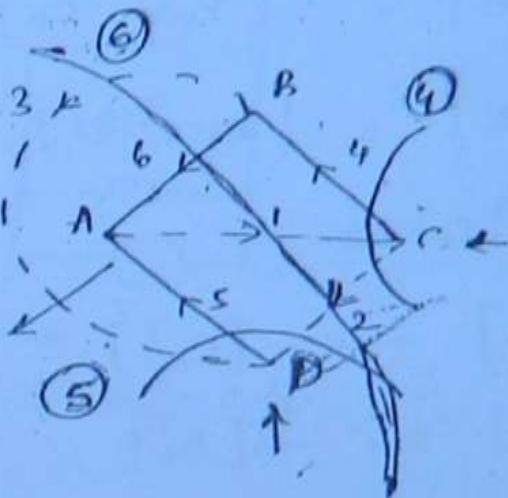
$$-i_3 - i_4 + i_8 = 0$$

3. Develop cut set matrix for the graph shown.



(233)

Assume 4, 5, 6 as tree branches.

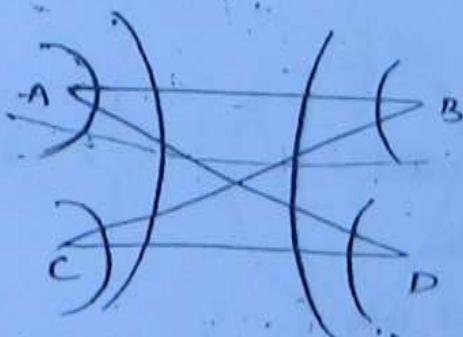


	1	2	3	4	5	6
④	-1	+1				
⑤		-1	-1	+1		
⑥	-1	+1	+1	+1	+1	+1

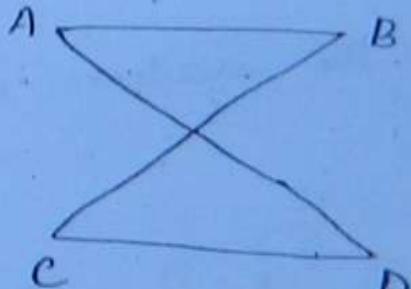
Identify total no. of cut-sets of the graph shown.

(a) 3 (b) 4

(c) 5 (d) 6



Inclusions



Total no. of possible trees = N^{N-2}

The set matrix is not unique, total no. of possible
Tree set matrices = N^{N-2} .

3. Cut-set matrix is not unique; Total no. of possible cut-set matrices = N^{N-2}

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~~4.~~ $[C] \xrightarrow{\text{Tieset}} [v : c_b]$

$[B] \xrightarrow{\text{Cut set}} [B_1 : v]$

$B_1 = -(c_b^T)$

$\checkmark [B_1] = -[c_b^T] \leftarrow$

$\checkmark [C_b] = -[B_1^T]$

5. The rank of the tie-set matrix = total no. of links.
Rank = $l = b - (N-1)$

6. Rank of the cut-set matrix = total no. of tree branches.

7. Rank of the incidence matrix = $(N-1)$

Duality

$R \leftrightarrow G$

series \longleftrightarrow parallel

$L \leftrightarrow C$

b.c \longleftrightarrow s.c

$V \leftrightarrow I$

Tieset \longleftrightarrow Cut-set

KVL \leftrightarrow KCL

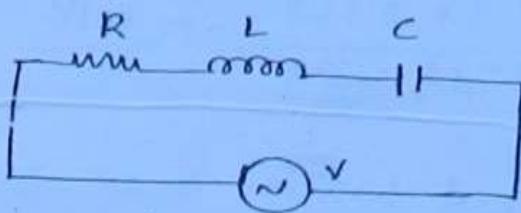
Thevenin's \longleftrightarrow Norton's

loop \longleftrightarrow node
(mesh)

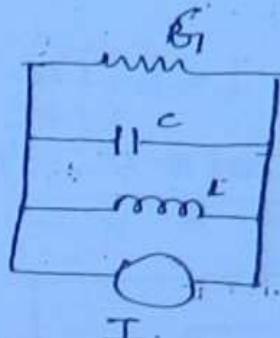
Foster I form \longleftrightarrow Foster II form

$\frac{dr}{dt} \longleftrightarrow \frac{di}{dt}$

Duality doesn't mean equivalence. But it means, mathematical representation of both the networks are identical.



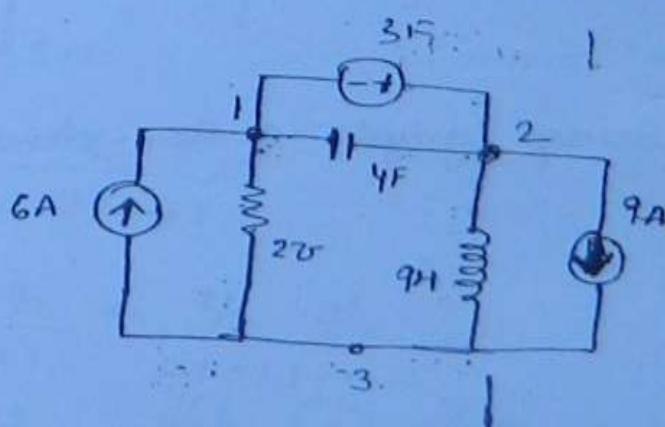
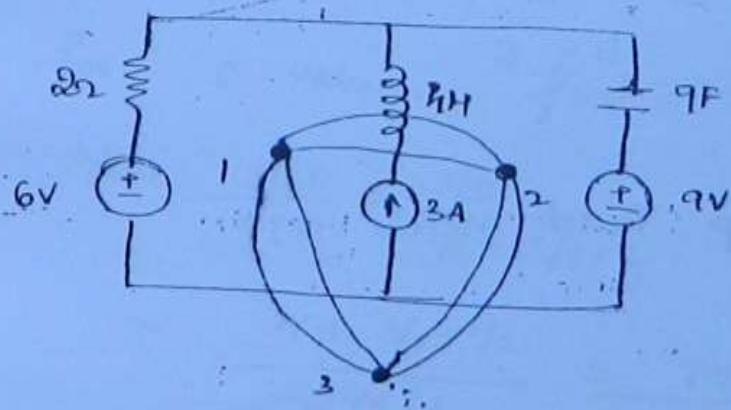
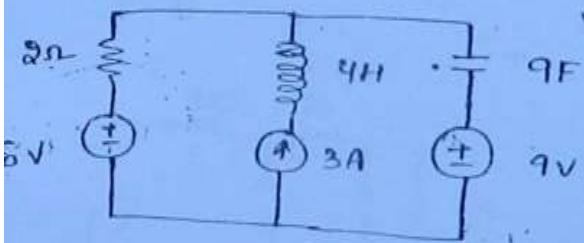
(235)



$$V = IR + L \frac{di}{dt} + \frac{1}{C} \int i dt \quad \hookrightarrow \text{KVL}$$

$$I = VG + \frac{C}{L} \frac{dv}{dt} + \frac{1}{L} \int v dt \rightarrow \text{KCL}$$

Draw the dual of the n/w shown.



When voltage source drives a current in cw direction
now mark off the current source as indicated towards
specive node.

when current source drives a current in C.W direction
 +ve sign is assigned to respective node.

