Solutions to Questions - Chapter 5
Adjustable and Floating Rate Mortgage Loans

Question 5-1

In the previous chapter, significant problems regarding the ability of borrowers to meet mortgage payments and the evolution of fixed interest rate mortgages with various payment patterns were discussed. Why didn't this evolution address problems faced by lenders? What have lenders done in recent years to overcome these problems?

These inadequacies stem from the fact that although payment patterns can be altered to suit borrowers as expectations change, the CPM, CAM, and GPM are all originated in fixed interest rates and all have predetermined payment patterns. Neither the interest rate nor the payment patterns will change, regardless of economic conditions. A variety of mortgages are now made with either adjustable interest rates or with variable payment provisions that change with economic conditions.

Question 5-2

How do inflationary expectations influence interest rates on mortgage loans?

Most savings institutions had been making constant payment mortgage loans with relatively long maturities, and the yields on those mortgages did not keep pace with the cost of deposits. These problems prompted savings institutions (lenders) to change the mortgage instruments to now make more mortgages with adjustable interest rate features that will allow adjustments in both interest rates and payments so that the yields on mortgage assets will change in relation to the cost of deposits.

Question 5-3

How does the price level adjusted mortgage (PLAM) address the problem of uncertainty in inflationary expectations? What are some of the practical limitations in implementing a PLAM program?

One concept that has been discussed as a remedy to the imbalance problems for savings institutions is the price level adjusted mortgage (PLAM). To help reduce interest rate risk, or the uncertainty of inflation and its effect on interest rates, it has been suggested that lenders should originate mortgages at interest rates that reflect expectations of the real interest rate plus a risk premium for the likelihood of loss due to default on a given mortgage loan.

Should prices of other goods, represented in the CPI increase faster than housing prices, indexing loan balances to the CPI could result in loan balances increasing faster than property values. If this occurred, borrowers would have an incentive to default. A second problem with PLAMs has to do with the relationship between mortgage payments and borrower incomes. Should inflation increase sharply, it is not likely that borrower incomes would increase at the same rate in the short run. During short periods, the payment burden may increase, and households may find it more difficult to make mortgage payments. A third problem with PLAMs is that the price level chosen for indexation is usually measured on a historical basis. In other words, the index is based on data collected in the previous period but published currently.

Question 5-4

Why do adjustable rate mortgages (ARMs) seem to be a more suitable alternative for mortgage lending than PLAMs?

An ARM provides for adjustments that are more timely for lenders than a PLAM because values for r, p, and f are revised at specific time intervals to reflect market expectations of future values for each component of i between adjustments dates.

Question 5-5

List each of the main terms likely to be negotiated in an ARM. What does pricing an ARM using these terms mean?

Initial interest rate, index, adjustment interval, margin, composite rate, interest rate caps, negative amortization, floors, assumability, discount points, prepayment privilege. Anytime the process of risk bearing is analyzed, individual borrowers and lenders differ in the degree to which they are willing to assume risk. Consequently, the market for ARMs contains a large set of mortgage instruments that differ with respect to how risk is to be shared between borrowers and lenders. The terms listed above are features that might be used in pricing an ARM and establishing the bearing of risk.

Question 5-6

What is the difference between interest rate risk and default risk? How do combinations of terms in ARMs affect the allocation of risk between borrowers and lenders?

Interest rate risk is the risk that the interest rate will change at some time during the life of the loan. Default risk is the risk to the lender that the borrower will not carry out the full terms of the loan agreement. The fact that ARMs shift all or part of the interest rate risk to the borrower means that the risk of default will generally increase to the lender, thereby reducing some of the benefits gained from shifting interest rate risk to borrowers.

Question 5-7

Which of the following two ARMs is likely to be priced higher, that is, offered with a higher initial interest rate? ARM A has a margin of 3 percent and is tied to a two-year index with payments adjustable every two years; payments cannot increase by more than 10 percent from the preceding period; the term is 30 years and no assumption or points will be allowed. ARM B has a margin of 3 percent and is tied to a one-year index with payments to be adjusted each year; payments cannot increase by more than 10 percent from the preceding period; the term is 30 years and no assumption or points are allowed.

ARM A is likely to be priced higher, because it has a longer-term index and adjustment period. Subsequently, the lender bears more risk and can expect a higher return.

Question 5-8

What are forward rates of interest? How are they determined? What do they have to do with indexes used to adjust ARM payments?

