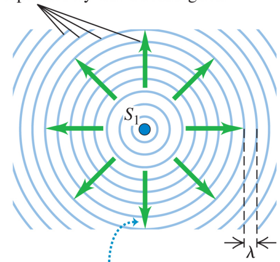
Chapter 35: Interference

Principle of Superposition

- The term **interference** refers to any situation in which two or more waves overlap in space.
- When this occurs, the total wave at any point at any instant of time is governed by the **principle of superposition.**
- **When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.**

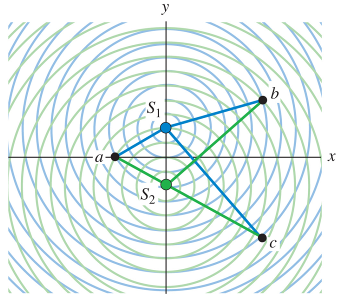
Wave Fronts From a Disturbance

- Interference effects are most easily seen when we combine sinusoidal waves with a single frequency and wavelength.
- Shown is a "snapshot" of a single source S_1 of sinusoidal waves and some of the wave fronts produced by this source.



Constructive and Destructive Interference

- Shown are two identical sources of monochromatic waves, S_1 and S_2.
- The two sources are permanently **in phase;** they vibrate in unison.
- Two monochromatic sources of the same frequency and with a constant phase relationship (not necessarily in phase) are said to be **coherent.**

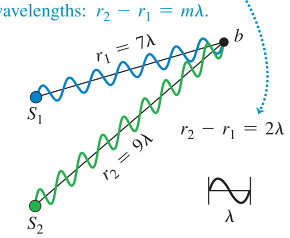


- The two distances from S_1 to a and from S_2 to a are *equal;* waves from the two sources thus require equal times to travel to a. Hence waves that leave S_1 and S_2 in phase arrive at a in phase, and the total amplitude at a is *twice* the amplitude of each individual wave.

Conditions for Constructive Interference (1 of 3)

- The distance from S_2 to point b is exactly two wavelengths greater than the distance from S_1 to b.
- In general, when waves from two or more sources arrive at a point *in phase*, they reinforce each other: The amplitude of the resultant wave is the *sum* of the amplitudes of the individual waves.
- This is called **constructive interference.**

Conditions for constructive interference: Waves interfere constructively if their path lengths differ by an integer number of wavelengths: $r_2 - r_1 = m\lambda$.



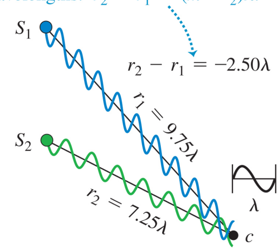
- Let the distance from S_1 to any point P be r_1, and let the distance from S_2 to P be r_2. For constructive interference to occur at P, the path difference $r_2 - r_1$ for the two sources must be an integer multiple of the wavelength λ:

$$r_2 - r_1 = m\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots).$$

Conditions for Constructive Interference (2 of 3)

- The distance from S_1 to point c is a **half-integer** number of wavelengths greater than the distance from S_2 to c.
- Waves from the two sources arrive at point c exactly a half-cycle out of phase. The resultant amplitude is the *difference* between the two individual amplitudes.
- This is called **destructive interference.**

Conditions for destructive interference: Waves interfere destructively if their path lengths differ by a half-integer number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$.

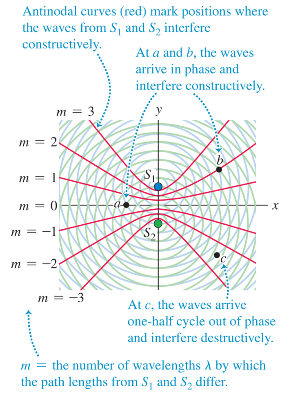


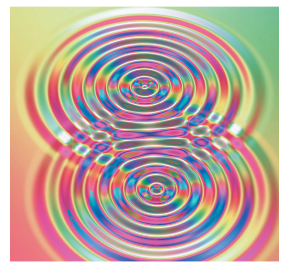
- For destructive interference to occur at P, the path difference $r_2 - r_1$ for the two sources must be an odd half-integer multiple of the wavelength λ:

$$r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots).$$

Conditions for Constructive Interference (3 of 3)

