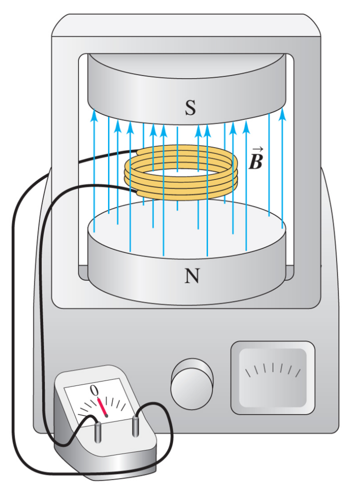
Chapter 29: Electromagnetic Induction

Induction Experiments

- Suppose you connected a coil of wire to a galvanometer and measured the current through the coil while it is placed in an electromagnet.
- If the magnetic field $\mathbf{B}$ is constant, and the shape, location, and orientation of the coil do not change, then no current is measured on the galvanometer.
- Changing any one of the aforementioned factors will result in a current through the coil!
- The reason why this happens is because the magnetic flux through the coil Φ_B changes whenever we alter the strength of the magnetic field, the shape of the coil, move the coil through a nonuniform magnetic field, or change the orientation of the coil.



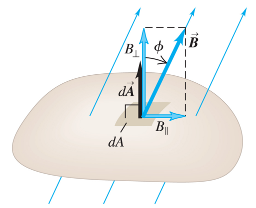
Magnetic Flux Review

- Recall the definition of magnetic flux through a surface:

$$\Phi_B = \int \mathbf{B} \cdot \mathrm{d}\mathbf{A} = \int B \,\mathrm{d}A \cos\phi.$$

- If $\mathbf{B}$ is uniform over a flat area $\mathbf{A}$, then

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\phi.$$

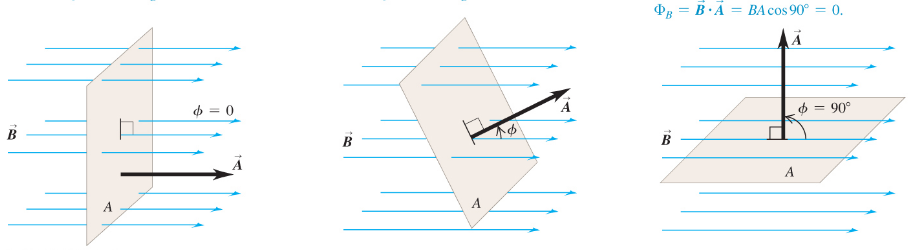


Magnetic flux through element of area $d\vec{A}$:
$d\Phi_B = \vec{B} \cdot d\vec{A} = B_\perp dA = B\,dA\cos\phi$

Surface is face-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are parallel (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.

Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{B}$ and $\vec{A}$ is ϕ.
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\phi$.

Surface is edge-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are perpendicular (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 90°$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos 90° = 0$.



Faraday's Law

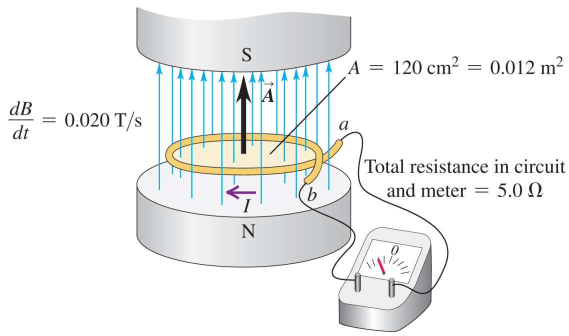
- **Faraday's law of induction** states that the induced emf $\mathcal{E}$ in a closed loop is equal to the negative time rate of change of the magnetic flux through the loop:

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}.$$

- This emf can drive a current around the loop.
- Recall that the unit of magnetic flux is the weber (Wb).
- $1\ \mathrm{T} \cdot \mathrm{m}^2 = 1$ Wb, so 1 V $= 1$ Wb/s.

Example 29.1: Emf and Current Induced in a Loop

The magnetic field between the poles of the electromagnet in the figure below is uniform at any time, but its magnitude is increasing at the rate 0.020 T/s. The area of the conducting loop in the field is 120 cm^2, and the total circuit resistance, including the meter, is $5.0\ \Omega$. (a) Find the induced emf and the induced current in the circuit. (b) If the loop is replaced by one made of an insulator, what effect does this have on the induced emf and induced current?



