CS 181 HW 8: TEJAS KAMTAM

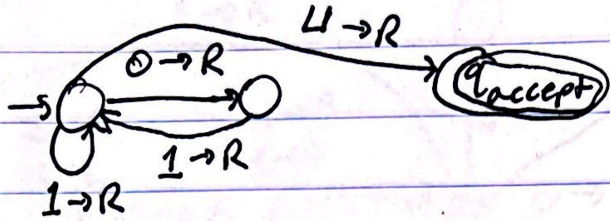
1a) bin. strs. every 0 followed by a 1

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, \sqcup\}$

given as unidirectional tape:

w_1	w_2	w_3	...	$\sqcup$	$\sqcup$	$\sqcup$	...



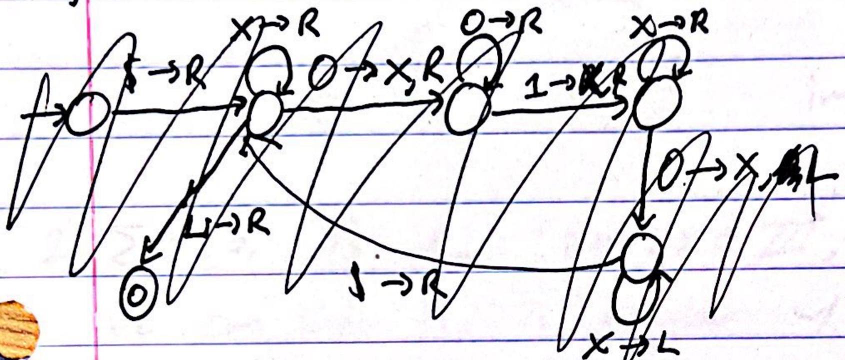
1b) bin. strs. of form $0^n 1 0^n$, $n \geq 0$

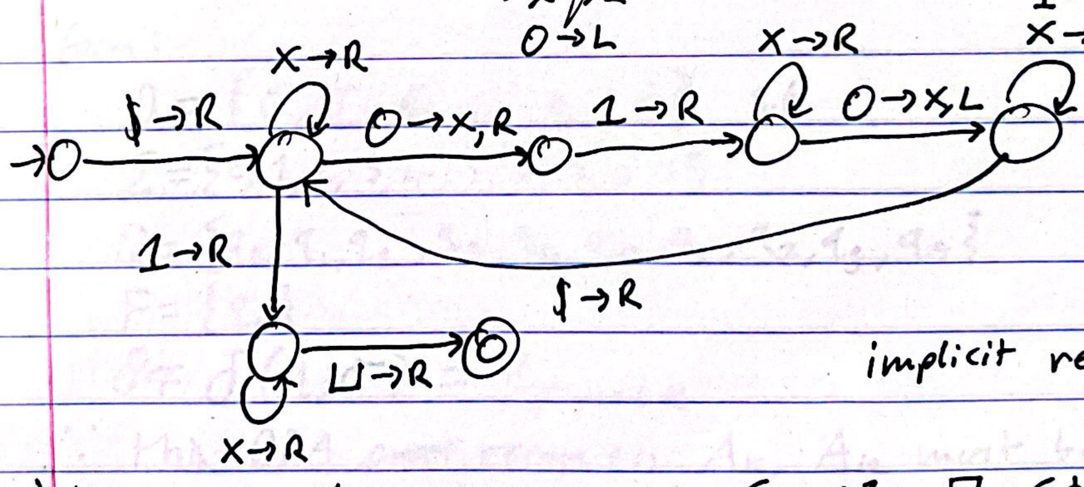
$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, \sqcup, X\} \cup \{\$\}$

given as unidirectional tape w/ first cell as \$

\$	w_1	w_2	...	$\sqcup$	$\sqcup$	...





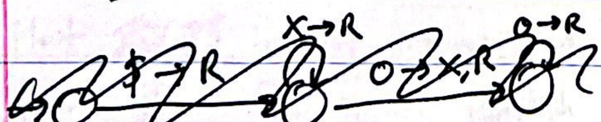
implicit rejection

1c) bin strs. $\#_0 = \#_1$ $\Sigma = \{0, 1\}$, $\Gamma = \{\$, 0, 1, \sqcup, X\}$

assuming given w/ unidirectional tape w/ first start as \$:

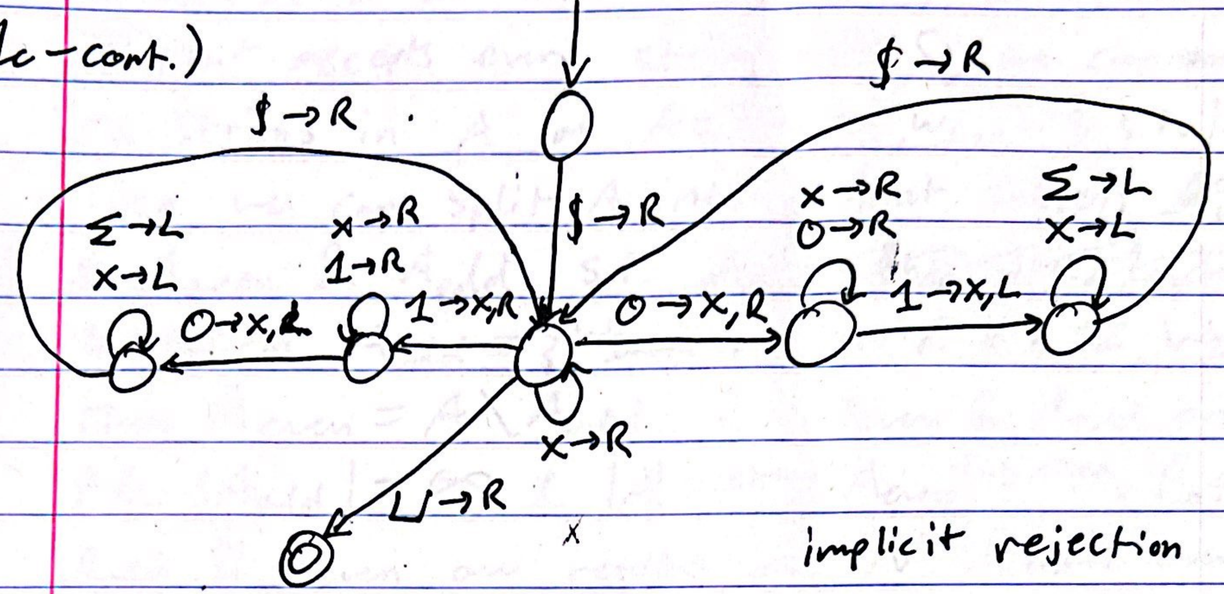
e.g.

\$	w_1	w_2	...	$\sqcup$	$\sqcup$	...



State transition diagram on next page →

1c - cont.)



implicit rejection

2) $\Sigma = \{0, \dots, 9\}$ $A_k = \{kn \mid \forall n \in \mathbb{Z}^+, ~~k n~~ k > 0 \ nk \% k = 0\}$

We can show A_k is regular w/ an automata of the form:

$D = \{Q, \Sigma, \cancel{\ }\ \delta, q_0, F\}$ s.t.

$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}$

$F = \{q_0\}$

$\cancel{\ }\ \delta(q, \sigma) = q_{\sigma \bmod k}$

$\therefore$ this DFA ~~must~~ recognizes A_k, A_k must be regular.

3) Given $B = \{w \in \Sigma^* :$ every computation of N accepts on $w\}$ for a given NFA N. Then, $\bar{B} = \{w \in \Sigma^* :$ no computation of N on w accepts$\}$. This means that $\cancel{\ }\bar{B}\}$ is all strings that reject on N i.e. $\cancel{\ }\bar{B}\} = \Sigma^* \setminus A$ then $\bar{\bar{B}} = \overline{\Sigma^* \setminus A}$ ~~which~~ $= B$ which is a regular language as reg. langs. are closed under set minus & complement and we know Σ^* & A to be regular.

1.63a) Given A is an infinite regular lang. $\exists$ an automaton N that accepts every string $\in A$. So, we can ~~not~~ index the strings in A as $A = \{w_0, w_1, w_2, \dots\}$ s.t. $|A| = \infty$. Then we can split A into disjoint subsets ~~A_1 & A_2~~ ~~s.~~ A_{even} & A_{odd} s.t. ~~$A_{odd} = \{w_k : \forall k \geq 1 \text{ for some}$~~ ~~integers in~~ $A_{odd} = \{w_{2k+1} : k \geq 0 \text{ \& } k \in \mathbb{Z}, w_{2k+1} \in A\}$ then $A_{even} = A \setminus A_{odd}$ $\therefore$ A_{even} & A_{odd} are disjoint. B/c $|A_{odd}| = \infty$ & $|A| = \infty \Rightarrow |A_{even}| = \infty$. Both A_{odd} & A_{even} are regular as N accepts every string in A_{odd} & A_{even} ~~[illegible]~~ therefor these langs. are indeed regular. $\therefore$ A_{odd}, A_{even} are disjoint, inf., reg. subsets

1.63b) Similar to the description above, consider D to be the infinite reg. lang. Then ~~[illegible]~~ Sps. B is the subset of ~~all~~ D that contains all strings of an index divisible by 4. Then ~~[illegible]~~ obviously $B \subseteq D$. Now consider C to be the subset of ~~[illegible]~~ ~~[illegible]~~ D containing all even indexed strings. Then $B \subseteq C \subseteq D$. We have shown C is reg. w/ 1.63a and B is regular with an analogous proof to 1.63a treating C as the larger set and B as the subset.