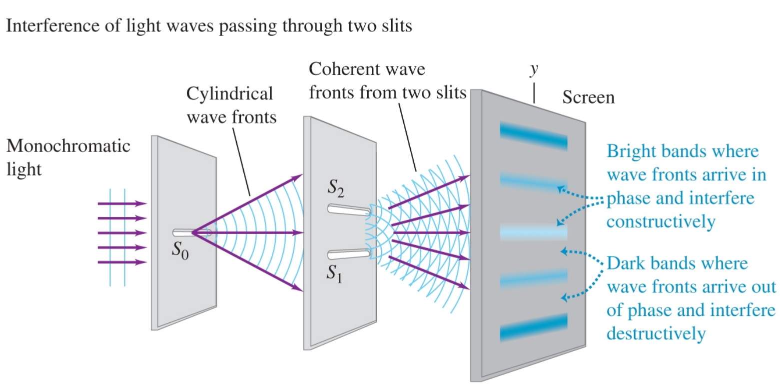
- Shown are two identical sources of monochromatic waves, S_1 and S_2, which are in phase.
- The red curves show all positions where constructive interference occurs, which are called **antinodal curves.**
- Not shown are the **nodal curves,** which are the curves that show where destructive interference occurs.
- The concepts of constructive interference and destructive interference apply to water waves as well as to light waves and sound waves.





Two-Source Interference of Light (1 of 2)

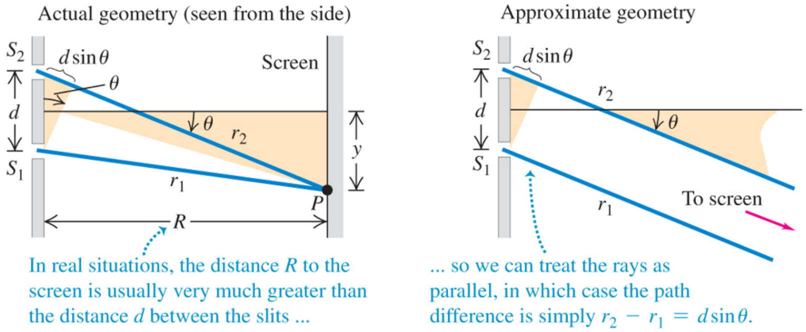
- Shown below is one of the earliest quantitative experiments to reveal the interference of light from two sources, first performed by Thomas Young in 1800.
- The interference of waves from slits S_1 and S_2 produces a pattern on the screen.



Two-Source Interference of Light (2 of 2)

- To simplify the analysis of Young's experiment, we assume that the distance R from the slits to the screen is large compared to the distance d between the slits.
- This leads to the approximation that the rays r_1 and r_2 originating from S_1 and S_2 are parallel.
- The difference in path length is then given by

$$r_2 - r_1 = d \sin\theta.$$



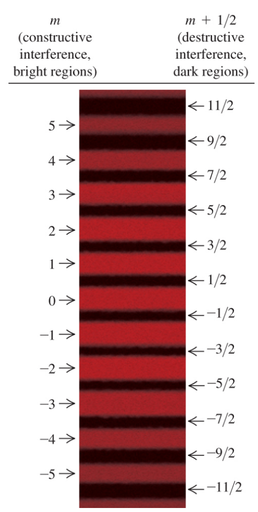
Interference From Two Slits (1 of 2)

- Constructive interference occurs at points where the path difference is an integer number of wavelengths, $m\lambda$.
- The bright regions on the screen occur at angles θ for which

$$d \sin\theta = m\lambda \quad (m = 0, \pm1, \pm2, \ldots).$$

- Similarly, destructive interference occurs, forming dark regions on the screen, at points for which the path difference is a half-integer number of wavelengths:

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm1, \pm2, \ldots).$$



Interference From Two Slits (2 of 2)