(a) We will take the area vector of the loop $\mathbf{A}$ to point in the same direction of the magnetic field, so that the magnetic flux through the loop is $\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA$. The time rate of change of the magnetic flux through the loop is then

$$\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(BA) = \frac{\mathrm{d}B}{\mathrm{d}t}A = (0.020\ \text{T/s})(0.012\ \text{m}^2) = 2.4\times10^{-4}\ \text{V} = 0.24\ \text{mV}.$$

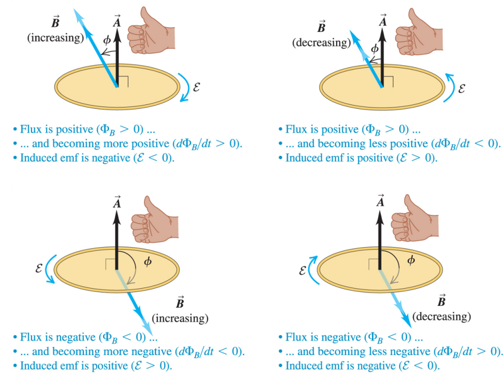
Ignoring the sign from Faraday's law, the induced current in the loop is

$$I = \frac{\mathcal{E}}{R} = \frac{2.4\times10^{-4}\ \text{V}}{5.0\ \Omega} = 4.8\times10^{-5}\ \text{A} = 0.048\ \text{mA}.$$

(b) If we change to an insulating loop, the resistance of the loop then becomes very high. There will still be an induced emf according to Faraday's law, and its value will be the same as before. But the induced current will drop significantly due to the increased resistance of the loop.

Direction of Induced Emf

- The direction of the current induced by the emf can be found as follows:
  1. Define a positive direction for the vector area $\mathbf{A}$.
  2. Determine the sign of Φ_B and its time rate of change $\mathrm{d}\Phi_B/\mathrm{d}t$.
  3. Determine the sign of $\mathcal{E}$ from Faraday's law.
  4. Curl your fingers on your right hand with your thumb pointing in the direction of $\mathbf{A}$. Your fingers point in the direction of a positive emf or current.



- If a coil has N identical turns and if the flux varies at the same rate through each turn the *total* rate of change through all N turns is N times that for a single turn:

$$\mathcal{E} = -N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}.$$

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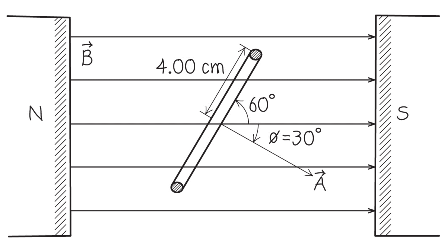
Example 29.2: Magnitude and Direction of an Induced Emf

A 500 loop circular wire coil with radius $r = 4.00$ cm is placed between the poles of a large electromagnet. The magnetic field is uniform and makes an angle of 60° with the plane of the coil; it decreases at 0.200 T/s. What are the magnitude and direction of the induced emf?



Since the plane of the coil makes an angle of 60° with the magnetic field, the angle ϕ between the area vector $\mathbf{A}$ and the magnetic field $\mathbf{B}$ is $\phi = 30^\circ$, where we have chosen $\mathbf{A}$ to point in the direction as indicated in the figure. The emf is then

$$\mathcal{E} = -N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -N\frac{\mathrm{d}B}{\mathrm{d}t}A\cos\phi = -500(-0.200\ \text{T/s})\pi(0.0400\ \text{m})^2\cos(30^\circ) = 0.435\ \text{V}.$$

Since the sign of the emf $\mathcal{E}$ is positive, if you point your right thumb in the direction of $\mathbf{A}$, the induced emf follows the direction of your fingers curled around your thumb. Thus, if viewing the coil from above with $\mathbf{A}$ pointing out of the page, the emf would be counterclockwise.

