Tejas Kamtam, 305749402

1.21

Part a

$$(a^* \cup ba^*b) \oplus ba^* = a^*ba^* \cup ba^*bba^*$$

Part b

$$(((a \cup b)a^*b) \oplus ((a(a \cup b) \cup b)a^*b)^* \oplus (a \cup \varepsilon)) \cup \varepsilon$$

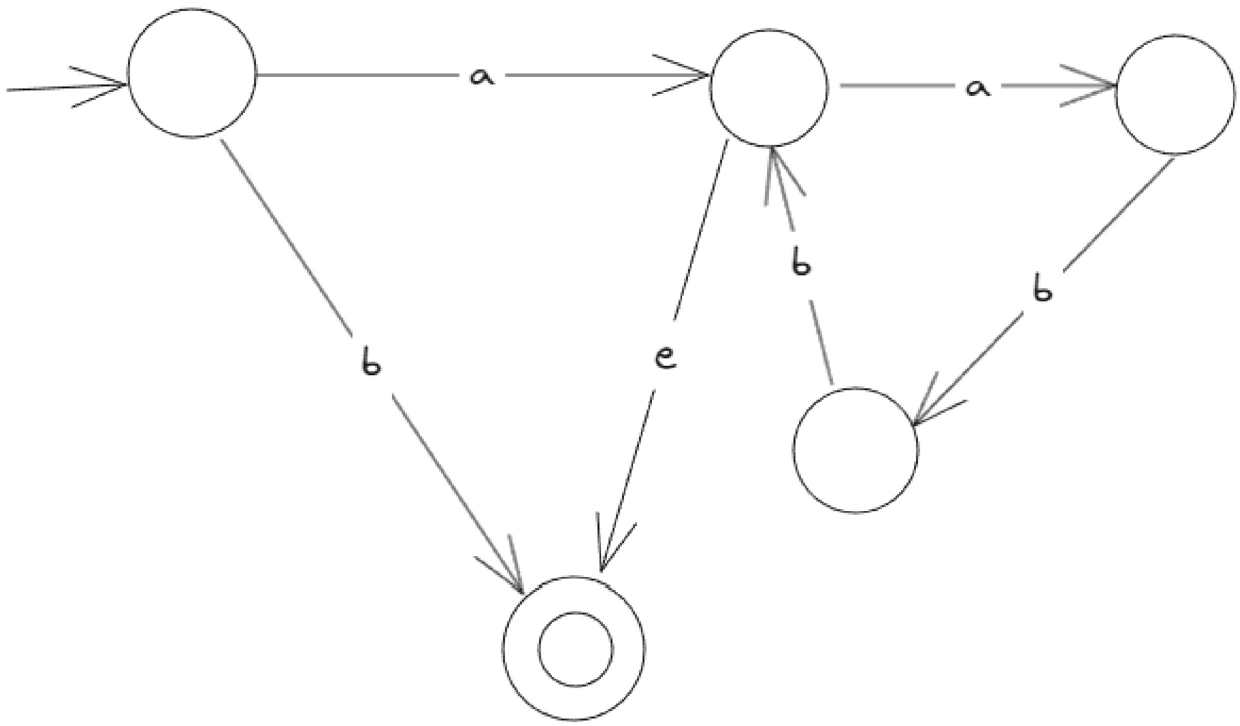
$$= ((aa^*b \cup ba^*b) \oplus (aaa^*b \cup aba^*b \cup ba^*b)^* \oplus (a \cup \varepsilon)) \cup \varepsilon$$

