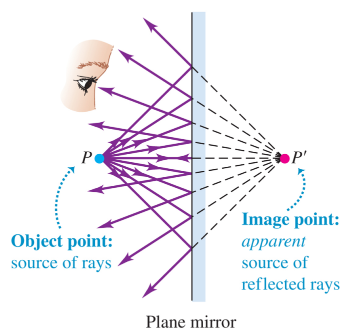
Chapter 34: Geometric Optics

Reflection at a Plane Surface

- An object that emits light rays but has no physical extent is called a **point object.**
  - These are the simplest objects we can consider when doing geometric optics.
  - Light rays from a point object at point P are reflected from a plane mirror.
  - The reflecting surface forms an **image** of point P, and the reflected rays entering the eye look as though they had come from image point P' behind the mirror.
- If the outgoing rays do not pass through the image point, then it forms a **virtual image.**
- If instead the outgoing rays pass through the image point, then it is **real image.**



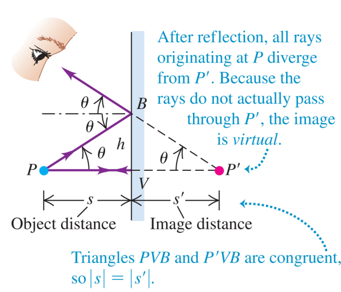
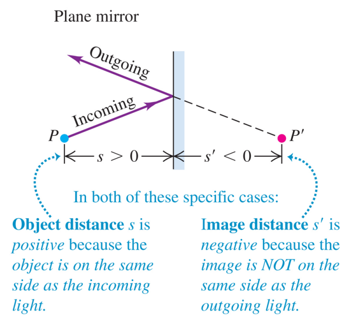


Image Formation by a Plane Mirror (1 of 2)

- We denote the **object distance** to the reflecting surface by s, and the **image distance** to the surface by s'.
- The distances and curvatures for the surfaces obey the following sign rules:
  1. When the object is on the same side of the reflecting or refracting surface as the incoming light, $s > 0$.
  2. When the image is on the same side of the reflecting or refracting surface as the outgoing light, $s' > 0$.
  3. When the center of curvature C is on the same side as the outgoing light, $C > 0$.



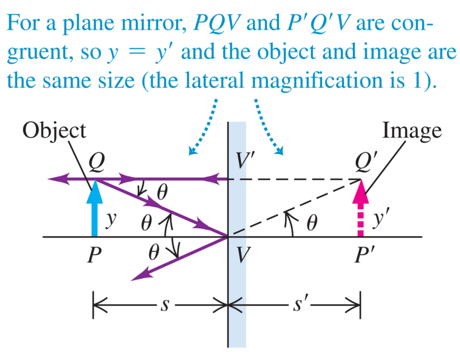
- For a plane mirror, s' is negative because it is not on the same side as the outgoing rays:

$$s = -s' \quad \text{(plane mirror)}.$$

Image Formation by a Plane Mirror (2 of 2)

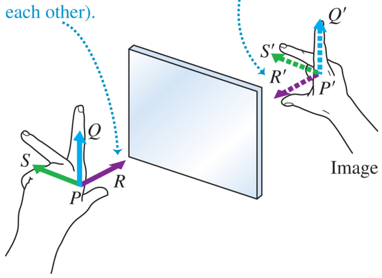
- In a plane mirror, the image is virtual, erect, reversed, and the same size as the object.
- The **lateral magnification** is defined by the object height y and the image height y':

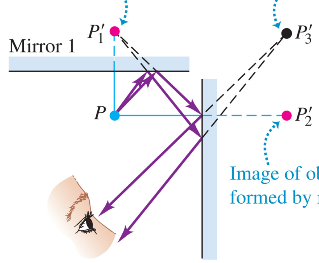
$$m = \frac{y'}{y}.$$



Characteristics of the Image from a Plane Mirror

- The image and object heights are equal for plane mirrors, so $y = y'$, and $m = +1$.
- Since the image formed by a plane mirror is reversed, the image of a right hand becomes a left hand.
- An image formed by one surface or device can also serve as the object for a second surface or device.

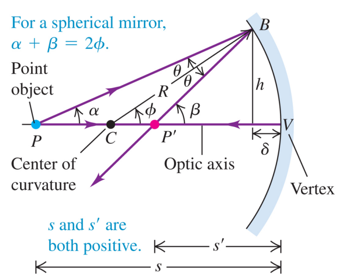




Reflection at a Spherical Surface (1 of 2)

- A spherical mirror with radius of curvature R forms a real image P' of the point object P.
- To locate the real image P', we must consider the triangles formed by the point object P, the image point P', the center of curvature C, the height h of point B where the rays from the point object bounce off of the mirror, and the horizontal distance δ from the vertex of the mirror to point B. The object distance from the vertex V is s, and the image distance from V is s'.

Construction for finding the position P' of an image formed by a concave spherical mirror



- We start by noting that the angles α, ϕ, β, and θ satisfy

$$\phi = \alpha + \theta, \quad \beta = \phi + \theta, \quad \rightarrow \quad \alpha + \beta = 2\phi.$$

Reflection at a Spherical Surface (2 of 2)

- The tangents of α, β, and ϕ give us

$$\tan\alpha = \frac{h}{s-\delta}, \quad \tan\beta = \frac{h}{s'-\delta}, \quad \tan\phi = \frac{h}{R-\delta}.$$

- To proceed, we make the following approximations:
  - The angle α is small, so β and ϕ are small as well, and we approximate their tangents using $\tan x \approx x$.
  - If α is small, then δ is small as well and $y - \delta \approx y$ for s, s', and R.
- The approximations yield

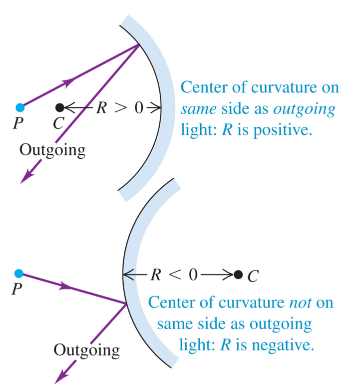
$$\alpha = \frac{h}{s}, \quad \beta = \frac{h}{s'}, \quad \phi = \frac{h}{R}.$$

- Using the fact that $\alpha + \beta = 2\phi$, we obtain the following object-image relationship:

$$\frac{h}{s} + \frac{h}{s'} = \frac{2h}{R} \quad \rightarrow \quad \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \quad \text{(spherical mirror)}.$$

Sign Conventions for Spherical Mirrors

- If the object point P is on the same side as the incident light, $s > 0$.
- If the image point P' is on the same side as the reflected light, $s' > 0$.
- If the center of curvature C is on the same side as the reflected light $R > 0$.



Focal Point and Focal Length (1 of 2)

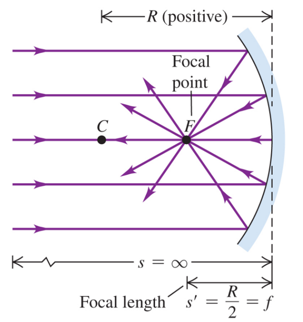
- When the object point P is very far from a spherical mirror ($s \to \infty$), the incoming rays are parallel. Using the object-image relationship, we get

$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \quad \to \quad s' = \frac{R}{2}.$$

- The beam of rays converges to a point F a distance $R/2$ from the vertex of the mirror. We call F the focal point, and the distance from the vertex to the focal point is called the focal length f. For a spherical mirror,

$$f = \frac{R}{2}.$$

All parallel rays incident on a spherical mirror reflect through the focal point.