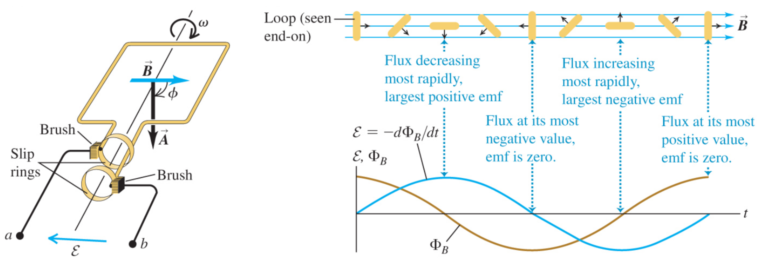
A Simple Alternator

- An **alternator** is a device that generates an emf by rotating a rectangular loop at an angular speed ω through a constant magnetic field $\mathbf{B}$.
- The emf can be determined by considering the magnetic flux as a function of time.
  - If the angle $\phi = 0$ with respect to $\mathbf{B}$ at $t = 0$, then the flux is

$$\Phi_B(t) = BA\cos\phi = BA\cos\omega t.$$

  - The emf is therefore

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}(BA\cos\omega t) = \omega BA\sin\omega t.$$



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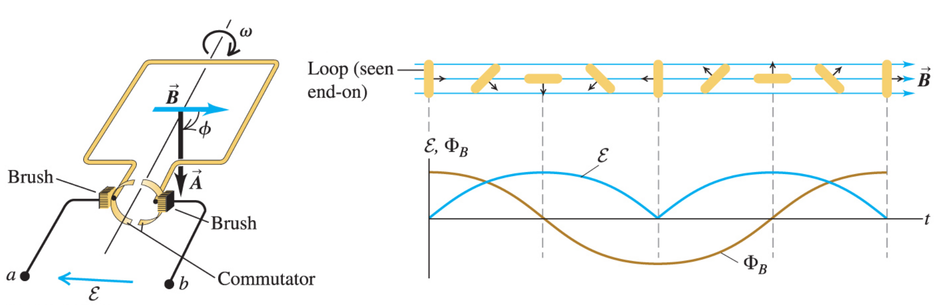
Example 29.4: A Dc Generator and Back Emf in a Motor

The alternator shown above produces a sinusoidally varying emf and hence an alternating current. The figure below shows a *direct-current* (dc) *generator* that produces an emf that always has the same sign. The arrangement of split rungs, called a *commutator*, reverses the connections to the external circuit at angular positions at which the emf reverses. The graph below shows the resulting emf. Commercial dc generators have a large number of coils and commutator segments, smoothing out the bumps in the emf so that the terminal voltage is not only one-directional, but also practically constant. This brush-and-commutator arrangement is the same as that in the direct-current motor discussed previously. The motor's *back emf* is just the emf induced by the changing magnetic flux through its rotating coil. Consider a motor with a square, 500 turn coil 10.0 cm on a side. If the magnetic field has magnitude 0.200 T, at what rotation speed is the *average* back emf of the motor equal to 112 V.



First, we start by noting that the absolute value of the back emf is

$$|\mathcal{E}| = N\left|\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}\right| = N\omega BA|\sin\omega t|.$$

The average value of $\sin\omega t$ can be determined by integrating it over half a cycle, from $t = 0$ to $t = T/2$, and dividing the result by the elapsed time π/ω. Thus, we have

$$(|\sin\omega t|)_{\text{avg}} = \frac{\omega}{\pi}\int_0^{\pi/\omega} \sin\omega t\,\mathrm{d}t = \frac{2}{\pi}.$$

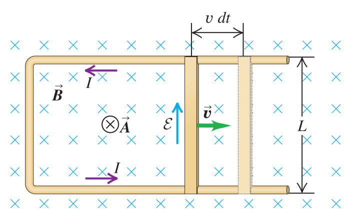
The average back emf is then

$$\mathcal{E}_{\text{avg}} = \frac{2N\omega BA}{\pi},$$

which gives

$$\omega = \frac{\pi\mathcal{E}_{\text{avg}}}{2NBA} = \frac{\pi(112\text{ V})}{2(500)(0.200\text{ T})(0.100\text{ m})^2} = 176\text{ rad/s}.$$

The Slidewire Generator



- A U-shaped conductor with a metal rod of length L that is allowed to slide along the rails in a uniform magnetic field is called a **slidewire generator.**
- The magnetic flux through the loop bounded by the rod changes as the rod moves with velocity v.
- The induced emf is

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -B\frac{\mathrm{d}A}{\mathrm{d}t} = -B\frac{Lv\,\mathrm{d}t}{\mathrm{d}t} = -BLv.$$

- The minus sign tells us that the emf is directed *counterclockwise* around the loop.