1.28

Part a

$$a(abb)^* \cup b =$$

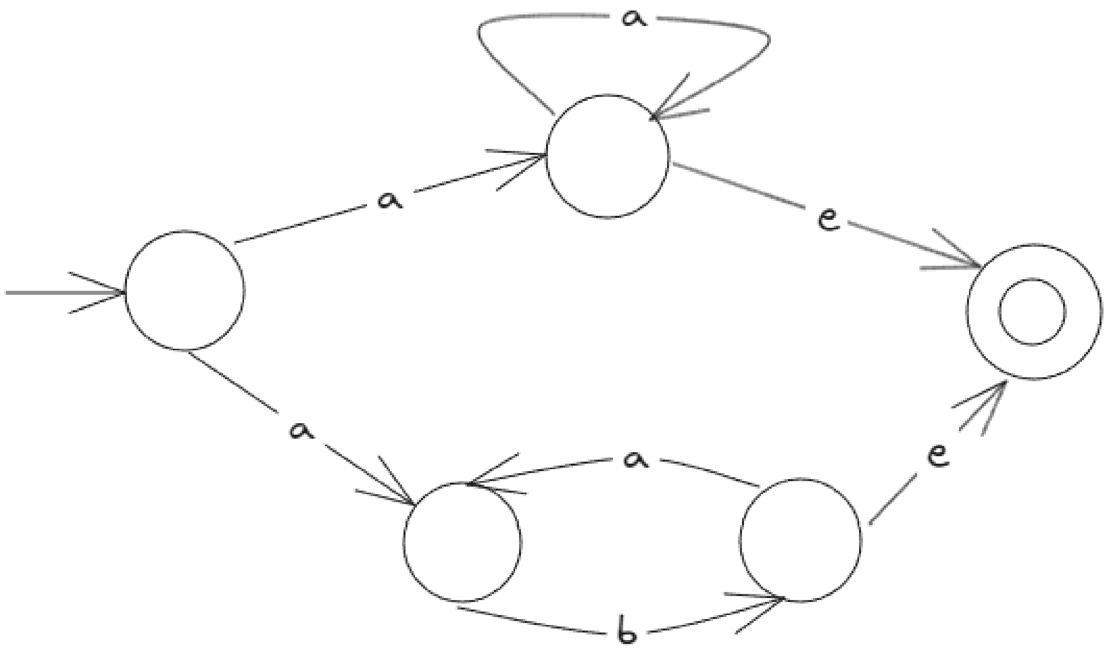
where $e = \varepsilon$:



Part b

$$a^+ \cup (ab)^+$$

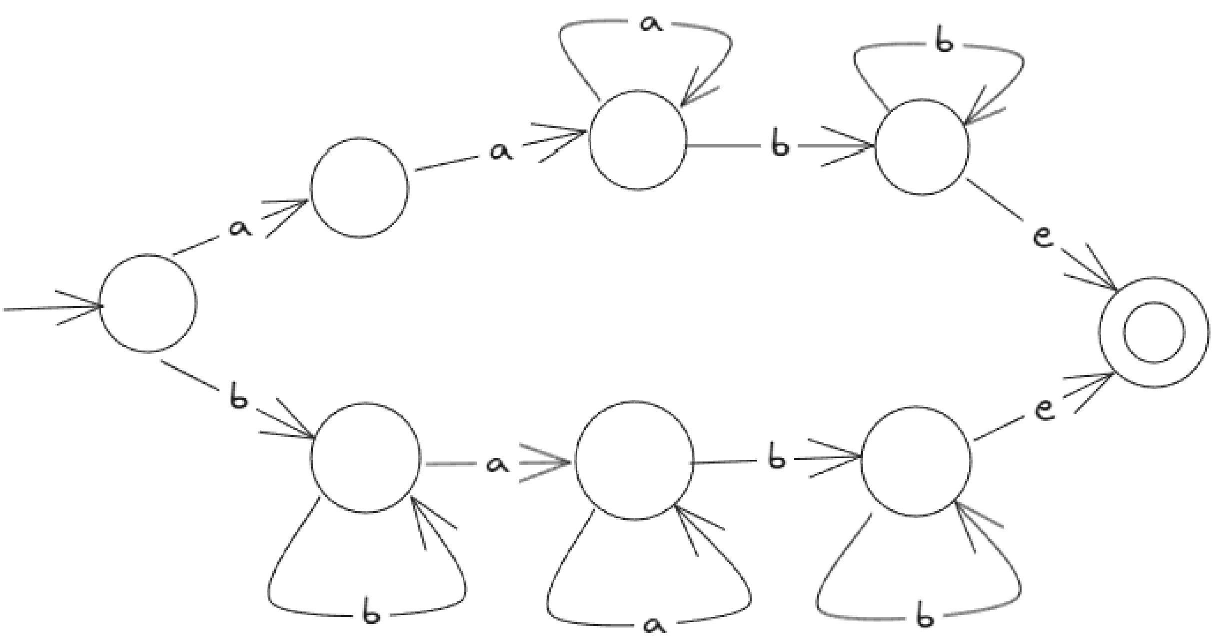
where $e = \varepsilon$:



Part c

$$(a \cup b^+)a^+b^+ = aa^+b^+ \cup b^+a^+b^+$$

where $e = \varepsilon$:



1.47

Consider the string

$$w = 1^p \# 1^{p!} \in L$$

For this string any choice of substring xy lies within 1^p so **pumping up** s.t.

$$w' = xyy^{\frac{p!-|y|}{|y|}}z \notin L$$

This is because $0 < |y| \leq p$ so $\frac{p!}{|y|} \in \mathbb{Z}^+$ and $|y| + |y|(\frac{p!-|y|}{|y|}) = p!$ so the string is not in the language. Thus, by the contrapositive corollary to the Pumping Lemma, L is nonregular.

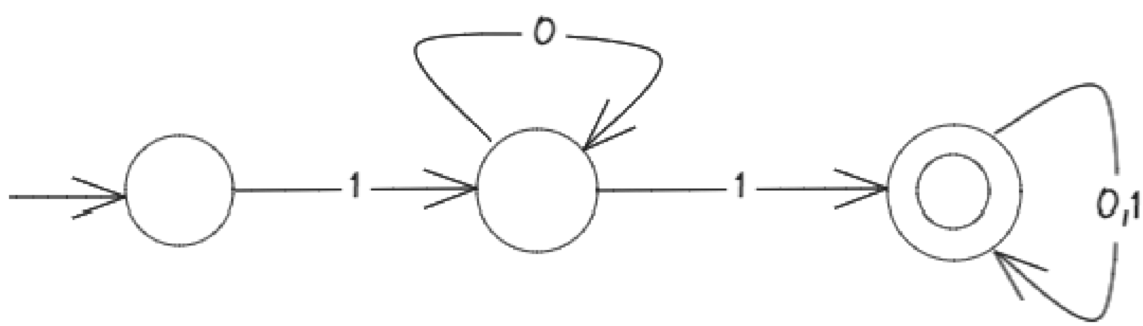
1.49

Part a

The language B can be described by the regular expression:

$$10^*1 \oplus \Sigma^*$$

Thus, the language is regular. An equivalent NFA that recognizes B is shown below:



Part b

Consider the string $w = 1^p01^p \in C$.
Then we can express the string as $w = xyz$ s.t. $xy \in 1^p$ and $|xy| \leq p$
Pumping down, such that we get a new string $w' = xz$ reduces the $\#_1$s by $|y|$ so the new string can be expressed as $1^{p-|y|}01^p$.

Now, based on the definition of the language C, $k = p - |y|$ and now (after pumping down) $\#_1(z) = p$
For the pumped string to be in the language, the following must be true: $p \leq k$
But because of pumping down: $p \not\leq k = p - |y| \because |y| \geq 1$
Thus, the new string $w' \notin C$. And, by the contrapositive of the P.L., C is nonregular.

1.53

Consider the string

$$w : xyz : 1^p = 1^p + 0^p \in ADD$$

However, choosing any $xy \in 1^p$ and pumping down, that is constructing the new string:

$$w' : xz \notin ADD$$

Because pumping down is equivalent to shifting right by some $|y|$ integers. So, the sum does not hold after pumping down. Thus, the language ADD is nonregular by the contrapositive to the P.L.

6

We have shown before that the language $A = 0^*1^*$ is regular. So,

$$A \cup B \text{ is nonregular} \iff B \text{ is non regular } \because A \text{ is regular}$$

By the closure principle of regular languages under union. So, to show the union is nonregular, we must show B is nonregular.
Consider the string

$$w = xyz = 0^p 1^p \in B$$

for some any choice of $xy \in 0^p$ s.t. $|xy| \leq p$, pumping down result in the new string:

$$w' = xz = 0^{p-|y|} 1^p \notin B$$

because $|y| \geq 1$ so $|0^{p-|y|}| \neq |1^p|$. So, this pumped-down string is not in the language. Thus, B is nonregular by the contrapositive of the P.L.

And, since we've shown B to be nonregular, the union $A \cup B$ must also be nonregular by closure principles.