Forward rates are based on future interest rate expectations that are implicit in the yield curve and reveal investor expectations of interest rates between any two maturity periods on the yield curve. For example, the yield for a security maturing one year from now is 8 percent, and the yield for a security that matures two years from now is 9 percent. Based on these two yields, we can compute a forward rate, or rate that an investor who invests in a one-year security can expect to reinvest funds for one additional year. This forward rate will be 10 percent because if investors have the opportunity to invest today in either the one- or the two-year security and are indifferent between the two choices, the investor buying a one-year security must be able to earn 10 percent on funds available for reinvestment at the end of year 1. This information is important and represents a reference point that may help lenders and borrowers when pricing ARMs and calculating expected yields at the time ARMs are made. Additionally, interest rate series, which may include forward rates of interest, comprise the indexes used to adjust ARMs. This is especially true if an index is long term in nature.

Question 5-9

Distinguish between the initial rate of interest and expected yield on an ARM. What is the general relationship between the two? How do they generally reflect ARM terms?

One important issue in ARMs is the yield to lenders, or cost to borrowers, for each category of loan. Given the changes in interest rates, payments, and loan balances, it is not obvious what these yields (costs) will be. This means that the costs of each category of loan will be added to the initial interest rate, thus producing an expected yield.

Question 5-10

If an ARM is priced with an initial interest rate of 8 percent and a margin of 2 percent (when the ARM index is also 8 percent at origination) and a fixed rate mortgage with constant payment is available at 11 percent, what does this imply about inflation and the forward rates in the yield curve at the time of origination? What is implied if a fixed rate mortgage were available at 10 percent? 12 percent?

The initial interest rate and expected yield for all ARMs should be lower than that of a fixed rate mortgage on the day of origination. The extent which the initial rate and expected yield on an ARM will be lower than that on a fixed rate mortgage or another ARM, depends on the terms relative to payments, caps, etc. One would expect the difference between interest rates at the point if origination to reflect expectations of inflation and forward rates as well. As a fixed rate mortgage's interest rate increases from 11 percent to 10 percent and 12 percent, greater inflation and/or greater uncertainty with respect to inflation is implied.

Solutions to Problems - Chapter 5
Adjustable Rate and Variable Payment Mortgages

Problem 5-1

(a) Compute the payments at the beginning of each year of the PLAM.

Principal	=	$95,000	Inflation Adjustment	=	6.00%
Term	=	30 years	Points	=	6.00%
Interest Rate	=	4.0%			

Year	(1) BOY Balance	(2) Annual Interest Rate	(3) Monthly Interest Rate (2)/12	(4) Payments	(5) Monthly Interest (3) x (1)	(6) Monthly Amort (4) - (5)	(7) Annual Amort	(8) EOY Balance (1) -(7)	(9) Inflation Adjusted EOY Balance
0	$95,000	4.00%	0.33%	$453.54	$316.67	$136.88	$1,672.98	$93,327	$98,927
1	98,927	4.00%	0.33%	480.76	329.76	151.00	1,845.61	97,081	102,906
3	102,906	4.00%	0.33%	509.60	343.02	166.58	2,036.05	100,870	106,922
4	106,922	4.00%	0.33%	540.18	356.41	183.77	2,246.15	104,676	110,956
5	110,956	4.00%	0.33%	572.59	369.85	202.73	2,477.92	108,479	114,987

(b) The loan balance at the end of the fifth year = $$108,479.

(c) **IRR (CF1, CF2, ….CFn)**

CF_j	n_j
-$89,300	
453.54	n = 12
480.76	n = 12
509.60	n = 12
540.18	n = 12
572.59	n = 11
572.59 + 114,987	n = 1

Solve for the *annual* IRR:

= 0.85% x 12 = 11.11%

Problem 5-2

Year	(1) BOY Balance	(2) Annual Interest Rate	(3) Monthly Interest Rate (2)/12	(4) Payments	(5) Monthly Interest (3) x (1)	(6) Monthly Amort (4) -(5)	(7) Annual Amort.	(8) EOY Balance (1) - (7)
0								
1	$200,000	6.00%	0.50%	$1,199.10	$1,000.00	$199.10	$2,456.02	$197,544
2	$197,544	7.00%	0.58%	$1,327.75	$1,152.34	$175.41	$2,173.82	$195,370