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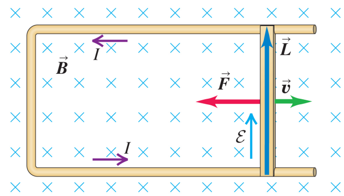
Work and Power in the Slidewire Generator

- If the rod has resistance R, then the induced current is $I = |\mathcal{E}|/R = BLv/R$, and power dissipated in circuit is



$$P_{\text{dissipated}} = I^2 R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2 L^2 v^2}{R}.$$

- To sustain the rod's motion, an external force must be applied to exactly cancel the magnetic force $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$ acting on the current in the rod. So the applied force's magnitude is

$$F = ILB = \frac{BLv}{R} LB = \frac{B^2 L^2 v}{R}.$$

- The rate at which work must be done to keep the rod moving is then

$$P_{\text{applied}} = Fv = \frac{B^2 L^2 v^2}{R}.$$

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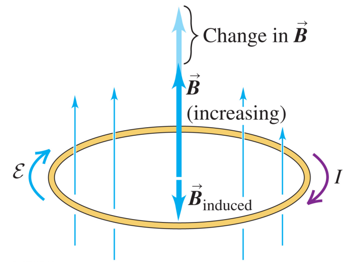
Lenz's Law

- Lenz's law is a convenient way of determining the direction of an induced current or emf.
- It states that **the direction of any magnetic induction effect is such as to oppose the cause of the effect.**



- Consider a loop in a magnetic field $\mathbf{B}$ that is pointed upwards and increasing in strength.
  - The increasing magnetic flux induces an emf $\mathcal{E}$, which in turn produces a clockwise (as viewed from above) current I.
  - The magnetic field $\mathbf{B}_{\text{induced}}$ produced by this current points in the direction opposite of $\mathbf{B}$, opposing the increasing strength of $\mathbf{B}$ in the upwards direction.
  - The strength of the current I (and, by extension, $\mathbf{B}_{\text{induced}}$) depends on the resistance R of the loop.

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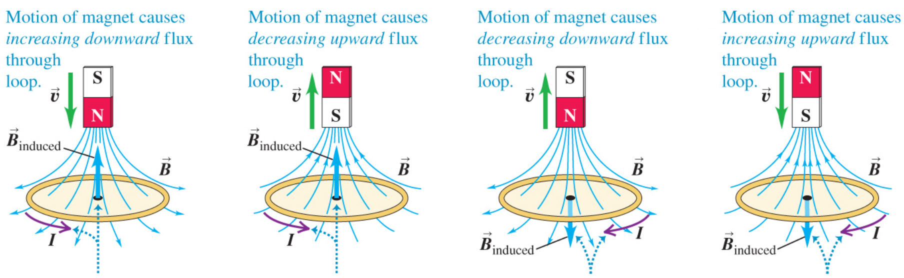
Lenz's Law and the Direction of Induced Current

- Another example of Lenz's law is the induced current in a loop due to the motion of a bar magnet moving towards or away from the loop.
- The direction of the induced current depends on both the orientation of the bar magnet, and which direction it is moving with respect to the loop.



The induced magnetic field is *upward* to oppose the flux change. To produce this induced field, the induced current must be *counterclockwise* as seen from above the loop.

The induced magnetic field is *downward* to oppose the flux change. To produce this induced field, the induced current must be *clockwise* as seen from above the loop.

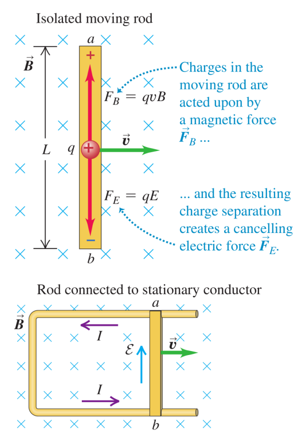
Motional Emf

- Consider a conducting rod of length L moving at speed v through a uniform magnetic field $\mathbf{B}$ perpendicular to the rod.
  - The magnetic force $F = qvB$ causes free charges in the rod to accumulate at the ends, with positive and negative charges on opposite sides.
  - This creates an electric field that eventually balances the magnetic force: $qE = qvB \to E = vB$.
  - If we connect the rod to a U-shaped conductor, the E field will produce a current I in a closed loop.
  - The potential difference across the bar due to E creates an emf, known as a **motional emf**, given by

$$|\mathcal{E}| = EL = vBL.$$

- This is the same result as with slidewire generator:

$$|\mathcal{E}| = \left|-\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}\right| = B\left|\frac{\mathrm{d}A}{\mathrm{d}t}\right| = BL\left|\frac{\mathrm{d}x}{\mathrm{d}t}\right| = BLv.$$



