Chapter 32: Electromagnetic Waves

James Clerk Maxwell and Electromagnetic Waves

- The Scottish physicist James Clerk Maxwell (1831-1879) was the first person to truly understand the fundamental nature of light.
- He proved in 1865 that an electromagnetic disturbance should propagate in free space with a speed equal to that of light.
- From this, he deduced correctly that light was an electromagnetic wave.

Electricity, Magnetism, and Light (1 of 2)

- The basic principles of electromagnetism can be expressed in terms of the four equations that we now call **Maxwell's equations:**

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0} \quad \text{(Gauss's law)},$$

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0 \quad \text{(Gauss's law for magnetism)},$$

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \quad \text{(Faraday's law)},$$

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 \left(i_{\mathrm{C}} + \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} \right)_{\text{enc}} \quad \text{(Ampère's law)}.$$

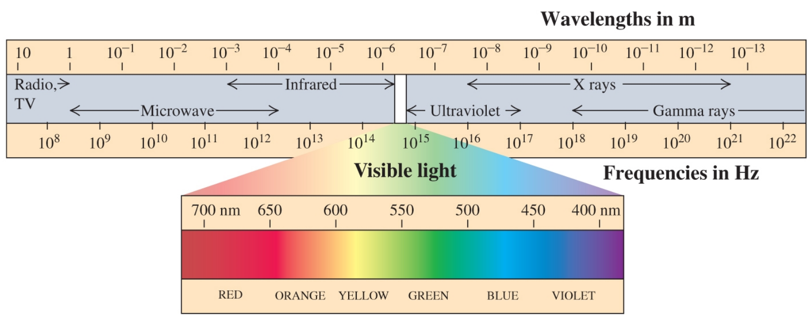
Electricity, Magnetism, and Light (2 of 2)

- According to Maxwell's equations, an accelerating electric charge must produce electromagnetic waves.
- For example, power lines carry a strong alternating current, which means that a substantial amount of charge is accelerating back and forth and generating electromagnetic waves.

- These waves can produce a buzzing sound from your car radio when you drive near the lines.

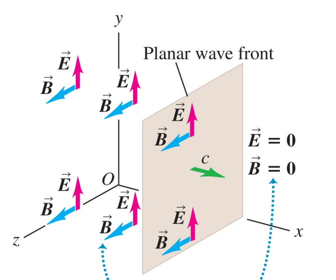
The Electromagnetic Spectrum

- The frequencies and wavelengths of electromagnetic waves found in nature extend over such a wide range that we have to use a logarithmic scale to show all important bands.
- In a vacuum, electromagnetic waves obey the relationship $\lambda f = c$, where λ is the wavelength and f is the frequency of the wave.
- The boundaries between bands are somewhat arbitrary. Visible light covers a wavelength range of roughly 380-750 nm.



A Simple Plane Electromagnetic Wave

- Suppose we divided all space into two regions by a plane perpendicular to the x-axis.
  - At every point to the left of the plane there are uniform electric and magnetic fields perpendicular to each other.
  - The boundary plane, which is called the **wave front,** moves in the $+x$-direction with constant speed c.
  - In front of the plane, the electric and magnetic fields are zero.

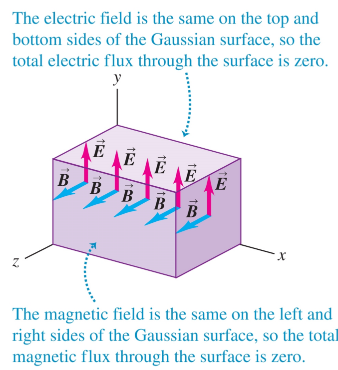


The electric and magnetic fields are uniform behind the advancing wave front and zero in front of it.

- This configuration defines a **plane wave,** and we will show that this type of wave is consistent with Maxwell's equations.

Gauss's Laws and the Simple Plane Wave

- First, we will take a look at the two Gauss's laws for electric and magnetic fields.
  - We may create a rectangular box for our Gaussian surface with sides parallel to the xy-, xz-, and yz-coordinate planes.
  - The box encloses no electric charge, and the total electric and magnetic flux through the box is zero, even if the box encloses part of the region where the fields are zero.



