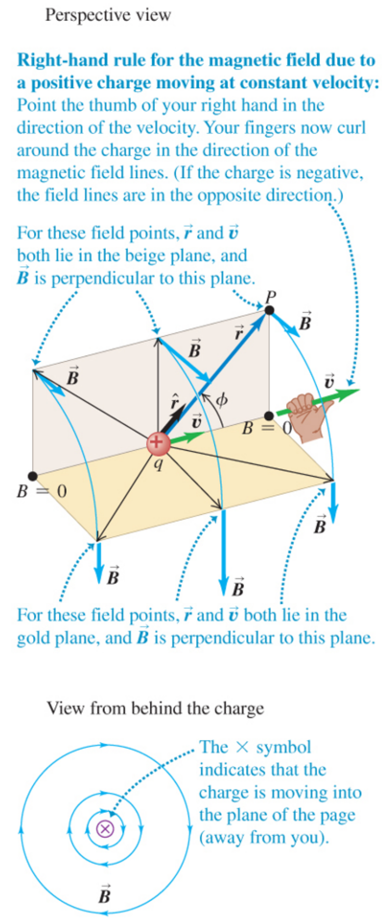
Sources of Magnetic Field (Ch. 28)

Magnetic field of a Moving Charge

- A moving charge generates a magnetic field that depends on the velocity of and distance from the charge:

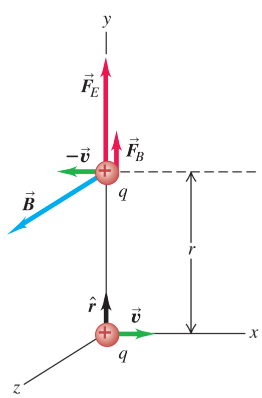
$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}.$$

  - We use the unit vector $\hat{\mathbf{r}}$ that points from the source of the field to the field point.
  - The **magnetic constant** $\mu_0 = 4\pi \times 10^{-7}$ T · m/A is analogous to ϵ_0, but for magnetic fields.
- The magnetic field lines are circles centered around the line of $\mathbf{v}$.



Example: Forces Between Two Moving Protons

Two protons move parallel to the x-axis in opposite directions at the same speed v. At the instant shown, find the electric and magnetic forces on the upper proton and compare their magnitudes



Coulomb's law tells us that the force on the upper proton due to the electric field of the lower proton is repulsive since they are both positively charged. The electric force on the upper proton therefore points in the positive y-direction, and is given by

$$\mathbf{F}_E = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \hat{\mathbf{j}}.$$

The velocity of the lower proton is $\mathbf{v} = v\,\hat{\mathbf{i}}$, so the magnetic field at the location of the upper proton is

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q(v\,\hat{\mathbf{i}}) \times \hat{\mathbf{j}}}{r^2} = \frac{\mu_0}{4\pi} \frac{qv}{r^2} \hat{\mathbf{k}}.$$

Meanwhile, the velocity of the upper proton is $-\mathbf{v} = -v\,\hat{\mathbf{i}}$ since it is going in the opposite direction. The magnetic force on the upper proton is therefore

$$\mathbf{F}_B = q(-\mathbf{v}) \times \mathbf{B} = \frac{\mu_0}{4\pi} \frac{q^2 v^2}{r^2} (-\,\hat{\mathbf{i}} \times \hat{\mathbf{k}}) = \frac{\mu_0}{4\pi} \frac{q^2 v^2}{r^2} \hat{\mathbf{j}}.$$

So both the electric force $\mathbf{F}_E$ and magnetic force $\mathbf{F}_B$ point in the positive y-direction.

The ratio of the magnitudes of the two forces is

$$\frac{F_B}{F_E} = \frac{\mu_0}{4\pi}(4\pi\epsilon_0)\frac{q^2v^2}{r^2}\frac{r^2}{q^2} = \mu_0\epsilon_0 v^2 = \frac{v^2}{c^2},$$

where we have used the fact that $c = 1/\sqrt{\epsilon_0\mu_0}$. Therefore, the strength of the magnetic force is much smaller than the electric force when v is much smaller than c.

