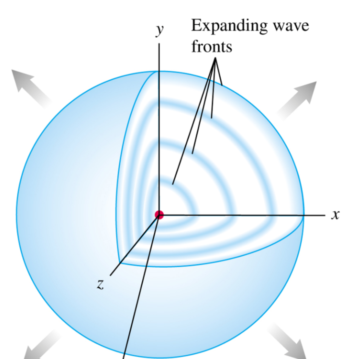
Chapter 33: The Nature and Propagation of Light

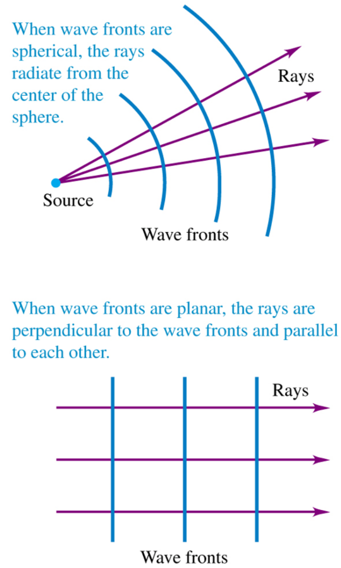
Waves and Wave Fronts

- A **wave front** is the locus of all adjacent points at which the **phase** of a wave is the same.
- Spherical wave fronts of sound spread out uniformly in all directions from a point source.
- Electromagnetic waves in vacuum also spread out as shown in the figure.



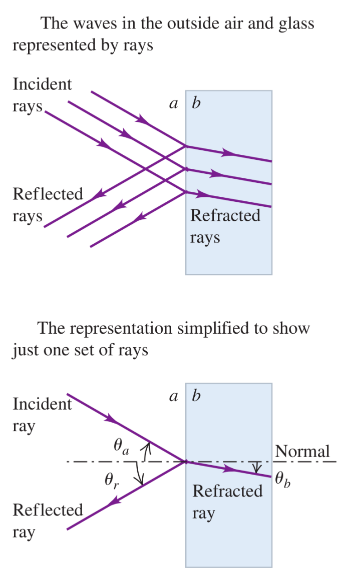
Wave Fronts and Rays

- It's often convenient to represent a light wave by **rays** rather than by wave fronts.
- A ray is an imaginary line along the direction of travel of the wave.
- When waves travel in a homogeneous isotropic material, the rays are always straight lines **normal** to the wave fronts.
- Far away from a source, where the radii of the spheres have become very large, a section of a spherical surface can be considered as a plane, and we have a **plane wave.**



Reflection and Refraction

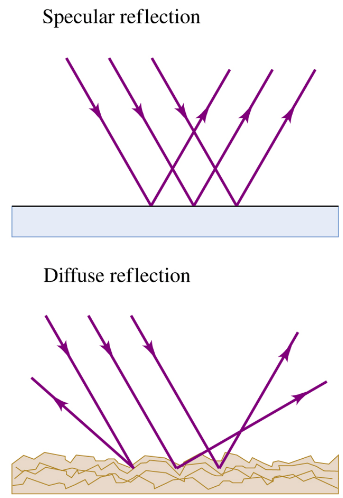
- When a light wave strikes a smooth interface separating two transparent materials, such as air and glass, the wave is in general partly reflected and partly refracted (transmitted) into the second material.
- The segments of plane waves can be represented by bundles of rays forming beams of light.
- For simplicity, we often draw only one ray in each beam.
- We describe the directions of the incident, reflected, and refracted rays at an interface between two optical materials in terms of the angles they make with the normal to the surface at the point of incidence.



Diffuse and Specular Reflection

- We will primarily concern ourselves with **specular reflection** from a very smooth surface such as highly polished glass or metal.
- Scattered reflection from a rough surface is called **diffuse reflection.**
- The vast majority of objects in your environment are visible to us because they reflect light in a diffuse manner.
- The index of refraction n plays a central role in geometric optics, which is defined as the ratio of the speed of light in vacuum c to the speed of light in a material v:

$$n = \frac{c}{v}.$$



The Laws of Reflection and Refraction

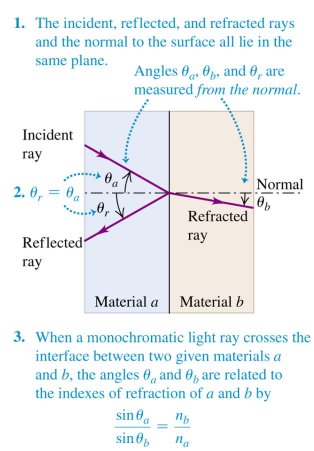
- There are three observations consider for specular reflection:
  1. **The incident, reflected, and refracted rays and the normal to the surface all lie in the same plane,** which is called the **plane of incidence.**
  2. **The angle of reflection θ_r is equal to the angle of incidence θ_a for all wavelengths and for any pair of materials,** as measured from the normal:

     $$\theta_r = \theta_a.$$

     This is called the **law of reflection.**
  3. For monochromatic light passing through materials with indexes of refraction n_a and n_b, with angles θ_a and θ_b with respect to the normal:

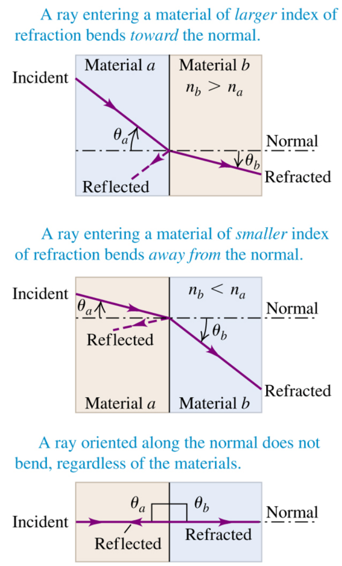
     $$n_a \sin\theta_a = n_b \sin\theta_b.$$

     This is called the **law of refraction**, or **Snell's law.**



Reflection and Refraction in Three Cases

- When a ray passes from one material into another material having a larger index of refraction $n_b > n_a$, then the relationship between the angles is $\theta_a > \theta_b$.
- On the other hand, when a ray passes from one material into another material with a smaller index of refraction $n_b < n_a$, the relationship between the angles is $\theta_a < \theta_b$.
- In the case of normal incidence, the transmitted ray is not bent at all.
  - In this case, $\theta_a = 0$ and $\sin\theta_a = 0$, so $\theta_b = 0$ as well, and hence the transmitted ray is also normal to the interface.
  - The angle θ_r is also equal to zero, so the reflected ray travels back along the same path as the incident ray.

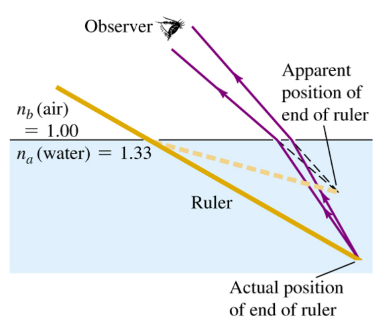


Why Does the Ruler Appear to be Bent?

- The law of refraction explains why an object partially submerged in water appears bent.
- Light rays coming from below the surface change in direction at the air-water interface, so the rays appear to be coming from a position above their actual point of origin.

A straight ruler half-immersed in water

Why the ruler appears bent



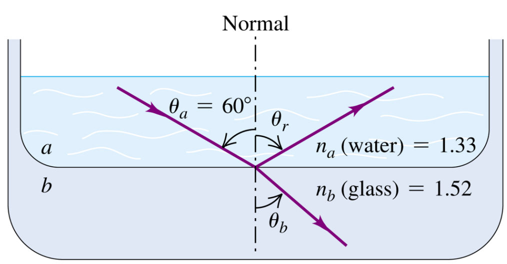
Index of Refraction and the Wave Aspects of Light

- The frequency of a wave does not change when passing from one material to another.
- In any material, $v = \lambda f$, and since f is the same in any material as in vacuum and v is always less than the wave speed c in vacuum, λ is also correspondingly reduced. The wavelength in the material λ is related to the wavelength in vacuum λ_0 by
$$\lambda = \frac{\lambda_0}{n}.$$
- When a wave passes from one material into a second material, the waves get "squeezed" (the wavelength gets shorter) if the wave speed decreases, and get "stretched" (the wavelength gets longer) if the wave speed increases.

8/21

Example 33.1: Reflection and Refraction

In the figure below, material a is water and material b is glass with index of refraction 1.52. The incident ray makes an angle of $60.0°$ with the normal; find the directions of the reflected and refracted rays.