236

W.B.

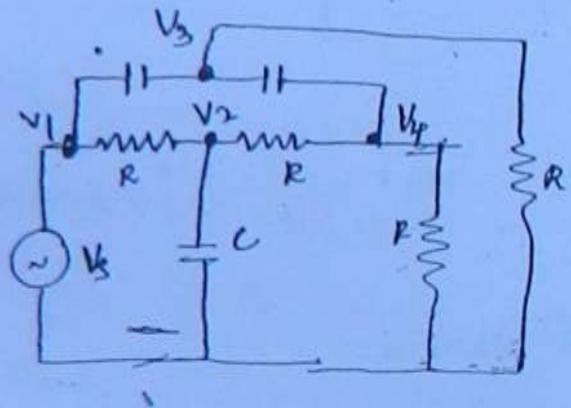
$$\textcircled{1} \quad f-\text{loops} = l = b - (N-1) = 6 - (4-1) = \underline{\underline{3}}$$

$$\textcircled{2} \quad \frac{N(N-1)}{2} = \frac{10(10-1)}{2} = 45$$

\textcircled{3}

\textcircled{4}

\textcircled{5}



$$e = M = 4$$

$$e = N-1 = 5-1 = \underline{\underline{4}}$$

$$\min \underline{\underline{3}} \quad (V_s = V_1 \text{ not considered})$$

$$\textcircled{6} \quad f\text{-cutset} = N-1 = 5-1 = \underline{\underline{4}}$$

$$\textcircled{7} \quad N^{N-2} = 4^{4-2} = \underline{\underline{16}}$$

$$\textcircled{8} \quad f\text{ loops} = l = b - (N-1) \Rightarrow 3 = b - (4-1)$$

$$b = 6$$

Conv.

1 Develop graph & then develop Tie-set matrix.

Network Synthesis W.B.

$$TF = \frac{1}{Rcs + 1} = \frac{1}{RC[s + \gamma_{RC}]} \quad (d)$$

237

$$\frac{Y(s)}{V(s)} = \frac{s+2 \cdot 5s+1}{s^2+4s+3} = \frac{(s+0 \cdot s)(s+2)}{(s+1)(s+3)}$$

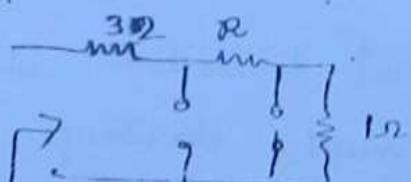
1. lowest critical freq \rightarrow pole $= \infty$ $\rightarrow 0 \cdot s \Rightarrow 2\omega_0$

2. alternate pole zero \checkmark

$\times 3$.

$$4. \frac{Y(s)}{V(s)} = \frac{0 \cdot s \times 3}{s+1} = 1/3$$

$$s=0, X_c = \frac{1}{sc} = \infty, c \rightarrow \infty$$



$$Z(s) = \frac{3 \times 8}{s+1} = 4 + R$$

$$\Rightarrow 8 = 4 + R \Rightarrow R = 4 \Omega$$

for C value, find $Z(s)|_{s=1}$

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1/s}{1/s + 1/s + \gamma_1} = \frac{\gamma s + 1}{2\gamma s + 1}$$

$$= \frac{\gamma s + \gamma_2 + \gamma_2}{2\gamma s + 1} = \frac{\gamma s + \gamma_2}{2\gamma s + 1} + \frac{\gamma_2}{2\gamma s + 1}$$

$$\Downarrow \text{ 2 } \quad \Downarrow \left(\frac{1}{2\gamma s + 1} \right) \rightarrow R_2$$

$$Z(s) = \frac{s+2+\frac{3}{s}}{L=1-R=2} \rightarrow C = 4 \Omega, Z(s) \text{ in PRF}$$

$$10. \quad Z(j\omega) = \frac{j\omega + \alpha}{j\omega + \beta} = \frac{\tan'(w/\alpha)}{\tan'(w/\beta)} = \tan'\left(\frac{w}{\alpha}\right) - \tan'\left(\frac{w}{\beta}\right)$$

$$\theta(j\omega) = \alpha - \beta$$

238

$$11. \quad \frac{s^2 + s + 1}{s^2 + 2s + 2} \left(\begin{array}{c} \xrightarrow{s} \\ 1 \end{array} \right) \xrightarrow{R}$$

$$\frac{s^2 + s + 1}{s + 1} \left(\begin{array}{c} \xrightarrow{s} \\ s + 1 \end{array} \right) \xrightarrow{C}$$

$$\frac{s^2 + s}{1) s + 1} \left(\begin{array}{c} \xrightarrow{s} \\ s \end{array} \right) \xrightarrow{Y}$$

$$\frac{s}{1) 1 (1} \xrightarrow{L}$$

$$\frac{1}{s} \xrightarrow{R}$$

(d)

$$12. \quad Z(s) = k \frac{(s+3)}{(s+j+1)(s+1-j)} = \frac{k(s+3)}{(s^2 + 2s + 2)}$$

$$b. \quad Z(s) = \frac{k(s^2 + 20^{\circ})(s^2 + 60^{\circ})}{s(s^2 + 40^{\circ})}$$

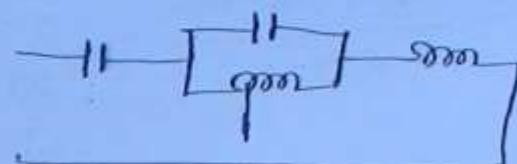
Note - Imaginary poles & zeros should be conjugate pair.

Only for LC network imaginary poles and zeros are present.

$$P \rightarrow \infty \rightarrow H(s)$$

$$P \rightarrow \text{origin} \rightarrow \frac{k_0}{s}$$

P → conjugate pair.



$$13. \quad Z(s) = \frac{(s+1)}{(s+s_2)(s+\frac{1}{s_2})} \Rightarrow Z(j\omega) = \frac{j\omega + 1}{j\omega + s_2 + \frac{1}{j\omega + s_2}}$$

$$f(t) = 8 \sin t \Rightarrow \omega = 1.$$

$$Z(jt) = \frac{1+jt}{(\sqrt{2}+j)(\frac{1}{\sqrt{2}}+j)}$$

239

$V = 12$

$$|Z| = \frac{\sqrt{2}}{(\sqrt{3})(\sqrt{3}/2)} = 2/\sqrt{3}$$

$$\therefore |V| = |Z| = 2/\sqrt{3}$$

Z_{RE}

1. lowest \rightarrow pole

(2) - high \rightarrow zero

2. alternate

(3)

In the continued fraction expansion, if all the quotients have the sign then it satisfies Hurwitz.

$$Q(s) = s^5 + 3s^3 + s$$

$$Q'(s) = 5s^4 + 9s^2 + 1$$

5	1	3
4	5	9
3	65	415

$$Q(s) = \frac{Q_1(s)}{Q'_1(s)}, \quad Q'_1(s) = 5s^4 + 9s^2 + 1 \quad s^5 + 3s^3 + s$$

