CS 181: HW 6

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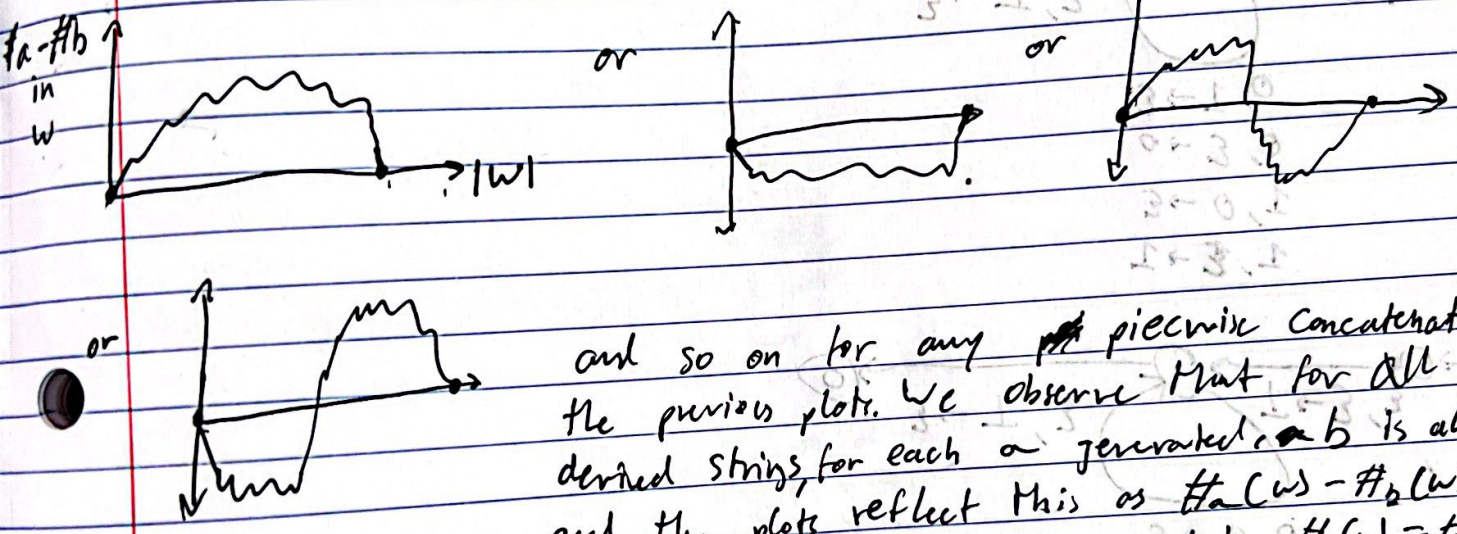
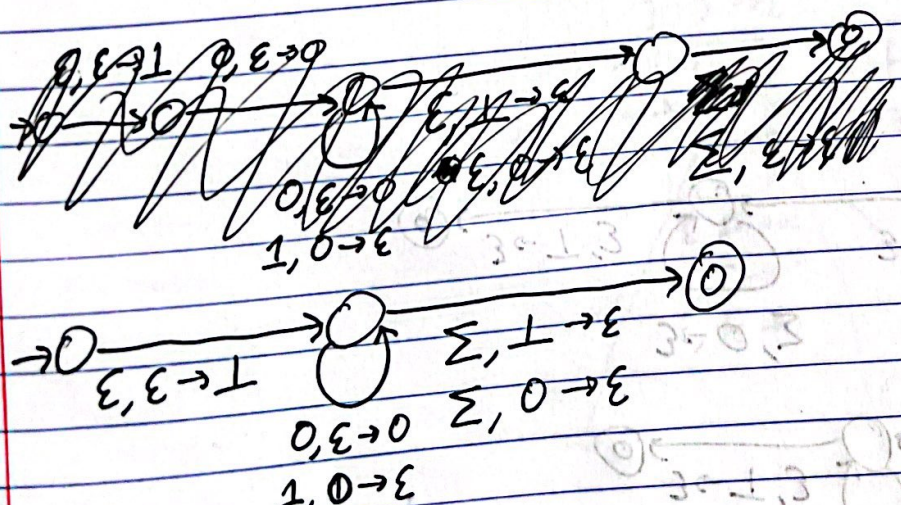
1) For the CFG w/ $R = \{S \to \varepsilon \mid aSbS \mid bSaS\}$ can be shown to derive strings of equally many a's as b's by observing the following case analysis of frequencies of a's to b's. Consider that given the ruleset, only the following "forms" of frequencies are possible:

