Math 170E Lecture 2
Practice Midterm 2

Instructions: Please read the following instructions **carefully** before starting the test:

• Unless otherwise stated, you need to justify your answer. Please show all of your work as partial credit will be given where appropriate, and there may be no credit given for problems where there is no work shown.

• You can use any theorem seen in class or in the weekly homework sheets, as long as you state it clearly and correctly.

• Note that it may be possible to successfully answer parts of a multi-part question without answering earlier parts.

• You have 50 minutes to complete the exam.

• The exam has 5 questions for a total of 0 points.

• You are allowed a single sheet of letter sized paper which can contain markings (by hand or printed) on both sides. No other notes or textbooks may be used in this exam.

• You may use a non-programmable calculator. No other electronics allowed on this exam. Make sure all cell phones are silenced, put away and out of sight. If you have a cell phone out at any point, for any reason, this will constitute a violation of test policy, and you may receive a zero on this exam.

• If asked, you must show us your bruin card.

• You may ask for scratch paper. You may use no other scratch paper. Please transfer all finished work onto the proper page in the test for us to grade there. We will not grade the work on the scratch paper. If you use the back of a page for your final answer, you must indicate so.

• Do not forget to write your name and UID in the space below. Good luck!
• You may use the following: $\Phi(-2.33) \sim 0.01$.

Name: _____
Student ID number: _____

Question:	1	2	3	4	5	Total
Points:	0	0	0	0	0	0
Score:						

1. A loss (in units of \$100000) due to a fire in a building has a pdf $f(x) = (1/6)e^{-x/6}$, $0 < x < \infty$. Given that the loss is greater than 5, what is the probability that it is greater than 8?

2. The "fill" problem is important in many industries, such as those making cereal, toothpaste, beer, and so on. If a company claims that is selling 12 ounces of its product in a container, it must have a mean greater than 12 ounces, or else the FDA will crack down, although the FDA will aloow a very small percentage of the containers to have less than 12 ounces.
   (a) If the weight of the content in a container, X, is normally distributed with mean 12.1 and the probability that X is less than 12 is 0.01, find an approximate value for the standard deviation of X.
   (b) If X has a $\mathcal{N}(\mu, (0.05)^2)$ distribution, find an approximate value for the mean μ so that the probability that X is less than 12 is 0.01.

3. Consider the following piecewise function with real parameters a and b:
$$f(x) = \begin{cases} 0 & \text{if} \quad x < -1, \\ ax^2 & \text{if} \quad -1 \leq x \leq 1, \\ b & \text{if} \quad x > 1. \end{cases}$$
   (a) For what values of a and b is f a pdf for a random variable?
   (b) Compute the mean and variance of the random variable.

4. Customers arrive at a coffee shop in the morning at a rate of 10 per hour. Assume the times between customer arrivals are independent and exponentially distributed.
   (a) What is the probability that at least 10 customers arrive in the first hour.
   (b) Suppose that each customer either spends \$5 on a latte with probability 2/3 or \$15 on a breakfast with probability 1/3 (no customer buys both a latte and a breakfast), independently of the other customers. Let X be the total amount of money spent by all the customers in the first hours. Find $\mathbb{P}(X \leq 15)$.
   (c) (Return to this question after Week 8) Find $\mathbb{E}[X]$ using conditional expectation.

5. A bin of 5 transistors is known to contain 2 that are defective. The transistors are to be tested, one at a time, until the defective ones are identified. Denote by N_1 the number of tests made until the first defective is identified and by N_2 the number of additional tests until the second defective is identified.
   (a) Find the joint probability mass function of N_1 and N_2.
   (b) Find $\mathbb{P}(N_2 - N_1 \geq 2)$
   (c) Find the marginal PMFs of N_1 and N_2
   (d) Are N_1 and N_2 independent? If so, prove it, if not, disprove it.
   (e) Find the mean and variance of the number of tests that must be done in order to identify both defective transistors.