Since the angle of the reflected ray is the same as the incident ray, we have that

$$\theta_r = \theta_a = 60°.$$

For the refracted ray, we may use Snell's law:

$$\begin{aligned} n_a \sin\theta_a &= n_b \sin\theta_b \\ \sin\theta_b &= \frac{n_a}{n_b}\sin\theta_a = \frac{1.33}{1.52}\sin 60° = 0.758 \\ \theta_b &= \arcsin(0.758) = 49.3°. \end{aligned}$$

Example 33.2: Index of Refraction in the Eye

The wavelength of the red light from a helium-neon laser is 633 nm in air but 474 nm in the aqueous humor inside your eyeball. Calculate the index of refraction of the aqueous humor and the speed and frequency of the light in it.

The index of refraction for air is very close to 1.0, so we shall assume that the wavelength of the light from the laser in vacuum λ_0 is the same as in air. Then we have that the index of refraction n for the aqueous humor is

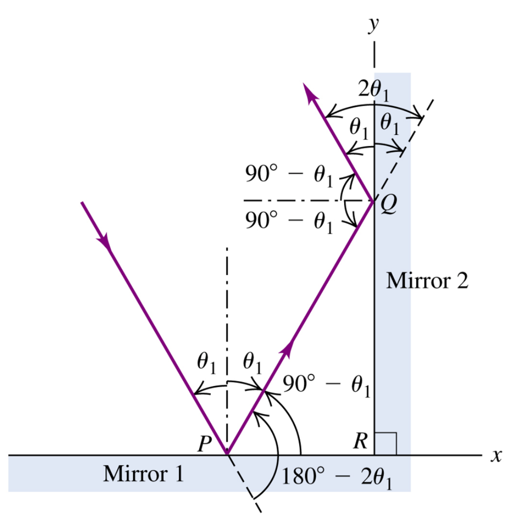
$$\lambda = \frac{\lambda_0}{n} \quad \rightarrow \quad n = \frac{\lambda_0}{\lambda} = \frac{633 \text{ nm}}{474 \text{ nm}} = 1.34.$$

The speed and frequency of the light in the aqueous humor is therefore

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.34} = 2.25 \times 10^8 \text{ m/s},$$
$$f = \frac{v}{\lambda} = \frac{2.25 \times 10^8 \text{ m/s}}{474 \times 10^{-9} \text{ m}} = 4.74 \times 10^{14} \text{ Hz}.$$

Example 33.3: A Twice-Reflected Ray

Two mirrors are perpendicular to each other. A ray traveling in a plane perpendicular to both mirrors is reflected from one mirror at P, then the other at Q, as shown in the figure below. What is the ray's final direction relative to its original direction?



For the first mirror, the angle of reflection is equal to the angle of incidence θ_1. Since the second mirror is perpendicular to the first mirror, the ray is incident upon the second mirror at an angle $90^\circ - \theta_1$ with respect to its normal. It then reflects at the same angle $90^\circ - \theta_1$ with respect to the normal of the second mirror. The total change in direction of the ray after both reflections is therefore $2(90^\circ - \theta_1) + 2\theta_1 = 180^\circ$, so the ray's final direction is opposite to its original.

Total Internal Reflection

- Under certain circumstances, **all** of the light can be reflected back from an interface, even though the second material is transparent.
- From Snell's law, we have that

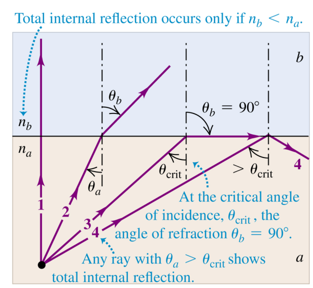
$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a.$$

- If $\sin\theta_b = 1$, then $\theta_b = 90^\circ$, which means that the light will not be transmitted through the interface.
- This leads to **total internal reflection,** and the angle beyond which this occurs is called the **critical angle:**

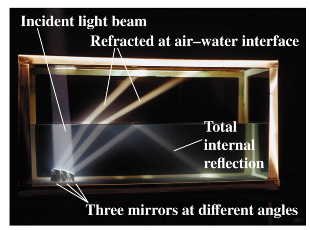
$$\sin\theta_{\text{crit}} = \frac{n_b}{n_a}.$$

- This situation occurs only when $n_b < n_a$.