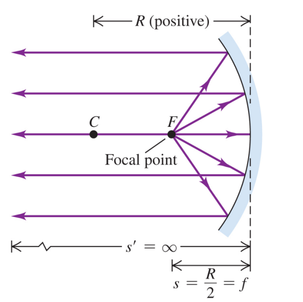
Focal Point and Focal Length (2 of 2)

- For the opposite situation where the object is placed at the focal point F, $s = f = R/2$, and the image distance becomes

$$\frac{2}{R} + \frac{1}{s'} = \frac{2}{R} \quad \to \quad \frac{1}{s'} = 0 \quad \to \quad s' = \infty.$$

- Thus, the reflected rays are parallel to the optic axis, and the image is formed at infinity.

Rays diverging from the focal point reflect to form parallel outgoing rays.



- The focal point F has the following properties:
  - Any incoming ray parallel to the optic axis is reflected through the focal point.
  - Any incoming ray that passes through the focal point is reflected parallel to the optic axis.
- We can also rewrite the object-image relationship using the fact that $f = R/2$:

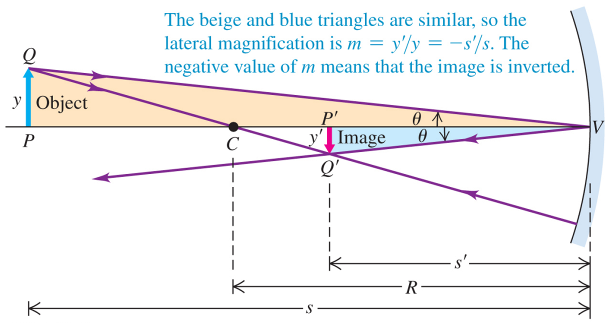
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(spherical mirror)}.$$

Image of an Extended Object: Spherical Mirror

- For a spherical mirror, the object and image heights obey the relationship $y/s = -y'/s'$.
- The lateral magnification is therefore

$$m = \frac{y'}{y} = -\frac{s'}{s} \quad \text{(spherical mirror)}.$$

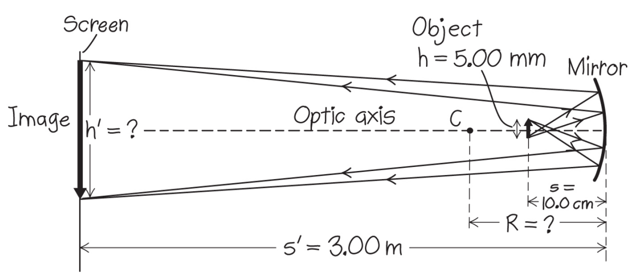
- If $m > 0$, the image is erect in comparison to the object.
- If $m < 0$, the image is inverted relative to the object.



10/44

Example 34.1: Image Formation by a Concave Mirror

A concave mirror forms an image, on a wall 3.00 m in front of the mirror, of a headlamp filament 10.0 cm in front of the mirror. (a) What are the radius of curvature and focal length of the mirror? (b) What is the lateral magnification? What is the image height if the object height is 5.00 mm?



(a) The object and the image are on the concave side of the mirror, so s and s' are positive. We have $s = 10.0$ cm and $s' = 300$ cm, and so

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \quad \rightarrow \quad R = 2\left(\frac{1}{10.0\text{ cm}} + \frac{1}{300\text{ cm}}\right)^{-1} = 19.4\text{ cm}.$$

The focal length is then

$$f = \frac{R}{2} = 9.68\text{ cm}.$$

(b) The lateral magnification is

$$m = -\frac{s'}{s} = -\frac{300 \text{ cm}}{10.0 \text{ cm}} = -30.0,$$

which means the image is inverted, and the height of the image is 30.0 times the height of the object:

$$y' = (-30.0)(5.00 \text{ mm}) = -150 \text{ mm}$$

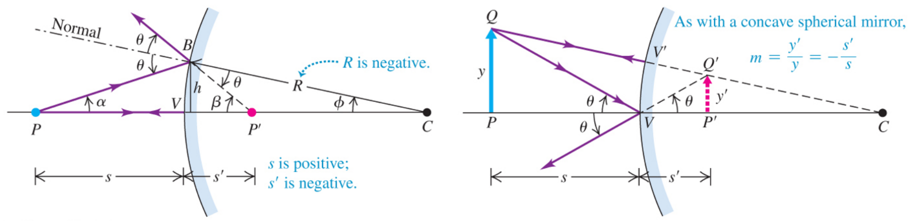
Convex Mirrors (1 of 2)

- If a mirror is convex, so that R is negative, the resulting image is virtual, erect, and smaller than the object.
- The same expressions for the object-image relationship and lateral magnification still hold:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}, \quad m = \frac{y'}{y} = -\frac{s'}{s}$$

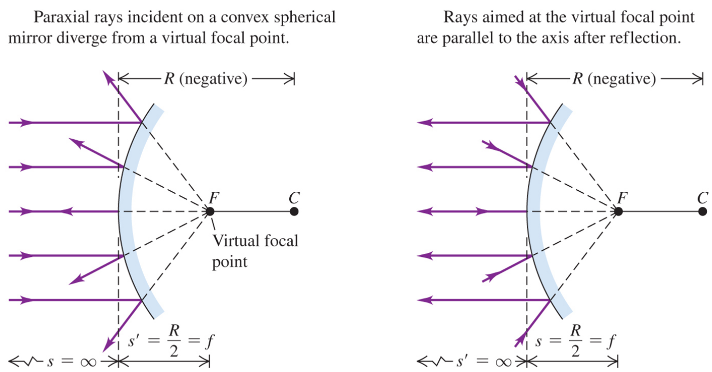
Construction for finding the position of an image formed by a convex mirror

Construction for finding the magnification of an image formed by a convex mirror



Convex Mirrors (2 of 2)

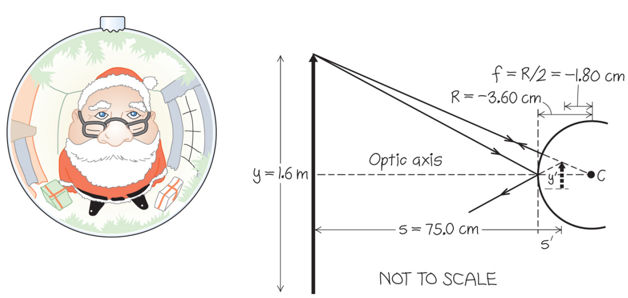
- When incoming rays that are parallel to the optic axis are reflected from a convex mirror, they diverge as though they had come from the **virtual focal point** F at a distance f behind the mirror.
- The corresponding image distance s' is negative, so both f and R are negative.
- When the incoming rays are converging as though they would meet at the virtual focal point F, then they are reflected parallel to the optic axis.



12/44

Example 34.3: Santa's Image Problem

Santa checks himself for soot, using his reflection in a silvered Christmas tree ornament 0.750 m away. The diameter of the ornament is 7.20 cm. Standard reference texts state that he is a "right jolly old elf," so we estimate his height to be 1.6 m. Where and how tall is the image of Santa formed by the ornament? Is it erect or inverted?



The radius of the mirror and focal length are

$$R = -\frac{(7.20 \text{ cm})}{2} = -3.60 \text{ cm}, \quad f = \frac{R}{2} = -\frac{3.60 \text{ cm}}{2} = -1.80 \text{ cm}.$$

Then the image distance s' is

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} \quad \rightarrow \quad s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = \left(\frac{1}{-1.80 \text{ cm}} - \frac{1}{75.0 \text{ cm}}\right)^{-1} = -1.76 \text{ cm}$$

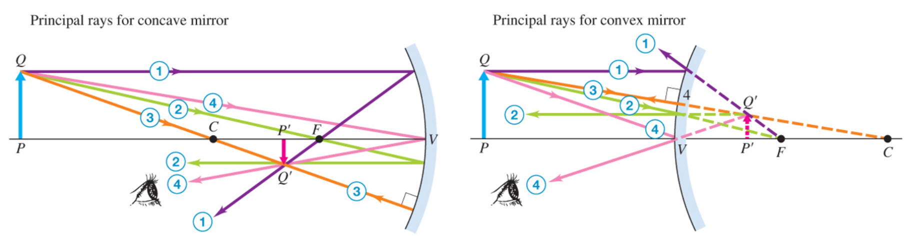
Since s' is negative the image is behind the mirror, and is therefore virtual.

