

Solutions to Questions—Chapter 4

Fixed Rate Mortgage Loans

Question 4-1

What are the major differences between the CAM, and CPM loans? What are the advantages to borrowers and risks to lenders for each? What elements do each of the loans have in common?

CAM - Constant Amortization Mortgage - Payments on constant amortization mortgages are determined first by computing a constant amount of each monthly payment to be applied to principal. Interest is then computed on the monthly loan balance and added to the monthly amount of amortization to determine the total monthly payment.

CPM - Constant Payment Mortgage - This payment pattern simply means that a level, or constant, monthly payment is calculated on an original loan amount at a fixed rate of interest for a given term.

CAM - lenders recognized that in a growing economy, borrowers could partially repay the loan over time, as opposed to reducing the loan balance in fixed monthly amounts.

CPM - At the end of the term of the mortgage loan, the original loan amount or principal is completely repaid and the lender has earned a fixed rate of interest on the monthly loan balance. However, the amount of amortization varies each month.

When both loans are originated at the same rate of interest, the yield to the lender will be the same regardless of when the loans are repaid (*i.e.*, early or at maturity).

Question 4-2

Define amortization. List the five types discussed in this chapter.

Amortization is rate at which the process of loan repayment occurs over the loan term.

Types of amortization are fully, partially, zero, negative and constant rates of amortization.

Question 4-3

Why do the monthly payments in the beginning months of a CPM loan contain a higher proportion of interest than principal repayment?

The reason for such a high interest component in each monthly payment is that the lender earns an annual percentage return on the outstanding monthly loan balance. Because the loan is being repaid over a long period of time, the loan balance is reduced only very slightly at first and monthly interest charges are correspondingly high.

Question 4-4

What are loan closing costs? How can they be categorized?

Closing costs are incurred in many types of real estate financing, including residential property, income property, construction, and land development loans.

Categories include statutory costs, third party charges, and additional finance charges.

Closing costs that do affect the cost of borrowing are additional finance charges levied by the lender. These charges constitute additional income to the lender and as a result must be included as a part of the cost of borrowing. Lenders refer to these additional charges as loan fees.

Question 4-5

In the absence of loan fees, does repaying a loan early ever affect the actual or true interest cost to the borrower?

No, the true interest rate always equals the contract rate of interest.

Question 4-6

Why do lenders charge origination fees, especially loan discount fees?

Lenders usually charge these costs to borrowers when the loan is made, or “closed”, rather than charging higher interest rates. They do this because if the loan is repaid soon after closing, the additional interest earned by the lender as of the repayment date may not be enough to offset the fixed costs of loan origination.

Question 4-7

What is the connection between the Truth-in-Lending Act and the annual percentage rate (APR)?

Truth-in-Lending Act - the lender must disclose to the borrower the annual percentage rate being charged on the loan.

The APR reflects origination fees and discount points and treats them as additional income or yield to the lender regardless of what costs the fees are intended to cover. The APR is always calculated assuming that the loan is repaid at maturity.

Question 4-8

What is the effective borrowing cost?

This differs from the contract rate because it includes financing fees (points, origination). It differs from the APR because the latter is calculated assuming that the loan is repaid at maturity. When calculating the effective cost, the expected repayment or payoff date must be used. The latter is usually sooner than the maturity date.

Question 4-9

What is meant by the “nominal rate” on a mortgage loan?

This rate is usually quoted as an annual rate, however the time intervals used to accrue interest is generally not quoted explicitly. Further, the rate generally does not specify the extent of any origination fees and/or discount points.

Question 4-10

What is the accrual rate and payment rate on a mortgage loan? What happens when the two are equal?

What happens when the accrual rate exceeds the payment rate? What if the payment rate exceeds the accrual rate?

The accrual rate is usually the nominal rate divided by the number of periods within a year that will be used to calculate interest. For example, if interest is to be accrued monthly, the nominal rate is divided by 12; if daily, the nominal rate is divided by 365. The payment rate, or “pay rate”, is the % of the loan to be paid at time intervals specified in the loan agreement. This rate is used to calculate payments which are usually made monthly (but could be quarterly, semi-annual, etc.) If the pay rate exceeds the accrual rate, this indicates that some loan repayment (amortization) is occurring. When it is equal to the accrual rate, amortization is not occurring. If the accrual rate is lower than the interest rate there will be negative amortization.

Question 4-11

An expected inflation premium is said to be part of the interest rate, what does this mean?

In general, the nominal interest rates for a specified period (say 10 years) is said to be a composite of three things; (a) real return-such as the growth rate in real GDP (underlying economic growth in the economy, (b) expected inflation, and (c) premium for risk. For example, if a lender quotes a 6% rate on a mortgage loan at a time when 10 year U.S. government bonds are yielding 3.6%, then the risk premium would be 2.4%. If at that same time growth in real GDP is 2.0% and is expected to continue at that rate for 10 years, then expected inflation can be estimated to be 1.6% (or $6\% - 2.4\% - 2.0\% = 1.6\%$). Alternatively, if 10 year U.S. Government Bonds that are indexed for inflation (TIPs) are currently yielding 2.0% and 10 year Treasuries not indexed for inflation are yielding 3.6%, the difference, or $3.6\% - 2.0\%$, or 1.6% is an estimate of expected inflation.

Question 4-12

A mortgage loan is made to Mr. Jones for \$30,000 at 10 percent interest for 20 years. If Mr. Jones has a choice between a

CPM and a CAM, which one would result in his paying a greater amount of total interest over the life of the mortgage?

Would one of these mortgages be likely to have a higher interest rate than the other? Explain your answer.

A CPM loan reduces the principal balance more slowly, as a result, if Mr. Jones chooses a CPM, he will pay a greater amount of interest over the life of the loan. As to the contract rate of interest, the borrower's

income constant, initial payments with the CAM will be higher and default risk will be greater. The initial monthly payments for a CPM are considerably less than those of a CAM. Because of lower initial payments with a CPM, this would reduce borrower default risk associated with a CPM loan. Additionally, lenders receive a greater portion of their return (interest earned) early with a CPM. By decreasing default risk a CPM may have lower rate of interest than a CAM.