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

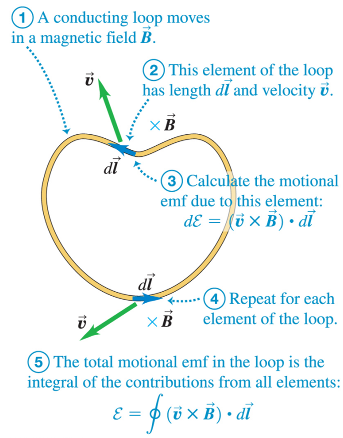
Motional Emf: General Form

- Motional emf can be generalized to conductors of any shape, and for non-uniform magnetic fields, provided that $\mathbf{B}$ does not vary with time.
- Each element $\mathrm{d}\mathbf{l}$ of the conductor contributes an emf $\mathrm{d}\mathcal{E}$ given by

$$\mathrm{d}\mathcal{E} = (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l}\,.$$

- For a closed conducting loop, we can integrate over all the emf contributions:

$$\mathcal{E} = \oint (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l}\,.$$



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Example 29.9: Motional Emf in the Slidewire Generator

Suppose the moving rod in the slidewire generator figure is 0.10 m long, the velocity v is 2.5 m/s, the total resistance of the loop is $0.030\ \Omega$, and B is 0.60 T. Find the motional emf, the induced current, and the force acting on the rod.

For the motional emf, we have

$$\mathcal{E} = vBL = (2.5 \text{ m/s})(0.60 \text{ T})(0.10 \text{ m}) = 0.15 \text{ V}.$$

The current induced in the loop is therefore

$$I = \frac{\mathcal{E}}{R} = \frac{0.15 \text{ V}}{0.030\ \Omega} = 5.0 \text{ A}.$$

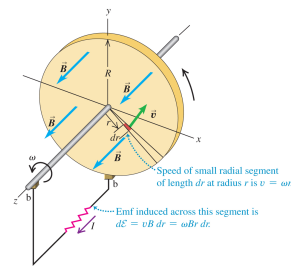
Finally, since the magnetic field $\mathbf{B}$ and the length vector $\mathbf{L}$ are perpendicular to each other, the magnitude of the force is

$$F = ILB = (5.0 \text{ A})(0.10 \text{ m})(0.60 \text{ T}) = 0.30 \text{ N},$$

with the force pointing in the direction opposite of the rod's motion.

The Faraday Disk Dynamo

- A **Faraday disk dynamo** is a conducting disk of radius R that rotates with constant angular velocity ω in a uniform magnetic field $\mathbf{B}$ perpendicular to the disk.
- This is another example of motional emf. An emf is generated between the center and the rim of the disk.
  - The radial velocity $\mathbf{v}$ and $\mathbf{B}$ are perpendicular, so $\mathrm{d}\mathcal{E} = (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l} = \omega r B \, \mathrm{d}r$.
  - We can integrate over the radius to get the emf:



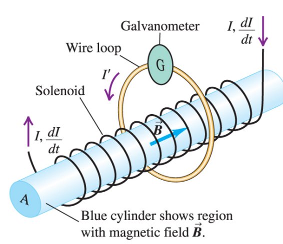
$$\mathcal{E} = \int_0^R \omega B r \, \mathrm{d}r = \frac{1}{2}\omega B R^2.$$

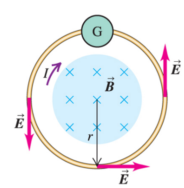
- The result is consistent with Faraday's law when considering the area A swept out by an angle θ:

$$A = \frac{\theta}{2}R^2 \quad \rightarrow \quad |\mathcal{E}| = \left| -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \right| = \left| \frac{\mathrm{d}A}{\mathrm{d}t} \right| B = \frac{1}{2} \left| \frac{\mathrm{d}\theta}{\mathrm{d}t} \right| B R^2 = \frac{1}{2}\omega B R^2.$$

Induced Electric Fields

- An induced emf also occurs whenever there is a changing flux through a stationary conductor.
- Suppose we measured the current through a wire loop with a solenoid passing through it, with a non-constant current through the solenoid.
  - If the solenoid has cross-sectional area A, n turns per unit length, and current $I(t)$, then the flux through the loop is $\Phi_B(t) = B(t)A = \mu_0 n A I(t)$.
  - The emf and current induced in the loop are then





