Math 170E: Homework 4

Due: Sat. 11th February by 11:59pm PDT via Gradescope

Submit answers to all problems via Gradescope. The reader will grade three problems each out of five points. Up to five further points will be awarded based on the proportion of the remaining problems that are completed.

Please make sure that your submission is readable. If your pencil is too faint, get a thicker one. If your handwriting is cramped and small, write bigger and use more paper. Please use simple plain paper or lined paper (e.g. please avoid graph paper etc.). It is your responsibility to ensure that your submission is readable. If we cannot read a solution, we may refuse to grade it. Thank you!

I encourage you to discuss and work on problems with other students in the class. Nevertheless, the solutions you present have to be your own. In particular, if the solution you present is identical to someone else's, or it is identical to some other resource (book, online, etc.), this will be considered cheating.

1. Customers arriving at a coffee shop ask for a hot drink with probability $p \in (0, 1)$ and an iced drink with probability $1 - p$, independently of other customers. If a customer asks for a hot drink, they ask for a tea with probability $q \in (0, 1)$ and a coffee with probability $1 - q$, independently of other customers. In a given day, the shop serves n customers.
   (a) How many iced drinks does the coffee shop expect to have served that day?
   (b) Let X be the number of customers that asked for a hot tea. What is the PMF of X?
   (c) How many hot coffees does the coffee shop expect to have served that day?

2. In a lab experiment involving inorganic syntheses of molecular precursors to organometallic ceramics, the final step of a five-step reaction involves the formation of a metal–metal bond. The probability of such a bond forming is 0.20. Let X equal the number of successful reactions out of 25 such experiments.
   (a) Find the probability that X is at most 4.
   (b) Find the probability that X is at least 5.
   (c) Give the mean, variance, and standard deviation of X.

3. A hospital obtains 40% of its flu vaccine from Company A, 50% from Company B, and 10% from Company C. From past experience, it is known that 3% of the vials from A are ineffective, 2% from B are ineffective, and 5% from C are ineffective. The hospital tests

five vials from each shipment. If at least one of the five is ineffective, find the conditional probability that the whole shipment came from Company C.

4. Find the expected value and the variance of the number of times one must throw a die until the outcome 1 has occurred 4 times.

5. For each question on a multiple-choice test, there are five possible answers, of which exactly one is correct. If a student selects answers at random, give the probability that the first question answered correctly is question 4.

6. At all times, a pipe-smoking mathematician carries 2 matchboxes—1 in their left-hand pocket and 1 in their right-hand pocket. Each time they need a match, they are equally likely to take it from either pocket. Consider the moment when the mathematician first discovers that one of their matchboxes is empty. If it is assumed that both matchboxes initially contained N matches, what is the probability that there are exactly k matches, $k = 0, 1, ..., N$, in the other box?
Hint: Let $E = \{\text{RH matchbox is empty and LH matchbox has k matches}\}$. What does E occurring mean?

7. Suppose that earthquakes occur in the western portion of the United States at a rate of 2 per week. Suppose further that the number of earthquakes that occur in a given week can be well-approximated by a Poisson random variable. Find the probability that at least 3 earthquakes occur during the next 2 weeks.

8. The Pegasus Insurance Company has introduced a policy that covers certain forms of personal injury with a standard payment of $\$100,000$. The yearly premium for the policy is $25. On average, 100 claims per year lead to payment. There are more than one million policyholders. What is the probability that more than 15 million dollars will have to be paid out in the space of a year?
(You might find it helpful to use an online Poisson distribution calculator to evaluate the probability.)