  - In order to satisfy the two Gauss's laws, the electric and magnetic fields cannot have an x-component since that would give rise to a non-zero flux through the left-hand side of the box, but not the right-hand side.
- To satisfy the two Gauss's laws, plane waves must have electric and magnetic fields perpendicular to the direction of propagation, which means the waves must be **transverse.**

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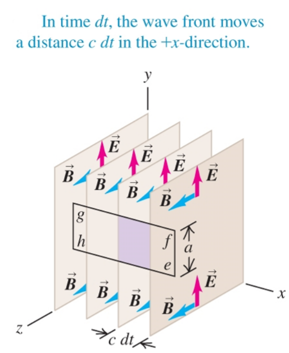
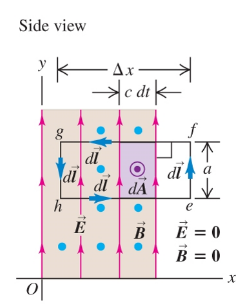
Faraday's Law and the Simple Plane Wave

- To evaluate Faraday's law, we define the loop of height a and width Δx in the xy-plane for the line integral of $\mathbf{E}$ so that it passes through the wave front and into the region where the fields are zero.



- The line integral of $\mathbf{E}$ is

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -Ea.$$



- For the magnetic flux, in a time $\mathrm{d}t$, the flux through the rectangle in the xy-plane increases by an amount $\mathrm{d}\Phi_B$. This increase equals the flux through the shaded rectangle with area $ac\,\mathrm{d}t$, so $\mathrm{d}\Phi_B = Bac\,\mathrm{d}t$.
- Faraday's law therefore says

$$\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = Bac \quad \rightarrow \quad E = cB.$$

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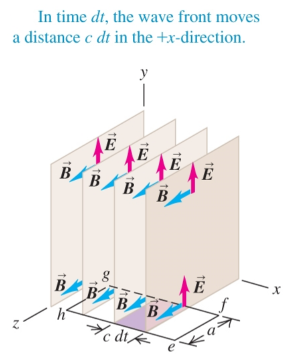
Ampère's Law and the Simple Plane Wave

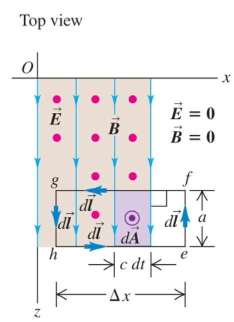
- The last equation to check is Ampère's law. We choose our Ampèrian loop so that the path has height a and width Δx in the xz-plane, and also passes through the wave front into the region where the fields are zero.
- The line integral of $\mathbf{B}$ is

$$\oint \mathbf{B} \cdot \mathrm{d}l = Ba.$$

- In a time $\mathrm{d}t$, the electric flux through the rectangle in the xz-plane increases by an amount $\mathrm{d}\Phi_E$. This increase equals the flux through the shaded rectangle with area $ac\,\mathrm{d}t$, so $\mathrm{d}\Phi_E = Eac\,\mathrm{d}t$.
- Ampère's law therefore gives us

$$\frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = Eac \quad \rightarrow \quad B = \epsilon_0\mu_0 cE.$$



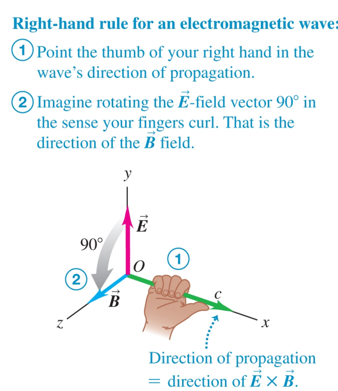


Properties of Electromagnetic Waves

- Some important features of all electromagnetic waves:
  1. Electromagnetic waves are transverse: both $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to the direction of propagation for the wave. The $\mathbf{E}$ and $\mathbf{B}$ fields are also perpendicular to each other, with the direction of propagation pointing in the direction of $\mathbf{E} \times \mathbf{B}$.
  2. The ratio between the magnitudes of $\mathbf{E}$ and $\mathbf{B}$ is $E = cB$.
  3. Electromagnetic waves travel in vacuum with a definite and unchanging speed:

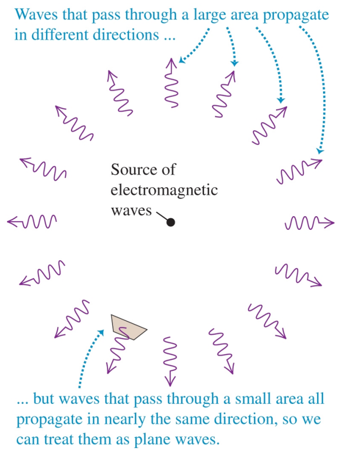
$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 2.99 \times 10^8 \text{ m/s}.$$

  4. Unlike mechanical waves, electromagnetic waves do not require a medium to propagate in.



Sinusoidal Electromagnetic Waves (1 of 2)

- Electromagnetic waves produced by an oscillating point charge are an example of sinusoidal waves that are not plane waves.
- But if we restrict our observations to a relatively small region of space far away enough from the source, these waves are well approximated by plane waves.
- We can describe such waves in the same way that we expressed waves on a string by using a sinusoidal function.

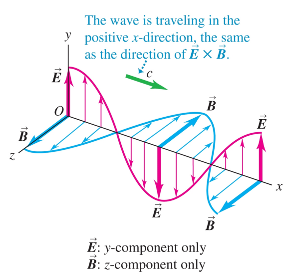


- Given an amplitude of oscillation A, the displacement in the y-direction of a string due to a transverse wave with wavenumber $k = 2\pi/\lambda$ and angular frequency ω is

$$y(x,t) = A\cos(kx - \omega t).$$

Sinusoidal Electromagnetic Waves (2 of 2)

- Suppose we have a sinusoidal electromagnetic wave traveling in the x-direction, with $\mathbf{E}$ pointing in the y-direction and $\mathbf{B}$ pointing in the z-direction.
  - Given maximum amplitudes of the electric and magnetic fields $E_{\max}$ and $B_{\max}$, wave number k, and angular frequency ω, the wavefunctions for E_y and B_z are



$$E_y(x,t) = E_{\max}\cos(kx - \omega t), \quad B_z(x,t) = B_{\max}\cos(kx - \omega t).$$

  - We can also represent the wavefunctions in vector form:

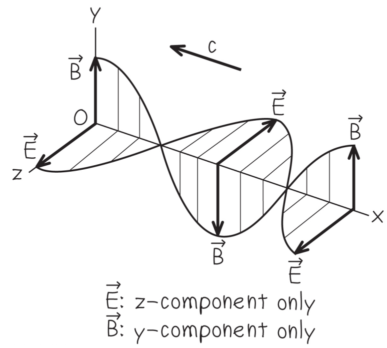
$$\mathbf{E}(x,t) = E_{\max}\cos(kx - \omega t)\,\hat{\mathbf{j}}, \quad \mathbf{B}(x,t) = B_{\max}\cos(kx - \omega t)\,\hat{\mathbf{k}}.$$

  - The amplitudes for the waves are related by $E_{\max} = cB_{\max}$, and the wavenumber and angular frequency are related to the speed c by

$$c = \frac{\omega}{k} = (2\pi f)\frac{\lambda}{2\pi} = f\lambda.$$

Example 32.1: Electric and Magnetic Fields of a Laser Beam

A carbon dioxide laser emits a sinusoidal electromagnetic wave that travels in vacuum in the negative x-direction. The wavelength is 10.6 μm and the $\mathbf{E}$ field is parallel to the z-axis, with $E_{\max} = 1.5$ MV/m. Write vector equations for $\mathbf{E}$ and $\mathbf{B}$ as functions of time and position.



Since the $\mathbf{E}$ field points parallel to the z-axis, and the wave propagates along the negative x-direction, the $\mathbf{B}$ field must point in the y-direction. It must also satisfy the fact that $\mathbf{E} \times \mathbf{B}$ points in the direction of propagation. Therefore, $\mathbf{B}$ must point in the positive y-direction.