We can find the height of the image y' by determining the lateral magnification:

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{(-1.76 \text{ cm})}{75.0 \text{ cm}} = 0.0234 \quad \to \quad y' = my = (0.0234)(1.6 \text{ m}) = 3.8 \times 10^{-2} \text{ m} = 3.8 \text{ cm}.$$

Graphical Methods for Mirrors

- The properties of an image can be determined using graphical methods.
- There are four rays that can be drawn for spherical mirrors, which are called **principal rays:**



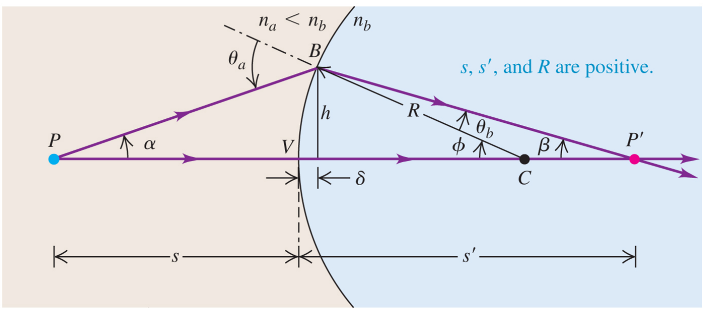
1 Ray parallel to axis reflects through focal point.
2 Ray through focal point reflects parallel to axis.
3 Ray through center of curvature intersects the surface normally and reflects along its original path.
4 Ray to vertex reflects symmetrically around optic axis.

1 Reflected parallel ray appears to come from focal point.
2 Ray toward focal point reflects parallel to axis.
3 As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
4 As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

Refraction at a Spherical Surface (1 of 3)

- Suppose we have a spherical surface of radius R that forms the interface between two materials with different indexes of refraction n_a and n_b. The surface forms an image P' of an object at P.
- Obtaining the object-image relationship is similar to how it was done for a spherical mirror, but now we need to take Snell's law into account because of refraction.
- First, we start by noting that the angles θ_a, θ_b, α, β, and ϕ are all related by

$$\theta_a = \alpha + \phi, \quad \phi = \beta + \theta_b.$$



Refraction at a Spherical Surface (2 of 3)

- Snell's law tells us that

$$n_a \sin \theta_a = n_b \sin \theta_b.$$

- The tangents of α, β, and ϕ are

$$\tan \alpha = \frac{h}{s + \delta}, \quad \tan \beta = \frac{h}{s' - \delta}, \quad \tan \phi = \frac{h}{R - \delta}.$$

- We make the approximation that θ_a and θ_b are small so that $\sin \theta \approx \theta$:

$$n_a \theta_a = n_b \theta_b.$$

- Substituting this result into the expressions for θ_a and ϕ gives us

$$\theta_b = \frac{n_a}{n_b}(\alpha + \phi) \quad \rightarrow \quad \phi = \beta + \frac{n_a}{n_b}(\alpha + \phi) \quad \rightarrow \quad n_a \alpha + n_b \beta = (n_b - n_a)\phi.$$

Refraction at a Spherical Surface (3 of 3)

- Now we use the approximation that $\tan x \approx x$ for α, β, and ϕ, along with the fact that δ is small compared to s, s', and R, which gives us

$$\alpha = \frac{h}{s}, \quad \beta = \frac{h}{s'}, \quad \phi = \frac{h}{R}.$$

- These can be substituted into the previous expression to get the object-image relationship:

$$n_a \frac{h}{s} + n_b \frac{h}{s'} = (n_b - n_a)\frac{h}{R}$$
$$\rightarrow \quad \frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \quad \text{(spherical refracting surface).}$$

- This result applies to both convex and concave refracting surfaces, provided the sign rules are applied consistently.

Image of an Extended Object: Spherical Refracting Surface (1 of 2)

- To obtain the lateral magnification, we first note that the ray diagram tells us that the angles θ_a and θ_b and the object and image heights are related by

$$\tan\theta_a = \frac{y}{s}, \quad \tan\theta_b = \frac{-y'}{s'}.$$

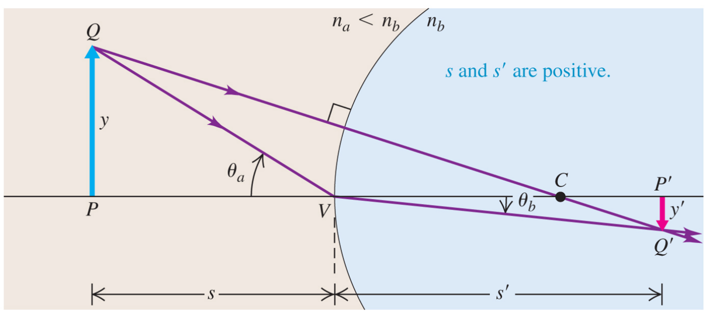


Image of an Extended Object: Spherical Refracting Surface (2 of 2)

- We can use Snell's law and the fact that $\sin\theta \approx \tan\theta$ for small θ to get

$$n_a \sin\theta_a = n_b \sin\theta_b \quad \rightarrow \quad n_a \tan\theta_a = n_b \tan\theta_b \quad \rightarrow \quad \frac{n_a y}{s} = -\frac{n_b y'}{s'}.$$

- The lateral magnification is therefore

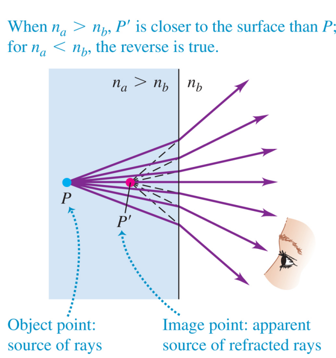
$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s} \quad \text{(spherical refracting surface)}.$$

Refraction at a Plane Surface

- We can use the object-image relationship for refraction at a spherical surface to obtain the object-image relationship for a plane refracting surface.
- A plane refracting surface is a spherical refracting surface with an infinite radius of curvature. In the limit $R \to \infty$, we get

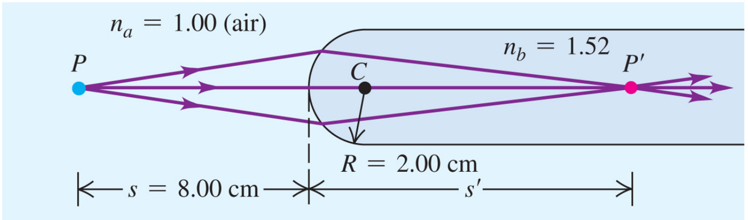
$$\frac{n_b - n_a}{R} \to 0, \quad \frac{n_a}{s} + \frac{n_b}{s'} = 0 \quad \text{(plane refracting surface)}.$$

- In this case, the magnification simply becomes $m = 1$.



Example 34.5: Image Formation by Refraction I

A cylindrical glass rod has index of refraction 1.52, and it is surrounded by air. One end is ground to a hemispherical surface with radius $R = 2.00$ cm, and a small object is placed on the axis of the rod, 8.00 cm left of the vertex. (a) Find the image distance. (b) Find the lateral magnification.



