Electromagnetic Radiation

Electromagnetic waves carry momentum p with momentum flow rate:

$$\frac{dv}{dt} = \frac{dp}{dt}$$

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{E_{max}B_{max}}{\mu_0 c} \qquad p = mv$$

Let the intensity of solar radiation near the earth be $S_{avg} = I$ and the speed of light be c. Calculate the velocity change in time t to a solar-wind-powered satellite of mass m if solar wind hits its surface as shown in the figure below where the inner surface has reflectivity ρ_i and the outer surface has reflectivity ρ_o, and the reflectivity of material i, ρ_i, is 0 if the material purely absorbs radiation and 1 if it purely reflects radiation:

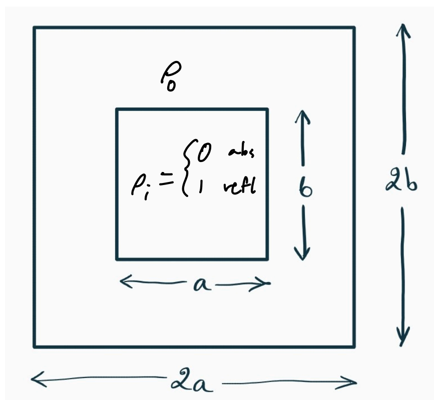


Figure 1: Electromagnetic Radiation - Satellite Surface

Solution:

m is const $\Rightarrow \dfrac{dv}{dt} = \dfrac{dp}{dt} = \dfrac{S}{c}A$

$\rho_i = 0 \Rightarrow P_{rad} = \dfrac{I}{c} = \dfrac{S}{c}$

$\rho_i = 1 \Rightarrow P_{rad} = \dfrac{2I}{c} = \dfrac{2S}{c}$

$$\boxed{\frac{dv}{dt} = \begin{cases} \frac{I}{c}ab + \frac{3I}{c}ab & \rho_i = 0 \\ \frac{2I}{c}ab + \frac{6I}{c}ab & \rho_o = 1 \end{cases}}$$

EM Waves in a Cavity

Consider electromagnetic standing waves in a cavity with two parallel highly conducting walls separated by a distance of L. For the fourth overtone, write the equations for E and B as functions of time and position.

Solution:

$$f_4 = \frac{4c}{2L} = \frac{2c}{L} \Rightarrow \omega_4 = 2\pi f_4 = \frac{4\pi c}{L}$$

$$\lambda_4 = \frac{2L}{4} = \frac{L}{2} \Rightarrow K_4 = \frac{2\pi}{\lambda_4} = \frac{4\pi}{L}$$

$$\Rightarrow \boxed{\begin{aligned} E_y(x,t) &= -2\, E_{max} \sin\left(\frac{4\pi}{L} x\right) \sin\left(\frac{4\pi c}{L} t\right) \\ B_z(x,t) &= -2\, B_{max} \cos\left(\frac{4\pi}{L} x\right) \cos\left(\frac{4\pi c}{L} t\right) \end{aligned}}$$

(Challenge Problem) Poynting Vector

Consider the electromagnetic wave in vacuum with an electric field:

$$\boldsymbol{E}(x,t) = E_{max}\cos(kx - \omega t)\hat{j}$$

and a magnetic field:

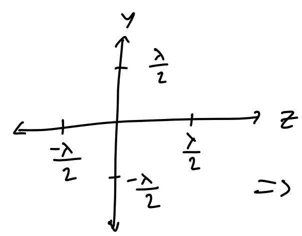
$$\boldsymbol{B}(x,t) = B_{max}\cos(kx - \omega t)\hat{k}.$$

Calculate the energy flow between $t_0 = 0$ and $t_1 = \frac{T}{2}$ passing through a square region at $x = \frac{\lambda}{2}$ from $y = -\frac{\lambda}{2}$ to $y = \frac{\lambda}{2}$ and from $z = -\frac{\lambda}{2}$ to $z = \frac{\lambda}{2}$.

Solution:

$k = \frac{2\pi}{\lambda}$ & $f = \frac{c}{\lambda} \Rightarrow \omega = \frac{2\pi c}{\lambda}$

total energy flow: $P = \oint \vec{S} \cdot d\vec{A}$



s.t. $\vec{S}(x,t) = \frac{E_{max} B_{max}}{2\mu_0} \cos^2(kx - \omega t)\,\hat{i}$

$\Rightarrow A = \left(\frac{\lambda}{2}\right)^2 = \frac{\lambda^2}{4}$

$$\Rightarrow P = \oint \vec{S} \cdot d\vec{A} = \frac{E_{max} B_{max}}{2\mu_0} \int_0^{T/2} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} \cos^2(kx - \omega t)\; dx\, dt$$

$$= \frac{E_{max} B_{max}}{2\mu_0} \int_0^{T/2} \frac{\sin(k\lambda)\cos(2\omega t)}{2k} + \frac{\lambda}{2}\; dt = \frac{E_{max} B_{max}}{2\mu_0} \cdot \frac{\lambda}{2} \int_0^{T/2} dt = \frac{E_{max} B_{max}}{2\mu_0} \cdot \frac{\lambda}{2} \cdot \frac{T}{2} = \boxed{\frac{E_{max} B_{max}\, \lambda T}{8\mu_0}}$$