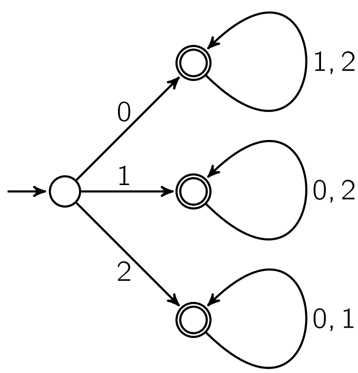
CS 181 PRACTICE MIDTERM 1A

You may state without proof any fact taught in lecture.

1 Give a simple verbal description of the language recognized by the following NFA with alphabet $\{0, 1, 2\}$:



2 Draw a finite automaton (deterministic or nondeterministic) for each of the following languages:

a. binary strings that start with a 1 or have a 1 in the third position from the end;

b. binary strings that contain 01 or 10 but not both.

3 Give a regular expression for each of the following languages over $\Sigma = \{0, 1\}$:

a. even-length strings that contain 01;

b. strings in which every 1 is adjacent to a 0.

4 For languages A and B over a given alphabet Σ, define $A \diamond B$ to be the set of all strings in A that do not contain a substring that is in B. Prove that regular languages are closed under the $\diamond$ operation.

5 Let L be a given regular language. Define $L^\dagger$ to be the set of all strings obtained by taking a nonempty string in L and removing its last symbol. Prove that $L^\dagger$ is regular.

6 Prove or disprove:

a. if L is a nonregular language and w a string, then the concatenation wL is nonregular;

b. if L is a nonregular language, then prefix(L) is also nonregular;

c. if L is a regular language, then the language L' of even-length strings whose first half is in L is also regular.

7 For each of the following languages L over the binary alphabet, determine whether it is regular and prove your answer:

a. even-length strings whose first half contains as many 0s as the second half;

b. strings w such that every prefix of w is equal to some suffix of w;

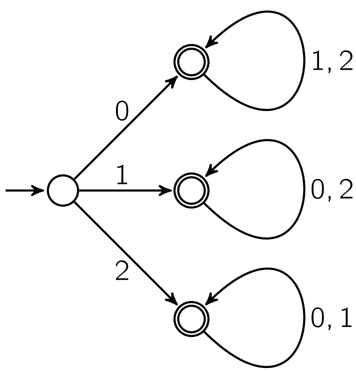
c. strings whose length, when expressed as a decimal integer, uses no digits other than 0 and 1.

SOLUTIONS

CS 181 PRACTICE MIDTERM 1A

You may state without proof any fact taught in lecture.

1 Give a simple verbal description of the language recognized by the following NFA with alphabet $\{0, 1, 2\}$:



Solution: nonempty strings in which the first symbol occurs only once.

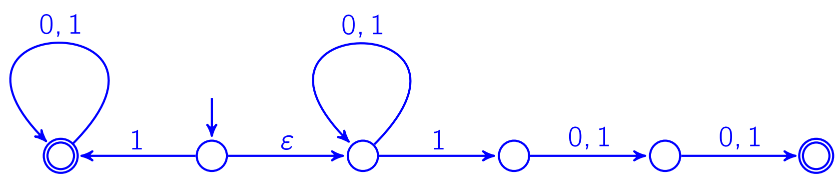
2 Draw a finite automaton (deterministic or nondeterministic) for each of the following languages:

a. binary strings that start with a 1 or have a 1 in the third position from the end;

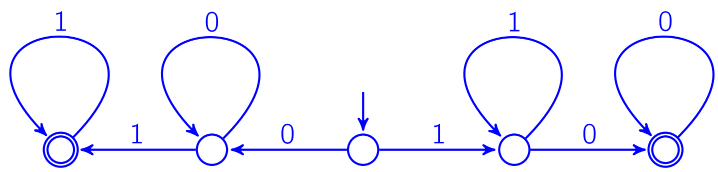
b. binary strings that contain 01 or 10 but not both.

Solution.

a.



b.



3 Give a regular expression for each of the following languages over $\Sigma = \{0, 1\}$:

a. even-length strings that contain 01;

b. strings in which every 1 is adjacent to a 0.

