Homework 5

2.1 Consider the following grammar over the alphabet $\{\texttt{+}, \texttt{x}, \texttt{(}, \texttt{)}, \texttt{a}\}$.

$$\begin{aligned} E &\rightarrow E \texttt{+} T \mid T \\ T &\rightarrow T \texttt{x} F \mid F \\ F &\rightarrow \texttt{(} E \texttt{)} \mid \texttt{a} \end{aligned}$$

Give parse trees and derivations for each string.

a. `a`

b. `a+a`

c. `a+a+a`

d. `((a))`

2.6 Give context-free grammars generating the following languages.

d. $\{x_1\texttt{\#}x_2\texttt{\#}\cdots\texttt{\#}x_k \mid k \geq 1$, each $x_i \in \{\texttt{a}, \texttt{b}\}^*$, and for some i and j, $x_i = x_j^{\mathcal{R}}\}$

2.9 Give a context-free grammar that generates the language

$$A = \{\texttt{a}^i\texttt{b}^j\texttt{c}^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

*2.19 Let CFG G be the following grammar.

$$\begin{aligned} S &\rightarrow \texttt{a}S\texttt{b} \mid \texttt{b}Y \mid Y\texttt{a} \\ Y &\rightarrow \texttt{b}Y \mid \texttt{a}Y \mid \varepsilon \end{aligned}$$

Let $L(G)$ denote the language of G. Give a simple description of $L(G)$ in English. Use that description to give a CFG for the complement of $L(G)$.

*2.27 Let $G = (V, \Sigma, R, \langle\text{STMT}\rangle)$ be the following grammar.

$$\begin{aligned} \langle\text{STMT}\rangle &\rightarrow \langle\text{ASSIGN}\rangle \mid \langle\text{IF-THEN}\rangle \mid \langle\text{IF-THEN-ELSE}\rangle \\ \langle\text{IF-THEN}\rangle &\rightarrow \texttt{if condition then } \langle\text{STMT}\rangle \\ \langle\text{IF-THEN-ELSE}\rangle &\rightarrow \texttt{if condition then } \langle\text{STMT}\rangle \texttt{ else } \langle\text{STMT}\rangle \\ \langle\text{ASSIGN}\rangle &\rightarrow \texttt{a:=1} \end{aligned}$$

$$\Sigma = \{\texttt{if}, \texttt{condition}, \texttt{then}, \texttt{else}, \texttt{a:=1}\}$$
$$V = \{\langle\text{STMT}\rangle, \langle\text{IF-THEN}\rangle, \langle\text{IF-THEN-ELSE}\rangle, \langle\text{ASSIGN}\rangle\}$$

G is a natural-looking grammar for a fragment of a programming language, but G is ambiguous.

a. Show that G is ambiguous.

b. Give a new unambiguous grammar for the same language.

*2.28 Give unambiguous CFGs for the following languages.

c. $\{w \mid$ the number of `a`'s is at least the number of `b`'s in $w\}$

7 Prove that every regular language has a context-free grammar. *Hint:* given a DFA, explain how to transform it into an equivalent grammar.