The fields are then given by

$$\mathbf{E}(x,t) = E_{\max}\cos(kx+\omega t)\,\hat{\mathbf{k}}, \qquad \mathbf{B}(x,t) = B_{\max}\cos(kx+\omega t)\,\hat{\mathbf{j}}.$$

Notice that the plus sign in the argument of the cosine functions indicates that the wave is traveling in the negative x-direction.

Since $E_{\max} = cB_{\max}$, we have

$$B_{\max} = \frac{E_{\max}}{c} = \frac{1.5\times 10^{6}\ \text{V/m}}{3.0\times 10^{8}\ \text{m/s}} = 5.0\times 10^{-3}\ \text{T}.$$

We are also given the wavelength, from which we can obtain the wavenumber and angular frequency:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\ \text{rad}}{10.6\times 10^{-6}\ \text{m}} = 5.93\times 10^{5}\ \text{rad/m},$$
$$\omega = ck = (3.00\times 10^{8}\ \text{m/s})(5.93\times 10^{5}\ \text{rad/m}) = 1.78\times 10^{14}\ \text{rad/s}$$

The wave functions are therefore

$$\mathbf{E}(x,t) = (1.5\times 10^{6}\ \text{V/m})\cos[(5.93\times 10^{5}\ \text{rad/m})x + (1.78\times 10^{14}\ \text{rad/s})t]\,\hat{\mathbf{k}},$$
$$\mathbf{B}(x,t) = (5.0\times 10^{-3}\ \text{T})\cos[(5.93\times 10^{5}\ \text{rad/m})x + (1.78\times 10^{14}\ \text{rad/s})t]\,\hat{\mathbf{j}}.$$

Electromagnetic Waves in Matter

- Electromagnetic waves can travel in both a vacuum and matter, such as glass, air, or water.
- When an electromagnetic wave travels in dielectric materials, the speed of the wave is not the same as the speed of light c in a vacuum.
  - Suppose we have a material with permittivity $\epsilon = K\epsilon_0$ and permeability $\mu = K_m\mu_0$. Then the wave travels in the material with speed v, and the electric and magnetic fields are related by

$$E = vB, \quad B = \epsilon\mu v E.$$

  - The speed of the wave v is then

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}}\frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{KK_m}}.$$

  - The ratio of the speed of light in vacuum c to the speed in the material v is defined as the **index of refraction** of the material:

$$\frac{c}{v} = n = \sqrt{KK_m}.$$

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Example 32.2: Electromagnetic Waves in Different Materials

(a) Visiting a jewelry store one evening, you hold a diamond up to the light of a sodium-vapor street lamp. The heated sodium vapor emits yellow light with a frequency of 5.09×10^{14} Hz. Find the wavelength in vacuum and the wave speed and wavelength in diamond, for which $K = 5.84$ and $K_m = 1.00$ at this frequency. (b) A 90.0 MHz radio wave (in the FM radio band) passes from vacuum into an insulating ferrite (a ferromagnetic material used in computer cables to suppress radio interference). Find the wavelength in vacuum and the wave speed and wavelength in the ferrite, for which $K = 10.0$ and $K_m = 1000$ at this frequency.

(a) The wavelength in vacuum of the yellow light is

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}} = 5.89 \times 10^{-7} \text{ m} = 589 \text{ nm}.$$

The wave speed and the wavelength in the diamond are

$$v_{\text{diamond}} = \frac{c}{\sqrt{KK_m}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(5.84)(1.00)}} = 1.24 \times 10^8 \text{ m/s},$$

$$\lambda_{\text{diamond}} = \frac{v_{\text{diamond}}}{f} = \frac{1.24 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}} = 2.44 \times 10^{-7} \text{ m} = 244 \text{ nm}.$$

(b) For the radio wave, we obtain

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}} = 3.33 \text{ m}.$$

In the ferrite, we have

$$v_{\text{ferrite}} = \frac{c}{\sqrt{KK_m}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(10.0)(1000)}} = 3.00 \times 10^6 \text{ m/s},$$
$$\lambda_{\text{ferrite}} = \frac{v_{\text{ferrite}}}{f} = \frac{3.00 \times 10^6 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}} = 3.33 \times 10^{-2} \text{ m} = 3.33 \text{ cm}.$$

