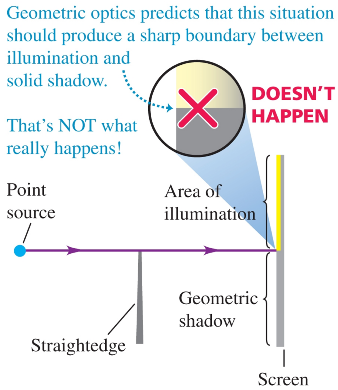
Chapter 36: Diffraction

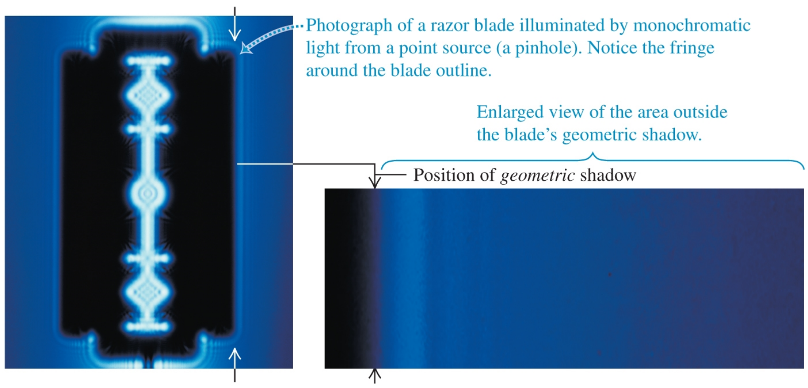
Diffraction (1 of 2)

- According to geometric optics, when an opaque object is placed between a point light source and a screen, the shadow of the object forms a perfectly sharp line.
- However, the wave nature of light causes interference patterns, which blur the edge of the shadow.
- This is one effect of **diffraction.**



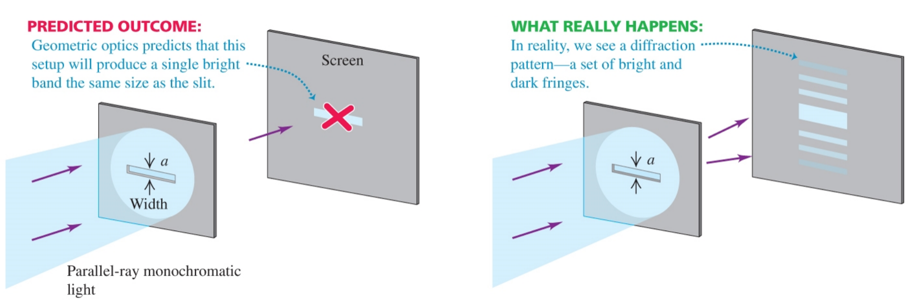
Diffraction (2 of 2)

- The photograph below was made by placing a razor blade halfway between a pinhole, illuminated by monochromatic light, and a photographic film.
- The film recorded the shadow cast by the blade.
- Note the fringe pattern around the blade outline, which is caused by diffraction.



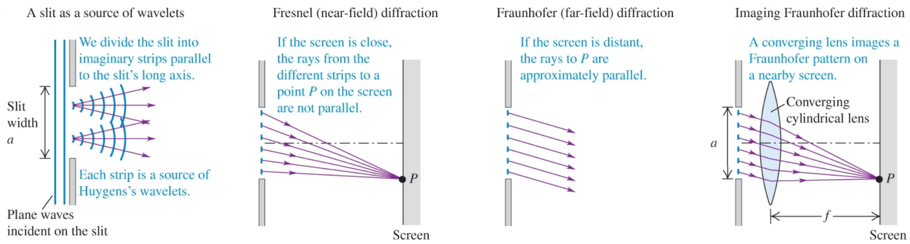
Diffraction from a Single Slit (1 of 2)

- If we shine light through a single slit with a screen behind it, a diffraction pattern is produced, with a set of bright and dark fringes.
- According to geometric optics, the transmitted beam should have the same cross section as the slit. What is *actually* observed is the diffraction pattern shown in the figure.
- The diffraction pattern consists of a central bright band, which may be much broader than the width of the slit, bordered by alternating dark and bright bands with rapidly decreasing intensity.