$S \to \varepsilon$

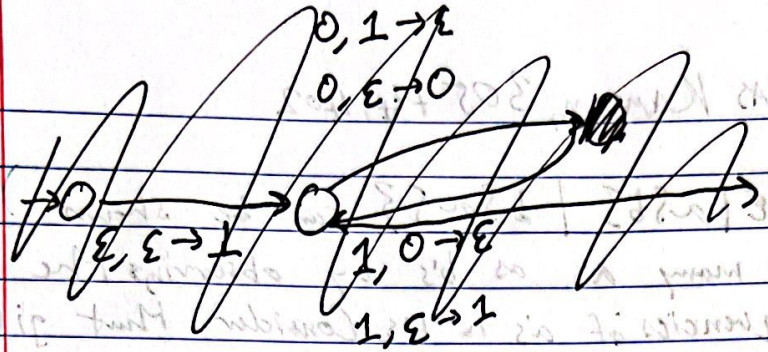


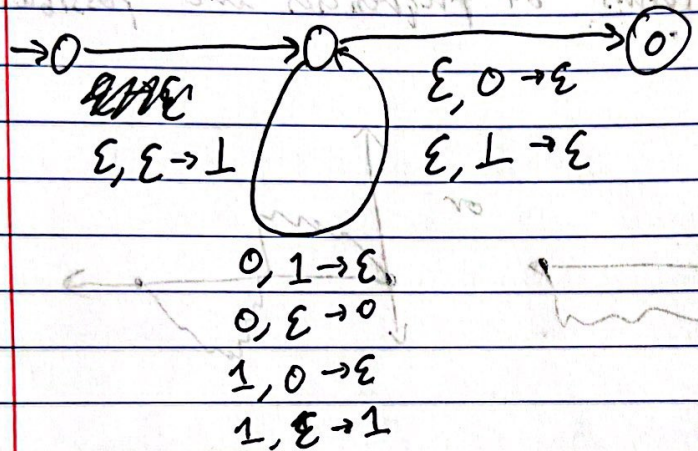
and so on for any piecewise concatenation of the previous plots. We observe that for all possible derived strings, for each a generated, a b is also generated and the plots reflect this as $\#_a(w) - \#_b(w)$ when returning to $x = 0$. Thus, there must be $\#_a(w) = \#_b(w)$.

2a)

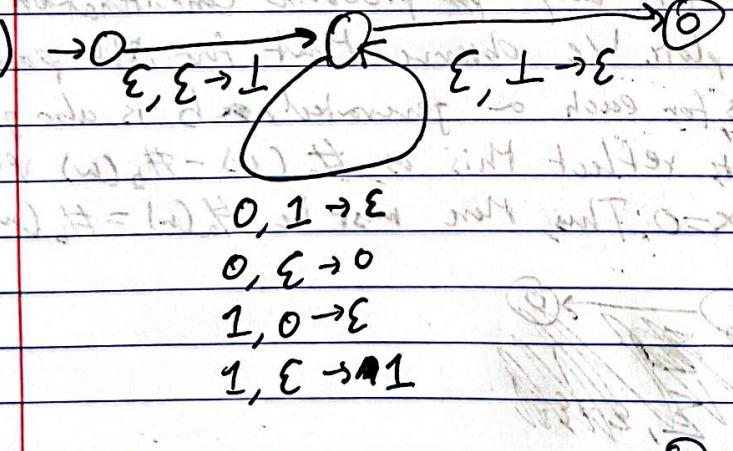


2b)