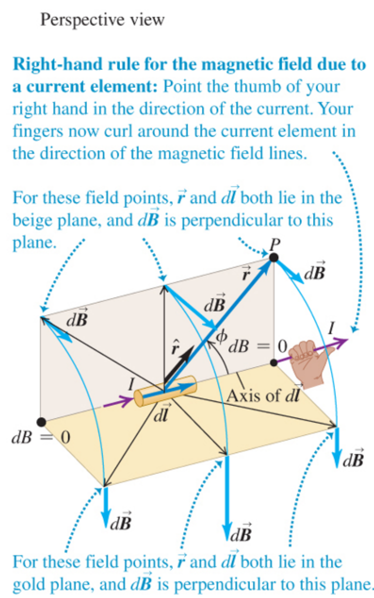
Magnetic Field of a Current Element

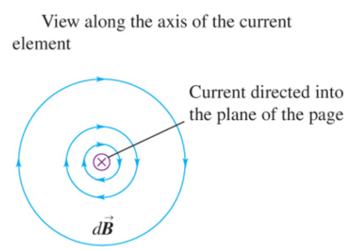
- For a current-carrying conductor segment with cross sectional area A, length $\mathrm{d}l$, n charges per unit volume, each with charge q, the total charge in the segment is $\mathrm{d}Q = nqA\,\mathrm{d}l$.
- We can take the vector sum of the field contributions of multiple moving charges to get the field contribution $\mathrm{d}\mathbf{B}$ due to a current $I = n|q|Av_\mathrm{d}$:

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi}\frac{\mathrm{d}Q\,\mathbf{v}_\mathrm{d}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\mathbf{l}\times\hat{\mathbf{r}}}{r^2}.$$

- Integrating over the current segments gives us the **Biot-Savart law:**

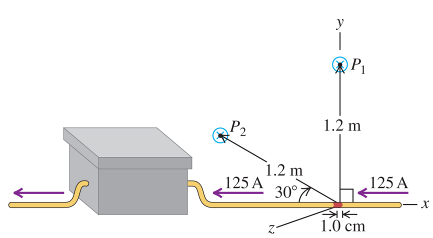
$$\mathbf{B} = \frac{\mu_0}{4\pi}\int\frac{I\,\mathrm{d}\mathbf{l}\times\hat{\mathbf{r}}}{r^2}.$$





Example: Magnetic Field of a Current Segment

A copper wire carries a steady 125 A current to an electroplating tank. Find the magnetic field due to a 1.0 cm segment of the wire at a point 1.2 m away from it, if the point is (a) point P_1, straight out to the side of the segment, and (b) point P_2, in the xy-plane and on a line at 30° to the segment.



(a) At point P_1, $\hat{\mathbf{r}} = \hat{\mathbf{j}}$, so the field due to the wire segment is

$$\begin{aligned}\mathbf{B} &= \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\mathbf{l}\times\hat{\mathbf{r}}}{r^2}\\ &= \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}l\,(-\hat{\mathbf{i}}\times\hat{\mathbf{j}})}{r^2}\\ &= -\frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}l}{r^2}\,\hat{\mathbf{k}}\\ &= -(10^{-7}\ \mathrm{T\cdot m/A})\frac{(125\ \mathrm{A})(1.0\times10^{-2}\ \mathrm{m})}{(1.2\ \mathrm{m})^2}\,\hat{\mathbf{k}}\\ &= -(8.7\times10^{-8}\ \mathrm{T})\,\hat{\mathbf{k}},\end{aligned}$$

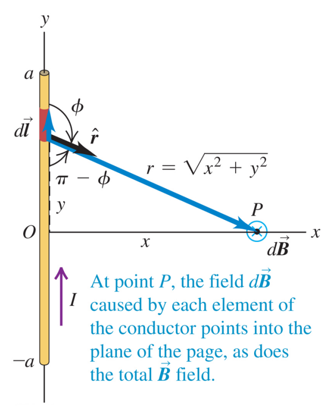
which points into the xy-plane.

(b) At the other point P_2, the unit vector is $\hat{\mathbf{r}} = -\cos(30°)\,\hat{\mathbf{i}} + \sin(30°)\,\hat{\mathbf{j}}$. The field is therefore

$$\begin{aligned}
\mathbf{B} &= \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\mathbf{l}\times\hat{\mathbf{r}}}{r^2} \\
&= \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}l\,(-\,\hat{\mathbf{i}})\times[-\cos(30°)\,\hat{\mathbf{i}} + \sin(30°)\,\hat{\mathbf{j}}]}{r^2} \\
&= -\frac{\mu_0 I}{4\pi}\frac{\mathrm{d}l\sin(30°)}{r^2}\,\hat{\mathbf{k}} \\
&= -(10^{-7}\ \mathsf{T\cdot m/A})\frac{(125\ \mathsf{A})(1.0\times 10^{-2}\ \mathsf{m})\sin(30°)}{(1.2\ \mathsf{m})^2}\,\hat{\mathbf{k}} \\
&= -(4.3\times 10^{-8}\ \mathsf{T})\,\hat{\mathbf{k}},
\end{aligned}$$

which also points into the xy-plane.