(a) We can use the object-image relationship for a spherical refracting surface to get

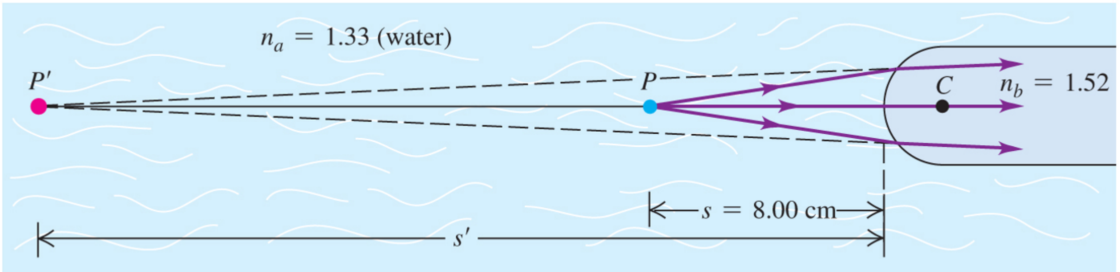
$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \quad \rightarrow \quad s' = n_b\left(\frac{n_b - n_a}{R} - \frac{n_a}{s}\right)^{-1} = (1.52)\left(\frac{1.52 - 1.00}{2.00 \text{ cm}} - \frac{1.00}{8.00 \text{ cm}}\right)^{-1} = 11.3 \text{ cm}.$$

(b) The lateral magnification is

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(11.3 \text{ cm})}{(1.52)(8.00 \text{ cm})} = -0.929.$$

Example 34.6: Image Formation by Refraction II

The glass rod of the previous example is immersed in water, which has index of refraction $n = 1.33$. The object distance is again 8.00 cm. Find the image distance and lateral magnification.



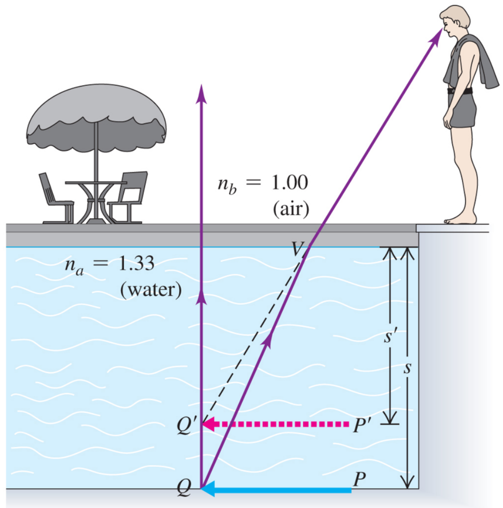
Now we have

$$s' = n_b\left(\frac{n_b - n_a}{R} - \frac{n_a}{s}\right)^{-1} = (1.52)\left(\frac{1.52 - 1.33}{2.00 \text{ cm}} - \frac{1.33}{8.00 \text{ cm}}\right)^{-1} = -21.3 \text{ cm}, \quad \rightarrow \quad m = -\frac{n_a s'}{n_b s} = 2.33.$$

In this case, we obtain a virtual image rather than a real one as in the previous example.

Example 34.7: Apparent Depth of a Swimming Pool

If you look straight down into a swimming pool where it is 2.00 m deep, how deep does it appear to be?



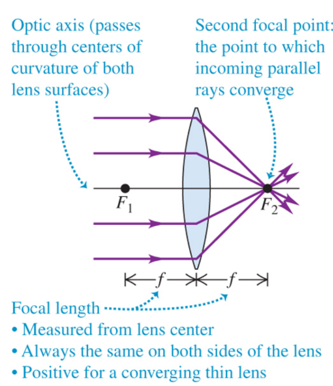
We can use the object-image relationship for a plane refracting surface:

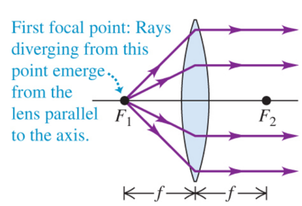
$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \quad \rightarrow \quad s' = -\frac{n_b s}{n_a} = -\frac{(1.00)(2.00 \text{ m})}{(1.33)} = -1.50 \text{ m}$$

The image distance is negative, which means the image is virtual. Furthermore, the apparent depth of the pool is 1.50 m, which is just 75% of its actual depth.

Thin Converging Lens

- A **lens** is an optical system with two refracting surfaces.
- The simplest lens has two spherical surfaces close enough together that we can ignore the distance between them, which is called a **thin lens.**
- Rays passing through the first focal point F_1 emerge from a **converging lens** as a beam of parallel rays.
- As with other optical systems, converging lenses have a focal length f.





20/44

Image Formed by a Thin Converging Lens (1 of 2)

- By considering the object and image heights y and y', we can obtain a relationship between them and the object and image distances s and s'.
- The triangles formed by the heights and distances are congruent, and so we obtain

$$\frac{y}{s} = -\frac{y'}{s'} \quad \rightarrow \quad \frac{y'}{y} = -\frac{s'}{s}.$$

- The two angles labeled β are equal, which means the triangle formed by y and f is congruent to the triangle formed by y' and $s' - f$:

$$\frac{y}{f} = -\frac{y'}{s' - f} \quad \rightarrow \quad \frac{y'}{y} = -\frac{s' - f}{f}.$$

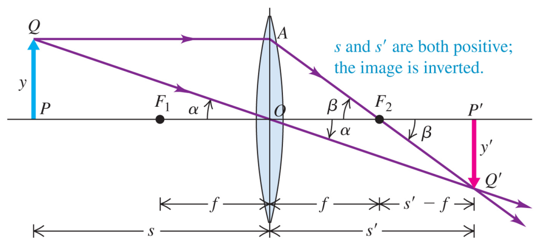


Image Formed by a Thin Converging Lens (2 of 2)

- Therefore, the object-image relationship is

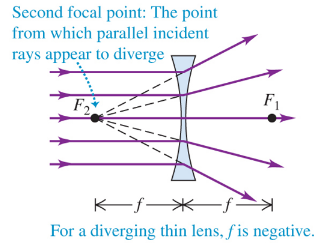
$$\frac{s'}{s} = \frac{s' - f}{f} \quad \rightarrow \quad \frac{s'}{s} + \frac{s'}{s'} = \frac{s'}{f} \quad \rightarrow \quad \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(thin lens).}$$

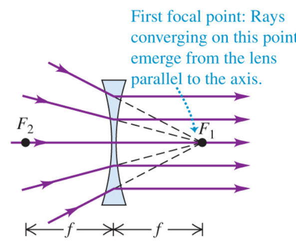
- Correspondingly, the lateral magnification is

$$m = \frac{y'}{y} = -\frac{s'}{s}.$$

Thin Diverging Lens

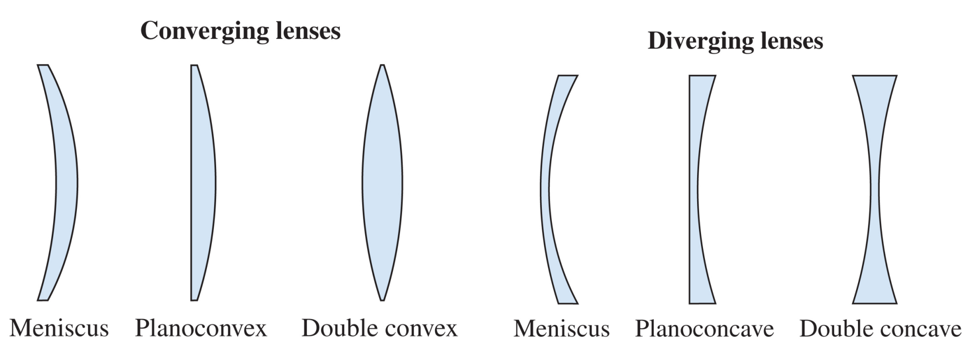
- When a beam of parallel rays is incident on a **diverging lens,** the rays diverge after refraction.
- The focal length f of a diverging lens is a negative quantity, and the lens is also called a **negative lens.**
- Incident rays converging toward the first focal point F_1 of a diverging lens emerge from the lens parallel to its axis.
- The object-image relationship and lateral magnification apply to both converging and diverging lenses.