Total internal reflection



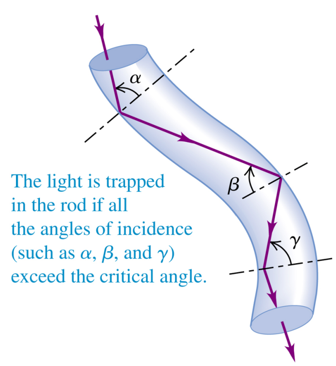
A light beam enters the top left of the tank, then reflects at the bottom from mirrors tilted at different angles. One beam undergoes total internal reflection at the air–water interface.



9/21

Fiber Optics

- When a beam of light enters at one end of a transparent rod, the light can be totally reflected internally if the index of refraction of the rod is greater than that of the surrounding material.
- The light is "trapped" within even a curved rod, provided that the curvature is not too great.
- A bundle of fine glass or plastic fibers behaves in the same way and has the advantage of being flexible.
- Fiber optics have a wide range of applications in medicine and communications.



10/21

Example 33.4: A Leaky Periscope

A submarine periscope uses two totally reflecting $45°$-$45°$-$90°$ prisms with total internal reflection on the sides adjacent to the $45°$ angles. Explain why the periscope will no longer work if it springs a leak and the bottom prism is covered with water.

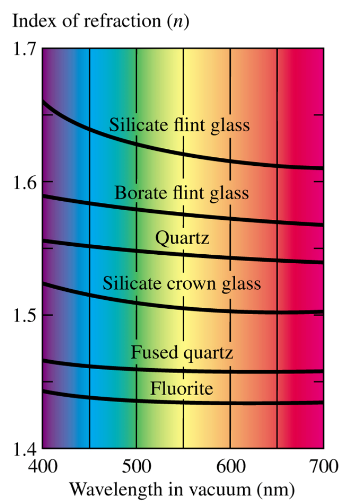
The critical angle for water ($n_b = 1.33$) on glass ($n_1 = 1.52$) is

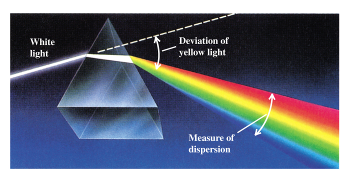
$$\theta_{\text{crit}} = \arcsin \frac{1.33}{1.52} = 61.0°$$

Thus, the $45°$ angle of incidence for a totally reflecting prism is smaller than the new $61°$ critical angle when submerged in water, so the total internal reflection does not occur.

Dispersion

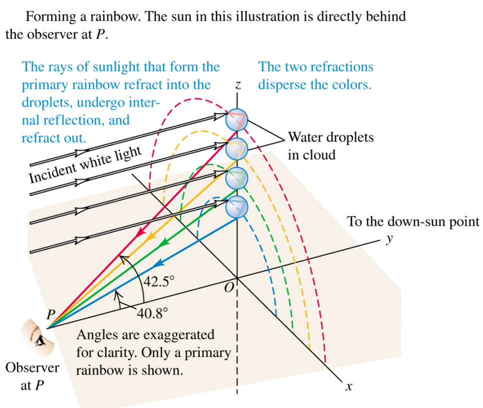
- The speed of light in vacuum is the same for all wavelengths, but the speed in a material substance is different depending on the wavelength of the light.
- The dependence of wave speed and index of refraction n on the wavelength is called **dispersion.**
- In most materials, the value of n **decreases** with increasing wavelength and decreasing frequency.
- Ordinary white light is a superposition of waves with all visible wavelengths.
- The band of dispersed colors is called a **spectrum.**

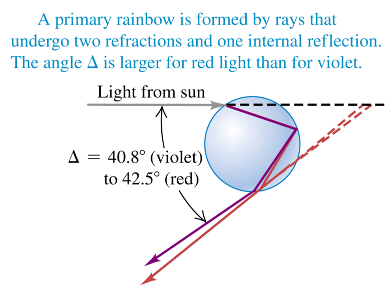




How Rainbows Form (1 of 2)

- When sunlight enters a spherical water droplet suspended in the air, it is (partially) reflected from the back surface of the droplet, and is refracted again upon exiting the droplet.
- A light ray that enters the middle of the raindrop is reflected straight back.
- All other rays exit the raindrop within an angle Δ of that middle ray, with many rays "piling up" at the angle Δ.