Question 4-13

What is *negative amortization*?

Negative amortization means that the loan balance owed increases over time because payments are less than interest due.

Question 4-14

What is *partial amortization*?

Partial amortization occurs when payments exceed interest due but not by enough to reduce the amount owed to zero at maturity.

Solutions to Problems—Chapter 4

Fixed Rate Mortgage Loans

Problem 4-1

A borrower makes a fully amortizing CPM mortgage loan.

Principal	=	\$125,000
Interest	=	6.00%
Term	=	10 years

CPM Payment:

The monthly payment for a CPM is found using the following formula:

Monthly payment	=	PMT (n,i,PV, FV)
Monthly payment	=	PMT (10 yrs, 6%, \$125,000 , \$0)
Payment	=	\$1,387.76

If the loan maturity is increased to 30 years the payment would be:

Monthly payment	=	PMT (n,i,PV, FV)
Monthly payment	=	PMT (30 yrs, 6%, \$125,000 , \$0)
Payment	=	\$749.44

Problem 4-2

(a) Monthly payment (PMT (n,i,PV, FV) = \$515.44

Solution:

n	=	25×12 or 300
i	=	6%/12 or .50
PV	=	\$80,000
FV	=	0

Solve for payment:

PMT	=	-\$515.44
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(b) Month 1:

interest payment:

$$\$80,000 \times (6\%/12) = \$400$$

principal payment:

$$\$515.44 - \$400 = \$115.44$$

(c) Entire 25 Year Period:

total payments:

$$\$515.44 \times 300 = \$154,632$$

total principal payment: \$80,000

total interest payments:

$$\$154,632 - \$80,000 = \$74,632$$

(d) Outstanding loan balance if repaid at end of ten years = \$61,081.77

Solution:

n	=	120 (pay off period)
i	=	6%/12 or 0.50
PMT	=	\$515.44
PV	=	\$80,000

Solve for FV:

FV	=	\$61,081.77
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(e) Through ten years:

total payments:

$$\$515.44 \times 120 = \$61,852.80$$

total principal payment (principal reduction):

$$\$80,000 - 61,081.77^* = \$18,918.23$$

*PV of loan at the end of year 10

total interest payment:

$$\$61,852.80 - \$18,918.23 = \$42,934.57$$

(f) Step 1, Solve for loan balance at the end of month 49:

$$n = 49$$

$$i = 6\%/12 \text{ or } 0.50$$

$$\text{PMT} = \$515.44$$

$$\text{PV} = -\$80,000$$

Solve for loan balance:

$$\text{PV} = \$73,608.28$$

Step 2, Solve for the interest payment at month 50:

interest payment:

$$\$73,608.28 \times (.06/12) = \$368.04$$

principal payment:

$$\$515.44 - \$368.04 = \$147.40$$

Problem 4-3

(a) Monthly payment PMT (n,i,PV, FV) = \$599.55

Solution:

$$n = 30 \times 12 \text{ or } 360$$

$$i = 6\%/12 \text{ or } 0.50$$

$$\text{PV} = -\$100,000$$

$$\text{FV} = 0$$

Solve for payment:

$$\text{PMT} = \$599.55$$

(b) Quarterly Payment PMT (n,i,PV, FV) = \$1,801.85

Solution:

$$n = 30 \times 4 \text{ or } 120$$

$$i = 6\%/4 \text{ or } 1.50$$

$$\text{PV} = -\$100,000$$

$$\text{FV} = 0$$

Solve for payment:

$$\text{PMT} = \$1,801.85$$

(c) Annual Payment PMT (n,i,PV, FV) = \$7,264.89

Solution:

$$n = 30$$

$$i = 6\%$$

$$\text{PV} = -\$100,000$$

$$\text{FV} = 0$$

Solve for payment:

$$\text{PMT} = \$7,264.89$$

(d) Weekly Payment (n,i,PV, FV) = \$138.26

Solution:

$$\begin{array}{rcl} n & = & 52 \times 30 \text{ or } 1,560 \\ i & = & 6\%/52 \text{ or } 0.12 \\ PV & = & -\$100,000 \\ FV & = & 0 \end{array}$$

Solve for payment:

$$PMT = \$138.26$$

Problem 4-4

Monthly:

total principal payment: \$100,000

total interest:

$$(\$599.55 \times 360) - \$100,000 = \$115,838$$

Quarterly:

total principal payment: \$100,000

total interest:

$$(\$1,801.85 \times 120) - \$100,000 = \$116,222$$

Annually:

total principal payment: \$100,000

total interest:

$$(\$7,264.89 \times 30) - \$100,000 = \$117,946.70$$

Weekly:

total principal payment: \$100,000

total interest:

$$(\$138.26 \times 1560) - \$100,000 = \$115,685.60$$

The greatest amount of interest payable is with the Annual Payment Plan because you are making payments less frequently. Therefore, the balance is reduced slower and interest is paid on a larger loan balance each period.

Problem 4-5

(a) Monthly Payment PMT (n,i,PV,FV):

Solution:

$$\begin{array}{rcl} n & = & 20 \times 12 \text{ or } 240 \\ i & = & 6\%/12 \text{ or } 0.50 \\ PV & = & -\$100,000 \\ FV & = & 0 \end{array}$$

Solve for payment:

$$PMT = \$716.43$$

(b) Entire Period:

Monthly Payment PMT (n,i,PV,FV):

total payment:

$$\$716.43 \times 240 = \$171,943.45$$

total principal payment: \$100,000

total interest:

$$\$171,943.45 - 100,000 = \$71,943.45$$

(c) Outstanding loan balance if repaid at end of year eight = \$73,415.98

Solution:

$$\begin{array}{rcl} n & = & 96 \\ i & = & 6\%/12 \text{ or } 0.50 \\ PMT & = & -\$716.43 \\ PV & = & \$100,000 \end{array}$$

Solve for mortgage balance:

$$FV = \$73,416.22$$

Total interest collected:

$$\begin{array}{l} \text{total payment} + \text{mortgage balance} - \text{principal} \\ \$716.43 \times (8 \times 12) + \$73,416.22 - 100,000 \\ \text{total interest collected} = \$42,193.50 \end{array}$$

(d) Step 1, Solve for the loan balance at the end of year 8:

$$\begin{array}{rcl} n & = & 96 \\ i & = & 6\%/12 \text{ or } 0.50 \\ PMT & = & -\$716.43 \\ PV & = & \$100,000 \end{array}$$

Solve for loan balance:

$$FV = \$73,416.22$$

After reducing the loan by \$5,000, the balance is:

$$\$73,416.22 - 5,000 = \$68,416.22$$

(e) The new loan maturity will be 131 months after the loan is reduced at the end of year 8.