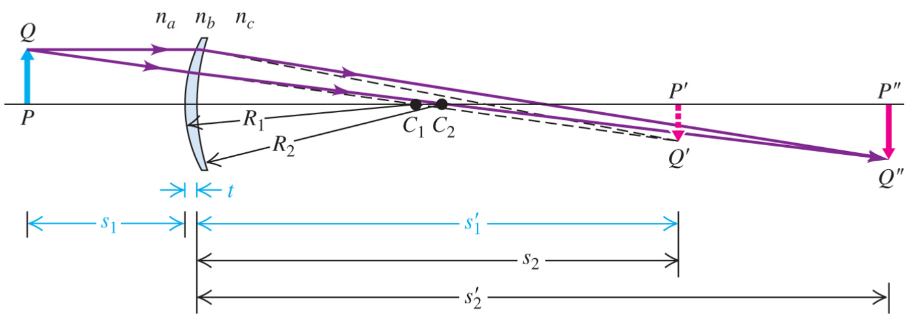
Types of Lenses

- Shown below are various types of lenses, both converging and diverging.
- Any lens that is thicker at its center than at its edges is a **converging** lens with positive focal length f.
- Any lens that is thicker at its edges than at its center is a **diverging** lens with negative focal length f.



The Lensmaker's Equation (1 of 3)

- We can obtain a relationship between the focal length f, index of refraction n, and radii of curvature R_1 and R_2 for the lens surfaces of a thin lens. We will do this by considering rays originating from a material with index of refraction n_a that passes through a lens with n_b, then refracted into a third material with n_c.
- For the first surface, the object and image distances are s_1 and s_1'. The second surface instead has s_2 and s_2'. If we ignore the thickness t, then s_1' and s_2 have equal magnitude and opposite sign: $s_2 = -s_1'$.



The Lensmaker's Equation (2 of 3)

- Using the equation for refraction at a spherical surface for both sides of the lens, we get

$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}, \quad \frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}.$$

- Assuming the lens is in air, $n_a = n_c = 1$, and the index of refraction for the lens n_b will be denoted by n for simplicity. Using the fact that $s_2 = -s_1'$, we have

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}, \quad -\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}.$$

- Adding the two equations together yields

$$\frac{1}{s_1} + \frac{1}{s_2'} = \frac{n-1}{R_1} + \frac{1-n}{R_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

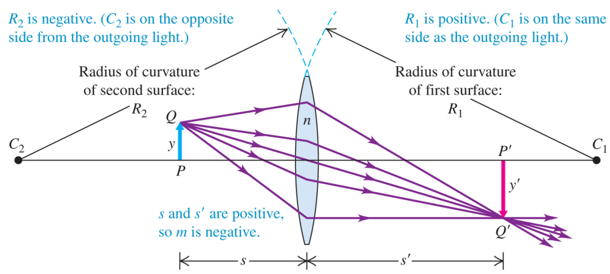
The Lensmaker's Equation (3 of 3)

- Since the lens is a single unit, we may instead refer to the object distance s_1 as s, and the image distance s_2' as s' to get

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

- The left-hand side of the previous expression is equal to f, from which we arrive at the lensmaker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$



27/44

Example 34.8: Determining the Focal Length of a Lens

(a) Suppose the absolute values of the radii of curvature of the lens surfaces of a double convex thin lens are $R_1 = R_2 = 10$ cm and the index of refraction for the lens is $n = 1.52$. What is the focal length f of the lens? (b) Suppose a double concave lens also has $n = 1.52$ and the absolute values of the radii of curvature of its lens surfaces are also both equal to 10 cm. What is the focal length of this lens?

(a) For a double convex lens, the center of curvature of the first surface C_1 is on the outgoing side of the lens, so R_1 is positive. The center of curvature for the second surface C_2 is on the incoming side, which means R_2 is negative. Thus, $R_1 = +10$ cm and $R_2 = -10$ cm. We have

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \rightarrow \quad f = \left[(1.52-1)\left(\frac{1}{10\text{ cm}} - \frac{1}{-10\text{ cm}}\right)\right]^{-1} = 9.6\text{ cm}.$$

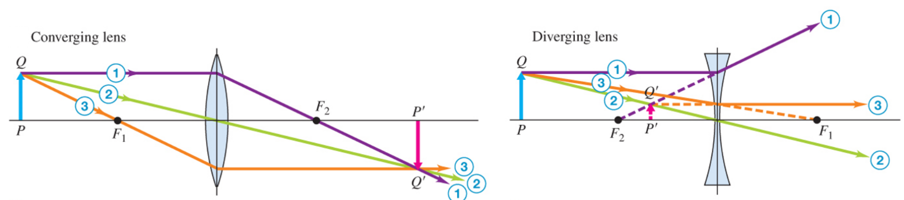
(b) In a double concave lens, the centers of curvature are on the sides opposite to that of a double convex lens, so the signs for both R_1 and R_2 are flipped, and we get

$$f = \left[(1.52-1)\left(\frac{1}{-10\text{ cm}} - \frac{1}{10\text{ cm}}\right)\right]^{-1} = -9.6\text{ cm}.$$

Notice that the resulting focal length is simply the negative of the focal length for the double convex lens.

Graphical Methods for Lenses

- As with spherical mirrors, we can use graphical methods to determine the properties of an image formed by a thin lens.
- In this case, there are only three principal rays to consider:



(1) Parallel incident ray refracts to pass through second focal point F_2.
(2) Ray through center of lens does not deviate appreciably.
(3) Ray through the first focal point F_1 emerges parallel to the axis.

(1) Parallel incident ray appears after refraction to have come from the second focal point F_2.
(2) Ray through center of lens does not deviate appreciably.
(3) Ray aimed at the first focal point F_1 emerges parallel to the axis.

28/44

Example 34.10: Image Formation by a Diverging Lens

A beam of parallel rays spreads out after passing through a thin diverging lens, as if the rays came from a point 20.0 cm from the center of the lens. You want to use this lens to form an erect, virtual image that is 1/3 the height of the object. (a) Where should the object be placed? Where will the image be? (b) Draw a principal ray diagram.

(a) We wish to have $m = 1/3$, and so

$$m = \frac{1}{3} = -\frac{s'}{s} \quad \rightarrow \quad s' = -s/3.$$

In this case, the focal length is $f = -20.0$ cm, so the image-object relationship gives us

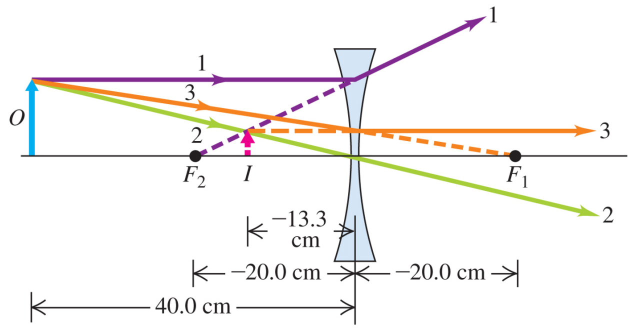
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{-s/3} = \frac{1}{s} - \frac{3}{s} = -\frac{2}{s} = \frac{1}{f} \quad \rightarrow \quad s = -2f = -2(-20.0 \text{ cm}) = 40.0 \text{ cm}.$$

Since the object distance is 40.0 cm, the image distance is

$$s' = -\frac{s}{3} = -\frac{40.0 \text{ cm}}{3} = -13.3 \text{ cm}.$$

Since s' is negative, the object and image are on the same side of the lens.