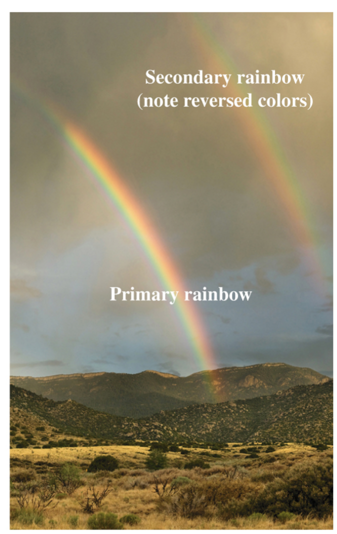




12/21

How Rainbows Form (2 of 2)

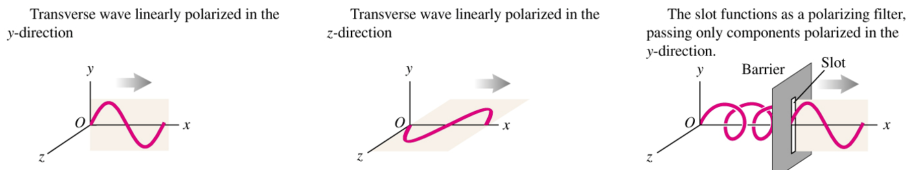
- In many cases you can see a second, larger rainbow.
- It is the result of two reflections from the back surface of the droplet.
- Just as a mirror held up to a book reverses the printed letters, so the second reflection reverses the sequence of colors in the secondary rainbow.



13/21

Polarization

- A transverse wave is linearly polarized if the displacement due to the wave has only one component.
- When a wave has only y-displacements, we say that it is linearly polarized in the y-direction.
- For mechanical waves, we can build a polarizing filter that only selects waves with a particular polarization:

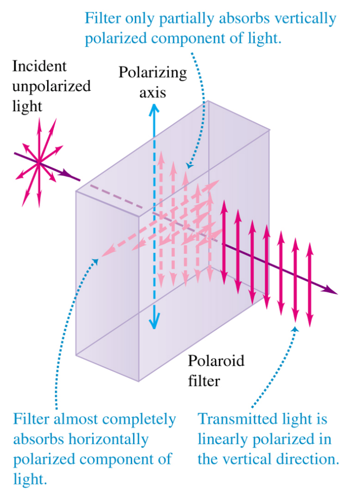


Polarizing Filters

- An electromagnetic wave is **linearly polarized** if the electric field has only one component.
  - We saw this before with the following electromagnetic wave:

$$\mathbf{E}(x,t) = E_{\max}\cos(kx-\omega t)\,\hat{\mathbf{j}},$$
$$\mathbf{B}(x,t) = B_{\max}\cos(kx-\omega t)\,\hat{\mathbf{k}}.$$

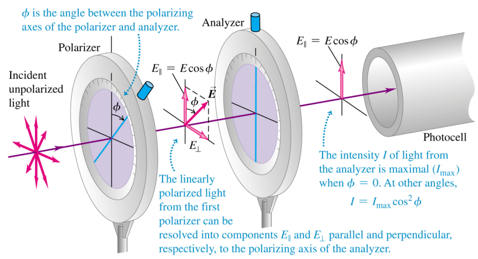
  - This is an example of a wave polarized in the y-direction.
- Light from most sources, such as from light bulbs, is a random mixture of waves linearly polarized in all possible transverse directions. Such light is called **unpolarized light** or **natural light.**
- A **polarizing filter** can convert unpolarized light to linearly polarized light.