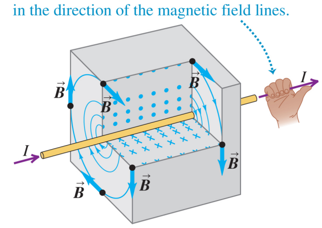
Magnetic Field of a Straight Current-Carrying Conductor



- The Biot-Savart law can be used to evaluate the magnitude of the magnetic field B a distance x away from the center of a conductor with current I and length $2a$. Using the fact that $r = \sqrt{x^2+y^2}$ and $|\,\mathrm{d}\mathbf{l}\times\hat{\mathbf{r}}| = \sin\phi\,\mathrm{d}y = x\,\mathrm{d}y\,/\sqrt{x^2+y^2}$, a trigonometric substitution gets us

$$\begin{aligned}
B &= \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x\,\mathrm{d}y}{(x^2+y^2)^{3/2}} \\
&= \frac{\mu_0 I}{4\pi}\int_{\mathrm{atan}(-a/x)}^{\mathrm{atan}(a/x)}\frac{\sec^2 u\,\mathrm{d}u}{x\sec^3 u} \\
&= \frac{\mu_0 I}{4\pi}\int_{\mathrm{atan}(-a/x)}^{\mathrm{atan}(a/x)}\frac{\cos u\,\mathrm{d}u}{x} \\
&= \frac{\mu_0 I}{4\pi}\frac{\sin u}{x}\bigg|_{\mathrm{atan}(-a/x)}^{\mathrm{atan}(a/x)} \\
&= \frac{\mu_0 I}{4\pi}\frac{y}{x\sqrt{x^2+y^2}}\bigg|_{-a}^{a} \\
&= \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}.
\end{aligned}$$

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.



- In the limit $a\to\infty$, the term in the denominator becomes $\sqrt{x^2+a^2}\to a$, and we obtain the equation for the magnitude of the magnetic field of an **infinitely long wire** at a distance r from the wire:

$$\lim_{a\to\infty}\frac{2a}{x\sqrt{x^2+a^2}} = \frac{2}{x} \quad\to\quad B = \frac{\mu_0 I}{2\pi r}.$$

Example: Magnetic Field of a Single Wire

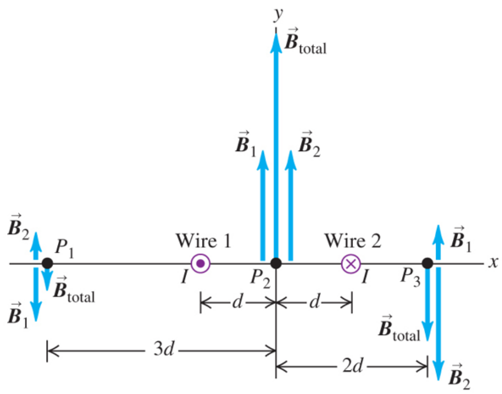
A long, straight conductor carries a 1.0 A current. At what distance from the axis of the conductor does the magnetic field have magnitude $B = 0.5\times 10^{-4}$ T?

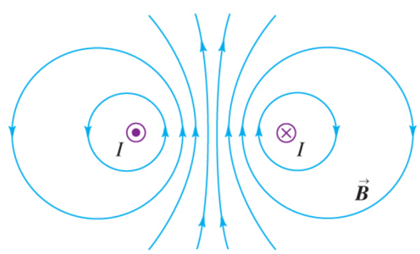
Using our result for the magnetic field due to an infinitely long wire, we get

$$B = \frac{\mu_0 I}{2\pi r} \quad \rightarrow \quad r = \frac{\mu_0 I}{2\pi B} = \frac{(4\pi \times 10^{-7}\ \mathsf{T \cdot m/A})(1.0\ \mathsf{A})}{2\pi(0.5 \times 10^{-4}\ \mathsf{T})} = 4 \times 10^{-3}\ \mathsf{m} = 4\ \mathsf{mm}.$$

Example: Magnetic Field of Two Wires

The figure below is an end-on view of two long, straight, parallel wires perpendicular to the xy-plane, each carrying a current I but in opposite directions. (a) Find $\mathbf{B}$ at points P_1, P_2, and P_3. (b) Find an expression for $\mathbf{B}$ at any point on the x-axis to the right of wire 2.