Diffraction from a Single Slit (2 of 2)

- There are two types of diffraction that can occur in this case.
- **Fresnel diffraction** occurs when the point source and the screen are relatively close to the obstacle forming the diffraction pattern.
- **Fraunhofer diffraction** takes place when the source, obstacle, and screen are far enough apart that we can consider all lines from the source, to the obstacle, and to the screen to be parallel.

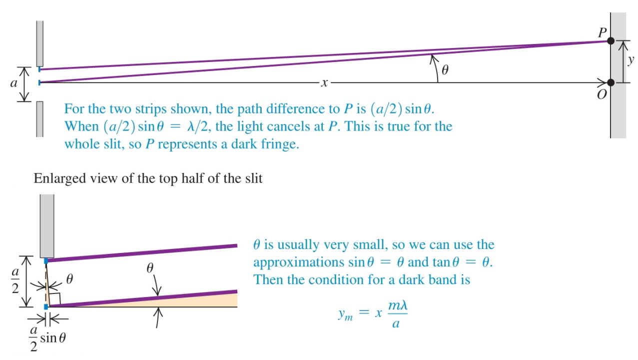


Locating the Dark Fringes (1 of 3)

- The Fraunhofer diffraction pattern is the easiest to consider. We can treat the single slit of width a as though there were a narrow slit at the top and another narrow slit at the center of single slit.
- The path difference between the rays in this case is

$$r_2 - r_1 = \frac{a}{2}\sin\theta,$$

and if the path difference is $\lambda/2$, the two rays destructively interfere.



Locating the Dark Fringes (2 of 3)

- This applies to any pair of rays from the top half of the strip to the bottom half of the strip. Thus, a dark fringe occurs whenever

$$\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2} \quad \rightarrow \quad \sin\theta = \pm\frac{\lambda}{a}.$$

- We can also use the same reasoning to divide the slit into quarters, sixths, and other integer values. A dark fringe therefore occurs when $\sin\theta = \pm 2\lambda/a, \pm 3\lambda/a$, and so on. Therefore, the condition for a dark fringe in single slit diffraction is

$$\sin\theta = \frac{m\lambda}{a}, \quad (m = \pm 1, \pm 2, \pm 3, \ldots).$$

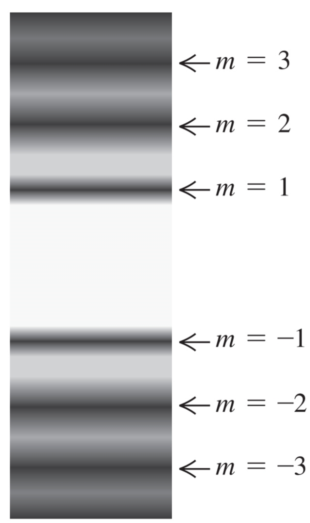
- Visible light typically has a wavelength that is much smaller than the slit width a. Therefore, the angle θ is small enough so that $\sin\theta \approx \theta$:

$$\theta = \frac{m\lambda}{a}, \quad (m = \pm 1, \pm 2, \pm 3, \ldots), \quad \text{(for small angles } \theta \text{ in radians)}.$$

Locating the Dark Fringes (3 of 3)

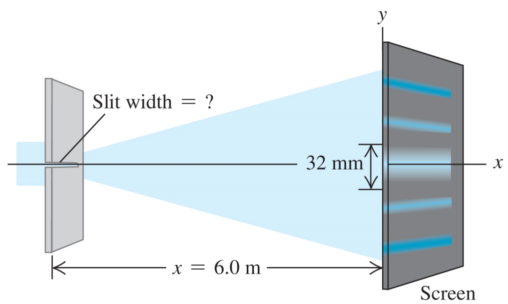
- Shown is the Fraunhofer diffraction pattern from a single horizontal slit.
- It is characterized by a central bright fringe centered at $\theta = 0$, surrounded by a series of dark fringes.
- The central bright fringe is twice as wide as the other bright fringes.
- If the distance to the screen is x, and the vertical distance of the mth dark band from the center of the pattern is y_m, then $\tan\theta = y_m/x$. For small θ, $\tan\theta \approx \theta$ and we get

$$y_m = x\frac{m\lambda}{a} \quad \text{(for } y_m \ll x\text{)}.$$



Example 36.1: Single-Slit Diffraction

You pass 633 nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm. How wide is the slit?