- If θ_m is the mth value of θ for the mth bright band with respect to $\theta = 0$, then the positions of the centers of the bright bands on the screen measured from the center of the pattern are

$$y_m = R \tan\theta_m.$$

- For the case where the distances y_m are much smaller than the distance R from the slits to the screen, we may use the approximation $\tan\theta_m \approx \sin\theta_m$ to obtain

$$y_m = R\sin\theta_m = R\frac{m\lambda}{d} \quad (m = 0, \pm1, \pm2, \ldots).$$

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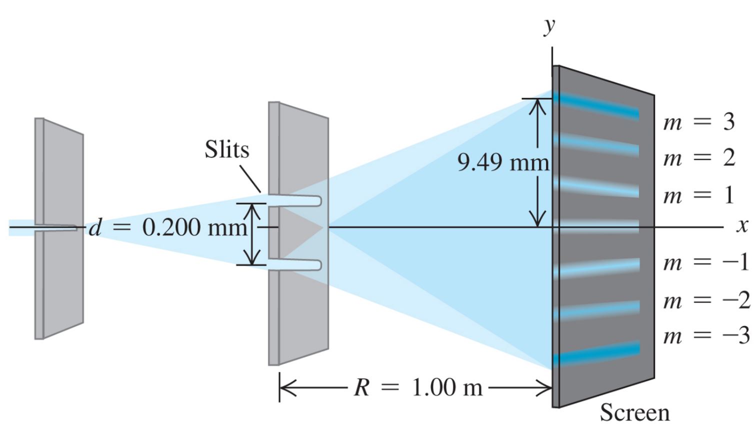
Example 35.1: Two-Slit Interference

The figure below shows a two-slit interference experiment in which the slits are 0.200 mm apart and the screen is 1.00 m from the slits. The $m = 3$ bright fringe in the figure is 9.49 mm from the central bright fringe. Find the wavelength of the light.

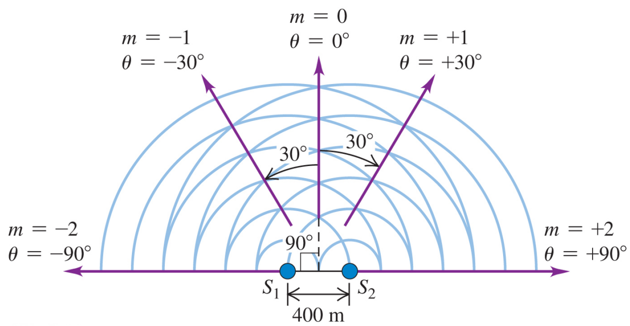


Plugging in $m = 3$ into our equation for y_m, we find that the wavelength is

$$\lambda = \frac{y_m d}{mR} = \frac{(9.49 \times 10^{-3}\text{ m})(0.200 \times 10^{-3}\text{ m})}{(3)(1.00\text{ m})} = 633 \times 10^{-9}\text{ m} = 633\text{ nm}.$$

Example 35.2: Broadcast Pattern of a Radio Station

It is often desirable to radiate most of the energy from a radio transmitter in particular directions rather than uniformly in all directions. Pairs or rows of antennas are often used to produce the desired radiation pattern. As an example, consider two identical vertical antennas 400 m apart, operating at 1500 kHz $= 1.5 \times 10^6$ Hz (near the top end of the AM broadcast band) and oscillating in phase. At distances much greater than 400 m, in what directions is the intensity from the two antennas greatest?



