Physics 1C - Discussion 2 Name: Tejas Kamtam UID: 305749402

Problem 1. Magnetic Fields due to Moving Charges

Calculate the magnetic field at the origin given two particles, one of charge $q_1 = +2.5$ C at $\mathbf{r}_1 = (2.0\,\hat{\imath}+3.5\,\hat{\jmath}-1.0\,\hat{k})$ m moving at a speed $\mathbf{v}_1 = (7.0\,\hat{\imath}+24.0\,\hat{\jmath})$ m/s and the other of charge $q_2 = -2.5$ C at $\mathbf{r}_2 = (1.0\,\hat{\imath}-4.0\,\hat{\jmath}+7.5\,\hat{k})$ m moving at a speed $\mathbf{v}_2 = (4.0\,\hat{\imath}-12.0\,\hat{\jmath}-3.0\,\hat{k})$ m/s.

$$r_1 = \sqrt{2.0^2+3.5^2+1.0^2} = \sqrt{17.25} \text{ m}$$

$$r_2 = \sqrt{1.0^2+4.0^2+7.5^2} = \sqrt{73.25} \text{ m}$$

$$\vec{B}_1 = \frac{\mu_0}{4\pi} q_1 \frac{\vec{v}_1 \times \vec{r}_1}{r_1^3} = \frac{2.5\mu_0}{4\pi} \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7.0 & 0 & 24.0 \\ 2.0 & 3.5 & -1.0 \end{vmatrix}}{r_1^3} = \frac{2.5\mu_0}{4\pi} \frac{(-84\,\hat{i} + 55\,\hat{j} + 24.5\,\hat{k})}{17.25\sqrt{17.25}} \quad [T]$$

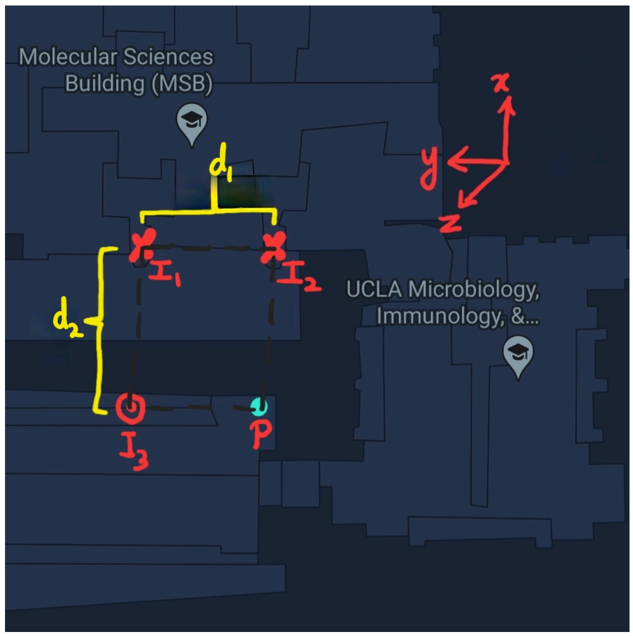
$$\vec{B}_2 = \frac{\mu_0}{4\pi} q_2 \frac{\vec{v}_2 \times \vec{r}_2}{r_2^3} = \frac{-2.5\mu_0}{4\pi} \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -12 & -3 \\ 1 & -4 & 7.5 \end{vmatrix}}{r_2^3} = \frac{-2.5\mu_0}{4\pi} \frac{(-102\,\hat{i} - 33\,\hat{j} - 4\,\hat{k})}{73.25\sqrt{73.25}} \quad [T]$$

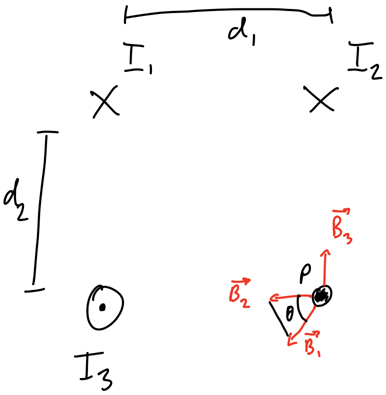
$$\boxed{\vec{B} = \vec{B}_1 + \vec{B}_2 \approx \frac{2.5\mu_0}{4\pi}\left(-1.00975\,\hat{i} + 0.82032\,\hat{j} + 0.34835\,\hat{k}\right) \quad [T]}$$

Problem 2. Current-carrying conductors

In order to conduct an experiment, Walter sets up three straight conductors carrying currents of I_1, I_2, and I_3. The currents are arranged as shown in Fig. 1. Here, I_1, I_2, and I_3 are pointing orthogonal to the plane of the page, with I_1 and I_2 pointing into the page (in the $-z$ direction) and I_3 pointing out of the page (in the $+z$ direction). Let us place the current I_1 at the origin, current I_2 at $-d_1\,\hat{y}$, current I_3 at $-d_2\,\hat{x}$ and point P at $-d_1\hat{y} + -d_2\hat{x}$.

