4.3.2-Method of Undetermined Coefficients

#UCLA #Y1Q3 #Math33B

Method of Undetermined Coefficients

Key Definitions

Method of Undetermined Coefficients - used to find particular sol. to 4.3.1-Inhomogeneous 2nd Order Linear Differentials if:

- p, q are *constant functions*

Trial Solution - arbitrary possible solution given by restraints:

- must include forcing term $g(t)$
- must be "closed" (similar) under derivation (e.g. trig funcs.)

Superposition Principle - used to deal with lin. combs. of forcing terms

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Method of Undetermined Coefficients

Given 4.3.1-Inhomogeneous 2nd Order Linear Differentials where p, q are constant and forcing term $g(t)$ is "closed" under derivation, we use a **trial solution** containing an undetermined coeff.

Selecting a Trial Function

The trial solution depends on the forcing term, if $g(t)$ *is not* a sol.:

1. $g(t) = e^{rt}$

$$y_p(t) = ae^{rt}$$

2. $g(t) = A\cos \omega t + B\sin \omega t$

$$y_p(t) = a\cos \omega t + b\sin \omega t$$

3. $g(t) = P(t)$

$$y_p(t) = p_0(t)$$

4. $g(t) = P(t)\cos \omega t$ or $g(t) = P(t)\sin \omega t$

$$y_p(t) = p_0(t)\cos \omega t + p_1(t)\sin \omega t$$

5. $g(t) = e^{rt}\cos \omega t$ or $g(t) = e^{rt}\sin \omega t$

$$y_p(t) = e^{rt}(a\cos \omega t + b\sin \omega t)$$

6. $g(t) = e^{rt}P(t)\cos \omega t$ or $g(t) = e^{rt}P(t)\sin \omega t$

$$y_p(t) = e^{rt}(p_0(t)\cos \omega t + p_1(t)\sin \omega t)$$

s.t. $A, B, a, b, r, \omega \in \mathbb{R}$ and $P(t), p_0(t), p_1(t)$ are polynomials of the same degree

if $g(t)$ *is* a sol. use

$$ty_p(t) \quad \text{or} \quad t^2y_p(t)$$

Attempting a Solution

Set the trial equal to the forcing term and solve for the undetermined coefficient to find that the trial function is a particular solution

$$y_p(t) = g(t)$$

Superposition Principle

if $y_f(t)$ is a part. sol. to $y'' + py' + qy = f(t)$ and $y_g(t)$ is a part. sol. to $y'' + py' + qy = g(t)$, and given:

$$y'' + py' + qy = \mathrm{a}\,f(t) + \mathrm{b}\,g(t)$$

then the general solution is:

$$y(t) = \mathrm{a}\,y_f(t) + \mathrm{b}\,y_g(t)$$