This is essentially the same configuration as a double-slit experiment, with the separation of the antennas acting as the slit separation d. The wavelength of the radio waves is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.5 \times 10^6 \text{ Hz}} = 200 \text{ m}.$$

Then the angles for the intensity maxima are given by

$$\sin\theta = \frac{m\lambda}{d} = \frac{m(200 \text{ m})}{400 \text{ m}} = \frac{m}{2}.$$

The only values of m for which this equation holds are $m = 0, \pm 1, \pm 2$. So the corresponding angles are

$$\theta = 0, \pm 30^\circ, \pm 90^\circ.$$

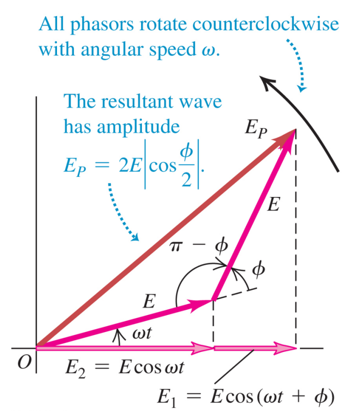
Intensity in Interference Patterns (1 of 6)

- To find the intensity at any point in a two-source interference pattern, we have to combine the two sinusoidally varying fields (from the two sources) at a point P.
- If the two sources are in phase, then the waves that arrive at P differ in phase by an amount ϕ that is proportional to the difference in their path lengths $r_2 - r_1$.
- The electric fields at P can be expressed by

$$E_1(t) = E\cos(\omega t + \phi),$$
$$E_2(t) = E\cos\omega t.$$

Intensity in Interference Patterns (2 of 6)

- To add the two sinusoidal functions with a phase difference, we can use the same phasor representation that we used for voltages and currents in ac circuits.
- Each sinusoidal function is represented by a rotating vector (phasor) whose projection on the horizontal axis at any instant represents the instantaneous value of the sinusoidal function.
- Both phasors have the same magnitude E, but one is ahead of the other by phase angle ϕ.



- To find the vector sum E_P, we can use the law of cosines and the trigonometric identity $\cos(\pi - \phi) = -\cos\phi$:

$$E_P^2 = E^2 + E^2 - 2E^2\cos(\pi - \phi) = 2E^2 + 2E^2\cos\phi = 2E^2(1 + \cos\phi).$$

Intensity in Interference Patterns (3 of 6)

- Using the fact that $1 + \cos\phi = 2\cos^2(\phi/2)$, we obtain

$$E_P^2 = 2E^2(1 + \cos\phi) = 4E^2\cos^2\left(\frac{\phi}{2}\right) \quad \rightarrow \quad E_P = 2E\left|\cos\frac{\phi}{2}\right|.$$

- When the two waves are in phase, $\phi = 0$ and $E_P = 2E$. But when they are exactly a half-cycle out of phase, $\phi = \pi$ and $\cos(\phi/2) = \cos(\pi/2) = 0$, so $E_P = 0$.
- We can use our result for E_P to find the intensity I at point P:

$$I = \frac{E_P^2}{2\mu_0 c} = \frac{1}{2}\epsilon_0 c E_P^2 = 2\epsilon_0 c E^2 \cos^2\frac{\phi}{2}.$$

- The maximum intensity $I_0 = 2\epsilon_0 c E^2$ occurs when the phase difference is zero $(\phi = 0)$, so we can also write I as

$$I = I_0\cos^2\frac{\phi}{2}.$$

Intensity in Interference Patterns (4 of 6)

- The phase difference between the two fields at any point P can also be determined by considering the path difference $r_2 - r_1$. The ratio of the phase difference ϕ to 2π is equal to the ratio of the path difference $r_2 - r_1$ to λ:

$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda} \quad \rightarrow \quad \phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1).$$

- If the material in the space between the sources and P has index of refraction n, and λ_0 and k_0 are the wavelength and wave number in vacuum, then

$$\lambda = \frac{\lambda_0}{n} \quad \rightarrow \quad k = nk_0.$$

- If P is far away from the sources in comparison to their separation d, the path difference is $r_2 - r_1 = d\sin\theta$, so the phase difference is

$$\phi = k(r_2 - r_1) = kd\sin\theta = \frac{2\pi d}{\lambda}\sin\theta.$$

Intensity in Interference Patterns (5 of 6)

- The complete expression for the intensity is then

$$I = I_0 \cos^2\left(\frac{1}{2}kd\sin\theta\right) = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right).$$

