Solutions to Questions - Chapter 3
Mortgage Loan Foundations: The Time Value of Money

Question 3-1

What is the essential concept in understanding compound interest?

The concept of earning interest on interest is the essential idea that must be understood in the compounding process and is the cornerstone of all financial tables and concepts in the mathematics of finance.

Question 3-2

How are the interest factors in Exhibit 3-3 developed? How may financial calculators be used to calculate interest factors in Exhibit 3-3?

Computed from the general formula for monthly compounding for various combinations of "i" and years. $FV = PV \times (1+i)^n$. Calculators can be used by entering $1 for PV, the desired values for n and i and solving for FV.

Question 3-3

What general rule can be developed concerning maximum values and compounding intervals within a year? What is an equivalent annual yield?

Whenever the nominal annual interest rates offered on two investments are equal, the investment with the more frequent compounding interval within the year will always result in a higher effective annual yield. An equivalent annual yield is a single, annualized discount rate that captures the effects of compounding (and if applicable, interest rate changes).

Question 3-4

What does the time value of money mean?

Time value simply means that if an investor is offered the choice between receiving $1 today or receiving $1 in the future, the proper choice will always be to receive the $1 today, because that $1 can be invested in some opportunity that will earn interest. Present value introduces the problem of knowing the future cash receipts for an investment and trying to determine how much should be paid for the investment at present. When determining how much should be paid today for an investment that is expected to produce income in the future, we must apply an adjustment called discounting to income received in the future to reflect the time value of money.

Question 3-5

How does discounting, as used in determining present value, relate to compounding, as used in determining future value? How would present value ever be used?

The discounting process is a process that is the opposite of compounding. To find the present value of any investment is simply to compound in a "reverse" sense. This is done by taking the reciprocal of the interest factor for the compound value of $1 at the interest rate, multiplying it by the future value of the investment to find its present value. Present value is used to find how much should be paid for a particular investment with a certain future value at a given interest rate.

Question 3-6

What are the interest factors in Exhibit 3-9? How are they developed? How may financial calculators be used to calculate interest factors in Exhibit 3-9?

Compound interest factors for the accumulation of $1 per period, e.g. $\$1 \times [1 + (1+i) + (1+i)^2 \ldots]$, etc. Calculators may be used by entering $1 values for PMT, entering the desired values for n and I, then solving for FV.

Question 3-7

What is an annuity? How is it defined? What is the difference between an ordinary annuity and an annuity due?

An annuity is a series of equal deposits or payments.
An ordinary annuity assumes payments or receipts occur at the end of a period.
An annuity due assumes deposits or payments are made at the beginning of the period.

Question 3-8

How must one discount a series of uneven receipts to find PV?

Each periodic cash receipt or payment must be discounted individually then summed to obtain present value. That is: $PV= CF_1 \div (1 + i)^1 + CF_2 \div (1+ i)^2 \ldots + CF_n \div (1 + i)^n$ where CF is cash inflow and i equals the discount rate.

Question 3-9

What is the sinking-fund factor? How and why is it used?
A sinking-fund factor is the reciprocal of interest factors for compounding annuities. These factors are used to determine the amount of each payment in a series needed to accumulate a specified sum at a given time. To this end, the specified sum is multiplied by the sinking-fund factor.

Question 3-10

What is an internal rate of return (IRR)? How is it used? How does it relate to the concept of compound interest?
The internal rate of return integrates the concepts of compounding and present value. It represents a way of measuring a return on investment over the entire investment period, expressed as a compound rate of interest. It tells the investor what compound interest rate the return on an investment being considered is equivalent to.

Solutions to Problems - Chapter 3

The Interest Factor in Financing

Problem 3-1

a)	Future Value	=	FV (n, i, PV, PMT)
		=	FV (7 years, 6%, $12,000, 0)
		=	$18,044 (annual compounding)
b)	Future Value	=	FV (n, i, PV, PMT)
		=	FV (28 quarters, 9% ÷ 4, $12,000, 0)
		=	$22,375 (quarterly compounding)

c) Equivalent annual yield: (consider one year only)

Future Value of (a)	=	FV (n, i, PV, PMT)
	=	FV (1 year, 6%, $12,000, 0)
	=	$12,720
($12,720 - $12,000) / $12,000 =		6.00% effective annual yield
Future Value of (b)	=	FV (n, i, PV, PMT)
	=	FV (1year, 9%, $12,000, 0)
	=	$13,117
($13,117 - $12,000) / $12,000 =		9.31% effective annual yield

Alternative (b) is better because of its higher effective annual yield.

