CS163: Deep Learning for Computer Vision

Lecture 6: Backpropagation

Courtesy of Prof. Justin Johnson at Umich and the team of Stanford CS231n for the slides

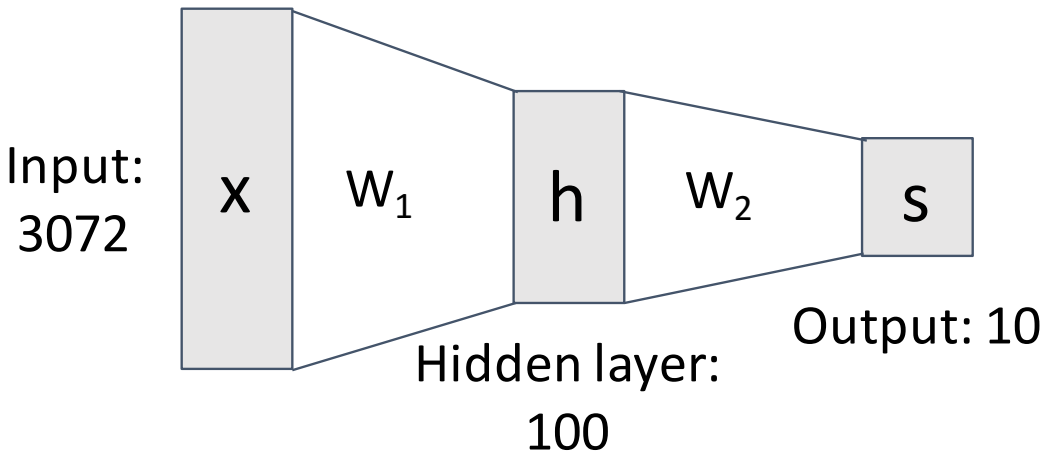
Announcement

- TA Tutorial on Course Project
- Deep Learning Hardware and Software

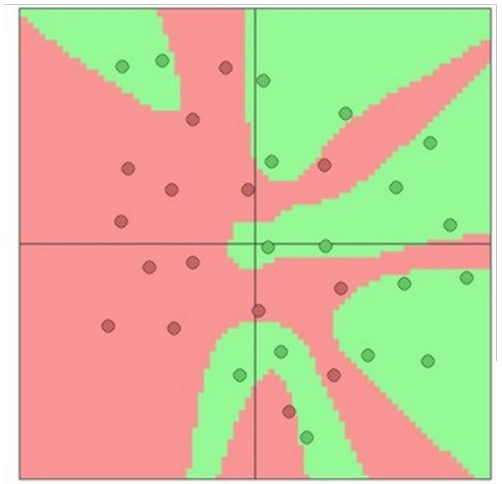
Last time: Neural Networks

From linear classifiers to fully-connected networks

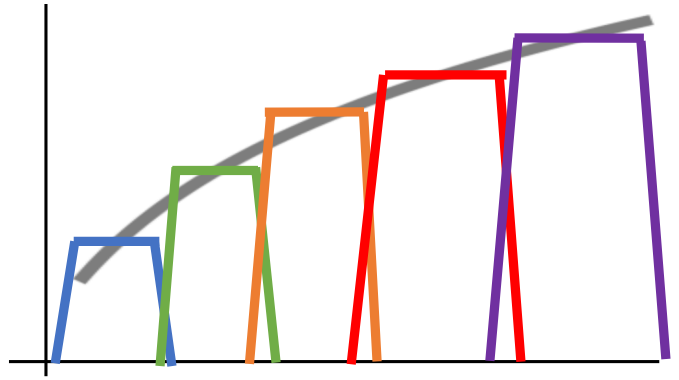
$$f(x) = W_2 \max(0, W_1 x + b_1) + b_2$$



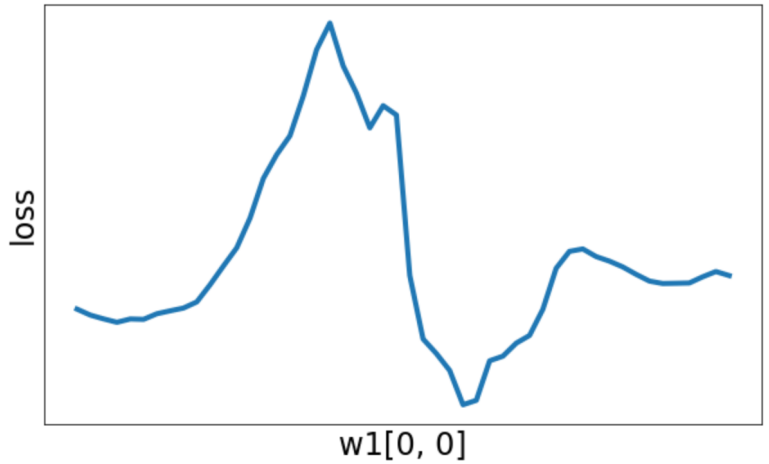
Space Warping



Universal Approximation



Nonconvex



Problem: How to compute gradients?

$s = W_2 \max(0, W_1 x + b_1) + b_2$ Nonlinear score function

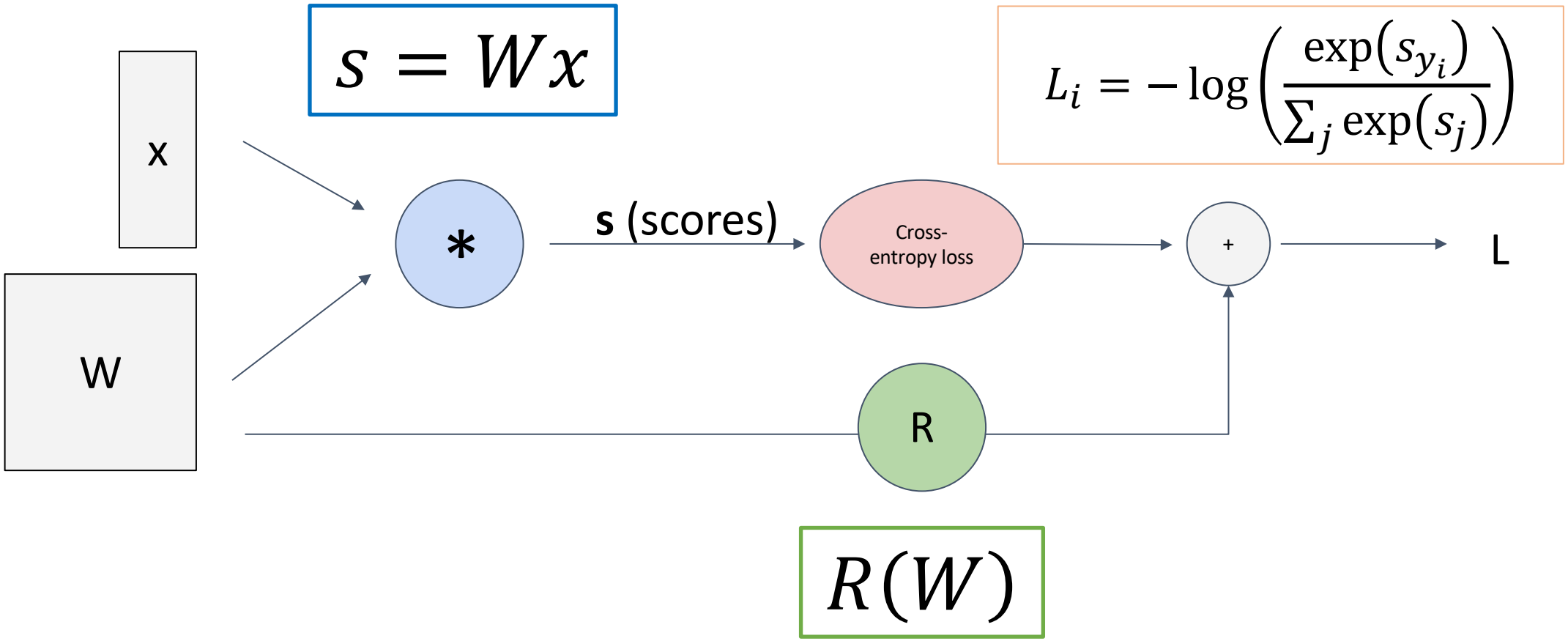
$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$ Softmax loss

$R(W) = \sum_k W_k^2$ L2 Regularization

$L(W_1, W_2, b_1, b_2) = \frac{1}{N}\sum_{i=1}^{N} L_i + \lambda R(W_1) + \lambda R(W_2)$ Total loss

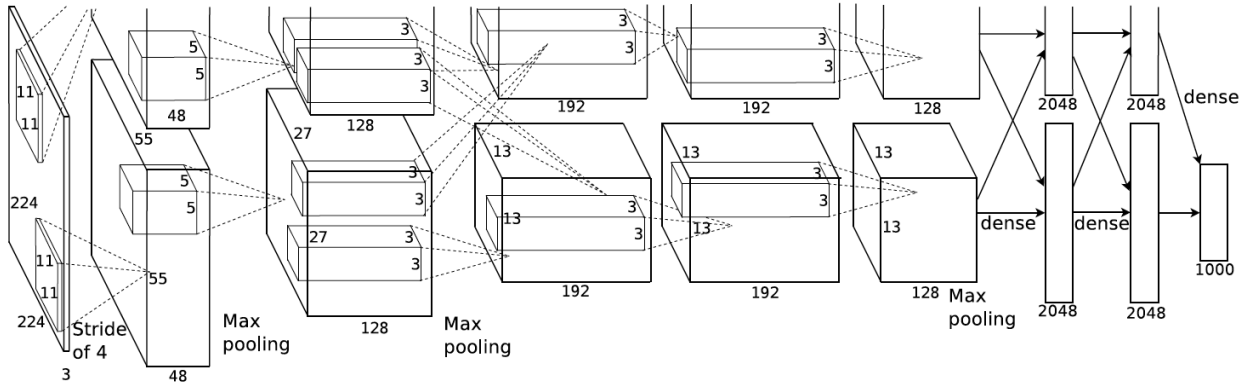
If we can compute $\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_2}, \frac{\partial L}{\partial b_1}, \frac{\partial L}{\partial b_2}$ then we can optimize with SGD

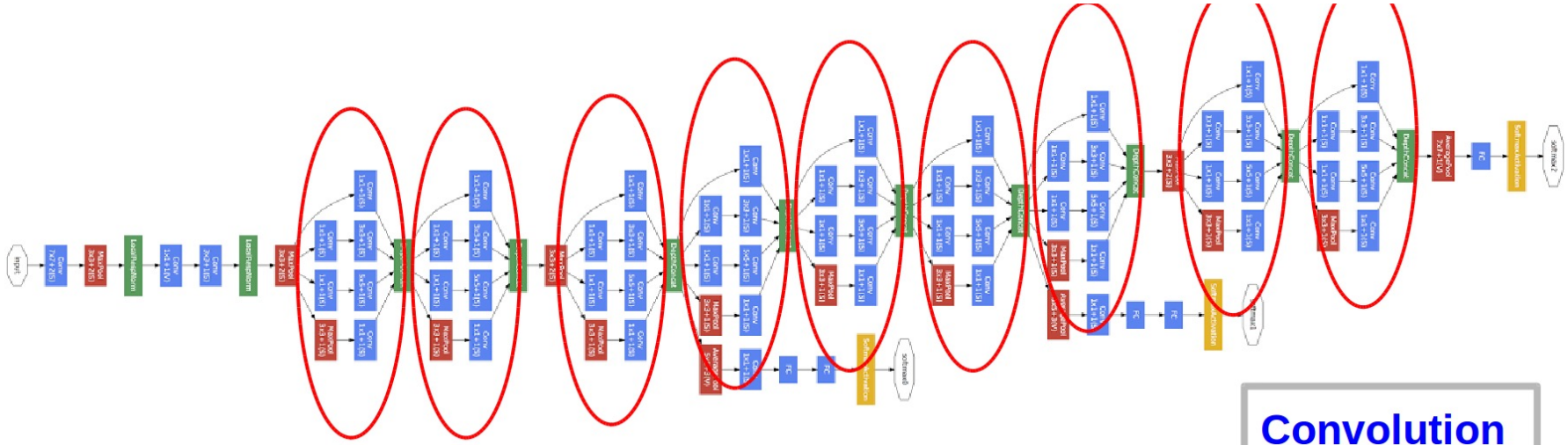
Computational Graphs



Simplified computational graph to denote neural networks

AlexNet



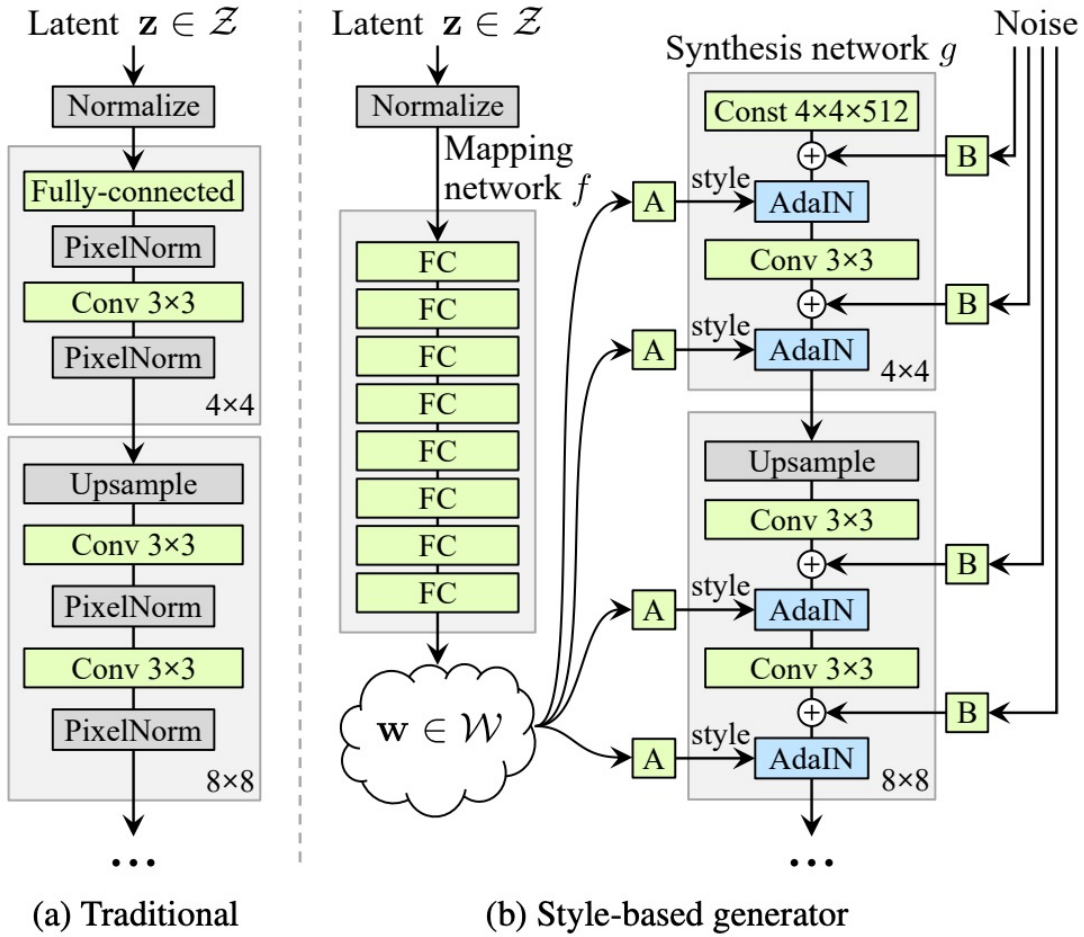


GoogleNet

Convolution
Pooling
Softmax
Concat/Normalize

Simplified computational graph to denote neural networks

StyleGAN



Transformer

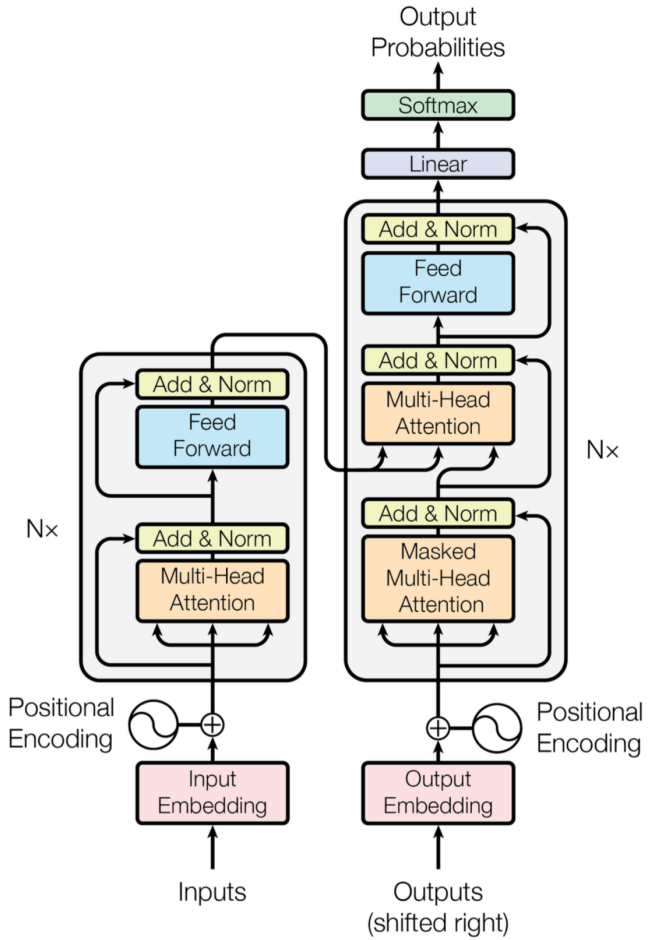
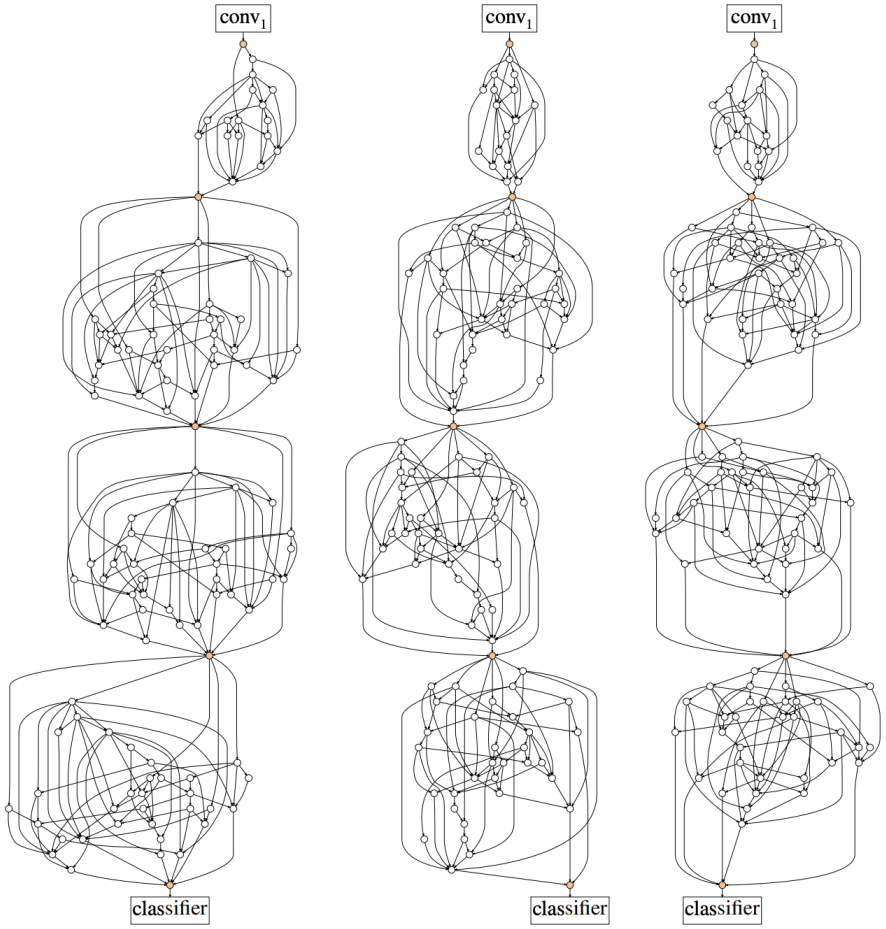


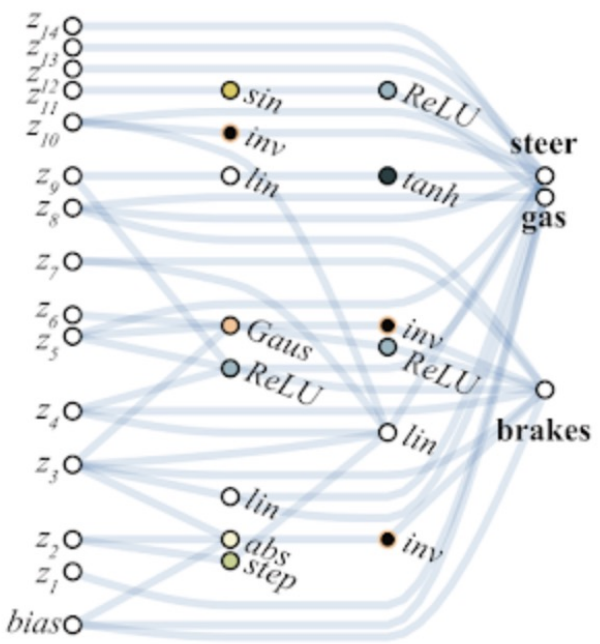
Figure 1: The Transformer - model architecture.

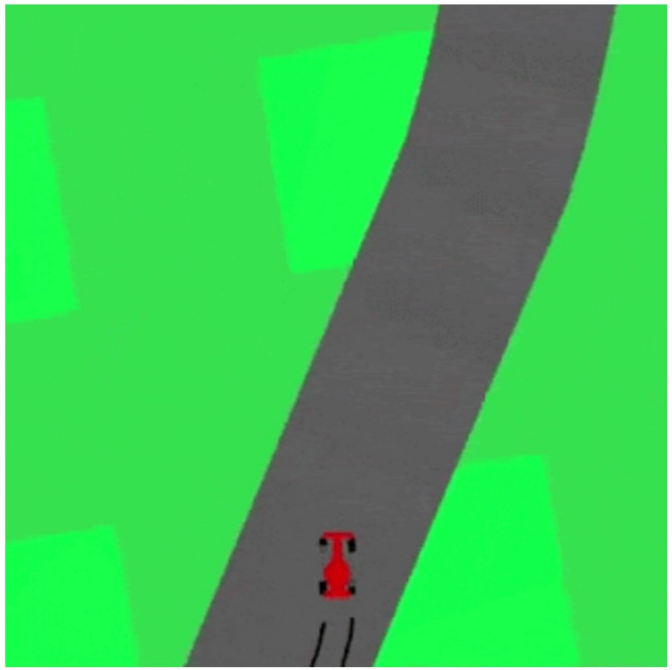
Simplified computational graph to denote neural networks



Xie et al. Random Wired Networks

Could be messy….





Weight agnostic neural networks by David Ha

Chain Rule

Suppose that functions $y = f(u)$ and $u = g(x)$ are both differentiable, then the chain rule states that

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

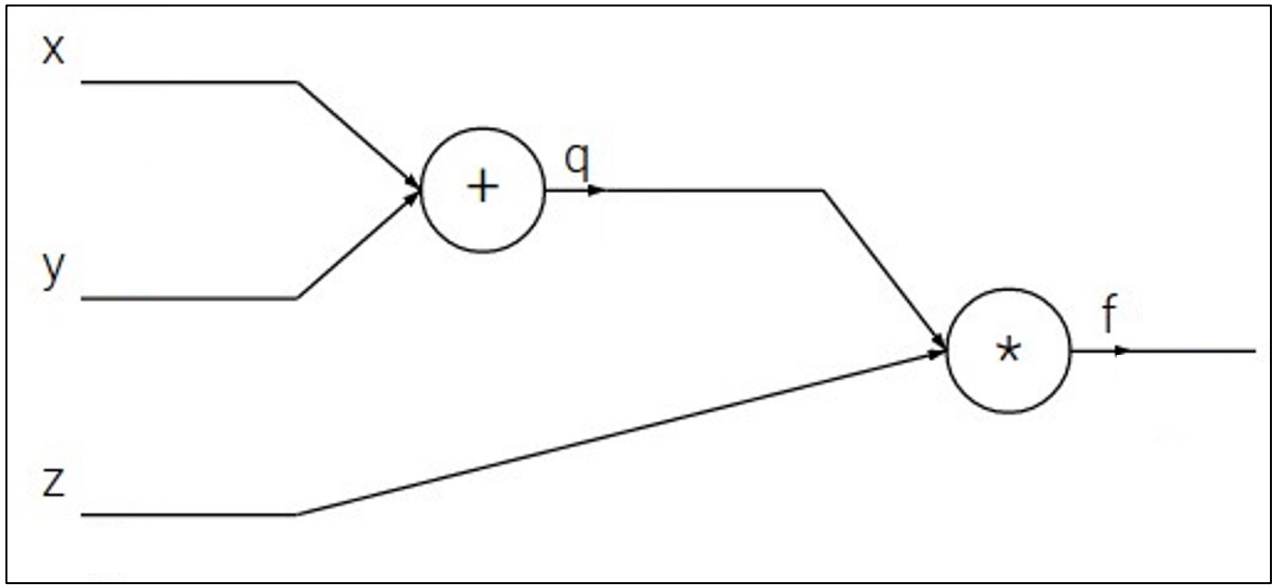
In more general scenario, suppose that y has variables $u_1, u_2, \dots, u_m$, where each differentiable function u_i has variables $x_1, x_2, \dots, x_n$. Then the chain rule gives

$$\frac{dy}{dx_i} = \frac{dy}{du_1}\frac{du_1}{dx_i} + \frac{dy}{du_2}\frac{du_2}{dx_i} + \dots + \frac{dy}{du_m}\frac{du_m}{dx_i}$$

for any $i = 1,2, \dots, n$

Backpropagation: Simple Example

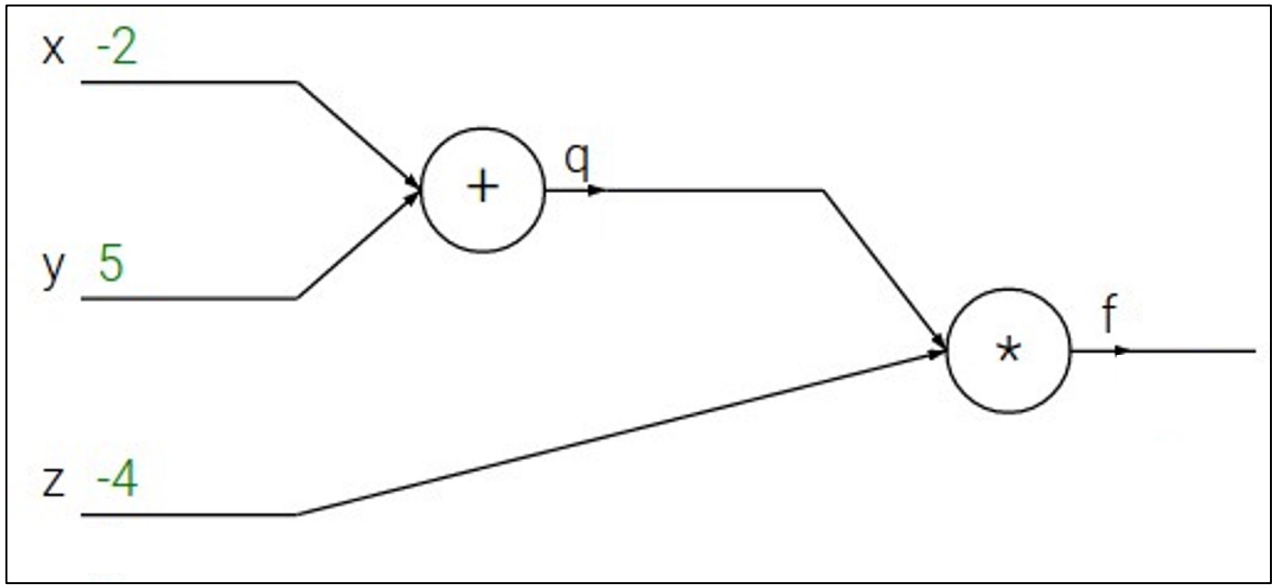
$$f(x, y, z) = (x + y) \cdot z$$



Backpropagation: Simple Example

$$f(x, y, z) = (x + y) \cdot z$$

e.g. x = -2, y = 5, z = -4



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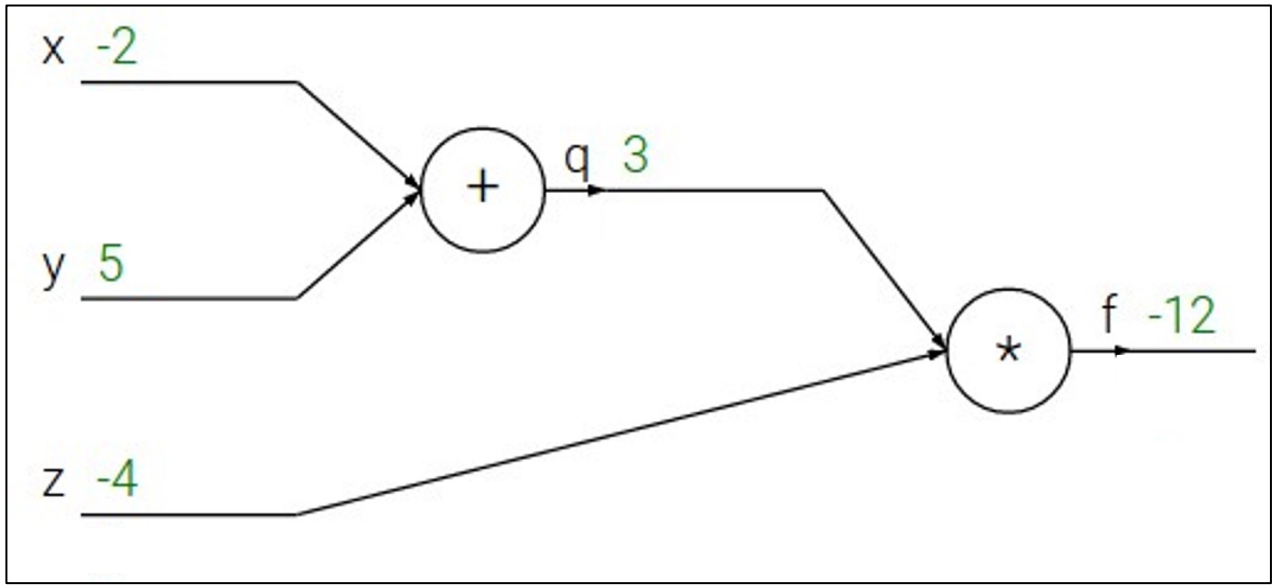

1. Forward pass: Compute outputs

$$q = x + y \quad f = q \cdot z$$

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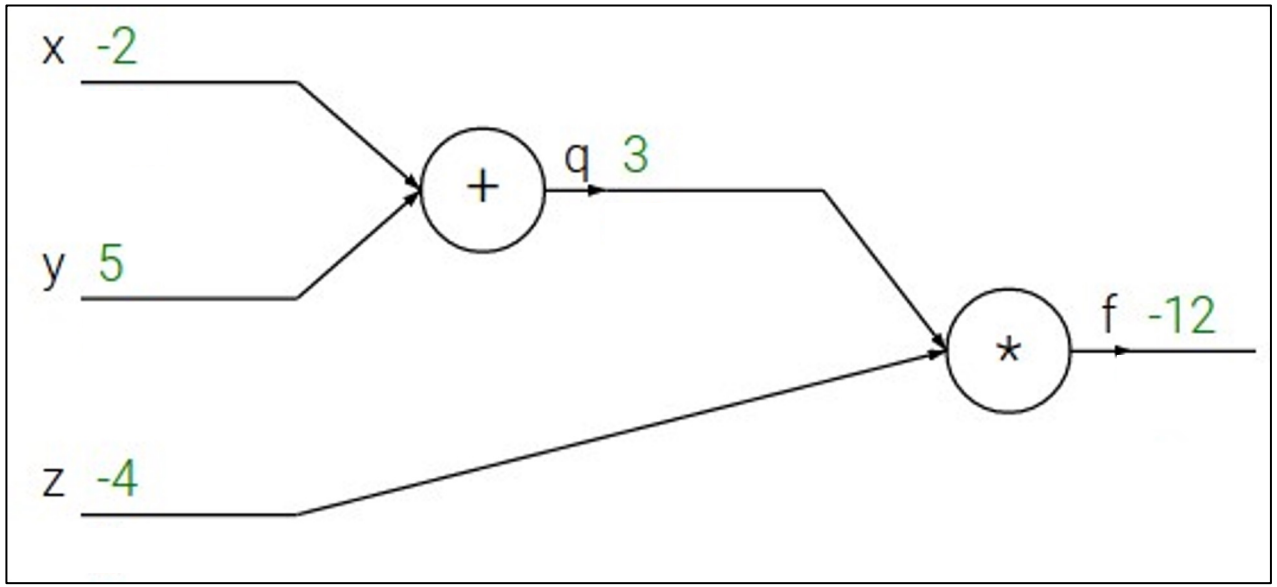

1. Forward pass: Compute outputs

$$q = x + y \quad f = q \cdot z$$

2. Backward pass: Compute derivatives

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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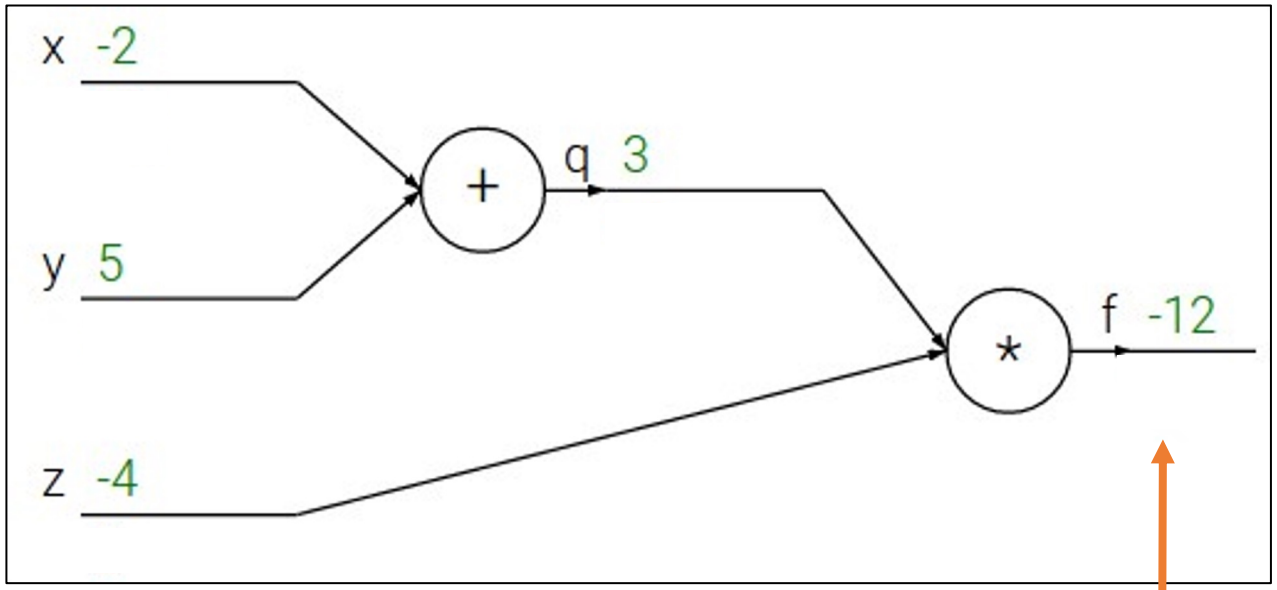