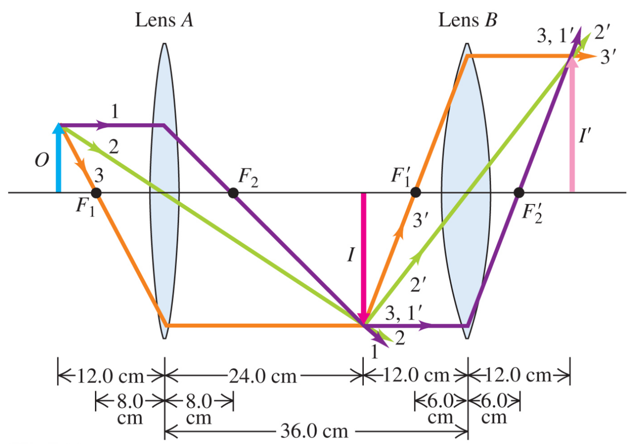
(b) The principal ray diagram looks as follows:



Example 34.11: An Image of an Image

Converging lenses A and B, of focal lengths 8.0 cm and 6.0 cm, respectively, are placed 36.0 cm apart. Both lenses have the same optic axis. An object 8.0 cm high is placed 12.0 cm to the left of lens A. Find the position, size, and orientation of the image produced by the lenses in combination.

The principal ray diagram looks as follows:



To begin, we must find the position and size of the first image I formed by lens A. We shall call this image distance $s'_{I,A}$, which gives us

$$\frac{1}{s}+\frac{1}{s'_{I,A}}=\frac{1}{f_A} \quad\rightarrow\quad s'_{I,A}=\left(\frac{1}{f_A}-\frac{1}{s}\right)^{-1}=\left(\frac{1}{8.0\text{ cm}}-\frac{1}{12.0\text{ cm}}\right)^{-1}=24.0\text{ cm}.$$

We find that the image I is 24.0 cm to the right of lens A, so the lateral magnification is

$$m_A=-\frac{s'_{I,A}}{s}=-\frac{24.0\text{ cm}}{12.0\text{ cm}}=-2.00,$$

which means I is inverted and twice as tall as the object O.

Now we must use the image I as the object for lens B. Since I is 36.0 cm $-$ 24.0 cm $=$ 12.0 cm to the left of lens B, so the object distance in this case is $s_{I,B}=12.0$ cm. Applying the image-object relationship again, we find that the final image distance $s'_{I',B}$ is

$$\frac{1}{s_{I,B}}+\frac{1}{s'_{I',B}}=\frac{1}{f_B} \quad\rightarrow\quad s'_{I',B}=\left(\frac{1}{f_B}-\frac{1}{s_{I,B}}\right)^{-1}=\left(\frac{1}{6.0\text{ cm}}-\frac{1}{12.0\text{ cm}}\right)^{-1}=12.0\text{ cm}.$$

The final image I' is 12.0 cm to the right of lens B, and the magnification is

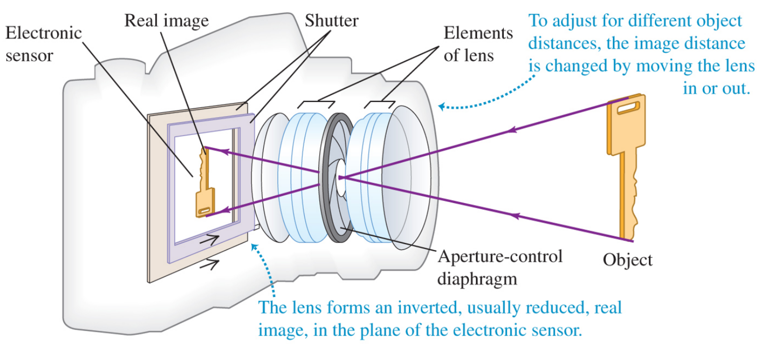
$$m_B = -\frac{s'_{I',B}}{s_{I,B}} = -\frac{12.0\text{ cm}}{12.0\text{ cm}} = -1.00.$$

The overall magnification of the lens system is the product of the two individual magnifications m_A and m_B, which gives us

$$m = m_A m_B = (-2.00)(-1.00) = 2.00.$$

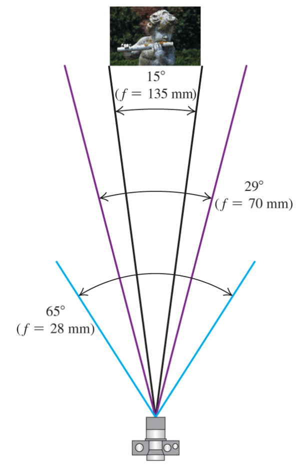
Cameras (1 of 4)

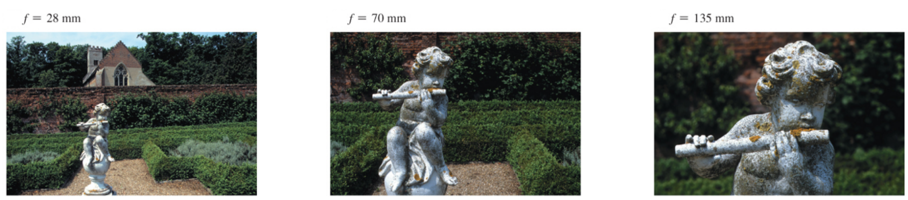
- A **camera** consists of a light-tight box, a converging lens, a shutter to open the lens, and a light-sensitive recording medium.
- When a camera is in proper focus, the position of the recording medium coincides with the position of the real image formed by the lens.
- To control the amount of light energy per unit area reaching the recording medium, a shutter and lens aperture control the exposure of the camera. A typical exposure time ranges from 1 s to 1 ms.



Cameras (2 of 4)

- The choice of focal length f for a camera depends on the size of the recording medium and the desired angle of view.
- A lens with a long focal length, called a *telephoto* lens, gives a narrow angle of view and a large image of a distant object.
- A lens with a short focal length gives a small image and a wide angle of view, and is called a *wide-angle lens.*

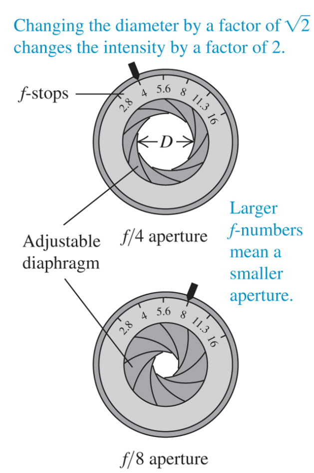




Cameras (3 of 4)

- The focal length f of a camera lens is the distance from the lens to the image when the object is infinitely far away.
- The effective area of the lens is controlled by means of an adjustable lens aperture, or diaphragm, a nearly circular hole with diameter D.
- Photographers commonly express the light-gathering capability of a lens in terms of the ratio f/D, called the ***f*-number** of the lens:

$$f\text{-number of a lens} = \frac{f}{D}.$$

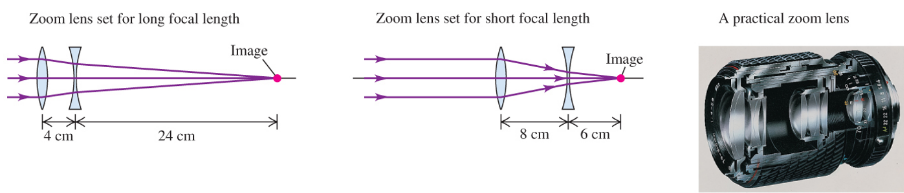


- Since the intensity of the light reaching the recording medium is proportional to D^2/f^2, the intensity is inversely proportional to the square of the f-number:

$$I \propto \frac{1}{(f\text{-number})^2}.$$

Cameras (4 of 4)

- Many cameras make use of a zoom lens, which consists of a complex collection of lens elements that allow for a continuously variable focal length f.
- A simple zoom lens uses a converging lens and a diverging lens in tandem.
  - When the two lenses are close together, the combination behaves like a single lens of long focal length.
  - If the two lenses are moved further apart, the combination behaves like a single lens of short focal length.



