CS 181: HW 7 TEJAS KAMTAM, 305749402

2.30a) $L = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Consider $w = uvxyz = 0^p 1^p 0^p 1^p \in L$, then pumping down will give us 2 cases (high level)

(1) if $vxy \in 0^p$ or $vxy \in 1^p$, then pumping down will result in $\#_0(w) \neq \#_1(w)$, thus the resulting string $\notin L$

(2) if vxy "straddles" $0^m 1^k$ for some m, k s.t. $m + k \leq p$, then pumping down also results in $\#_0(w) \neq \#_1(w)$ due to the other pair of $0^p 1^p$.

Thus, ~~it is not in~~ L is not context-free by the P.L.

2.30b) $L = \{w\#t \mid w \in t \text{ s.t. } w, t \in \{a,b\}^*\}$

Consider the string $c = w\#t = uvxyz = a^p b^p \# a^p b^p \in L$, then we have a few cases:

(1) $vxy \in w$, then pumping up will result in either $\#_a(w) > \#_a(t)$ or $\#_b(w) > \#_b(t)$ making $w \notin t$ therefore the resulting ~~str~~ string $\notin L$.

(2) vxy straddles $b^p \# a^p$:

(a) if $\# \in v$ or $\# \in y$ then pumping down will result in a string without "#" thus the resulting string is not in L

(b) otherwise, pumping up will result in $\#_b(w) > \#_b(t)$, thus the resulting string $\notin L$

(3) $vxy \in t$, then pumping down results in a string where either $\#_a(w) > \#_a(t)$ or $\#_b(w) > \#_b(t)$, thus the resulting string $\notin L$.

Therefore, L is not context-free by the P.L.

2.30d) $L = \{t_1 \# t_2 \# \ldots \# t_k \mid k \geq 2, t_i \in \{a,b\}^*, t_i = t_j \text{ for some } i \neq j\}$

Consider the string $w = t_1 \# t_2 = uvxyz = a^p \# a^p$, then we have 2 cases when pumping down

(1) if $vxy \in a^p$ then pumping down will result in a string s.t. $\#_a(t_1) \neq \#_a(t_2)$, thus this string $\notin L$

(2) if $\# \in v$ or $\# \in y$, then pumping down will result in a string s.t. $\# \notin w$ thus this string $\notin L$

Therefore this language L is not context-free by P.L.

2.31) $B = \{$all palindromes over $\{0,1\}$ s.t. $\#_0 = \#_1\}$

Consider $w = uvxyz = 0^p 1^{2p} 0^p$, then pumping down has 2 cases

(1) if $vxy \in 0^p$ or $vxy \in 1^{2p}$, s.t. $|vxy| \leq p$ then, pumping down ~~either~~ results in a string s.t. $\#_0 \neq \#_1$ thus this pumped down string $\notin B$

(2) if vxy straddles $0^m 1^k$ or $1^k 0^m$ for some m, k s.t. $m+k \leq p$, then pumping down results in a string that is not a palindrome b/c $\#0$s on the LHS of the 1s $\neq \#0$s on RHS of the 1s, thus this pumped string $\notin B$

Therefore, L is not context-free by the P.L.

2.43a) Consider that this lang. can be simplified, since $|\Sigma| = 2$, to all strings $w' \in \text{SCRAMBLE}(A)$ for which, $w \in A$, $\#_0(w') = \#_0(w)$ and $\#_1(w') = \#_1(w)$. Then we can recognize the scramble by considering that A is a reg. lang. $\therefore \exists$ DFA D that recognizes A. We will transform D into PDAs P_1, P_2 by changing the transitions s.t. given $D = (Q, \Sigma, \delta, q_0, F)$,

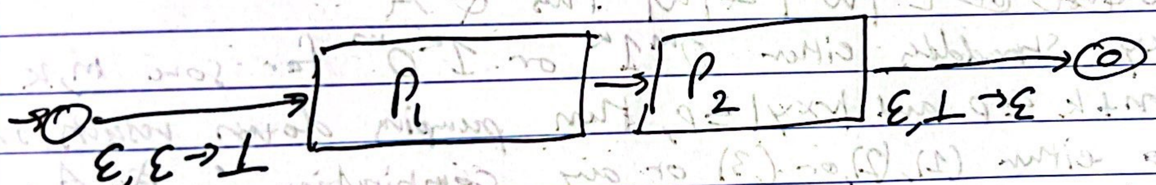
$P_1 = (Q, \Sigma, \Sigma, \delta_1, q_0, F)$ and $P_2 = (Q, \Sigma, \Sigma, \delta_2, q_0, F)$