Solution:

$$\begin{array}{rcl} i & = & 6\%/12 \text{ or } 0.50 \\ PMT & = & -\$716.43 \\ PV & = & \$68,416.22 \\ FV & = & 0 \end{array}$$

Solve for maturity:

$$n = 131 \text{ (months)}$$

(f) The new payment would be \$667.64

Solution:

$$\begin{array}{rcl} i & = & 6\%/12 \text{ or } 0.50 \\ n & = & 12 \times 12 \text{ or } 144 \\ PV & = & \$68,416.22 \\ FV & = & 0 \end{array}$$

Solve for payment:

$$PMT = -\$667.64$$

Problem 4-6

Step 1, Solve for the original monthly payment:

$$\begin{aligned}i &= 6\%/12 \text{ or } 0.50 \\n &= 30 \times 12 \text{ or } 360 \\PV &= -\$75,000 \\FV &= 0\end{aligned}$$

Solve for payment:

$$PMT = \$449.66$$

Step 2, Solve for current balance:

$$\begin{aligned}i &= 6\%/12 \text{ or } 0.50 \\n &= 10 \times 12 \text{ or } 120 \\PV &= -\$75,000 \\PMT &= \$449.66\end{aligned}$$

Solve for mortgage balance:

$$FV = \$62,764.29$$

(a) New Monthly Payment = \$378.02

Solution:

$$\begin{aligned}i &= 6\%/12 \text{ or } 0.50 \\n &= 12 \times 20 \text{ or } 240 \\PV &= \$52,764.29^* \\FV &= 0\end{aligned}$$

Solve for payment:

$$PMT = \$378.02$$

(b) New Loan Maturity = 178 months

Solution:

$$\begin{aligned}i &= 6\%/12 \text{ or } 0.50 \\PMT &= -\$449.66 \\PV &= \$52,764.29^* \\FV &= 0\end{aligned}$$

Solve for maturity:

$$n = 178$$

$$^*\$62,764.29 - \$10,000.00$$

Problem 4-7

The loan will be repaid in 145 months.

Solution: n (PMT,i,PV,FV)

$$\begin{aligned}i &= 6.5\%/12 \text{ or } 0.54 \\PMT &= \$1,000 \\PV &= \$100,000 \\FV &= 0\end{aligned}$$

Solve for maturity:

$$n = 145$$

Problem 4-8

The interest rate on the loan is 12.96%.

Solution:

$$\begin{aligned}
 n &= 25 \times 12 \text{ or } 300 \\
 PMT &= -\$900 \\
 PV &= \$80,000 \\
 FV &= 0
 \end{aligned}$$

Solve for the *annual* interest rate:

$$i = 1.08 (\times 12) \text{ or } 12.96\%$$

Problem 4-9

(a) Monthly Payments = \$656.70

Solution:

$$\begin{aligned}
 n &= 10 \times 12 \text{ or } 120 \\
 i &= 7\%/12 \text{ or } 0.58 \\
 PV &= -\$60,000 \\
 FV &= \$20,000
 \end{aligned}$$

Solve for monthly payment:

$$PMT = \$581.10$$

(b) Loan balance at the end of year five = \$43,454.81

Solution:

$$\begin{aligned}
 n &= 5 \times 12 \text{ or } 60 \\
 i &= 7\%/12 \text{ or } 0.58 \\
 PMT &= \$581.10 \\
 FV &= \$20,000
 \end{aligned}$$

Solve for the loan balance:

$$PV = -\$43,454.81$$

Problem 4-10

(a) Monthly Payments = \$400.00

Solution:

$$\begin{aligned}
 n &= 10 \times 12 \text{ or } 120 \\
 i &= 6\%/12 \text{ or } 0.00500 \\
 PV &= -\$80,000 \\
 FV &= \$80,000
 \end{aligned}$$

Solve for monthly payments:

$$PMT = \$400.00$$

(b) Loan balance = \$80,000

Solution:

$$\begin{aligned}
 n &= 12 \times 5 \text{ or } 60 \\
 i &= 6\%/12 \text{ or } 0.0050 \\
 PV &= -\$80,000 \\
 PMT &= \$400.00
 \end{aligned}$$

Solve for loan balance:

$$FV = \$80,000$$

The solution does not have to be calculated because the loan balance will be the same as initial loan amount. This is because it is an interest only loan and there is no loan amortization or reduction of principal.

(c) Yield to the lender $i(n, PV, PMT, FV) = 6\%$

Solution:

$$\begin{array}{rcl} n & = & 12 \times 5 \text{ or } 60 \\ PMT & = & \$400.00 \\ PV & = & -\$80,000 \\ FV & = & \$80,000 \end{array}$$

Solve for the *annual* yield:

$$i = 0.0050 (\times 12) \text{ or } 6\%$$

(d) Yield to the lender $i(n, PV, PMT, FV) = 8\%$

Solution:

$$\begin{array}{rcl} n & = & 12 \times 10 \text{ or } 120 \\ PMT & = & \$400.00 \\ PV & = & -\$80,000 \\ FV & = & \$80,000 \end{array}$$

Solve for the *annual* yield:

$$i = 0.0050 (\times 12) \text{ or } 6\%$$

Problem 4-11

Monthly Payments $PMT(n, i, PV, FV) = \$877.14$

Solution:

$$\begin{array}{rcl} n & = & 10 \times 12 \text{ or } 120 \\ i & = & 6\% / 12 \text{ or } 0.50 \\ PV & = & \$90,000 \\ FV & = & -\$20,000 \end{array}$$