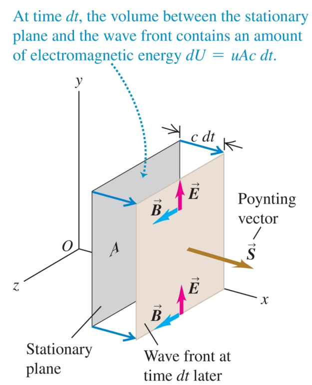
Energy in Electromagnetic Waves (1 of 2)

- Electromagnetic waves carry energy that comes from the electric and magnetic fields.
  - The energy density u in a region of empty space with $\mathbf{E}$ and $\mathbf{B}$ fields is

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2.$$

  - Since $B = E/c = \sqrt{\epsilon_0\mu_0}E$, we can express u as

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}(\sqrt{\epsilon_0\mu_0}E)^2 = \epsilon_0 E^2.$$



  - We can compute the energy flowing into a volume $\mathrm{d}V$ by considering a stationary plane of area A, with the wave traversing a distance $c\,\mathrm{d}t$:

$$\mathrm{d}U = u\,\mathrm{d}V = (\epsilon_0 E^2)(Ac\,\mathrm{d}t).$$

  - The energy flow per unit time per unit area S is therefore

$$S = \frac{1}{A}\frac{\mathrm{d}U}{\mathrm{d}t} = \epsilon_0 c E^2 = \frac{EB}{\mu_0}.$$

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Energy in Electromagnetic Waves (2 of 3)

- We can define a vector quantity $\mathbf{S}$ that describes the magnitude and direction of the energy flow rate, called the **Poynting vector:**

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}.$$

- The Poynting vector is the energy flow per unit area and per unit time through a cross-sectional area perpendicular to the propagation direction.
- The total energy flow per unit time out of any closed surface is the integral of $\mathbf{S}$ over the surface:

$$P = \oint \mathbf{S} \cdot \mathrm{d}\mathbf{A}\,.$$

Energy in Electromagnetic Waves (3 of 3)

- In terms of sinusoidal waves with $\mathbf{E}$ pointing in the y-direction and $\mathbf{B}$ pointing in the z-direction, $\mathbf{S}$ is given by

$$\begin{aligned}\mathbf{S}(x,t) &= \frac{1}{\mu_0}\mathbf{E}(x,t) \times \mathbf{B}(x,t) \\ &= \frac{1}{\mu_0}\Big[E_{\max}\cos(kx-\omega t)\,\hat{\mathbf{j}}\Big] \times \Big[B_{\max}\cos(kx-\omega t)\,\hat{\mathbf{k}}\Big] \\ &= \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx-\omega t)\,\hat{\mathbf{i}}.\end{aligned}$$

- The magnitude of the average of $\mathbf{S}$ is defined as the **intensity** of the wave:

$$I = S_{\mathrm{avg}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{1}{2}\epsilon_0 c E_{\max}^2.$$

Example 32.3: Energy in a Nonsinusoidal Wave

For the nonsinusoidal wave described earlier when showing that a plane wave satisfies Maxwell's equations, suppose that $E = 100$ V/m $= 100$ N/C. Find the value of B, the energy density u, and the rate of energy flow per unit area S.

The strength of the magnetic field is

$$B = \frac{E}{c} = \frac{100 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-7} \text{ T}.$$

For the energy density, we have

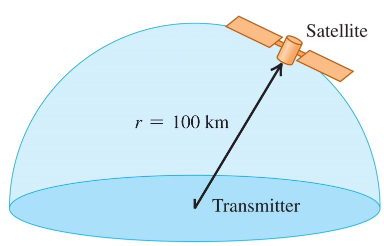
$$u = \epsilon_0 E^2 = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(100 \text{ N/C})^2 = 8.85 \times 10^{-8} \text{ J/m}^3.$$

Finally, the magnitude of the Poynting vector is

$$S = \frac{EB}{\mu_0} = \frac{(100 \text{ V/m})(3.33 \times 10^{-7} \text{ T})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = 26.5 \text{ W/m}^2.$$

Example 32.4: Energy in a Sinusoidal Wave

A radio station on the Earth's surface emits a sinusoidal wave with average total power 50 kW. Assuming the transmitter radiates equally in all directions above the ground (which is unlikely in real situations), find the electric field and magnetic field amplitudes $E_{\max}$ and $B_{\max}$ detected by a satellite 100 km from the antenna.