Malus's Law

- Unpolarized light passing through a polarizer results in polarized light with exactly half the intensity of the unpolarized light.
- When polarized light is incident on a polarizing filter used as an analyzer, the intensity I of the light transmitted through the analyzer depends on the angle ϕ between the polarization direction of the incident light and the polarizing axis of the analyzer.
- If the maximum intensity of the light passing through the first polarizer is $I_{\max}$, then the intensity of polarized light passed through the analyzer is given by **Malus's law:**

$$I = I_{\max} \cos^2 \phi.$$



16/21

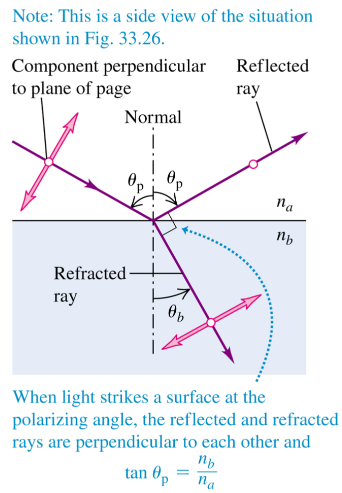
Example 33.5: Two Polarizers in Combination

In the figure above the incident unpolarized light has intensity I_0. Find the intensities transmitted by the first and second polarizers if the angle between the axes of the two filters is 30°.

The incident light is unpolarized, so the intensity of the linearly polarized light that emerges from the first polarizer is $I_0/2$. Since $\phi = 30^\circ$, we have that the transmitted light that has gone through both polarizers is

$$I = \frac{I_0}{2} \cos^2 30^\circ = \frac{I_0}{2} \frac{3}{4} = \frac{3}{8} I_0.$$

Polarization by Reflection



- Unpolarized light can be polarized, either partially or totally, by reflection.
- At one particular angle of incidence, called the **polarizing angle,** the light for which $\mathbf{E}$ lies in the plane of incidence is not reflected at all but is completely refracted.
- When the angle of incidence is equal to the polarizing angle θ_p, the reflected ray and the refracted ray are perpendicular to each other:

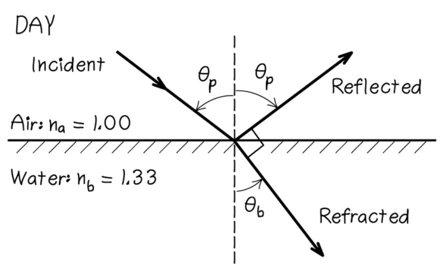
$$n_a \sin\theta_p = n_b \sin(90^\circ - \theta_p) = n_b \cos\theta_p.$$

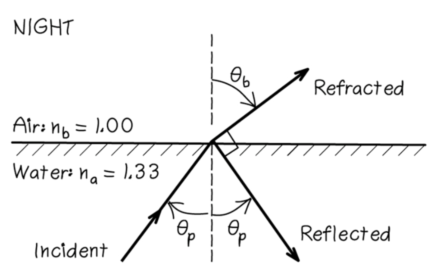
- The relationship between the polarizing angle and the indices of refraction for the two media is known as **Brewster's law:**

$$\tan\theta_p = \frac{n_b}{n_a}.$$

Example 33.6: Reflection from a Swimming Pool's Surface

Sunlight reflects off of the smooth surface of a swimming pool. (a) For what angle of reflection is the reflected light completely polarized? (b) What is the corresponding angle of refraction? (c) At night, an underwater floodlight is turned on in the pool. Repeat parts (a) and (b) for rays from the floodlight that strike the surface from below.





(a) During the day the light moves in air towards water, so $n_a = 1.00$, and $n_b = 1.33$. The polarizing angle is therefore

$$\theta_\mathrm{p} = \arctan\frac{n_b}{n_a} = \arctan\frac{1.33}{1.00} = 53.1^\circ.$$

(b) The incident light is at the polarizing angle, so the reflected and refracted rays are perpendicular, and thus

$$\theta_b = 90^\circ - \theta_\mathrm{p} = 90^\circ - 53.1^\circ = 36.9^\circ.$$

(c) At night the light moves in water towards air, so now $n_a = 1.33$ and $n_b = 1.00$. Therefore,

$$\theta_\mathrm{p} = \arctan\frac{1.00}{1.33} = 36.9^\circ, \qquad \theta_b = 90^\circ - 36.9^\circ = 53.1^\circ.$$

Circular Polarization (1 of 2)

