MIDTERM 1 FORMULA SHEET

- $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ for $n \in \mathbb{Z}_{\geq 0}$
- $(x+y) \mod n = [(x \mod n) + (y \mod n)] \mod n$
- $(x \cdot y) \mod n = [(x \mod n) \cdot (y \mod n)] \mod n$
- $x \mod n = 0 \iff x$ is divisible by $n \iff x = k \cdot n$ for some $k \in \mathbb{Z}$ (assuming $n \in \mathbb{Z}_{>0}$ throughout)
- $P(n,r) = \frac{n!}{(n-r)!}$ for $r \leq n \in \mathbb{Z}_{\geq 0}$
  - ways to select r-permutations from an n element set
- $C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$ for $r \leq n \in \mathbb{Z}_{\geq 0}$
  - ways to select r-combinations from an n element set
- $\binom{k+t-1}{k}$ for $k, t \in \mathbb{Z}_{\geq 0}$
  - ways to make k unordered selections from a set containing t elements, allowing repetition
- $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ for $n, k \in \mathbb{Z}_{\geq 0}$
  - different ways to choose k-combinations of an $n+1$-element set
- $\frac{n!}{n_1! n_2! \cdots n_k!}$ for $n, n_i \in \mathbb{Z}_{\geq 0}$ for $1 \leq i \leq k$ and $\sum_{i=1}^{k} n_i = n$
  - The number of orderings of n elements where n_1 elements are of type 1, n_2 elements are of type 2, ..., and n_k elements are of type k
- $\binom{k+t-1}{k} = \binom{k+t-1}{t-1}$ for $k, t \in \mathbb{Z}_{\geq 0}$.
  - The number of ways to select k elements from a set X of t elements, allowing repetition.
- $\binom{n+m}{n} = \binom{n+m}{m}$.
  - The number of grid walks from $(0,0)$ to (n,m) for $n, m \in \mathbb{Z}_{\geq 0}$.
- For X, Y finite sets, $|X \cup Y| = |X| + |Y| - |X \cap Y|$.
  - Inclusion-Exclusion Principle
  - when $|X \cap Y| = \emptyset$, this is the Addition Principle
- $|X| = |X_1| \cdot |X_2| \cdot \ldots \cdot, |X_k|$
  - Multiplication Principle
  - when analyzing that X breaks down into independent steps $X_1, X_2, \ldots X_k$
- For X, Y finite sets, let $f : X \to Y$ where $\left\lceil \frac{|X|}{|Y|} \right\rceil = k$. Then there is a k-subset $\{x_1, x_2, \ldots, x_k\} \subseteq X$ such that $f(x_1) = f(x_2) = \ldots f(x_k)$.
  - Generalized Pigeonhole Principle

Note: you do not need to re-prove statements, principles, and theorems given in class. You need only justify *why* they apply to the situations where you are using them.