Physics 1C - Discussion 2 Name: UID:

Problem 1. Analyzing an LRC Circuit

A 200 Ω resistor, a 0.400 H inductor, and a 6.00 μF capacitor are connected to form a series RLC circuit with a voltage source that has voltage amplitude 30.0 V and an angular frequency of 250 rad/s. (a) What is the impedance of the circuit? (b) What is the current amplitude? (c) What is the phase angle of the source voltage with respect to the current? Does the source voltage lag or lead the current? (d) What are the voltage amplitudes across the resistor, inductor, and capacitor? (e) Explain how it is possible for the voltage amplitude across the capacitor to be greater than the voltage amplitude across the source.

a)

$$|\tilde{Z}| = \sqrt{200^2 + \left[(250)(0.4) - \frac{1}{(250)(6\times10^{-6})}\right]^2} \quad \boxed{\approx 600.9252\ [\Omega]}$$

b)

$$I_0 = \frac{30.0}{600.9252} \quad \boxed{\approx 4.9923 \times 10^{-2}\ [A]}$$

c)

$$\phi = \arctan\left[\frac{250(0.4) - \frac{1}{250(6\times10^{-6})}}{200}\right] \quad \boxed{\approx -1.2315\ [\text{rad}]}$$

d)

$$V_R \approx (4.9923\times10^{-2})\,200 \quad \boxed{= 9.9846\ [V]}$$

$$V_L \approx (4.9923\times10^{-2})(250)(0.4) \quad \boxed{= 4.9923\ [V]}$$

$$V_C \approx (4.9923\times10^{-2})\left(\frac{1}{250(6\times10^{-6})}\right) \quad \boxed{= 33.282\ [V]}$$

e) This is possible b/c these are the maxima of the oscillating voltages and for any given instance are "impeded" by some other impedance depending on the point of measurement. At the EOD, $V \approx V_0$

Problem 2. Phase angle of LRC circuit

An LRC series circuit has source voltage amplitude $V = 240$ V, and the voltage amplitudes for the inductor and capacitor are $V_L = 310$ V and $V_C = 180$ V. What is the phase angle ϕ?

$$\phi = \arctan\left[\frac{X_L - X_C}{R}\right]$$

$$V = V_R + V_L + V_C = V_R + 480 = 240 \quad \Rightarrow \quad V_R = -250 \text{ [V]}$$

$$X_L = \frac{V_L}{I_0} \qquad X_C = \frac{V_C}{I_0} \qquad R = \frac{V_R}{I_0}$$

$$\frac{X_L - X_C}{R} = \frac{\frac{V_L - V_C}{I_0}}{\frac{V_R}{I_0}} = \frac{V_L - V_C}{V_R} = \frac{310 - 180}{-250} = -0.52$$

$$\Rightarrow \phi = \arctan(-0.52) \boxed{\approx -0.4795 \text{ [rad]}}$$

Problem 3. RMS voltages

In an LRC series circuit, the rms voltage across the resistor is 30.0 V, across the capacitor it is 90.0 V, and across the inductor it is 50.0 V. What is the rms voltage of the source?

$$V_{RMS} = \frac{V}{\sqrt{2}}$$

$V_{RMS,R} = 30\ [V] \Rightarrow V_R = 30\sqrt{2}$

$V_{RMS,C} = 90\ [V] \Rightarrow V_C = 90\sqrt{2}$

$V_{RMS,L} = 50\ [V] \Rightarrow V_L = 50\sqrt{2}$

Kirchhoff's:

$V_0 - V_R - V_C - V_L = 0 = V_0 - 170\sqrt{2} \Rightarrow V_0 = 170\sqrt{2}$

$$\boxed{\therefore V_{RMS,0} = 170\ [V]}$$

Problem 4. (Challenge Problem) Potentials across circuit elements

A 200 Ω resistor, a 0.900 H inductor, and a 6.00 μF capacitor are connected in series across a voltage source that has voltage amplitude 30.0 V and an angular frequency of 250 rad/s. (a) What are v, v_R, v_L, and v_C at $t = 20$ ms? Compare $v_R + v_L + v_C$ to v at this instant. (b) What are V_R, V_L, and V_C? Compare V to $V_R + V_L + V_C$. Explain why these two quantities are not equal.

a) $V_0 = 30 \Rightarrow 30 = I_0 |\tilde{Z}| = I_0 \left[\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \right] \Rightarrow |\tilde{Z}| \approx 301.0399$ [Ω]

$I_0 = \frac{V_0}{|\tilde{Z}|} \approx \frac{30}{301.0399} = 9.9655 \times 10^{-2}$ [A]

$V_R \approx 19.931$ [V] $\quad V_C \approx 66.437$ [V] $\quad V_L \approx 22.422$ [V]

$v(t=.02) = V_0 \cos \omega t = 30 \cos (250)(0.02) \boxed{\approx 29.8858 \text{ [V]}}$

$v_R(t=0.02) = V_R \cos \omega t \boxed{\approx 19.8552 \text{ [V]}}$

$v_L(t=.02) = V_L \cos \omega t \boxed{\approx 22.3367 \text{ [V]}}$

$v_C(t=.02) = V_C \cos \omega t \boxed{\approx 66.1842 \text{ [V]}}$

$\sum_{j \neq 0} v_j = 108.3761 \neq v_0$

b) $V_R \approx \boxed{19.931 \text{ [V]}}$

$V_C \approx \boxed{66.437 \text{ [V]}}$

$V_L \approx \boxed{22.422 \text{ [V]}}$

$\sum_j V_j = 108.79 \neq V_0$

They do not capture the phase angle which would cause some wave superposition that results in true voltages $\sim V_0, v_0$ respectively