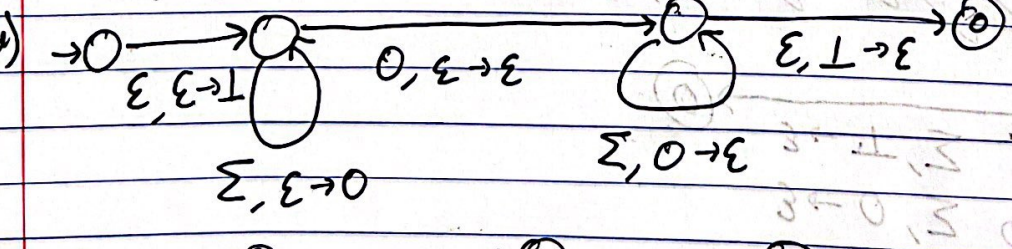

0, 1 → ε
0, ε → 0
ε, ε → ⊥
1, 0 → ε
1, ε → 1


ε, ε → ⊥
ε, 0 → ε
ε, ⊤ → ε
0, 1 → ε
0, ε → 0
1, 0 → ε
1, ε → 1

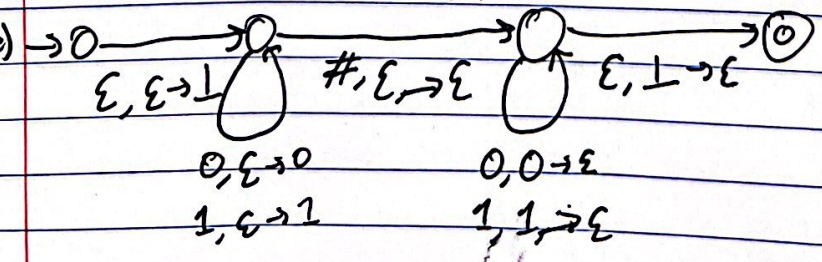
2c)


ε, ε → ⊥
ε, ⊤ → ε
0, 1 → ε
0, ε → 0
1, 0 → ε
1, ε → 1

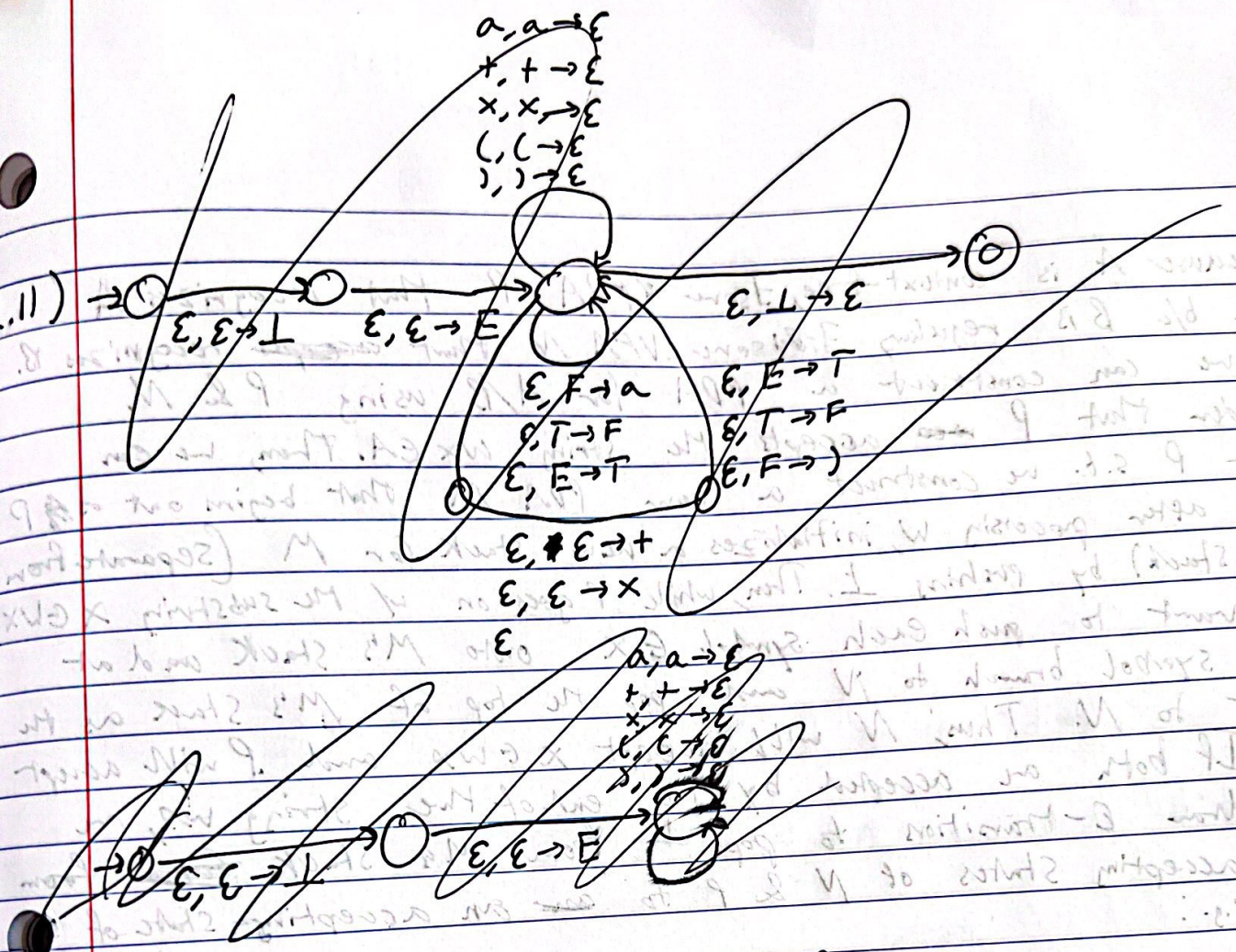
2d)