Q satisfies Hurwitz

$$F(s) = \frac{P(s)}{Q(s)} = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)}$$

$$F(s) = \frac{m_1 + n_1}{m_2 + n_2}$$

$$F(s) = \frac{m_1 m_2 - n_1 n_2}{m_1^2 - n_1^2} + \frac{n_1 m_2 - m_1 n_2}{m_1^2 - n_1^2}$$

240.

$$= \text{even } F(s) + \text{odd } F(s)$$

$$S_{\text{even}} \cdot S_{\text{even}} = S_{\text{even}}$$

$$= \text{Real } F(s) + \text{Im } F(s)$$

$$S_{\text{odd}} \cdot S_{\text{odd}} = S_{\text{odd}}$$

$$S_{\text{even}} \cdot S_{\text{odd}} = S_{\text{odd}}$$

also $F(s) = \text{PRF}$

$$S_{\text{even}} = (j\omega)^4 = \omega^4 \text{ Re.}$$

$$S_{\text{odd}} = (j\omega)^2 = -j\omega^2 \text{ Im.}$$

$$\frac{m_1 m_2 - n_1 n_2}{m_1^2 - n_1^2} \geq 0$$

$$m_1 m_2 - n_1 n_2 = 0$$

$$\left| \begin{array}{l} \frac{m_1 (\text{even})}{n_2 (\text{odd})} = \frac{n_1 (\text{odd})}{m_2 (\text{even})} \end{array} \right|$$

**

$$23. \quad S^3 + 4S \quad 2S^4 + \dots \quad (2S + \dots) \quad 2S \xrightarrow{L} Z$$

$$24. \quad Z_1 = \frac{(R_2 + Ls) Z_1}{R_1 + R_2 + Ls}, \quad Z_{22} = \frac{R_2 (R_2 + Ls)}{R_1 + R_2 + Ls} \quad \Rightarrow L = 2H$$

equal denominators.

$$Z_2(s) = \text{PRF} \quad | \text{Re } Z_2(\omega) > R_{\text{neg}}$$

$$Z(s) = \frac{2s+1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{1}{s} + \frac{1}{s+1}$$



$$A = \frac{V_1}{V_2} \quad \left| \begin{array}{l} I_2 = 0 \\ \Rightarrow A = \frac{Z_{IL}}{Z_{21}} \end{array} \right.$$

$$D = -V_1$$

31

$$\therefore B = - \left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}} \right) = \textcircled{241}$$

$$D = \left. \frac{-x_1}{x_2} \right\} \text{at } v_2=0 \quad , \quad D_i = \left. \frac{-y_{ii}}{v_{2i}} \right\} = \begin{pmatrix} \frac{z_{22}/z_A}{z_{21}/z_A} \\ \vdots \\ \frac{z_{22}}{z_{21}} \end{pmatrix} =$$

Conv.

$$Z(s) = \frac{s^{\gamma} + 1}{s(s^{\gamma} + 2^2)}$$

$$\rho \rightarrow \infty$$

$P \rightarrow$ origin ✓

P → conjugate pair ✓

$$Z(s) = \frac{k_0}{s} + \frac{\bar{Q}ks}{s^2 + \omega^2}$$

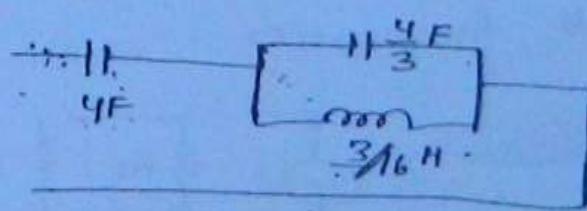
$$\frac{s^v + 1}{s^v(s^v + 4)} = \frac{k_0}{s} + \frac{aks}{s^2 + 4}$$

$$k_0 \rightarrow \gamma_\varphi$$

$$2k \rightarrow -3/4$$

$$Z(s) = \frac{1/4}{s} + \frac{3s/4}{s+4}$$

$$Z(s) = \frac{1}{4s} + \frac{1}{4s/3 + 16/3s}$$



1. (b)

$$Z(s) = \frac{(s+1)(s+3)}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} + C$$

2.

$$Z_D(s) = \frac{K(s+2)(s+4)}{s(s+4)(s+8)}$$

(242)

$$Z_D(s) \Big|_{s=-3} = +1 \Rightarrow K=5$$

$$Z_D(s) = \frac{1 \cdot 8 \cdot 4}{s} + \frac{1 \cdot 25}{s+4} + \frac{1 \cdot 8 \cdot 25}{s+8}$$

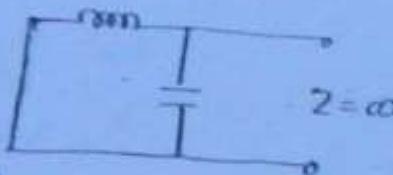
Resistance

1. $\omega_0 = \frac{\omega_0 L}{R} = \frac{X_L}{R} = \frac{1000}{0.1} = 10^4$

$$B_W = \frac{-f_0}{\omega_0} = \frac{10 \times 10^3}{10^4} = 1 \text{ kHz}$$

2. Find Z_R

Ans

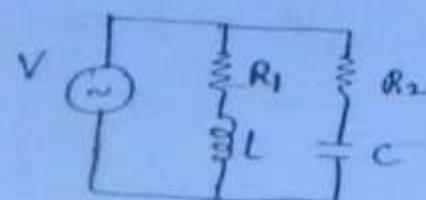

 $Z = \infty$ for ideal
current source

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1 \text{ rad/sec} \rightarrow \text{Remnant condition}$$

* Q. In the ckt shown, at what value of R_1, R_2
circuit resonant for all frequencies.

$$B_L = B_C$$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$



$$\frac{\omega L}{R_1^2 + (\omega L)^2} = \frac{1/\omega C}{R_2^2 + (1/\omega C)^2}$$

$$\frac{1}{\frac{R_1 + \omega L}{\omega t}} = \frac{1}{R_2^2 \omega C + \frac{1}{\omega C}}$$

243

$$w \rightarrow -L = R_2^2 C \Rightarrow -R_2 = \sqrt{\frac{L}{C}}$$

$$-\frac{1}{\omega} \rightarrow \frac{R_1^2}{L} = \frac{1}{C} \Rightarrow R_1 = \sqrt{\frac{L}{C}}$$

$$R_1 = R_2 = \sqrt{\frac{L}{C}}$$

$$Y(s) = \frac{s^2 + 0.5s + 100}{5s} = \frac{s}{5} + 0.1 + \frac{20}{s}$$

$$L = \frac{1}{20} H, \quad G = 0.1, \quad C = 15 \text{ mF}$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$15 = \sqrt{V_R^2 + 9^2} \Rightarrow V_R = 12V$$

$$20 = \sqrt{V_R^2 + V_L^2} \Rightarrow 20^2 = 12^2 + V_L^2 \Rightarrow V_L = 16V$$