Problem 3-2

Investment A: 6% compounded monthly

Future Value of A	=	FV (n, i, PV, PMT)
	=	FV (12 months, 6% ÷ 12, $25,000, 0)
	=	$26,542 (monthly compounding)

Investment B: 7% compounded annually

Future Value of B	=	FV (n, i, PV, PMT)
	=	FV (1 year, 7%, $25,000, 0)
	=	$26,750 (annual compounding)

Investment B should be chosen over A. Investment B pays 7% compounded annually and is the better choice because it provides the greater future value.

Problem 3-3

Find the future value of 24 deposits of $5,000 made at the end of each 6 months. Deposits will earn an annual rate of 8.0%, compounded semi-annually.

Future Value	=	FV (n, i, PV, PMT)
	=	FV (24 periods, 8% ÷ 2, 0, $5,000)
	=	$195,413

Note: Total cash deposits are $5,000 x 24 = $120,000. Total interest equals $75,413 or ($195,413 - $120,000). The $120,000 represents the return of capital (initial principal) while the $75,413 represents the interest earned on the capital contributions.

Find the future value of 24 *beginning-of-period* payments of $5,000 at an annual rate of 8.0%, compounded semi-annually based on an annuity due.

Future Value	=	FV (n, i, PV, PMT)
	=	FV (25 periods, 8% ÷ 2, 0, $5,000)
	=	**$208,230**

Note: n is changed to 25 because the deposits are made at the beginning of each period. Therefore, the first deposit will be compounded 25 times whereas if the 1st deposit was made at the end of the period it would be compounded only 24 times. This pattern holds true for each deposit made. The second deposit would be compounded 24 times and the last deposit would be compounded once. This example illustrates the difference between an annuity due (beginning of period deposits) and an ordinary annuity (end of period deposits).

Problem 3-4

Find the future value of quarterly payments of $1,250 for four years, each earning an interest rate of 10 percent annually, compounded quarterly.

Future Value	=	FV (n, i, PV, PMT)
	=	FV (16 periods, 10% ÷ 4, 0, $1,250)
	=	$24,225

Problem 3-5

End of Year	Amount Deposited	FV (n, i, PV, PMT)	Future Value
1	$2,500	FV (4 years, 15%, $2,500, 0)	$4,373
2	$0	FV (3 years, 15%, 0, 0)	$0
3	$750	FV (2 years, 15%, $750, 0)	$992
4	$1,300	FV (1 year, 15%, $1,300, 0)	$1,495
5	$0		$0
		Total Future Value =	$6,860

The investor will have $6,860 on deposit at the end of the 5th year.
*Each deposit is made at the end of the year.

Problem 3-6

a) Find the present value of 96 monthly payments of $750 (end-of-month) discounted at an interest rate of 15 percent compounded monthly.

Present Value	=	PV (n, i, PMT, FV)
	=	PV (96 periods, 15% ÷ 12, $750, 0)
	=	$41,793 should be paid today

b) The total sum of cash received over the next 8 years will be:
8 years x 12 payments per year x $750 per month = $72,000

c)	Total cash received by the investor	$72,000
	Initial price paid by the investor	$41,793

Difference: Interest Earned	$30,207

The difference represents the total interest earned by the investor on the initial investment of $41,793 if each $750 payment is discounted at 15 percent compounded monthly.

Problem 3-7

Find the present value of 10 end-of-year payments of $2,150 discounted at an annual interest rate of 12 percent.

Present Value	=	PV (n, i, PMT, FV) - ordinary annuity
	=	PV (10 years, 12%, $2,150, 0)
	=	$12,148 should be paid today

Find the present value of 10 beginning-of-year payments of $2,150 discounted at an annual interest rate of 12 percent.