$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -\mu_0 n A \frac{\mathrm{d}I}{\mathrm{d}t}, \qquad I_{\text{ind}} = \frac{|\mathcal{E}|}{R}.$$

- There's no magnetic field outside of the solenoid, so why do charges in the loop move? There is a *non-conservative* electric field induced in the wire, and its closed line integral is equal to $\mathcal{E}$:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = \mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \quad \rightarrow \quad E = \frac{1}{2\pi r} \left| \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \right|.$$

Example 29.11: Induced Electric Fields

Suppose the long solenoid in the figure has 500 turns per meter and cross sectional area 4.0 cm^2. The current in the windings is increasing at 100 A/s. (a) Find the magnitude of the induced emf in the wire loop outside the solenoid (b) Find the magnitude of the induced electric field within the loop if its radius is 2.0 cm.

(a) The induced emf is

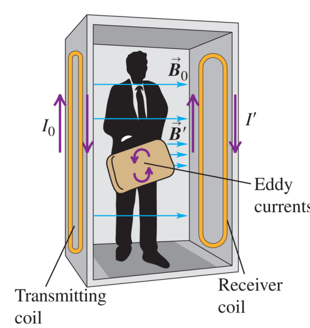
$$\begin{aligned}\mathcal{E} &= -\mu_0 n A \frac{\mathrm{d}I}{\mathrm{d}t} \\ &= -(4\pi \times 10^{-7}\ \text{Wb/A}\cdot\text{m})(500\ \text{turns/m})(4.0 \times 10^{-4}\ \text{m}^2)(100\ \text{A/s}) \\ &= -25 \times 10^{-6}\ \text{V}.\end{aligned}$$

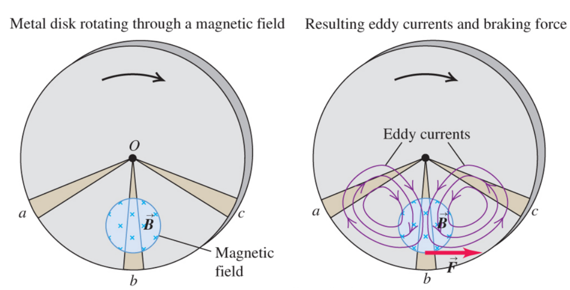
(b) Using the expression previously obtained for the electric field due to a changing magnetic flux, we have

$$E = \frac{|\mathcal{E}|}{2\pi r} = \frac{25 \times 10^{-6}\ \text{V}}{2\pi(2.0 \times 10^{-2}\ \text{m})} = 2.0 \times 10^{-4}\ \text{V/m}.$$

Eddy Currents

- A conducting material moving through a magnetic field or located in a changing magnetic field can experience induced currents that flow throughout the material.
- Such currents are known as **eddy currents**, because they resemble swirling eddies in flowing water.
- The effect is used in devices such as metal detectors, which detect eddy currents in metal objects.





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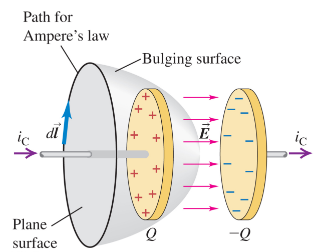
Displacement Current (1 of 3)

- Ampère's law is incomplete, as can be shown by considering the process of charging a capacitor.
- We may draw an Ampèrian loop around the wire connected to the capacitor, and find that the current enclosed I_{enc} is equal to the conduction current i_{C} flowing through the conductor.



- But we can also consider another surface bounded by our Ampèrian loop that bulges away from the wire and goes into the gap between the capacitors.
- If we consider the current through the bulging surface, we get no current, which leads to a contradiction.