(a)
Monthly Payment = $1,199.10

(b)
Loan balance at EOY 1 = $197,544

(c)
Monthly Payment = $1,327.75

(d)
Loan balance at EOY 2 = $195,370

(e)
Monthly Payment for year 1= $1,000

Problem 5-3

Year	(1) BOY Balance	(2) Annual Interest Rate	(3) Monthly Interest Rate (2)/12	(4) Payments	(5) Monthly Interest (3) x (1)	(6) Monthly Amort (4) -(5)	(7) Annual Amort.	(8) EOY Balance (1) - (7)
0								
1	$150,000	7.00%	0.58%	$997.95	$875.00	$122.95	$1,523.71	$148,476
2	148,476	7.00%	0.58%	$997.95	$866.11	$131.84	$1,633.86	$146,842
3	146,842	7.00%	0.58%	$997.95	$856.58	$141.37	$1,751.98	$145,090
4	145,090	6.00%	0.50%	$905.34	$725.45	$179.89	$2,219.06	$142,871

(a)
Monthly Payment = $997.95
Loan Balance EOY 3 = $145,090

(b)
New Monthly Payment = $905.34

(c)
Interest only monthly payment = $875.00
Monthly payments in year 4 = $935.98

Problem 5-4

Year	(1) BOY Balance	(2) Annual Interest Rate	(3) Monthly Interest Rate (2)/12	(4) Payments	(5) Monthly Interest	(6) Monthly Amort (4) -(5)	(7) Annual Amort.	(8) EOY Balance (1) - (7)
0								
1	$100,000	2.00%	0.17%	$423.85	$166.67	$257.19	$3,114.70	$96,885
2	96,885	6.00%	0.50%	$635.55	$484.43	$151.12	$1,864.15	$95,021

(a)
Monthly payment during 1 year = $423.85

(b)
Monthly payment in 2 year = $635.55

(c)
Percentage increase in monthly payment = 50%

(d)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	BOY Balance	Annual Interest Rate	Monthly Interest Rate (2)/12	Payments	Monthly Interest (3) x (1)	Monthly Amort (4) -(5)	Annual Amort.	EOY Balance (1) - (7)
0								
1	$100,000	2.00%	0.17%	$423.85	$166.67	$257.19	$3,114.70	$96,885
2	96,885	2.00%	0.17%	$423.85	$161.48	$262.38	$3,177.57	$93,708
3	93,708	2.00%	0.17%	$423.85	$156.18	$267.67	$3,241.71	$90,466
4	90,466	6.00%	0.50%	$617.95	$452.33	$165.62	$2,043.02	$88,423

Monthly payments at beginning of year 4 = $ 617.95

Problem 5-5

(a) Interest only payments for the 1 year = $833.33

(b) The loan balance is $200,000. To reset the interest rate at 6% and to amortize the loan over the remaining 27 years (or 324 months) we have:

	PV	=	-$200,000
	i	=	6 ÷ 12
	FV	=	0
	n	=	324
Solve	PMT	=	$1,247.97

Problem 5-6
Compute the payments, loan balance, and yield for an unrestricted ARM

Principal	=	$150,000
Points	=	2.00%
Term	=	30 years
Initial Rate	=	6.0%

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Annual Interest Rate	Monthly Interest Rate (2)/12		Monthly Interest (3) x (1)	Monthl y Amort	Annual Amort.	EOY Balance (1) - (7)

Year	BOY Balance			Payments		(4) -(5)		
0								
1	$150,000	6.00%	0.50%	$899.33	$750.00	$149.33	$1,842.02	$148,158
2	148,158	9.00%	0.75%	$1,200.31	$1,111.18	$89.13	$1,114.78	$147,043
3	147,043	10.50%	0.88%	$1,359.42	$1,286.63	$72.79	$916.79	$146,126
4	146,126	11.50%	0.96%	$1,467.12	$1,400.38	$66.74	$844.50	$145,282
5	145,282	13.00%	1.08%	$1,630.42	$1,573.89	$56.53	$720.27	$144,562

IRR (CF1, CF2, ….CFn)

CF_j	**n_j**
-$147,000	
899.33	n = 12
1200.31	n = 12
1359.42	n = 12
1467.12	n = 12
1630.42	n = 11
1630.42 + 144,562	n = 1

Solve for the IRR:

= 0.85% x 12 = 10.16% (annual rate, compounded monthly)

Problem 5-7

Compute the payments, loan balances, and yield for an ARM that has a maximum 5% annual payment cap and allows negative amortization.