The first minimum corresponds to $m = 1$, and the distance y_1 from the central maximum to the first minimum on either side is half the distance between the two first minima. Therefore,

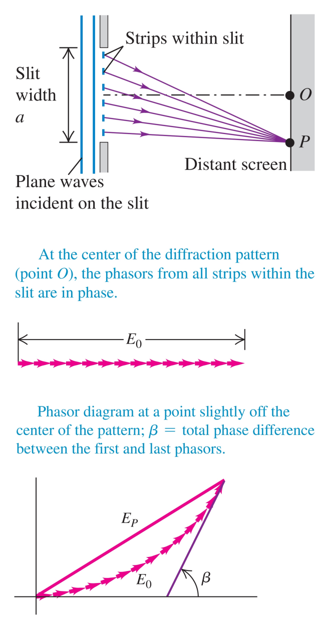
$$y_1 = \frac{32 \text{ mm}}{2} = 16 \text{ mm}.$$

Since $y_1 = x\lambda/a$, we have

$$a = \frac{x\lambda}{y_1} = \frac{(6.0 \text{ m})(633 \times 10^{-9} \text{ m})}{16 \times 10^{-3} \text{ m}} = 2.4 \times 10^{-4} \text{ m} = 0.24 \text{ mm}.$$

Intensity in the Single-Slit Pattern (1 of 5)

- We can derive an expression for the intensity distribution for the single-slit diffraction pattern by using phasor-addition. We imagine a plane wave front at the slit subdivided into a large number of strips.
- At the point O, the phasors are all in phase.
- Now consider wavelets arriving from different strips at point P. Because of the differences in path length, there are now phase differences between wavelets coming from adjacent strips.
- The vector sum of the phasors is now part of the perimeter of a many-sided polygon.



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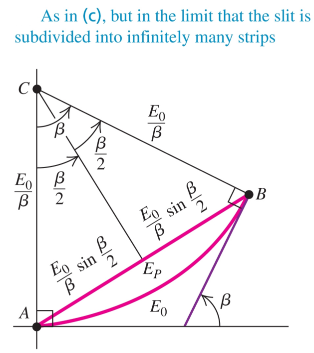
Intensity in the Single-Slit Pattern (2 of 5)

- We can bisect the triangle formed by the radial lines and E_P, which forms two triangles with angles $\beta/2$ at the center of curvature, giving us

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}.$$

- The intensity at any point along the screen is proportional to E_P^2, so if I_0 is the intensity at $\theta = 0$ (and hence $\beta = 0$), then the intensity at any point is

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2.$$



- Since β is the total phase shift between E_P and E_0, and the phase difference is $2\pi/\lambda$ times the path difference, we have

$$\beta = \frac{2\pi}{\lambda} a \sin\theta.$$

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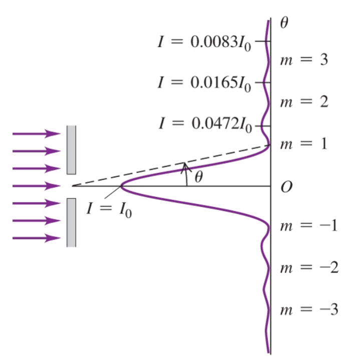
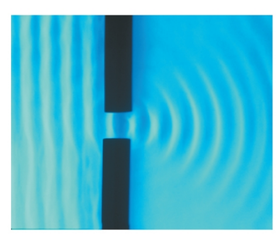
Intensity in the Single-Slit Pattern (3 of 5)

- The intensity at angle θ can be written in terms of the intensity I_0 at $\theta = 0$ as

$$I = I_0 \left\{ \frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda} \right\}^2.$$

- Most of the wave power goes into the central intensity peak (between the $m = 1$ and $m = -1$ intensity minima).
- The behavior is similar to that of water waves passing through a small aperture.





