# Inductance

## -Mutual Inductance

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B,2}}{dt} = -M\frac{di_1}{dt}$$

$$M = \frac{N_2 \Phi_{B,2}}{i_1} = \frac{N_1 \Phi_{B,1}}{i_2}$$

## -Self inductance

$$L = \frac{N\Phi_B}{i}$$

$$\mathcal{E} = -N\frac{d\Phi_B}{i} = -L\frac{di}{dt}$$

## -Inductor Circuit Element

$$\vec{E}_c + \vec{E}_n = 0$$

$$\mathcal{E} = \int_a^b \vec{E}_n \cdot d\vec{l} = -L\frac{di}{dt}$$

$$V_{ab} = V_a - V_b = L\frac{di}{dt}$$

## -Magnetic Field Energy

$$P_{in} = V_{ab}\, i = L i \frac{di}{dt}$$

$$U_{total} = L\int_0^I i\, di = \frac{1}{2}LI^2$$

## -Magnetic Energy Density

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{2\pi r}$$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{2\pi r} I^2$$

$$u = \frac{U}{2\pi r A} = \frac{\mu_0}{2}\frac{N^2 I^2}{(2\pi r)^2} = \frac{B^2}{2\mu_0}$$

$$\mu = K_m \mu_0 \Rightarrow u = \frac{B^2}{2\mu}$$

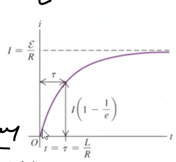
## -Kirchhoff's Loop Rule

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

## -RL Circuit

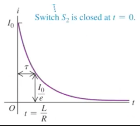
$$i = \frac{\mathcal{E}}{R}\left[1 - e^{-(R/L)t}\right]$$

$$\tau = L/R$$



- Voltage Decay

$$i = I_0 e^{-(R/L)t}$$




## -LC Circuit

| Mass-Spring System | Inductor-Capacitor Circuit |
|---|---|
| Kinetic energy $= \frac{1}{2}mv_x^2$ | Magnetic energy $= \frac{1}{2}Li^2$ |
| Potential energy $= \frac{1}{2}kx^2$ | Electrical energy $= q^2/2C$ |
| $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ | $\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$ |
| $v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$ | $i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$ |
| $v_x = dx/dt$ | $i = dq/dt = -\omega Q\sin(\omega t + \phi)$ |
| $\omega = \sqrt{\frac{k}{m}}$ | $\omega = \sqrt{\frac{1}{LC}}$ |
| $x = A\cos(\omega t + \phi)$ | $q = Q\cos(\omega t + \phi)$ |

## -LRC Circuit

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$q = Ae^{-(R/2L)t}\cos(\omega' t + \phi).$$

Underdamped: $R^2 < 4L/C$

Critical: $R^2 = 4L/C \rightarrow \omega' = 0$

Overdamped: $R^2 > 4L/C$

# AC Current

## -phasors

$v = V\cos\omega t \quad i = I\cos\omega t$

$$I_{rms} = \frac{I}{\sqrt{2}} \quad V_{rms} = \frac{V}{\sqrt{2}}$$

## -Resistors ($V_R = IR$)

$$v_R = iR = IR\cos\omega t$$

## -Inductor ($V_L = I\omega L = IX_L$)

$$v_L = L\frac{di}{dt} = -I\omega L\cos(\omega t + \phi)$$

$$\phi_L = \phi + \pi/2$$

## -Capacitor ($V_c = \frac{I}{\omega C} = IX_c$)

$$v_C = \frac{q}{C} = \frac{I}{\omega C}\cos(\omega t + \phi)$$

$$\phi_c = \phi - \pi/2$$

| Circuit Element | Amplitude Relationship | Circuit Quantity | Phase of $v$ |
|---|---|---|---|
| Resistor | $V_R = IR$ | $R$ | In phase with $i$ |
| Inductor | $V_L = IX_L$ | $X_L = \omega L$ | Leads $i$ by 90° |
| Capacitor | $V_C = IX_C$ | $X_C = 1/\omega C$ | Lags $i$ by 90° |