(a) At the point P_1, the fields due to the two wires are

$$\mathbf{B}_1 = \frac{\mu_0 I}{2\pi(2d)}(-\hat{\mathbf{j}}), \quad \mathbf{B}_2 = \frac{\mu_0 I}{2\pi(4d)}\hat{\mathbf{j}}.$$

The total magnetic field is therefore

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = -\frac{\mu_0 I}{4\pi d}\hat{\mathbf{j}} + \frac{\mu_0 I}{8\pi d}\hat{\mathbf{j}} = -\frac{\mu_0 I}{8\pi d}\hat{\mathbf{j}}.$$

For P_2, we have

$$\mathbf{B}_1 = \frac{\mu_0 I}{2\pi d}\hat{\mathbf{j}}, \quad \mathbf{B}_2 = \frac{\mu_0 I}{2\pi d}\hat{\mathbf{j}} \quad \rightarrow \quad \mathbf{B} = \frac{\mu_0 I}{2\pi d}\hat{\mathbf{j}} + \frac{\mu_0 I}{2\pi d}\hat{\mathbf{j}} = \frac{\mu_0 I}{\pi d}\hat{\mathbf{j}}.$$

Lastly, at P_3 we obtain

$$\mathbf{B}_1 = \frac{\mu_0 I}{2\pi(3d)}\hat{\mathbf{j}}, \quad \mathbf{B}_2 = \frac{\mu_0 I}{2\pi d}(-\hat{\mathbf{j}}) \quad \rightarrow \quad \mathbf{B} = \frac{\mu_0 I}{6\pi d}\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi d}\hat{\mathbf{j}} = -\frac{\mu_0 I}{3\pi d}\hat{\mathbf{j}}.$$

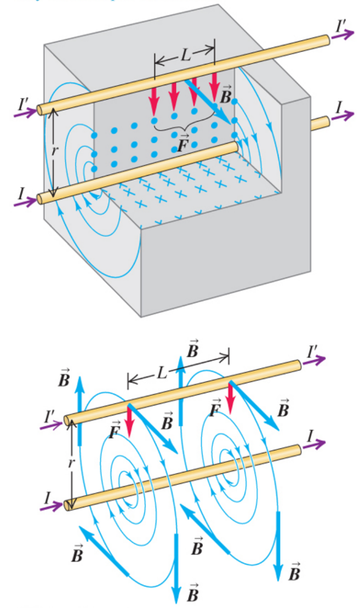
(b) At any point on the x-axis to the right of wire 2, the magnetic fields $\mathbf{B}_1$ and $\mathbf{B}_2$ are in the same directions as at the point P_3. Such a point is a distance $x+d$ from wire 1 and $x-d$ from wire 2, which gives us a total magnetic field of

$$\mathbf{B} = \frac{\mu_0 I}{2\pi(x+d)}\,\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi(x-d)}\,\hat{\mathbf{j}} = -\frac{\mu_0 I d}{\pi(x^2-d^2)}\,\hat{\mathbf{j}}.$$

Force Between Parallel Conductors

The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.



- Given two infinitely long wires with currents I and I' running in the same direction, we can determine the force per unit length F/L using $\mathbf{F} = I\mathbf{L}\times\mathbf{B}$.
  - The field due to the bottom current (I) exerts a *downward* force on the upper current (I') given by

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r}.$$

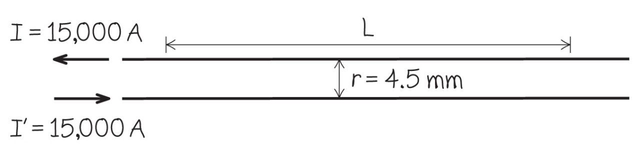
  - The resulting force per unit length is

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}.$$

- The forces are equal and opposite on the wires, so the top wire exerts an *upward* force on the bottom wire.
- Parallel conductors carrying currents in the same direction **attract** each other, and if the currents are in opposite directions they **repel** each other.

Example: Forces Between Parallel Wires

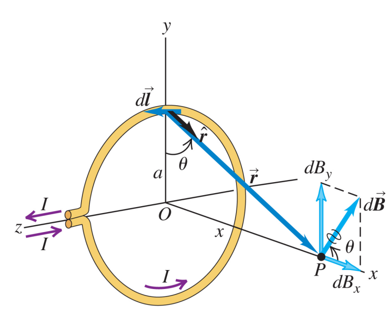
Two straight, parallel, superconducting wires 4.5 mm apart carry equal currents of 1.5×10^4 A in opposite directions. What force, per unit length, does each wire exert on the other?