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$\frac{\partial f}{\partial f}$

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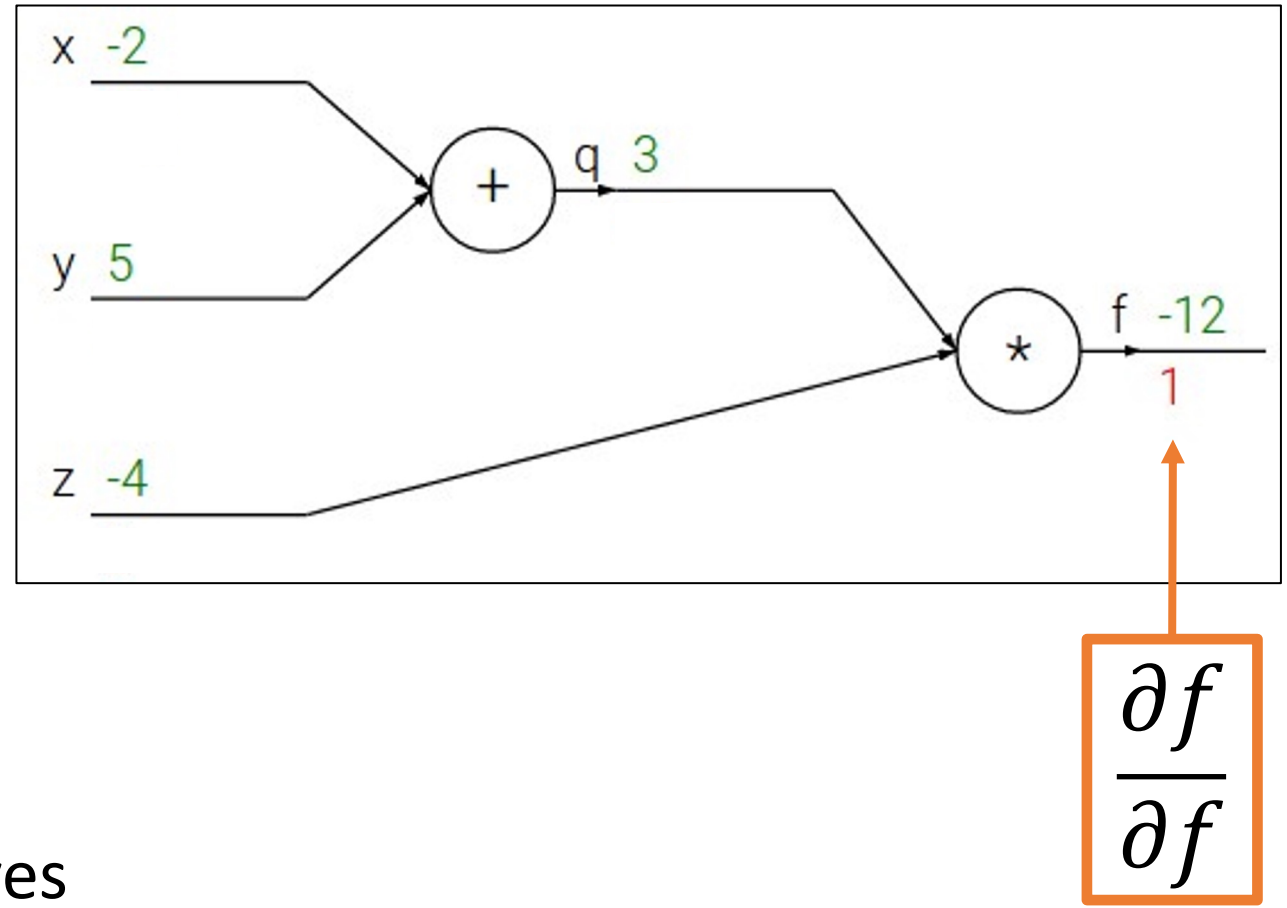

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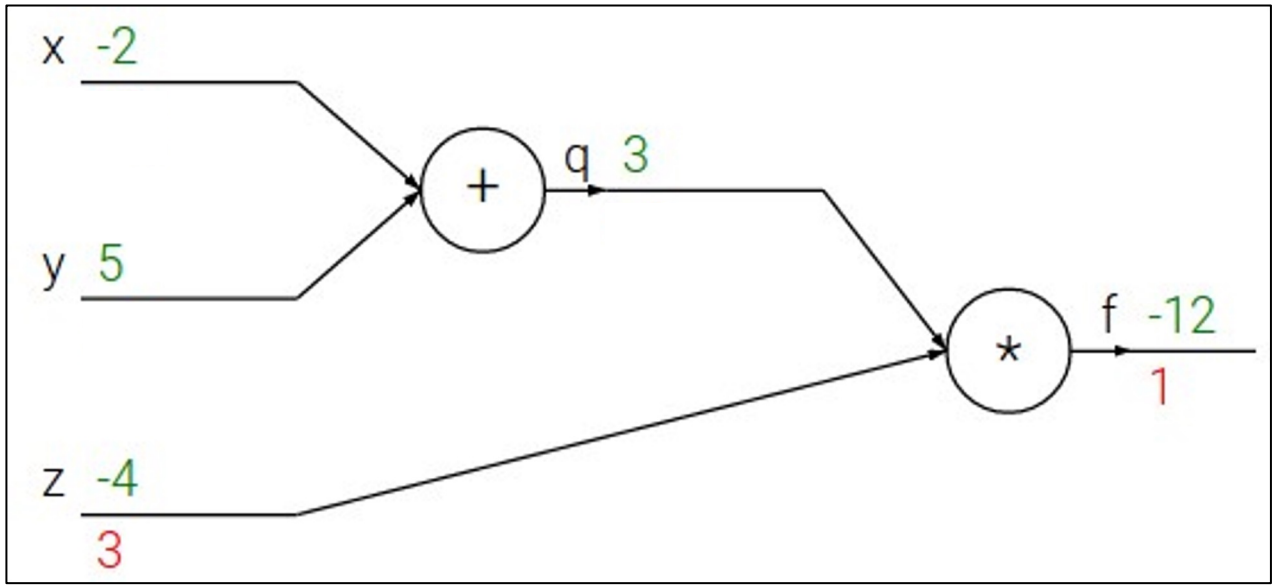

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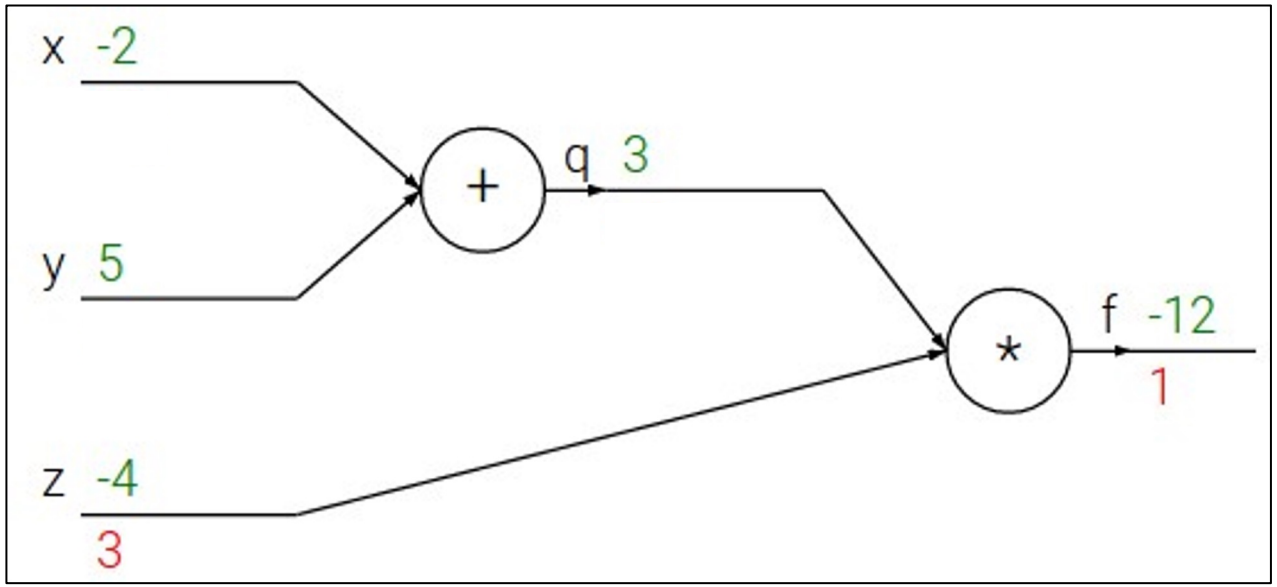

1. Forward pass: Compute outputs

$$q = x + y \qquad f = q \cdot z$$

2. Backward pass: Compute derivatives

$$\frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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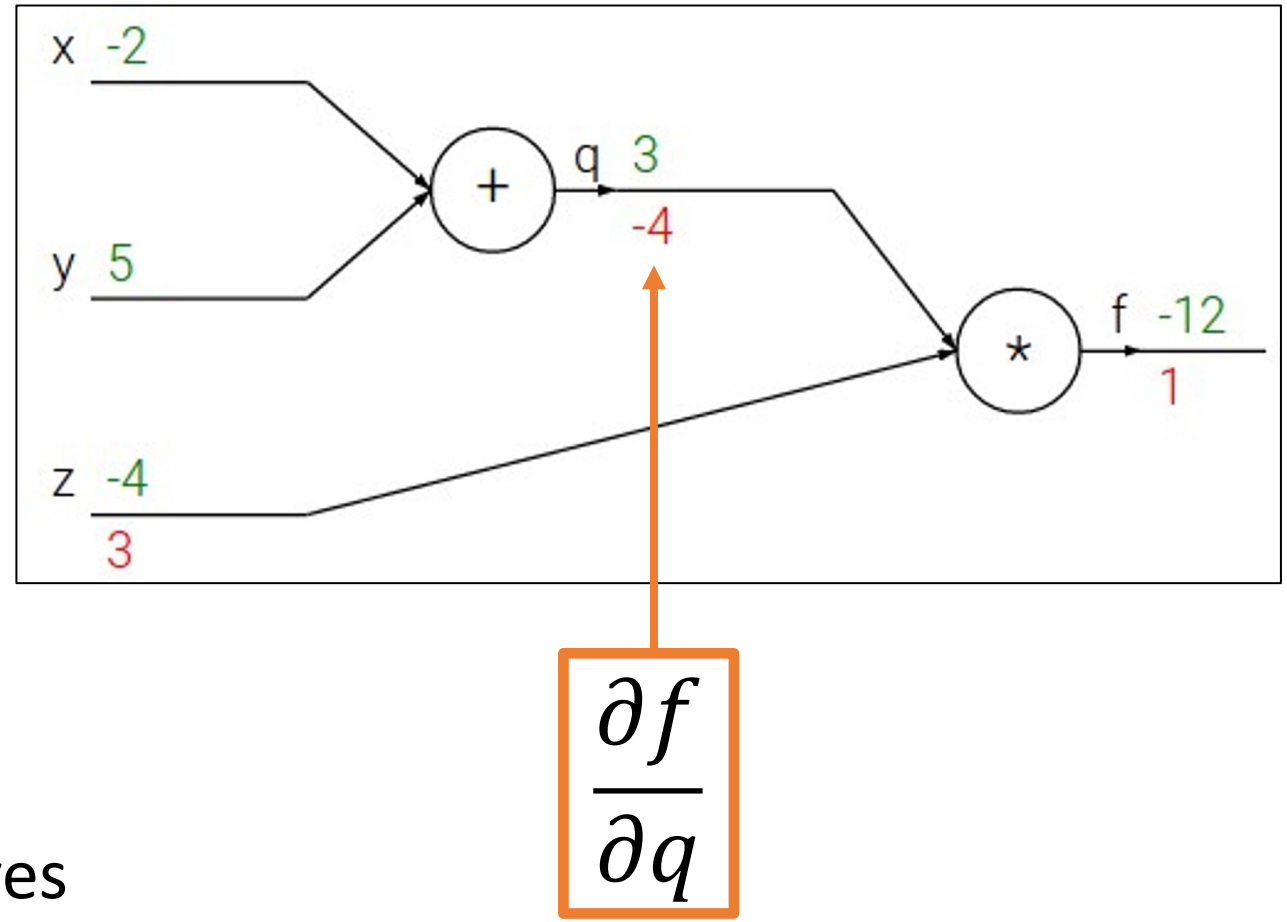

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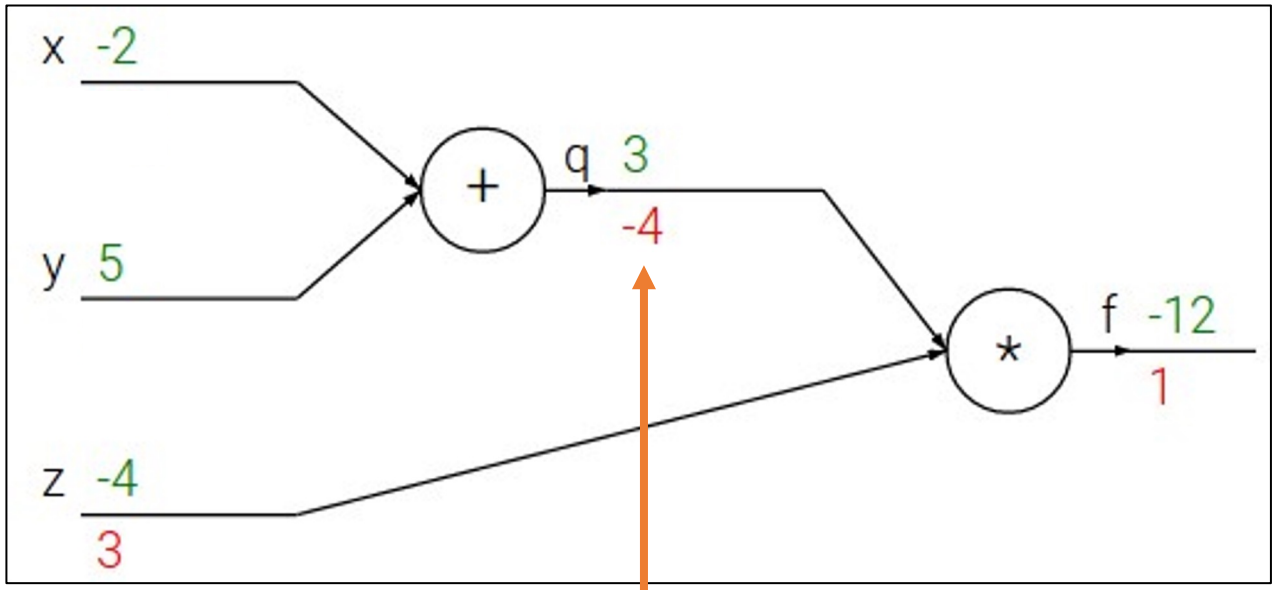

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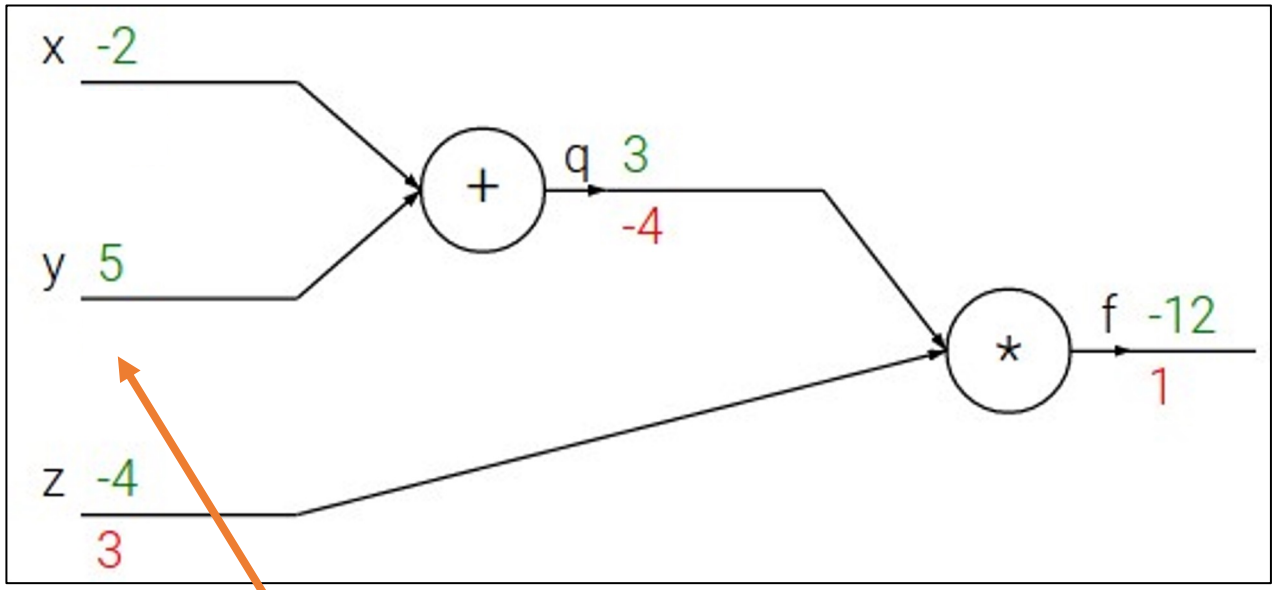

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$\frac{\partial f}{\partial y}$

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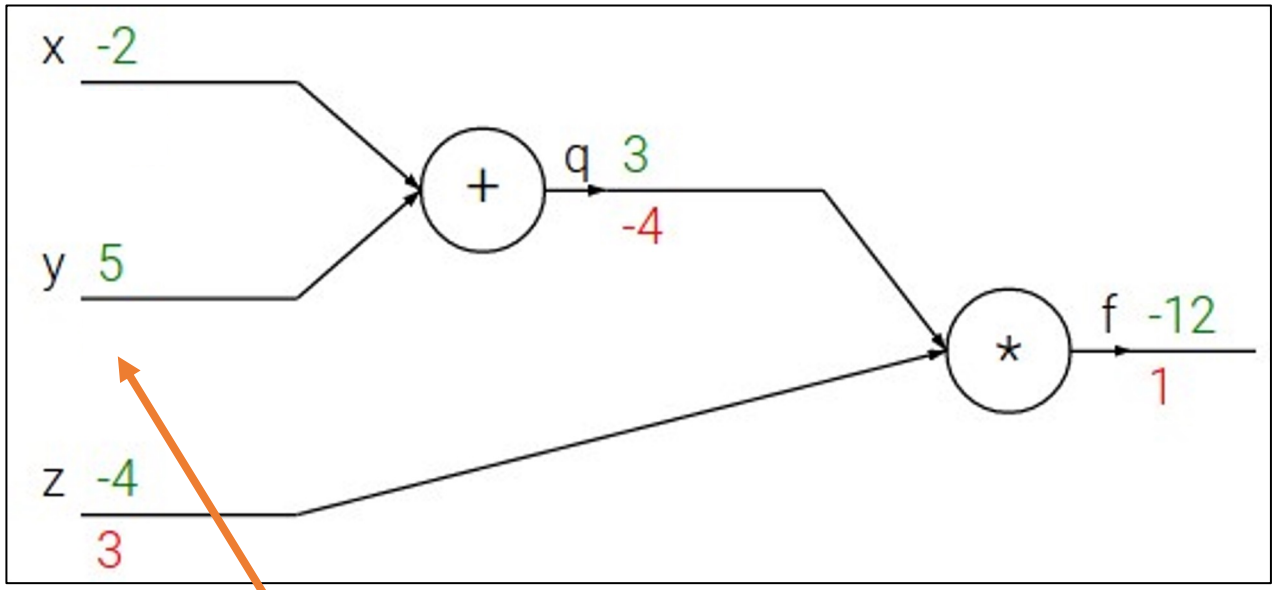

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Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

Chain Rule

$$\frac{\partial f}{\partial y} = \frac{\partial q}{\partial y}\frac{\partial f}{\partial q}$$

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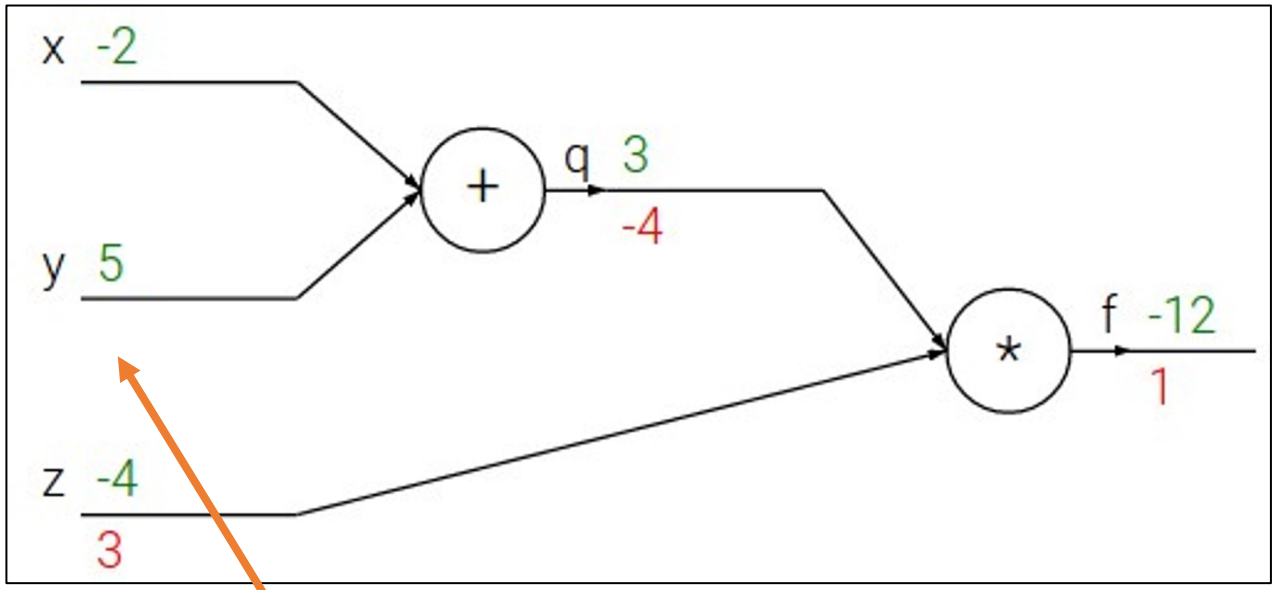

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Chain Rule

$$\frac{\partial f}{\partial y} = \frac{\partial q}{\partial y}\frac{\partial f}{\partial q}$$

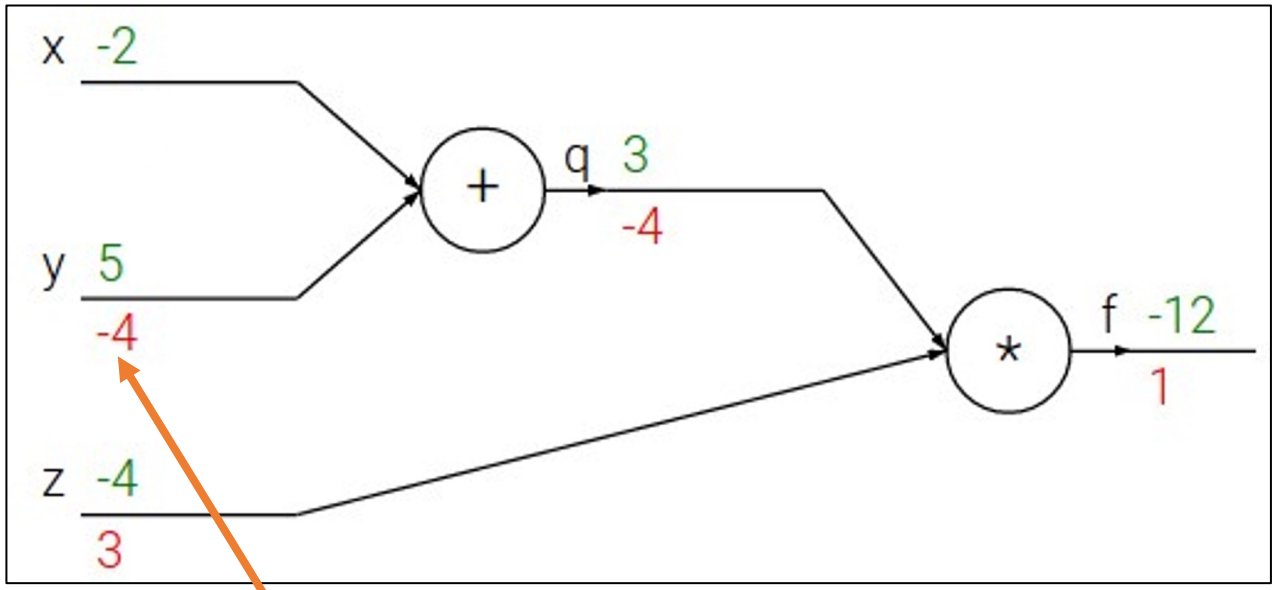
$$\frac{\partial q}{\partial y} = 1$$

Downstream Gradient **Local Gradient** **Upstream Gradient**

Backpropagation: Simple Example

$$f(x, y, z) = (x + y) \cdot z$$

e.g. x = -2, y = 5, z = -4



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Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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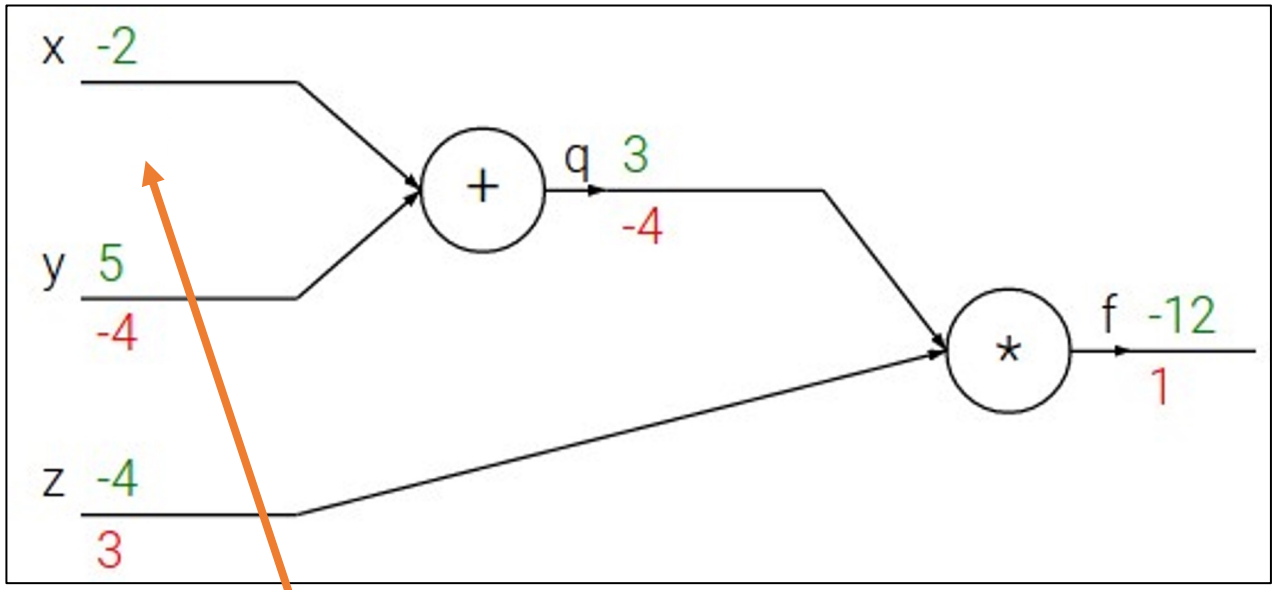

1. Forward pass: Compute outputs

$$q = x + y \qquad f = q \cdot z$$

2. Backward pass: Compute derivatives

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

Chain Rule

$$\frac{\partial f}{\partial x} = \frac{\partial q}{\partial x}\frac{\partial f}{\partial q}$$

$$\frac{\partial q}{\partial x} = 1$$

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Backpropagation: Simple Example

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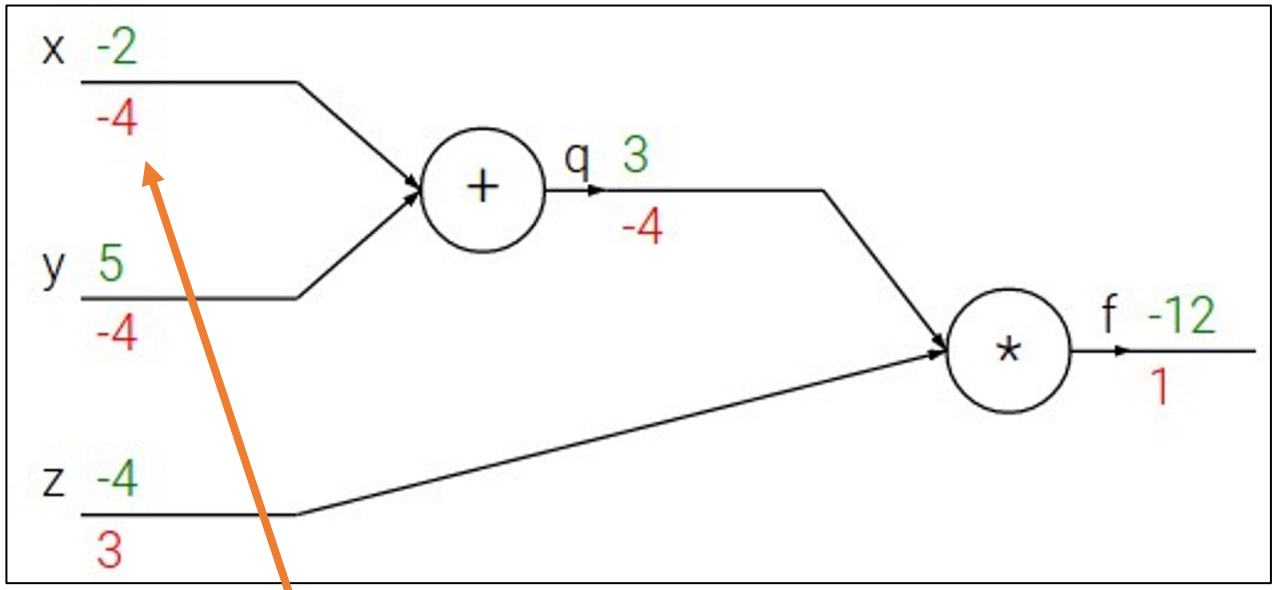

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$$q = x + y \qquad f = q \cdot z$$

