Name(last, first): _____

ID (rightmost 4 digits): _ _ _ _

U C L A Computer Science Department

CS 180 Algorithms & Complexity

Final Exam **Total Time: 2:45 hours** **June 6th, 2022**

***** Write all algorithms in bullet form (as done in the past) *****

You need to prove EVERY answer that you provide.

There are a total of 8 pages including this page.

You need to upload ONE file in PDF to Gradescope.

You can include at most 15 pages in your PDF.

Name(last, first): _____

1. (20 points: each part has 10 points)

a. Consider an instance of the weighted interval scheduling problem (find a maximum weighted non-overlapping intervals). Design an **O(nlogn)** time algorithm to solve the problem. Prove its correctness and analyze its time complexity.

b. Design an **O(n)** time algorithm that merges two sorted lists of size **n**.

2. **(15 points)** A Hamiltonian path in a DAG is a path that contains each vertex exactly once. Is the problem of finding a Hamiltonian path in a DAG NP-Complete? Either prove it or design an **O(e+n)** time algorithm for finding a Hamiltonian path in a DAG.

3. **(15 points)** Given an **nxn** matrix where every row is sorted in increasing order, design an algorithm that outputs the smallest common element in all rows. If there is no common element return -1. Analyze the time complexity of your algorithm.

Input: mat = [[1,2,3,4,5,8], [2,4,5,8,10], [3,5,7,8,9,11], [1,3,5,7,8,9]] **Output: 5**

Name(last, first): _____

4. **(15 points)** In a technical interview, you've have been given an array of **n** numbers and you need to find a pair of numbers which are equal to given target value **L**. Numbers can be either positive, negative or both. An $O(n^2)$ time algorithm is trivial so you need to do better than that. Analyze the time complexity of your algorithm.

sequence = [8, 10, 2, 9, 7, 5] Target L= 11 Answer (9,2)

5. **(15 points)** Consider a given set of airline travel segments in a day. For example (Los Angeles, SanFrancisco)(8:00, 9:15) means there is a flight that leaves Los Angeles at 8 am and arrives in San Francisco at 9:15 am. We also know the number of passengers that can travel on each flight. Design an efficient algorithm that finds the maximum number of passengers that can travel from LosAngeles to NewYork in a day. What is the time complexity of your algorithm.

6. **(20 points)** Prove that finding a vertex cover (VC) in a graph is NP-complete. You can assume that finding a maximum independent set (MIS) in a graph is known to be NP-complete.VC is the minimum number of vertices that contains at least one end point of every edge. MIS is the maximum number of vertices that are pairwise disconnected (there is no edge between them).