32/44

Example 34.12: Photographic Exposures

A telephoto lens for a 35 mm film camera has a focal length of 200 mm. The f-stops for the camera range from $f/2.8$ to $f/22$. (a) What is the corresponding range of aperture diameters? (b) What is the corresponding range of intensities on the film?

(a) The diameter ranges from

$$D = \frac{f}{f\text{-number}} = \frac{200\text{ mm}}{2.8} = 71\text{ mm}$$

to

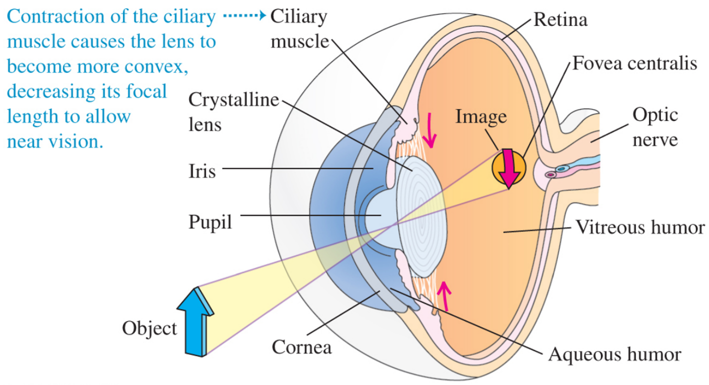
$$D = \frac{200\text{ mm}}{22} = 9.1\text{ mm}.$$

(b) Since the intensity is proportional to D^2, the ratio of the intensity at $f/2.8$ to $f/22$ is

$$\left(\frac{71\text{ mm}}{9.1\text{ mm}}\right)^2 = \left(\frac{22}{2.8}\right)^2 = 62.$$

The Eye (1 of 2)

- The optical behavior of the eye is similar to that of a camera.
- The front portion of the eye is a tough, transparent membrane called the *cornea.* Eyes contain a *crystalline lens* that has an average index of refraction of 1.437, with the *aqueous humor* in front of the lens and the *vitreous humor* behind the lens—both of which have indexes of refraction of 1.336.

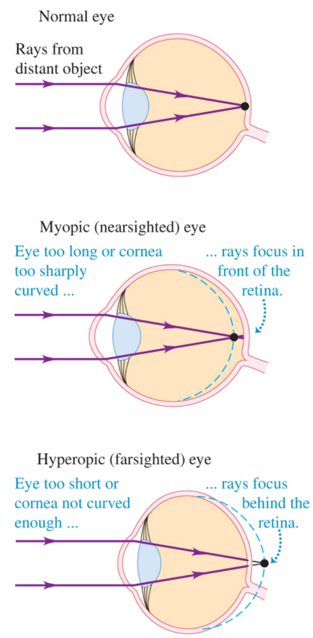


The Eye (2 of 2)

- Refraction at the cornea and surfaces of the lens produce a real image of the object that forms on the *retina.* The retina is analogous to the recording medium in a camera.
- In front of the lens is the *iris,* which has a variable diameter aperture called the *pupil.* The pupil can expand or contact to control light intensity.
- For an object to be seen sharply, the image must be formed exactly at the location of the retina. The eye adjusts to different object distances s by changing the focal length f of the lens, as the lens-to-retina distance s' does not change.
- The focal length f of the lens changes based on the tension in the *ciliary muscle.* At low muscle tension, when the eye is viewing an object at infinity, the lens has a larger radius of curvature and hence a lower focal length f. For closer objects, the tension in the ciliary muscle increases, which lowers the curvature and decreases f. This process is known as *accommodation.*

Defects of Vision

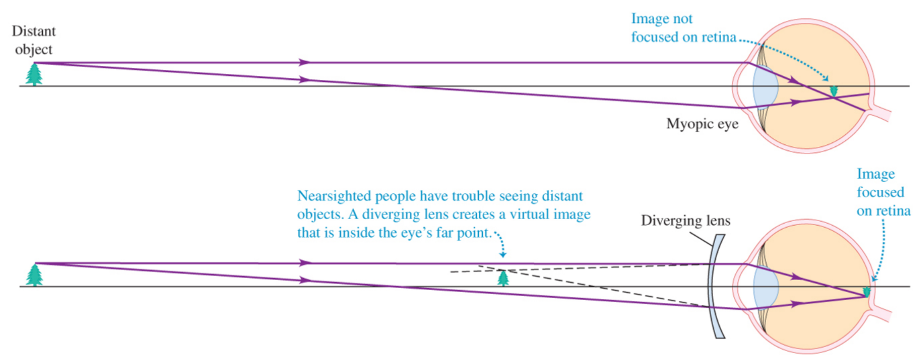
- The extremes of the range over which distinct vision is possible are known as the *far point* and the *near point* of the eye. The far point of an normal eye is at infinity, and the near point is often taken to be at 25 cm, but varies with age.
- A normal eye forms an image on the retina of an object at infinity when the eye is relaxed.
- In the *myopic* (nearsighted) eye, the eyeball is too long from front to back in comparison with the radius of curvature of the cornea, and rays from an object at infinity are focused in front of the retina.
- In a *hyperopic* (farsighted) eye, the eyeball is too short or the cornea is not curved enough, and the image of an infinitely distant object is behind the retina.



35/44

Nearsighted Correction

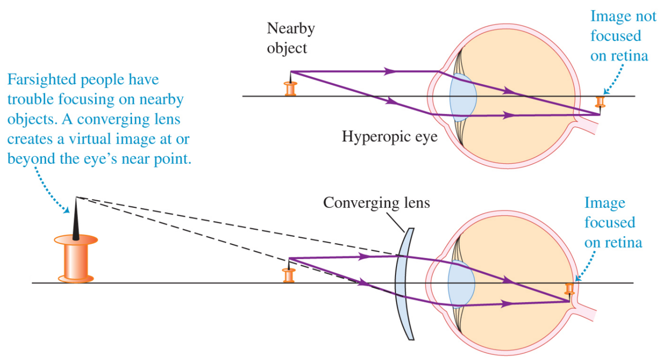
- A diverging lens can be used to create an image closer to the eye than the actual object is so the wearer can see it clearly.



36/44

Farsighted Correction

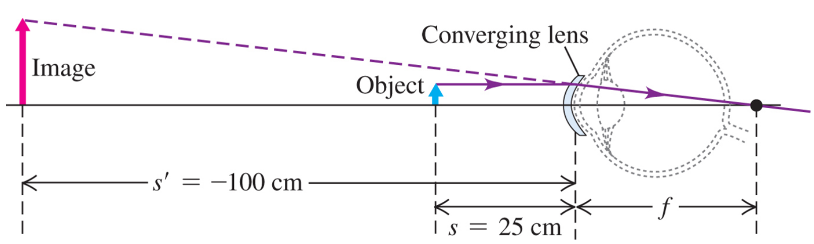
- A converging lens can be used to create an image far enough away from the hyperopic eye at a point where the wearer can see it clearly.



37/44

Example 34.13: Correcting for Farsightedness

The near point of a certain hyperopic eye is 100 cm in front of the eye. Find the focal length and power of the contact lens that will permit the wearer to see clearly an object that is 25 cm in front of the eye.