- The *maximum* intensity occurs when the cosine has the values ±1—that is, when

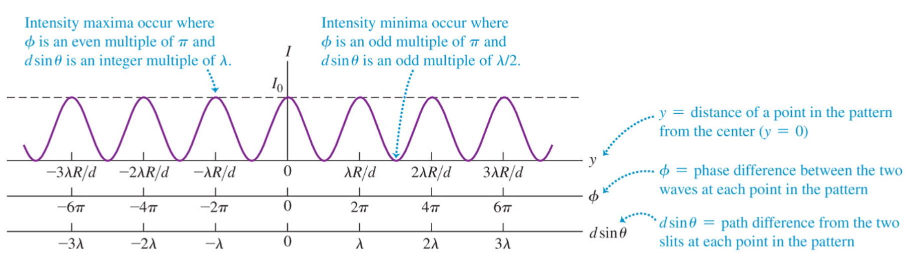
$$\frac{\pi d}{\lambda}\sin\theta = m\pi \quad \rightarrow \quad d\sin\theta = m\lambda \quad (m = 0, \pm1, \pm2, \ldots).$$

- This is equivalent to the condition obtained before for the locations of the maxima in the interference pattern.
- We can also write the intensity in terms of the distance y along the screen where the interference pattern is projected. For the case where $y \ll R$, $\sin\theta \approx y/R$, and we obtain

$$I = I_0 \cos^2\left(\frac{kdy}{2R}\right) = I_0 \cos^2\left(\frac{\pi dy}{\lambda R}\right).$$

Intensity in Interference Patterns (6 of 6)

- All of the peaks in the interference pattern have the same intensity, but in a real experiment each successive peak fades away as we get further from the central maximum. We will explore this when we introduce diffraction later on.



Example 35.3: A Direction Transmitting Antenna Array

Suppose the two identical radio antennas of the previous example are moved to be only 10.0 m apart and the broadcast frequency is $f = 60.0$ MHz. At a distance of 700 m from the point midway between the antennas and in the direction of $\theta = 0$, the intensity is $I_0 = 0.020$ W/m^2. At this same distance, find (a) the intensity in the direction $\theta = 4.0°$; (b) the direction near $\theta = 0$ for which the intensity is $I_0/2$; and (c) the directions in which the intensity is zero.

(a) The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.00 \times 10^7 \text{ Hz}} = 5.00 \text{ m},$$

and the spacing is $d = 10.0$ m, so $d/\lambda = 2.00$, and the intensity is therefore

$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = I_0 \cos^2\left[(2.00\pi \text{ rad})\sin\theta\right].$$

When $\theta = 4.0°$,

$$I = I_0 \cos^2\left[(2.00\pi \text{ rad})\sin 4.0°\right] = 0.82 I_0 = (0.82)(0.020 \text{ W/m}^2) = 0.016 \text{ W/m}^2.$$

(b) The intensity I equals $I_0/2$ when the cosine function has the value $\pm 1/\sqrt{2}$. The smallest angles for which this occurs correspond to $2.00\pi\sin\theta = \pm\pi/4$ rad. Thus,

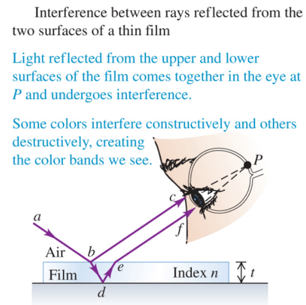
$$\sin\theta = \pm\frac{1}{8.00} = \pm 0.125 \quad \rightarrow \quad \theta = \pm 7.2°.$$

(c) The intensity is zero when $\cos\left[(2.00\pi \text{ rad})\sin\theta\right] = 0$. This occurs when $2.00\pi\sin\theta = \pm\pi/2, \pm 3\pi/2$, and so

$$\sin\theta = \pm 0.250, \pm 0.750 \quad \rightarrow \quad \theta = \pm 14.5°, \pm 48.6°.$$

Interference in Thin Films (1 of 4)

- The bright bands of color from light reflecting off of a thin layer of oil on water or from a soap bubble are the result of interference.
- If we have a thin film of thickness t on top of a reflecting surface, some light will be reflected at the interface between the film and the air, while some will pass through the film and be reflected back into the air.
- The light that is reflected off of the surface of the film interferes with the light that has passed through the film and reflected back into the air, creating color bands of visible light.



