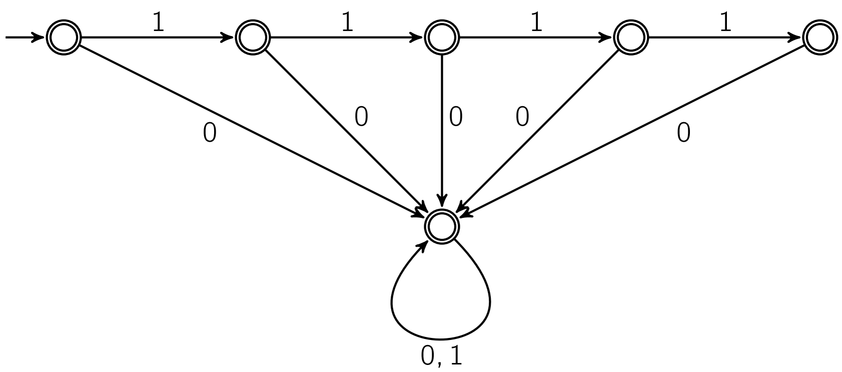
CS 181 PRACTICE MIDTERM 1B

You may state without proof any fact taught in lecture.

1 Give a simple verbal description of the language recognized by the following NFA:



2 Draw a finite automaton (deterministic or nondeterministic) for each of the following languages:

a. binary strings that have odd length or contain at most one 1;

b. binary strings in which every 0 is immediately preceded and immediately followed by a 1.

3 Give a regular expression for each of the following languages:

a. binary strings other than 01;

b. binary strings that do not contain 100 as a substring;

c. strings over the alphabet $\{a, b, c\}$ that contain all three alphabet symbols.

4 For a binary string w, its *bitwise complement* is denoted $\overline{w}$ and defined as the string obtained by flipping every bit of w. Prove that for every regular language L over the binary alphabet, the language $L' = \{\overline{w} : w \in L\}$ is also regular.

5 Let L be a regular language. Define $L^\dagger$ to be the set of all strings that can be obtained by concatenating one or more nonempty strings in L. Prove that $L^\dagger$ is regular.

6 Prove or disprove: for any nonregular language L, the palindromes in L also form a nonregular language.

7 For a language L, let permute(L) denote the set of all strings that can be obtained by taking a string in L and keeping it as is or reordering its symbols. For example, permute($\{\epsilon, ab, abb\}$) $=$ $\{\epsilon, ab, ba, abb, bab, bba\}$. Prove that regular languages are not closed under the permute operation.

8 For each of the following languages L over the binary alphabet, determine whether it is regular and prove your answer:

a. strings of the form wwu, where $w \in \{0,1\}^+$ and $u \in \{0,1\}^*$;

b. strings that are palindromes or contain 00 as a substring;

c. strings that contain a nonempty substring with equally many 0s and 1s.

SOLUTIONS

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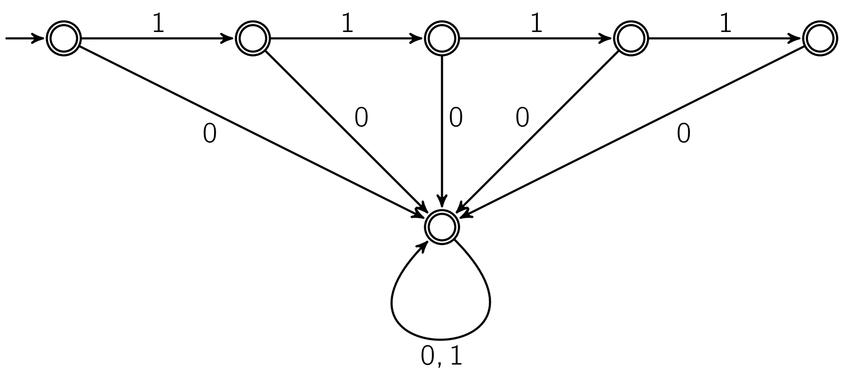
You may state without proof any fact taught in lecture.

1 Give a simple verbal description of the language recognized by the following NFA:



Solution: binary strings that do not start with 11111.

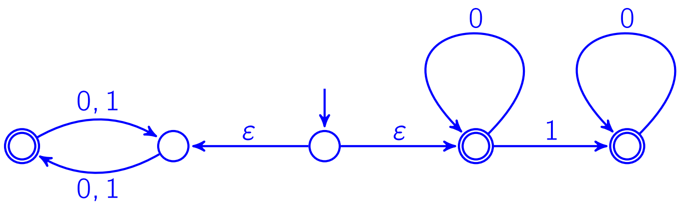
2 Draw a finite automaton (deterministic or nondeterministic) for each of the following languages:

a. binary strings that have odd length or contain at most one 1;

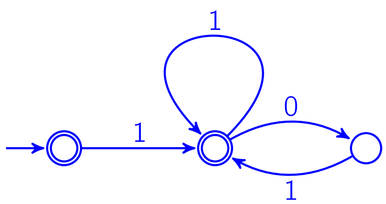
b. binary strings in which every 0 is immediately preceded and immediately followed by a 1.