Solve for monthly payments:

$$PMT = \$877.14$$

Yield to the lender $i(n, PV, PMT, FV) = 6.39\%$

Solution:

$$\begin{array}{rcl} n & = & 12 \times 10 \text{ or } 120 \\ PMT & = & \$877.14 \\ PV & = & -\$88,200^* \\ FV & = & \$20,000 \end{array}$$

Solve for the *annual* yield:

$$i = 6.39\%$$

$$^* -\$90,000 \times (100 - 2)\% = -\$88,200 \text{ (amount disbursed)}$$

Step 1, Solve the loan balance if repaid in four years:

Solution:

$$\begin{array}{rcl} n & = & 4 \times 12 \text{ or } 48 \\ i & = & 6\% / 12 \text{ or } 0.50 \\ PV & = & -\$90,000 \\ PMT & = & \$877.14 \end{array}$$

Solve for the loan balance:

$$\text{FV} = \$66,892.65$$

Step 2, Solve for the yield:

Solution:

$$n = 12 \times 4 \text{ or } 48$$

$$\text{PMT} = \$877.14$$

$$\text{PV} = -\$88,200^*$$

$$\text{FV} = \$66,892.65$$

Solve for the *annual* yield:

$$i = i(n, \text{PV}, \text{PMT}, \text{FV})$$

$$i = 6.64\%$$

$$^* -\$90,000 \times (100-2)\% = -\$88,200$$

Problem 4-12

(a) At the end of year ten \$94,622.86 will be due:

Solution:

$$n = 12 \times 10 \text{ or } 120$$

$$i = 8\%/12 \text{ or } 0.67$$

$$\text{PV} = -\$50,000$$

$$\text{PMT} = 0$$

Solve for loan balance:

$$\text{FV} = \$110,982.01$$

(b) Step 1, the loan yield remains 8%, this can be “proved” by solving for loan balance at end of year eight.

Solution:

$$n = 8 \times 12 \text{ or } 96$$

$$i = 8\%/12 \text{ or } 0.67$$

$$\text{PV} = -\$50,000$$

$$\text{PMT} = 0$$

Solve for loan balance:

$$\text{FV} = \$94,622.86$$

Step 2, Solve for the yield:

Solution:

$$n = 8 \times 12 \text{ or } 96$$

$$\text{PMT} = 0$$

$$\text{PV} = -\$50,000$$

$$\text{FV} = \$94,622.86$$

Solve for the *annual* yield:

$$i = .67 (\times 12) \text{ or } 8\%$$

Note: because there were no points, the yield must be the same as the initial interest rate of 8% so no calculations were really necessary.

(c) Yield to lender with one point charged = 8.13%

Solution:

$$n = 8 \times 12 \text{ or } 96$$

$$\begin{aligned}
 \text{PMT} &= 0 \\
 \text{PV} &= -\$49,500^* \\
 \text{FV} &= \$94,622.86 \\
 \text{Solve for the annual yield:} \\
 i &= .68 (\times 12) \text{ or } 8.13\% \text{ (annual rate, compounded monthly)}
 \end{aligned}$$

$$^*-\$50,000 \times (100-1)\% = -\$49,500$$

Problem 4-13

(a)

Property value	=	\$105,000
Principal	=	\$84,000
Interest rate	=	6.00%
Maturity	=	30 years
Loan origination fee	=	\$3,500

Lender will disburse \$84,000.00 less the loan origination fee of \$3,500.00 or \$80,500.00

(b) Monthly payments are based on the loan amount of \$84,000 and would be PMT (n,i,PV,FV):

$$\begin{aligned}
 \text{Monthly Payment} &= \text{PMT (n,i,PV,FV)} \\
 n &= 360 \\
 i &= 6\% \div 12 \\
 \text{FV} &= 0 \\
 \text{PV} &= -\$84,000
 \end{aligned}$$

$$\text{Monthly Payment} = \$503.62$$

The effective interest rate would be:

$$\begin{aligned}
 \text{Effective Interest rate} &= i(n,\text{PV},\text{PMT},\text{FV}) \\
 n &= 360 \\
 \text{PMT} &= \$503.62 \\
 \text{FV} &= 0 \\
 \text{PV} &= \$80,500
 \end{aligned}$$

$$\text{Effective Interest rate} = .533472 \times 12 = 6.40\%$$

(c) Assuming the loan payoff occurs after 5 years, determine the mortgage balance:

Mortgage balance = PV of 300 monthly payments of \$503.62 discounted at 6.00%

$$\begin{aligned}
 \text{PV} &= \text{PV (n,i,PMT,FV)} \\
 n &= 60 \\
 \text{PMT} &= \$503.62 \\
 \text{FV} &= 0 \\
 i &= 6 \div 12 \\
 \text{PV} &= \$78,165.66
 \end{aligned}$$

The effective interest rate would be:

n	=	60
PMT	=	\$503.62
PV	=	-\$80,500
FV	=	\$78,165.66
i	=	i(n,PV,PMT,FV)
i	=	7.02%

The effective interest rate in this part is different from the APR because the loan origination fee is amortized over a much shorter period (5 years instead of 30 years).

(d) With a prepayment penalty of 2% on the outstanding loan balance of \$78,165.66, the penalty would be \$1,563.30.

The effective interest cost would be:

n	=	60
PMT	=	\$503.62
PV	=	-\$80,500
FV	=	\$79,728.96 (\$78,165.66+\$1,563.30)
i	=	i(n,PV,PMT,FV)
i	=	7.35%

This rate is different from the APR because penalty points are not used in the calculation of the APR.

Note: Penalty equals $\$78,165.66 \times .02 = \$1,563.30$

Problem 4-14

Solution: Loan fees are now being loaned by adding \$3,500 to \$84,000. Amount borrowed is now \$87,500.