The conductors repel each other since the currents are in opposite directions. The force per unit length is

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.5\times10^4\ \text{A})^2}{2\pi(4.5\times10^{-3}\ \text{m})} = 1.0\times10^4\ \text{N/m}.$$

Magnetic Field of a Circular Current Loop

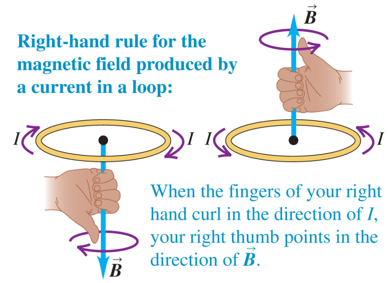


- We can use the Biot-Savart law to find the magnitude of the magnetic field along the axis of a current loop:

$$\mathrm{d}B = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}l}{(x^2+a^2)}.$$

- The radial components of the field cancel each other out, and we are left only with a component along the axis of the loop (x-axis):

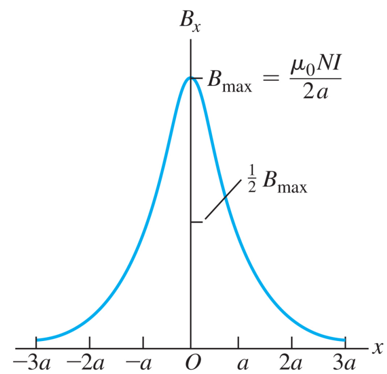
$$\mathrm{d}B_x = \mathrm{d}B\cos\theta = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}l}{(x^2+a^2)}\frac{a}{(x^2+a^2)^{1/2}}.$$



- Integrating $\mathrm{d}B_x$ over the whole loop, we get:

$$\begin{aligned} B_x = \int \mathrm{d}B_x &= \frac{\mu_0 I}{4\pi}\int\frac{a\,\mathrm{d}l}{(x^2+a^2)^{3/2}} \\ &= \frac{\mu_0 I}{4\pi}\frac{a}{(x^2+a^2)^{3/2}}\int \mathrm{d}l \\ &= \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}. \end{aligned}$$

Magnetic Field on the Axis of a Coil



- If we have N such conducting loops closely stacked on top of each other so that the plane of each loop is essentially the same distance x from the field point P, then the total field is simply N times the result for a single loop:

$$B_x = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}.$$

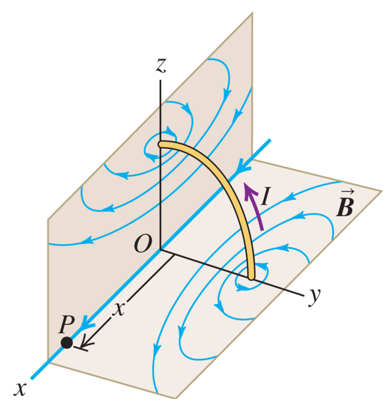
- At the center of the loops, the field strength is at its maximum since $x = 0$, and we have

$$B_{\text{max}} = B_x(x=0) = \frac{\mu_0 N I}{2a}.$$

- The magnetic moment of a single loop is $\mu = I\pi a^2$, so for N loops, $\mu = NI\pi a^2$. The magnetic field along the axis can be written as

$$B_x = \frac{\mu_0 \mu}{2\pi(x^2+a^2)^{3/2}}.$$

Magnetic Field Lines of a Circular Current Loop



- The figure shows some of the magnetic field lines surrounding a circular current loop (magnetic dipole) in planes through the axis.
- The field lines for the circular current loop are closed curves that encircle the conductor; they are not circles, however.

Example: Magnetic Field of a Coil

A coil consisting of 100 circular loops with radius 0.60 m carries a 5.0 A current. (a) Find the magnetic field at a point along the axis of the coil, 0.80 m from the center. (b) Along the axis, at what distance from the center of the coil is the field magnitude 1/8 as great as it is at the center?

(a) At 0.80 m from the center along the axis of the loops, the field strength is

$$B_x = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(100)(5.0\ \text{A})(0.60\ \text{m})^2}{2[(0.80\ \text{m})^2+(0.60\ \text{m})^2]^{3/2}} = 1.1\times10^{-4}\ \text{T}.$$

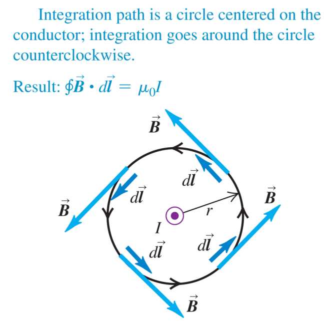
(b) We wish to find a value of x such that

$$\frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}} = \frac{1}{8}\frac{\mu_0 N I}{2a} \quad \rightarrow \quad \frac{1}{(x^2+a^2)^{3/2}} = \frac{1}{8a^3}.$$