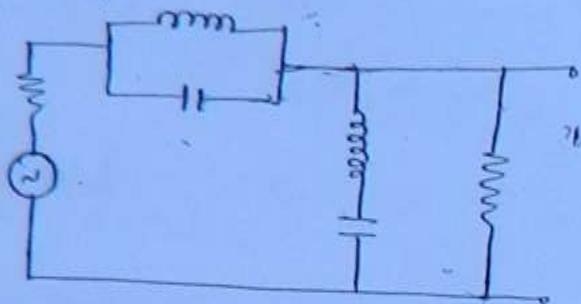
$$V_L - V_C = 9$$

$$V_C = 7$$

Low pass filter

Q. Identify type of n/w for the n/w shown.

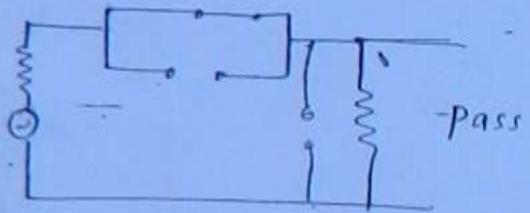
244



at $f=0$,

$$X_C = \infty \rightarrow C \rightarrow 0.C$$

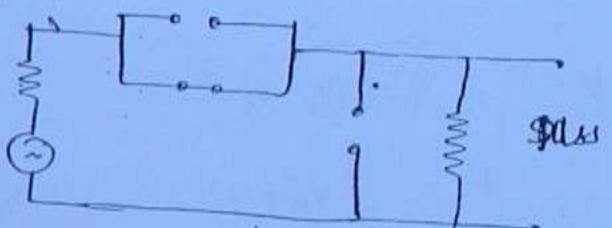
$$X_L = 0 \rightarrow L \rightarrow S.C$$



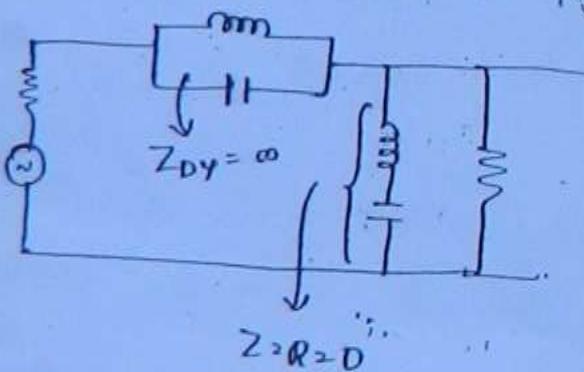
at $f \rightarrow \infty$,

$$X_C = 0 \rightarrow C = S.C$$

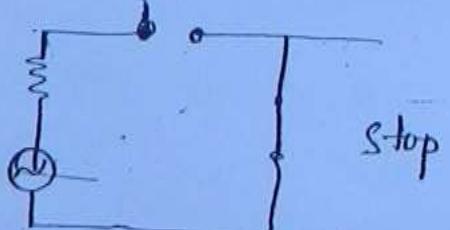
$$X_L = \infty \rightarrow L = 0.C$$



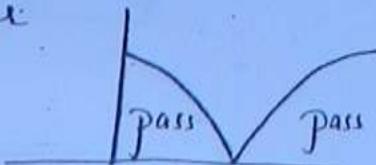
To verify whether BEF or all pass filter we have to check resonant freq.



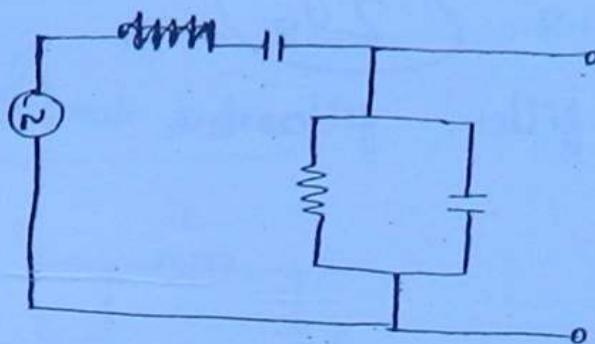
at $f = f_0$



When BEF eliminates only few frequencies, then it is also called as notch filter.



Identify type of the n/w for the n/w shown.



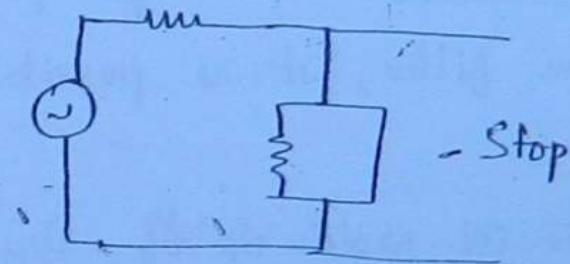
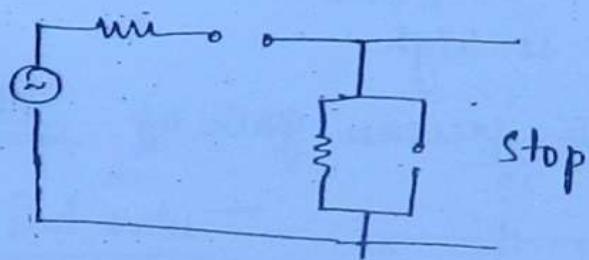
245

at $f=0$,

$$X_C \rightarrow \infty \Rightarrow C = 0.C.$$

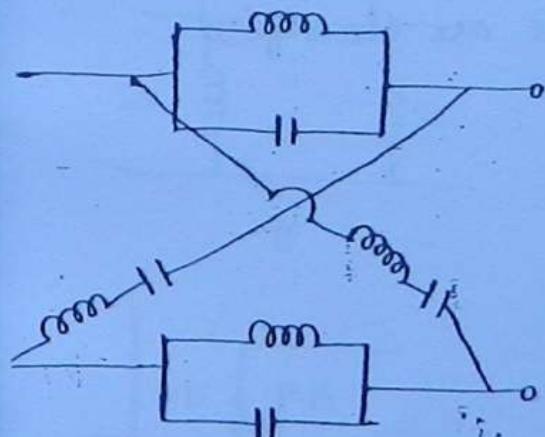
at $f \rightarrow \infty$,

$$X_C \rightarrow 0, S.C \rightarrow C.$$



\Rightarrow Band Pass filter.

Identify type of the filter of the n/w shown.



- (a) LPF
- (b) BPF
- (c) BEF
- (d) all pass filter

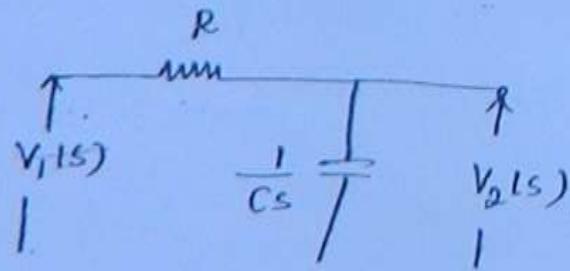
Based on components present in the filter, filters are classified as

1. Active filter
2. Passive filter.

- Active filters are made up of op-amp & capacitor.
- Generally inductor is not used in the active filter, since size of the inductor is bulky & cost is high.
- In the active filter, it is possible to increase gain of the system.
- Passive filter is made up of series and parallel LC section (reactive n/w)
- In passive filter it is not possible to increase gain of the system.
- Based on frequency of operation, filters are classified as
 1. LPF
 2. HPF
 3. BPF
 4. BEF (BSF)
 5. All pass filter.