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Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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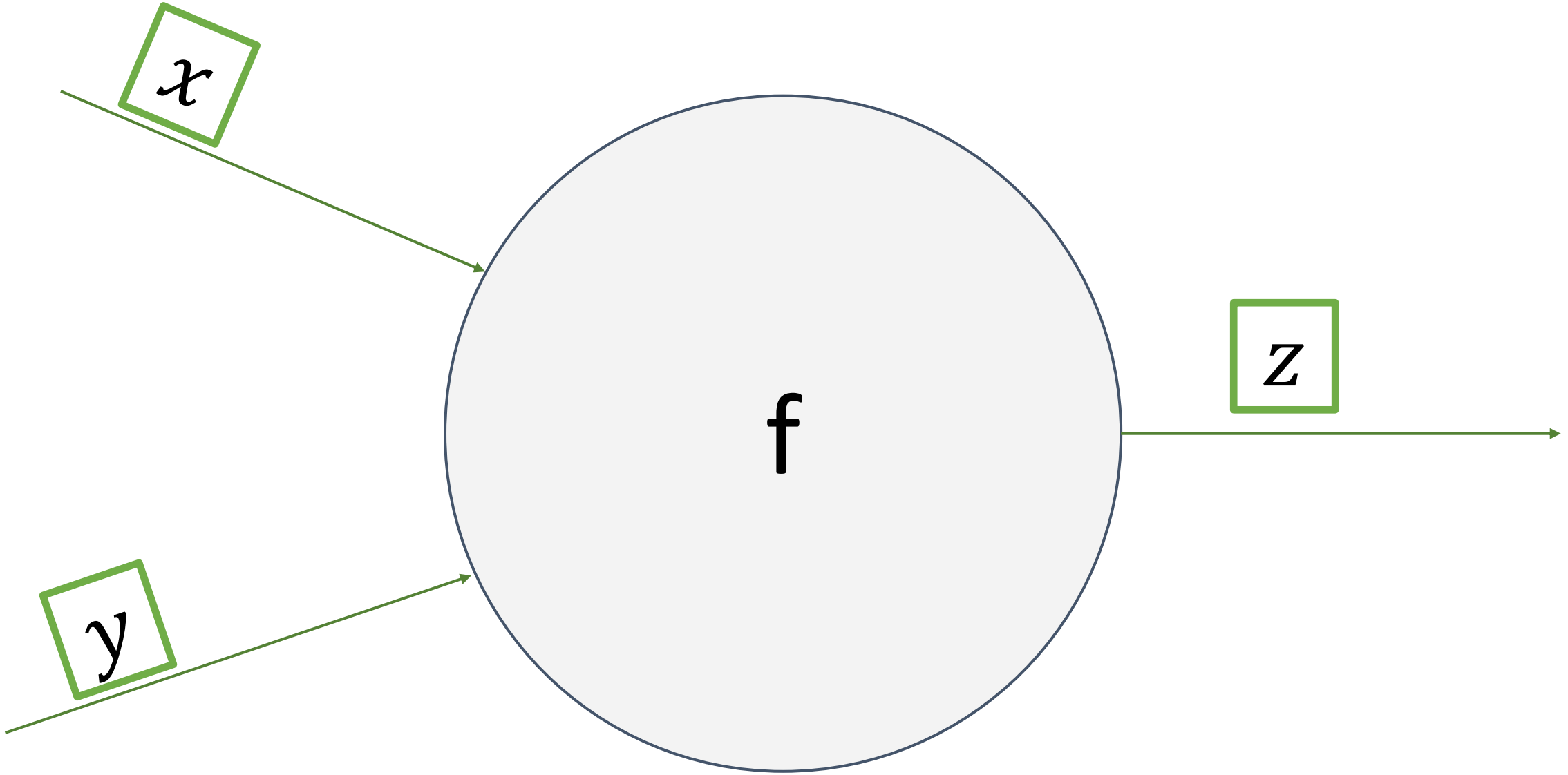
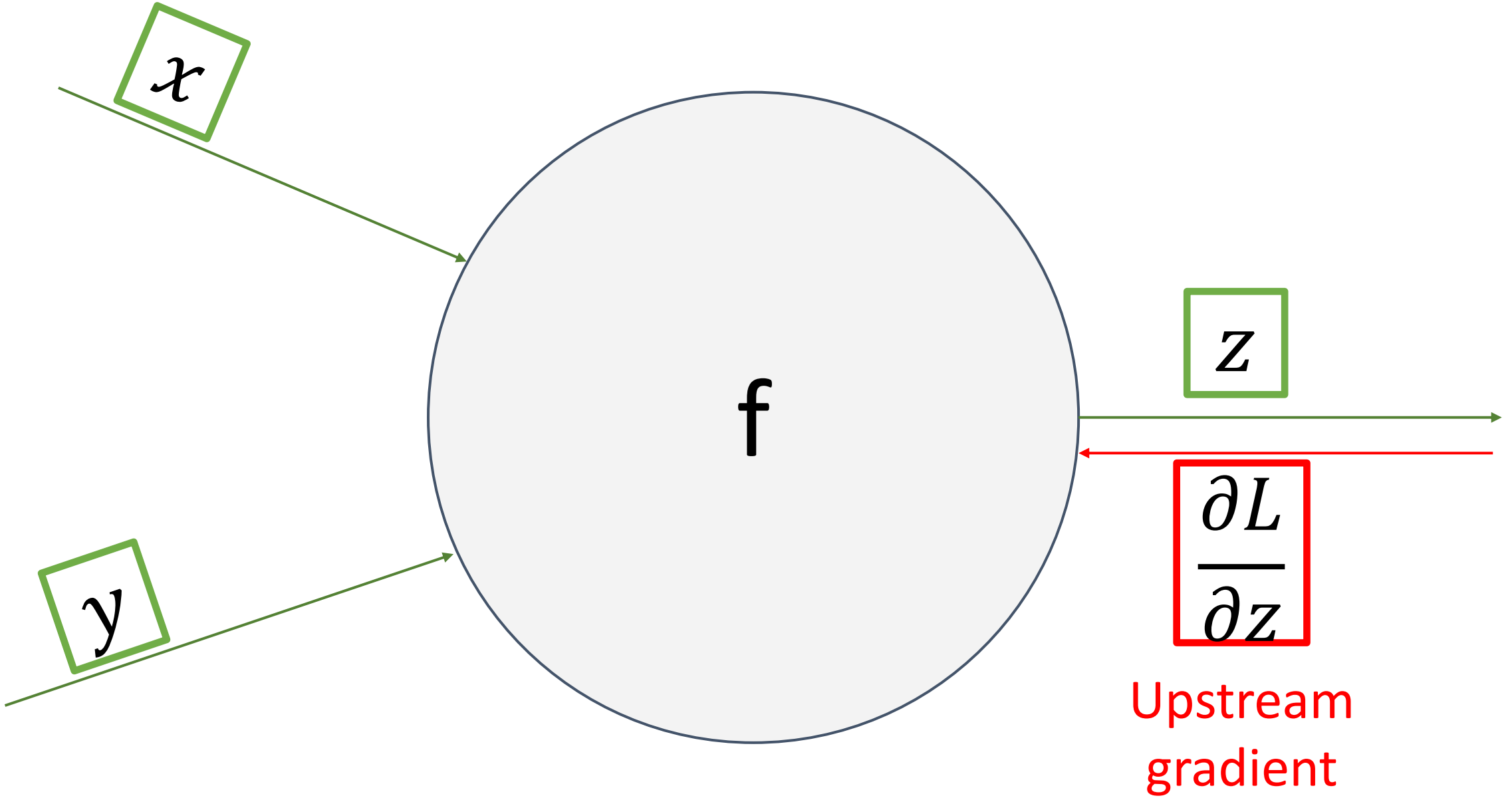
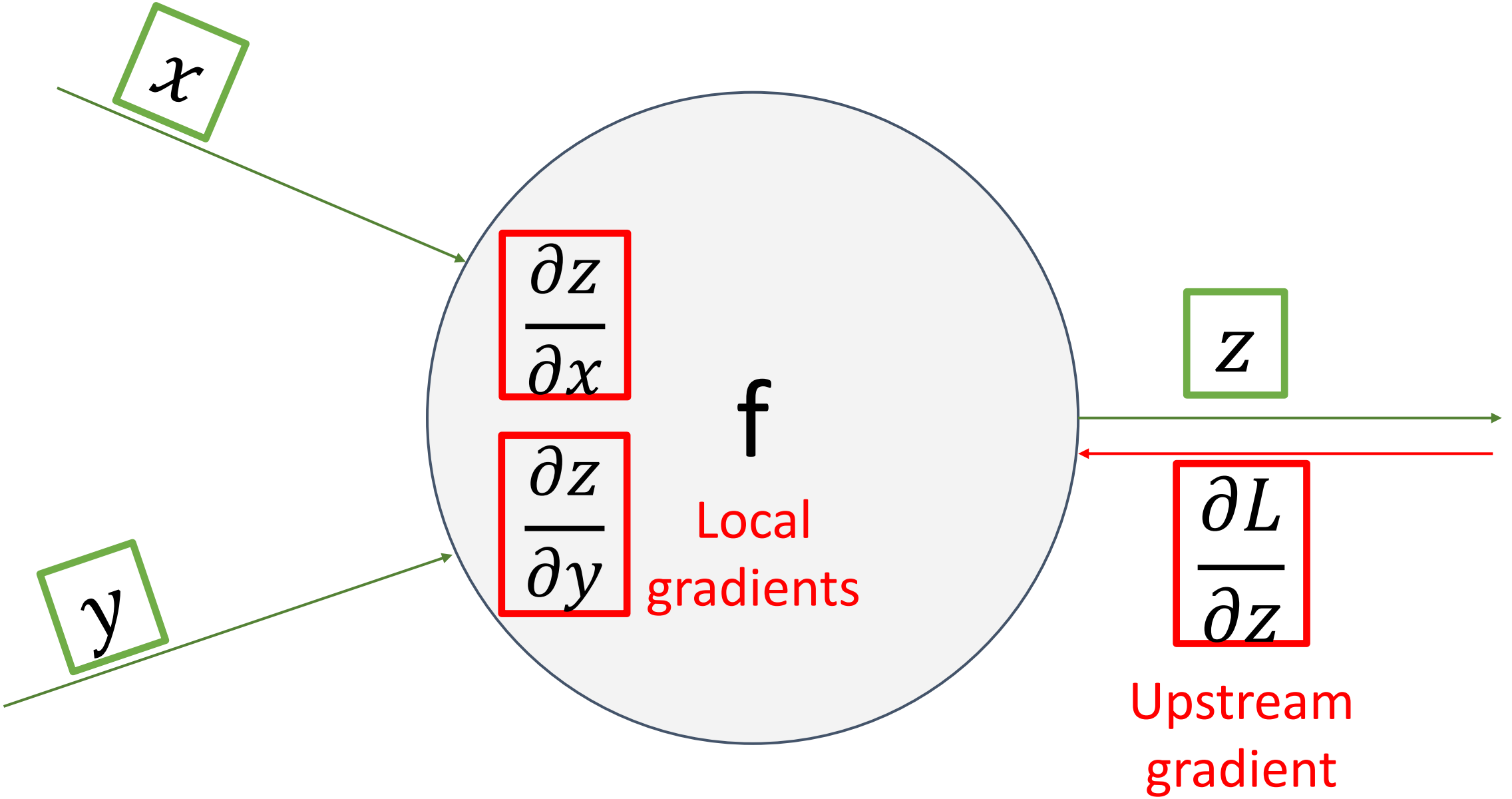
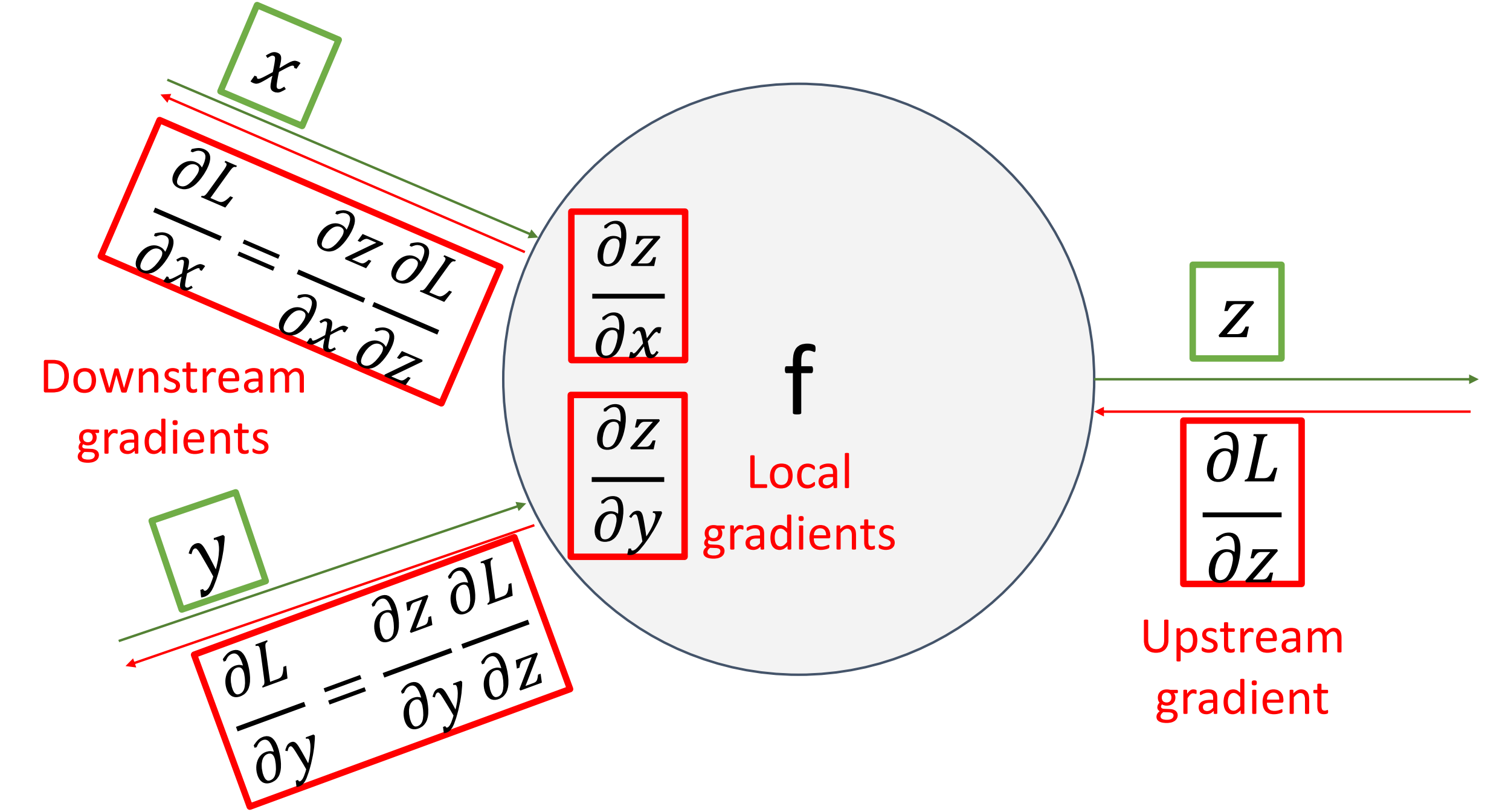
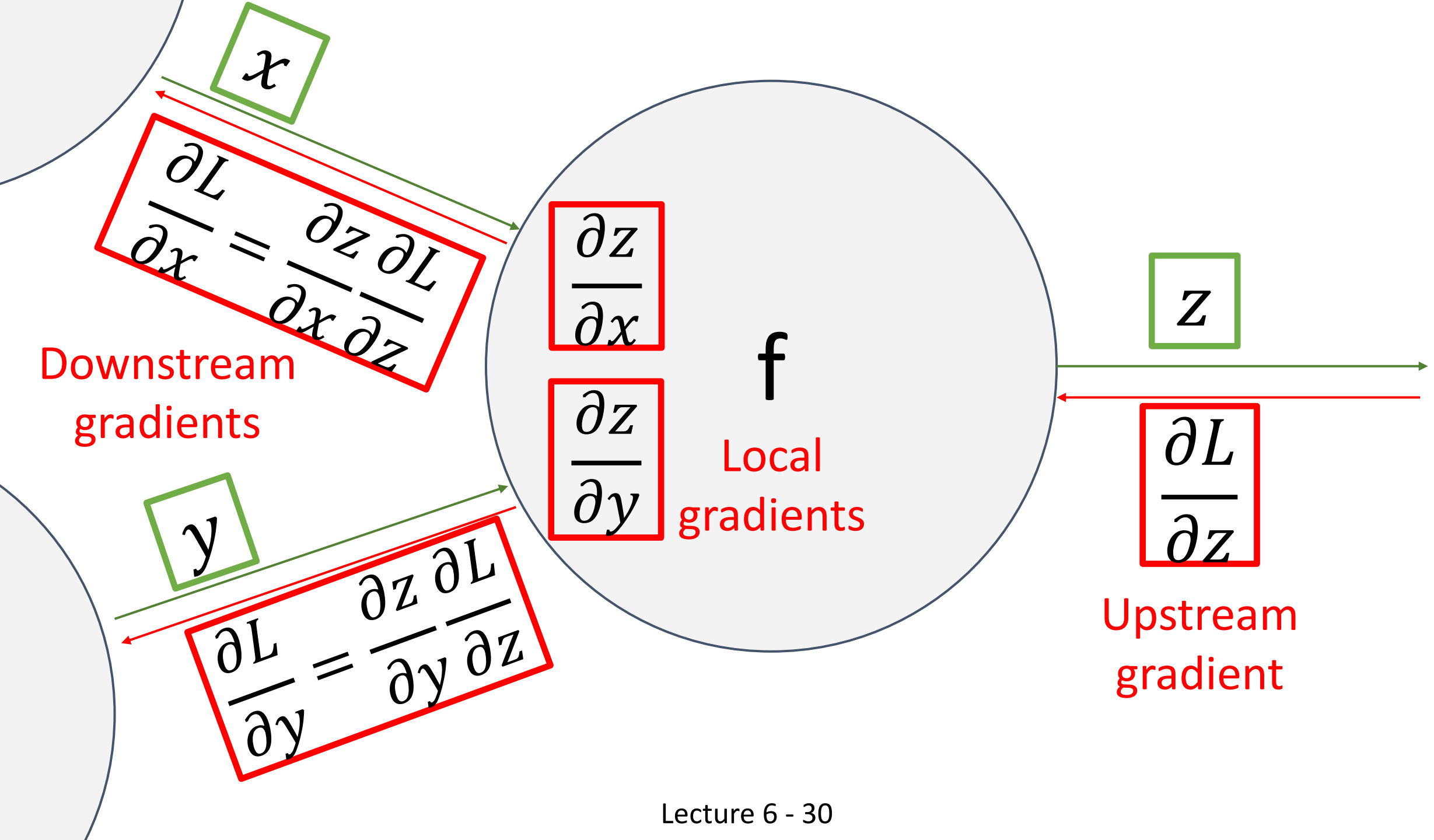

x
y
f
z


x
y
f
z
$\frac{\partial L}{\partial z}$
Upstream gradient


x
y
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
Local gradients
z
$\frac{\partial L}{\partial z}$
Upstream gradient


x
$\frac{\partial L}{\partial x} = \frac{\partial z}{\partial x}\frac{\partial L}{\partial z}$
Downstream gradients
y
$\frac{\partial L}{\partial y} = \frac{\partial z}{\partial y}\frac{\partial L}{\partial z}$
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
Local gradients
z
$\frac{\partial L}{\partial z}$
Upstream gradient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Downstream gradients
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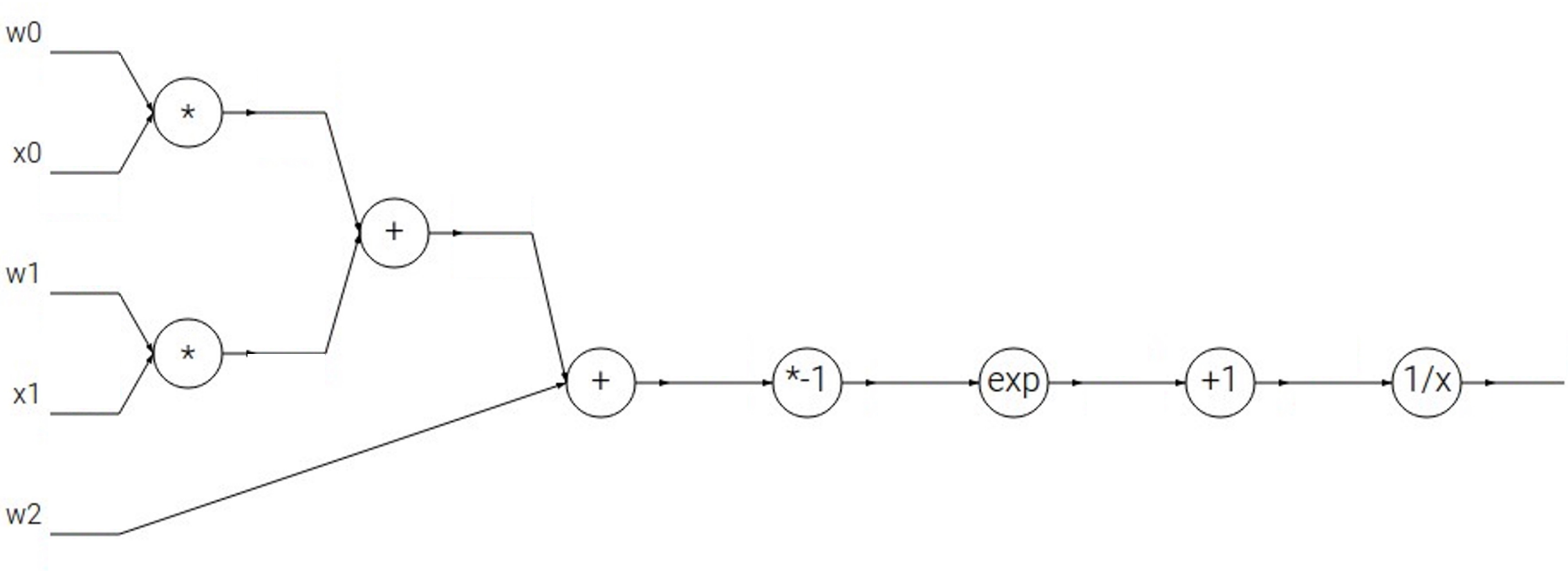
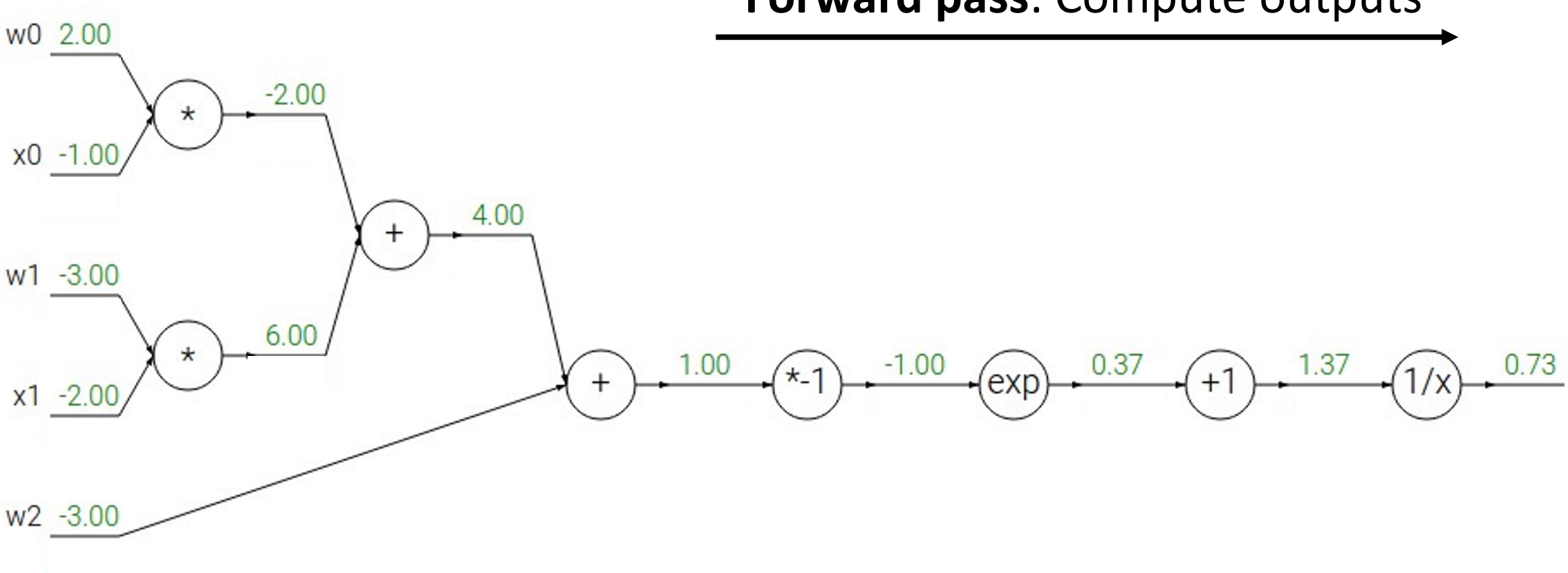
$\frac{\partial z}{\partial y}$
f
Local gradients
z
$\frac{\partial L}{\partial z}$
Upstream gradient

Another Example $f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



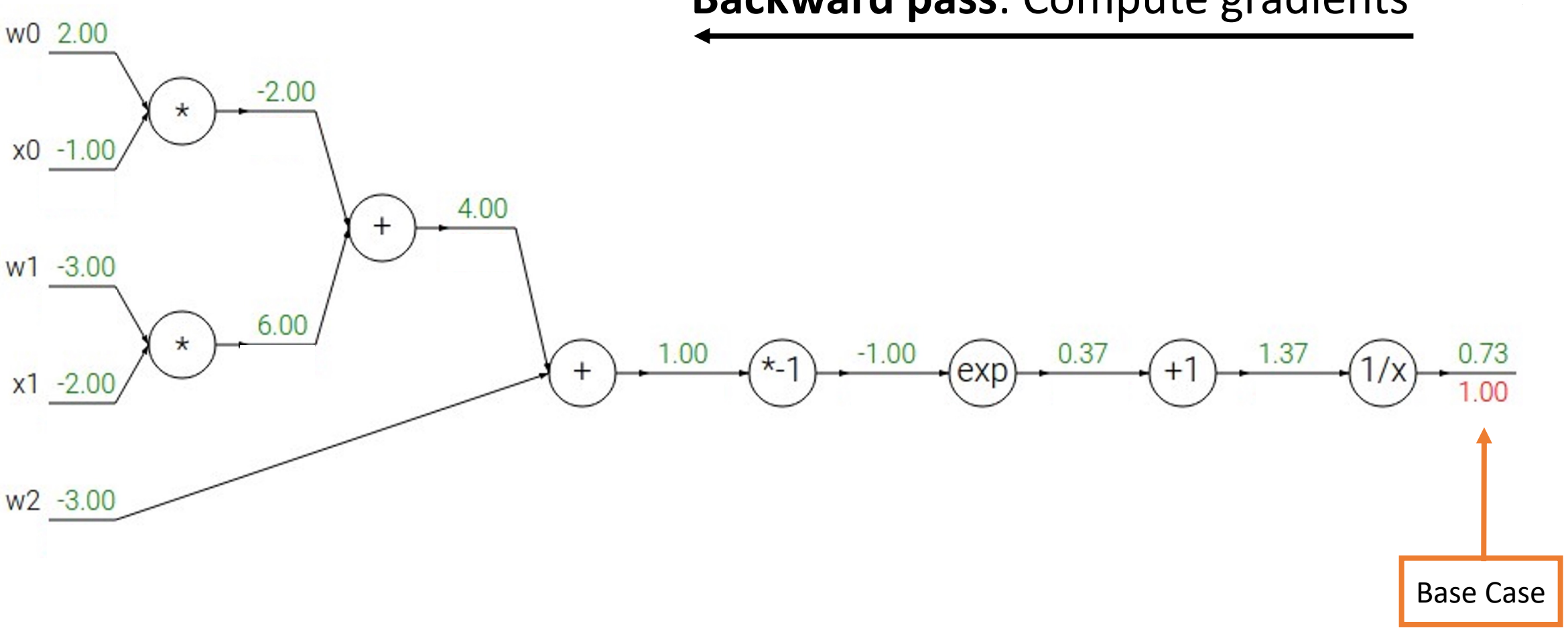
Another Example $f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$

Forward pass: Compute outputs



Another Example $f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$

Backward pass: Compute gradients



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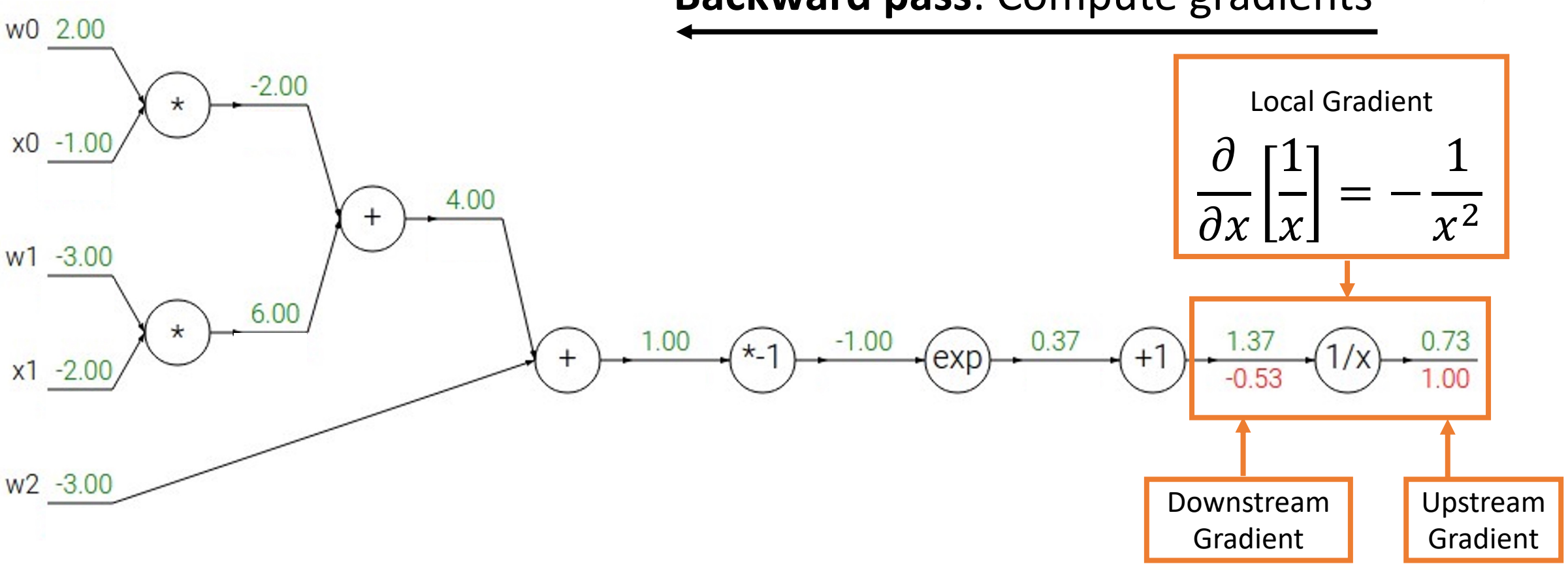
Backward pass: Compute gradients



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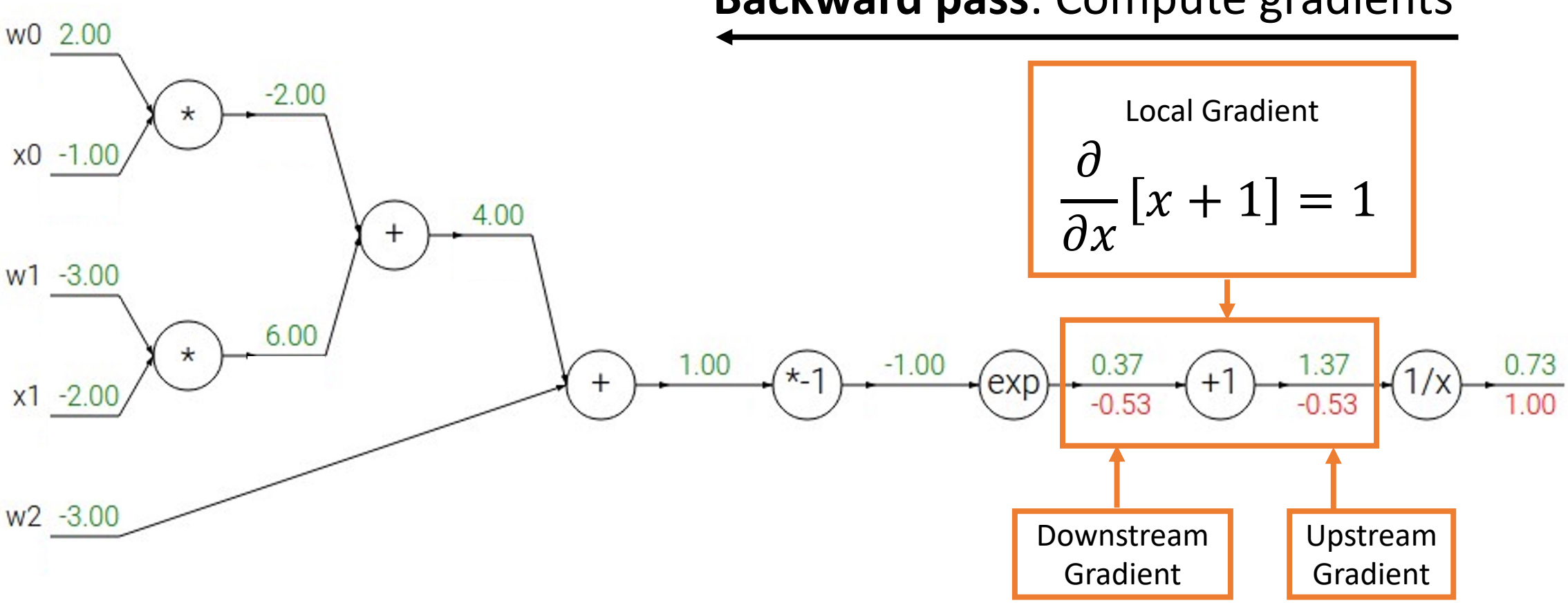
Backward pass: Compute gradients



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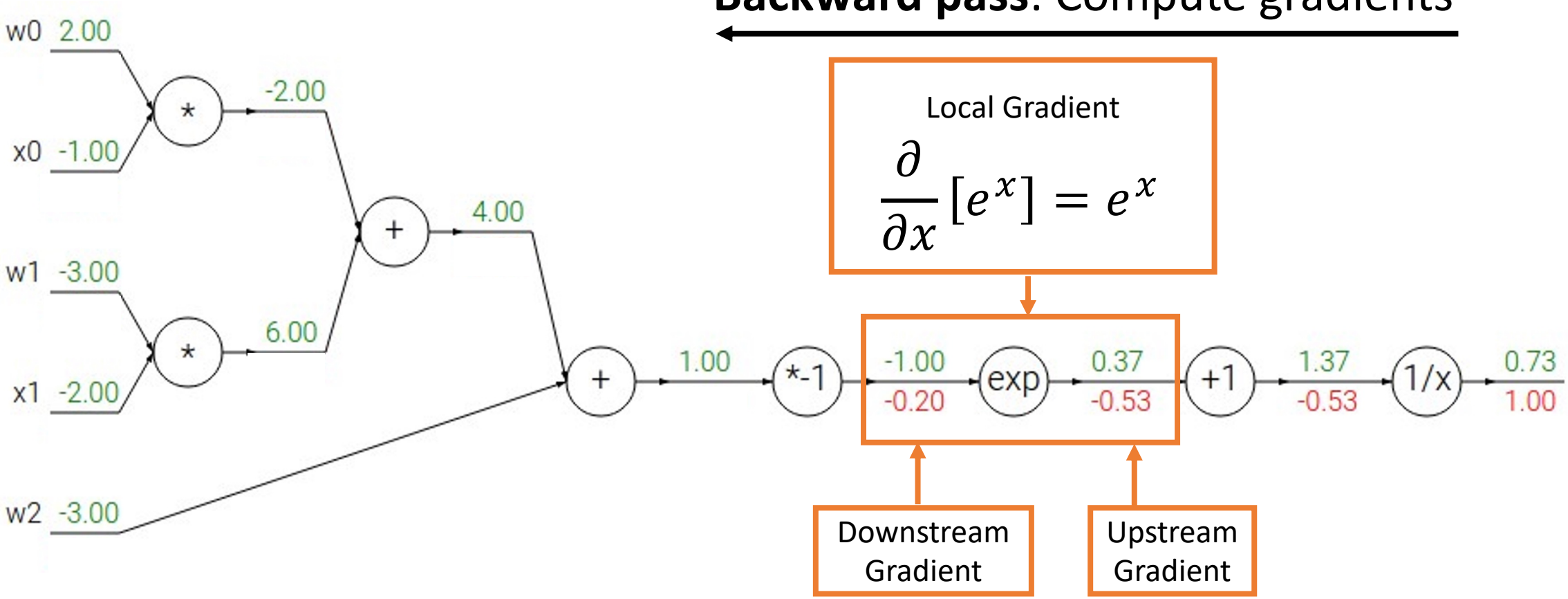
Backward pass: Compute gradients