Thus,

$$(x^2+a^2)^{3/2} = 8a^3 \quad \rightarrow \quad x^2+a^2 = 4a^2 \quad \rightarrow \quad x = \pm\sqrt{3}a = \pm 1.04 \text{ m}.$$

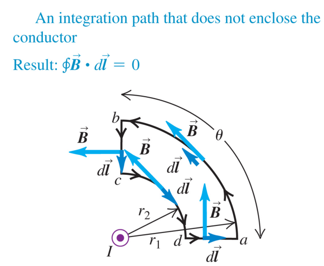
Ampère's law



- Ampère's law is similar to Gauss' law and can be used for finding $\mathbf{B}$ in highly symmetric situations, but is instead based on a **line integral** rather than flux.
- It relates the current I to the line integral of the magnetic field $\mathbf{B}$ around a **closed path.**
- Take the example of an infinitely long conductor with current I:

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \oint B_\parallel \, \mathrm{d}l = B \oint \mathrm{d}l = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I.$$

- If instead there is no current enclosed within the path, the line integral is zero:

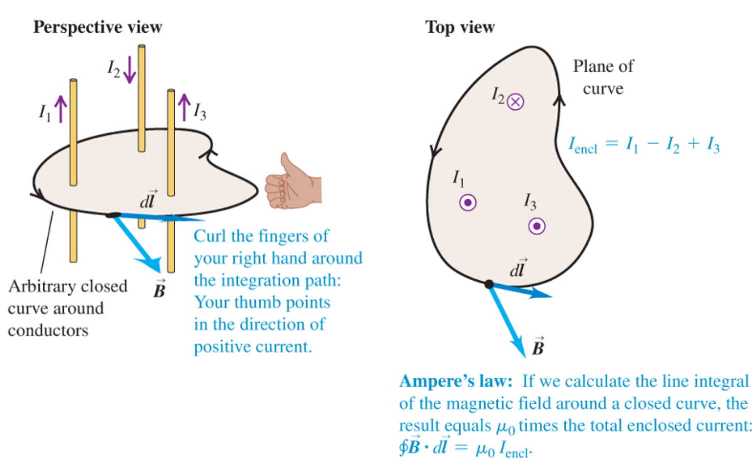


$$\begin{aligned}\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} &= \oint B_\parallel \, \mathrm{d}l \\ &= B_1 \int_a^b \mathrm{d}l + (0)\int_b^c \mathrm{d}l + (-B_2)\int_c^d \mathrm{d}l + (0)\int_d^a \mathrm{d}l \\ &= \frac{\mu_0 I}{2\pi r_1}(r_1\theta) - \frac{\mu_0 I}{2\pi r_2}(r_2\theta) \\ &= 0.\end{aligned}$$

- The general statement of Ampère's law is that the line integral of $\mathbf{B}$ around a closed path is proportional to the net current **enclosed by the path** I_{enc}:

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \mu_0 I_{\text{enc}}$$

- The statement is true regardless of the shape of the path, and it is one of the fundamental equations of electrodynamics.



- Be careful! If the line integral of $\mathbf{B}$ is zero, that **does not** necessarily mean $\mathbf{B} = \mathbf{0}$. It only means that $I_{\text{enc}} = 0$.
- This version of Ampère's law is only valid if the currents are steady and there are no time-varying electric fields present. More on that later...

Applications of Ampère's Law

- Having obtained Ampère's law, we are now able to determine the magnetic field due to highly symmetric current distributions in much the same way that we can obtain the electric field from highly symmetric charge distributions using Gauss's law.
- The next few examples show that in such situations you can exploit the symmetry to calculate the line integral of $\mathbf{B}$ around a simple path.

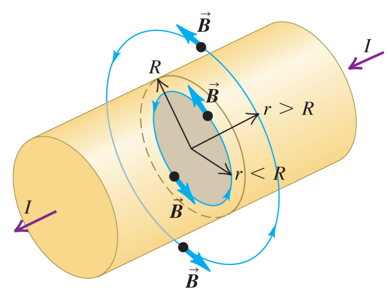
Example: Field of a Long, Straight, Current-Carrying Conductor

Previously we derived Ampère's law from the equation for the field $\mathbf{B}$ of a long, straight, current-carrying conductor. Reverse this process, and use Ampère's law to find $\mathbf{B}$ for this situation.