Solution.

a.



b.



3 Give a regular expression for each of the following languages:

a. binary strings other than 01;

b. binary strings that do not contain 100 as a substring;

c. strings over the alphabet $\{a, b, c\}$ that contain all three alphabet symbols.

Solution.

a. $\varepsilon \cup \Sigma \cup 1\Sigma \cup \Sigma 0 \cup \Sigma^3\Sigma^*$

b. $0^*(1 \cup 10)^*$

c. $a\Sigma^*b\Sigma^*c\Sigma^* \cup a\Sigma^*c\Sigma^*b\Sigma^* \cup b\Sigma^*a\Sigma^*c\Sigma^* \cup b\Sigma^*c\Sigma^*a\Sigma^* \cup c\Sigma^*a\Sigma^*b\Sigma^* \cup c\Sigma^*b\Sigma^*a\Sigma^*$

4 For a binary string w, its *bitwise complement* is denoted $\overline{w}$ and defined as the string obtained by flipping every bit of w. Prove that for every regular language L over the binary alphabet, the language $L' = \{\overline{w} : w \in L\}$ is also regular.

Solution. Starting with a DFA for L, swap the edge labels 0 and 1 out of every state. The resulting DFA recognizes L'. Formally, let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA for L. Then L' is recognized by $(Q, \Sigma, \delta', q_0, F)$, where $\delta'(q, \sigma) = \delta(q, \neg\sigma)$.

5 Let L be a regular language. Define $L^\dagger$ to be the set of all strings that can be obtained by concatenating one or more nonempty strings in L. Prove that $L^\dagger$ is regular.

Solution. Note that $L^\dagger = L^* \setminus \{\varepsilon\}$, where L and $\{\varepsilon\}$ are regular. Since regular languages are closed under Kleene star and set difference, $L^\dagger$ is also regular.

An incorrect solution. Starting with a DFA for L, make the initial state rejecting and add ε-transitions from every accepting state back to the initial state. It is tempting to claim that the resulting automaton recognizes $L^\dagger$. This is incorrect in general. The new automaton may not accept all strings in $L^\dagger$.

6 Prove or disprove: for any nonregular language L, the palindromes in L also form a nonregular language.

Solution. False. Let $L = \{0^n 1^n : n \geqslant 0\}$. We proved in class that L is nonregular. Yet, the palindromes in L form the regular language $\{\varepsilon\}$.

7 For a language L, let permute(L) denote the set of all strings that can be obtained by taking a string in L and keeping it as is or reordering its symbols. For example, permute$(\{\epsilon, ab, abb\}) = \{\epsilon, ab, ba, abb, bab, bba\}$. Prove that regular languages are not closed under the permute operation.

Solution. Consider the regular language $L = (01)^*$. Then permute(L) is the language of binary strings with as many 0s as 1s, which we proved in class is nonregular.

8 For each of the following languages L over the binary alphabet, determine whether it is regular and prove your answer:

a. strings of the form wwu, where $w \in \{0,1\}^+$ and $u \in \{0,1\}^*$;

b. strings that are palindromes or contain 00 as a substring;

c. strings that contain a nonempty substring with equally many 0s and 1s.

Solution.

a. Nonregular. We will first prove that the language $L' = L \cap 01^+01^+$ is nonregular. Let $p \geqslant 1$ be arbitrary. Consider the string $w = 01^p01^p \in L'$. Let $w = xyz$ be any decomposition such that y is nonempty and is contained within the first p symbols. If y contains the leading 0, then xz contains only one 0 and therefore $xz \notin L'$. If y is composed entirely of 1s, then the pumped-up string xy^2z is not even in L and therefore also not in L'. By the pumping lemma, L' is nonregular. Now, if L were regular, the closure properties would force the intersection $L \cap 01^+01^+$ to be a regular language as well, we which just showed is not the case. Therefore, L is nonregular.

b. Nonregular. Let $p \geqslant 1$ be arbitrary. Consider the string $w = 1^p01^p \in L$. Let $w = xyz$ be any decomposition such that y is nonempty and is contained within the first p ones. Then pumping up results in the string $1^{p+|y|}01^p$, which is neither a palindrome nor does it contain 00. Thus, $xy^2z \notin L$. By the pumping lemma, L is nonregular.

c. Regular, with regular expression $\Sigma^*(01 \cup 10)\Sigma^*$. Indeed, L by definition contains every string that has 01 or 10 as a substring. Conversely, any string $w \in L$ must contain both 0s and 1s, which means that w must contain 01 or 10.