4.2-Homogenous 2nd Order Linear

#UCLA #Y1Q3 #Math33B

Homogenous 4.1-2nd Order Linear Differentials with Constant Coefficients

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Key Definitions

Characteristic Polynomial - given $y'' + py' + qy = 0$, the char. pol. is:

$$F(\Lambda) = \Lambda^2 + P\Lambda + Q$$

s.t. the roots are called the **characteristic roots**
Note: the discriminant of the quadratic eq. of the char. pol. can be distinct-real, same-real, or distinct-complex

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Homogenous 2nd Order Solutions

Given diff. eq. of form:

$$Y'' + PY' + QY = 0$$

and char. pol.:

$$F(\Lambda) = \Lambda^2 + P\Lambda + Q$$

having roots of 3 different outcomes:

Distinct Real Roots

If the char. pol. gives distinct, real roots, $\lambda_1, \lambda_1 \in \mathbb{R}$, then the general solution is:

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Repeated Real Roots

If the char. pol. gives repeated, real roots, $\lambda_1 \in \mathbb{R}$, then the general solution is:

$$y(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t}$$

Distinct Complex Roots

If the char. pol. gives repeated, real roots, $\lambda_1 = a + bi, \lambda_2 = a - bi$, then the general solutions are:

Complex Solution

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Real Solution

$$y(t) = C_1 e^{at} \cos bt + C_2 e^{at} \sin bt$$