We will draw an Ampèrian loop that is centered about the conductor and oriented so that $\mathrm{d}\mathbf{l}$ points in the same direction as $\mathbf{B}$ everywhere. The resulting expression exactly matches what we obtained with the Biot-Savart law:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \oint B_{\|}\, \mathrm{d}l = B(2\pi r) = \mu_0 I \quad \rightarrow \quad B = \frac{\mu_0 I}{2\pi r}.$$

Field of a Long Cylindrical Conductor



- Consider a cylindrical conductor with radius R and total current I uniformly distributed across the cross-sectional area of the conductor.
  - Draw an Ampèrian loop of radius $r < R$ on the inside of the conductor. The total current enclosed is $I_{\text{enc}} = (\pi r^2)I/(\pi R^2)$. The magnetic field **on the inside** is therefore:

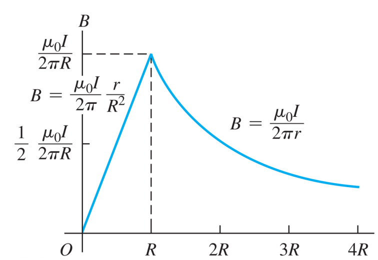
$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = B \oint \mathrm{d}l = B(2\pi r) = \mu_0 I \frac{r^2}{R^2}$$

$$\rightarrow \quad B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2} \quad (r < R).$$

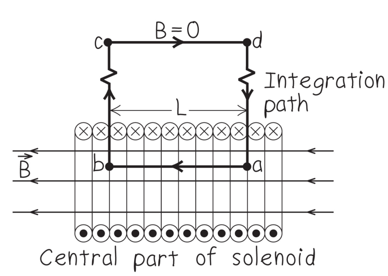
  - **On the outside** of the conductor, the total current enclosed is I, so the field is:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = B \oint \mathrm{d}l = B(2\pi r) = \mu_0 I$$

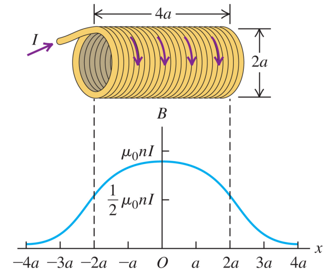
$$\rightarrow \quad B = \frac{\mu_0 I}{2\pi r} \quad (r > R).$$



Field of a Solenoid



- We can find the magnitude of the magnetic field in the center of a solenoid with n turns per unit length and current I passing through each turn by drawing an Ampèrian loop that passes through a length L of the solenoid.
  - We approximate the solenoid as infinitely long, so that the field lines in the solenoid are straight, and there is no magnetic field on the outside.
  - The only contribution to the line integral comes from the inside of the solenoid:



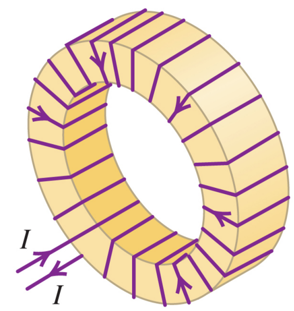
$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = B \int_a^b \mathrm{d}l + (0) \int_b^c \mathrm{d}l + (0) \int_c^d \mathrm{d}l + (0) \int_d^a \mathrm{d}l$$
$$= BL.$$

– The enclosed current is $I_{\text{enc}} = nLI$, so we get:

$$BL = \mu_0 nLI \quad \rightarrow \quad B = \mu_0 nI.$$

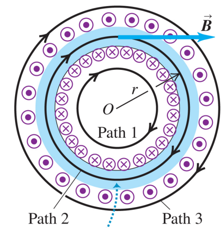
- For a real solenoid (i.e., finite in length), the field strength at each end drops to about half of the strength in the center.

Field of a Toroidal Solenoid



- A **toroidal solenoid** has N turns of current-carrying wire tightly wound around a doughnut-shaped loop.
- The magnetic field lines are circles that are concentric with the solenoid's axis, so we choose our Ampèrian loops to be circles of radius r about the axis of the solenoid.
  - We approximate the turns of the current-carrying wire as circular loops.
  - In the hollow region of the loop, the total current enclosed is zero, so $B = 0$.
  - Inside of the solenoid, we are enclosing all N turns of wire carrying current I, so the enclosed current is $I_{\text{enc}} = NI$:

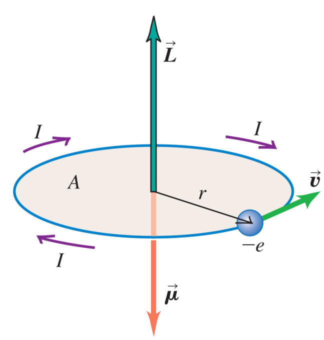
$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = B \oint \mathrm{d}l = B(2\pi r) \quad \rightarrow \quad B = \frac{\mu_0 NI}{2\pi r}$$



The magnetic field is confined almost entirely to the space enclosed by the windings (in blue).

