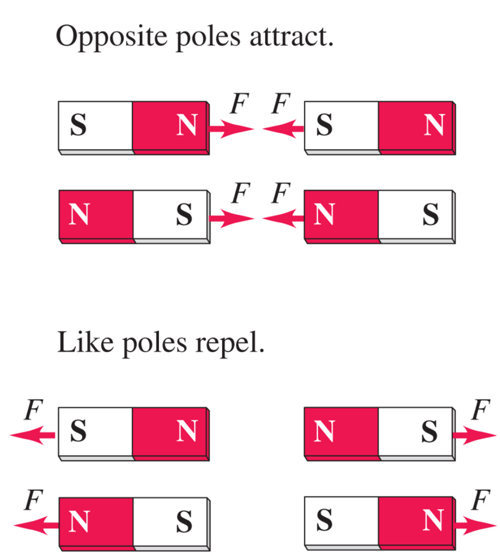
Chapter 27: Magnetic Field and Magnetic Forces

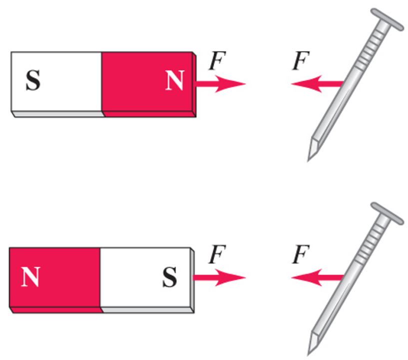
Magnetic Poles

- If a bar-shaped permanent magnet, or bar magnet, is free to rotate, one end points north; this end is called a **north pole** or **N pole.**
- The other end is a **south pole** or **S pole.**
- Opposite poles attract each other, and like poles repel each other, as shown.



Magnetism and Certain Metals

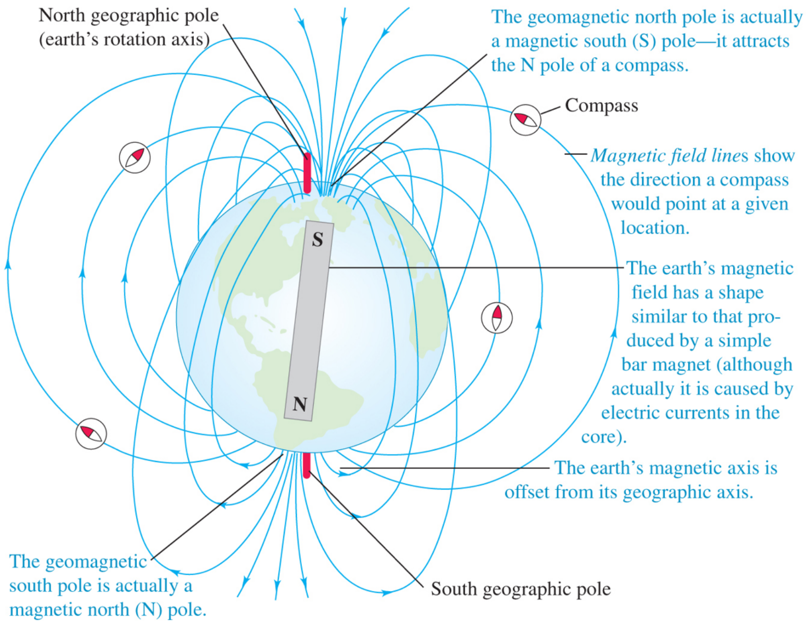
- An object that contains iron but is not itself magnetized (that is, it shows no tendency to point north or south) is attracted by either pole of a permanent magnet.
- This is the attraction that acts between a magnet and the unmagnetized steel door of a refrigerator.



Magnetic Field of the Earth (1 of 2)

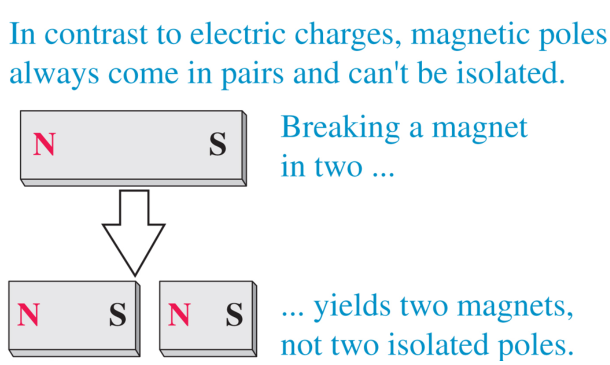
- The Earth itself is a magnet.
- Its north geographic pole is close to a magnetic **south** pole, which is why the north pole of a compass needle points north.
- The Earth's magnetic axis is not quite parallel to its geographic axis (the axis of rotation), so a compass reading deviates somewhat from geographic north.
- This deviation, which varies with location, is called **magnetic declination** or **magnetic variation.**
- Also, the magnetic field is not horizontal at most points on the Earth's surface; its angle up or down is called **magnetic inclination.**

Magnetic Field of the Earth (2 of 2)



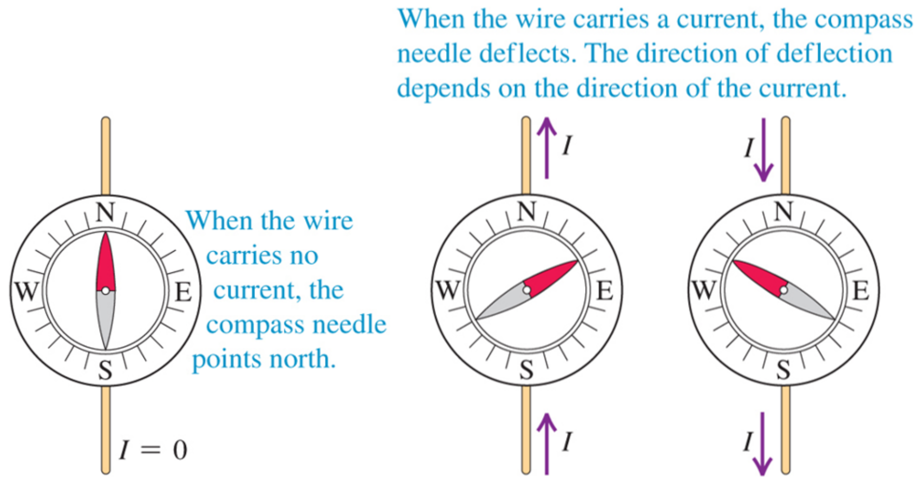
Magnetic Monopoles

- Magnetic poles always come in pairs.
- There is **no** experimental evidence for **magnetic monopoles.**



Electric Current and Magnets

- A compass near a wire with no current points north.
- However, if an electric current runs through the wire, the compass needle deflects somewhat.



6/35

The Magnetic Field

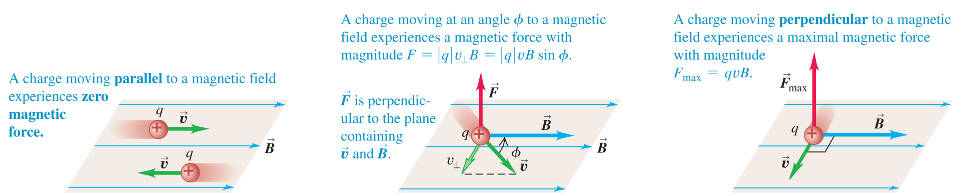
- A moving charge (or current) creates a **magnetic field** in the surrounding space.
- The magnetic field exerts a force on any other moving charge (or current) that is present in the field.
- Like an electric field, a magnetic field is a vector field—that is, a vector quantity associated with each point in space.
- We will use the symbol $\mathbf{B}$ for magnetic field.
- At any position the direction of $\mathbf{B}$ is defined as the direction in which the north pole of a compass needle tends to point.