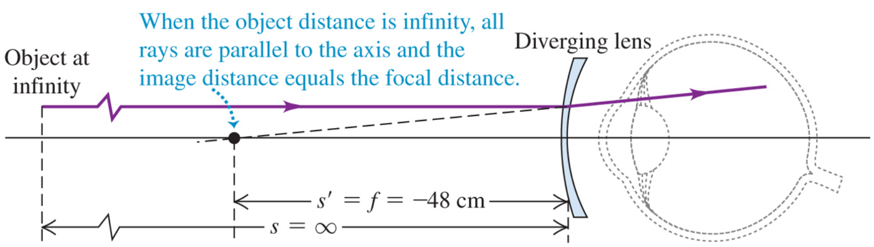
From the image-object relationship, we get

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \rightarrow \quad f = \left(\frac{1}{s} + \frac{1}{s'}\right)^{-1} = \left(\frac{1}{25 \text{ cm}} + \frac{1}{-100 \text{ cm}}\right)^{-1} = 33 \text{ cm}.$$

We therefore need a converging lens with focal length $f = 33$ cm, which corresponds to a power of $1/(0.33 \text{ m}) = 3.0$ diopters.

Example 34.14: Correcting for Nearsightedness

The far point of a certain myopic eye is 50 cm in front of the eye. Find the focal length and power of the eyeglass lens that will permit the wearer to see clearly an object at infinity. Assume that the lens is worn 2 cm in front of the eye.



Using the image-object relationship,

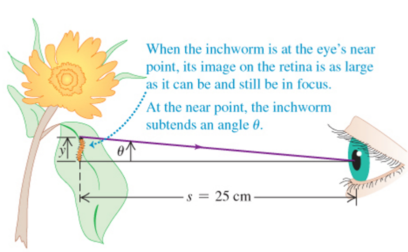
$$f = \left(\frac{1}{s} + \frac{1}{s'}\right)^{-1} = \left(\frac{1}{\infty} + \frac{1}{-48\text{ cm}}\right)^{-1} = -48\text{ cm}.$$

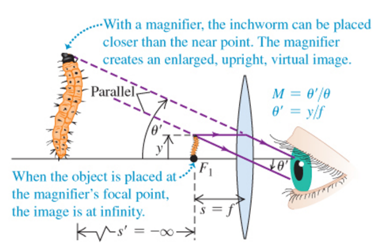
We therefore need a diverging lens with focal length $f = -48$ cm, which corresponds to a power of $1/(-0.48\text{ m}) = -2.1$ diopters.

The Magnifier (1 of 2)

- The apparent size of an object depends on the angle θ subtended by the object at the eye, which is called the **angular size.**
- The maximum angular size of an object viewed at a comfortable distance is the angle θ it subtends at a distance of 25 cm.
- A converging lens can be used to form a virtual image that is larger and further from the eye than the object itself. Such a lens is called a **magnifier.**
- The **angular magnification** is defined as the ratio of the angle θ' subtended by the image of the magnifier to the angle θ subtended without the magnifier:

$$M = \frac{\theta'}{\theta}.$$





38/44

The Magnifier (2 of 2)

- To find M in terms of the focal length f of the magnifier, we approximate the angles as sufficiently small so that for each angle $\sin x \approx \tan x \approx x$. Then θ and θ' in radians are

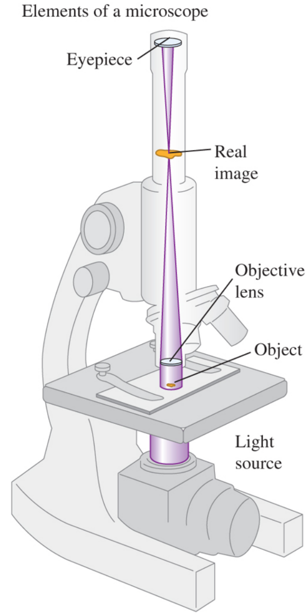
$$\theta = \frac{y}{25\text{ cm}}, \quad \theta' = \frac{y}{f}.$$

- The angular magnification is therefore

$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25\text{ cm}} = \frac{25\text{ cm}}{f}.$$

Microscopes (1 of 2)

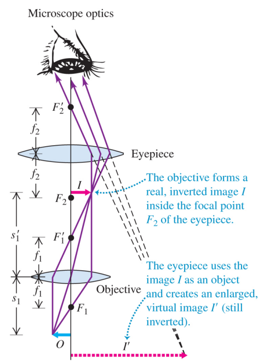
- **Microscopes** use two converging lenses in succession to magnify an image, with the first lens being called the **objective**, which forms a real image. The second lens is a magnifier that is used to make an enlarged, virtual image, called the **eyepiece.**
- The object to be viewed is placed just beyond the first focal point F_1 of the converging objective lens. A real image I is formed just outside the focal point F_2 of the eyepiece, which forms a final virtual image I' of I.



Microscopes (2 of 2)

- The overall magnification M of the microscope is the product of the two magnifications m_1 and M_2 of the lenses.
- The first factor m_1 is the lateral magnification of the objective, which determines the size of the real image I. Since the object is usually very close to the focal point, $s_1 \approx f$:

$$m_1 = -\frac{s_1'}{s_1} = -\frac{s_1'}{f_1}.$$



- The second factor is the angular magnification M_2 of the eyepiece:

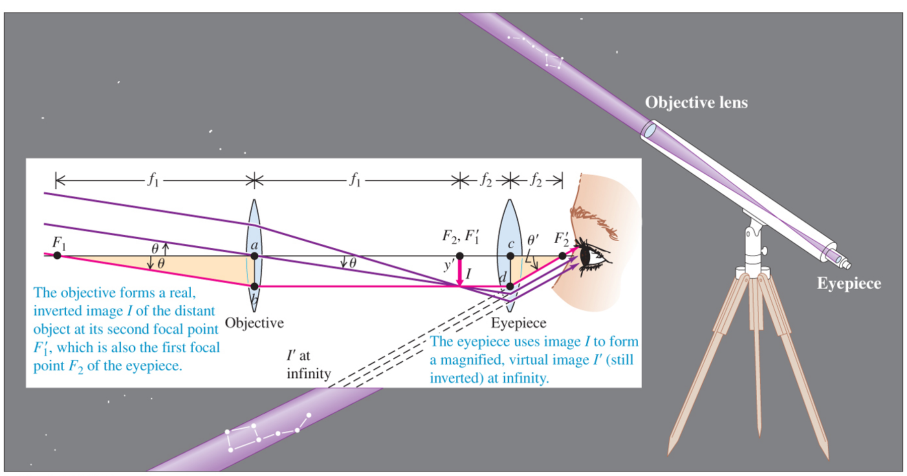
$$M_2 = \frac{25\text{ cm}}{f_2}.$$

- The total magnification is the product of m_1 and M_2, which gives us

$$M = m_1 M_2 = \frac{(25\text{ cm})s_1'}{f_1 f_2}.$$

Telescopes (1 of 3)

- **Telescopes** operate in a manner similar to that of compound microscopes.
- Rays from a distant object form an image I at the focal point of the objective lens, which coincides with the focal length of the eyepiece and forms a final image I' of I at infinity. The distance between the objective and the eyepiece is therefore $f_1 + f_2$.



Telescopes (2 of 3)

- The angles are related to the image heights and focal lengths by

$$\tan\theta = -\frac{y'}{f_1}, \quad \tan\theta' = \frac{y'}{f_2}.$$

- Because the angles θ and θ' are small, we may approximate them as $\tan x \approx x$:

$$\theta = -\frac{y'}{f_1}, \quad \theta' = \frac{y'}{f_2}.$$

- The angular magnification of a telescope is therefore

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2}.$$

Telescopes (3 of 3)

- In a reflecting telescope, the objective lens is replaced by an objective concave mirror.
- Secondary mirrors can also be used to reflect the image from the objective mirror into an eyepiece.
- A cage containing a camera at the focal point of the objective mirror is sometimes used instead of an eyepiece.