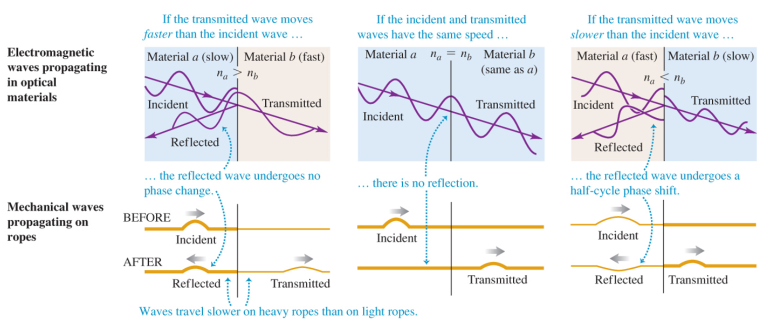
Colorful reflections from a soap bubble

Interference in Thin Films (2 of 4)

- At normal incidence with incident amplitude E_i, the amplitude E_r of the reflected wave at the interface between two materials with indexes of refraction n_a and n_b is

$$E_\mathrm{r} = \frac{n_a - n_b}{n_a + n_b} E_\mathrm{i}.$$

- There are three cases to consider:
  - If $n_a > n_b$, there is no phase shift for the reflected wave.
  - If $n_a = n_b$, there is no reflected wave ($E_\mathrm{r} = 0$).
  - If $n_a < n_b$, there is a half-cycle phase shift the for reflected wave.



Interference in Thin Films (3 of 4)

- We can determine the conditions for constructive or destructive reflection by considering the path difference between the light reflected at the interface and the light that has traveled a distance $2t$ in the thin film before recombining with the light reflected at the interface.
- For light of normal incidence on a thin film with wavelength λ in the film, in which neither or both of the reflected waves have a half-cycle phase shift:

Constructive reflection: $$2t = m\lambda \quad (m = 0, 1, 2, \ldots).$$

Destructive reflection: $$2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots).$$

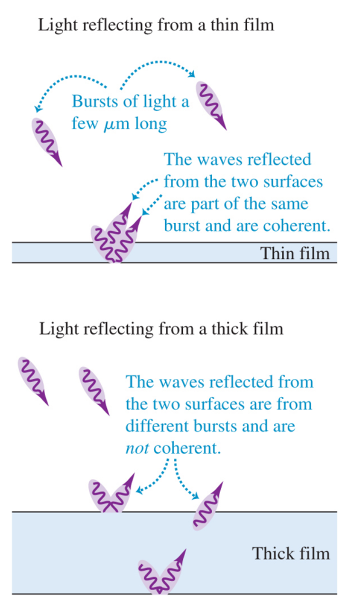
- When only one of the reflected waves has a half-cycle phase shift:

Constructive reflection: $$2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots).$$

Destructive reflection: $$2t = m\lambda \quad (m = 0, 1, 2, \ldots).$$

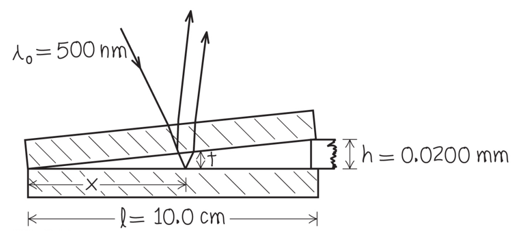
Interference in Thin Films (4 of 4)

- So far we have dealt with interference in thin films, but what about thick films?
- For waves to cause a steady interference pattern, the waves must be coherent, with a definite and constant phase relationship.
- If the film is too thick, the two reflected waves will have no definite phase relationship, and the two waves will be incoherent.
- Thus, thick films do not produce an interference pattern.
- This is why we can see interference colors in bubbles only a few micrometers thick, but not in glass panes that are a few millimeters thick.