7/35

The Magnetic Force on a Moving Charge

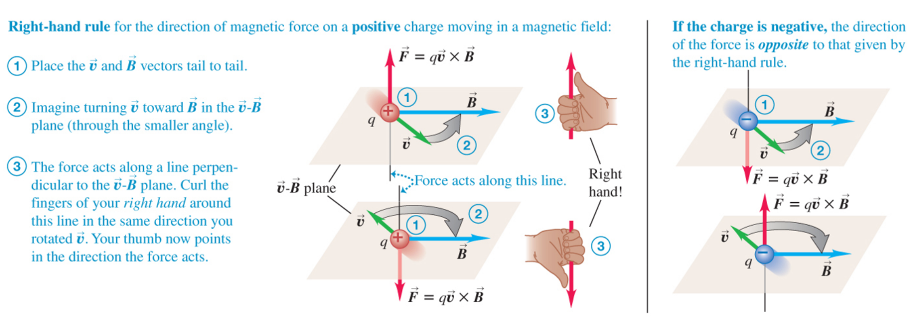
- The magnitude of the magnetic force on a moving particle is proportional to the component of the particle's velocity **perpendicular to the field.**
- If the particle is at rest, or moving parallel to the field, it experiences **zero** magnetic force.
- The magnetic force is best represented as a vector product:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}.$$



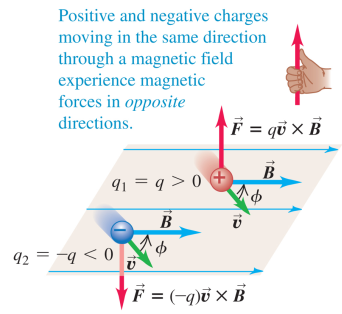
Right-Hand Rule for Magnetic Force

- The right-hand rule gives the direction of the force on a **positive** charge.
- If the charge is negative, the direction of the force is **opposite** to that given by the right-hand rule.
- The figure shows three steps involved in applying the right-hand rule:



Equal Velocities But Opposite Signs

- Imagine two charges of the same magnitude but opposite sign moving with the same velocity in the same magnetic field.
- The magnetic forces on the charges are equal in magnitude but opposite in direction.



Units of Magnetic Field

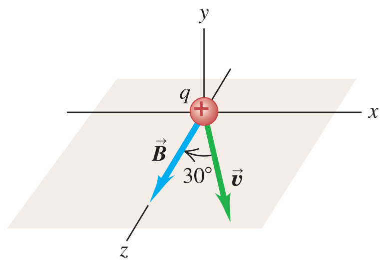
- The SI unit of **magnetic field** B is called the tesla (1 T), in honor of Nikola Tesla:

$$1 \text{ T} = 1 \text{ N/A} \cdot \text{m}.$$

- Another unit of B, the gauss (1 G $= 10^{-4}$ T) is also in common use.
- The magnetic field of the Earth is on the order of 10^{-4} T or 1 G.

Example 27.1: Magnetic Force on a Proton

A beam of protons ($q = 1.6 \times 10^{-19}$ C) moves at 3.0×10^5 m/s through a uniform 2.0 T magnetic field directed along the z-axis. The velocity of each proton lies in the xz-plane and is directed at 30° to the $+z$-axis. Find the force on a proton.



The magnitude of the force can be evaluated using the magnitude of the cross product:

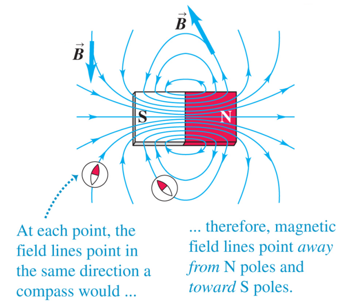
$$F = qvB\sin\phi = (1.6 \times 10^{-19}\ \text{C})(3.0 \times 10^5\ \text{m/s})(2.0\ \text{T})\sin(30^\circ) = 4.8 \times 10^{-14}\ \text{N}.$$

We can also explicitly compute the vector cross product to get

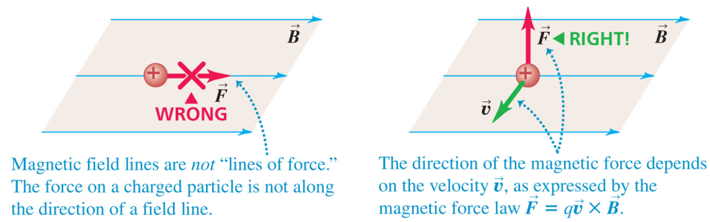
$$\begin{aligned}
\mathbf{F} &= q\mathbf{v} \times \mathbf{B} \\
&= q(v_x\,\hat{\mathbf{i}} + v_z\,\hat{\mathbf{k}}) \times (B\,\hat{\mathbf{k}}) \\
&= qv_xB(\hat{\mathbf{i}} \times \hat{\mathbf{k}}) + qv_zB(\hat{\mathbf{k}} \times \hat{\mathbf{k}}) \\
&= qv_xB(-\hat{\mathbf{j}}) \\
&= -q(v\sin\phi)B\,\hat{\mathbf{j}} \\
&= -(4.8 \times 10^{-14}\ \text{N})\,\hat{\mathbf{j}}.
\end{aligned}$$

Magnetic Field Lines

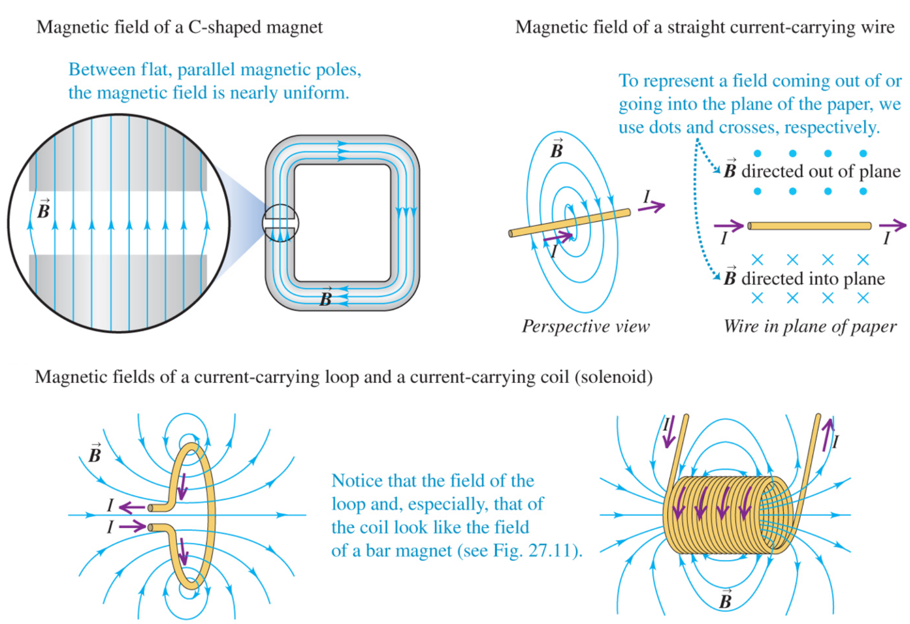
- We can represent any magnetic field by **magnetic field lines.**
- We draw lines so that the line through any point is tangent to the magnetic field vector at that point.
- Field lines never intersect.
- It is important to remember that magnetic field lines are **not** lines of magnetic force.