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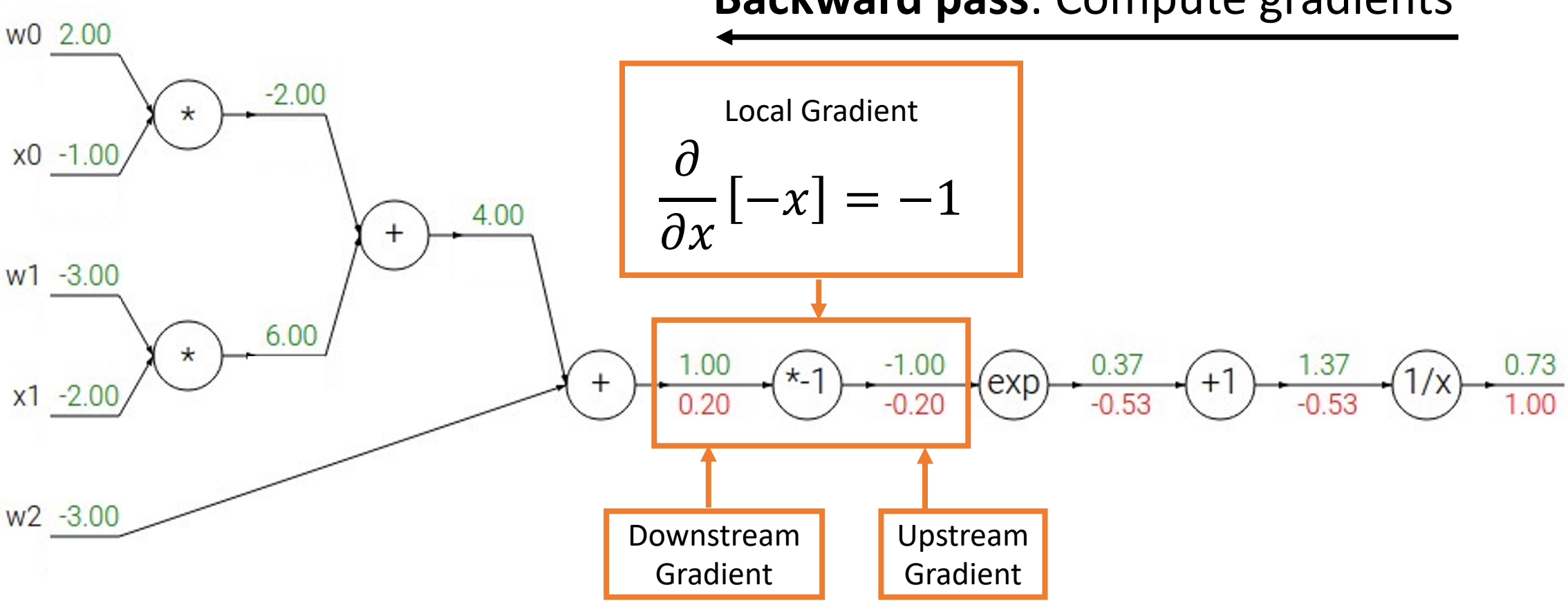
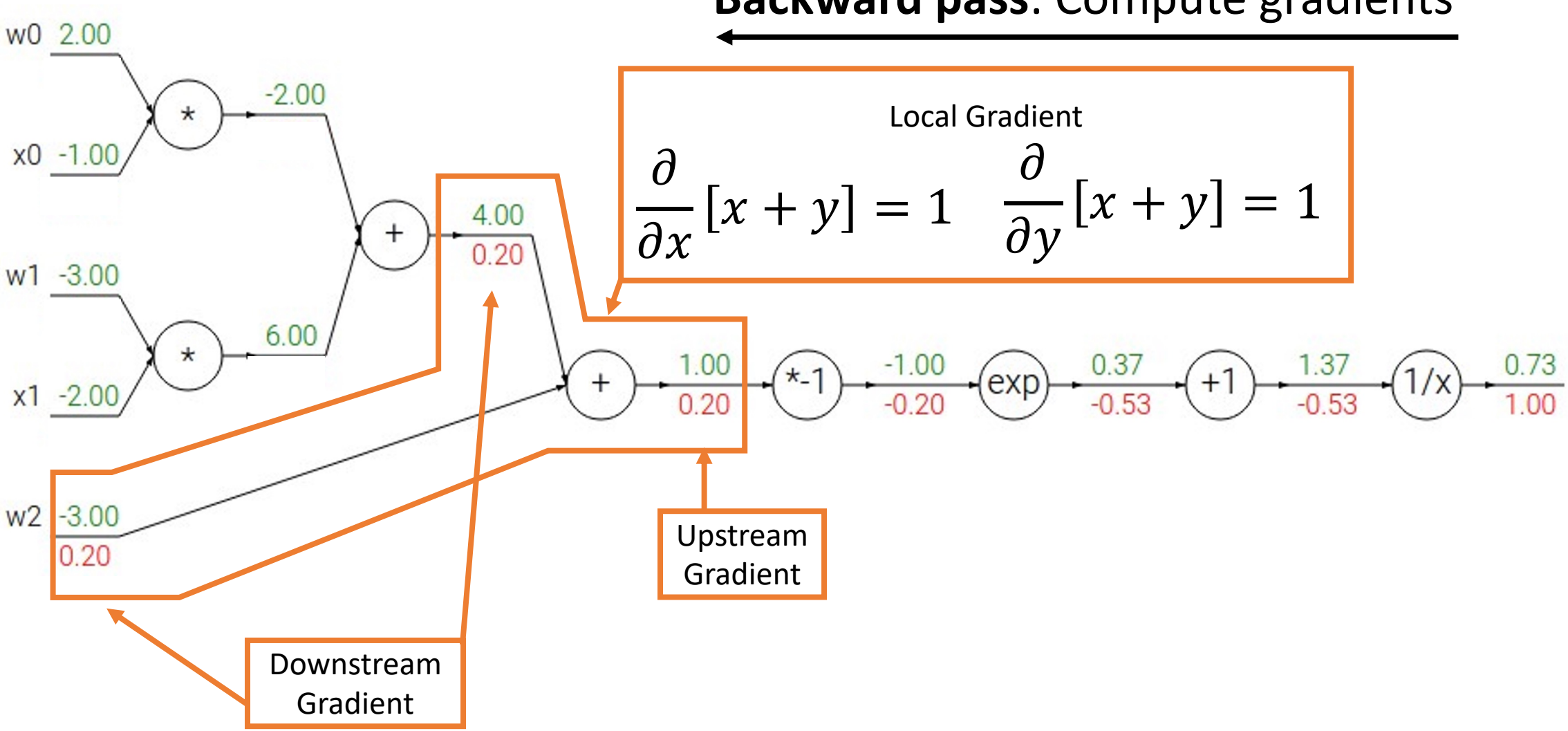
Backward pass: Compute gradients



Another Example

$$f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

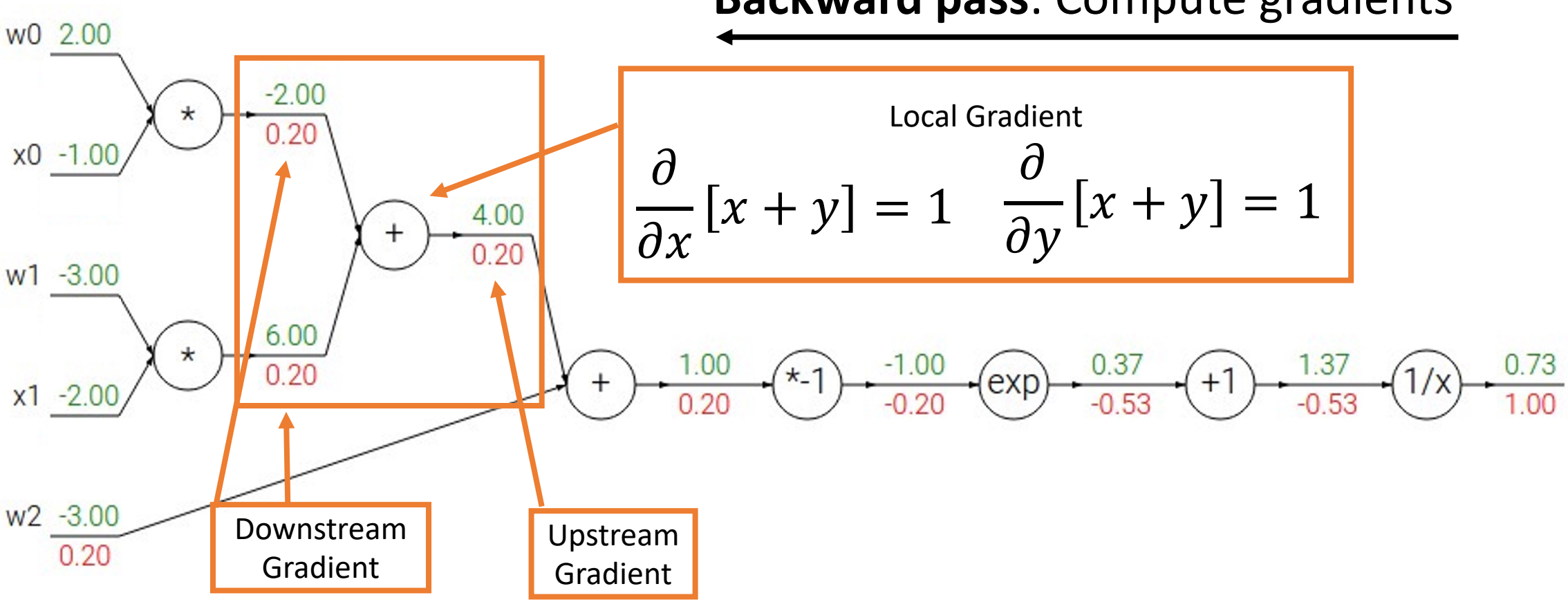
Backward pass: Compute gradients



Another Example

$$f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

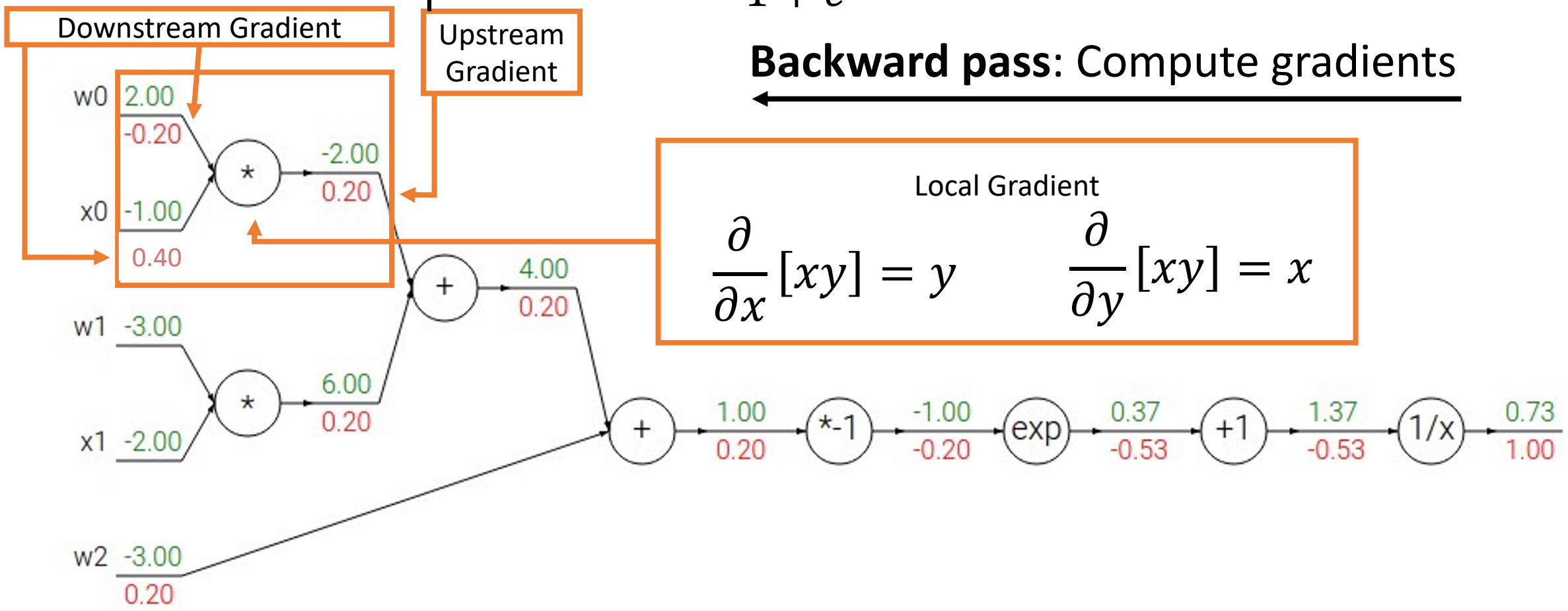
Backward pass: Compute gradients



Another Example

$$f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

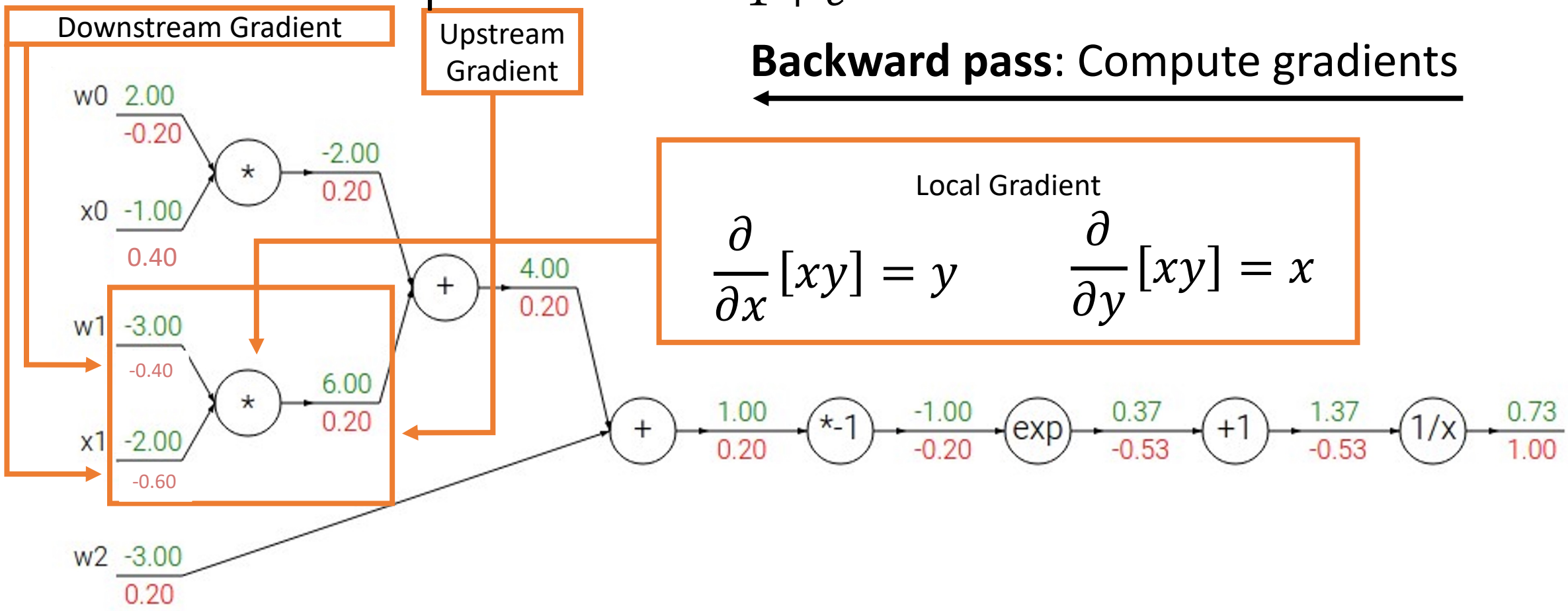
Backward pass: Compute gradients



Another Example

$$f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

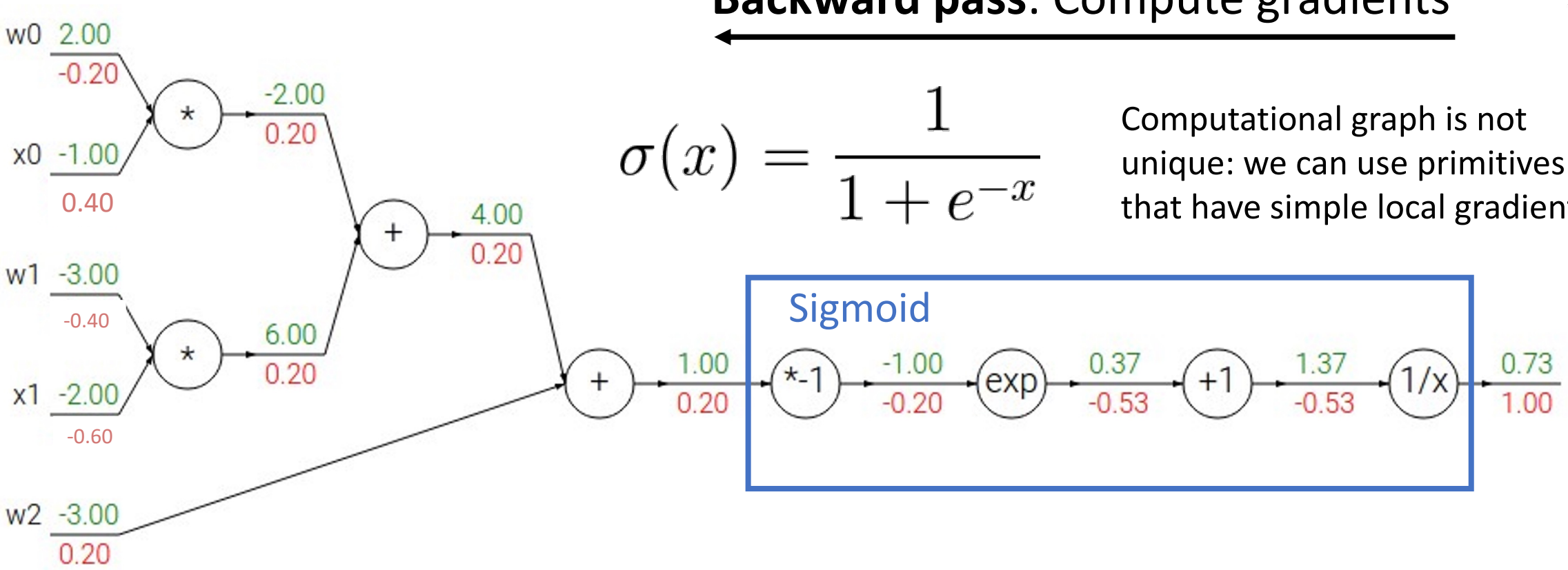
Backward pass: Compute gradients



Another Example

$$f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}} = \boxed{\sigma(w_0x_0 + w_1x_1 + w_2)}$$

Backward pass: Compute gradients



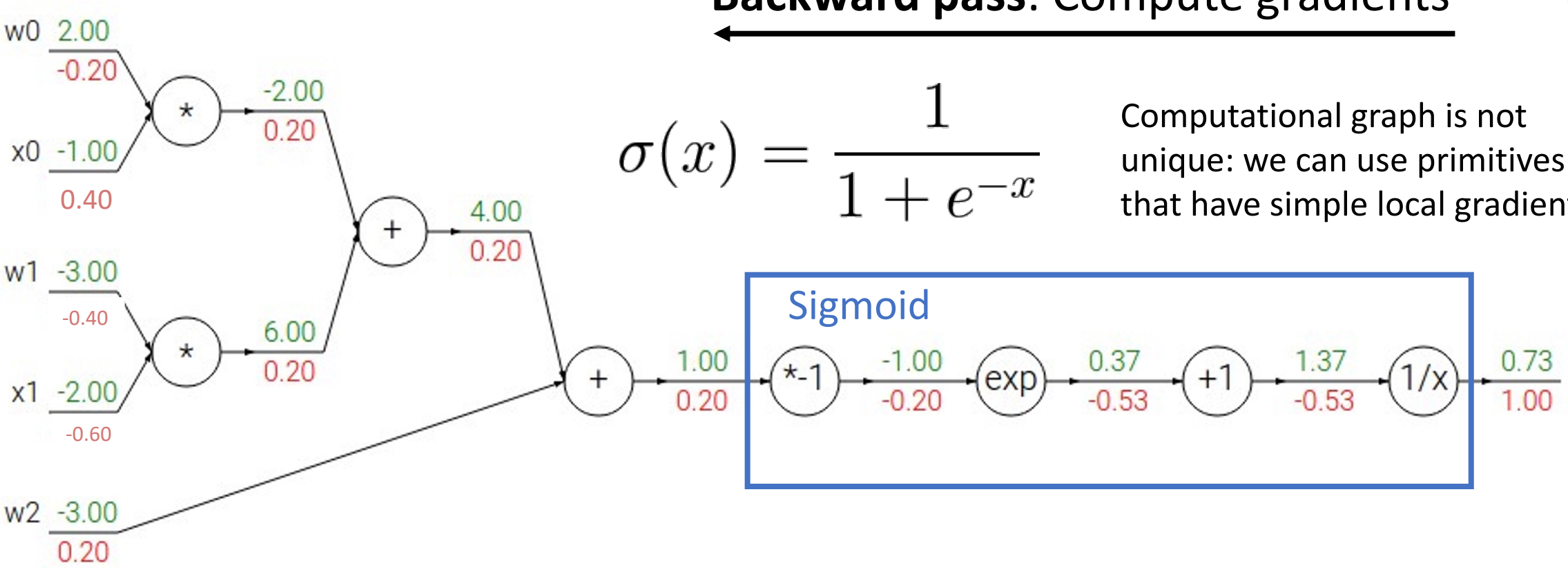
$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Computational graph is not unique: we can use primitives that have simple local gradients

Another Example

$$f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}} = \boxed{\sigma(w_0x_0 + w_1x_1 + w_2)}$$

Backward pass: Compute gradients



$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Computational graph is not unique: we can use primitives that have simple local gradients

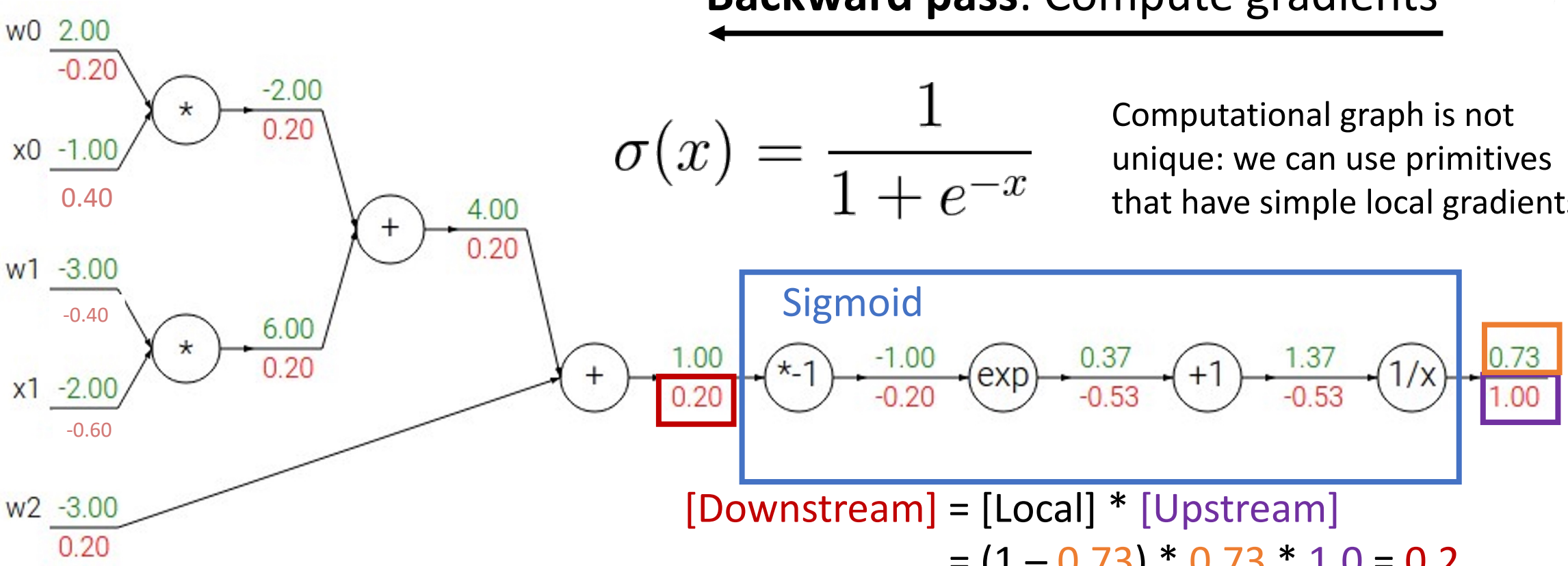
Sigmoid local gradient:

$$\frac{\partial}{\partial x}[\sigma(x)] = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = \big(1-\sigma(x)\big)\sigma(x)$$

Another Example

$$f(x,w) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}} = \sigma(w_0x_0 + w_1x_1 + w_2)$$

Backward pass: Compute gradients



$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Computational graph is not unique: we can use primitives that have simple local gradients

[Downstream] = [Local] * [Upstream]

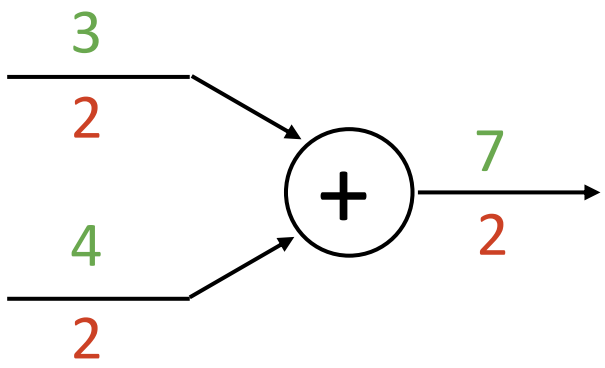
= (1 – 0.73) * 0.73 * 1.0 = 0.2

Sigmoid local gradient:

$$\frac{\partial}{\partial x}[\sigma(x)] = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = \big(1-\sigma(x)\big)\sigma(x)$$

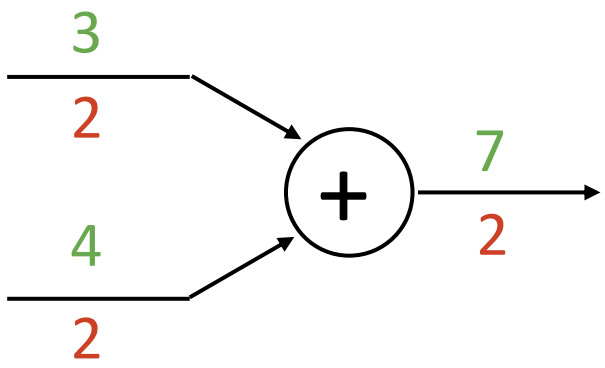
Patterns in Gradient Flow

add gate: gradient distributor

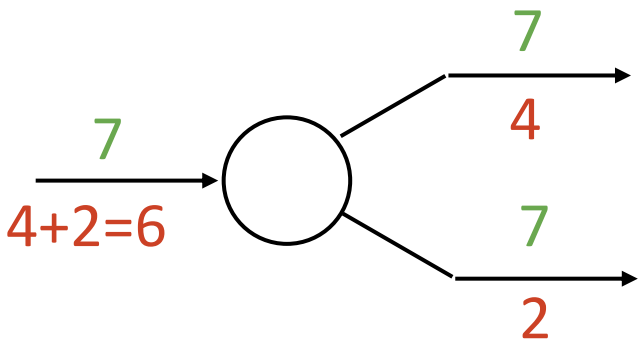


Patterns in Gradient Flow

add gate: gradient distributor

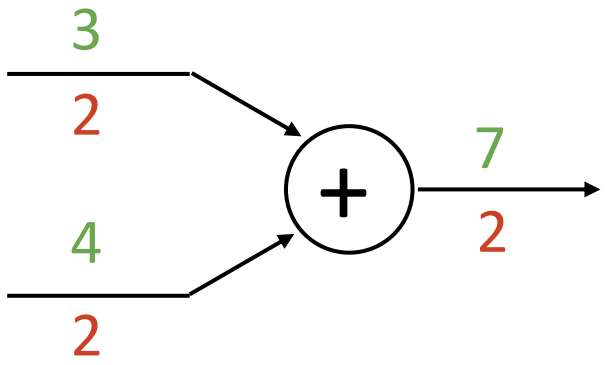


copy gate: gradient adder

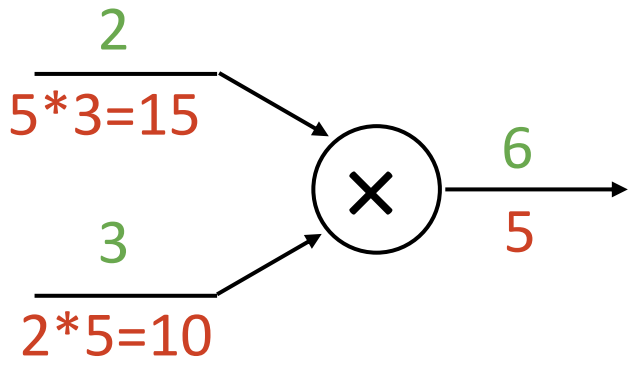


Patterns in Gradient Flow

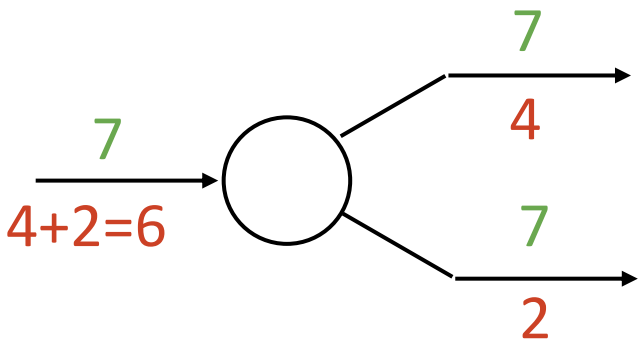
add gate: gradient distributor



mul gate: "swap multiplier"

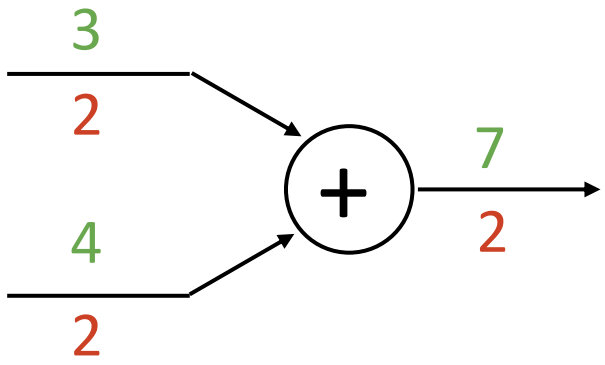


copy gate: gradient adder



Patterns in Gradient Flow

add gate: gradient distributor



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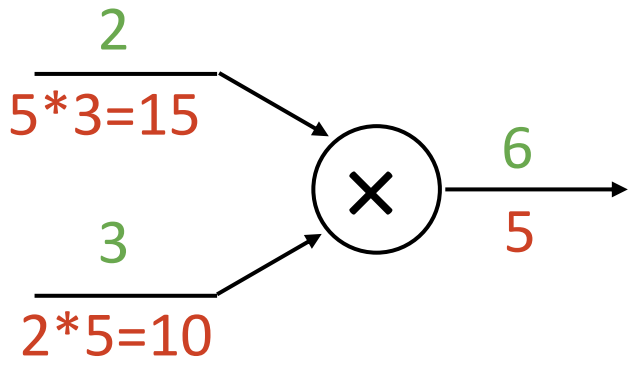
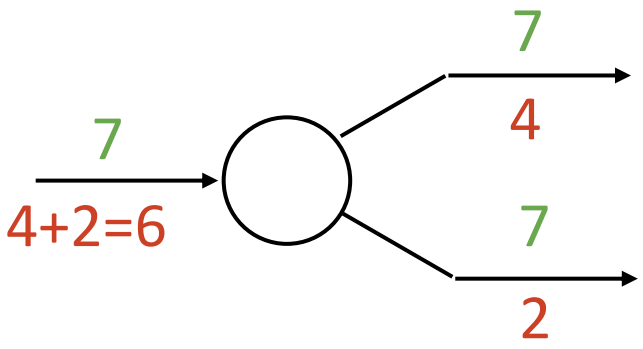
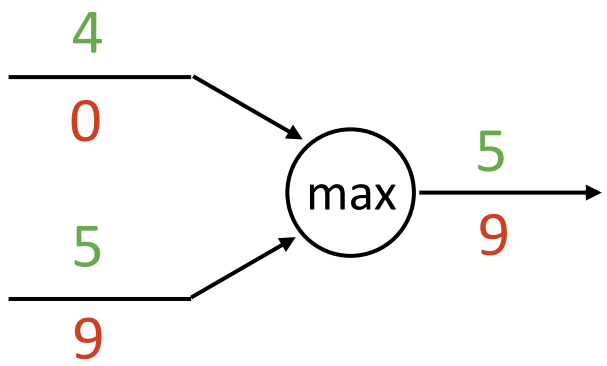


copy gate: gradient adder



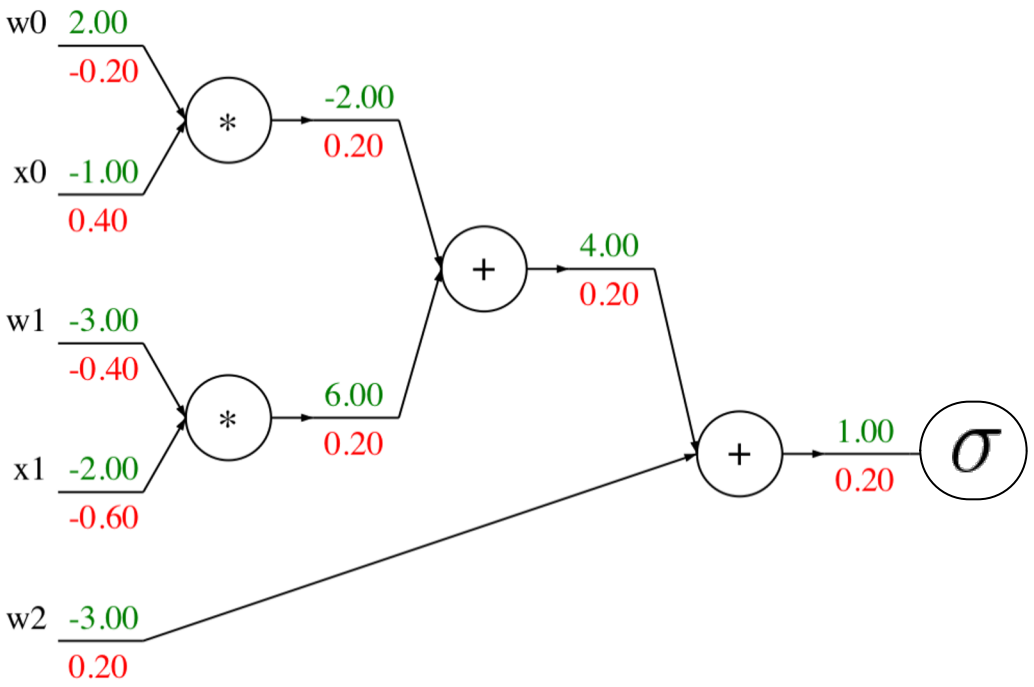
max gate: gradient router



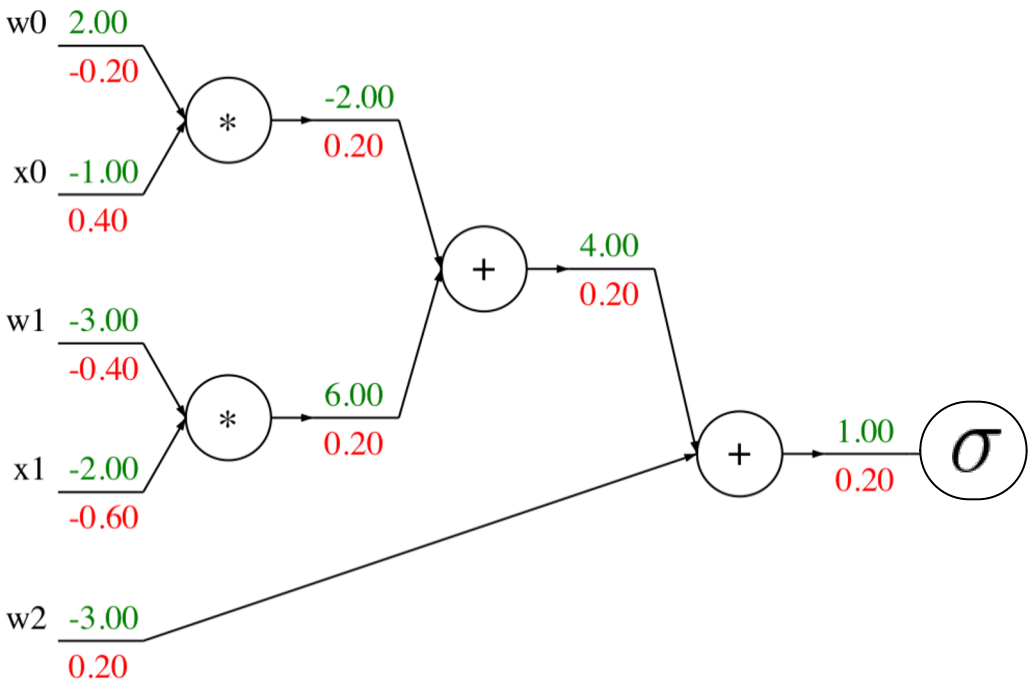
Backprop Implementation: "Flat" gradient code:

Forward pass:
Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```



Backprop Implementation: "Flat" gradient code:



Forward pass:
Compute output

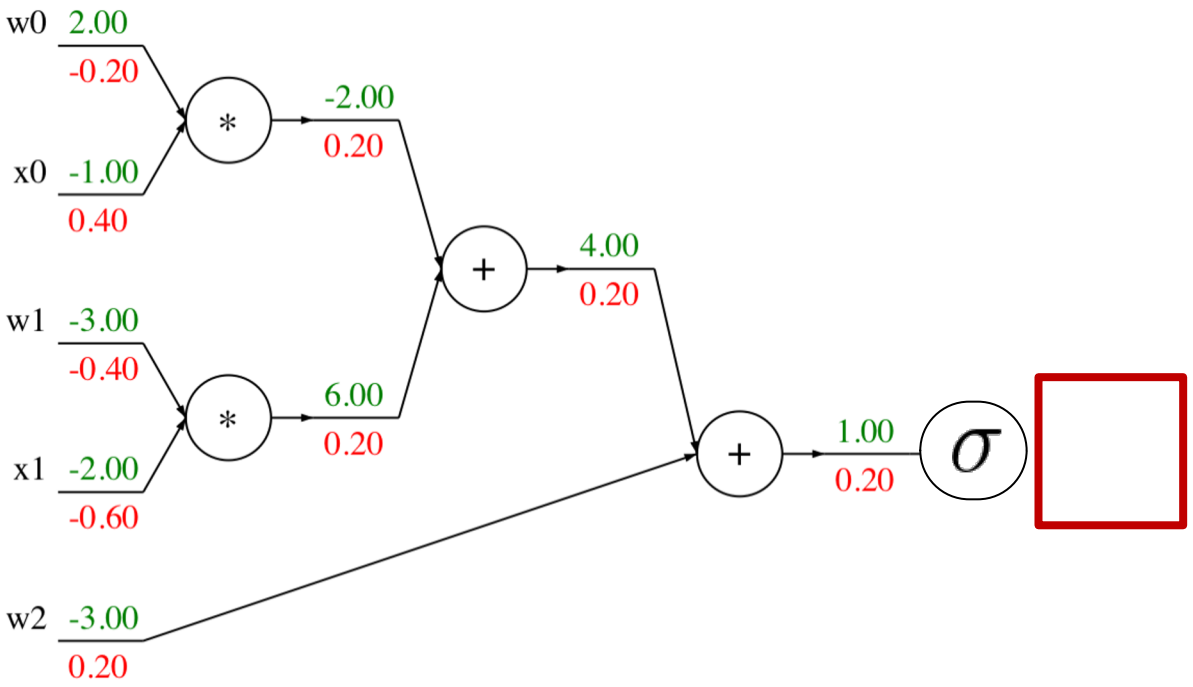
```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

Backward pass:
Compute grads

```
grad_L = 1.0
grad_s3 = grad_L * (1 - L) * L
grad_w2 = grad_s3
grad_s2 = grad_s3
grad_s0 = grad_s2
grad_s1 = grad_s2
grad_w1 = grad_s1 * x1
grad_x1 = grad_s1 * w1
grad_w0 = grad_s0 * x0
grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" gradient code:

Forward pass: Compute output



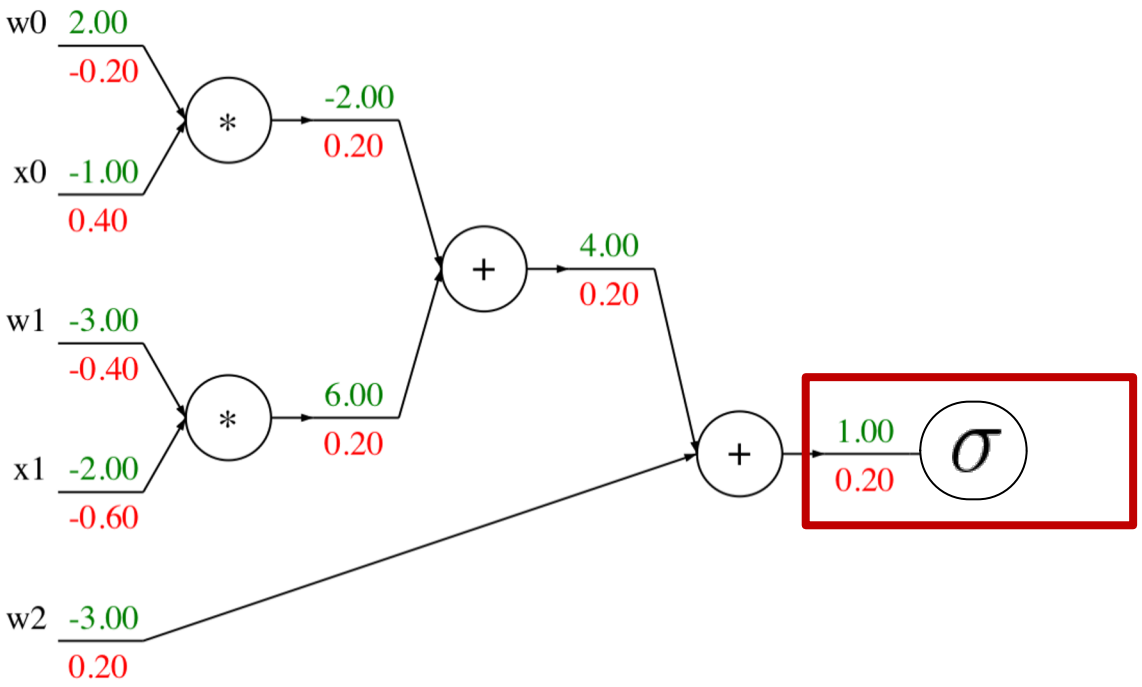

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

Base case

```
  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" gradient code:

Forward pass:
Compute output

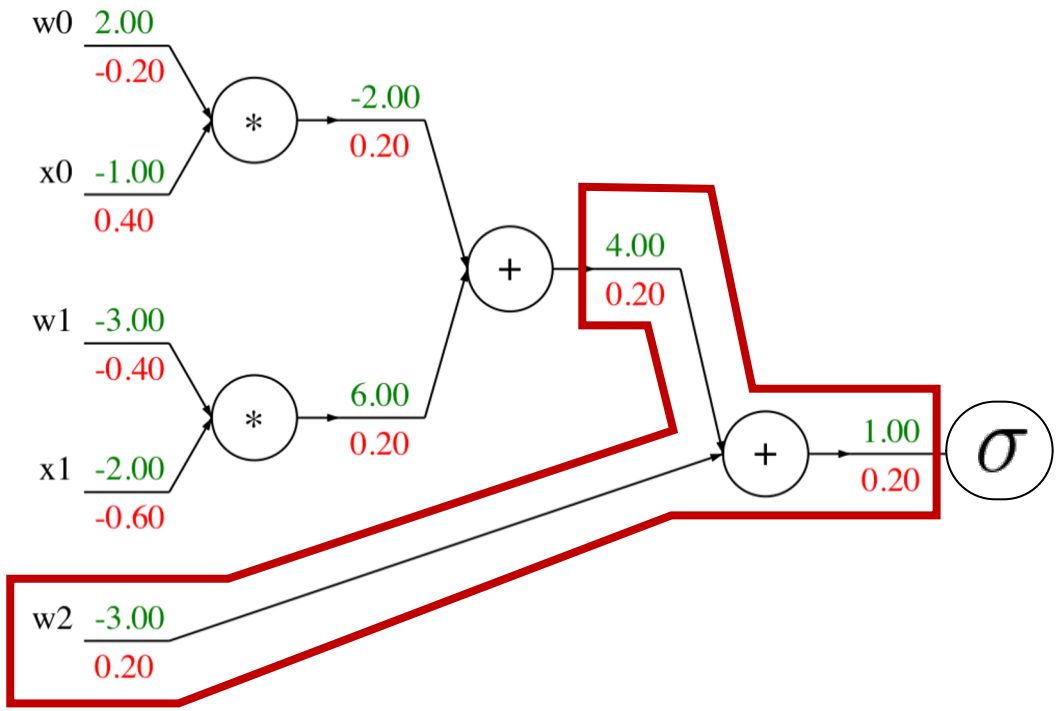



```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

Sigmoid

```
  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" gradient code:



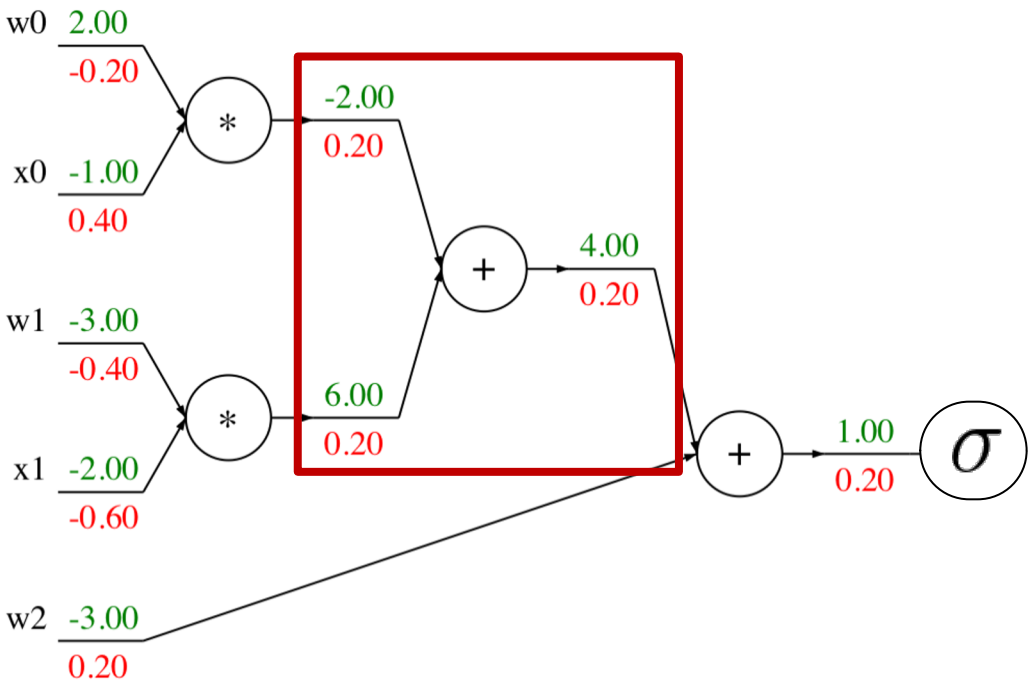
Forward pass:
Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

Add

```
  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" gradient code:



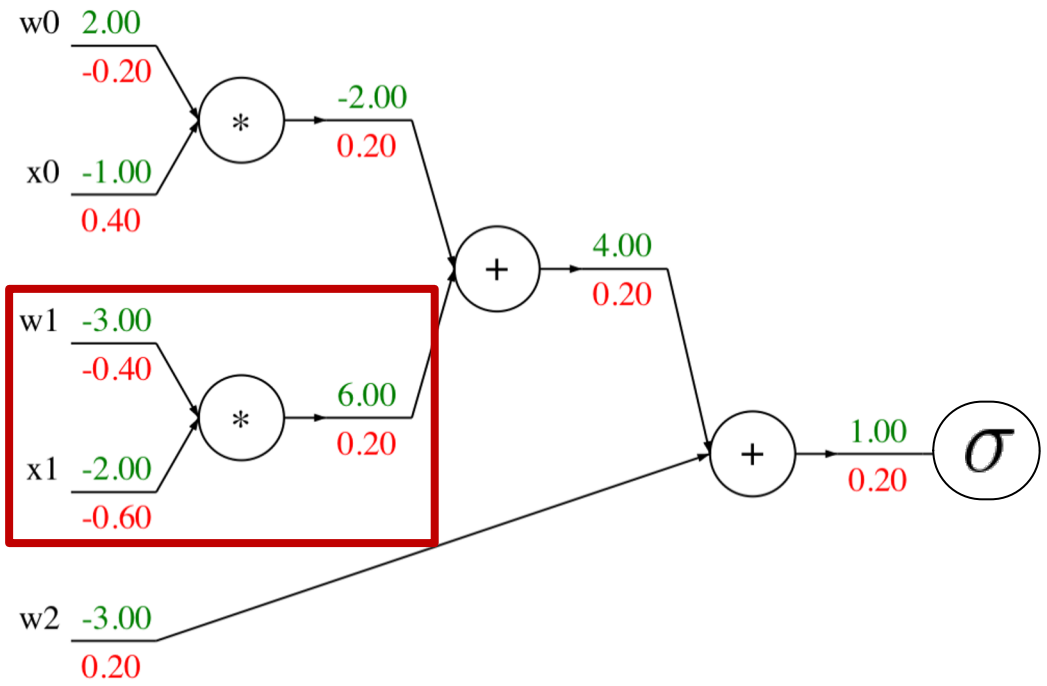
Forward pass:
Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

Add

```
  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" gradient code:



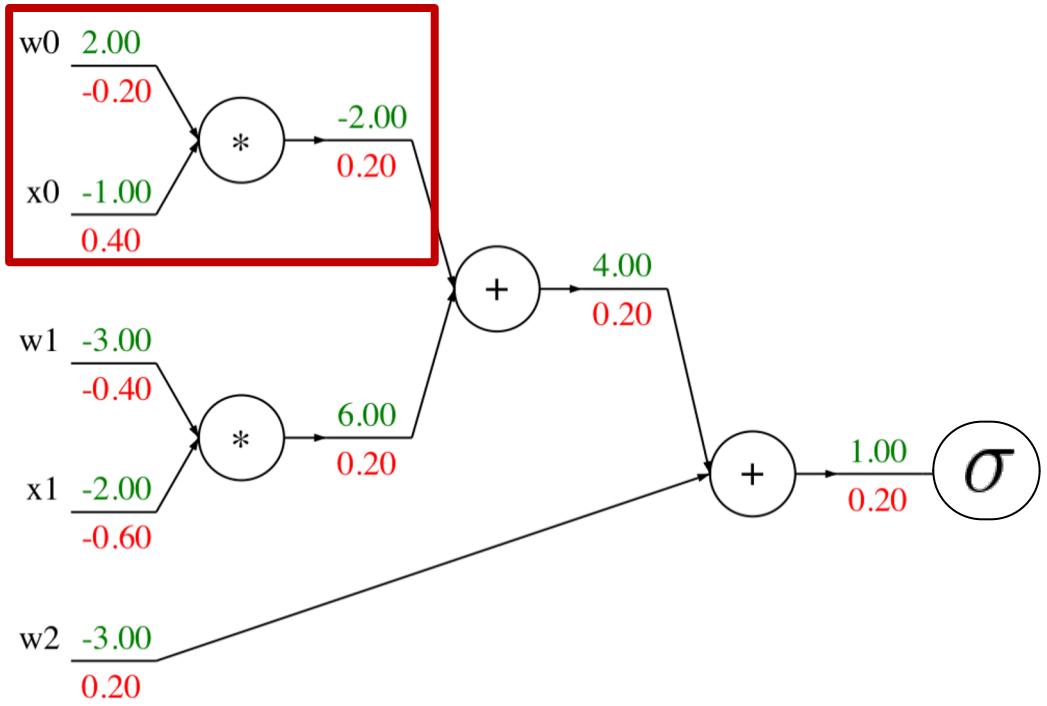
Forward pass: Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

Multiply

```
  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" gradient code:



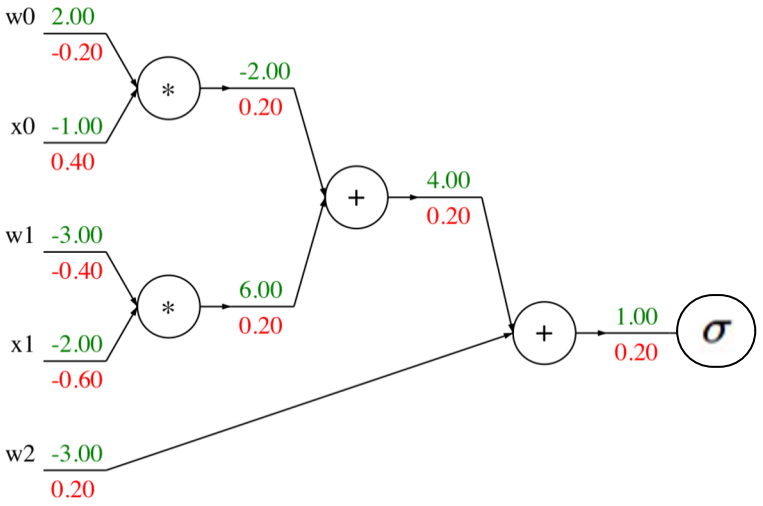
Forward pass: Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

```
  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Multiply

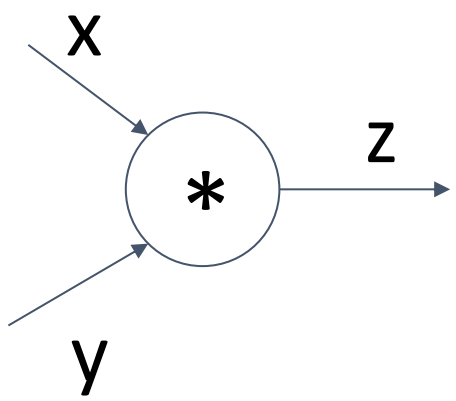
Backprop Implementation: Modular API



Graph (or Net) object *(rough pseudo code)*

```
class ComputationalGraph(object):
    #...
    def forward(inputs):
        # 1. [pass inputs to input gates...]
        # 2. forward the computational graph:
        for gate in self.graph.nodes_topologically_sorted():
            gate.forward()
        return loss # the final gate in the graph outputs the loss
    def backward():
        for gate in reversed(self.graph.nodes_topologically_sorted()):
            gate.backward() # little piece of backprop (chain rule applied)
        return inputs_gradients
```

Example: PyTorch Autograd Functions



(x,y,z are scalars)

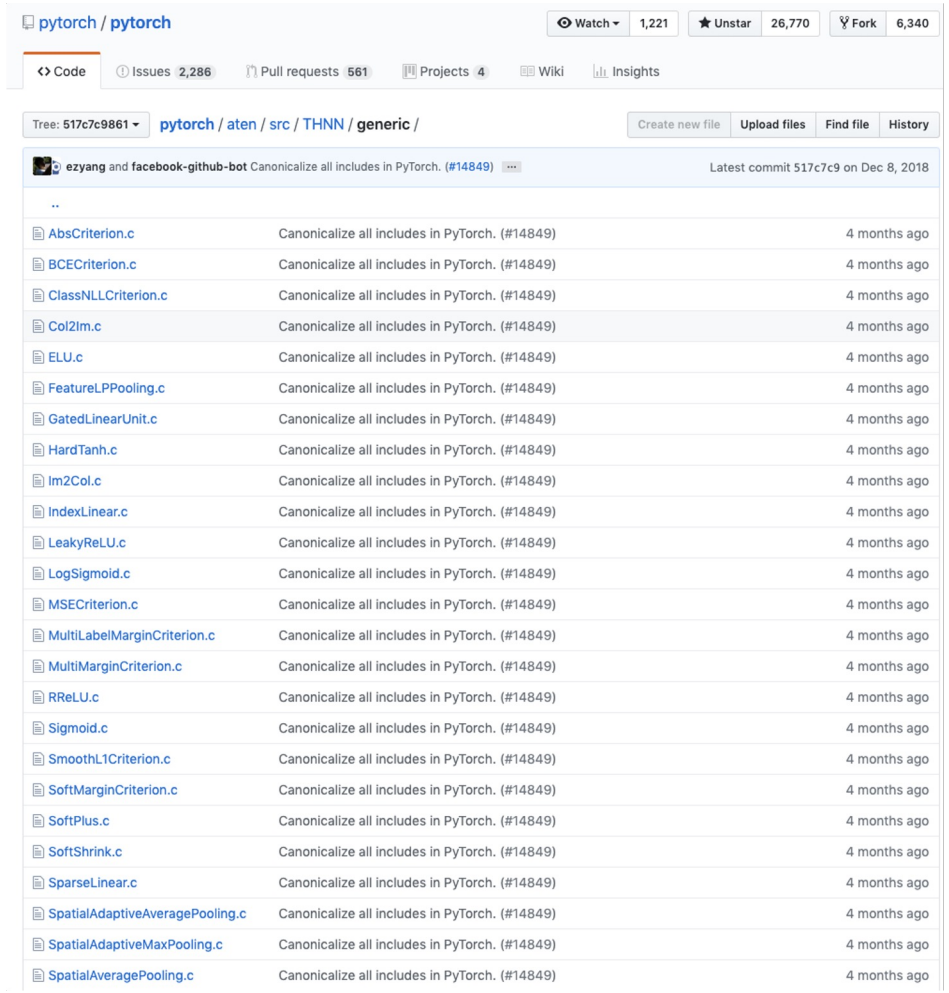
```
class Multiply(torch.autograd.Function):
  @staticmethod
  def forward(ctx, x, y):
    ctx.save_for_backward(x, y)
    z = x * y
    return z
  @staticmethod
  def backward(ctx, grad_z):
    x, y = ctx.saved_tensors
    grad_x = y * grad_z   # dz/dx * dL/dz
    grad_y = x * grad_z   # dz/dy * dL/dz
    return grad_x, grad_y
```

Need to stash some values for use in backward

Upstream gradient

Multiply upstream and local gradients

Example: PyTorch operators



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ezyang and facebook-github-bot Canonicalize all includes in PyTorch. (#14849) ··· Latest commit 517c7c9 on Dec 8, 2018

..

AbsCriterion.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
BCECriterion.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
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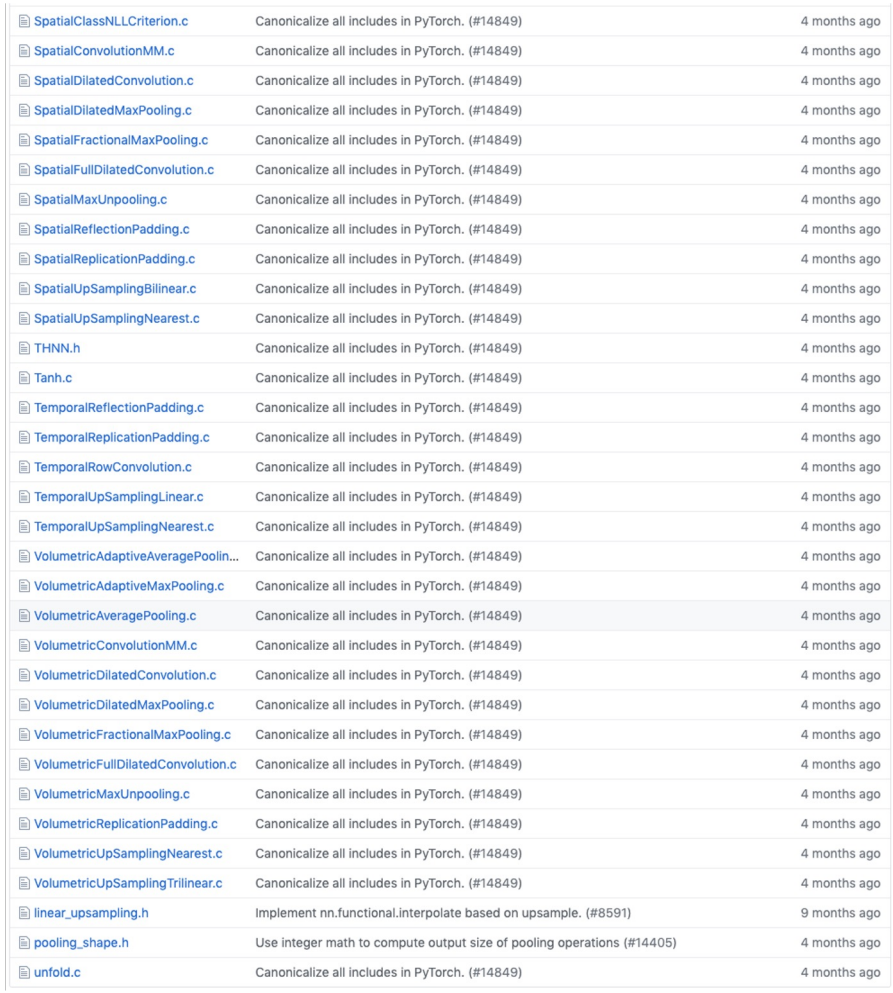
SpatialAveragePooling.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago



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linear_upsampling.h	Implement nn.functional.interpolate based on upsample. (#8591)	9 months ago
pooling_shape.h	Use integer math to compute output size of pooling operations (#14405)	4 months ago
unfold.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago

PyTorch sigmoid layer

```
#ifndef TH_GENERIC_FILE
#define TH_GENERIC_FILE "THNN/generic/Sigmoid.c"
#else

void THNN_(Sigmoid_updateOutput)(
          THNNState *state,
          THTensor *input,
          THTensor *output)
{
  THTensor_(sigmoid)(output, input);
}

void THNN_(Sigmoid_updateGradInput)(
          THNNState *state,
          THTensor *gradOutput,
          THTensor *gradInput,
          THTensor *output)
{
  THNN_CHECK_NELEMENT(output, gradOutput);
  THTensor_(resizeAs)(gradInput, output);
  TH_TENSOR_APPLY3(scalar_t, gradInput, scalar_t, gradOutput, scalar_t, output,
    scalar_t z = *output_data;
    *gradInput_data = *gradOutput_data * (1. - z) * z;
  );
}

#endif
```

Source

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          THTensor *gradOutput,
          THTensor *gradInput,
          THTensor *output)
{
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  TH_TENSOR_APPLY3(scalar_t, gradInput, scalar_t, gradOutput, scalar_t, output,
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    *gradInput_data = *gradOutput_data * (1. - z) * z;
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}

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```

Forward

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

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  );
}

#endif
```

Forward

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

```
static void sigmoid_kernel(TensorIterator& iter) {
  AT_DISPATCH_FLOATING_TYPES(iter.dtype(), "sigmoid_cpu", [&]() {
    unary_kernel_vec(
        iter,
        [=](scalar_t a) -> scalar_t { return (1 / (1 + std::exp((-a)))); },
        [=](Vec256<scalar_t> a) {
          a = Vec256<scalar_t>((scalar_t)(0)) - a;
          a = a.exp();
          a = Vec256<scalar_t>((scalar_t)(1)) + a;
          a = a.reciprocal();
          return a;
        });
  });
}
```

Source

PyTorch sigmoid layer

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{
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}

void THNN_(Sigmoid_updateGradInput)(
          THNNState *state,
          THTensor *gradOutput,
          THTensor *gradInput,
          THTensor *output)
{
  THNN_CHECK_NELEMENT(output, gradOutput);
  THTensor_(resizeAs)(gradInput, output);
  TH_TENSOR_APPLY3(scalar_t, gradInput, scalar_t, gradOutput, scalar_t, output,
    scalar_t z = *output_data;
    *gradInput_data = *gradOutput_data * (1. - z) * z;
  );
}

#endif
```

Forward

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Backward

$$(1-\sigma(x))\,\sigma(x)$$

Source

Life is easy in PyTorch (no need to write backpropagation because of Autograd)

```
class NestMLP(nn.Module):
    def __init__(self):
        super().__init__()
        self.net = nn.Sequential(nn.Linear(20, 64), nn.ReLU(),
                                 nn.Linear(64, 32), nn.ReLU())
        self.linear = nn.Linear(32, 16)

    def forward(self, X):
        return self.linear(self.net(X))

