Math 170E Lecture 1
Midterm 1 Practice

Instructions: Please read the following instructions **carefully** before starting the test:

• Unless otherwise stated, you need to justify your answer. Please show all of your work as partial credit will be given where appropriate, and there may be no credit given for problems where there is no work shown.

• You can use any theorem seen in class or in the weekly homework sheets.

• Note that it may be possible to successfully answer parts of a multi-part question without answering earlier parts.

• You have 50 minutes to complete the exam.

• You are allowed a single sheet of letter sized paper which can contain markings (by hand or printed) on both sides. No other notes or textbooks may be used in this exam.

• You may use a non-programmable calculator. No other electronics allowed on this exam. Make sure all cell phones are silenced, put away and out of sight. If you have a cell phone out at any point, for any reason, this will constitute a violation of test policy, and you may receive a zero on this exam.

• If asked, you must show us your bruin card.

• You may ask for scratch paper. You may use no other scratch paper. Please transfer all finished work onto the proper page in the test for us to grade there. We will not grade the work on the scratch paper. If you use the back of a page for your final answer, you must indicate so.

• Do not forget to write your name and UID in the space below. Good luck!

Name: _____
Student ID number: _____

1. You have three white balls and two blue balls in an urn to your left, and two blue balls and one white ball in an urn to your right. When you pick a ball from an urn, each ball is picked equally likely. You pick balls without replacement. When pick an urn randomly, both urns are chosen equally likely.
   (a) You pick two balls from the left urn. What is the probability that you picked precisely one white ball?
   (b) You pick two balls from the left urn. What is the probability that you picked at least one white ball?
   (c) You pick a ball from a randomly chosen urn. What is the probability that you pick a white ball?
   (d) You pick a ball from a randomly chosen urn. Given the you chose a white ball, what is the probability that it came from the left urn?

2. It costs you \$1 to play a game. If you win, you get \$2 and if you lose, you get \$0. There is a probability $\frac{1}{4}$ of winning each game, independently of all the other games. Let X be your profit in dollars after 20 games.
   (a) Explain why X must take values in the set $S = \{-20, -18, -16, \ldots, 18, 20\}$.
   (b) Find the PMF of X.

3. Let X be a discrete random variable with PMF
$$p_X(-1) = \frac{1}{2}, \quad p_X(1) = \frac{1}{2}. \tag{1}$$
   (a) What is the cumulative distribution function of X?
   (b) Find the mean and standard deviation of X.
   (c) Find a function $g : \mathbb{R} \to \mathbb{R}$ such that $g(X) \sim \text{Bernoulli}(1/2)$.

4. In the game of bridge, the entire deck of 52 cards is dealt out to 4 players. Compute the following probabilities. You do not need to simplify your answers.
   (a) one of the players receives all 13 hearts
   (b) each player receives 1 ace.