LPF

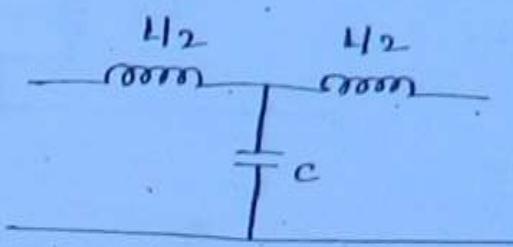
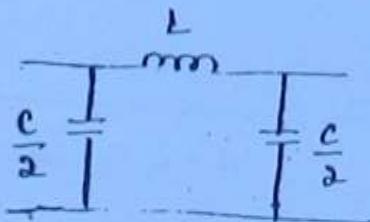
First order



$$V_2(s) = V_1(s) \frac{1/cs}{R + 1/cs} = \frac{-V_1}{1 + Rcs}$$

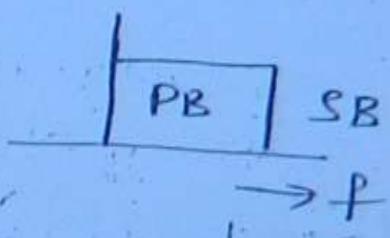
Second order

(247)



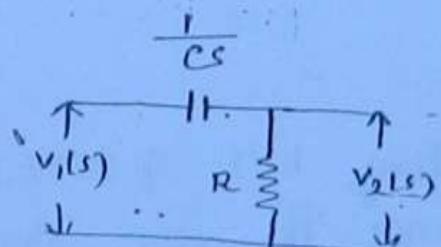
$$X_C = \frac{1}{2\pi f C}$$

$$X_L = 2\pi f L$$



LPF

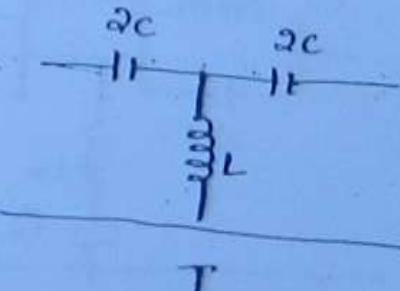
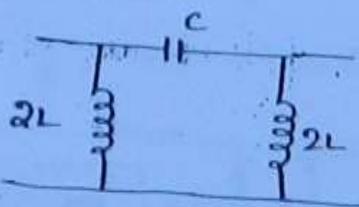
first order



$$V_2(s) = V_1(s) \frac{R}{R + 1/cs}$$

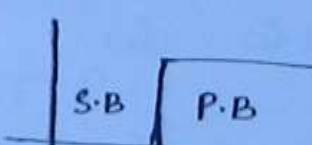
$$\frac{V_2(s)}{V_1(s)} = \frac{Rcs}{1 + Rcs}$$

cond order

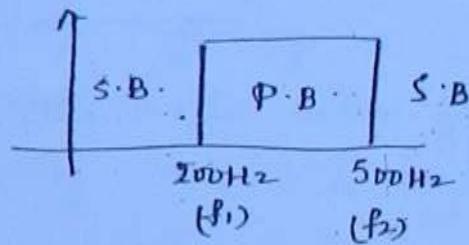
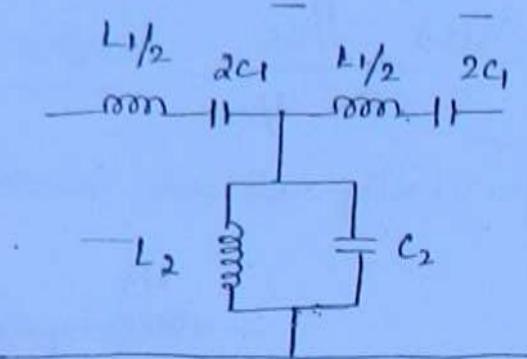
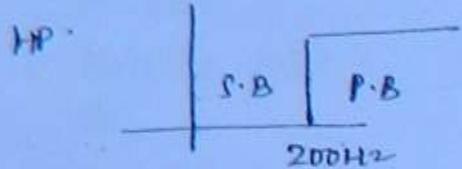
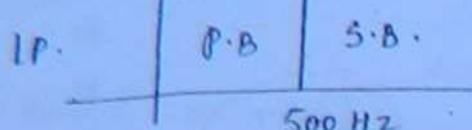


$$X_C = \frac{1}{2\pi f C}$$

$$X_L = 2\pi f L$$

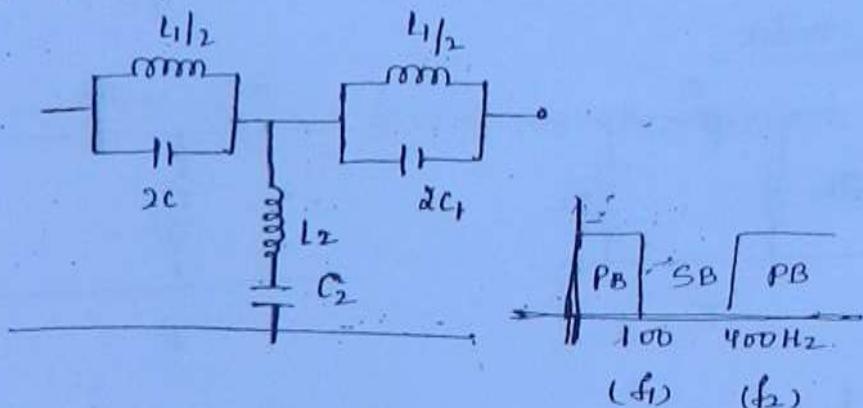
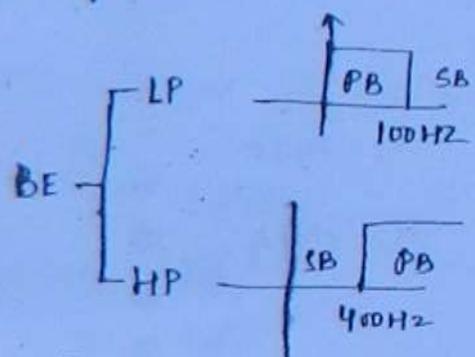


SPF can be obtained with the combination of LPF & HPF
cut-off freq. of LPF should be greater than, cut-off
freq. of HPF.



BEF / BSF

BEF can be obtained by connecting LPF & HPF in parallel and cut off frequency of the HPF should be greater than cut off freq. of LPF.



First Order filter T.F

$$\frac{1}{1 + \gamma_s} \rightarrow \text{LPF}$$

$$\frac{\gamma_s}{1 + \gamma_s} \rightarrow \text{HPF}^{\gamma_s \text{RC}}$$

$$\frac{1 - \gamma_s}{1 + \gamma_s} \rightarrow \text{All pass}$$

Note :-

1. In All pass filter poles are present in LHP & zeros are present in right half plane.
2. Poles and zeros are in symmetric about $j\omega$ axis.