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Intensity in the Single-Slit Pattern (4 of 5)

- The dark fringes occur where $I = 0$, which corresponds to $\sin[\pi a(\sin\theta)/\lambda] = 0$. For this to happen, we require $\pi a(\sin\theta)/\lambda = m\pi$, where m is an integer. Thus,

$$\frac{a\sin\theta}{\lambda} = m \quad \rightarrow \quad \sin\theta = \frac{m\lambda}{a}, \quad (m = \pm 1, \pm 2, \ldots).$$

- At first glance, we might expect that the maxima occur when $\sin(\beta/2) = \pm 1$, which would give us

$$\beta \approx \pm(2m+1)\pi, \quad (m = 0, 1, 2, \ldots).$$

  However, the factor of $(\beta/2)^2$ in the denominator ensures that the maxima are not quite at these points.

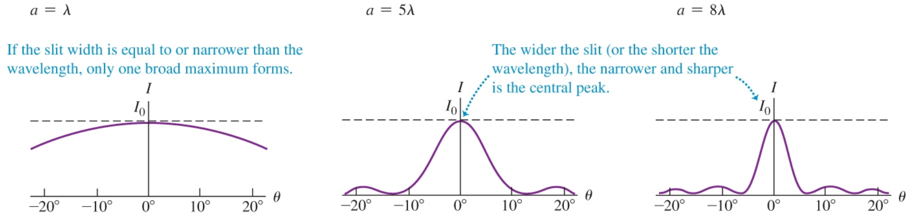
- Instead, the first maximum after $\beta = 0$ is at $\beta = \pm 2.850\pi$, and the second is at $\beta = \pm 4.918\pi$.

Intensity in the Single-Slit Pattern (5 of 5)

- The intensities of the peaks are approximately

$$I_m \approx \frac{I_0}{(m+1/2)^2\pi^2}.$$

- The angular spread is therefore inversely proportional to the slit width a. Furthermore, if the slit width is much larger than the wavelength λ, the peaks become narrower and sharper.



Example 36.2: Single-Slit Diffraction Intensity I

(a) The intensity at the center of a single-slit diffraction pattern is I_0. What is the intensity at a point in the pattern where there is a 66 radian phase difference between wavelets from the two edges of the slit? (b) If this point is $7.0°$ away from the central maximum, how many wavelengths wide is the slit?

(a) We have $\beta/2 = 33$ rad, so

$$I = I_0 \left[\frac{\sin(33 \text{ rad})}{33 \text{ rad}} \right]^2 = (9.2 \times 10^{-4}) I_0.$$

(b) For the slit, we have

$$\frac{a}{\lambda} = \frac{\beta}{2\pi \sin\theta} = \frac{66 \text{ rad}}{(2\pi \text{ rad}) \sin(7.0°)} = 86.$$

Example 36.3: Single-Slit Diffraction Intensity II

In the experiment described in a previous example where we passed 633 nm laser light through a narrow slit and produce a diffraction pattern on a screen 6.0 m away, with a slit width of $a = 0.24$ mm, the intensity at the center of the pattern is I_0. What is the intensity at a point on the screen 3.0 mm away from the center of the pattern?

We have $y = 3.0$ mm and $x = 6.0$ m, so

$$\tan\theta = \frac{y}{x} = \frac{3.0 \times 10^{-3} \text{ m}}{6.0 \text{ m}} = 5.0 \times 10^{-4},$$

which is so small that $\tan\theta \approx \sin\theta \approx \theta$. Then we have that