Solution.

a. $(\Sigma\Sigma)^*01(\Sigma\Sigma)^* \cup \Sigma(\Sigma\Sigma)^*01\Sigma(\Sigma\Sigma)^*$

b. $(0 \cup 01 \cup 10 \cup 101)^*$

4 For languages A and B over a given alphabet Σ, define $A \diamond B$ to be the set of all strings in A that do not contain a substring that is in B. Prove that regular languages are closed under the $\diamond$ operation.

Solution. Let A and B be regular. We have $A \diamond B = A \setminus (\Sigma^* B \Sigma^*)$. Here A and B are regular by hypothesis, and Σ is regular because it is finite. Since regular languages are closed under Kleene star, concatenation, and set difference, we conclude that $A \diamond B$ is regular.

5 Let L be a given regular language. Define $L^{\dagger}$ to be the set of all strings obtained by taking a nonempty string in L and removing its last symbol. Prove that $L^{\dagger}$ is regular.

Solution. Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA for L. Then $L^{\dagger}$ is recognized by the DFA $(Q, \Sigma, \delta, q_0, F^{\dagger})$, where $F^{\dagger} = \{q \in Q : \delta(q, \sigma) \in F \text{ for some } \sigma\}$. The new DFA operates like the old one but accepts the input if and only if it can be extended by a single character to a string that D would accept.

6 Prove or disprove:

a. if L is a nonregular language and w a string, then the concatenation wL is nonregular;

b. if L is a nonregular language, then prefix(L) is also nonregular;

c. if L is a regular language, then the language L' of even-length strings whose first half is in L is also regular.

Solution.

a. True. If wL were regular with a DFA $D = (Q, \Sigma, \delta, q_0, F)$, then L would be recognized by the DFA $(Q, \Sigma, \delta, q, F)$, where q is the end state of D after processing w.

b. False. Consider the language L of strings that contain as many 0s as 1s. We showed in class that L is nonregular. But prefix(L) $= \Sigma^*$, which is a regular language because it is given by a regular expression.

c. False. Consider the regular language $L = 0^*$. Then L' is the set of even-length binary strings whose first half does not contain a 1. However, we will show that L' is nonregular. Let $p \geqslant 1$ be arbitrary. Consider the string $w = 0^p1^p \in L'$. Let $w = xyz$ be any decomposition such that y is nonempty and is contained within the first p zeroes. Then the string xz either has odd length, or contains a 1 in its first half, or both. Therefore, $xz \notin L'$. By the pumping lemma, L' is nonregular.

7 For each of the following languages L over the binary alphabet, determine whether it is regular and prove your answer:

a. even-length strings whose first half contains as many 0s as the second half;

b. strings w such that every prefix of w is equal to some suffix of w;

c. strings whose length, when expressed as a decimal integer, uses no digits other than 0 and 1.

Solution.

a. Nonregular. Let $p \geqslant 1$ be arbitrary. Consider the string $w = 0^p110^p \in L$. Let $w = xyz$ be any decomposition such that y is nonempty and is contained within the first p zeroes. Then xy^3z is an even-length string whose first half contains only zeroes and whose second half contains both ones and zeroes. Therefore, $xy^3z \notin L$. By the pumping lemma, L is nonregular.

b. Regular, with regular expression $L = 0^* \cup 1^*$. Indeed, L by definition contains every string in $0^* \cup 1^*$. Conversely, let $w = w_1w_2 \ldots w_n$ be any string in L. Then the prefix $w_1w_2 \ldots w_{n-1}$ must be equal to some suffix of w. But w's only suffix of length $n-1$ is $w_2w_3 \ldots w_n$. This means that $w_1w_2 \ldots w_{n-1} = w_2w_3 \ldots w_n$, which simplifies to $w_1 = w_2 = \cdots = w_n$ and thus $w \in 0^* \cup 1^*$.

c. Nonregular. Let $p \geqslant 1$ be arbitrary. Take $w \in L$ to be any string of length 10^p. Now, let $w = xyz$ be any decomposition such that y is nonempty and has length at most p. Then $|y| \leqslant p \leqslant 10^{p-1}$. Therefore, the length of the pumped-down string xz is at least

$$10^p - 10^{p-1} = 9\underbrace{00\ldots0}_{p-1}$$

and at most

$$10^p - 1 = \underbrace{99\ldots9}_{p}.$$

This means that the length of xz in decimal begins with the digit 9, forcing $xz \notin L$. By the pumping lemma, L is nonregular.