- The force on a charged particle is not along the direction of a field line.

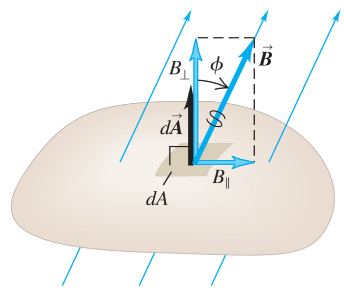


Some Common Examples of Sources of Magnetic Fields



Magnetic Flux

- To define the **magnetic flux**, we can divide any surface into elements of area $\mathrm{d}A$.
- The magnetic flux through the area element is defined to be $\mathrm{d}\Phi_B = B_\perp \,\mathrm{d}A$.
- The **total** magnetic flux through the surface is the sum of the contributions from the individual area elements:



$$\Phi_B = \int B\cos\phi \,\mathrm{d}A = \int B_\perp \,\mathrm{d}A = \int \mathbf{B}\cdot \mathrm{d}\mathbf{A}\,.$$

- The SI unit of **magnetic flux** Φ_B is called the weber (1 Wb), in honor of Wilhelm Weber:

$$1\ \mathrm{Wb} = 1\ \mathrm{T}\cdot\mathrm{m}^2.$$

Gauss's Law for Magnetism

- In Gauss's law the total electric flux through a closed surface is proportional to the total electric charge enclosed by the surface:

$$\oint \mathbf{E}\cdot \mathrm{d}\mathbf{A} = \frac{q_{\mathrm{enc}}}{\epsilon_0}.$$

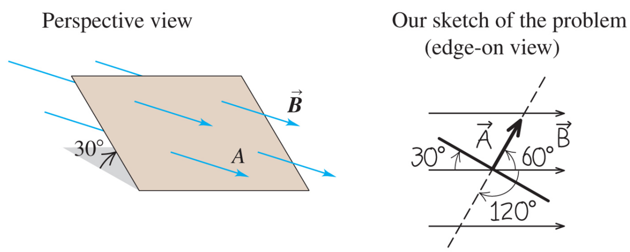
- If the closed surface encloses an electric dipole, the total electric flux is zero because the total charge is zero.
- If there were such a thing as a single magnetic charge (magnetic monopole), the total magnetic flux through a closed surface would be proportional to the total magnetic charge enclosed.
- Since no magnetic monopole has ever been observed, the magnetic flux through any closed surface is zero:

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{A} = 0 \quad \text{(Gauss's law for magnetism)}.$$

- **Magnetic field lines always form closed loops.**

Example 27.2: Magnetic Flux Calculations

The figure below is a perspective view of a flat surface with area $3.0\ \text{cm}^2$ in a uniform magnetic field $\mathbf{B}$. The magnetic flux through this surface is $+0.90$ mWb. Find the magnitude of the magnetic field and the direction of the area vector $\mathbf{A}$.



Since the flux through the area is positive, the area vector $\mathbf{A}$ and $\mathbf{B}$ must have components that are parallel. Because the area makes a $30°$ angle with the field lines for $\mathbf{B}$, and the flux Φ_B is positive, the area vector $\mathbf{A}$ must be at a $90° - 30° = 60°$ angle with respect to $\mathbf{B}$. Therefore, $\phi = 60°$, and the strength of the magnetic field is

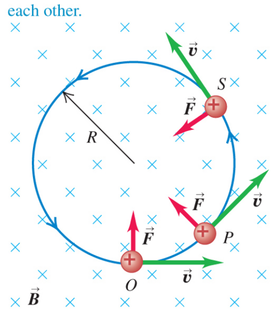
$$B = \frac{\Phi_B}{A\cos\phi} = \frac{0.90\times 10^{-3}\ \text{Wb}}{(3.0\times 10^{-4}\ \text{m}^2)\cos(60°)} = 6.0\ \text{T}.$$

Motion of Charged Particles in a Magnetic Field

- When a charged particle moves in a magnetic field, it is acted on by the magnetic force.
- The force is always perpendicular to the velocity, so it cannot change the speed of the particle.
- The particle moves in a circle of radius R with constant speed v:

$$F = |q|vB = \frac{mv^2}{R} \quad \rightarrow \quad R = \frac{mv}{|q|B}.$$

A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.



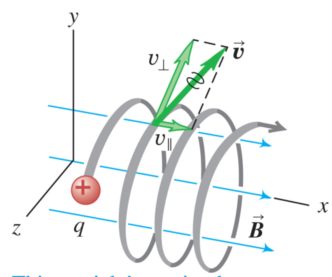
- The angular speed ω can be found from $v = R\omega$ to get the **cyclotron frequency** f:

$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m} \quad \rightarrow \quad f = \frac{|q|B}{2\pi m}.$$

16/35

Helical Motion

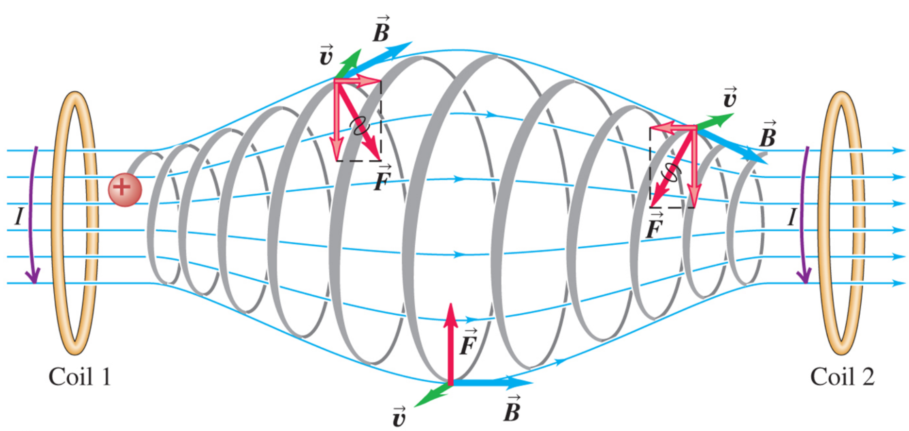
- If the particle has velocity components parallel to and perpendicular to the field, its path is a **helix.**
- The speed and kinetic energy of the particle remain constant.



This particle's motion has components both parallel ($v_\parallel$) and perpendicular ($v_\perp$) to the magnetic field, so it moves in a helical path.

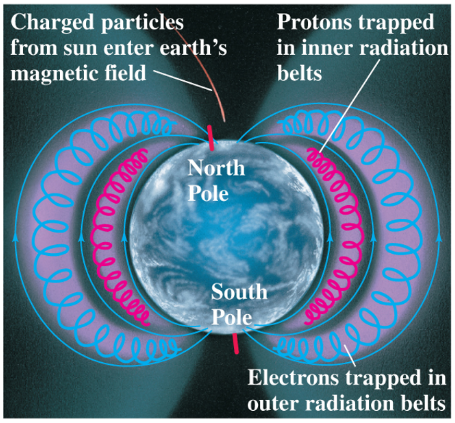
Nonuniform Magnetic Fields

- The motion of charges in a nonuniform magnetic field is more complex.
- Charged particles can be trapped in **magnetic bottles.**
- The field shape causes particles to oscillate between the end of a magnetic bottle. This can be used to contain hot ionized gases (plasmas).



