Homework 7

2.30 Use the pumping lemma to show that the following languages are not context free.

      a. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

      ^{A}**c.** $\{w \# t \mid w$ is a substring of t, where $w, t \in \{a, b\}^* \}$

      d. $\{t_1 \# t_2 \# \cdots \# t_k \mid k \geq 2, $ each $t_i \in \{a, b\}^*,$ and $t_i = t_j$ for some $i \neq j\}$

2.31 Let B be the language of all palindromes over $\{0,1\}$ containing equal numbers of 0s and 1s. Show that B is not context free.

2.43 For strings w and t, write $w \doteq t$ if the symbols of w are a permutation of the symbols of t. In other words, $w \doteq t$ if t and w have the same symbols in the same quantities, but possibly in a different order.

For any string w, define $SCRAMBLE(w) = \{t \mid t \doteq w\}$. For any language A, let $SCRAMBLE(A) = \{t \mid t \in SCRAMBLE(w)$ for some $w \in A\}$.

      a. Show that if $\Sigma = \{0,1\}$, then the $SCRAMBLE$ of a regular language is context free.

      b. What happens in part (a) if Σ contains three or more symbols? Prove your answer.

***2.45** Let $A = \{wtw^{\mathcal{R}} \mid w, t \in \{0,1\}^*$ and $|w| = |t|\}$. Prove that A is not a CFL.

2.38 For languages A and B, let the **shuffle** of A and B be the language

$$\{w \mid w = a_1 b_1 \cdots a_k b_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma^*\}.$$

Show that the class of context-free languages is not closed under shuffle.

Hint: see bottom of page.

2.41 Recall the definitions of NOPREFIX and NOEXTEND from Homework 3.

      a. Show that context-free languages are not closed under NOPREFIX.

      b. Show that context-free languages are not closed under NOEXTEND.

Hint: see bottom of page.

Hints