

Math 170E: Homework 9

Due: Tue 21st March by 11:59pm PDT via Gradescope

The reader will grade three problems each out of five points. Up to five further points will be awarded based on the proportion of the remaining problems that are completed.

Please make sure that your submission is readable. If your pencil is too faint, get a thicker one. If your handwriting is cramped and small, write bigger and use more paper. Please use simple plain paper or lined paper (e.g. please avoid graph paper etc.). It is your responsibility to ensure that your submission is readable. If we cannot read a solution, we may refuse to grade it. Thank you!

I encourage you to discuss and work on problems with other students in the class. Nevertheless, the solutions you present have to be your own. In particular, if the solution you present is identical to someone else's, or it is identical to some other resource (book, online, etc.), this will be considered cheating.

1. Please go to myUCLA and complete the class evaluations. I really appreciate your feedback. Thank you!
2. Let X_1, X_2 be independent random variables representing lifetimes (in hours) of two key components of a device that fails when and only when both components fail. Say each X_j has an exponential distribution with mean 1000. Let $Y_1 = \min(X_1, X_2)$ and $Y_2 = \max(X_1, X_2)$, so that the space of Y_1, Y_2 is $0 < y_1 < y_2 < \infty$.
 - (a) Find $G(y_1, y_2) = \mathbb{P}(Y_1 \leq y_1, Y_2 \leq y_2)$
 - (b) Compute the probability that the device fails after 1200 hours; that is, compute $\mathbb{P}(Y_2 > 1200)$.
3. A company provides earthquake insurance. The premium X is modelled by the pdf

$$f_X(x) = \frac{x}{25}e^{-\frac{x}{5}}, \quad 0 < x < \infty,$$

while the claims Y have the PDF

$$f_Y(y) = \frac{1}{5}e^{-\frac{y}{5}}, \quad 0 < y < \infty,$$

If X and Y are independent, find the PDF of $Z = X/Y$.

4. Let X_1, X_2, X_3 be three independent random variables with binomial distributions Binomial(4, 1/2), Binomial(6, 1/3) and Binomial(12, 1/6), respectively. Find:

- (a) $\mathbb{P}(X_1 = 2, X_2 = 2, X_3 = 5)$
 - (b) $\mathbb{E}[X_1 X_2 X_3]$
 - (c) The mean and variance of $Y = X_1 + X_2 + X_3$.
5. Two components operate in parallel in a device, so the device fails when and only when both components fail. The lifetimes, X_1 and X_2 , of the respective components are independent and identically distributed with an exponential distribution with $\theta = 2$. The cost of operating the device is $Z = 2Y_1 + Y_2$, where $Y_1 = \min(X_1, X_2)$ and $Y_2 = \max(X_1, X_2)$. Compute $\mathbb{E}(Z)$.
6. If X is a random variable with mean 33 and variance 16, use Chebyshev's inequality to find:
- (a) A lower bound for $\mathbb{P}(23 < X < 43)$.
 - (b) An upper bound for $\mathbb{P}(|X - 33| \geq 14)$.
7. If the distribution of Y is Binomial($n, 1/4$), give a lower bound for $\mathbb{P}(|Y/n - 0.25| < 0.05)$ when $n = 100, 1000, 10000$.