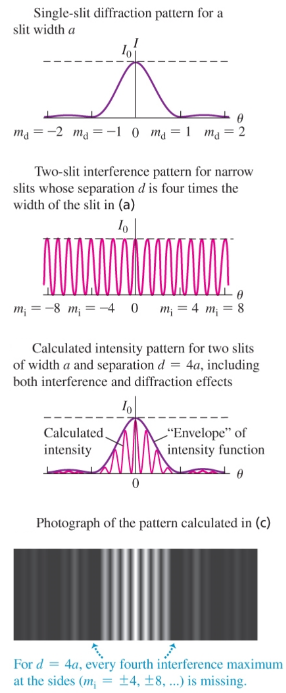
$$\frac{\beta}{2} = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi(2.4 \times 10^{-4} \text{ m})(5.0 \times 10^{-4})}{6.33 \times 10^{-7} \text{ m}} = 0.60 \quad \rightarrow \quad I = I_0 \left[\frac{\sin(0.60)}{0.60} \right]^2 = 0.89 I_0.$$

Two Slits of Finite Width

- Our original analysis of the two-slit interference pattern did not take the finite width of the slits into account, and the intensity of each peak is the same.
- If we take the finite width of the slits into account, then the two-slit peaks are in the same positions as before, but their intensities are modulated by the single-slit pattern, which acts as an "envelope" for the intensity function.
- The resulting intensity arises from the combined effects of interference and diffraction:

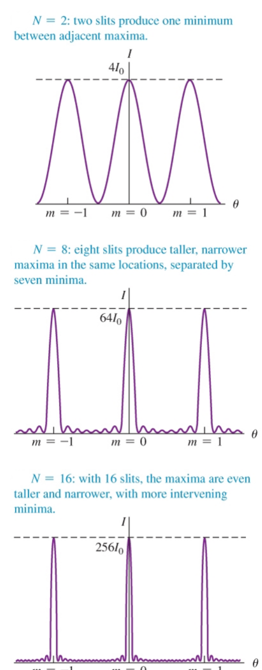
$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 .$$

- In general, there will be missing maxima whenever the slit spacing d is an integer multiple of the slit width a.



Interference Pattern of Several Slits

- If we have several slits, constructive interference occurs for rays at angle θ to the normal that arrive at a point P with a path difference between adjacent slits equal to an integer number of wavelengths.
- Shown are the results of the intensities for $N = 2, 8, 16$ slits.
- The large maxima, called the principal maxima, are in the same positions as for a two-slit pattern, but are much narrower.
- In general, when there are N slits, there are $N - 1$ minima between each pair of principle maxima. The height of each principal maximum is proportional to N^2, so from energy conservation, the width of each principal maximum must be proportional to $1/N$.



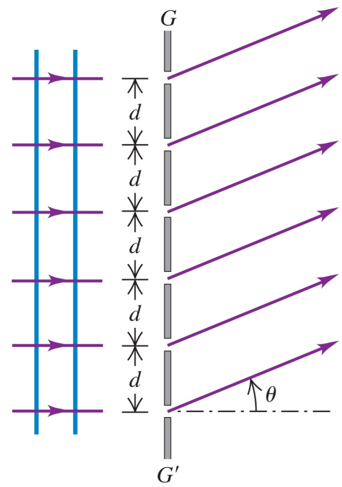
The Diffraction Grating

- An array of a large number of parallel slits, all with the same width a and spaced equal distance d between centers, is called a **diffraction grating.**
- For Fraunhofer (far-field) diffraction, the positions of the maxima are:

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

- More slits in a diffraction grating leads to a sharper intensity pattern.
- A typical grating can have hundreds or thousands of slits. Gratings for visible light typically have about 1000 slits per millimeter, so

$$d \approx \frac{1}{1000}\ \text{mm} = 1000\ \text{nm}.$$



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The Reflection Grating

- In a **reflection grating,** the array of equally spaced slits is replaced with ridges or grooves on a reflective screen.
- The rainbow-colored reflections from the surface of a DVD are a reflection grating effect.
- The "grooves" are tiny pits 0.12 μm deep in the surface of the disc, with a uniform radial spacing of 0.74 μm $= 740$ nm.
- Information is coded on the DVD by varying the length of the pits.
- The reflection-grating aspect of the disc is merely an aesthetic side benefit.

