Math 170E: Homework 1

Due: Fri 20th Jan. by 11:59pm PDT via Gradescope

Submit answers to all problems via Gradescope. The reader will grade three problems each out of five points. Up to five further points will be awarded based on the proportion of the remaining problems that are completed.

Please make sure that your submission is readable. If your pencil is too faint, get a thicker one. If your handwriting is cramped and small, write bigger and use more paper. Please use simple plain paper or lined paper (e.g. please avoid graph paper etc.). It is your responsibility to ensure that your submission is readable. If we cannot read a solution, we may refuse to grade it. Thank you!

I encourage you to discuss and work on problems with other students in the class. Nevertheless, the solutions you present have to be your own. In particular, if the solution you present is identical to someone else's, or it is identical to some other resource (book, online, etc.), this will be considered cheating.

1. Consider the trial on which a 3 is first observed in successive rolls of a six-sided die. Let A be the event that 3 is observed on the first trial. Let B be the event that at least two trials are required to observe a 3. Assuming that each side has probability $1/6$, find:

   (a) $\mathbb{P}(A)$

   (b) $\mathbb{P}(B)$

   (c) $\mathbb{P}(A \cup B)$

2. An insurance company looks at its auto insurance customers and finds that (a) all insure at least one car, (b) 85% insure more than one car, (c) 23% insure a sports car, and (d) 17% insure more than one car, including a sports car. Find the probability that a customer selected at random insures exactly one car and it is not a sports car.

3. Let A and B be independent events with $\mathbb{P}(A) = 0.7$ and $\mathbb{P}(B) = 0.2$. Compute:

   (a) $\mathbb{P}(A \cap B)$

   (b) $\mathbb{P}(A \cup B)$

   (c) $\mathbb{P}(A' \cup B')$

4. If $\mathbb{P}(A) = 0.8$, $\mathbb{P}(B) = 0.5$, and $\mathbb{P}(A \cup B) = 0.9$, are A and B independent events? Why or why not?

5. Show that if A, B, and C are mutually independent, then the following pairs of events are independent: A and $(B \cap C)$, A and $(B \cup C)$, A' and $(B \cap C')$. Show also that A', B', and C' are mutually independent.

6. Let A and B be two events.

   (a) If the events A and B are mutually exclusive, are A and B always independent? If the answer is no, can they ever be independent? Explain.

   (b) If $A \subseteq B$, can A and B ever be independent events? Explain. (Recall that $A \subseteq B$ means either $A \subset B$ or $A = B$).

7. Each of three football players will attempt to kick a field goal from the 25-yard line. Let A_j denote the event that the field goal is made by player j, $j = 1, 2, 3$. Assume that A_1, A_2, A_3 are mutually independent and that $\mathbb{P}(A_1) = 0.5$, $\mathbb{P}(A_2) = 0.7$, $\mathbb{P}(A_3) = 0.6$.

   (a) Compute the probability that exactly one player is successful.

   (b) Compute the probability that exactly two players make a field goal (i.e., one misses).

(Hint: you may find the results of Problem 6 helpful).

8. Let $A, B, C \subseteq \Omega$ be events. Show that

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C).$$

9. Let A, B be independent events. Prove that A' and B' are independent events.

10. An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. The classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

    (a) If a student is chosen randomly, what is the probability that they are not in any of the language classes?

    (b) If a student is chosen randomly, what is the probability that they are taking exactly one language class?

    (c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class?