chimera = nn.Sequential(NestMLP(), nn.Linear(16, 20), FixedHiddenMLP())
chimera(X)
```

So far: backprop with scalars

What about vector-valued functions?

Recap: Vector Derivatives

$x \in \mathbb{R}, y \in \mathbb{R}$

Regular derivative:

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

If x changes by a small amount, how much will y change?

Recap: Vector Derivatives

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$x \in \mathbb{R}^N, y \in \mathbb{R}$

Derivative is **Gradient**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N,$$

$$\left(\frac{\partial y}{\partial x}\right)_i = \frac{\partial y}{\partial x_i}$$

For each element of x, if it changes by a small amount then how much will y change?

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For each element of x, if it changes by a small amount then how much will y change?

$x \in \mathbb{R}^N, y \in \mathbb{R}^M$

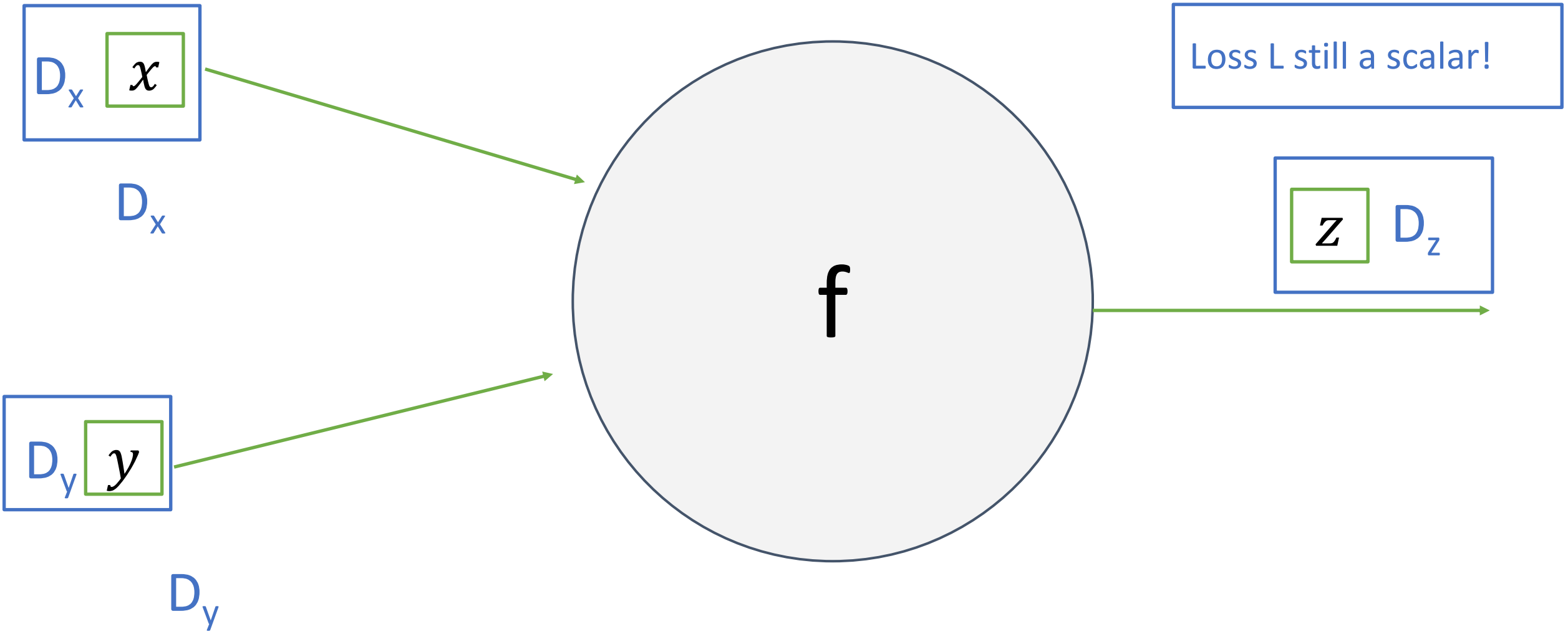
Derivative is **Jacobian**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^{M \times N}$$

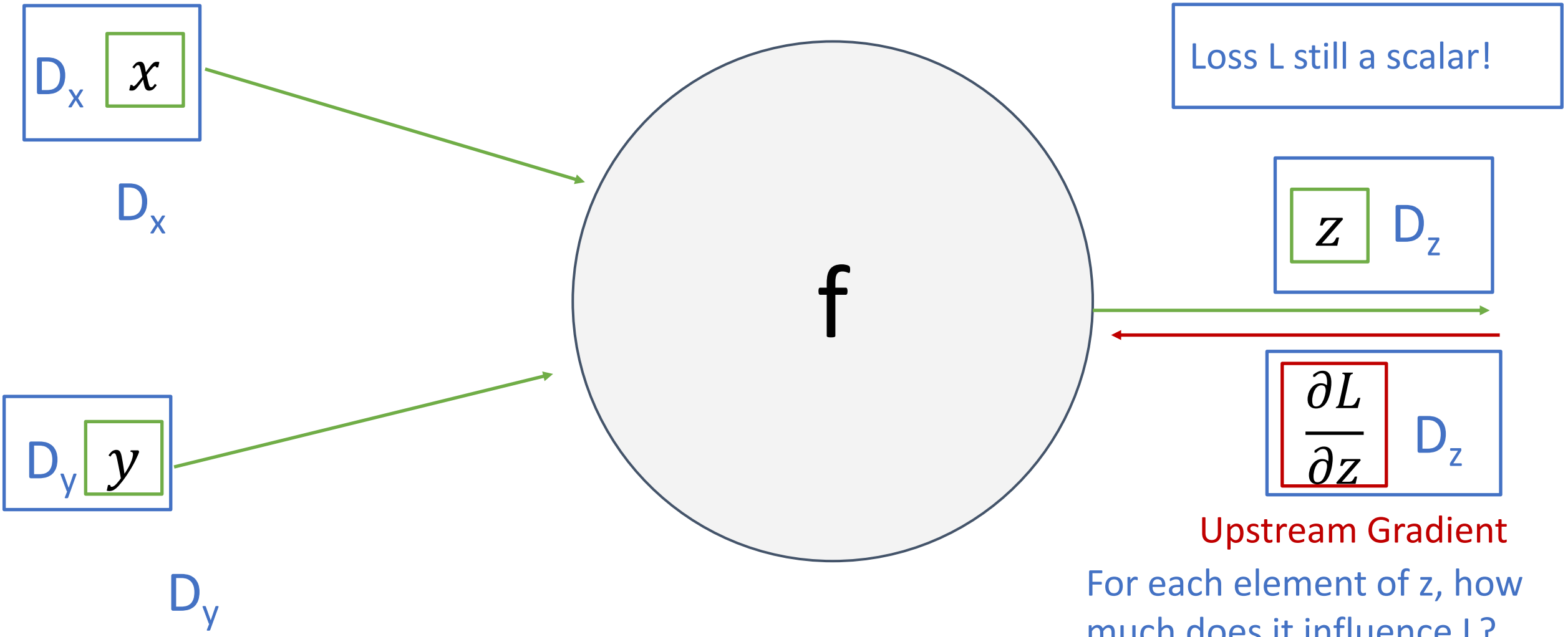
$$\left(\frac{\partial y}{\partial x}\right)_{j,i} = \frac{\partial y_j}{\partial x_i}$$

For each element of x, if it changes by a small amount then how much will each element of y change?

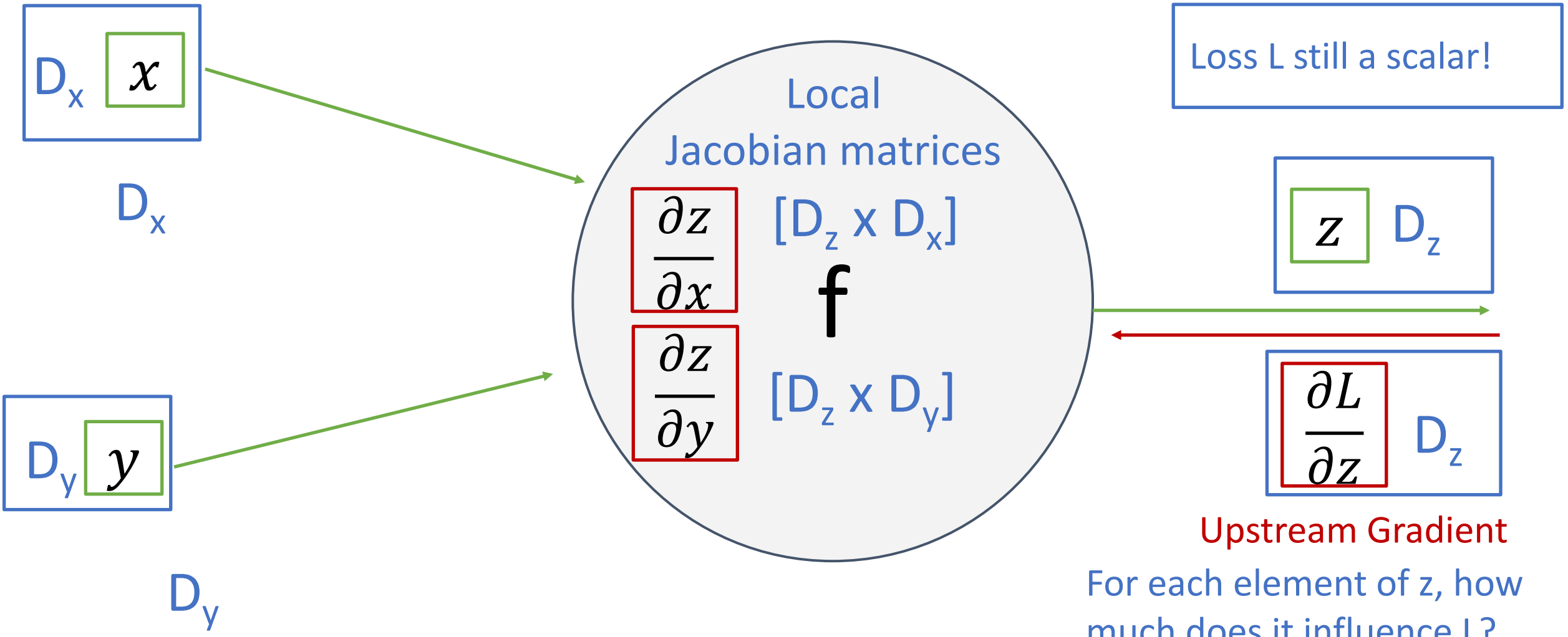
Backprop with Vectors



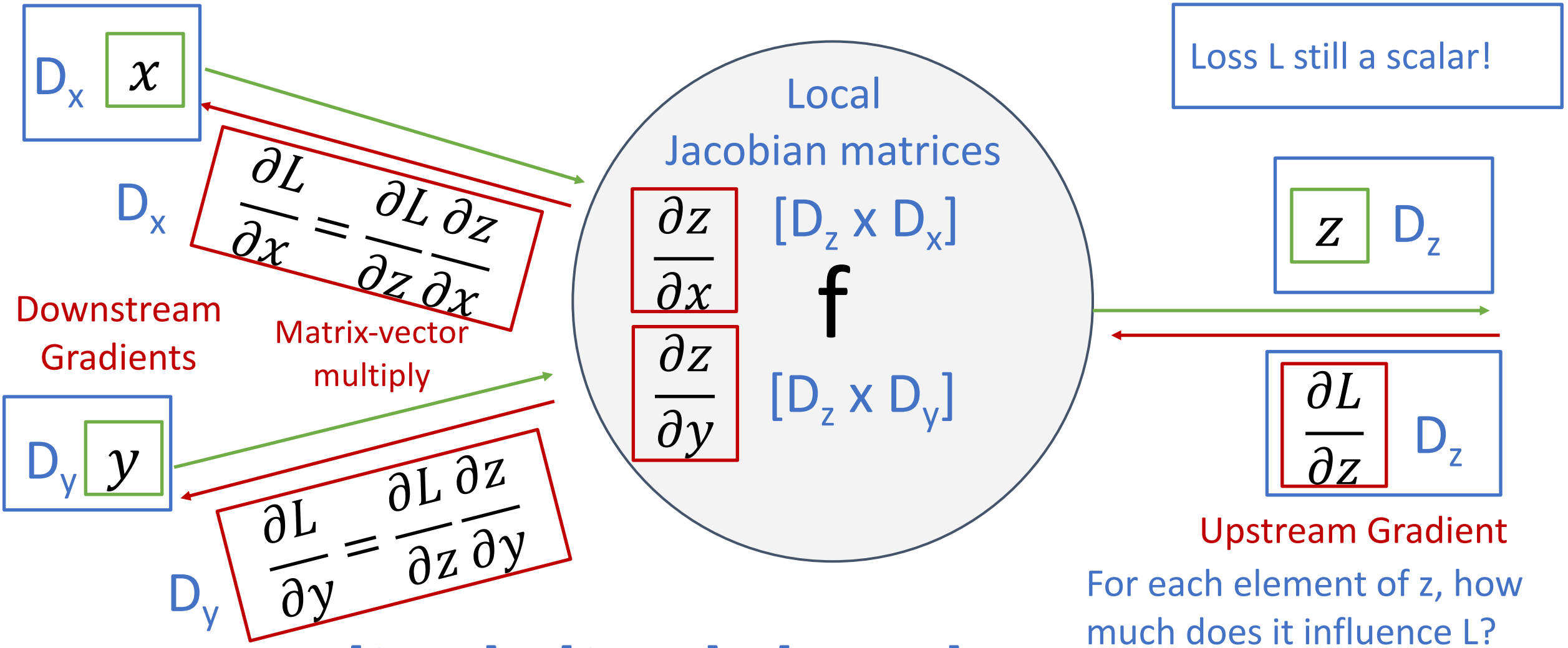
Backprop with Vectors



Backprop with Vectors

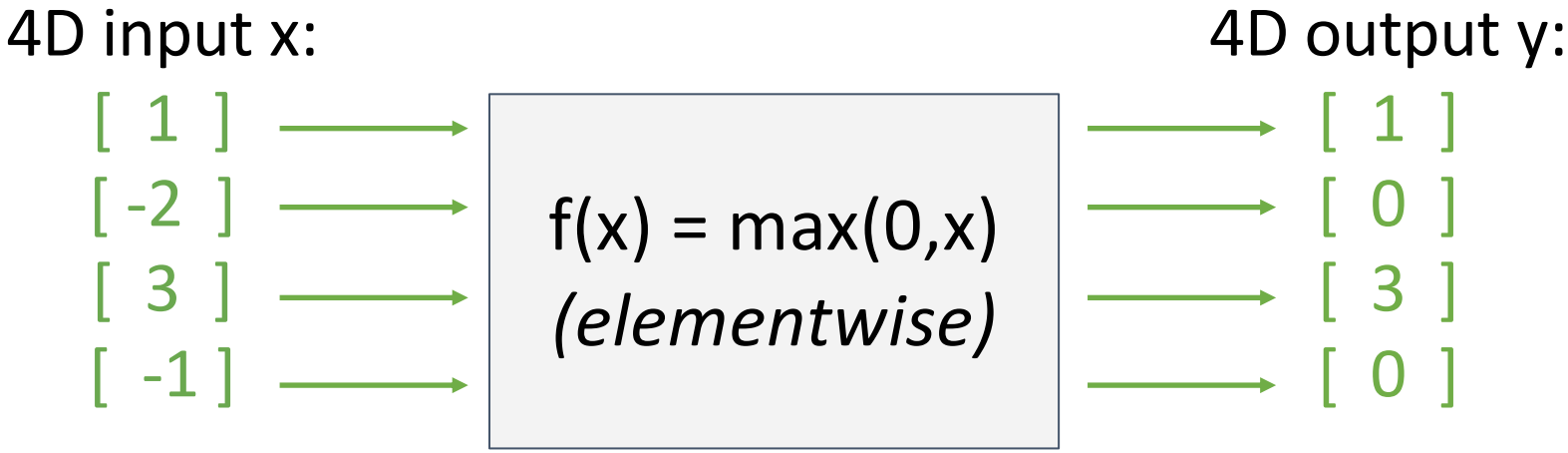


Backprop with Vectors

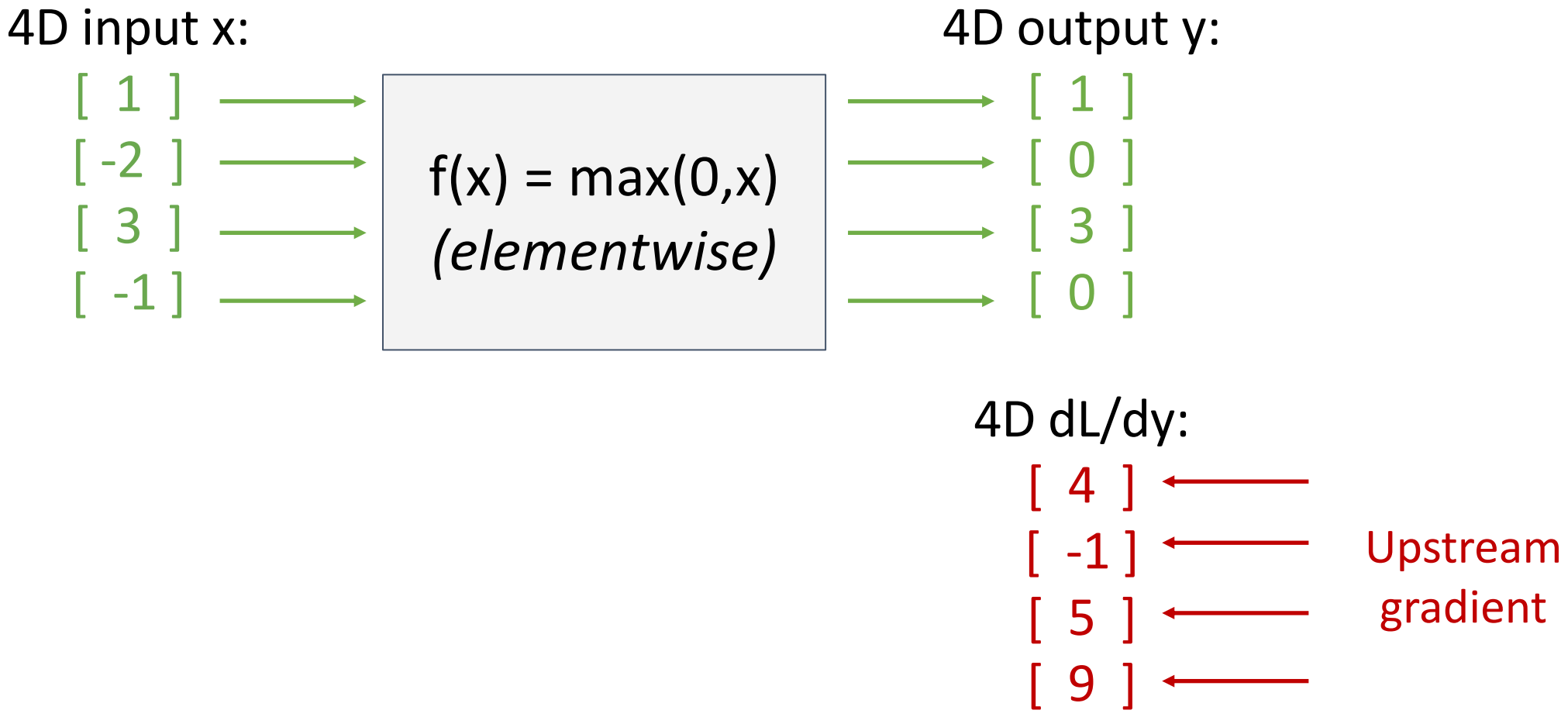


$$[1 \times D_y] = [1 \times D_z] \times [D_z \times D_y]$$

Backprop with Vectors



Backprop with Vectors



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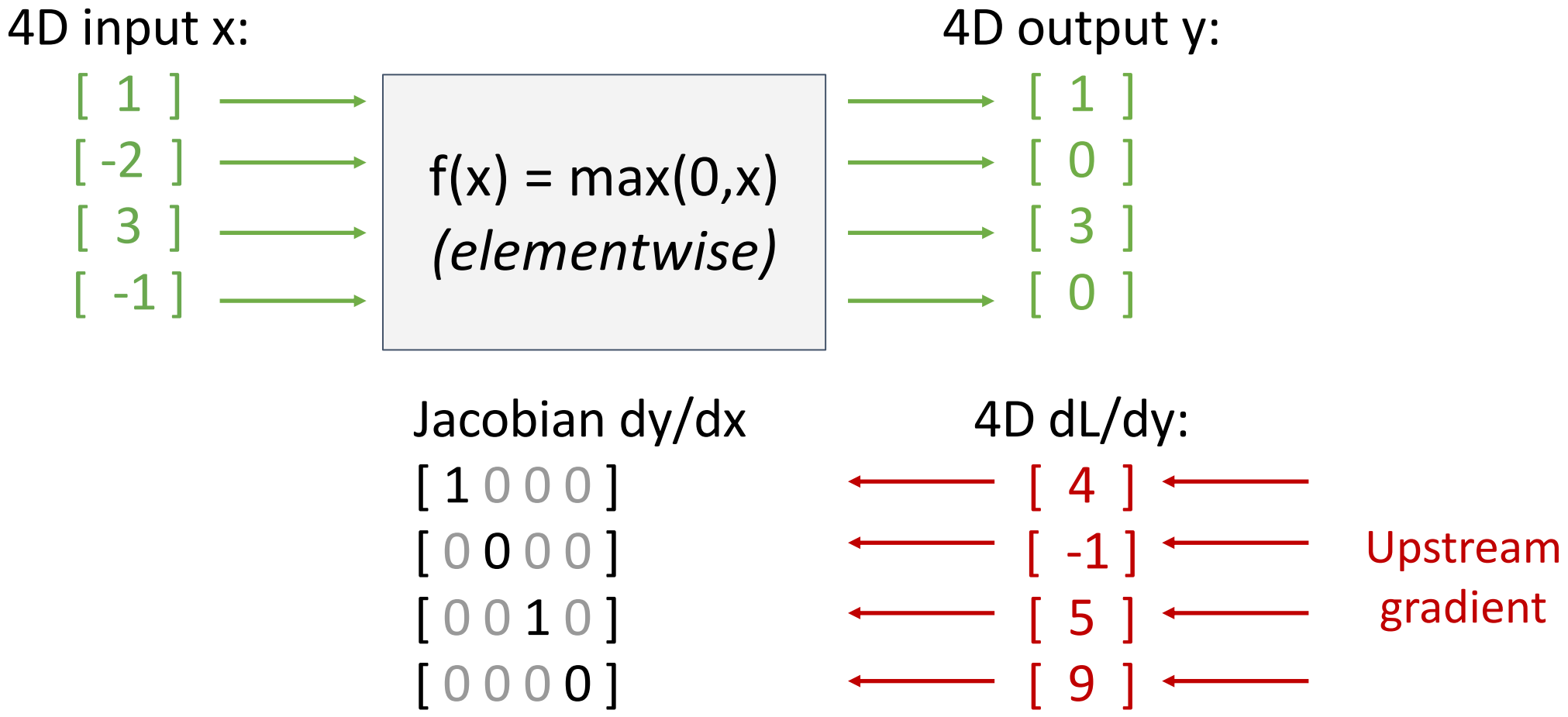

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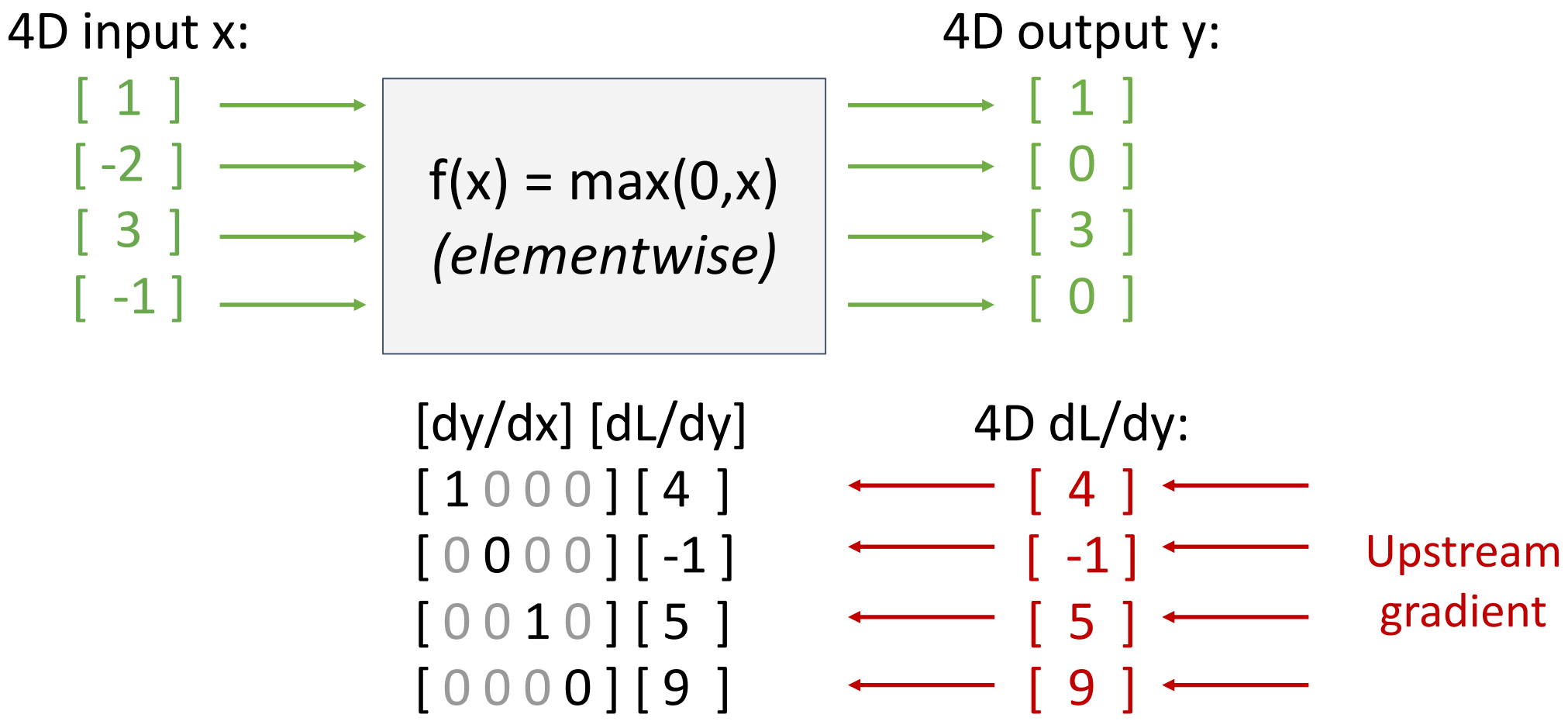
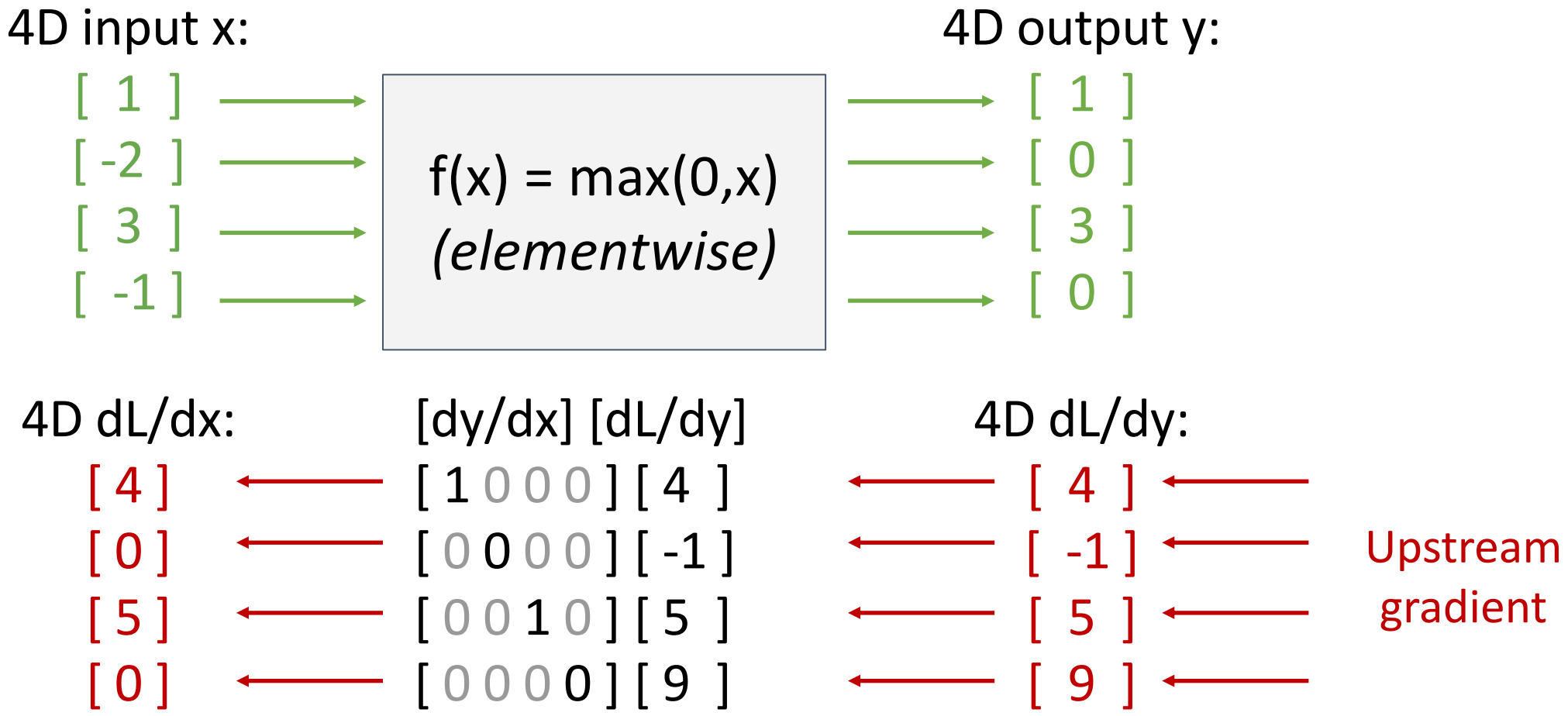
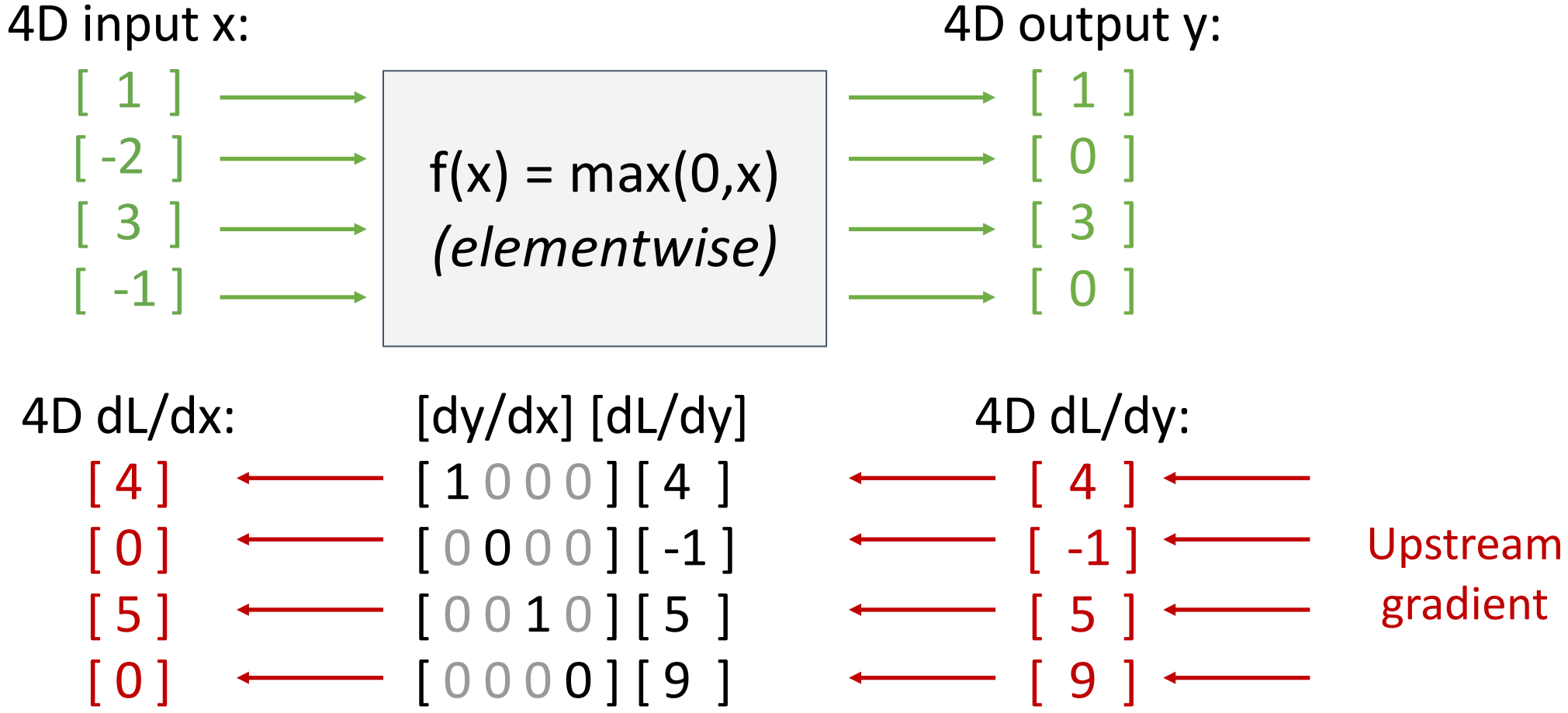


Backprop with Vectors



Backprop with Vectors

Jacobian is **sparse**: off-diagonal entries all zero! Never **explicitly** form Jacobian; instead use **implicit** multiplication



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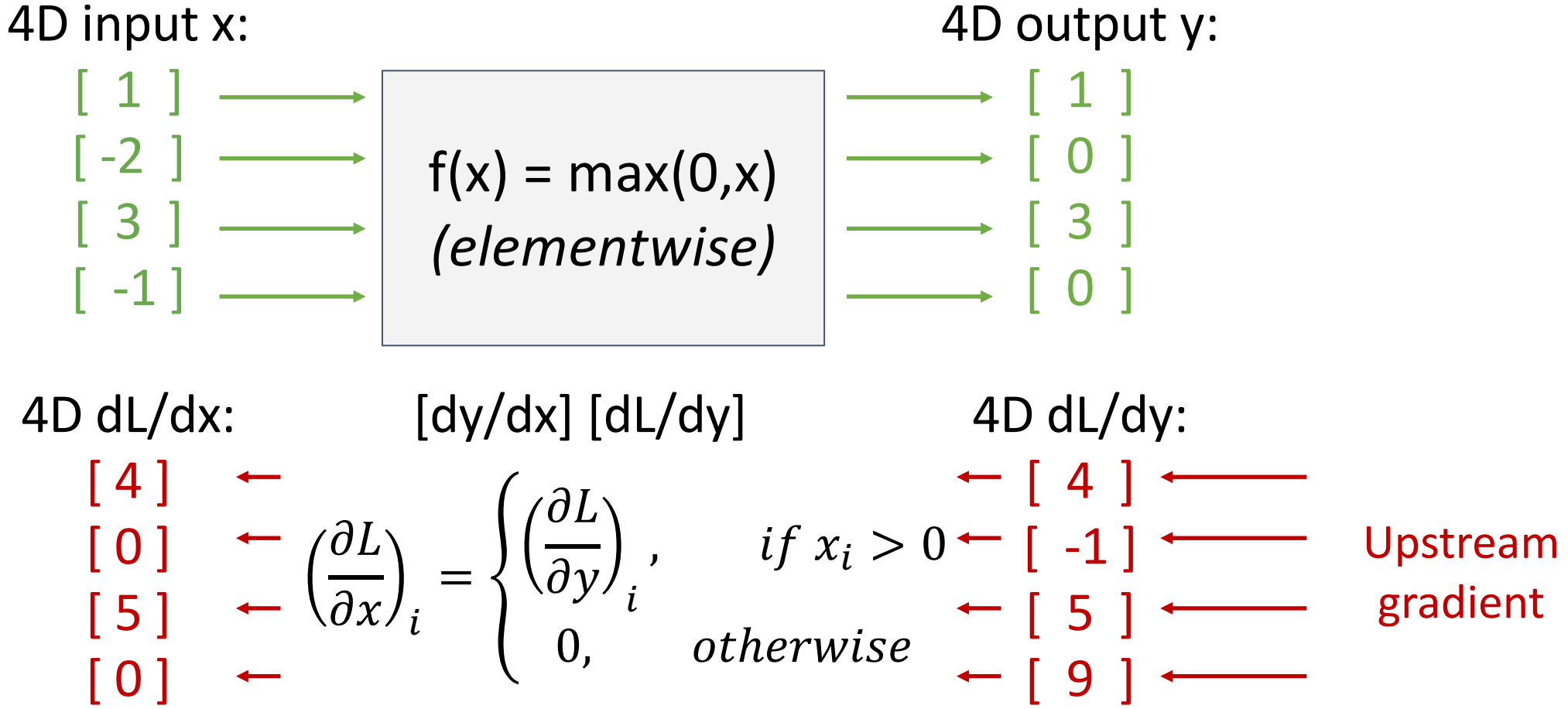
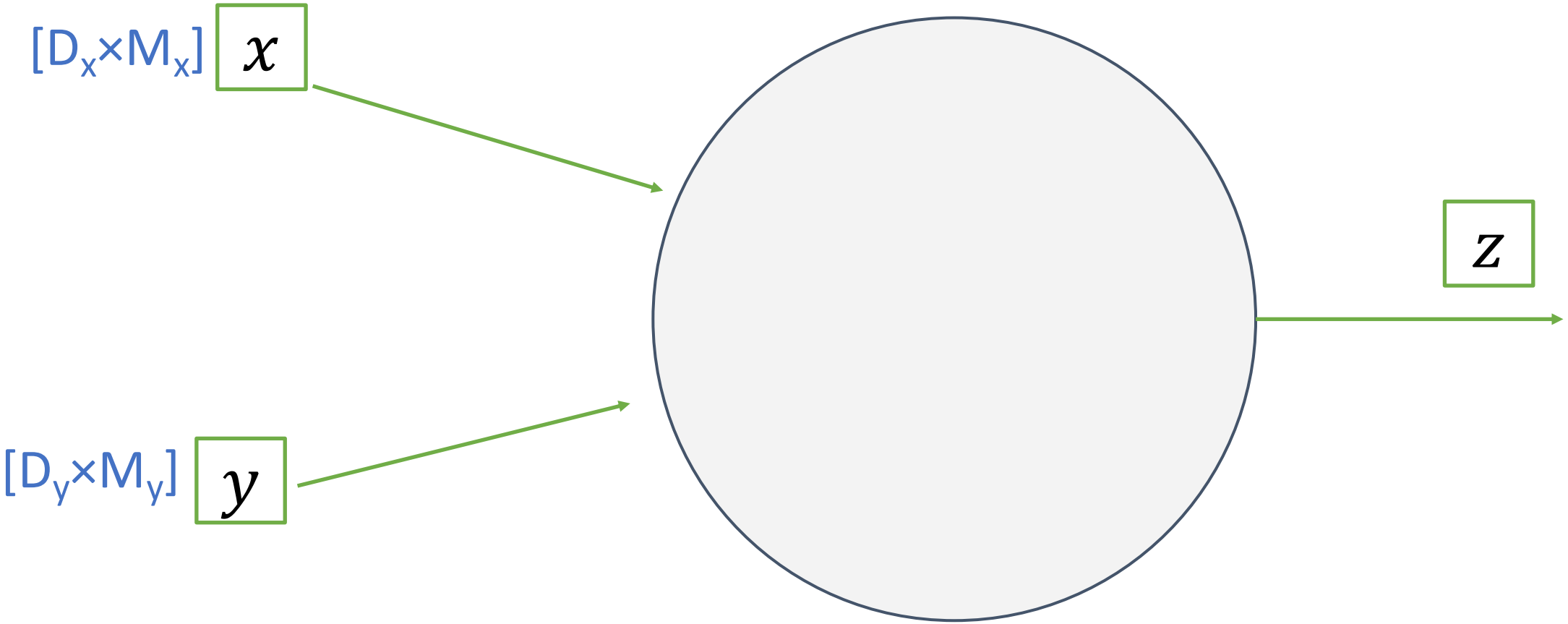


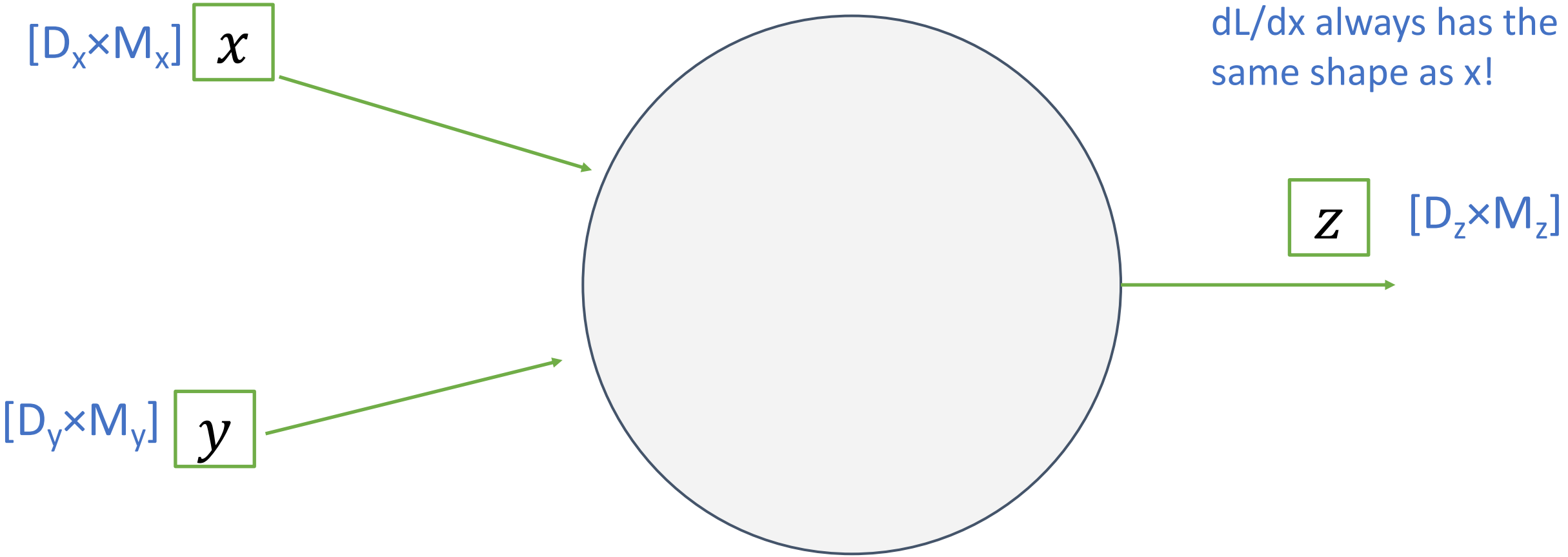
Backprop with Matrices (or Tensors):



Backprop with Matrices (or Tensors):

Loss L still a scalar!

dL/dx always has the same shape as x!



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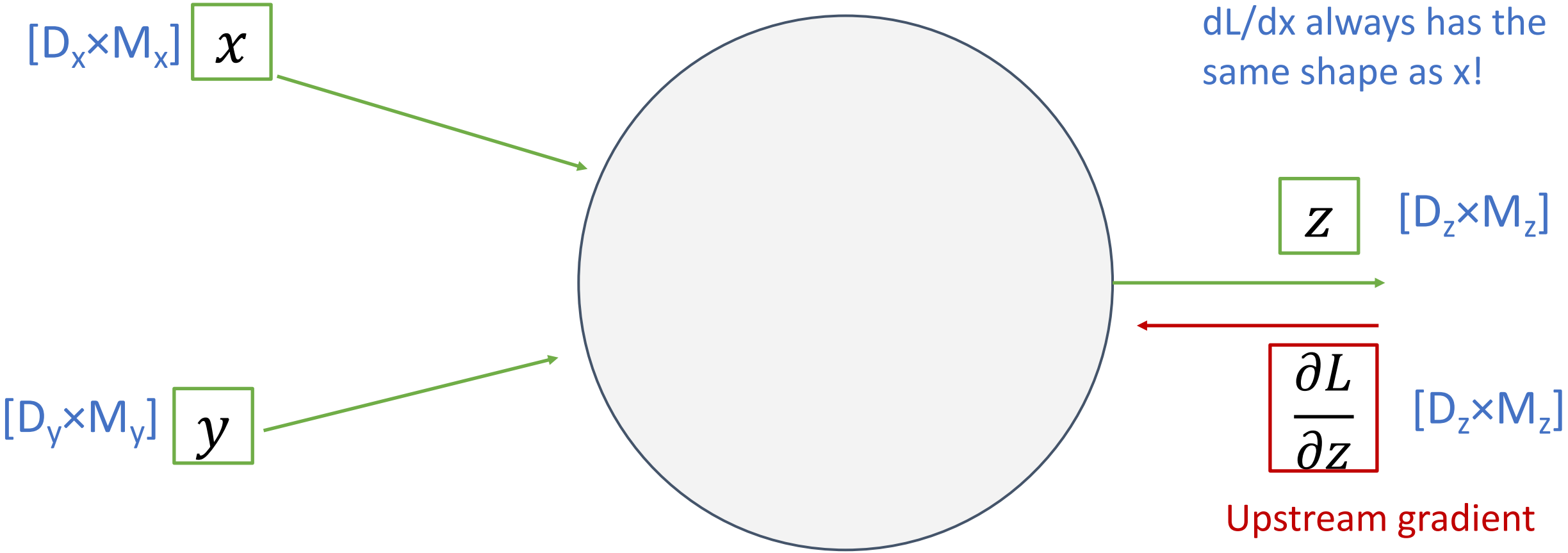

Upstream gradient

For each element of z, how much does it influence L?

Backprop with Matrices (or Tensors):

Loss L still a scalar!

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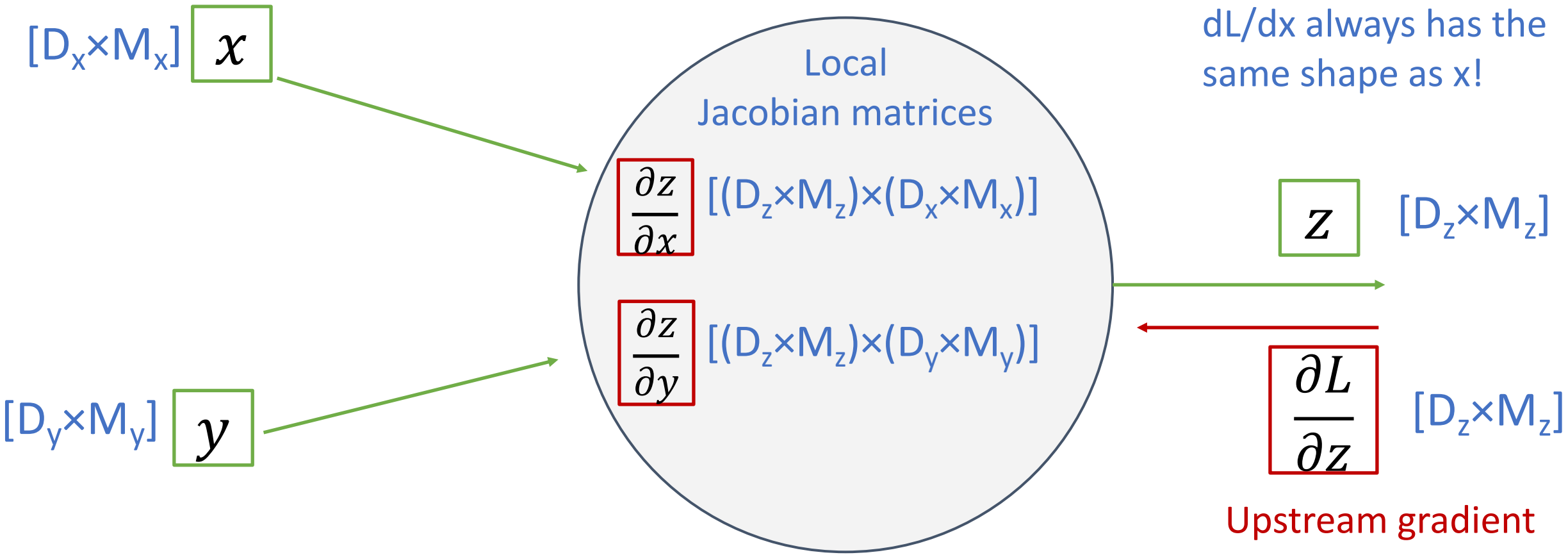

For each element of y, how much does it influence each element of z?

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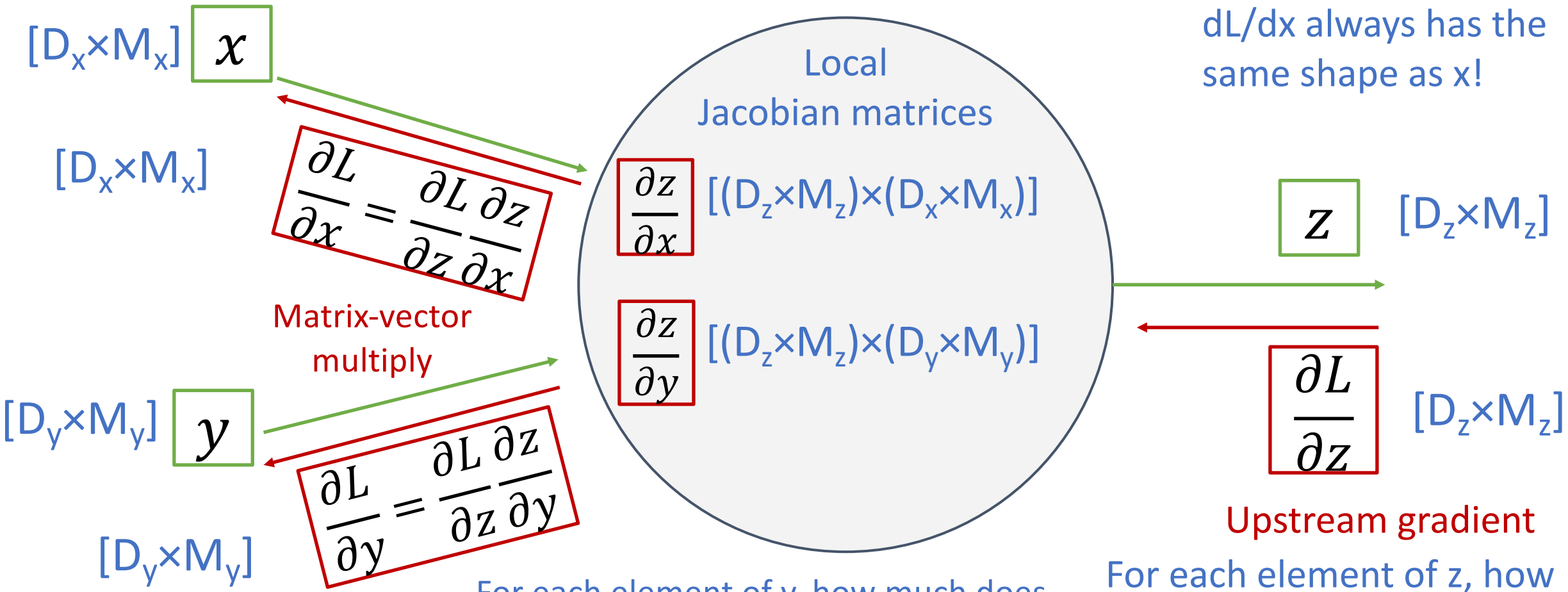