(a) Lender will now disburse \$87,500 less the loan fees of \$3,500 or \$84,000

(b) Payment calculation is based on new loan amount \$87,500 and new PMT:

PV	=	\$-87,500
n	=	360
FV	=	\$0
i	=	6% ÷ 12
Solve: PMT	=	\$524.61 (vs \$503.62 in problem 13)

APR is now:

PV	=	\$-84,000
PMT	=	\$524.61
N	=	360
FV	=	0
Solve: i	=	6.39% APR

This can be compared to 6.40% in 13(b).

(c) If the loan is repaid after 5 years, the effective interest rate can be calculated as follows:

Solve for mortgage balance:

Part I:

PMT	=	\$524.61
i	=	6% ÷ 12
N	=	60
PV	=	\$87,500
FV	=	\$81,422.56 at end of month 60

Part II: Solve for i or effective interest rate.

$$\begin{array}{rcl} \text{PV} & = & \$-84,000 \\ n & = & 60 \\ \text{FV} & = & \$81,422.56 \\ \text{PMT} & = & \$524.61 \end{array}$$

$$\text{Solve for (i)} = 6.98\%$$

(d) Include prepayment penalty of 2% of \$81,422.56 or \$1,628.45

Solution i:

$$\begin{array}{rcl} \text{PV} & = & \$-84,000 \\ \text{PMT} & = & \$524.61 \\ n & = & 60 \\ \text{FV} & = & \$83,051.01 \end{array}$$

$$i = 7.31\% \text{ effective rate}$$

Problem 4-15

Points required to achieve a yield to 10% for the 25 year loan. Fv is now \$83,186.41 = \$1,663.73 = \$84,850.14

Monthly payments PMT (n,i,PV,FV):

$$\begin{array}{rcl} n & = & 300 \\ i & = & 9\% \div 12 \\ \text{PV} & = & \$95,000 \\ \text{FV} & = & \$0 \end{array}$$

Solve for monthly payments:

$$\text{PMT} = \$797.24$$

PV (n,i,PMT,FV) of 300 payments of \$797.24 discounted at 10% = \$87,733.67

Subtracting \$87,733.67 from \$95,000.00, we get \$7,266.33

The loan origination fee should be \$7,266.33 if the loan is to be repaid after 25 years and the lender requires a 10% yield.

If the loan is expected to be repaid after 10 years, the loan balance at the end of 10 years must be determined:

$$\begin{array}{rcl} n & = & 120 \\ i & = & 9\% \\ \text{PMT} & = & \$797.24 \\ \text{PV} & = & \$95,000 \end{array}$$

Solve for FV:

$$\text{FV} = \$78,601.60$$

Loan balance after 10 years = \$78,601.60

Discounting \$797.24 monthly for 120 months and \$78,601.60 at the end of the 120th month by the desired yield of 10% gives:

Present value = \$89,364.06

Subtracting \$89,364.06 from \$95,000.00, we get \$5,635.94.

The loan origination fee should be \$5,635.94 if the loan is to be repaid after 10 years, and the lender requires a yield of 10%.

Problem 4-16

(a) In order to find which loan is the better choice after 20 years, the effective interest rate for each loan must be calculated.

	<u>Loan A</u>	<u>Loan B</u>
Principal	\$75,000	\$75,000
Nominal interest rate	6.00%	7.00%
Term (years)	30	30
Points	6	2
Payment	\$449.66	\$498.98
Loan Balance after 20 years	\$40,502.43	\$42,975.33
Loan Balance after 5 years	\$69,790.32	\$70,599.14

Loan A

n	=	240
PMT	=	\$449.66
PV	=	-\$70,500
FV	=	\$40,502.43
i	=	i(n,PV,PMT,FV)
i	=	.5525% × 12 = 6.63%

Loan B

n	=	240
PMT	=	\$498.98
PV	=	-\$73,500
FV	=	\$42,975.33
i	=	i(n,PV,PMT,FV)
i	=	.6008% × 12 = 7.21%

Loan A is the better alternative if the loan is repaid after 20 years.

(b) This part is solved the same as (a) except using the assumption that the loan is repaid after 5 years.

Loan A

n	=	60
PMT	=	\$449.66
PV	=	-\$70,500
FV	=	\$69,790.32
i	=	i(n,PV,PMT,FV)
i	=	.623917% × 12 = 7.49%

Note: Balance at the end
of 60 months = \$69,790.32

Loan B

n	=	60
---	---	----

Note: Balance at the end
of 60 months = \$70,599.14

PMT	=	\$498.98
PV	=	-\$73,500
FV	=	\$70,599.14
i	=	i(n,PV,PMT,FV)
i	=	.624417 \times 12 = 7.49%

The borrower would be indifferent between the two loans if the repayment period is 5 years.

Problem 4-17

(a) Monthly Payments = \$1,830.61 to be made to the borrower

Solution:

n	=	10 \times 12 or 120
i	=	6%/12 or 0.005
PV	=	0
FV	=	-\$300,000

Solve for monthly payments:

PMT	=	\$1,830.61
-----	---	------------

(b) The borrower will have received monthly payments of \$1,830.61 during months 1 to 36

Solve for loan balance at the end of month 36

Solution:

n	=	36
i	=	6%/12 or 0.005
PV	=	0
PMT	=	\$1,830.61

Solve for loan balance*:

FV	=	-\$72009.27
----	---	-------------

*Note that this is equivalent to finding the Future Value of a \$1,830.61 monthly ordinary annuity at an annual rate of 6%, compounded monthly.