Principal	=	$150,000
Term	=	30 years
Points	=	2.00%
Initial Rate	=	7.0%

	(1)	(2)	(3)	(4)	(5)
Year	BOY Balance	Uncapped Rate	Payment Uncapped	Payment Capped	EOY Balance
1	$150,000	7.00%	$997.95	$997.95	$148,476
2	$148,476	9.00%	$1,202.89	$1,047.85	$149,298
3	$149,298	10.50%	$1,380.27	$1,100.24	$151,894
4	$151,894	11.50%	$1,525.03	$1,155.26	$155,695
5	$155,695	13.00%	$1,747.28	$1,213.02	$161,731
6	$161,731				

Note: EOY Balance is calculated by using: FV (n, i, PV, PMT)

PV = Loan amount
n = 12 months
i = Uncapped rate
PMT = Capped payment
FV =

Calculator: **IRR (CF1, CF2, ….CFn)**

CF_j	n_j
-$147,000	
997.95	n = 12
1047.85	n = 12
1100.24	n = 12
1155.26	n = 12
1213.02	n = 11
1213.02 + 161,731	n = 1

Solve for the IRR:

= 0.8706% x 12 = 10.45% (annual rate, compounded monthly)

Problem 5-8

Compute the payments, loan balances, and yield for an ARM that has a 1% annual and 3% lifetime interest rate cap and does not accumulate negative amortization.

Principal	=	$150,000
Points	=	2.00%
Term	=	30 years
Initial Rate	=	7.5%

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	BOY Balance	Uncapped Interest Rate	Capped Interest Rate	Monthly Interest Rate (3) /12	Payment (@ Capped Rate)	Monthly Interest (1) x (3)/12	Monthly Amort (5) - (6)	Annual Amort	EOY Balance (1) - (8)
0									
1	$150,000	7.50%	7.50%	0.63%	$1,048.82	$937.50	$111.32	$1,382.75	$148,617
2	148,617	9.00%	8.50%	0.71%	$1,151.44	$1,052.71	$98.74	$1,232.11	$147,385
3	147,385	10.50%	9.50%	0.79%	$1,255.55	$1,166.80	$88.75	$1,112.59	$146,273
4	146,273	11.50%	10.50%	0.88%	$1,360.78	$1,279.88	$80.89	$1,018.84	$145,254
5	145,254	13.00%	10.50%	0.88%	$1,360.78	$1,270.97	$89.81	$1,131.12	$144,123
	144,123								

Calculator: **IRR (CF1, CF2, ….CFn)**

CF_j	n_j
-$147,000	
1048.82	n = 12
1151.44	n = 12
1255.55	n = 12
1360.78	n = 12
1360.78	n = 11
1360.78 + 144,123	n = 1

Solve for the IRR:

= 0.80% x 12 = 9.65% (annual rate, compounded monthly)

Problem 5-9

(a) Compute the payments, loan balances, and yield for a Stable Home Mortgage which is comprised of a fixed and adjustable rate component.

Loan Amount	=	$95,000
Points	=	2.00%
Fixed Rate Portion:		75.00% of the loan balance
		10.50% annual interest rate
		30 year term

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	BOY Balance	Annual Interest Rate	Monthly Interest Rate (2)/12	Payments	Monthly Interest (3) x (1)	Monthly Amort (4) -(5)	Annual Amort.	EOY Balance (1) - (7)
0								
1	$71,250	10.50%	0.88%	$651.75	$623.44	$28.31	$356.61	$70.893
2	70,893	10.50%	0.88%	651.75	620.32	31.43	395.91	70.497
3	70,497	10.50%	0.88%	651.75	616.85	34.90	439.54	70,058
4	70,058	10.50%	0.88%	651.75	613.01	38.74	487.98	69,570
5	69,570	10.50%	0.88%	651.75	608.74	43.01	541.75	69,028

Adjustable Rate Portion:	25.00% of the loan balance
	9.00% initial interest rate
	2.00% margin

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	BOY Balance	Annual Interest Rate	Monthly Interest Rate (2)/12	Payments	Monthly Interest (3) x (1)	Monthly Amort (4) -(5)	Annual Amort	EOY Balance (1) - (7)
0								
1	$23,750	9.00%	0.75%	$191.10	$178.13	$12.97	$162.26	$23,588
2	23,588	12.00%	1.00%	243.51	235.88	7.63	96.80	23,491
3	23,491	13.00%	1.08%	261.49	254.49	7.00	89.19	23,402
4	23,402	10.00%	0.83%	209.23	195.01	14.22	178.68	23,223
5	23,223	14.00%	1.17%	278.40	270.94	7.46	95.55	23,128

MORTGAGE SUMMARY:

YEAR	BOY Balance	Payments	EOY Balance
0			
1	$95,000.00	$842.85	$94,481.13
2	94,481.13	895.26	93,988.42
3	93,988.42	913.24	93,459.69
4	93,459.69	860.99	92,793.03
5	82,793.03	930.15	92,155.73