Example 35.4: Thin-Film Interference I

Suppose we have two microscope slides that are 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Is the fringe at the line of contact bright or dark? Assume monochromatic light with a wavelength in air of $\lambda = \lambda_0 = 500$ nm.



Only one of the reflected waves undergoes a phase shift since the air in the gap between the slides has a lower index of refraction than the glass. Therefore, the condition for destructive interference is

$$2t = m\lambda_0 \quad (m = 0, 1, 2, \ldots).$$

We can use the fact that thickness t of the air wedge at each point is proportional to the distance x from the line of contact, and since the triangles formed by t and x are similar to the triangle formed by l and h, we have

$$\frac{t}{x} = \frac{h}{l}.$$

Then the spacing in the interference fringes is

$$\frac{2xh}{l} = m\lambda_0 \quad \rightarrow \quad x = m\frac{l\lambda_0}{2h} = m\frac{(0.100 \text{ m})(500 \times 10^{-9} \text{ m})}{(2)(0.0200 \times 10^{-3} \text{ m})} = m(1.25 \text{ mm}).$$

Thus, the fringes are spaced by 1.25 mm.

When $m = 0$, we get $x = 0$, which is where the two microscope slides are in contact. This means there is a dark fringe at the line of contact.

Example 35.5: Thin-Film Interference II

Suppose the glass plates of the previous example have $n = 1.52$ and the space between plates contains water ($n = 1.33$) instead of air. What happens now?

In the film of water, the wavelength is now

$$\lambda = \frac{\lambda_0}{n_{\text{water}}} = \frac{500 \text{ nm}}{1.33} = 376 \text{ nm}.$$

The locations of the dark fringes become

$$x = m\frac{l\lambda}{2h} = m\frac{(0.100 \text{ m})(376 \times 10^{-9} \text{ m})}{(2)(0.0200 \times 10^{-3} \text{ m})} = m(0.940 \text{ mm}).$$

For $m = 0$, $x = 0$ still, so the dark fringe at the line of contact remains.

Example 35.6: Thin-Film Interference III

Suppose the upper of the two plates of the previous example is a plastic with $n = 1.40$, the wedge is filled with a silicone grease with $n = 1.50$, and the bottom plate is a dense flint glass with $n = 1.60$. What happens now?

In this case, half-cycle phase shifts occur at both surfaces of the grease wedge, so there is no relative phase shift between the two outgoing light rays. The condition for dark fringes is now

$$2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots).$$

Now the wavelength in the wedge is

$$\lambda = \frac{\lambda_0}{n_{\text{grease}}} = \frac{500 \text{ nm}}{1.50} = 333 \text{ nm}.$$

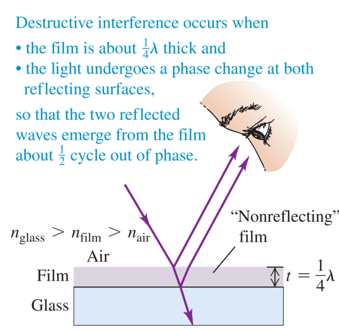
Therefore,

$$x = \left(m + \frac{1}{2}\right)\frac{l\lambda}{2h} = \left(m + \frac{1}{2}\right)\frac{(0.100 \text{ m})(333 \times 10^{-9} \text{ m})}{(2)(0.0200 \times 10^{-3} \text{ m})} = \left(m + \frac{1}{2}\right)(0.83 \text{ mm}),$$

so the fringe spacing is 0.83 mm, and there is now a bright fringe at the line of contact.

Nonreflective Coatings

- **Nonreflective coatings** for lens surfaces make use of thin-film interference.
  - Suppose we have light incident on a thin film on top of a layer of glass, with $n_{\text{glass}} > n_{\text{film}} > n_{\text{air}}$.
  - If the film thickness is one-fourth of the wavelength in the film, then the total path difference is a half-wavelength.
  - As a result, the light reflected at the interface is a half-cycle out of phase with light that has traveled through the film and is reflected back towards the interface, resulting in destructive interference.