Suppose that the current values of I_1 and I_2 were tuned such that $I_1 = I\sqrt{d_1^2 + d_2^2}$, $I_2 = Id_2$, and I_3 was a free parameter. Here, I is a convenient constant with units of A/m so that I_1 and I_2 have the correct units of amperes.





Find $\vec{B}$ @ P

Figure 1: Current-carrying conductors

If the experiment requires a magnetic field with magnitude $|\mathbf{B}| = B_0$ at point P, what should be the magnitude of I_3 need to be?

Ampere's Law : $|\vec{B}| = \dfrac{\mu_0 I}{2\pi R}$ $\Rightarrow$ $|\vec{B}_1| = \dfrac{\mu_0 I_1}{2\pi\sqrt{d_1^2+d_2^2}}$, $|\vec{B}_2| = \dfrac{\mu_0 I_2}{2\pi\, d_2}$, $|\vec{B}_3| = \dfrac{\mu_0 I_3}{2\pi d_1}$

$|\vec{B}| = |\vec{B}_1 + \vec{B}_2 + \vec{B}_3| = \sqrt{B_1^2 + B_2^2 + B_3^2} = B_0 \Rightarrow B_3 = \sqrt{B_0^2 - B_1^2 - B_2^2}$

$\Rightarrow I_3 = \dfrac{2\pi d_1}{\mu_0}\sqrt{B_0^2 - B_1^2 - B_2^2}$ s.t. B_0 is a given constant, $B_1 = \dfrac{\mu_0 I_1}{2\pi\sqrt{d_1^2+d_2^2}}$, $B_2 = \dfrac{\mu_0 I_2}{2\pi\, d_2}$

Problem 3. (Challenge Problem) Current-carrying conical loop

Consider the conical loop in the following image. Assuming the second configuration for simplicity of calculation, calculate the magnetic field at the four points P_1, P_2, P_3 (at a distance r from P_1), and P_4 (at a distance r from P_2). Let the current flowing through each ring be I and assume $r \gg L$ and $r \gg a, b$.

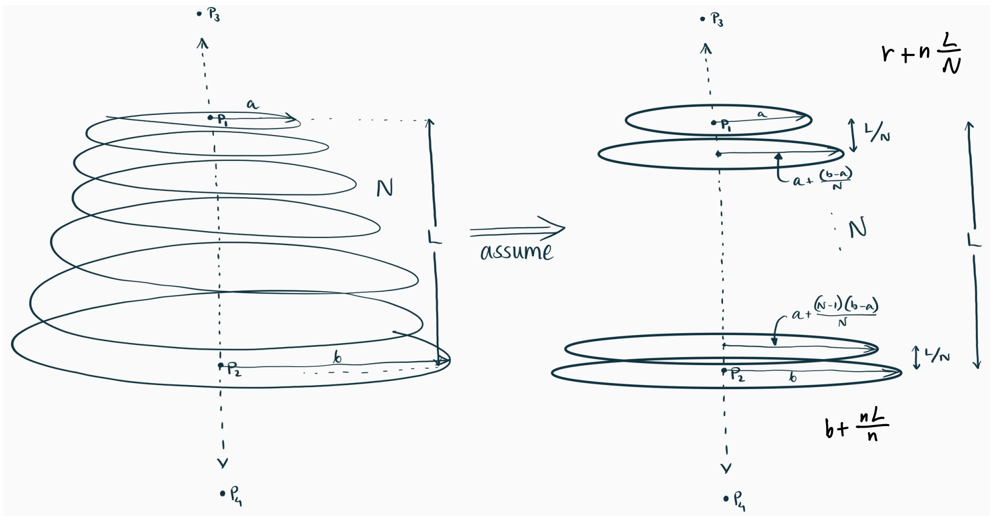


Figure 2: Current-carrying conical loop

P.S.: It will suffice to represent net magnetic field as a summation of terms for points P_1 and P_2. Show entire calculation for points P_3 and P_4

Assuming $\vec{I}$ moves counter-clockwise up the coil

$$B_R = \frac{\mu_0 I R^2}{2(R^2+h^2)^{3/2}}$$

$$\Rightarrow B_1 = B_2 = \sum_{n=0}^{N} \frac{\mu_0 I}{2\left[a+\frac{(N-n)(b-a)}{N}\right]}$$

$$\Rightarrow B_3 = \sum_{n=0}^{N} \frac{\mu_0 I \left[a+\frac{(N-n)(b-a)}{N}\right]^2}{2\left(\left[a+\frac{nL}{N}\right]^2+\left[a+\frac{(N-n)(b-a)}{N}\right]^2\right)^{3/2}}$$

$$B_4 = \sum_{n=N>0}^{0} \frac{\mu_0 I \left[a+\frac{(N-n)(b-a)}{N}\right]^2}{2\left(\left[b+\frac{nL}{N}\right]^2+\left[a+\frac{(N-n)(b-a)}{N}\right]^2\right)^{3/2}}$$