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Example 36.4: Width of a Grating Spectrum

The wavelengths of the visible spectrum are approximately 380 nm (violet) to 750 nm (red). (a) Find the angular limits of the first-order visible spectrum produced by a plane grating with 600 slits per millimeter when white light falls normally on the grating. (b) Do the first-order and second-order spectra overlap? What about the second-order and third-order spectra?

(a) First, we must find the grating spacing d, which is

$$d = \frac{1}{600 \text{ slits/mm}} = 1.67 \times 10^{-6} \text{ m}.$$

Then the angle θ for a given wavelength is

$$\theta = \arcsin \frac{m\lambda}{d}.$$

Therefore, for $m = 1$, the angular deviations θ_{v1} and θ_{r1} for violet and red light are

$$\begin{aligned}\theta_{\mathrm{v1}} &= \arcsin\left(\frac{380 \times 10^{-9} \text{ m}}{1.67 \times 10^{-6} \text{ m}}\right) = 13.2°,\\ \theta_{\mathrm{r1}} &= \arcsin\left(\frac{750 \times 10^{-9} \text{ m}}{1.67 \times 10^{-6} \text{ m}}\right) = 26.7°.\end{aligned}$$

(b) For $m = 2$ and $m = 3$, the deviations for the 380 nm violet light are

$$\begin{aligned}\theta_{\mathrm{v2}} &= \arcsin\left[\frac{2(380 \times 10^{-9} \text{ m})}{1.67 \times 10^{-6} \text{ m}}\right] = 27.1°,\\ \theta_{\mathrm{v3}} &= \arcsin\left[\frac{3(380 \times 10^{-9} \text{ m})}{1.67 \times 10^{-6} \text{ m}}\right] = 43.0°.\end{aligned}$$

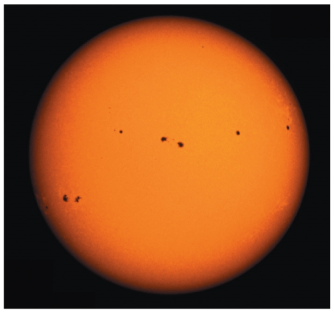
Meanwhile, for 750 nm red light, we have

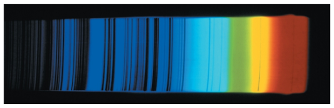
$$\begin{aligned}\theta_{\mathrm{r2}} &= \arcsin\left[\frac{2(750 \times 10^{-9} \text{ m})}{1.67 \times 10^{-6} \text{ m}}\right] = 63.9°,\\ \theta_{\mathrm{r3}} &= \arcsin\left[\frac{3(750 \times 10^{-9} \text{ m})}{1.67 \times 10^{-6} \text{ m}}\right] = \arcsin(1.35) \rightarrow \text{undefined.}\end{aligned}$$

Therefore, the second-order spectrum extends from $27.1°$ to $63.9°$, while the third-order spectrum extends from $43.0°$ to $90°$ since θ_{r3} is undefined. Thus, the first-order spectrum (from $13.2°$ to $26.7°$) does not overlap with the second-order spectrum, but the second-order spectrum does overlap with the third-order spectrum.

Spectroscopy

- Diffraction gratings are widely used to measure the spectrum of light emitted by a source.
- The process of measuring such a spectrum is known as **spectroscopy.**
- This technique is commonly used in astronomy.
  - Specific wavelengths of light generated by the sun get absorbed in the atmosphere.
  - The light that gets absorbed shows up in the spectrum of a diffraction grating as dark absorption lines.
- Different types of atoms absorb and emit different wavelengths of light.
- The spectrum observed through a diffraction grating can be used to deduce the chemical composition of the source that emitted the light.