Calculator: Calculator: **IRR (CF1, CF2, ….CFn)**

CF_j	n_j
-$93,100	
842.85	n = 12
895.26	n = 12
913.24	n = 12

	860.99	n = 12
	930.15	n = 11
	930.15 + 92,155.73	n = 1

Solve for the IRR:

= 0.94% x 12 = 11.26% (annual rate, compounded monthly)

(b) Adjustable rate portion now has an initial rate of 9.5% and an annual interest rate cap of 1%

Loan Amount	=	$95,000
Points	=	2.00%
Fixed Rate Portion:		75.00% of the loan balance
		10.50% annual interest rate
		30 year term

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	BOY Balance	Annual Interest Rate	Monthly Interest Rate (2)/12	Payments	Monthly Interest (3) x (1)	Monthly Amort (4) -(5)	Annual Amort.	EOY Balance (1) - (7)
0								
1	$71,250	10.50%	0.88%	$651.75	$623.44	$28.31	$356.61	$70.893
2	70,893	10.50%	0.88%	651.75	620.32	31.43	395.91	70.497
3	70,497	10.50%	0.88%	651.75	616.85	34.90	439.54	70,058
4	70,058	10.50%	0.88%	651.75	613.01	38.74	487.98	69,570
5	69,570	10.50%	0.88%	651.75	608.74	43.01	541.75	69,028

Adjustable Rate Portion:	25.00% of the loan balance
	9.00% initial interest rate
	2.00% margin

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	BOY Balance	Capped Interest Rate	Monthly Interest Rate (2)/12	Payments	Monthly Interest (3) x (1)	Monthly Amort (4) -(5)	Annual Amort	EOY Balance (1) - (7)
0								
1	$23,750	9.50%	0.791%	$199.70	See note.	See note.	See note.	$23,604
2	23,604	10.50%	0.875%	217.00				23,472
3	23,472	11.50%	0.958%	234.45				23,351
4	23,351	10.00%	0.833%	208.78				23,173
5	23,173	11.00%	0.917%	225.50				23,008

Note: Although shown above, these columns are not necessary if a financial calculator are used to get the ending loan balance as was done here. To get the EOY balance, simply enter the BOY Balance as PV, the monthly interest rate in effect during that year as i, the monthly payment as PMT, 12 (for 12 months) as n, and solve for the FV. For example, in year 1 we have: PV = -$23,750, i = 9.50%/12, PMT = $199.70, n=12, solve for FV. Answer is $23,604.

MORTGAGE SUMMARY:

YEAR	BOY Balance	Payments	EOY Balance
0			
1	$95,000.	$851.45	$94,497.
2	94,497.	868.75	93,969.
3	93,969.	886.20	93,409.
4	93,409.	860.53	92,743.

5	92,473.	877.25	92,036.

Yield: Using a financial calculator, the yield is now 11.01%.

Problem 5-10

(a) Loan Balance at the end of year five is $116,333.93

Solution:

n = 5x12 or 60
i = 12/12 or 1
PV = -$100,000
PMT = $800

Solve for the loan balance:

FV = $116,333.93

Loan Balance at the end of year 30 is $798,992.83

Solution:

n = 30x12 or 360
i = 12/12 or 1
PV = -$100,000
PMT = $800

Solve for the loan balance:

FV = $798,992.83

(b) Year 1:

Interest paid will be $800 per month, and interest accrued as negative amortization will be $200 per month compounded monthly at 12% over 12 months a year (12%/12), for one year we have -$2,836.50.

Solution:

n = 12
i = 12/12 or 1
PV = 0
PMT = $200

Solve for the future value:

FV = -$2,536.50

Therefore, interest paid equals $9,600 (or $800x12=$9,600) in year one and interest accrued is $2,536.50.

Year 5:

Balance at the beginning of year 5:

n = 4x12 or 48
i = 12/12 or 1
PV = -$100,000
PMT = $800

Solve for the loan balance:

FV = $112,244.52

Interest due during first month of year 5:

$112,244.52 x (12%/12) = $1,122.45

Negative Amortization during year 5:

n = 12
i = 12/12 or 1
PV = 0
PMT = $322.45

Solve for negative amortization:

FV = -$4,089.41

Therefore, total interest paid during year five is $9,600 (or $800x12=$9,600) in year five and interest accrued is $4,089.41.

Problem 5-11
The effective cost increases to 14.13%

Problem 5-12
The effective cost increase to 13.96%. The interest rate is limited to the lifetime cap of 5% which is why the rate is the same as the prior year.

Problem 5-13
The yield increases to 17.84%. The payment cap limits the payment increase but not the effective cost because interest is still paid at the market interest rate. The unpaid interest accrues (negative amortization) and is reflected in the higher loan balance.