ε, ε → ⊥
0, ε → ε
ε, ⊤ → ε
ε, ε → 0
ε, 0 → ε

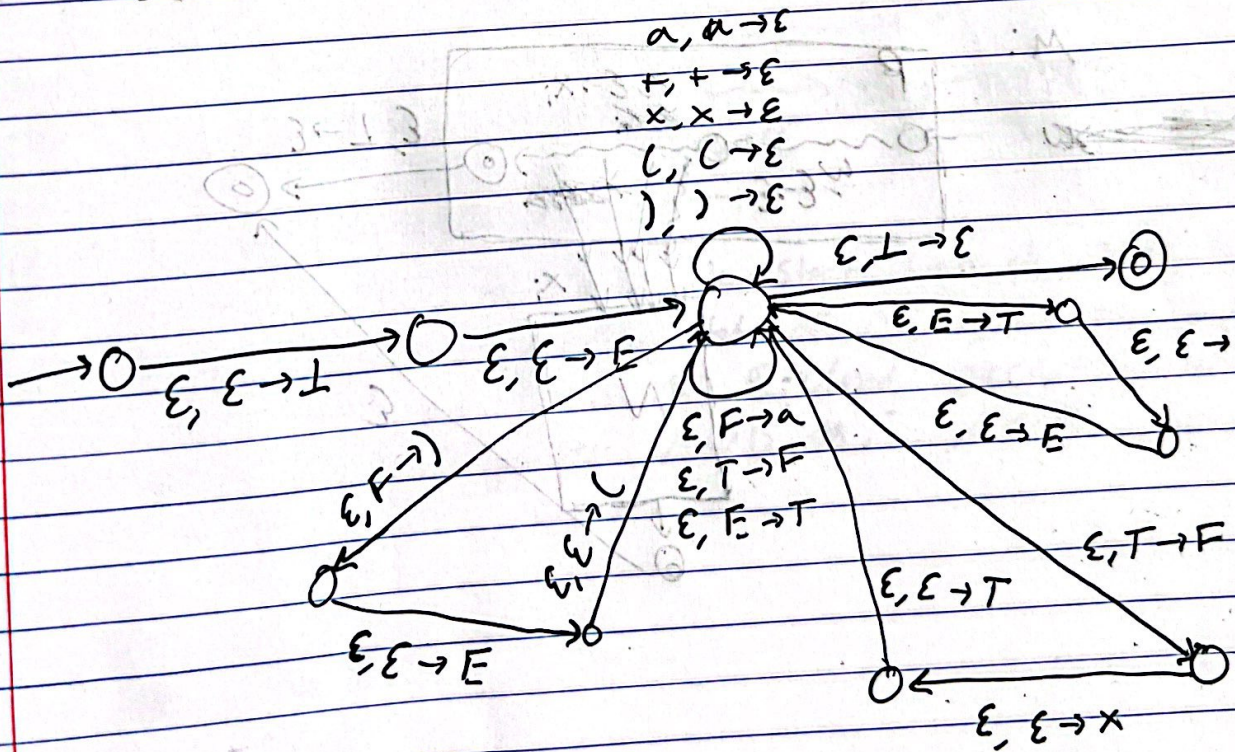
2e)