For each element of y, how much does it influence each element of z?

For each element of z, how much does it influence L?

Example: Matrix Multiplication

x: [N×D]

[2 1 -3]
[-3 4 2]

w: [D×M]

[3 2 1 -1]
[2 1 3 2]
[3 2 1 -2]

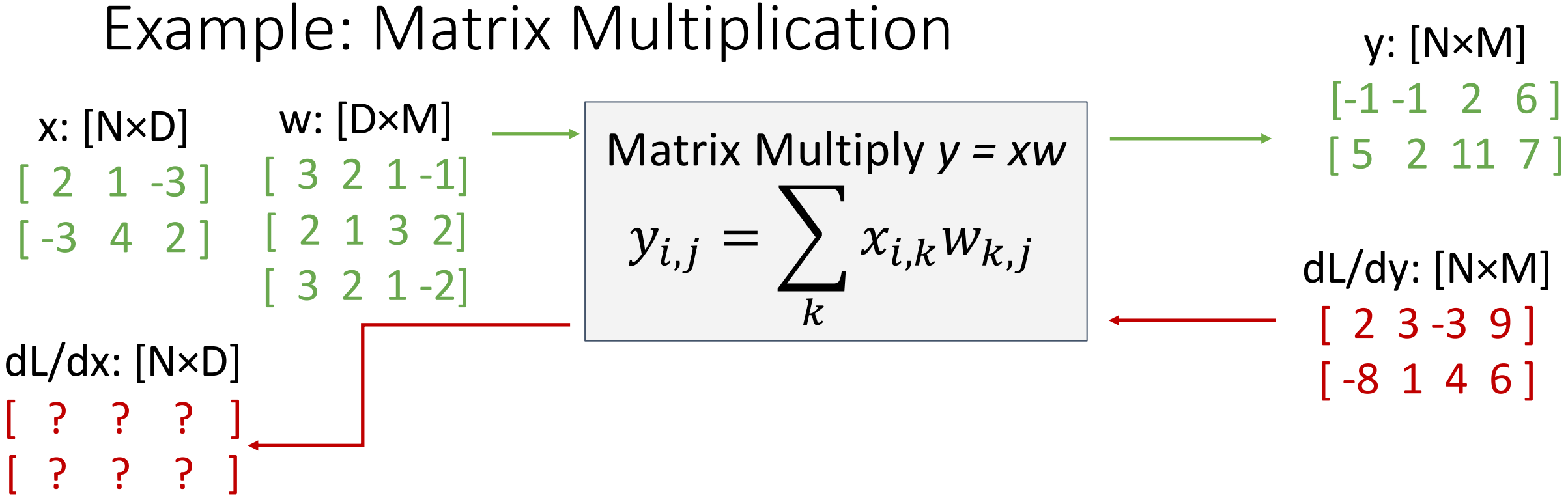
Matrix Multiply $y = xw$

$$y_{i,j} = \sum_k x_{i,k} w_{k,j}$$

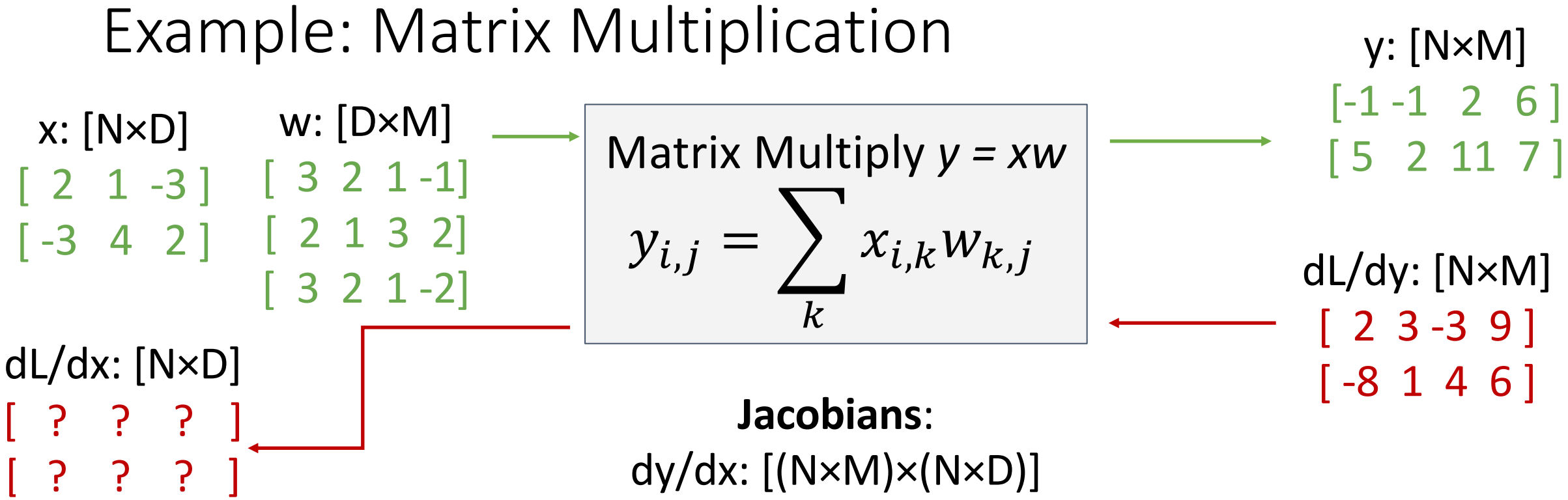
y: [N×M]

[-1 -1 2 6]
[5 2 11 7]

Example: Matrix Multiplication



Example: Matrix Multiplication



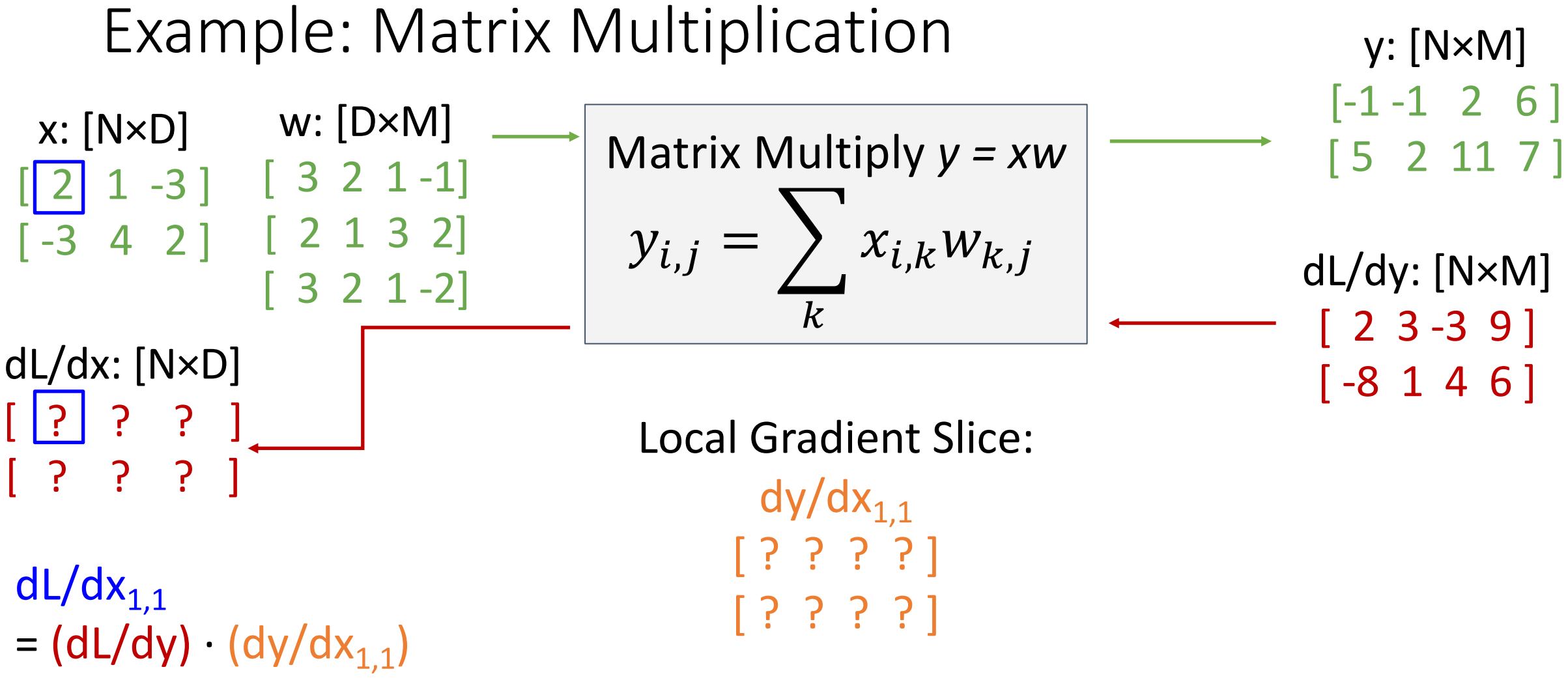
Jacobians:

dy/dx: [(N×M)×(N×D)]

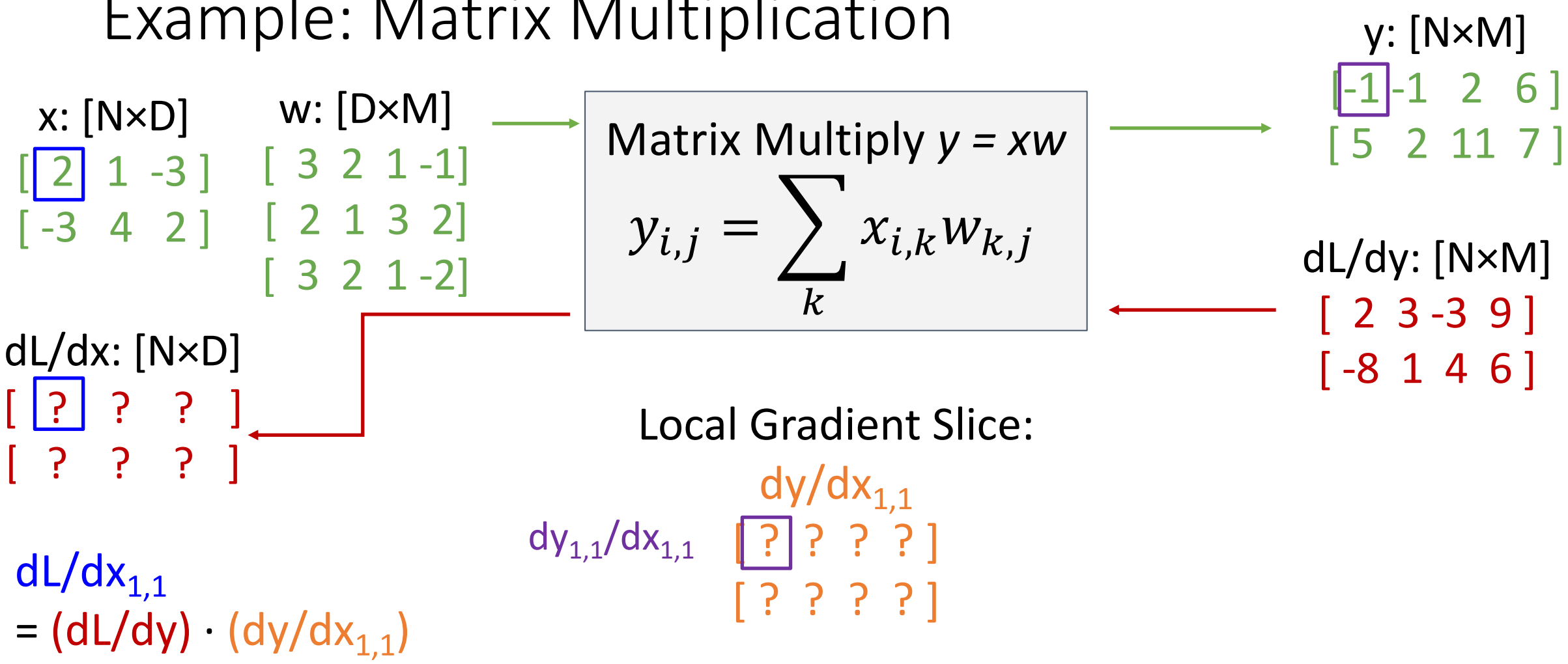
dy/dw: [(N×M)×(D×M)]

For a neural net we may have
N=64, D=M=4096
Each Jacobian takes 256 GB of memory! Must work with them implicitly!

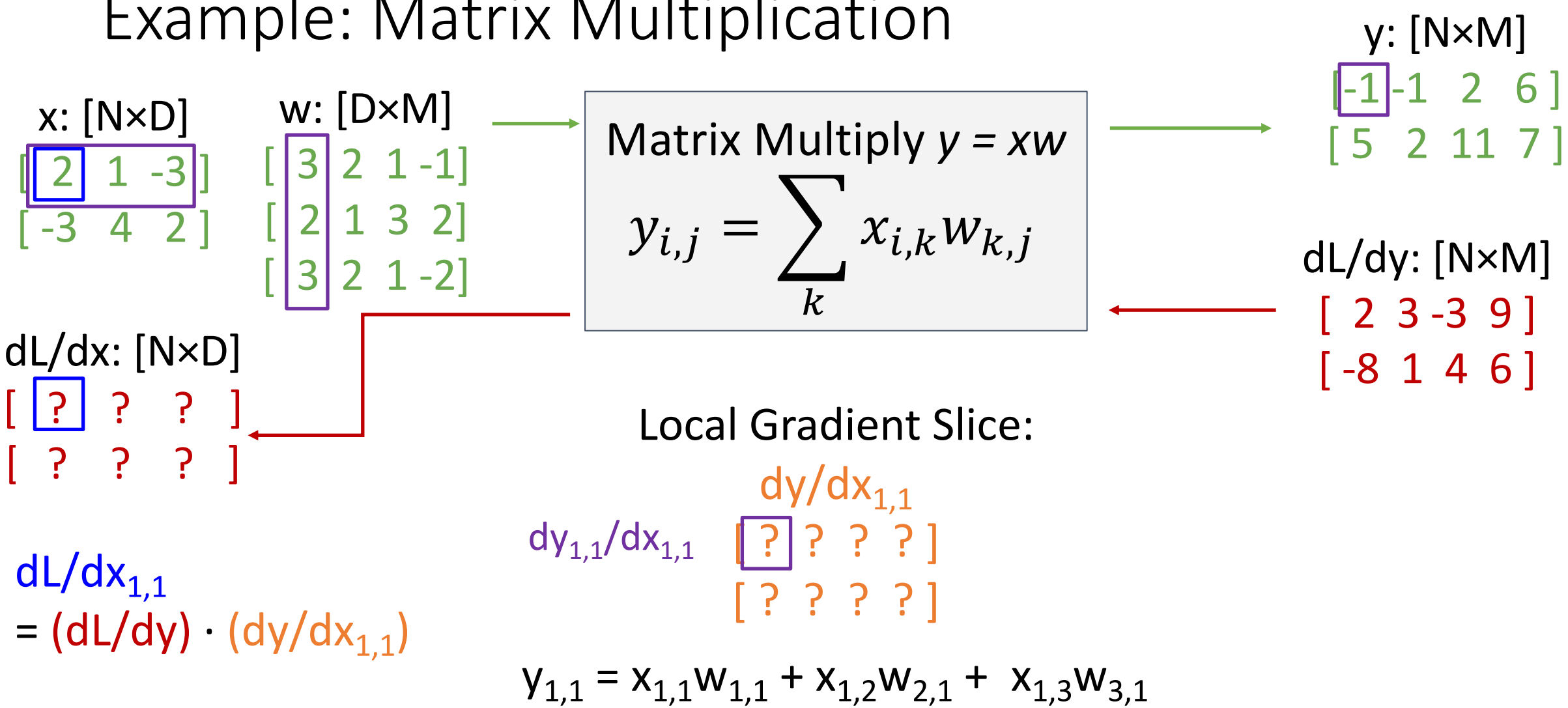
Example: Matrix Multiplication



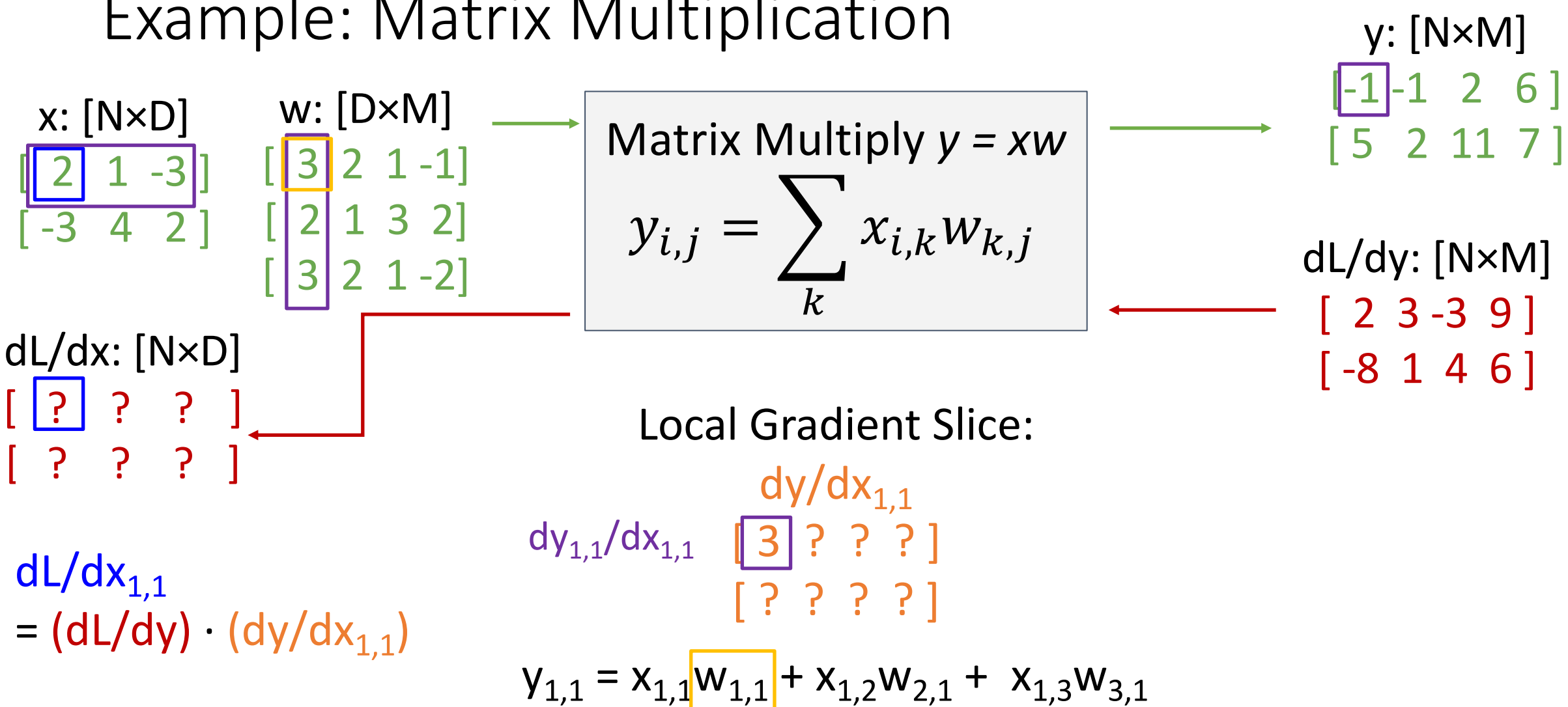
Example: Matrix Multiplication



Example: Matrix Multiplication



Example: Matrix Multiplication



$dL/dx_{1,1}$
$= (dL/dy) \cdot (dy/dx_{1,1})$

Local Gradient Slice:

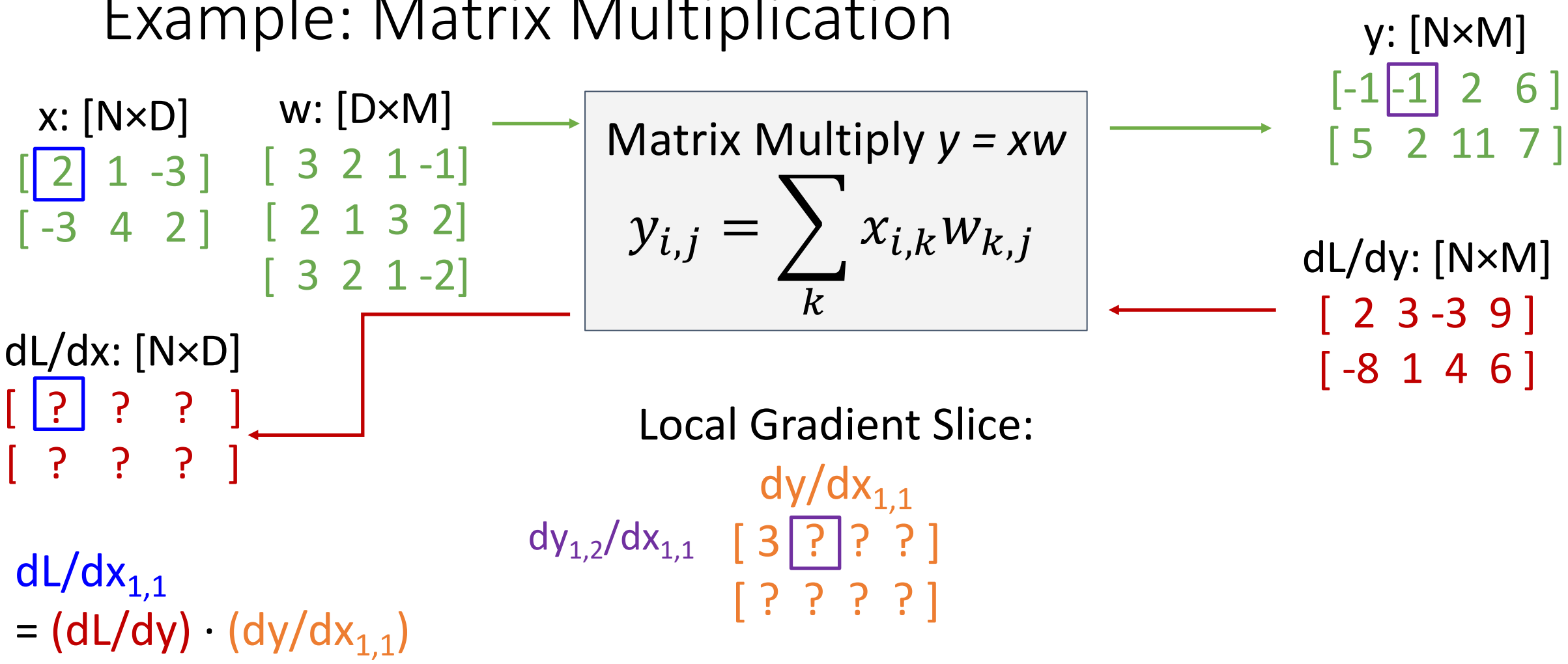
$dy/dx_{1,1}$

$dy_{1,1}/dx_{1,1}$ [3 ? ? ?]
[? ? ? ?]

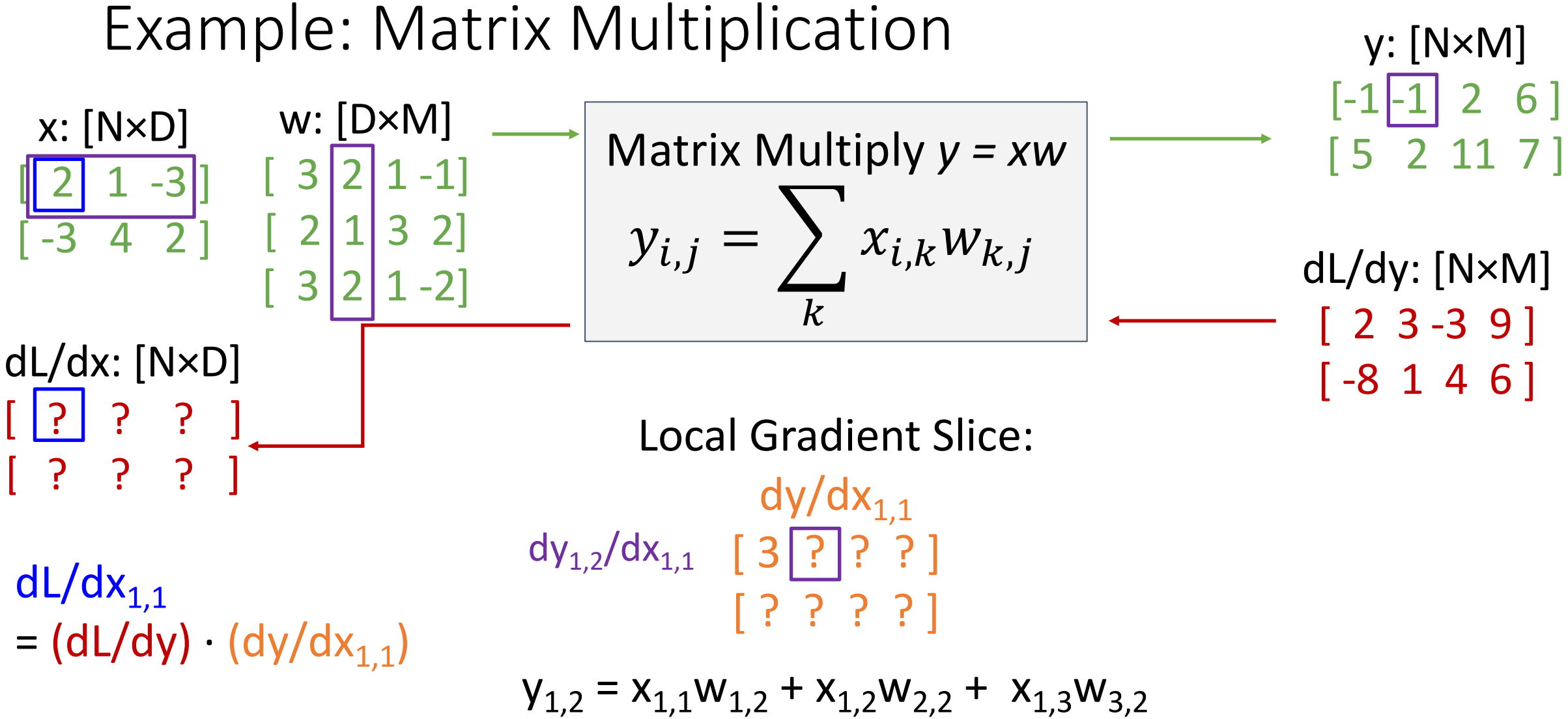
$y_{1,1} = x_{1,1}w_{1,1} + x_{1,2}w_{2,1} + x_{1,3}w_{3,1}$

$\Rightarrow dy_{1,1}/dx_{1,1} = w_{1,1}$

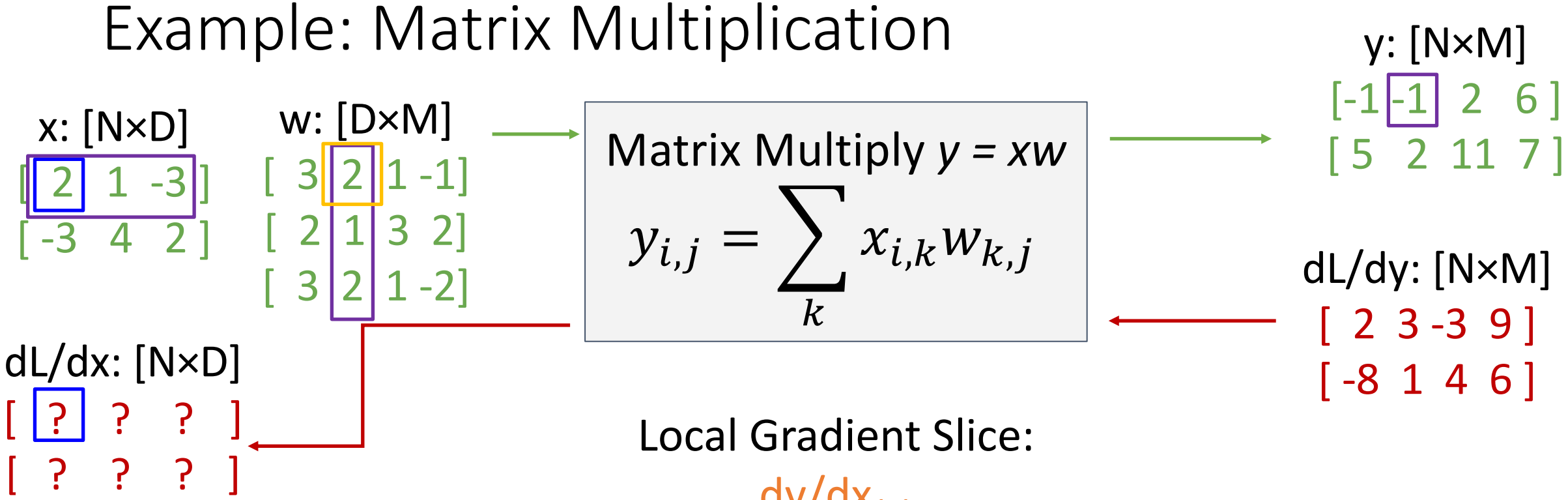
Example: Matrix Multiplication



Example: Matrix Multiplication



Example: Matrix Multiplication



$dL/dx_{1,1}$
$= (dL/dy) \cdot (dy/dx_{1,1})$

Local Gradient Slice:

$dy/dx_{1,1}$

$dy_{1,2}/dx_{1,1}$ [3 2 ? ?]
[? ? ? ?]

$y_{1,2} = x_{1,1}w_{1,2} + x_{1,2}w_{2,2} + x_{1,3}w_{3,2}$
$\Rightarrow dy_{1,2}/dx_{1,1} = w_{1,2}$

Example: Matrix Multiplication

x: [N×D]

[2 1 -3]
[-3 4 2]

w: [D×M]

[3 2 1 -1]
[2 1 3 2]
[3 2 1 -2]

Matrix Multiply $y = xw$

$$y_{i,j} = \sum_k x_{i,k} w_{k,j}$$

y: [N×M]

[-1 -1 2 6]
[5 2 11 7]

dL/dy: [N×M]

[2 3 -3 9]
[-8 1 4 6]

dL/dx: [N×D]

[? ? ?]
[? ? ?]

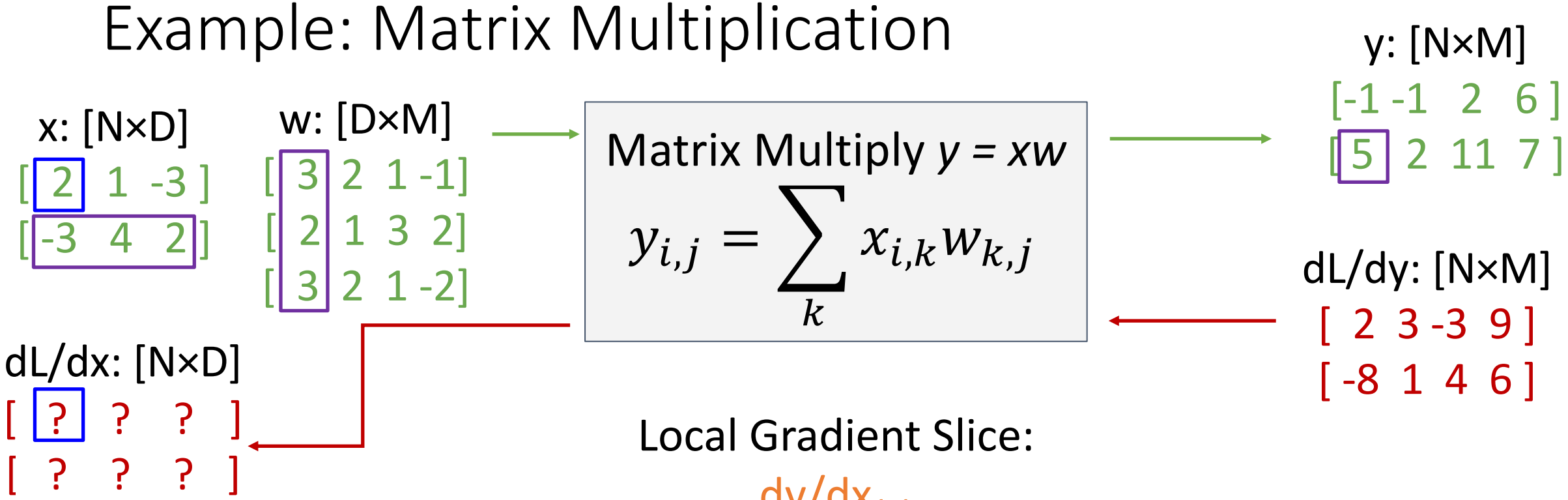
Local Gradient Slice:

$dy/dx_{1,1}$

$dy_{1,2}/dx_{1,1}$ [3 2 1 -1]
[? ? ? ?]

$dL/dx_{1,1}$
$= (dL/dy) \cdot (dy/dx_{1,1})$

Example: Matrix Multiplication



Local Gradient Slice:

$dy/dx_{1,1}$

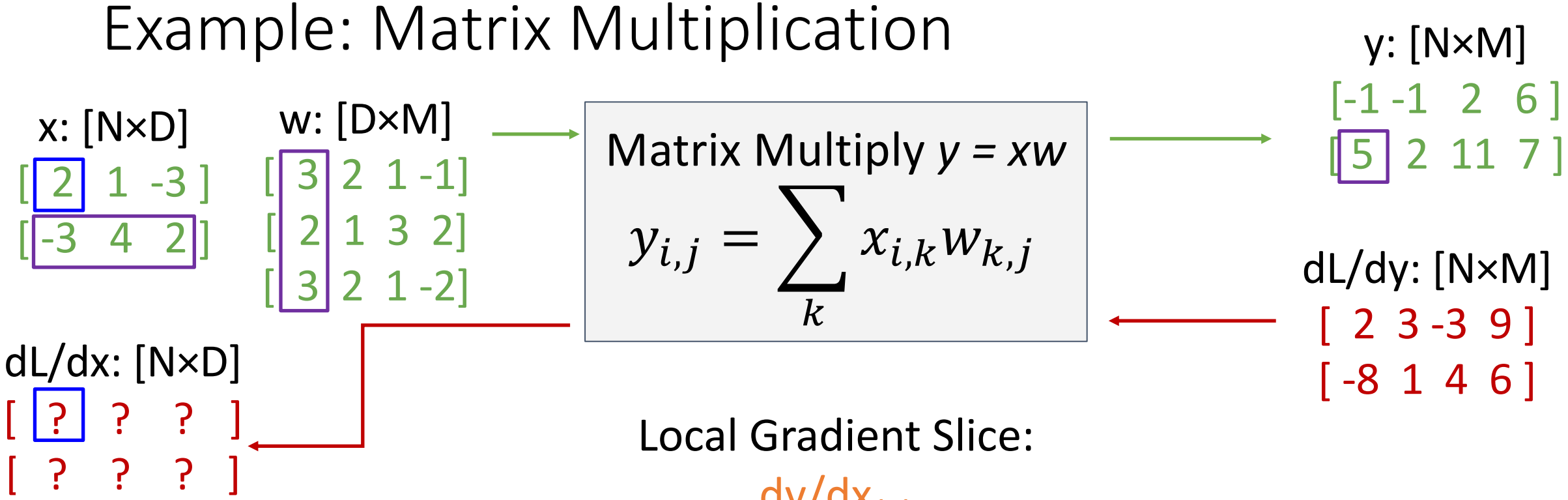
$dy_{1,2}/dx_{1,1}$ [3 2 1 -1]

[? ? ? ?]

$dL/dx_{1,1}$
$= (dL/dy) \cdot (dy/dx_{1,1})$

$y_{2,1} = x_{2,1}w_{1,1} + x_{2,2}w_{2,1} + x_{2,3}w_{3,1}$

Example: Matrix Multiplication



Local Gradient Slice:

$dy/dx_{1,1}$

$dy_{1,2}/dx_{1,1}$ [3 2 1 -1]

[0 ? ? ?]

$dL/dx_{1,1}$
$= (dL/dy) \cdot (dy/dx_{1,1})$

$y_{2,1} = x_{2,1}w_{1,1} + x_{2,2}w_{2,1} + x_{2,3}w_{3,1}$
$\Rightarrow dy_{2,1}/dx_{1,1} = 0$

Example: Matrix Multiplication

x: [N×D]

[2 1 -3]
[-3 4 2]

w: [D×M]

[3 2 1 -1]
[2 1 3 2]
[3 2 1 -2]

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[-1 -1 2 6]
[5 2 11 7]

dL/dy: [N×M]

[2 3 -3 9]
[-8 1 4 6]

dL/dx: [N×D]

[? ? ?]
[? ? ?]

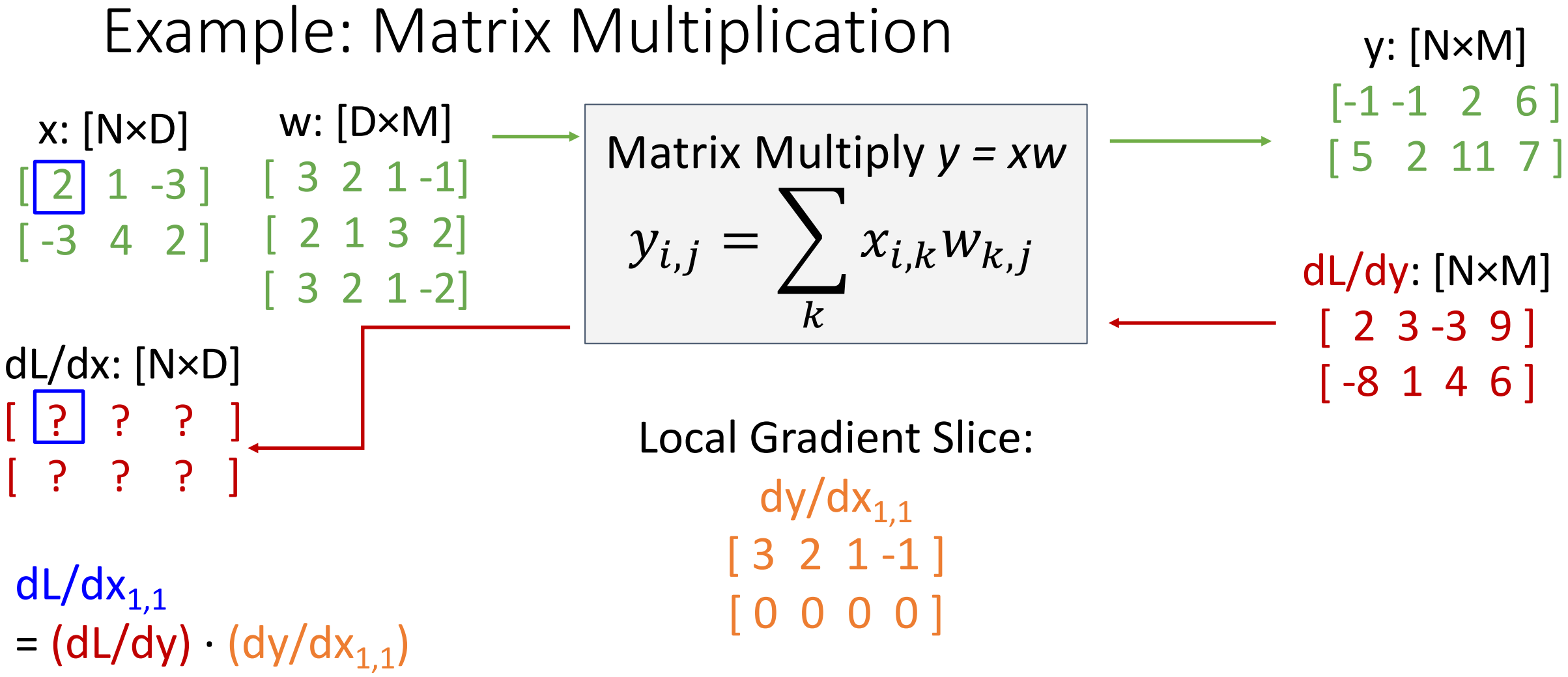
Local Gradient Slice:

$dy/dx_{1,1}$

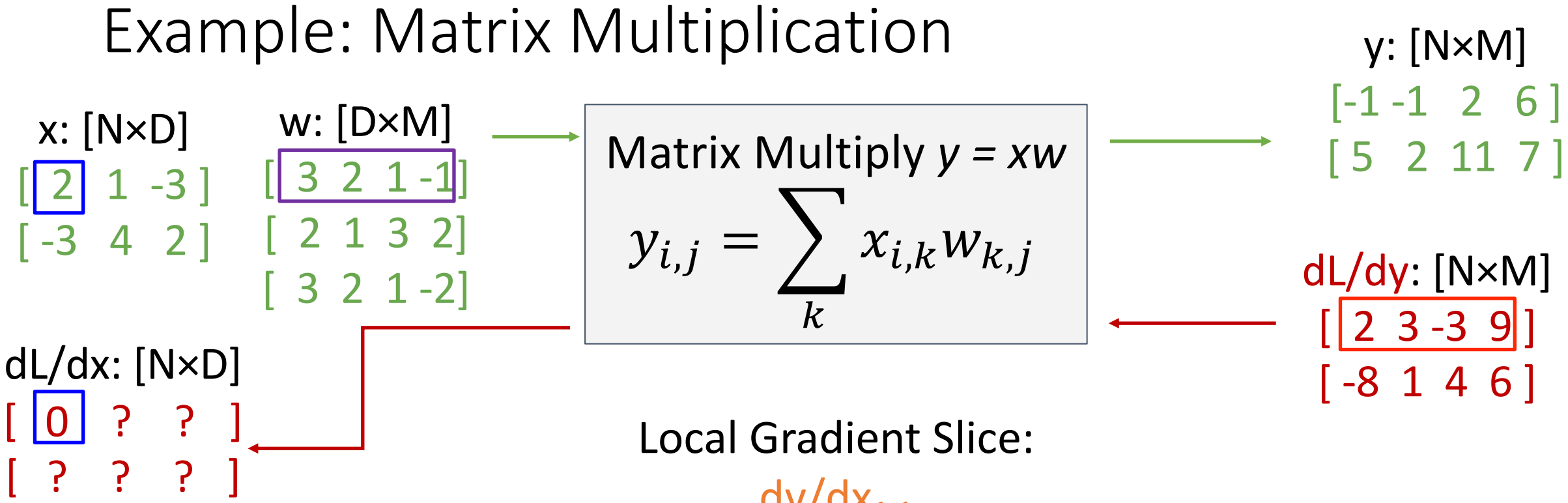
$dy_{1,2}/dx_{1,1}$ [3 2 1 -1]
[0 0 0 0]

$dL/dx_{1,1}$
$= (dL/dy) \cdot (dy/dx_{1,1})$

Example: Matrix Multiplication

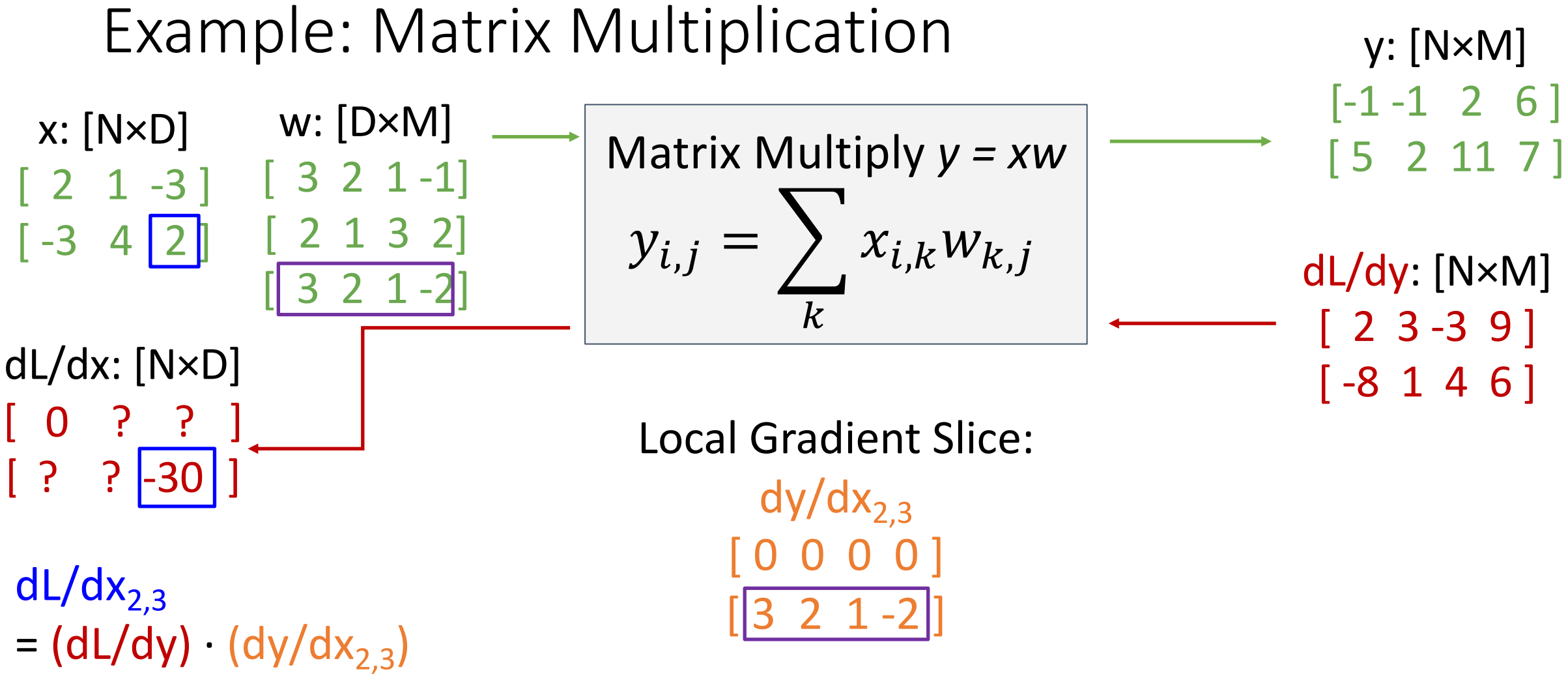


Example: Matrix Multiplication



$dL/dx_{1,1}$
$= (dL/dy) \cdot (dy/dx_{1,1})$
$= (dL/dy_{1,:}) \cdot (w_{1,:})$
$= 3*2 + 2*3 + 1*(-3) + (-1)*9 = 0$

Example: Matrix Multiplication



Example: Matrix Multiplication

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[2 1 -3]
[-3 4 2]

w: [D×M]

[3 2 1 -1]
[2 1 3 2]
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Matrix Multiply $y = xw$

$$y_{i,j} = \sum_k x_{i,k} w_{k,j}$$

y: [N×M]

[-1 -1 2 6]
[5 2 11 7]

dL/dy: [N×M]

[2 3 -3 9]
[-8 1 4 6]

dL/dx: [N×D]

[0 ? ?]
[? ? -30]

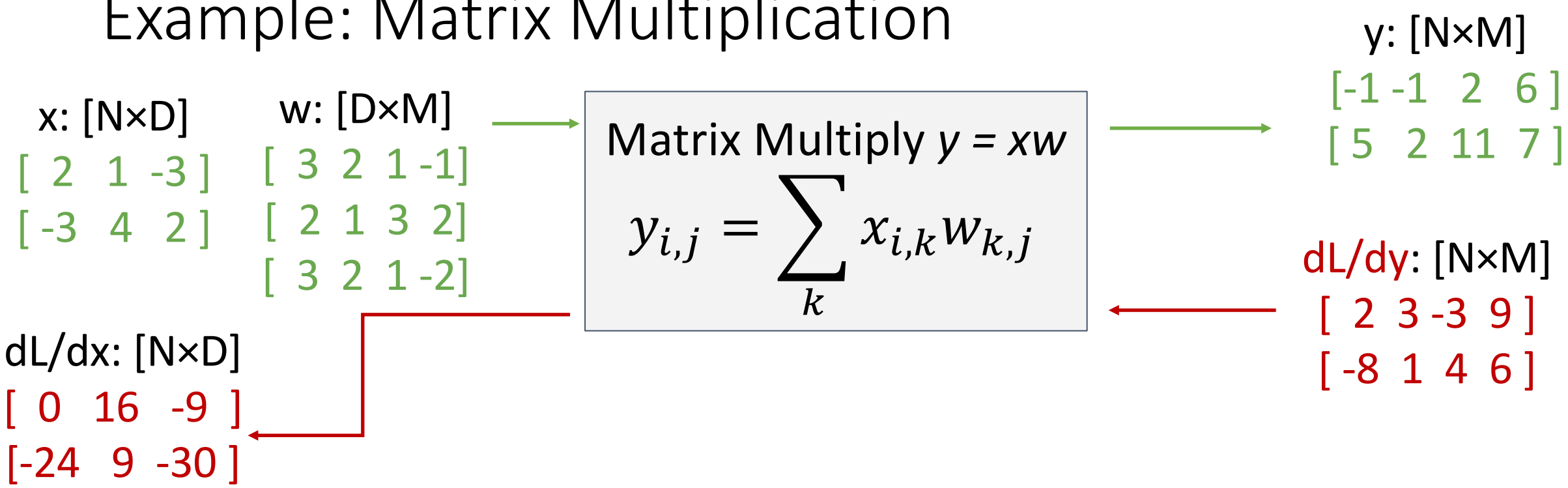
Local Gradient Slice:

$dy/dx_{2,3}$

[0 0 0 0]
[3 2 1 -2]

$dL/dx_{2,3}$
$= (dL/dy) \cdot (dy/dx_{2,3})$
$= (dL/dy_{2,:}) \cdot (w_{3,:})$
= 3*(-8) + 2*1 + 1*4 + (-2)*6 = -30

Example: Matrix Multiplication



$dL/dx_{i,j}$
$= (dL/dy) \cdot (dy/dx_{i,j})$
$= (dL/dy_{i,:}) \cdot (w_{j,:})$

Example: Matrix Multiplication

x: [N×D]
[2 1 -3]
[-3 4 2]

w: [D×M]
[3 2 1 -1]
[2 1 3 2]
[3 2 1 -2]

Matrix Multiply $y = xw$

$$y_{i,j} = \sum_k x_{i,k} w_{k,j}$$

y: [N×M]
[-1 -1 2 6]
[5 2 11 7]

dL/dy: [N×M]
[2 3 -3 9]
[-8 1 4 6]

dL/dx: [N×D]
[0 16 -9]
[-24 9 -30]

It is easy to derive $dy/dx = w^T$

$$dL/dx = (dL/dy)\, w^T$$

[N x D] [N x M] [M x D]

$dL/dx_{i,j}$
$= (dL/dy) \cdot (dy/dx_{i,j})$
$= (dL/dy_{i,:}) \cdot (w_{j,:})$

Easy way to remember:
It's the only way the shapes work out!

Example: Matrix Multiplication

x: [N×D]
[2 1 -3]
[-3 4 2]

w: [D×M]
[3 2 1 -1]
[2 1 3 2]
[3 2 1 -2]

Matrix Multiply $y = xw$

$$y_{i,j} = \sum_k x_{i,k} w_{k,j}$$

y: [N×M]
[-1 -1 2 6]
[5 2 11 7]

dL/dy: [N×M]
[2 3 -3 9]
[-8 1 4 6]

dL/dx: [N×D]
[0 16 -9]
[-24 9 -30]

dL/dx

$$dL/dx = (dL/dy)\, w^T$$
[N x D] [N x M] [M x D]

$$dL/dw = x^T\, (dL/dy)$$
[D x M] [D x N] [N x M]

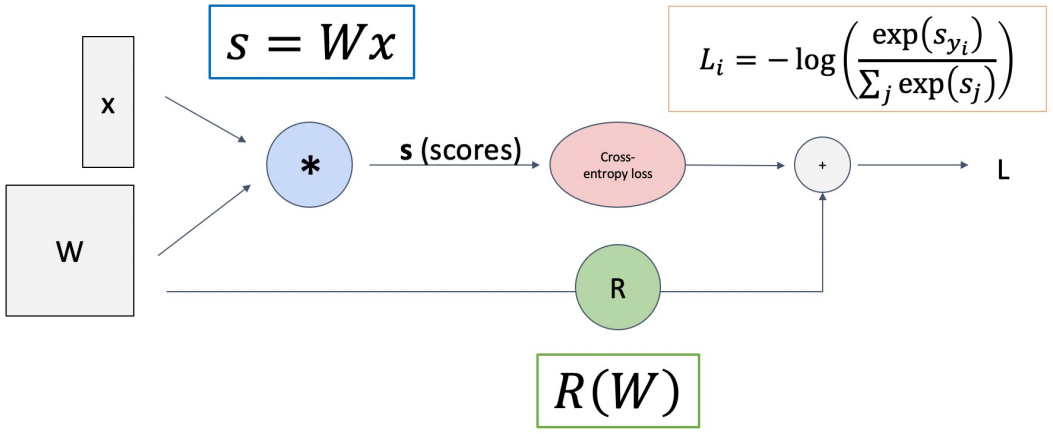
Easy way to remember:
It's the only way the shapes work out!

Reading and Practice

Derive by yourself on the gradient as a practice!
Read D2L textbook (https://d2l.ai/d2l-en.pdf) : Chapter 2.4, Chapter 2.5, Chapter 5.3
Notebooks in D2L textbook: Chapter 5. Multilayer Perceptrons

Summary

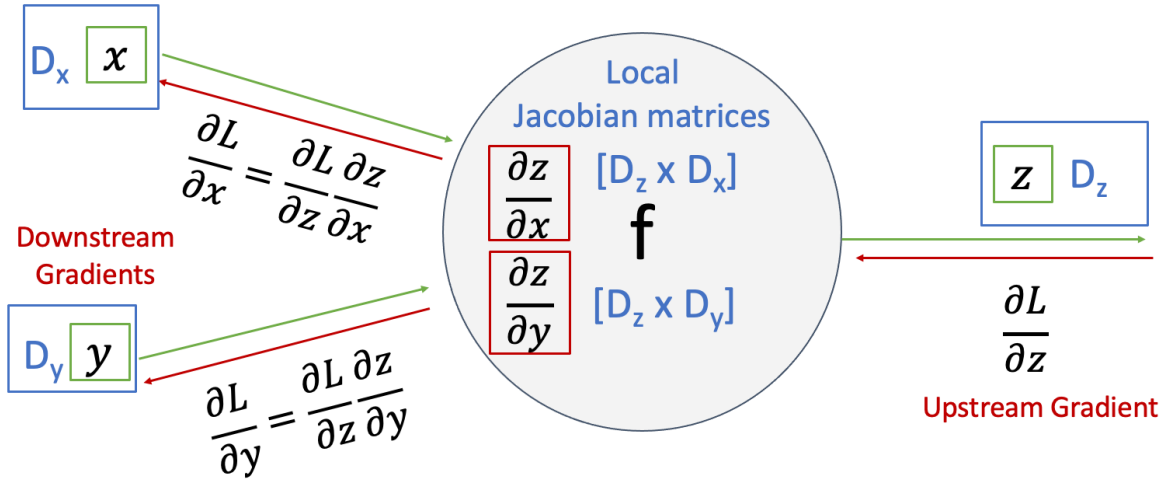
Represent complex expressions as **computational graphs**



Forward pass computes outputs

Backward pass computes gradients

During the backward pass, each node in the graph receives **upstream gradients** and multiplies them by **local gradients** to compute **downstream gradients**

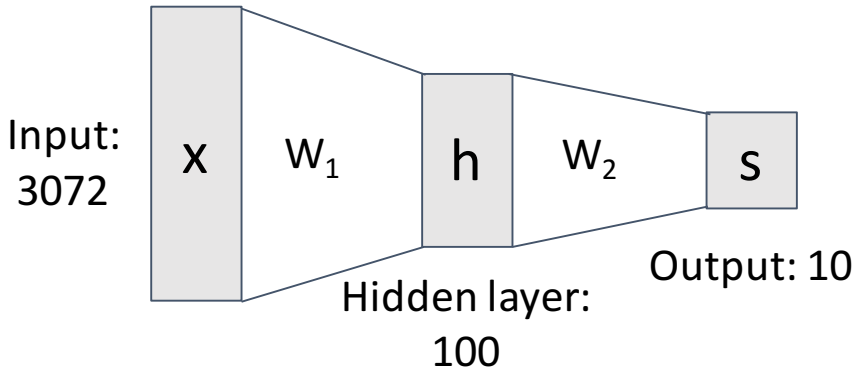


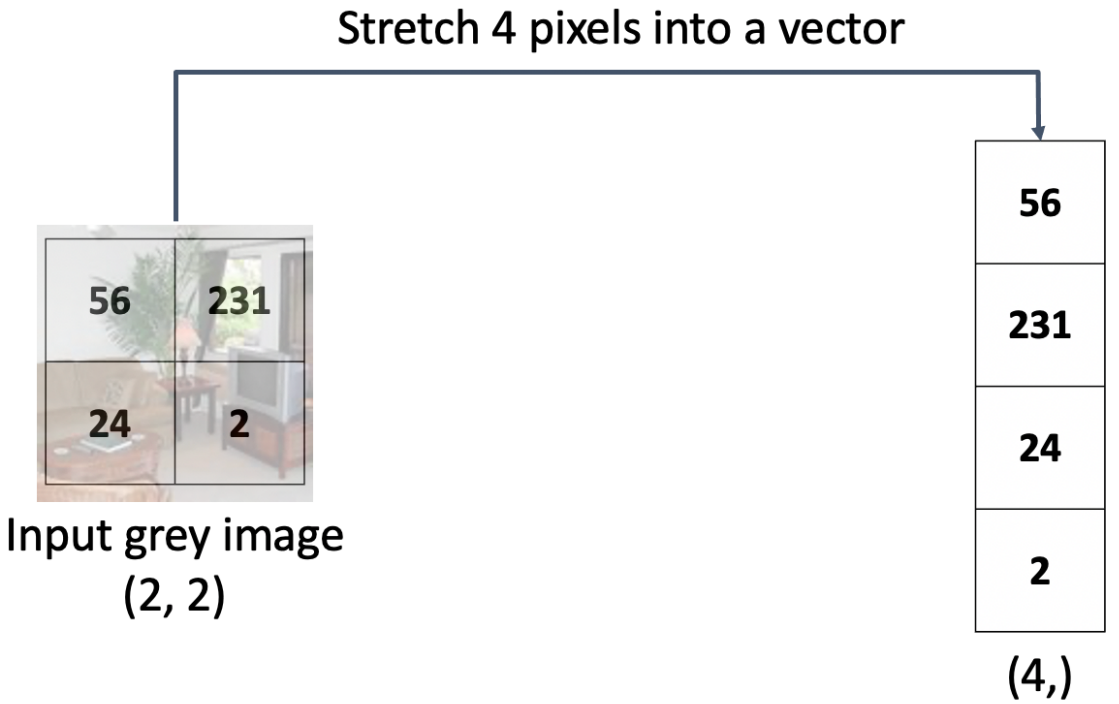
Problem: So far our classifiers don't respect the spatial structure of images!

Linear NN: $f = W_1 x$

Mult-Layer Perceptron:

$f = W_2 \max(0, W_1 x)$





Next time:
Convolutional Neural Networks