Present Value	=	PV (n, i, PMT, FV)
	=	PV (9 years,12%, $2,150, 0) + $2,150
	=	$13,606 should be paid today

Note: The 1st payment of $2,150 is not discounted because it is received immediately or at the beginning of year 1. The remaining 9 payments are discounted at 12% annually. This problem illustrates an annuity due.

Problem 3-8

Find the present value of $45,000 received at the end of 6 years, discounted at a 9% annual rate compounded quarterly.

Present Value	=	PV (n, i, PMT, FV)
	=	PV (24 quarters, 9% ÷ 4, $0, $45,000)
	=	$26,381 should be paid today

Note that a quarterly interest factor is used in this problem because the investor indicates that an annual rate of 9% compounded quarterly is desired.

Problem 3-9

Year	Amount Received	PV (n, i, PMT, FV)	Present Value
1	$12,500	PV (1 year, 12%, 0, $12,500)	$11,161
2	$10,000	PV (2 years, 12%, 0, $10,000)	$7,972
3	$7,500	PV (3 years, 12%, 0, $7,500)	$5,338
4	$5,000	PV (4 years, 12%, 0, $5,000)	$3,178
5	$2,500	PV (5 years, 12%, 0, $2,500)	$1,419
6	$0	PV (6 years, 12%, 0, $0)	$0
7	$12,500	PV (7 years, 12%, 0, $12,500)	$5,654
		Total Present Value =	$34, 722

* Each deposit is made at the end of the year

The investor should pay no more than $34,722 for the investment in order to earn the 12% annual interest rate compounded annually.

Problem 3-10

Find the present value of $15,000 discounted at an annual rate of 8% for 10 years.

Present Value	=	PV (n, i, PMT, FV)
	=	PV (10 years, 8%, 0, $15,000)
	=	$6,948 (annual compounding)

The investor should not purchase the lot because the present value of the lot (discounted at the appropriate interest rate) is less than the current asking price of $7,000.

Problem 3-11
What will be the rate of return (yield) on a project that initially costs $100,000 and is expected to pay out $15,000 per year for the next ten years?

Interest/IRR	=	i (n, PV, PMT, FV)
Interest/IRR	=	i (10 years, -$100,000, $15,000, 0)
Interest/IRR	=	8.14%

It is a good investment for DDC because the IRR of 8.14% exceeds DDC's desired return of 8%.

Problem 3-12
What will be the rate of return (yield) on a project that initially costs $75,000 and is expected to pay out $1,000 per month for the next 25 years?

Interest/IRR	=	i (n, PV, PMT, FV)
Interest/IRR	=	i (300 months, -$75,000, $1,000, 0)
Interest/IRR	=	15.67%

The total cash received will be: $1,000 x 25 years x 12 months = $300,000

How much is profit and how much is return on capital?

Total Amount Received	$300,000
Total Capital Invested (returned)	$75,000
Total Profit (interest earned)	$225,000

The total cost of the investment, $75,000, is capital recovery.
The difference between the total amount received and the capital recovery is total profit earned.

Problem 3-13
(a)

Year	Amount Received*	PV (n, i, PMT, FV)	Present Value
1	$5,500	PV (1 year, 12%, 0, $5,500)	$4,911
2	$7,500	PV (2 years, 12%, 0, $7,500)	$5,979
3	$9,500	PV (3 years, 12%, 0, $9,500)	$6,762
4	$12,500	PV (4 years, 12%, 0, $12,500)	$7,944
		Total Present Value =	$25,596

The investor should pay not more than $25,596 for investment in order to earn the 12 percent annual interest rate compounded annually.

(b)

End of Month	Amount Received*	PV (n, i, PMT, FV)	Present Value
12	$5,500	PV (12 months, 12% ÷ 12, 0, $5,500)	$4,881
24	$7,500	PV (24 months, 12% ÷ 12, 0, $7,500)	$5,907
36	$9,500	PV (36 months, 12% ÷ 12, 0, $9,500)	$6,639
48	$12,500	PV (48 months, 12% ÷ 12, 0, $12,500)	$7,753
		Total Present Value =	$25,180

The investor should not pay more than $25,180 for the investment in order to earn the 12 percent annual interest rate compounded monthly. (Note: the periods (n) above should be calculated using a monthly number i.e 1 year = 12 periods)

(c) These two amounts are different because the return demanded in part (b) is compounded monthly. The greater compounding frequency results in a lower present value.