ε, ε → ⊥
#, ε → ε
ε, ⊤ → ε
0, ε → 0
1, ε → 1
0, 0 → ε
1, 1 → ε

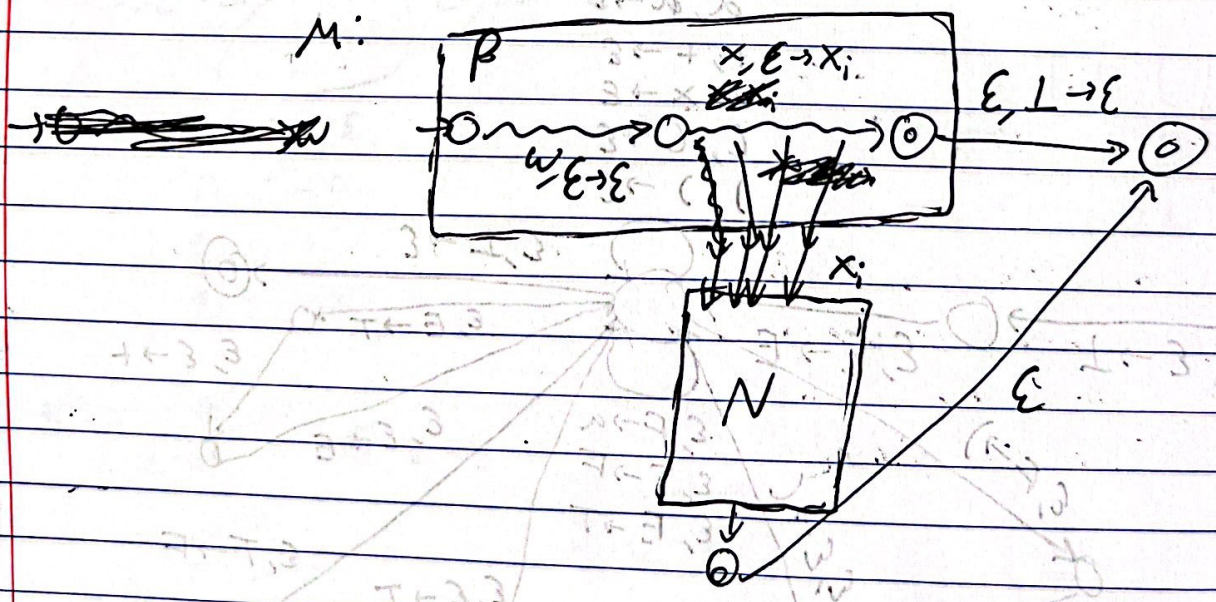
2.11)


a, a → ε
+, + → ε
x, x → ε
(, (→ ε
),) → ε
ε, ε → ⊥
ε, ε → E
ε, ⊥ → ε
ε, F → a
ε, T → F
ε, E → T
ε, E → T
ε, T → F
ε, F →)
ε, # ε → +
ε, ε → x
ε

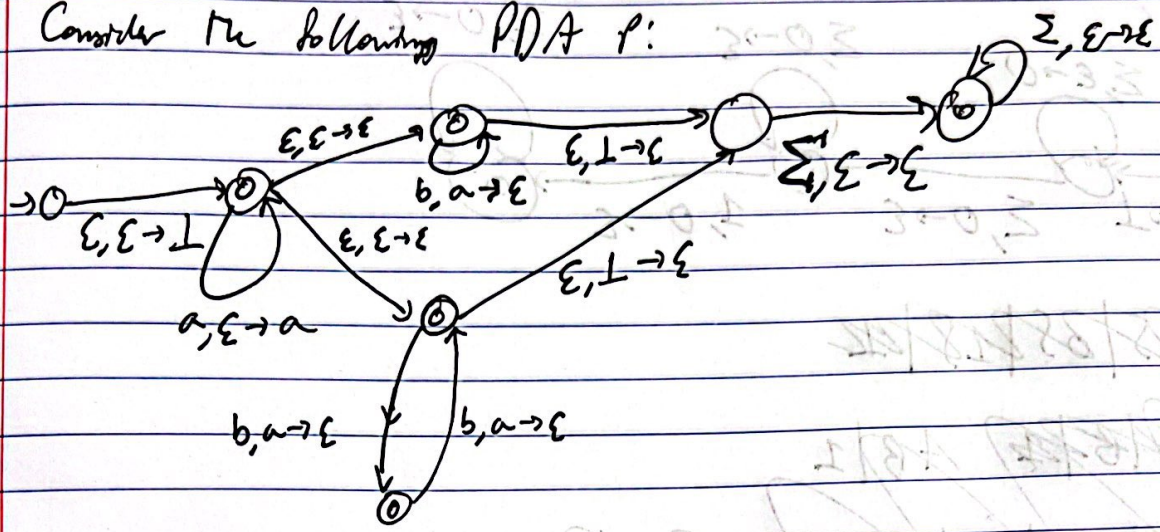
a, a → ε
+, + → ε
x, x → ε
(, (→ ε
),) → ε
ε, ε → ⊥
ε, ε → E


a, a → ε
+, + → ε
x, x → ε
(, (→ ε
),) → ε
ε, ε → ⊥
ε, ε → E
ε, ⊥ → ε
ε, E → T
ε, ε → +
ε, ε → E
ε, F → a
ε, T → F
ε, E → T
ε, F →)
ε, ε → E
ε, T → F
ε, ε → T
ε, ε → x