- **Circular polarization** occurs when the $\mathbf{E}$ vector has a constant magnitude but rotates around the direction of propagation.
- When the wave is propagating towards you and the $\mathbf{E}$ vector appears to be rotating clockwise, it is called a **right circularly polarized** electromagnetic wave.
- If instead the $\mathbf{E}$ vector of a wave coming towards you appears to be rotating counterclockwise, it is called a **left circularly polarized** electromagnetic wave.
- The lenses of the special glasses you wear to see a 3-D movie are circularly polarizing filters.
- This results when the components of the electric field are not in phase with each other. An example of this is an electromagnetic wave with $\mathbf{E}$ given by

$$\mathbf{E}(x,t) = E_\mathrm{max}\cos(kx - \omega t)\,\hat{\mathbf{j}} + E_\mathrm{max}\sin(kx - \omega t)\,\hat{\mathbf{k}}.$$

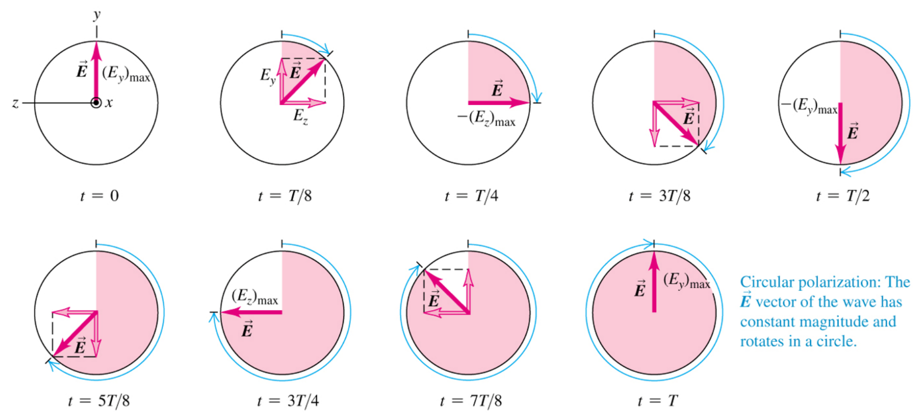
18/21

Circular Polarization (2 of 2)

- For simplicity, we will consider the wave at the point $x = 0$, so that $\mathbf{E}$ is

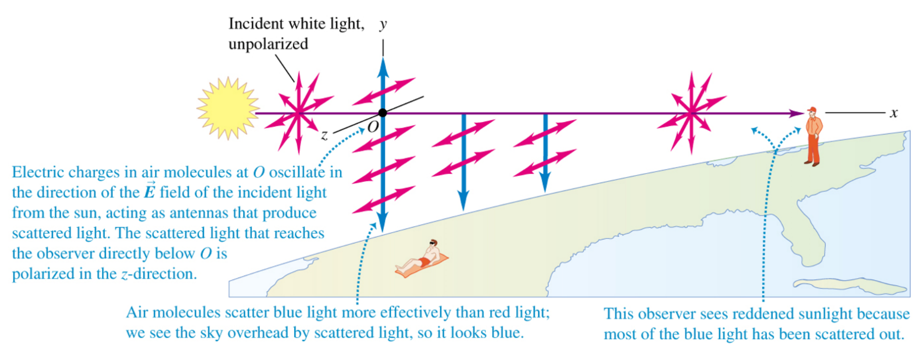
$$\mathbf{E}(0,t) = E_{\max}\cos(\omega t)\,\hat{\mathbf{j}} - E_{\max}\sin(\omega t)\,\hat{\mathbf{k}}.$$

- Then for various values of t in terms of the period T, we get the following for $\mathbf{E}$:



Scattering of Light (1 of 2)

- When you look at the daytime sky, the light that you see is sunlight that has been absorbed and then re-radiated in a variety of directions.
- This process is called **scattering.**
- Light scattered by air molecules contains 15 times as much blue light as red, and that's why the sky is blue.
- Clouds contain a high concentration of suspended water droplets or ice crystals, which scatter light of all wavelengths equally, so clouds look white.



Scattering of Light (2 of 2)

- Sunlight, which is unpolarized, causes the molecules of earth's atmosphere to oscillate in the direction of the $\mathbf{E}$ field from the light. The oscillations from the molecules produce polarized light, which is emitted in directions that are perpendicular to the motion of the charges.
- This specific scattering process is known as Rayleigh scattering, and the intensity of the electromagnetic waves produced by the scattering obey the following relation:

$$I \propto \frac{1}{\lambda^4}.$$

- Thus, shorter wavelengths dominate the spectrum for the scattered light.