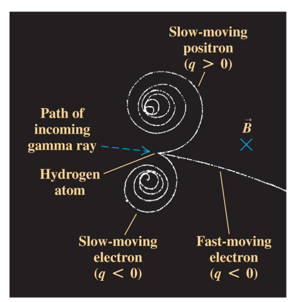
The Van Allen Radiation Belts

- Near the poles, charged particles from these belts can enter the atmosphere, producing the aurora borealis ("northern lights") and aurora australis ("southern lights").



Bubble Chamber

- The figure shows a chamber filled with liquid hydrogen and with a magnetic field directed into the plane of the photograph.
- The bubble tracks show that a high-energy gamma ray (which does not leave a track) collided with an electron in a hydrogen atom.
- The electron flew off to the right at high speed.



- Some of the energy in the collision was transformed into a second electron and a positron.

Example 27.3: Electron Motion in a Magnetron

A magnetron in a microwave oven emits electromagnetic waves with frequency $f = 2450$ MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?

The angular velocity that corresponds to this frequency is

$$\omega = 2\pi f = (2\pi)(2450 \times 10^6 \text{ Hz}) = 1.54 \times 10^{10} \text{ rad/s},$$

so the magnetic field strength must be

$$B = \frac{m\omega}{|q|} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.54 \times 10^{10} \text{ rad/s})}{1.60 \times 10^{-19} \text{ C}} = 0.0877 \text{ T}.$$

Example 27.4: Helical Particle Motion in a Magnetic Field

In a situation like that shown in the figure of helical motion of a charge, the charged particle is a proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) and the uniform, 0.500 T magnetic field is directed along the x-axis. At $t = 0$ the proton has velocity components $v_x = 1.50 \times 10^5$ m/s, $v_y = 0$, and $v_z = 2.00 \times 10^5$ m/s. Only the magnetic force acts on the proton. (a) At $t = 0$, find the force on the proton and its acceleration. (b) Find the radius of the resulting helical path, the angular speed of the proton, and the *pitch* of the helix (the distance traveled along the helix axis per revolution).

(a) We have that $\mathbf{B} = B\,\hat{\mathbf{i}}$, and $\mathbf{v} = v_x\,\hat{\mathbf{i}} + v_z\,\hat{\mathbf{k}}$, so the force on the proton is

$$\begin{aligned}
\mathbf{F} &= q\mathbf{v} \times \mathbf{B} \\
&= q(v_x\,\hat{\mathbf{i}} + v_z\,\hat{\mathbf{k}}) \times (B\,\hat{\mathbf{i}}) \\
&= qv_zB\,\hat{\mathbf{j}} \\
&= (1.60 \times 10^{-19} \text{ C})(2.00 \times 10^5 \text{ m/s})(0.500 \text{ T})\,\hat{\mathbf{j}} \\
&= (1.60 \times 10^{-14} \text{ N})\,\hat{\mathbf{j}}.
\end{aligned}$$

The resulting acceleration is then

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{1.60 \times 10^{-14} \text{ N}}{1.67 \times 10^{-27} \text{ kg}}\,\hat{\mathbf{j}} = (9.58 \times 10^{12} \text{ m/s}^2)\,\hat{\mathbf{j}}.$$

(b) The radius of the orbit depends only on the component of $\mathbf{v}$ perpendicular to $\mathbf{B}$, and so we have

$$R = \frac{mv_z}{|q|B} = \frac{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} = 4.18 \times 10^{-3} \text{ m} = 4.18 \text{ mm}.$$

Meanwhile, angular speed is

$$\omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})}{1.67 \times 10^{-27} \text{ kg}} = 4.79 \times 10^7 \text{ rad/s}.$$

Using this result, we can compute the pitch of the helix. Since the orbital period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{4.79 \times 10^7 \text{ rad/s}} = 1.31 \times 10^{-7} \text{ s},$$

the distance traveled along the x-axis in one orbital period is

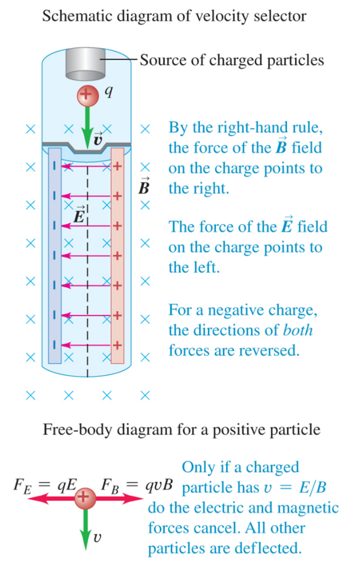
$$d = v_xT = (1.50 \times 10^5 \text{ m/s})(1.31 \times 10^{-7} \text{ s}) = 0.0197 \text{ m} = 19.7 \text{ mm}.$$

Velocity Selector

- A **velocity selector** uses perpendicular electric and magnetic fields to select particles of a specific speed from a beam.
- If q is positive, the electric force is to the left, with magnitude qE, and the magnetic force is to the right, with magnitude qvB.
- Particles travel in a straight line with constant velocity if $qE = qvB$, so the speed v for which there is no deflection is

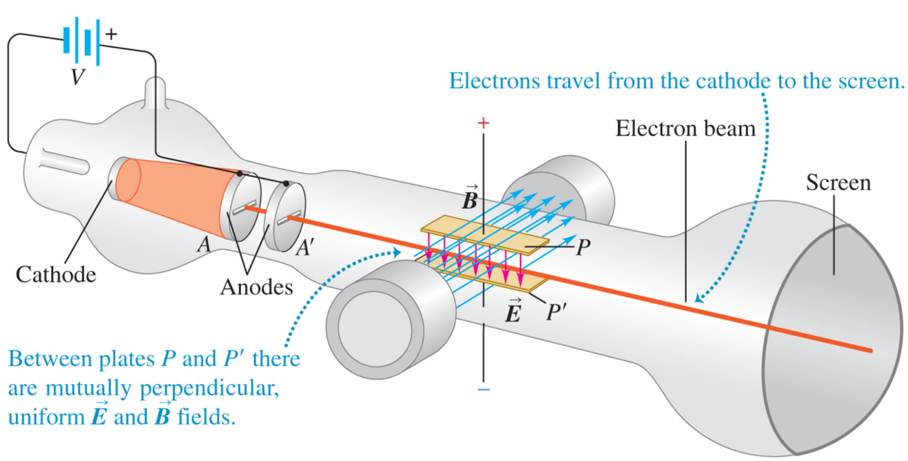
$$v = \frac{E}{B}.$$

- Since v does not depend on q, a velocity selector for positively charged particles also works for negatively charged particles.