Given the total power P, the intensity I is the total power per unit area. Since the power is distributed over a hemisphere of radius $r = 100$ km, we have

$$I = \frac{P}{A} = \frac{P}{2\pi R^2} = \frac{5.00 \times 10^4 \text{ W}}{6.28 \times 10^{10} \text{ m}^2} = 7.96 \times 10^{-7} \text{ W/m}^2.$$

Since $I = S_{\text{avg}} = E_{\max}^2/2\mu_0 c$, we have

$$E_{\max} = \sqrt{2\mu_0 c S_{\text{avg}}} = \sqrt{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(3.0 \times 10^8 \text{ m/s})(7.96 \times 10^{-7} \text{ W/m}^2)} = 2.45 \times 10^{-2} \text{ V/m},$$

$$B_{\max} = \frac{E_{\max}}{c} = 8.17 \times 10^{-11} \text{ T}.$$

Electromagnetic Radiation Pressure

- The Poynting vector shows us how electromagnetic waves carry energy, but they also carry momentum p, with a momentum density given by

$$\frac{\mathrm{d}p}{\mathrm{d}V} = \frac{EB}{\mu_0 c^2} = \frac{S}{c^2}.$$

- We can determine the momentum flow rate by considering the volume $\mathrm{d}V = Ac\,\mathrm{d}t$ occupied by an electromagnetic wave passing through an area A:

$$\frac{1}{A}\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{S}{c} = \frac{EB}{\mu_0 c}.$$

- This momentum is responsible for **radiation pressure** p_{rad}, which can be absorbed or reflected by a surface:

$$p_{\mathrm{rad}} = \frac{S_{\mathrm{avg}}}{c} = \frac{I}{c} \quad \text{(wave totally absorbed)},$$
$$p_{\mathrm{rad}} = \frac{2S_{\mathrm{avg}}}{c} = \frac{2I}{c} \quad \text{(wave totally reflected)}.$$

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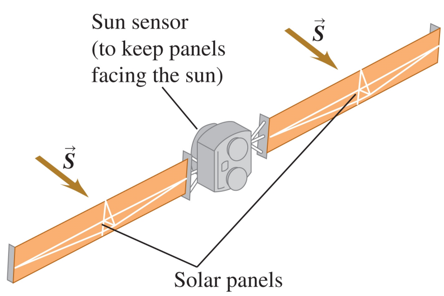
Example 32.5: Power and Pressure from Sunlight

An Earth-orbiting satellite has solar energy-collecting panels with a total area of 4.0 m^2. If the sun's radiation is perpendicular to the panels and is completely absorbed, find the average solar power absorbed and the average radiation-pressure force.



The power absorbed is

$$P = IA = (1.4 \times 10^3 \text{ W/m}^2)(4.0 \text{ m}^2) = 5.6 \times 10^3 \text{ W}.$$

Since the wave is totally absorbed, the radiation pressure is

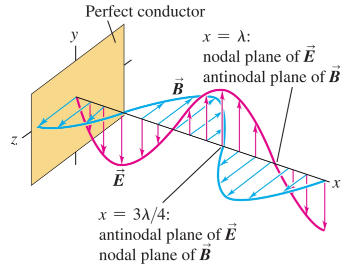
$$p_{\mathrm{rad}} = \frac{I}{c} = \frac{1.4 \times 10^3 \text{ W/m}^2}{3.0 \times 10^8 \text{ m/s}} = 4.7 \times 10^{-6} \text{ N/m}^2.$$

Then the total force is

$$F = p_{\mathrm{rad}}A = (4.7 \times 10^{-6} \text{ N/m}^2)(4.0 \text{ m}^2) = 1.9 \times 10^{-5} \text{ N}.$$

Standing Electromagnetic Waves (1 of 2)

- Much like with waves on a string, electromagnetic waves can also produce **standing waves.**
  - A standing wave is a superposition of an incident and reflected wave, and conductors or dielectrics can be used reflect electromagnetic waves.
  - If we have a perfect conductor struck with an incident electromagnetic wave, then the conductor will produce a reflected wave.