~~for every transition in D,~~
~~$\delta(q,\sigma) \Rightarrow 0, \varepsilon \to 0 \in \delta_1$ i.e. pushing on a 0 for every~~

where we transform every transition in D over Σ s.t. for a transition over $\sigma \in \Sigma$ in D, we set the transition ~~[illegible]~~ to $0, \varepsilon \to 0$ if $\sigma = 0$ else $1, \varepsilon \to \varepsilon$ in P_1

Then, similarly for P_2, for every transition over $\sigma \in \Sigma$ in D, we set $0, 0 \to \varepsilon$ if $\sigma = 0$ else $1, \varepsilon \to \varepsilon$ in P_2.

This basically allows P_1 to track #0s & P_2 to dump that # of 0s. Then we ~~[illegible]~~ consider the string wt where $w \overset{\circ}{=} t$ is accepted by the model:



The stack tracking # of 0s allows us to ensure $\#_0(w) = \#_0(t)$ and the states being the same # allows us to track 1s since $\Sigma = \{0, 1\}$ s.t. $|\Sigma| = 2$. Thus since we can ~~determine~~ recognize SCRAMBLE(A), ~~this ones~~ this lang is context free

2.43b) If $|\Sigma| \geq 3$, wlog. $\Sigma = \{0, 1, 2\}$ then ~~[illegible]~~ SCRAMBLE(A) would NOT be context-free. ~~This~~ We can show this by ~~[illegible]~~ observing that strings of the form $0^n 1^n 2^n$ are a subset of SCRAMBLE(A) for a lang. A over $\Sigma = \{1, 2, 3\}$. We know $0^n 1^n 2^n$ to be not context-free, thus $\nexists$ a PDA that accepts $\therefore \nexists$ a PDA that recognizes SCRAMBLE(A) as for it to recognize, the PDA for the lang must also recognize strings of $0^n 1^n 2^n$ which is not possible.

2.45) $A = \{wtw^R \mid w, t \in \{0,1\}^*, |w| = |t|\}$

Conceptually, we know to check $|w| = |t|$ requires writing and reading from the stack then to check that w^R is indeed the reverse of ~~w~~ w, requires reading the stack precisely after writing w to the stack, which is not possible due to us checking $|w| \overset{?}{=} |t|$ "dirtying" the stack.

Using the P.L., consider $a = wtw^R = uvxyz = 0^p 1^p 0^p \in A$.

Then pumping has a few cases:

(1) ~~If vxy ∈ 0^p, then pumping down~~ if $vxy \in w$, then pumping down results in ~~~~ $|w| \neq |t| \therefore \notin A$

(2) if $vxy \in t$, the same as (1) occurs

(3) if $vxy \in w^R$, then pumping down results in ~~w^R not being the reverse of w~~ $|w^R| \neq |w|$ thus $\notin A$

(4) if vxy straddles either $0^m 1^k$ or $1^k 0^m$ for some m, k s.t. $m + k \leq p$ and $|vxy| \leq p$, then pumping down results in ~~~~ either (1), (2), or (3) or any combination $\therefore \notin A$

Therefore, A is not context-free by the P.L.

2.38) $L = \text{Shuffle}(A, B) = \{w \mid w = a_1 b_1 \cdots a_k b_k, a_1 \ldots a_k \in A, b_1 \ldots b_k \in B, a_i, b_i \in \Sigma^*\}$

Consider $A = \{a^n b^n : n \geq 0\}$, $B = \{c^m d^m : m \geq 0\}$, then

$L = \{w \mid a_1 c_1 \ldots a_n c_m b_1 d_1 \ldots b_n d_m = w\}$.

Then consider ~~a string~~ the string: $w = uvxyz = a_1 c_1 \ldots a_p c_p b_1 d_1 \ldots b_p d_p$

Then we have a few cases of vxy:

(1) $vxy \in (ac)^p$ or $vxy \in (bd)^p$ then pumping down results in a string s.t. $\#_a(w) \neq \#_b(w)$ or $\#_c(w) \neq \#_d(w) \therefore \notin L$

(2) $vxy \in (ac)^m (bd)^k$ for some m, k s.t. $m + k \leq p$ (i.e. vxy straddles $acbd$, then pumping down results in a string for which $\#a(w) \neq \#_b(w)$ or $\#_c(w) \neq \#_d(w) \therefore \notin L$

Therefore, L is not a CFL by P.L.