(c) The borrower will receive \$2,000 per month for 50 months and then will receive monthly payments of \$626.22 during months 51 to 120. This is calculated as follows:

Step 1, Solve for loan balance at the end of month 50

Solution:

n	=	50
i	=	6%/12 or 0.005
PV	=	0
PMT	=	\$2,000

Solve for loan balance at the end of month 50:

FV	=	-\$113,290.33
----	---	---------------

Step 2, Solve for payments during months 51 to 120

Solution:

n	=	120-50 or 70
i	=	6%/12 or 0.005
PV	=	\$113,290.33
FV	=	-\$300,000

Solve for monthly payments beginning in month 51 through 120 or for the next 70 months:

PMT	=	\$1,667.82
-----	---	------------

Problem 4-18

Find the balance at the end of 5 years for a fully amortizing \$200,000, 5% mortgage with a 25 year amortization schedule:

$$\begin{array}{ll} \text{PV} & = -200,000 \\ i & = 5\% \div 12 \\ n & = 300 \end{array} \qquad \begin{array}{ll} \text{FV} & = 0 \\ \text{Solve PMT} & = \$1,169.18 \end{array}$$

Solve for balance at end of 5 years:

$$\begin{array}{ll} i & = 5\% \\ n & = 240 \end{array} \qquad \begin{array}{ll} \text{PMT} & = \$1,169.18 \\ \text{FV} & = 0 \\ \text{Solve PV} & = -\$177,160.38 \end{array}$$

Problem 4- 19

CAM loan:

(a) Calculate constant monthly amortization:

$$\$125,000 \div 240 \text{ months} = \$520.83 \text{ per month}$$

Calculate Monthly Interest:

Month	Beg. Balance	Rate	Interest	Amortization	Total Payment	End Balance
1	125,000	*6%/12	625.00	520.83	1145.83	124,479.17
2	124,479.17	*6%/12	622.40	520.83	1143.23	123,958.34
3	123,958.34	*6%/12	619.79	520.83	1140.62	123,437.51
4	123,437.51	*6%/12	617.18	520.83	1138.01	122,916.68
5	122,916.68	*6%/12	614.58	520.83	1135.41	122,395.85
6	122,395.85	*6%/12	611.98	520.83	1132.80	121,875.02

(b) For a constant payment loan (CPM) we have:

$$\begin{array}{ll} \text{PV} & = -\$125,000 \\ n & = 240 \\ i & = 6\% \div 12 \\ \text{FV} & = 0 \end{array}$$

Solve PMT = \$895.54

(c) In the absence of point and origination fees, the effective interest rates on both loans will be an annual rate of 6%, compounded monthly. This is true regardless of when either of the loans are repaid. Monthly payments are different, however i is the same for both loans.

Problem 4-20

- (a) Determine monthly payments based on interest being accrued daily.

Solve for interest due at the end of month one:

$$\begin{array}{rcl}
 \text{PV} & = & \$50,000 \\
 i & = & 6\% \div 360 \\
 n & = & 360 / 12 = 30^* \\
 \text{Solve for FV} & & \\
 \text{FV} & = & \$50,250.61 \quad \text{Thus the interest that accrues each month is } \$250.61
 \end{array}$$

*Assumes a 360 day year to have an even number of months. Answer will be slightly different if you use a 365 day year.

Because this is an “interest only” loan, payments of \$250.61 will be due at the end of each month for 360 months.

- (b) The loan balance will be \$50,000 at the end of each month for the life of the loan. At the end of 30 years it also will be \$50,000.

- (c) The equivalent annual rate will be:

$$\begin{array}{rcl}
 \text{FV} & = & \$50,000 \\
 n & = & 360 \\
 \text{PV} & = & -\$50,000 \\
 \text{PMT} & = & 250.61
 \end{array}$$

Solve for $i = .5012 \times 12 = 6.01\%$ (annual rate, compounded monthly)Or $(\$50,250.61 - \$50,000) / \$50,000 = .5012 \times 12 = 6.01\%$ **Problem 4- 21 Comprehensive Review Problem**

Loan = 100,000, 6% interest, 20 years

A. Monthly payments if

- (1) Fully amortizing:

$$\begin{array}{rcl}
 \text{PV} = -100,000 & n & = 240 \\
 i = 6\% & \text{Solve PMTs} & = \$716.43 \\
 \text{FV} = 0 & &
 \end{array}$$

- (2) Partial amortizing:

$$\begin{array}{rcl}
 \text{PV} = -100,000 & n & = 240 \\
 i = 6\% & \text{Solve PMTs} & = \$608.22 \\
 \text{FV} = \$50,000 & &
 \end{array}$$

- (3) Interest only

$$\begin{array}{rcl}
 \text{PV} = 100,000 & n & = 240 \\
 i = 6\% & \text{Solve PMTs} & = \$500.00 \\
 \text{FV} = 100,000 & &
 \end{array}$$

- (4) Negative amortization:

$$\begin{array}{rcl}
 \text{PV} = -100,000 & n & = 240 \\
 i = 6\% & \text{Solve PMTs} & = \$391.78 \\
 \text{FV} = 150,000 & &
 \end{array}$$

B. Loan Balances for A.1. – A.4 after 5 years

A.1	PMTs = \$716.43	FV	= 0
	i = 6%	<u>Solve</u> PV	= \$84,899.60
	n = 240		
A.2	PMTs = \$608.22	FV	= 50,000
	i = 6%	<u>Solve</u> PV	= \$92,450.33
	n = 180		
A.3	PMTs = \$500.00	FV	= 100,000
	i = 6%	<u>Solve</u> PV	= \$100,000.00
	n = 180		
A.4	PMTs = \$391.78	FV	= 150,000
	i = 6%	<u>Solve</u> PV	= \$107,549.67
	n = 180		

C. Interest at the end of month 61 for A.1 – A.4

A.1	\$84,899.60 * .005	= \$424.50
A.2	\$92,450.33 * .005	= \$479.36
A.3	\$100,000.00 * .005	= \$500.00
A.4	\$107,549.67 * .005	= \$537.75

D. APR* for loans in A.1 – A.4

A.1	PV = -97,000, PMT = \$716.43, FV = 0, n = 240 <u>Solve</u> i = 6.38%
A.2	PV = -97,000, PMT = \$608.22, FV = 50,000, n = 240 <u>Solve</u> i = 6.31%
A.3	PV = -97,000, PMT = \$500.00, FV = 100,000, n = 240 <u>Solve</u> i = 6.26%
A.4	PV = -97,000, PMT = \$391.78, FV = 150,000, n = 240 <u>Solve</u> i = 6.23%

*Solution shown based on calculation – final answers may be rounded to nearest 1/4%

E. Effective yield if loan prepaid EOY₅. Balances must be calculated at EOY₅ for each loan (not shown see solution 4-21B).