Thomson's e/m Experiment (1 of 2)

- Thomson's experiment measured the ratio e/m for the electron using a velocity selector (where e is the magnitude of the electron charge).
- His apparatus is shown below:



Thomson's e/m Experiment (2 of 2)

- Electrons from a hot cathode are accelerated and formed into a beam by a potential difference V between the two anodes A and A'.
- The gained kinetic energy equals the lost potential energy:

$$\frac{1}{2}mv^2 = eV \quad \rightarrow \quad v = \sqrt{\frac{2eV}{m}}.$$

- Since $v = E/B$ for a velocity selector,

$$\frac{E}{B} = \sqrt{\frac{2eV}{m}} \quad \rightarrow \quad \frac{e}{m} = \frac{E^2}{2VB^2}.$$

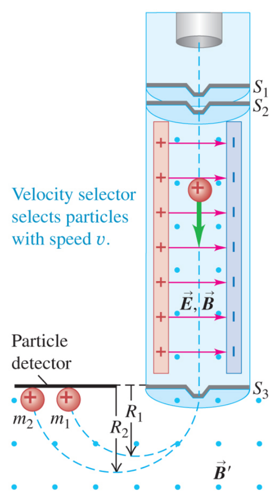
- This experiment resulted in only one value of e/m, demonstrating that the particles in the beam, which we now call electrons, are a common constituent of all matter:

$$e/m = 1.758820024(11) \times 10^{11} \text{ C/kg}.$$

Mass Spectrometers

- Velocity selectors can be combined with a second magnetic field $\mathbf{B}'$ in an outside region to make a **mass spectrometer.**
- These are used to determine mass of charged particles with known charge q.
- They select charged particles with velocity $v = E/B$, as before.
- The charges enter the region with magnetic field $\mathbf{B}'$ and undergo circular motion with radius $R = mv/qB'$.
- We can solve for the mass by measuring the orbit:

$$m = \frac{qB'R}{v} = \frac{qBB'R}{E}.$$



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

Example 27.5: An e/m Demonstration Experiment

You set out to reproduce Thomson's e/m experiment with an accelerating potential of 150 V and a deflecting electric field of magnitude 6.0×10^6 N/C. (a) How fast do the electrons move? (b) What magnetic field magnitude will yield zero beam deflection? (c) With this magnetic field, how will the electron beam behave if you increase the accelerating potential above 150 V?

(a) The electron gains kinetic energy as it passes through the 150 V potential difference, so by conservation of energy,

$$\frac{1}{2}mv^2 = |q|V \quad \to \quad v = \sqrt{\frac{2|q|V}{m}} = \sqrt{\frac{2(1.6\times10^{-19}\ \mathsf{C})(150\ \mathsf{V})}{9.11\times10^{-31}\ \mathsf{kg}}} = 7.27\times10^6\ \mathsf{m/s}.$$

(b) The required field strength is

$$B = \frac{E}{v} = \frac{6.0\times10^6\ \mathsf{N/C}}{7.27\times10^6\ \mathsf{m/s}} = 0.83\ \mathsf{T}.$$

(c) Increasing the accelerating potential will increase the speed v of the electrons. The electric force will still be the same since $F_E = qE$, but the magnetic force $F_B = qvB$ will increase proportionally with v, so the electrons will be deflected in the direction of the magnetic force.

Example 27.6: Finding Leaks in a Vacuum System

There is almost no helium in ordinary air, so helium sprayed near a leak in a vacuum system will quickly show up in the output of a vacuum pump connected to such a system. You are designing a leak detector that uses a mass spectrometer to detect He^+ ions (charge $+e = +1.60\times10^{-19}$ C, mass 6.65×10^{-27} kg). Ions emerge from the velocity selector with a speed of 1.00×10^5 m/s. They are curved in a semicircular path by a magnetic field B' and are detected at a distance of 10.16 cm from the slit S_3 in the figure of the mass spectrometer. Calculate the magnitude of the magnetic field B'.

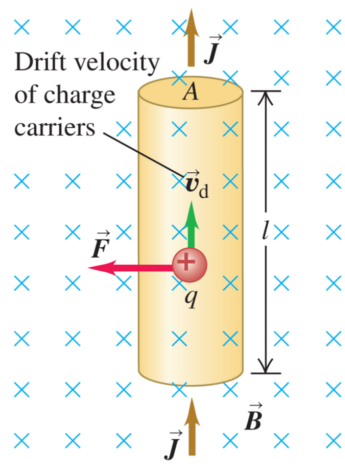
The distance d in this case corresponds to the diameter of the He^+ orbits in the magnetic field B', so the radius R of the orbit is $R = (1/2)(10.16\times10^{-2}\ \mathsf{m}) = 5.08\times10^{-2}$ m. Therefore, the magnetic field strength is

$$B' = \frac{mv}{qR} = \frac{(6.65\times10^{-27}\ \mathsf{kg})(1.00\times10^5\ \mathsf{m/s})}{(1.60\times10^{-19}\ \mathsf{C})(5.08\times10^{-2}\ \mathsf{m})} = 0.0818\ \mathsf{T}.$$

Magnetic Force on a Current-Carrying Conductor (1 of 2)

- The figure shows a straight segment of a conducting wire, with length l and cross sectional area A.
- The magnitude of the force on a single charge is $F = qv_\mathrm{d}B$.
- If the number of charges per unit volume is n, then the **total force** on all the charges in this segment is

$$F = (nAl)(qv_\mathrm{d}B) = (nqv_\mathrm{d}A)(lB).$$



Magnetic Force on a Current-Carrying Conductor (2 of 2)

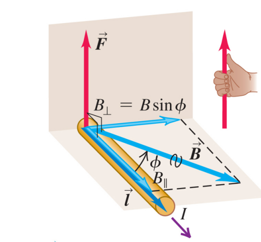
- The force is always perpendicular to both the conductor and the field, with the direction determined by the same right-hand rule we used for a moving positive charge.
- The magnetic force on a segment of a straight wire can be represented as a vector product:

$$\mathbf{F} = I\mathbf{l} \times \mathbf{B},$$

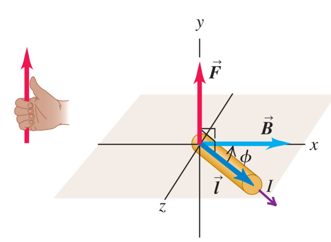
with $\mathbf{l}$ pointing along the wire in the direction of the current.

- If the conductor is not straight, we can divide it into infinitesimal segments $\mathrm{d}\mathbf{l}$. The force $\mathrm{d}\mathbf{F}$ on each segment is

$$\mathrm{d}\mathbf{F} = I\,\mathrm{d}\mathbf{l} \times \mathbf{B} \quad \rightarrow \quad \mathbf{F} = \int I\,\mathrm{d}\mathbf{l} \times \mathbf{B}.$$

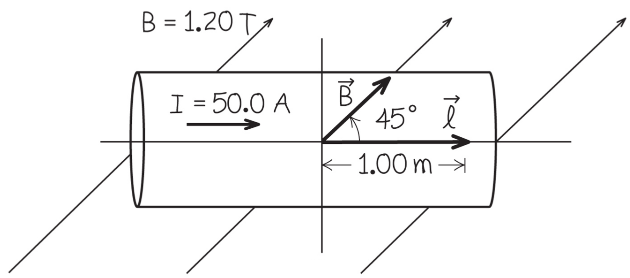


Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:
- Magnitude is $F = IlB_\perp = IlB\sin\phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.



Example 27.7: Magnetic Force on a Straight Conductor

A straight horizontal copper rod carries a current of 50.0 A from west to east in a region between the poles of a large electromagnet. In this region there is a horizontal magnetic field toward the northeast (that is, $45°$ north of east) with magnitude 1.20 T. (a) Find the magnitude and direction of the force on a 1.00 m section of rod. (b) While keeping the rod horizontal, how should it be oriented to maximize the magnitude of the force? What is the force magnitude in this case?