Second Order Filters T.P

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$$\frac{P}{s^2 + as + b} \rightarrow \text{LPF}$$

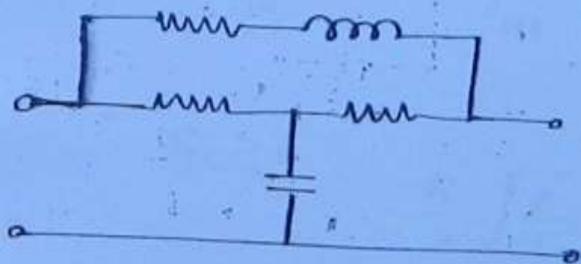
$$\frac{Ps^2}{s^2 + as + b} \rightarrow \text{HPF}$$

$$\frac{Ps}{s^2 + as + b} \rightarrow \text{BPF}$$

$$\frac{Ps^2 + q}{s^2 + as + b} \rightarrow \text{BEP}$$

$$\frac{s^2 - Ps + q}{s^2 + as + b} \rightarrow \text{All pass filter.}$$

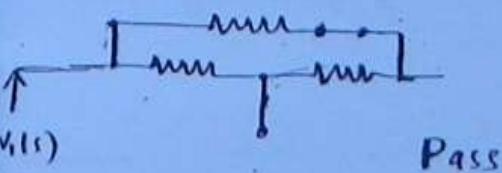
Identify type of the filter for the network shown.



$$f=0,$$

$$X_C = \frac{1}{2\pi f C} = \infty \Rightarrow C \rightarrow 0 \cdot \text{c}$$

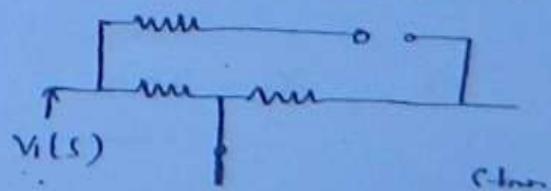
$$X_L = 2\pi f L = 0 \Rightarrow L \rightarrow \text{s.c.}$$



$$\text{at } f=\infty,$$

$$X_C = 0 \Rightarrow C \rightarrow \text{s.c.}$$

$$X_L = \infty \Rightarrow L \rightarrow 0 \cdot \text{c.}$$



filter WB

(250)

① denominator - charac. eqn. $\sim (1 + \gamma s)^2$

$$\sim \gamma s^2 + 2\gamma s + 1$$

$$\Rightarrow s^2 + \frac{2}{\gamma} s + \frac{1}{\gamma^2} \sim s^2 + 2\zeta \omega_n s + \omega_n^2$$

$$\omega_n = 1/\gamma$$

$$2\zeta \cdot \frac{1}{\gamma} = \frac{2}{\gamma} \Rightarrow \zeta = 1 \Rightarrow \text{critically damped.}$$

4. $B_1 = R/L_1$

$$B_2 = R/L_2 = \frac{R}{L_1/4} = \frac{4R}{L_1} \Rightarrow \frac{B_2}{B_1} = 1/4$$

Resonance ω_B cond-

$$\text{In: } \left(D + \frac{R}{L} s + \frac{1}{LC} \right) = 0 \Rightarrow s^2 + 2\zeta s + 10^6 = 0$$

$$\omega_2 - \omega_1 = B \omega = \frac{R}{L} = 20, \quad \omega_0 = 10^6$$

$$\theta_1 = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{10^3}{20} = 50$$

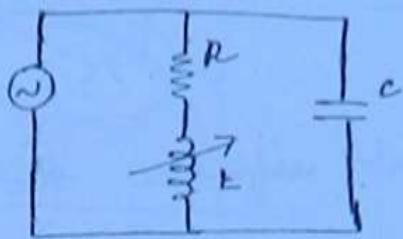
5. $V_L = 2V_C$

$$I \times L = 2 I \times C \Rightarrow X_C = \frac{X_L}{2}$$

$$\tan \theta = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow \omega_B 45^\circ = \frac{20}{\sqrt{20^2 + \left(X_L - \frac{X_L}{2}\right)^2}}$$

$$\Rightarrow X_L = 40 \Omega$$

(d)



25)

$$Z_{eq} = \frac{(R + jx_L)(-jx_C)}{R + jx_L - jx_C}$$

$$Z_{eq} = \frac{x_L x_C - jR x_C}{R + j(x_L - x_C)} \Rightarrow \frac{(x_L x_C - jR x_C)}{R^2 + (x_L - x_C)^2} [R - j(x_L - x_C)]$$

$$= \frac{R x_L x_C - R x_C (x_L - x_C) - j[R x_C + x_L x_C (x_L - x_C)]}{()} \text{ mho.}$$

$$\text{Im}(Z_{eq}) = 0 \quad R^2 - x_L x_C + x_L^2 = 0$$

$$x_L^2 - x_L x_C + R^2 = 0$$

$$s^2 + 2\zeta \omega_n s + \omega_n^2 = 0$$

$$\omega_n = R, \quad 2\zeta \omega_n = -x_C \Rightarrow \zeta = \frac{-x_C}{2R}$$

$$-\zeta < -1$$

$$\frac{-x_C}{2R} < -1 \Rightarrow \frac{x_C}{2R} > 1 \Rightarrow x_C > 2R$$

$$\underline{\underline{x_C > 10}} \quad (b)$$

3.

$$\zeta \geq 1$$

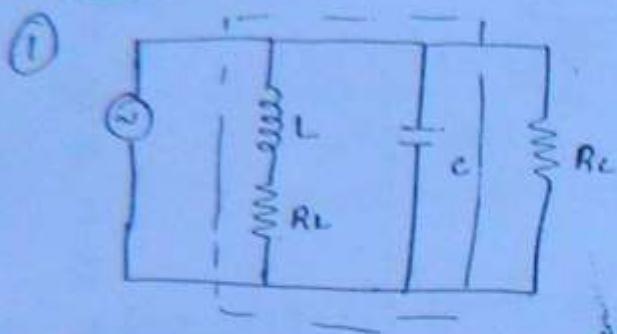
$$-\frac{R}{2} \int \frac{1}{L} \geq 1 \Rightarrow @$$

Job 1 ① In over damped system, no oscillations are present.

In the underdamping system more than one oscillation are present.

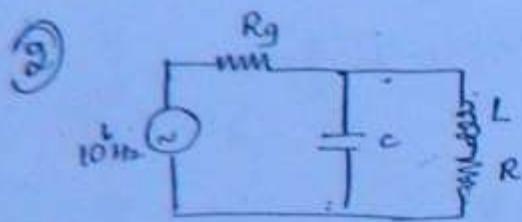
Conv.

(252)



R_C doesn't influence the imaginary part of Z_{eq} .
Hence can be neglected.