A.1	PV = -97,000, PMT = \$716.43, FV = 84,899.60 n = 60 <u>Solve</u> i = 6.76%
A.2	PV = -97,000, PMT = \$608.22, FV = 92,450.33 n = 60 <u>Solve</u> i = 6.73%
A.3	PV = -97,000, PMT = \$500.00, FV = 100,000.00 n = 60 <u>Solve</u> i = 6.70%
A.4	PV = -97,000, PMT = \$391.78, FV = 107,549.67 n = 60 <u>Solve</u> i = 6.68%

F. “Interest only” monthly payments in A.1 = \$100,000 × (6% ÷ 12) or \$500 per month for 36 mos. What must payments be from yr. 4-17 to fully amortize the loan at the end of 240 mos.?

Part 1:

PV = -100,000

i = 6%

n = 36

PMT = \$500.00

Solve FV = \$100,000

Part 2:

PV = -100,000

i = 6% ÷ 12

$n = 204$
 $FV = 0$
Solve $PMT = \$783.10$

- G. (1) Total PMTs = $(391.78 \times 240) + 150,000 = \$244,027.20$
Interest = 144,027.20
Amortization (Principal) = 100,000

(2) $n = 204$ $FV = 150,000$
 $PMTs = \$391.78$ $i = 6\%$
Solve $PV = \$104,256.20$ balance

(3) 12% because there are no points

(4) 4 points charged, loan payoff 36 months, what is effective interest rate?

$PV = -96,000$ $PMT = 783.10$
 $n = 36$ Solve $i = .621989\% \times 12 = 7.46\%$
 $FV = 107,550$

Problem 4-22

The effective cost is now 12.64% versus 12.82%.

Problem 4-23

The loan balance is now \$61,680 versus \$63,793.

Chapter 4 Appendix

Questions

Question 4-A1

Why do monthly mortgage payments increase so sharply during periods of inflation? What does the tilt effect have to do with this?

In order to receive the full interest necessary to leave enough for a real return and risk premium over the life of the loan, more “real dollars” must be collected in the early years of the loan (payments collected toward the end of the life of the mortgage will be worth much less in purchasing power.)

Tilting - the real payment stream in the early years have to make up for the loss in purchasing power in later years.

Question 4-A2

As inflation increases, the impact of the tilt effect is said to become even more burdensome on borrowers. Why is this so?

With the general rate of inflation and growth in the economy, borrower incomes will grow gradually or on a year-by-year basis. However, as expected inflation increases, lenders must build estimates of the full increase into current interest rates “up front” or when the loan is made. This causes a dramatic increase in required real monthly payments relative to the borrower’s current real income.

Problems

Problem 4A-1

(a)	Year	Payment
	2	$\$498.57 \times 1.075 = \535.96
	3	$\$535.96 \times 1.075 = \576.16
	4	$\$576.16 \times 1.075 = \619.37
	5	$\$619.37 \times 1.075 = \665.82

(b) Loan payment with a CPM

$$\begin{aligned} \text{Monthly payment: } & \$63,000.00 \times (\text{MLC}, 12\%, 30 \text{ years}) = \text{monthly payment} \\ & \$63,000.00 \times 0.010286 = \$648.03 \end{aligned}$$

(c) Effective yield of the GPM is the loan is originated with 2 discount points

$$\begin{aligned} CF_0 &= 61,740 \\ CF_j &= 498.57 \\ n_j &= 12 \\ CF_j &= 535.96 \\ n_j &= 12 \\ CF_j &= 576.16 \\ n_j &= 12 \\ CF_j &= 619.76 \\ n_j &= 12 \\ CF_j &= 665.82 \\ n_j &= 12 \\ CF_j &= 715.76 \\ n_j &= 11^* \\ CF_j &= 66,982.97 + 715.76 \\ n_j &= 1^* \end{aligned}$$

* Note the last scheduled payment and the loan balance are received by the lender at the same time or at the end of month 84. Therefore they must be added.

The effective yield is the internal rate of return that equates the payment stream with the initial loan amount.
Effective Yield = 12.41%

Problem 4A-2

Excel template Ch4 GPM from course website modified to solve this problem.

Chapter 4				
Loan Balance on GPM				
Spreadsheet Limitations: Projections for 5 years				
Input Assumptions				
Loan Amount	\$100,000			
Interest Rate	9.00%			
Loan Term	25	years		
Pmt Increase	7.50%			
Inc. Years	3			
Points	4			
Lender's Yield (after 7 yrs)				
				10.02%
INITIAL PAYMENT CALCULATION:				
(1)	(2)	(3)	(4)	(5)
Payment Period	Payment	MPVIFA	MPVIF	(2x3x4)
1	1.00000	11.43491	1.00000	11.43491
2	1.07500	11.43491	0.91424	11.23830
3	1.15563	11.43491	0.83583	11.04507
4	1.24230	11.43491	0.76415	10.85516
5	1.24230	11.43491	0.69861	9.92420
6-25	1.24230	111.14495	0.63870	88.18848
			Pmt factor =	142.68613
Initial Payment:				
	\$700.84	<=====	\$100,000	142.68613
			Loan Amt	/ Pmt Factor

- Calculation of initial payment is shown above. Payments for years 2 through 5 are shown in the exhibit below.
- The loan balance at the end of year 3 is shown in the exhibit below.