2.20) Because A is context-free, $\exists$ some PDA P that recognizes A and b/c B is regular $\exists$ some NFA N that recognizes B. So, we can construct a PDA for A/B using P & N. Consider that P accepts the string $wx \in A$. Then, we can alter P s.t. we construct a new PDA M that begins out as P but after processing w, initializes a new stack for M (separate from P's stack) by pushing $\perp$. Then, while P goes on of the substring $x \in wx$, we want to push each symbol $\in x$ onto M's stack and at each symbol branch to N and pop the top of M's stack as the input to N. Thus, N will accept $x \in wx$ and P will accept wx. If both are accepted by the end of the string wx, we can draw ε-transitions to pop $\perp$ from M's stack from the accepting states of N & P to an accepting state of M. e.g.:

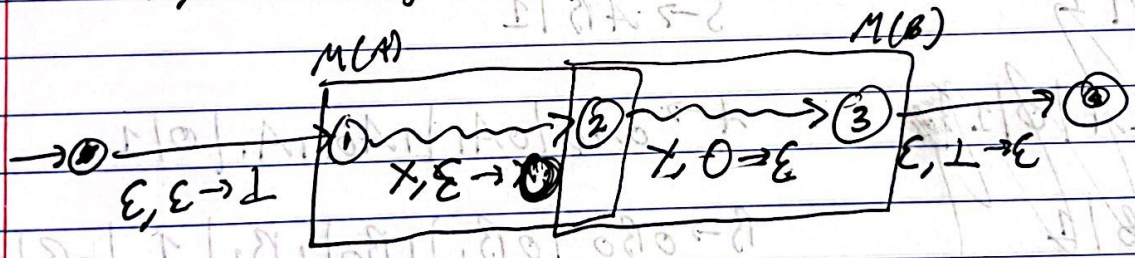


2.24) Consider the following PDA P:



This PDA recognizes the lang. so it is context-free.

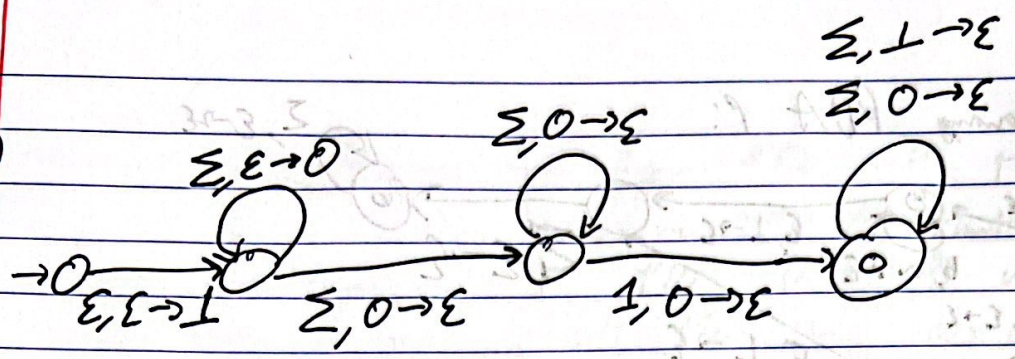
2.44) We can design a PDA P for A ∘ B given the M(A) & M(B) since A, B are regular langs. s.t. P =



As shown above, states (1) is the start state of M(A), state (2) is the ε-closed accepting state of M(A) and the start state of M(B), and state (3) is the ε-closed accepting state of M(B). This PDA recognizes A∘B, ∴ A∘B is context-free.

2.47)

a)



b) ~~S → SS | 0S1 | 1S0 | ε~~

S → ~~ABAB~~ AB | 1

A → 0A | 1A | ε | ~~00A | 01A | 10A | 11A~~

~~B → 0B | 1B | 1~~

~~S → AB | 1~~

~~A → 0A | 1A | 0 | 1~~

~~B → 0B | 1B | 1~~

S → AB | 1

A → 0A0 | 0A1 | 1A0 | 1A1 | 0 | 1

B → 0B0 | 0B1 | 1B0 | 1B1 | 1 | 0B | 1B