  - The sum of the incident and reflected waves produces a standing wave:

$$E_y(x,t) = E_{\max}[\cos(kx+\omega t) - \cos(kx-\omega t))],$$
$$B_z(x,t) = B_{\max}[-\cos(kx+\omega t) - \cos(kx-\omega t)].$$

  - Using the trigonometric identity $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$, these become

$$E_y(x,t) = -2E_{\max} \sin kx \sin \omega t,$$
$$B_z(x,t) = -2B_{\max} \cos kx \cos \omega t.$$

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Standing Electromagnetic Waves (2 of 2)

- At the boundaries of a standing wave on a conductor, the electric field is always zero because $\mathbf{E}$ cannot have a component parallel to the conducting surface.
- As with standing waves on a string, the nodes for the electric field occur at the **nodal planes** where $\sin kx = 0$, which are given by

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \quad \text{(nodal planes of } \mathbf{E}\text{).}$$

- The nodal planes for the magnetic field occur at the points where $\cos kx = 0$:

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \quad \text{(nodal planes of } \mathbf{B}\text{).}$$

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Standing Waves in a Cavity

- If we have a standing wave confined between two conductors separated by a distance L, then the allowed wavelengths and frequencies for the standing wave are

$$\lambda_n = \frac{2L}{n}, \quad f_n = \frac{c}{\lambda_n} = n\frac{c}{2L} \quad (n = 1, 2, 3, \ldots).$$

- A typical microwave oven sets up a standing electromagnetic wave with $\lambda = 12.2$ cm, a wavelength that is absorbed by the water in food.
- Because the wave has nodes spaced $\lambda/2 = 6.1$ cm apart, the food must be rotated while cooking. Otherwise, the portion that lies at a node—where the electric field amplitude is zero—will remain cold.

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Example 32.6: Intensity in a Standing Wave

Calculate the intensity of the standing wave represented by the above equations.

The Poynting vector $\mathbf{S}$ is

$$\begin{aligned}
\mathbf{S}(x,t) &= \frac{1}{\mu_0}\mathbf{E}(x,t) \times \mathbf{B}(x,t) = \frac{1}{\mu_0}\left[-2E_{\max}\sin kx \sin\omega t\,\hat{\mathbf{j}}\right] \times \left[-2B_{\max}\cos kx \cos\omega t\,\hat{\mathbf{k}}\right] \\
&= \frac{E_{\max}B_{\max}}{\mu_0}(2\sin kx\cos kx)(2\sin\omega t\cos\omega t)\,\hat{\mathbf{i}} \\
&= \frac{E_{\max}B_{\max}}{\mu_0}\sin 2kx \sin 2\omega t\,\hat{\mathbf{i}}
\end{aligned}$$

The average value of a sinusoid over a full period is zero, so the average of $\mathbf{S}$ is also zero. Thus, $I = S_{\text{avg}} = 0$.

Example 32.7: Standing Waves in a Cavity

Electromagnetic standing waves are set up in a cavity with two parallel, highly conducting walls 1.50 cm apart. (a) Calculate the longest wavelength λ and lowest frequency f of these standing waves. (b) For a standing wave of this wavelength, where in the cavity does $\mathbf{E}$ have the maximum magnitude? Where is $\mathbf{E}$ zero? Where does $\mathbf{B}$ have maximum magnitude? Where is $\mathbf{B}$ zero?

(a) for $n = 1$, the wavelength and frequency are

$$\lambda_1 = 2L = 2(1.50 \text{ cm}) = 3.00 \text{ cm},$$

$$f_1 = \frac{c}{2L} = \frac{(3.00 \times 10^8 \text{ ms})}{2(1.50 \times 10^{-2} \text{ m})} = 1.00 \times 10^{10} \text{ Hz}.$$

(b) With $n = 1$, there is a single half-wavelength between the conducting walls. Thus, the electric field has nodal planes at the walls and an anti-nodal plane in the midway between the walls. Conversely, the magnetic field has anti-nodal planes at the walls and a nodal plane midway between them.