(a) Since the angle between $\mathbf{l}$ and $\mathbf{B}$ is $\phi = 45°$, the magnitude of the force on the conductor is

$$F = ILB\sin\phi = (50.0 \text{ A})(1.00 \text{ m})(1.20 \text{ T})\sin(45°) = 42.4 \text{ N}.$$

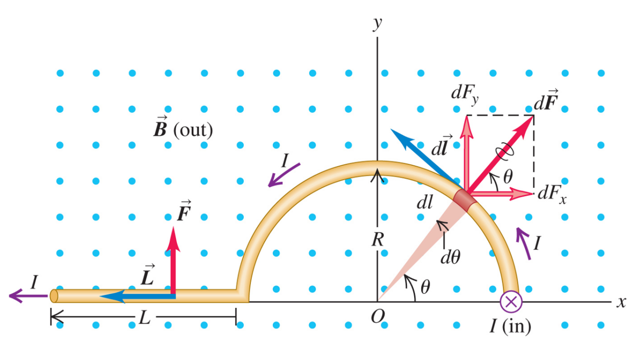
The direction of the force is perpendicular to the plane of the current and the field, and by the right-hand rule, the force is pointing vertically upward (out of the plane of the page).

(b) Since $F = ILB\sin\phi$, the force F is maximized when $\phi = 90°$, so that $\mathbf{l}$ and $\mathbf{B}$ are perpendicular. Therefore, if we rotate the rod clockwise by $45°$, the maximal force exerted on it is

$$F = ILB = (50.0 \text{ A})(1.00 \text{ m})(1.20 \text{ T}) = 60.0 \text{ N}.$$

Example 27.8: Magnetic Force on a Curved Conductor

In the figure below the magnetic field $\mathbf{B}$ is uniform and perpendicular to the plane of the figure, pointing out of the page. The conductor, carrying current I to the left, has three segments: (1) a straight segment with length L perpendicular to the plane of the figure, (2) a semicircle with radius R, and (3) another straight segment with length L parallel to the x-axis. Find the total magnetic force on this conductor.



For segment (1), the length vector is $\mathbf{L} = -L\,\hat{\mathbf{k}}$, so the magnetic force on this portion is

$$\mathbf{F}_1 = I\mathbf{L} \times \mathbf{B} = -ILB(\hat{\mathbf{k}} \times \hat{\mathbf{k}}) = \mathbf{0}.$$

For segment (2), we must first determine the direction of the force contribution $\mathrm{d}\mathbf{F}_2$ along every element of the semicircle. Since $\mathrm{d}\mathbf{l}$ points in the same direction as the current I, $\mathrm{d}\mathbf{l}$ points in the counter-clockwise direction. Every element has a force contribution $\mathrm{d}\mathbf{F}_2 = I\,\mathrm{d}\mathbf{l} \times \mathbf{B}$, and since $\mathbf{B}$ is in the z-direction, the force contribution $\mathrm{d}\mathbf{F}_2$ must point radially away from the center of curvature for segment (2). Furthermore, since every element of segment (2) is a distance R away from the center of curvature, the magnitude of the line element $\mathrm{d}\mathbf{l}$ is $|\,\mathrm{d}\mathbf{l}\,| = R\,\mathrm{d}\theta$, where θ is the angle as measured between the x-axis and the radial line from the center of curvature to any point along the semicircle. Therefore, the magnitude of $\mathrm{d}\mathbf{F}_2$ is

$$|\,\mathrm{d}\mathbf{F}_2\,| = I|\,\mathrm{d}\mathbf{l} \times \mathbf{B}| = IBR\,\mathrm{d}\theta\,.$$

Now we may decompose $\mathrm{d}\mathbf{F}_2$ into x and y-components. We have

$$\mathrm{d}F_{2,x} = \cos\theta\,\mathrm{d}F_2 = IRB\cos\theta\,\mathrm{d}\theta\,, \qquad \mathrm{d}F_{2,y} = \sin\theta\,\mathrm{d}F_2 = IRB\sin\theta\,\mathrm{d}\theta\,.$$

To find the total contributions in the x and y-directions for $\mathbf{F}_2$, we simply integrate these expressions with respect to θ. Segment (2) of the conductor subtends the angles from $\theta = 0$ to $\theta = \pi$, and so the contribution of the force $\mathbf{F}_2$ in the x-direction is

$$F_{2,x} = \int_0^\pi \mathrm{d}F_{2,x} = \int_0^\pi IRB\cos\theta\,\mathrm{d}\theta = IRB\,\sin\theta\bigg|_0^\pi = 0,$$

while the y-component is

$$F_{2,y} = \int_0^\pi \mathrm{d}F_{2,y} = \int_0^\pi IRB\sin\theta\,\mathrm{d}\theta = -IRB\,\cos\theta\bigg|_0^\pi = 2IRB.$$

Therefore, the force on segment (2) is

$$\mathbf{F}_2 = 2IRB\,\hat{\mathbf{j}}.$$

It is worth mentioning here that we should expect $\mathbf{F}_2$ to only point in the y-direction, as the x-components of $\mathbf{F}_2$ cancel due to the symmetry of the semicircular portion about the y-axis.

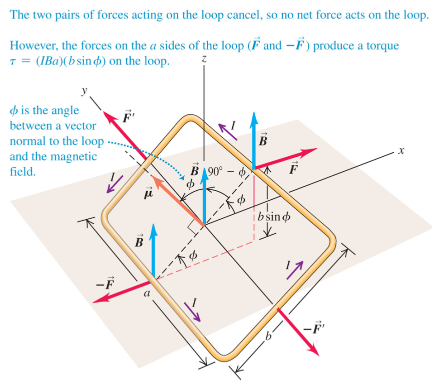
Finally, for segment (3), $\mathbf{L} = -L\,\hat{\mathbf{i}}$, so the force is simply

$$\mathbf{F}_3 = -ILB(\hat{\mathbf{i}} \times \hat{\mathbf{k}}) = ILB\,\hat{\mathbf{j}}.$$

Therefore, the total force on the conductor is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 2IRB\,\hat{\mathbf{j}} + ILB\,\hat{\mathbf{j}} = IB(2R + L)\,\hat{\mathbf{j}}.$$

Force and Torque on a Current Loop



- In a uniform magnetic field, the total force on a current loop is zero. But the field can induce a **torque** on the loop.
  - For a loop with side lengths a and b, the forces on the sides with length a are $F = IaB$ in opposite directions. The forces on the sides with length b are $F' = IbB\sin(90° - \phi) = IbB\cos\phi$, also in opposite directions. The net force is therefore zero.
  - Two forces $\mathbf{F}$ and $-\mathbf{F}$ provide a torque about the y-axis, with each contributing a torque $\tau_F = F(b/2)\sin\phi$. The net torque on loop with area $A = ab$ is

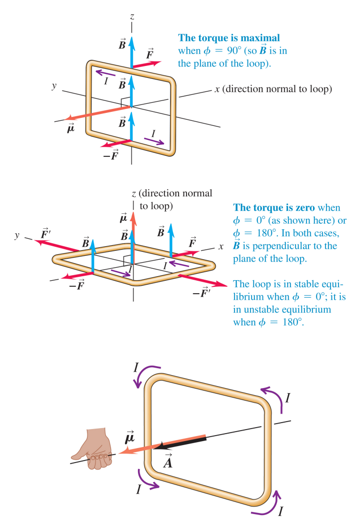
$$\tau = 2\tau_F = (IBa)(b\sin\phi) = IBA\sin\phi.$$