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$



$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

R_g can be neglected.
It doesn't influence the $\text{Im}(Z_{eq})$.

$$10^6 = \frac{i}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \rightarrow ①$$

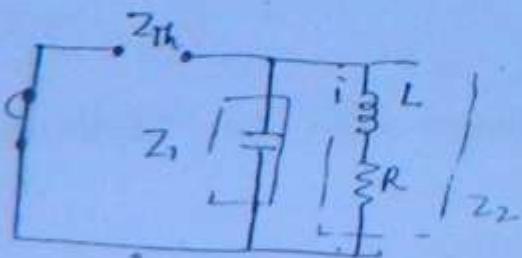
$$\theta = \frac{\omega L}{R} \Rightarrow \frac{R}{L} = \frac{\omega}{\theta}$$

$$\frac{R}{L} = \frac{2\pi f}{\theta} \Rightarrow \frac{R}{L} = \frac{2\pi \times 10^6}{\theta} \rightarrow ②$$

Sub. ② in ①.

$$L = 0.2 \text{ mH}$$

$$R = 180 \Omega$$

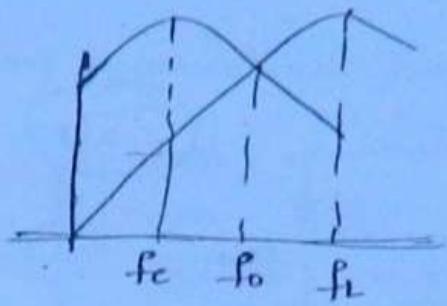


$$Z_1 = -jX_C$$

$$Z_2 = R + jX_L$$

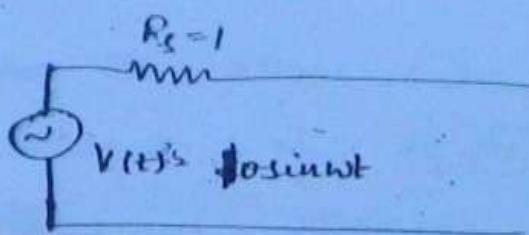
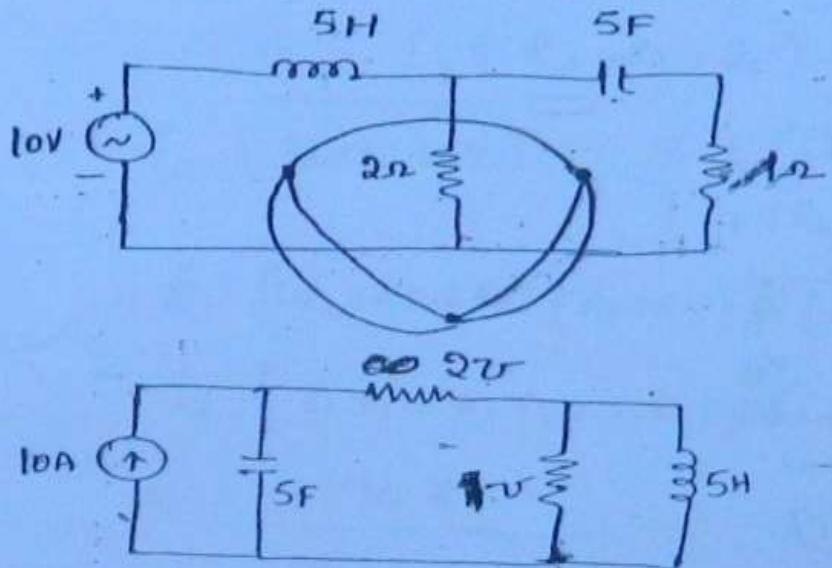
$$Z_m = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$|Z_m| = 5.3 \text{ k}\Omega$$



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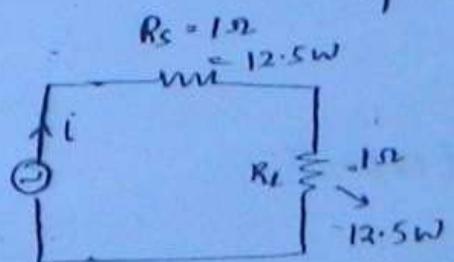
heoreus conv.



$$P_{\max} = \frac{V_s^2}{4R_L}$$

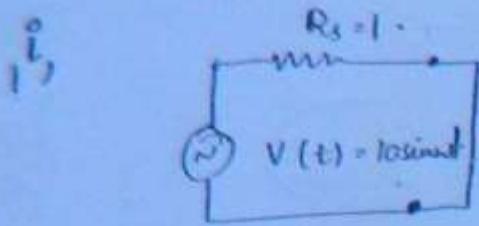
$$= \frac{(10/\sqrt{2})^2}{4} = \frac{100}{8} = 12.5 \text{ W}$$

$$R_L = R_S = 1 \Omega$$



$$P_T = 12.5 + 12.5$$

$$P_T = 25 \text{ W}$$

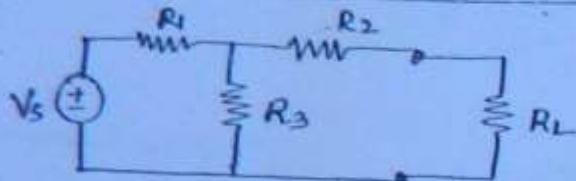


$$P_m = \frac{V_s^2}{R_s}$$

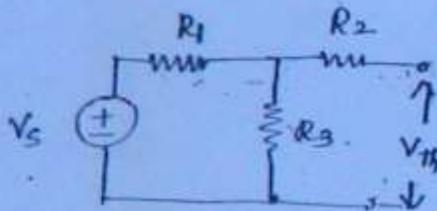
$$= \frac{(10/\sqrt{2})^2}{1} = \underline{\underline{50W}}$$

(254)

Proof of Thévenin's theorem.

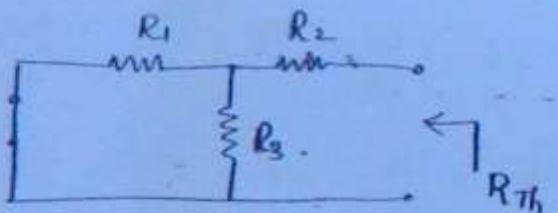


Case 1: (V_{Th})



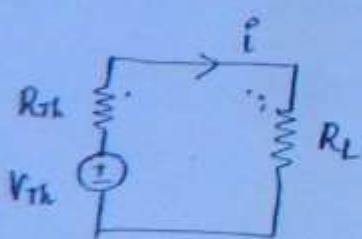
$$V_{Th} = \frac{V_s \cdot R_3}{R_1 + R_3} \rightarrow \textcircled{1}$$

Case 2:



$$R_{Th} = R_2 + \frac{R_1 R_3}{R_1 + R_3} \rightarrow \textcircled{2}$$

Case 3:

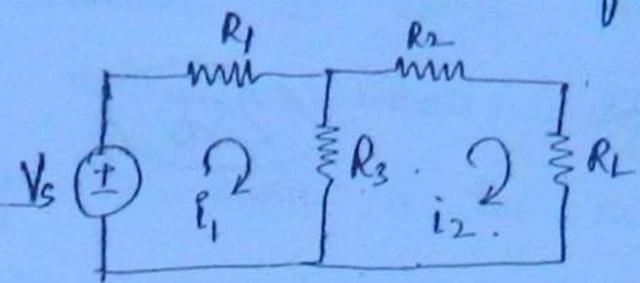


$$I_L = \frac{V_{Th}}{R_L + R_{Th}}$$

$$I_L = \frac{V_s \cdot R_3 / (R_1 + R_3)}{(R_L + R_2)(R_1 + R_3) + R_1 R_3 / (R_1 + R_2)}$$

$$I_L = \frac{V_s R_3}{(R_L + R_2)(R_1 + R_3) + R_1 R_3 / (R_1 + R_2)} \rightarrow \textcircled{3}$$

Using conventional method find out i_L .