- If a quarter-wavelength thickness of material with an index of refraction $n_{\text{film}} > n_{\text{glass}}$ is deposited on the glass, then there is a half-cycle phase shift from reflection at the glass.
  - This results in constructive interference with the light reflected at the film.
  - Such a material is called a **reflective coating.**

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Example 35.7: A Nonreflective Coating

A common lens coating material is magnesium fluoride (MgF_2), with $n_{\text{MgF}_2} = 1.38$. What thickness should a nonreflective coating have for 550 nm light if it is applied to glass with $n_{\text{glass}} = 1.52$?

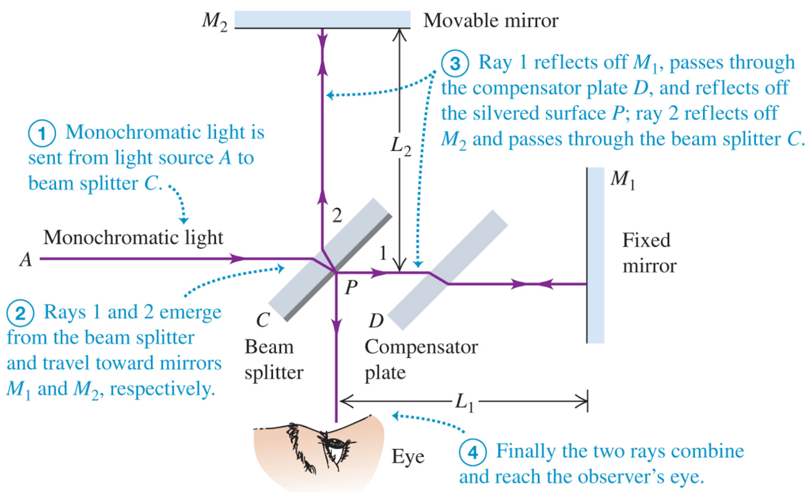
The wavelength in air is $\lambda_0 = 550$ nm, so the wavelength in the MgF_2 coating is

$$\lambda = \frac{\lambda_0}{n_{\mathrm{MgF_2}}} = \frac{550 \text{ nm}}{1.38} = 400 \text{ nm}.$$

The coating must have a thickness that is one-fourth of this value, so $t = \lambda/4 = 100$ nm.

The Michelson Interferometer (1 of 3)

- A **Michelson interferometer** is used to make precise measurements of wavelengths and very small distances.



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The Michelson Interferometer (2 of 3)

- A Michelson interferometer takes monochromatic light from a single source and divides it into two waves that follow different paths using a beam splitter. The beams are then recombined and produce an interference pattern.
- The compensator plate ensures that both rays pass through the same thickness of glass, preserving the phase relationship of the waves.
- The mirrors can be finely adjusted so that they are not exactly perpendicular to each other, which will result in interference fringes that can be observed through a telescope.
- The fringe positions y_m are given by

$$y_m = m\frac{\lambda}{2} \quad \rightarrow \quad \lambda = \frac{2y_m}{m}.$$

- If m is on the order of several thousand, then y_m is large enough to be easily measured. On the other hand, if the wavelength λ is known, then a small distance comparable to the wavelength of light can be measured by counting the fringes when one of the mirrors is moved.

The Michelson Interferometer (3 of 3)

- This apparatus was famously used in the Michelson-Morley experiment in an attempt to detect the motion of the Earth through the supposed ether, which was a proposed medium that light propagated in.
- If such a medium existed, then the speed of light should change depending on the relative motion of the Earth through the ether. This would produce fringe shifts relative to the positions that the fringes would have if the interferometer were at rest in the ether.
- When the entire device is rotated $90°$, the fringe shifts would be in the opposite direction, allowing for the measurement of the change in the observed speed of light for each beam.
- No such fringe shifts were observed, and despite the orbital motion of the Earth around the sun—and hence through the supposed ether—no such evidence of the ether was found.
- This issue would not be resolved until Einstein developed his theory of special relativity in 1905, which eventually led to the abandonment of the ether.