- We can define the **magnetic moment** $\mu = IA$ so that the torque is

$$\tau = \mu B\sin\phi.$$

27/35

Magnetic Torque: Vector Form



- The magnetic moment can also be treated as a vector by defining $\boldsymbol{\mu} = I\mathbf{A}$, where $\mathbf{A}$ is perpendicular to the current loop. The direction is determined by right-hand rule.
  - A current loop or any object with a magnetic moment $\boldsymbol{\mu}$ is known as a **magnetic dipole.**
  - The magnetic moment $\boldsymbol{\mu}$ is analogous to the electric dipole moment $\mathbf{p}$. The magnetic torque can then be written as

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}.$$

  - The magnetic field exerts a torque that causes $\boldsymbol{\mu}$ to align with $\mathbf{B}$.
- We can define the potential energy U for the magnetic moment by considering the work done due to torque:

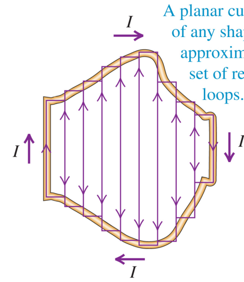
$$U = -\mu B\cos\phi = -\boldsymbol{\mu}\cdot\mathbf{B}.$$

28/35

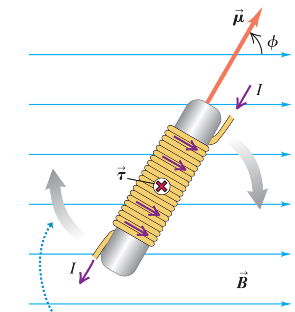
Magnetic Torque: Loops and Coils

- The magnetic moment can be defined for any planar current loop by approximating it as a set of rectangular current loops.
  - Currents passing through the area of the loop cancel each other out. Only the currents on the boundary contribute to the net force and torque on the loop.
  - Any planar current loop therefore has a magnetic moment $\boldsymbol{\mu} = I\mathbf{A}$.
- We can also consider the magnetic moment of a coil consisting of N planar loops stacked on top of each other, which is called a **solenoid.**
  - A solenoid with N turns, cross-sectional area A, and current I has a magnetic moment $\mu = NIA$. It experiences a torque in a magnetic field B given by

$$\tau = NIAB\sin\phi.$$



A planar current loop of any shape can be approximated by a set of rectangular loops.

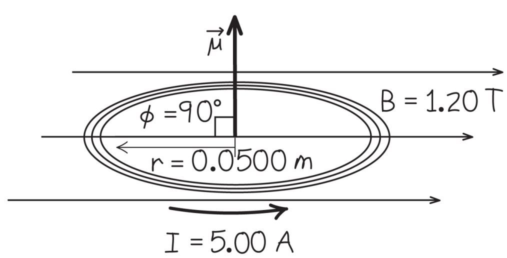


The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\mu}$ with field $\vec{B}$.

29/35

Example 27.9: Magnetic Torque on a Circular Coil

A circular coil 0.0500 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a counterclockwise (as viewed from above) current of 5.00 A. The coil is in a uniform 1.20 T magnetic field directed toward the right. Find the magnitudes of the magnetic moment and the torque on the coil.



The area of the coil is $A = \pi r^2$, so the total magnetic moment of all 30 turns is

$$\mu = NIA = 30(5.00\ \mathrm{A})\pi(0.0500\ \mathrm{m})^2 = 1.18\ \mathrm{A}\cdot\mathrm{m}^2.$$

The angle ϕ between $\mathbf{B}$ and $\boldsymbol{\mu}$ is $\phi = 90^\circ$, so the torque is

$$\tau = \mu B\sin\phi = (1.18\ \mathrm{A}\cdot\mathrm{m}^2)(1.20\ \mathrm{T})\sin(90^\circ) = 1.41\ \mathrm{N}\cdot\mathrm{m}.$$

Example 27.10: Potential Energy for a Coil in a Magnetic Field

If the coil from the previous example rotates from its initial orientation to one in which its magnetic moment $\boldsymbol{\mu}$ is parallel to $\mathbf{B}$, what is the change in potential energy?

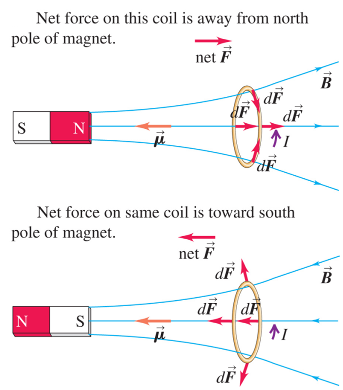
Let $\phi_1 = 90^\circ$ denote the initial angle of the coil, and $\phi_2 = 0$ be the final angle of the coil. Then the change in potential energy is

$$\begin{aligned}
\Delta U &= U_2 - U_1 \\
&= -\mu B \cos\phi_2 - (-\mu B \cos\phi_1) \\
&= -\mu B(\cos\phi_2 - \cos\phi_1) \\
&= -(1.18\ \mathsf{A}\cdot\mathsf{m}^2)(1.20\ \mathsf{T})[\cos(0^\circ) - \cos(90^\circ)] \\
&= -1.41\ \mathsf{J}.
\end{aligned}$$

As expected, the change in potential energy is negative because the magnetic moment has aligned with the magnetic field.

Magnetic Dipole in a Nonuniform Magnetic Field

- In a **nonuniform** magnetic field, a magnetic dipole can experience a non-zero force.
  - Placing a current loop near the pole of a magnet will result in a net force either towards or away from the magnet depending on how $\boldsymbol{\mu}$ of the current loop is oriented.
  - If the current loop is near the **north pole** of the magnet and has $\boldsymbol{\mu}$ pointing towards the magnet, the net force will be **away** from the magnet.
  - If the current loop is near the **south pole** of the magnet and has $\boldsymbol{\mu}$ pointing towards the magnet, the net force will be **towards** the magnet.

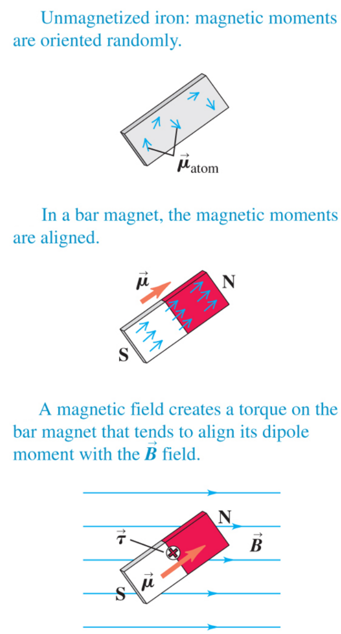


- The magnetic field generated a current loop is very similar to a bar magnet. The "north" pole of the current loop is attracted to the south pole of the bar magnet, and vice versa.