## -Impedances (Z)

$$V = I\left(R + i\omega L + \frac{1}{i\omega C}\right)$$

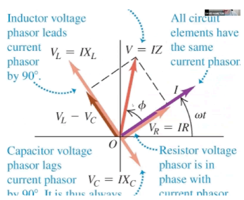
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$V_0 = I_0 Z \quad V_{rms} = I_{rms} Z$

$V_R = I_0 R \quad V_L = I_0 X_L \quad V_C = I_0 X_C$

## -Phase Angle ($\phi$)

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$




## -Power ($p = vi$)

$P_{avg} = I_{rms}^2 R = V_{rms} I_{rms}$

$P_R = I^2 R\cos^2\omega t$

$P_L = -I^2\omega L\sin\omega t\cos\omega t$

$P_C = \frac{I^2}{\omega C}\sin\omega t\cos\omega t$

} Circuit Elements

$p = vi = V\cos(\omega t + \phi) I\cos\omega t$

$P_{avg} = \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi$

} Total

## -Resonance ($\omega = \omega_0$)

$Z = R \Rightarrow X_L = X_C$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

## -Transformers

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

Step up: $N_2 > N_1$, $V_2 > V_1$

Step down: $N_1 > N_2$, $V_1 > V_2$

$V_1 I_1 = V_2 I_2 \Rightarrow P_1 = P_2$

$$\frac{V_1}{I_1} = \frac{R}{(N_2/N_1)^2}$$

# EM Waves & Light

$\boxed{E = cB} \quad B = \epsilon_0\mu_0 c E$

$$c = \lambda f = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

## -Properties:

- travel in $\vec{E}\times\vec{B}$ @ $c$
- $E = cB$
- both vacuum/matter

## -Sinusoidal Waves

$k = \frac{2\pi}{\lambda}$ (wave number)

$$y(x,t) = \cos(kx - \omega t)$$

$$\vec{E}(x,t) = \vec{E}_{max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \vec{B}_{max}\cos(kx - \omega t)$$

$$c = \frac{\omega}{k} \Rightarrow \omega = ck$$

## -Through Matter

$\epsilon = K\epsilon_0 \quad \mu = K_m\mu_0$

$E = vB \quad B = \epsilon\mu v E$

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{KK_m}}$$

$n = \frac{c}{v} = \sqrt{KK_m}$ (index of refract)

## -Energy Density

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2 = \epsilon_0 E^2$$

$$dU = u\,dV = (\epsilon_0 E^2)(Ac)\,dt$$

## -Energy Flow Rate (Poynting)

$$S = \frac{1}{A}\frac{dU}{dt} = \epsilon_0 c E_m^2 = \frac{E_m B_m}{\mu_0}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{E_m B_m}{2\mu_0}\cos^2(kx - \omega t)$$

$$I = S_{avg} = \frac{E_m B_m}{2\mu_0} = \frac{1}{2}\epsilon_0 c E_{max}^2$$

$P = \oint \vec{S}\cdot d\vec{A}$ (power)

## -Radiation Pressure ($P_r$)

$\frac{dp}{dV} = \frac{EB}{\mu_0 c^2} = \frac{S}{c^2}$ (momentum)

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$$

$P_r = \frac{S_{avg}}{c} = \frac{I}{c}$ (absorbed)

$P_r = \frac{2S_{avg}}{c} = \frac{2I}{c}$ (reflected)

$P = IA$ (Power)

$F = P_r A$ (force)

$F_g = G\frac{Mm}{r^2}$

## -Standing Waves

$E = -2E_m\sin kx\sin\omega t$

$B = -2B_m\cos kx\cos\omega t$

## -Nodal Planes ($\sin kx = 0$)

$x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$ ($\vec{E}$)

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$ ($\vec{B}$)

- Bounded distance ($L$)

$\lambda_n = \frac{2L}{n} \quad \forall n \in \mathbb{Z}^+$

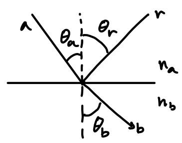
$$f_n = \frac{c}{\lambda_n} = \frac{nc}{2L}$$