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Displacement Current (2 of 3)

- The key to resolving this issue is to consider how the electric field between the plates is changing. When a capacitor is charging, the electric field and flux between the plates is increasing.
- We can write the charge q on the plates in terms of the electric flux Φ_E through the bulging surface bounded by our Ampèrian loop by considering the capacitance $C = \epsilon A/d$ and the potential difference V across the capacitor:

$$q = CV = \frac{\epsilon A}{d}(Ed) = \epsilon E A = \epsilon \Phi_E.$$

- Since $i_{\text{C}} = \mathrm{d}q/\mathrm{d}t$, we can take the derivative of the previous expression for q to get

$$i_{\text{C}} = \frac{\mathrm{d}q}{\mathrm{d}t} = \epsilon \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}.$$

- We can therefore define the **displacement current** i_{D} as

$$i_{\text{D}} = \epsilon \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} \quad \rightarrow \quad j_{\text{D}} = \epsilon \frac{\mathrm{d}E}{\mathrm{d}t}.$$

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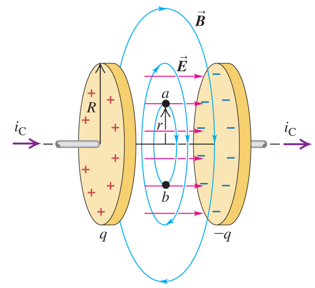
Displacement Current (3 of 3)

- Now Ampère's law is complete and reads

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \mu_0 (i_\mathrm{C} + i_\mathrm{D})_\mathrm{enc} = \mu_0 \left(i_\mathrm{C} + \epsilon \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} \right)_\mathrm{enc}.$$



- We can use this result to determine the magnetic field between the capacitor plates due to the changing electric field:

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} = B(2\pi r) = \mu_0 \frac{r^2}{R^2} i_\mathrm{C} \quad \rightarrow \quad B = \frac{\mu_0}{2\pi}\frac{r}{R^2} i_\mathrm{C}.$$

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Maxwell's Equations of Electromagnetism

- All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell's equations:**

$$\oint \mathbf{E}\cdot \mathrm{d}\mathbf{A} = \frac{q_\mathrm{enc}}{\epsilon_0} \quad \text{(Gauss's law)},$$

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{A} = 0 \quad \text{(Gauss's law for magnetism)},$$

$$\oint \mathbf{E}\cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \quad \text{(Faraday's law)},$$

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \mu_0 \left(i_\mathrm{C} + \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} \right)_\mathrm{enc} \quad \text{(Ampère's law)}.$$

- Combined with the Lorentz force $\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$, we have *all* of the fundamental relationships of electrodynamics.

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Maxwell's Equations in Empty Space

- In empty space, where there is no charge ($q_{\text{enc}} = 0$) and no current ($i_{\text{C}} = 0$), Maxwell's equations exhibit a unique symmetry:

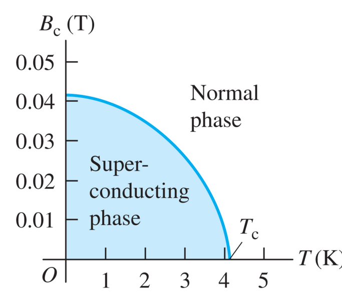
$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = 0, \qquad \oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0,$$

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \mathbf{B} \cdot \mathrm{d}\mathbf{A}\,, \qquad \oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 \epsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \int \mathbf{E} \cdot \mathrm{d}\mathbf{A}\,.$$

- The two Gauss's laws for electric and magnetic fields tell us that in empty space, electric and magnetic field lines must form closed loops.
- Faraday's law and Ampère's law tell us that a changing magnetic flux creates an electric field, and a changing electric flux creates a magnetic field.

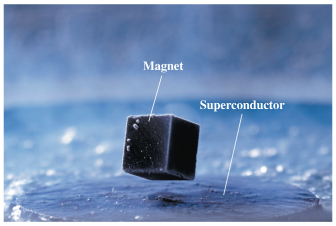
Superconductivity

- A **superconductor** loses all of its electrical resistance when it is cooled below its critical temperature T_c.
- The critical temperature changes when the material is placed in an externally produced magnetic field.
- As the magnitude of the external field B_0 increases, the superconducting transition occurs at lower and lower temperatures.



- The minimum magnitude of B_0 required to ensure that no superconducting transition occurs below T_c is called the critical field, denoted by B_c.

The Meissner Effect



- Superconductors have unusual magnetic properties when brought to the critical temperature.
  - When a superconducting material is in an external magnetic field $\mathbf{B}_0$ at $T > T_c$, there is a magnetic field present in the material.
  - When $T < T_c$, the superconductor expels all the magnetic flux, and $\mathbf{B} = \mathbf{0}$ inside of the superconductor.
  - A superconductor with $T < T_c$ therefore exerts a repulsive force on magnets.