Homework 6

1. Give a rigorous proof that the grammar $S \to \varepsilon \mid aSbS \mid bSaS$ generates every string with equally many a's and b's.

2. Give PDAs for the following languages:

a. binary strings in which every prefix contains at least as many 0s as 1s;
b. binary strings that contain at least as many 0s as 1s;
c. binary strings that contain equally many 0s and 1s;
d. odd-length binary strings with middle symbol 0;
e. strings of the form $v\#w$, where v and w are binary strings and w contains v^R.

2.11 Consider the following grammar over the alphabet $\{\mathtt{+}, \mathtt{\times}, \mathtt{(}, \mathtt{)}, \mathtt{a}\}$.

$$\begin{aligned} E &\to E \mathtt{+} T \mid T \\ T &\to T \mathtt{x} F \mid F \\ F &\to \mathtt{(} E \mathtt{)} \mid \mathtt{a} \end{aligned}$$

Convert this grammar into an equivalent PDA.

2.20 Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A is context free and B is regular, then A/B is context free.

***2.24** Let $E = \{\mathtt{a}^i\mathtt{b}^j \mid i \neq j \text{ and } 2i \neq j\}$. Show that E is a context-free language.

2.44 If A and B are languages, define $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. Show that if A and B are regular languages, then $A \diamond B$ is a CFL.

2.47 Let $\Sigma = \{\mathtt{0,1}\}$ and let B be the collection of strings that contain at least one $\mathtt{1}$ in their second half. In other words, $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^*\mathtt{1}\Sigma^* \text{ and } |u| \geq |v|\}$.

a. Give a PDA that recognizes B.
b. Give a CFG that generates B.