30/35

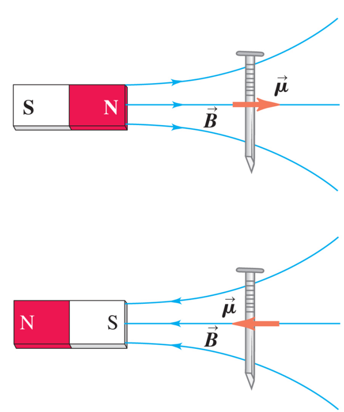
How Magnets Work (1 of 2)

- Magnets originate from the electrons that orbit the nucleus of an atom. Electrons have a magnetic moment that arises from quantum mechanical effects.
- An individual atom obtains a magnetic moment when a substantial fraction of the magnetic moments of its electrons align in the same direction.
- In an unmagnetized piece of iron, the magnetic moments of the individual atoms are randomly oriented and there is no net magnetic moment.
- A bar magnet is the result of a significant number of atomic magnetic moments aligning in the same direction.



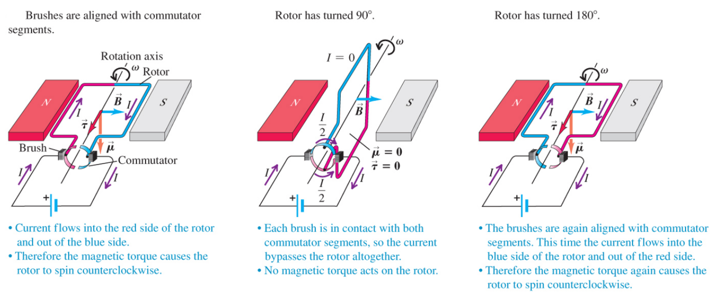
How Magnets Work (2 of 2)

- A bar magnet attracts an unmagnetized iron nail in two steps:
    1. The magnetic field of the bar magnet gives rise to a net magnetic moment in the nail.
    2. Because the field of the bar magnet is not uniform, this magnetic dipole is attracted toward the magnet.
- The attraction is the same whether the nail is closer to the magnet's north pole or the magnet's south pole.



The Direct-Current Motor

- Below is a schematic diagram of a simple dc motor.
- The **rotor** is a wire loop that is free to rotate about an axis; the rotor ends are attached to the two curved conductors that form the **commutator**.
- Current flows into the red side of the rotor and out of the blue side.
- Therefore, the magnetic torque causes the rotor to spin counterclockwise.
- If the potential difference between its terminals is V_{ab} and the current is I, then the power input is $P = V_{ab}I$.
- For a series motor with internal resistance r and emf $\mathcal{E}$ in the rotor, the potential difference is $V_{ab} = \mathcal{E} + Ir$.



33/35

Example 27.11: A Series Dc Motor

A dc motor with its rotor and field coils connected in series has an internal resistance of $r = 2.00\ \Omega$. When running at full load on a 120 V line, it draws a 4.00 A current. (a) What is the emf in the rotor? (b) What is the power delivered to the motor? (c) What is the rate of dissipation of energy in the internal resistance? (d) What is the mechanical power developed? (e) What is the motor's efficiency? (f) What happens if the machine being driven by the motor jams, so that the rotor stops turning?

(a) Solving for the emf, we have

$$\mathcal{E} = V_{ab} - Ir = 120\text{ V} - (4.00\text{ A})(2.00\ \Omega) = 112\text{ V}.$$

(b) The power delivered to the motor from the source is

$$P_{\text{in}} = V_{ab}I = (120\text{ V})(4.00\text{ A}) = 480\text{ W}.$$

(c) The power dissipated in the resistance r is

$$P_{\text{dissipated}} = I^2 r = (4.00\text{ A})^2(2.00\ \Omega) = 32\text{ W}.$$

(d) The mechanical power developed is the power input P_{in} minus the power dissipated in the rotor's internal resistance:

$$P_{\text{out}} = P_{\text{in}} - P_{\text{dissipated}} = 480\text{ W} - 32\text{ W} = 448\text{ W}.$$

(e) The efficiency e is the ratio of the mechanical power output to the electric power input:

$$e = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{448\ \text{W}}{480\ \text{W}} = 0.93 = 93\%.$$

(f) If the rotor stalls, then the back emf $\mathcal{E}$ goes to zero, and the current becomes

$$I = \frac{V_{ab}}{r} = \frac{120\ \text{V}}{2.00\ \Omega} = 60\ \text{A}.$$

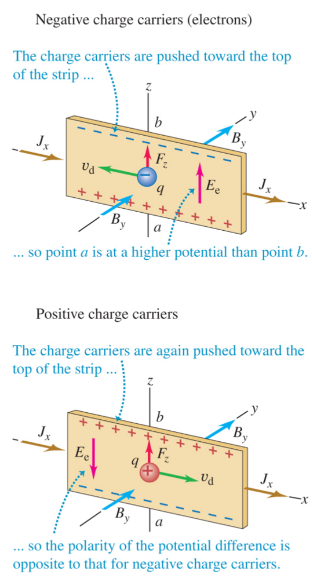
The power dissipated in the internal resistance is now

$$P_{\text{dissipated}} = I^2 r = (60\ \text{A})^2 (2.00\ \Omega) = 7200\ \text{W}.$$

The Hall Effect (1 of 2)

- Passing a current $\mathbf{J} = nq\mathbf{v}_\text{d}$ through a conductor while placed in a magnetic field $\mathbf{B}$ produces the **Hall effect.**
- Charges in the current are deflected due to the magnetic field. The deflection is the same for positive and negative charges.
- As charges accumulate in the conductor in the transverse direction of the current, an electric field $\mathbf{E}$ is generated that counteracts the magnetic force.
- Once the electric field becomes strong enough to cancel out the magnetic force, the charges can now pass through undeflected:

$$qE_z + qv_\text{d}B_y = 0 \quad \rightarrow \quad E_z = -v_\text{d}B_y.$$



34/35

The Hall Effect (2 of 2)

- The potential difference across the conductor due to the electric field is called the **Hall voltage.** The polarity of the potential difference tells us the sign of the charge carriers.
- Since the current density is $J_x = nqv_{\rm d}$, and the drift velocity is $v_{\rm d} = -E_z/B_y$, the charge per unit volume in the conductor is

$$\frac{J_x}{nq} = -\frac{E_z}{B_y} \quad \rightarrow \quad nq = -\frac{J_x B_y}{E_z}.$$

35/35

Example 27.12: A Hall-Effect Measurement

You place a strip of copper, 2.0 mm thick and 1.50 cm wide, in a uniform 0.40 T magnetic field as shown in the previous figure. When you run a 75 A current in the $+x$-direction, you find that the potential at the bottom of the slab is 0.81 μV higher than at the top. From this measurement, determine the concentration of mobile electrons in copper.

First, we must find the current density J_x and the electric field E_z generated by the charge deposition. For the current density, we have

$$J_x = \frac{I}{A} = \frac{75 \text{ A}}{(2.0 \times 10^{-3} \text{ m})(1.50 \times 10^{-2} \text{ m})} = 2.5 \times 10^6 \text{ A/m}^2,$$

while for the electric field,

$$E_z = \frac{V}{d} = \frac{0.81 \times 10^{-6} \text{ V}}{1.5 \times 10^{-2} \text{ m}} = 5.4 \times 10^{-5} \text{ V/m}.$$

Therefore, the concentration of electrons is

$$n = -\frac{J_x B_y}{qE_z} = -\frac{(2.5 \times 10^6 \text{ A/m}^2)(0.40 \text{ T})}{(-1.60 \times 10^{-19} \text{ C})(5.4 \times 10^{-5} \text{ V/m})} = 11.6 \times 10^{28} \text{ m}^{-3}.$$