(25)

$$V_s = (R_1 + R_3) i_1 - i_2 R_3.$$

$$i_1 R_3 = i_2 (R_2 + R_3 + R_L).$$

$$i_2 (R_2 + R_3 + R_L) = \frac{V_s + i_2 R_3}{(R_1 + R_3)} \cdot R_3$$

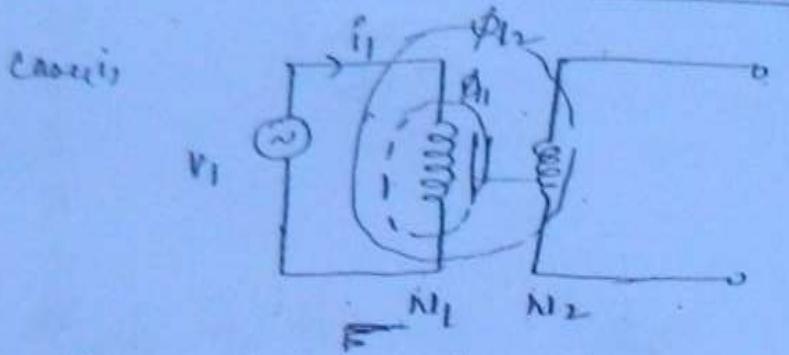
$$i_2 [(R_2 + R_3 + R_L)(R_1 + R_3) - (R_1 R_3)] = V_s \cdot R_3.$$

$$i_2 [R_2 R_1 + R_2 R_3 + R_1 R_2 + R_3^2 + R_L R_1 + R_L R_2 - R_3^2] = V_s R_2.$$

$$i_2 = \frac{V_s R_3}{(R_1 + R_3)(R_L + R_2) + R_1 R_3} = i_L \rightarrow \textcircled{8}$$

from the above calculation it is concluded that load current of eq. (4) & eq. (8) are equal. Hence
Therminius theorem is proved.

Magnetic Coupled circuits



(25b)

$I \rightarrow \phi_1 \leftarrow \phi_{11} \rightarrow$ leakage flux
 $\phi_{12} \rightarrow$ useful flux (or) mutual flux.

$$e_1 \propto \frac{d\phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\phi_1}{di_1} \cdot \frac{di_1}{dt} \quad \left(L = \frac{\mu N \phi}{I} \right)$$

$$\boxed{e_1 = -L_1 \frac{di_1}{dt}} \rightarrow \text{self induced emf.}$$

$$e_2 \propto \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{di_1} \cdot \frac{di_1}{dt}$$

$$\boxed{M_{21} = \frac{N_2 \phi_{12}}{i_1}}$$

$$\boxed{e_2 = -M_{21} \frac{di_1}{dt}} \rightarrow \text{mutual induced emf.}$$

current is either entering or leaving at other dotted terminals, sign of the mutual induced voltage is same as the sign of the self induced voltage.

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When one current is entering and other current is leaving at dotted terminal, sign of the mutual induced voltage is opposite to the sign of self induced voltage.

The amount of magnetic coupling between the inductors is expressed by coefficient of coupling (K).

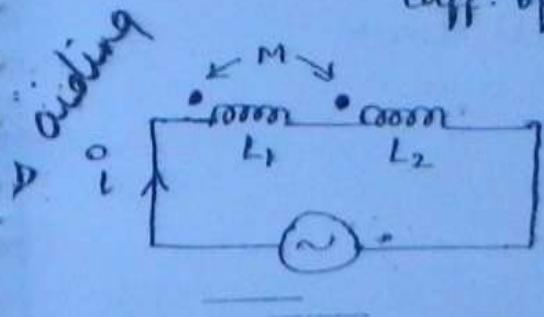
$$K = \frac{\text{useful flux}}{\text{Total flux}} = \frac{\phi_B}{\phi_1} = \frac{\phi_{B1}}{\phi_2}$$

For ideal system $K=1$. (\because leakage $\phi_L=0 \Rightarrow \phi_B=\phi_1, \phi_{B2}$)

For practical system, the range of K is $0 < K < 1$

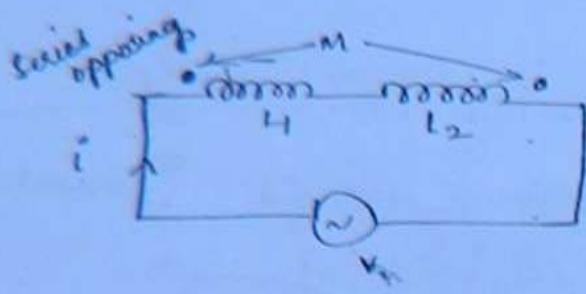
$$M = K \sqrt{L_1 L_2}$$

mutual inductance \downarrow $\xrightarrow{\text{coeff. of coupling}}$ self inductance



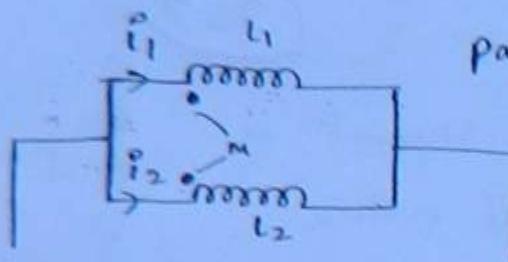
$$V = L_1 \frac{di_1}{dt} + \frac{M di_2}{dt} + L_2 \frac{di_2}{dt} + \frac{M di_1}{dt}$$

$$\text{Req. } \frac{di}{dt} = (L_1 + L_2 + 2M) \frac{di}{dt}$$



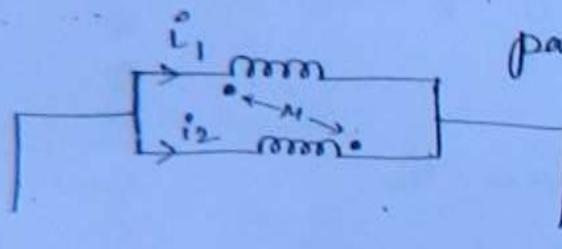
$$L_{eq} = L_1 + L_2 - 2M$$

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parallel aiding

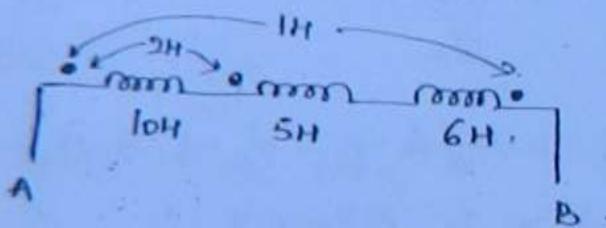
$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$



parallel opposing

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Q. Find eq. inductance wrt A & B.



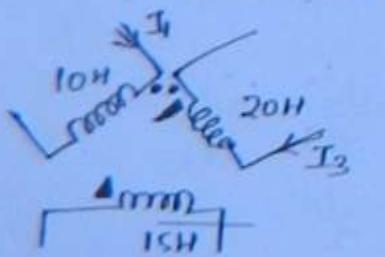
$$15 + 4 + 19$$

$$19 + 6 - 2 = \underline{\underline{23H}}$$

$$L_{eq} = L_1 + L_2 + L_2 \pm 2M_1 \pm 2M_2 \pm 2M_3$$

$$L_{eq} = 10 + 5 + 6 + 4 - 2 = \underline{\underline{23H}}$$

Develop inductance matrix for the network shown.



$$\bullet 14$$

$$\Delta 24$$