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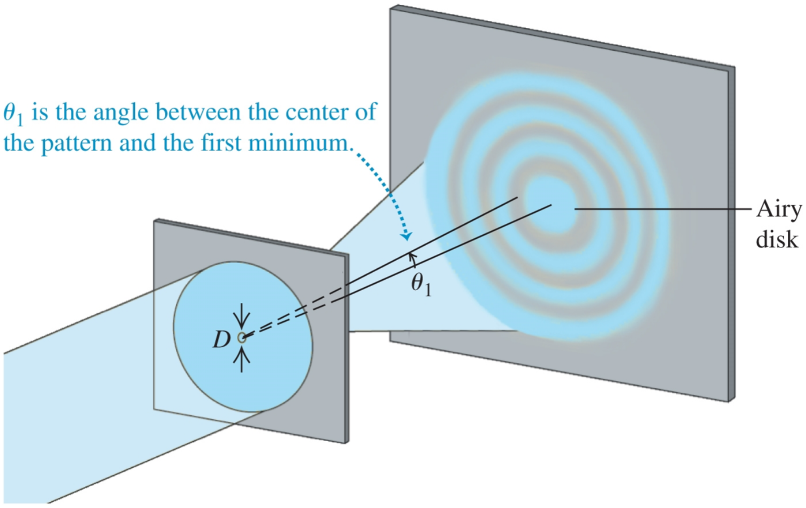
Circular Apertures

- The diffraction pattern formed by a circular aperture consists of a central bright spot surrounded by a series of bright and dark rings.



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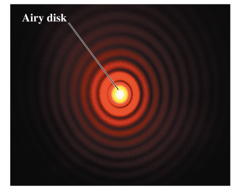
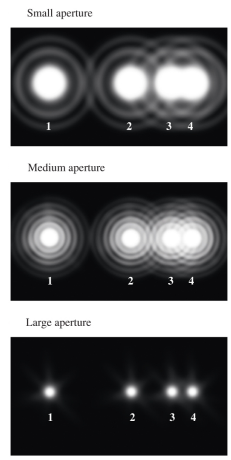
Diffraction by a Circular Aperture

- The central bright spot in the diffraction pattern of a circular aperture is called the Airy disk.
- We can describe the radius of the Airy disk by the angular radius θ_1 of the first dark ring given aperture diameter D and wavelength λ:

$$\sin\theta_1 = 1.22\frac{\lambda}{D}.$$

- Diffraction limits the resolution of optical equipment, such as telescopes.
- The larger the aperture, the better the resolution.
- A widely used criterion of two point objects is **Rayleigh's criterion:**
  - Two objects are just barely resolved if the center of one diffraction pattern coincides with the first minimum of the other.





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Example 36.6: Resolving Power of a Camera Lens

A camera lens with focal length $f = 50$ mm and maximum aperture $f/2$ forms an image of an object 9.0 m away emitting $\lambda = 500$ nm light. (a) If the resolution is limited by diffraction, what is the minimum distance between two points on the object that are barely resolved? What is the distance between image points? (b) How does the situation change if the lens is "stopped down" to $f/16$?

(a) The aperture diameter is

$$D = \frac{f}{f\text{-number}} = \frac{50 \text{ mm}}{2} = 25 \text{ mm}.$$

The angular separation θ of the two object points that are barely resolved is therefore

$$\theta \approx \sin\theta = 1.22\frac{\lambda}{D} = 1.22\frac{500\times10^{-9}\text{ m}}{25\times10^{-3}\text{ m}} = 2.4\times10^{-5}\text{ rad}.$$

Up to a minus sign, we have that

$$\frac{y}{s} = \frac{y'}{s'} = \theta,$$

so for the distance between two points on the object, we obtain

$$y = s\theta = (9.0\text{ m})(2.4\times10^{-5}) = 2.2\times10^{-4}\text{ m} = 0.22\text{ mm}.$$

Because the object distance s is much greater than the focal length $f = 50$ mm, the image distance s' is approximately equal to f. Therefore, the distance between image points is

$$y' = s'\theta = (50\text{ mm})(2.4\times10^{-5}) = 1.2\times10^{-3}\text{ mm}.$$

(b) If the lens is stopped down to $f/16$, then the aperture diameter is now

$$D = \frac{50 \text{ mm}}{16} = 3.125 \text{ mm}.$$

This is one-eighth as large as before, so the angular separation between barely resolved points is now eight times as large, and the values of y and y' are also eight times as large:

$$y = 1.8 \text{ mm}, \quad y' = 0.0096 \text{ mm}$$