Problem 3-14
What will be the internal rate of return (IRR) on a project that initially costs $100,000 and is expected to receive $1,600 per month for the next 5 years and, at the end of the five years, return the initial investment of $100,000?

Interest/IRR	=	i (n, PV, PMT, FV)
Interest/IRR	=	i (60 months, -$100,000, $1,600, $100,000)
Interest/IRR	=	1.6% monthly and 1.6% x 12 = 19.2% annual IRR

Problem 3-15
Annual sinking fund payments required to accumulate $60,000 after ten years

Payment	=	Payment (n, i, PV, FV)
Payment	=	Payment (10 years, 10%, 0, $60,000)
	=	$3,765 per year

Note to Instructor: In problem 3-15(b), the text indicates that annual payments be calculated. However, the text should read: monthly payments.

Monthly sinking fund payments required to accumulate $60,000 after ten years.

Payment	=	Payment(n,i,PV, FV)
Payment	=	Payment(120 periods, 10%/12, 0, $60,000)
	=	$292.90 per month

Problem 3-16
a) Find the ENAR for 12% EAY given Monthly Compounding.

ENAR	=	[(1 + EAY) ^ (1/m) - 1] x m
	=	[(1 + 0.12) ^ (1/12) -1] x 12
	=	[1.00948879 - 1] x 12
	=	[0.00948879] x 12
	=	0.1138655 or 11.39%

b) Find the ENAR for 12% EAY given Quarterly Compounding

ENAR	=	[(1 + EAY) ^ (1/m) x m
	=	[(1 + 0.12) ^ (1/4) -1] x 4
	=	[1.0287373 - 1] x 4
	=	[0.0287373] x 4
	=	0.1149493 or 11.49%

Problem 3-17
Part 1, calculate annual returns compounded annually: (Note: calculator should be set for one payment per period)

The Annual Rate compounded Monthly:
Solution:

N	=	28
PMT	=	$1,200
PV	=	-24,000
FV	=	0

Solve for the yield:

i	=	2.486% (x12) = 29.83%

The monthly rate can now be used to calculate the equivalent annual rate as follows:

The Annual Rate compounded annually:
Solution:

PV	=	-1
i	=	29.83% ÷ 12
PMT	=	0
N	=	12

Solve for the future value:

FV	=	1.34266

The annual rate of interest (compounded annually) needed to provide a return equivalent to that of an annual rate compounded monthly is:

FV - PV = 1.34266 - 1.0 = 34.2660%

This return is far greater than the annual rate compounded monthly or 29.830%

This tells us that an investor would have to find an investment yielding 34.3% if compounding occurred on an annual basis (once per year) in order for it to be equivalent to an investment that provides an annual rate of 29.8% compounded monthly.

Problem 3-18

Goa1: To show the relationship between IRRs, compound interest, recovery of capital and cash flows.

a) Note: the sum of all cash flows is $17,863.65. The investment is $13,000, therefore $4,863.65 must be interest (profit). The goal is (1) to determine the annual breakdown between interest (profit), recovery of capital (principal) from the cash flows and (2) show that compound interest is being earned on the investment balance at an interest rate equal to the IRR. This exercise should prove that the IRR is equivalent to an interest rate of 10% compounded annually. It should also demonstrate the equivalence between an IRR and compound interest.

(b) IRR = 10% (annual rate, compounded annually)

(c) Proof:

Beginning of Year	Investment	10% Interest	Cash Flow	Recovery of Capital (ROC)	End of Year (Balance)
1	13,000.00	$1,300.00	$ 5,000.00	$ 3,700.00	$ 9,300.00
2	9,300.00	930.00	1,000.00	70.00	9,230.00
3	9,230.00	923.00	-0-	-0-	10,153.00*
4	10,153.00	1,015.30	5,000.00	3,984.70	6,168.30
5	6,168.30	616.83	6,000.00	5,383.17	785.13
6	785.13	78.51	863.65	785.14	-0-
		$4,863.65	$17,863.76	$13,000.00	

* Note: Because the cash flow in year 3 is zero, interest must be accrued on the balance of $9,230 during year 3 and added to the investment balance.