4.1-2nd Order Linear Differentials

#UCLA #Y1Q3 #Math33B

2nd Order Linear Differentials

Key Definitions

Second-Order Linear Differential Equations - diff. eq. of the form:

$$y''(t) + p(t)y' + q(t)y = g(t)$$

Where p, q, g are **coefficient functions** and $g(t)$ is the **forcing term**

If $g(t) = 0$, the diff. eq. is **homogenous**

E.g. *Simple Harmonic Motion*:

$$y'' + \omega^2 y = 0$$

$$y_1(t) = \cos \omega t \quad \text{and} \quad y_2(t) = \sin \omega t$$

$$y(t) = C_1 \cos \omega t + C_2 \sin \omega t$$

Linear Combination - lin. comb. of 2 func. y_1, y_2:

$$C_1 y_1 + C_2 y_2 : I \to \mathbb{R}$$

Linearly Independent - $y_1, y_2 : I \to \mathbb{R}$ are lin. indep. if:

$$C_1 y_1 + C_2 y_2 = 0$$

for all $t \in I$ else the funcs. are **linearly dependent**

Fundamental Set of Solutions - if y_1, y_2 are **lin. indep.** solutions to some 2nd order lin. diff. eq., and they "generate" all other sols., then the general solution is:

$$y(t; C_1, C_2) = C_1 y_1 + C_2 y_2$$

Existence and Uniqueness Theorem: 2nd, Linear

Sps. $p, q, g : I \to \mathbb{R}$ are **cont.** w/ domain interval $I \subseteq \mathbb{R}$. Then, given $t_0 \in I$ and any $y_0, y_1 \in \mathbb{R}$ there is a unique func. $y : I \to \mathbb{R}$ which satisfies:

- $y'' + py' + q = g$
- $y(t_0) = y_0 \quad \text{and} \quad y'(t_0) = y_1$

Wronskian

Sps. $u, v : I \to \mathbb{R}$ are two *diff.* func. on interval $I \subseteq \mathbb{R}$. Then, the Wronskian of the two funcs. is $W : I \to \mathbb{R}$ s.t.

$$W(t) := \det \begin{bmatrix} u(t) & v(t) \\ u'(t) & v'(t) \end{bmatrix} := u(t)v'(t) - v(t)u'(t)$$

for all $t \in I$ s.t. if:

- $W(t_0) = 0$ then u, v are **lin. dep.**
- $W(t_0) \neq 0$ then u, v are **lin. indep.**