LOAN AMORTIZATION SCHEDULE:					
Month	Payment	Interest	Principle	Balance	CPM Payment
0	(\$96,000)	<==Amount Dispersed		\$100,000	
1	\$700.84	\$750.00	(\$49.16)	\$100,049	\$839.20
2	\$700.84	\$750.37	(\$49.53)	\$100,099	\$839.20
3	\$700.84	\$750.74	(\$49.90)	\$100,149	\$839.20
4	\$700.84	\$751.11	(\$50.28)	\$100,199	\$839.20
5	\$700.84	\$751.49	(\$50.65)	\$100,250	\$839.20
6	\$700.84	\$751.87	(\$51.03)	\$100,301	\$839.20
7	\$700.84	\$752.25	(\$51.42)	\$100,352	\$839.20
8	\$700.84	\$752.64	(\$51.80)	\$100,404	\$839.20
9	\$700.84	\$753.03	(\$52.19)	\$100,456	\$839.20
10	\$700.84	\$753.42	(\$52.58)	\$100,509	\$839.20
11	\$700.84	\$753.81	(\$52.98)	\$100,562	\$839.20
12	\$700.84	\$754.21	(\$53.37)	\$100,615	\$839.20
EOY 1					
13	\$753.40	\$754.61	(\$1.21)	\$100,616	\$839.20
14	\$753.40	\$754.62	(\$1.22)	\$100,617	\$839.20
15	\$753.40	\$754.63	(\$1.23)	\$100,619	\$839.20
16	\$753.40	\$754.64	(\$1.24)	\$100,620	\$839.20
17	\$753.40	\$754.65	(\$1.25)	\$100,621	\$839.20
18	\$753.40	\$754.66	(\$1.26)	\$100,622	\$839.20
19	\$753.40	\$754.67	(\$1.27)	\$100,624	\$839.20
20	\$753.40	\$754.68	(\$1.27)	\$100,625	\$839.20
21	\$753.40	\$754.69	(\$1.28)	\$100,626	\$839.20
22	\$753.40	\$754.70	(\$1.29)	\$100,627	\$839.20
23	\$753.40	\$754.71	(\$1.30)	\$100,629	\$839.20
24	\$753.40	\$754.72	(\$1.31)	\$100,630	\$839.20
EOY 2					
25	\$809.91	\$754.73	\$55.18	\$100,575	\$839.20
26	\$809.91	\$754.31	\$55.60	\$100,519	\$839.20
27	\$809.91	\$753.89	\$56.01	\$100,463	\$839.20
28	\$809.91	\$753.47	\$56.43	\$100,407	\$839.20
29	\$809.91	\$753.05	\$56.86	\$100,350	\$839.20
30	\$809.91	\$752.62	\$57.28	\$100,293	\$839.20
31	\$809.91	\$752.19	\$57.71	\$100,235	\$839.20
32	\$809.91	\$751.76	\$58.14	\$100,177	\$839.20
33	\$809.91	\$751.33	\$58.58	\$100,118	\$839.20
34	\$809.91	\$750.89	\$59.02	\$100,059	\$839.20
35	\$809.91	\$750.44	\$59.46	\$100,000	\$839.20
36	\$809.91	\$750.00	\$59.91	\$99,940	\$839.20
EOY 3					
37	\$870.65	\$749.55	\$121.10	\$99,819	\$839.20
38	\$870.65	\$748.64	\$122.01	\$99,697	\$839.20
39	\$870.65	\$747.73	\$122.92	\$99,574	\$839.20
40	\$870.65	\$746.80	\$123.85	\$99,450	\$839.20
41	\$870.65	\$745.87	\$124.78	\$99,325	\$839.20
42	\$870.65	\$744.94	\$125.71	\$99,199	\$839.20
43	\$870.65	\$744.00	\$126.65	\$99,073	\$839.20
44	\$870.65	\$743.05	\$127.60	\$98,945	\$839.20
45	\$870.65	\$742.09	\$128.56	\$98,817	\$839.20
46	\$870.65	\$741.12	\$129.53	\$98,687	\$839.20
47	\$870.65	\$740.15	\$130.50	\$98,557	\$839.20
48	\$870.65	\$739.17	\$131.48	\$98,425	\$839.20
EOY 4					
49	\$870.65	\$738.19	\$132.46	\$98,293	\$839.20
50	\$870.65	\$737.20	\$133.46	\$98,159	\$839.20
51	\$870.65	\$736.19	\$134.46	\$98,025	\$839.20
52	\$870.65	\$735.19	\$135.46	\$97,889	\$839.20
53	\$870.65	\$734.17	\$136.48	\$97,753	\$839.20
54	\$870.65	\$733.15	\$137.50	\$97,615	\$839.20
55	\$870.65	\$732.11	\$138.54	\$97,477	\$839.20
56	\$870.65	\$731.08	\$139.57	\$97,337	\$839.20
57	\$870.65	\$730.03	\$140.62	\$97,197	\$839.20
58	\$870.65	\$728.97	\$141.68	\$97,055	\$839.20
59	\$870.65	\$727.91	\$142.74	\$96,912	\$839.20
60	\$97,639.01	\$726.84	\$143.81	\$96,768	\$839.20
EOY 5					

c.

Input Assumptions		
Loan Amount	\$100,000	
Interest Rate	9.00%	
Loan Term	25	years
Pmt Increase	7.50%	
Inc. Years	3	
Points	4	
Lender's Yield (after 7 yrs)		
10.02%		

Problem 4A-3

The initial payment would now be \$518.09 as shown below.

Chapter 4				
Loan Balance on GPM				
Spreadsheet Limitations: Projections for 7 years				
Input Assumptions				
Loan Amount	\$60,000			
Interest Rate	12.00%			
Loan Term	30	years		
Pmt Increase	5.00%			
Inc. Years	5			
Points	0			
Lender's Yield (after 7 yrs)				
12.00%				
INITIAL PAYMENT CALCULATION:				
(1)	(2)	(3)	(4)	(5)
Payment Period	Payment	MPVIFA	MPVIF	(2x3x4)
1	1.00000	11.25508	1.00000	11.25508
2	1.05000	11.25508	0.88745	10.48773
3	1.10250	11.25508	0.78757	9.77269
4	1.15763	11.25508	0.69892	9.10640
5	1.21551	11.25508	0.62026	8.48555
6-25	1.27628	94.94655	0.55045	66.70268
			Pmt factor =	115.81012
Initial Payment:				
\$518.09 <=====				
<div> <div>\$60,000</div> <div>115.81012</div> <div>Loan Amt / Pmt Factor</div> </div>				