## -Reflection/Refraction

$n = \frac{c}{v}$ (index of refraction)

$\theta_a = \theta_r$ (reflection)

$n_a\sin\theta_a = n_b\sin\theta_b$ (refraction)




$\lambda = \frac{\lambda_0}{n}$ ← vacuum, ← index

$f = f_0$

- Total Internal Reflection

$\sin\theta_{crit} = \frac{n_b}{n_a} \quad n_b < n_a$

## -Polarization

- Light is polarized in direction of electric field $\vec{E}$
- unpolarized → polarized

$$I = \frac{1}{2}I_0$$

## -Malus' Law

- Polarized → polarized

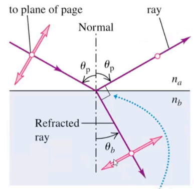
$$I_2 = I_1\cos^2\phi$$

- relative angle to previous filter ($\phi$)

## -Brewster's Law

- polarized from reflection
- in direction ⊥ to plane
- $\theta_r = \frac{\pi}{2} - \theta_b$

$$\tan\theta_p = \frac{n_b}{n_a}$$




$\Phi_B = BA\cos\phi$

$$\mathcal{E} = -\mu_0 n A\frac{dI}{dt}$$

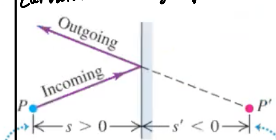
| Trig | d/dt | ∫dt |
|---|---|---|
| sin | cos | −cos |
| cos | −sin | sin |
| tan | $\sec^2$ | ln\|sec\| |

# Geometric Optics

object on incoming side: $s > 0$

image on outgoing side: $s' > 0$ $\begin{cases} s' > 0: \text{real} \\ s' < 0: \text{virtual} \end{cases}$

curvature on outgoing: $c > 0$




$$m = \frac{y'}{y} = -\frac{s'}{s}$$

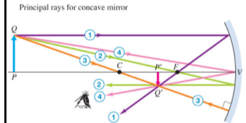
$m > 0$: erect; $m < 0$: inverted

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = \frac{2}{R}$

$f > 0$: concave, converging

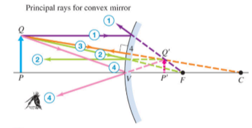
$f < 0$: convex, diverging

Principal rays for concave mirror



1. Ray parallel to axis reflects through focal point.
2. Ray through focal point reflects parallel to axis.
3. Ray through center of curvature intersects the surface normally and reflects along its original path.
4. Ray to vertex reflects symmetrically around optic axis.

Principal rays for convex mirror



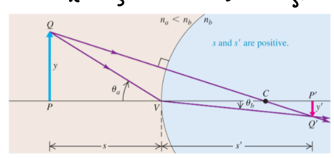
1. Reflected parallel ray appears to come from focal point.
2. Ray toward focal point reflects parallel to axis.
3. As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
4. As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

## -Spherical Refraction

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

$\tan\theta_a = \frac{y}{s} \quad \tan\theta_b = -\frac{y'}{s'}$

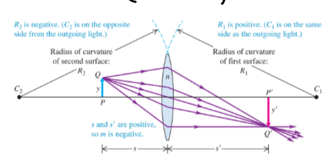



## -plane refraction

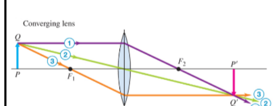
$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \quad m = 1 \quad \frac{n_b - n_a}{R} = 0$

## -Lens-maker's Equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

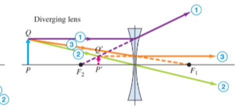



Converging lens



1. Parallel incident ray refracts to pass through second focal point $F_2$.
2. Ray through center of lens does not deviate appreciably.
3. Ray through the first focal point $F_1$ emerges parallel to the axis.

Diverging lens



1. Parallel incident ray appears after refraction to have come from the second focal point $F_2$.
2. Ray through center of lens does not deviate appreciably.
3. Ray aimed at the first focal point $F_1$ emerges parallel to the axis.