  - Outside of the solenoid, the total current enclosed is zero, so $B = 0$.
- In a real toroidal solenoid, the turns of wire are helical rather than circular loops, and the field on the outside is small but non-zero.

The Bohr Magneton



- Magnetism in materials starts at the atomic level, with magnetic moments being formed by the movement of electrons.
- We can model the electron as moving in a circular orbit of radius r with speed v.
  - This gives rise to a current $I = e/T$, where e is the elementary charge and T is the period of the orbit:

$$I = \frac{e}{T} = \frac{ev}{2\pi r}.$$

- The resulting magnetic moment $\mu = IA$ is

$$\mu = \frac{ev}{2\pi r}(\pi r^2) = \frac{evr}{2}.$$

- Since the angular momentum is $L = mvr$, we can write μ as

$$\mu = \frac{e}{2m} L.$$

- The atomic angular momentum is **quantized:** its component in any particular direction is always an integer multiple of $\hbar = h/2\pi$, where $h = 6.626 \times 10^{-34}$ J · s is **Planck's constant.**

- For $L = h/2\pi$, the magnetic moment is

$$\mu = \frac{e}{2m}\left(\frac{h}{2\pi}\right) = \frac{eh}{4\pi m}.$$

- This quantity is denoted as the **Bohr magneton** μ_B, which has a numerical value of $\mu_\mathrm{B} = 9.274 \times 10^{-24}$ A · m^2.
- Electrons also have an intrinsic angular momentum called **spin**, and the magnitude of the magnetic moment generated by an electron's spin is almost exactly equal to μ_B.

Paramagnetism and Diamagnetism

- Most orbital and spin magnetic moments from the electrons in an atom cancel out. But some materials are made of atoms that can have a net magnetic moment on the order of μ_B.
- If we place such a material in an external field $\mathbf{B}_0$, the individual magnetic moments will tend to align with $\mathbf{B}_0$. This in turn can produce another magnetic field that is proportional to total magnetic moment $\boldsymbol{\mu}_\mathrm{total}$ of the material.
  - The **magnetization** of a material is defined as the net magnetic moment $\boldsymbol{\mu}_\mathrm{total}$ per unit volume V:

$$\mathbf{M} = \frac{\boldsymbol{\mu}_\mathrm{total}}{V}.$$

  - The total magnetic field $\mathbf{B}$ in the material is then

$$\mathbf{B} = \mathbf{B}_0 + \mu_0 \mathbf{M}.$$

  - Such a material that exhibits this behavior is known as **paramagnetic.** The magnetic field on the inside of the material is stronger than the field on the outside.
- The result of paramagnetism is that the magnetic field inside of the material is *greater* than an equivalent magnetic field in a vacuum by a factor K_m, which is called the **relative permeability** of the material.
  - The value of K_m depends on the material.
  - We can use this to define the **permeability** μ of the material: $\mu = K_\mathrm{m}\mu_0$ (Careful! This μ is different from the magnetic moment!)
  - The corresponding equations that were used to derive the magnetic field $\mathbf{B}$ for various current distributions (Ampère and Biot-Savart laws) can be used for magnetic materials by simply replacing μ_0 with μ.
  - The amount by which the relative permeability differs from 1 is called the **magnetic susceptibility:**

$$\chi_\mathrm{m} = K_\mathrm{m} - 1.$$

- In some materials, we instead end up with a net magnetic moment that points in the direction *opposite* to that of the external magnetic field $\mathbf{B}_0$.
  - This behavior is called **diamagnetism**, and such materials have a relative permeability $K_\mathrm{m} < 1$.
  - In such a material, the magnetic field on the inside is weaker than the field on the outside.
- The table shows some magnetic susceptibilities of materials at $T = 20$ °C.

Material	$\boldsymbol{\chi_\mathrm{m} = K_\mathrm{m} - 1\ (\times 10^{-5})}$
Paramagnetic	
Iron ammonium alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	−16.6
Mercury	−2.9
Silver	−2.6
Carbon (diamond)	−2.1
Lead	−1.8
Sodium chloride	−1.4
Copper	−1.0

Example: Magnetic Dipoles in a Paramagnetic Material

Nitric oxide (NO) is a paramagnetic compound. The magnetic moment of each NO molecule has a maximum component in any direction of about one Bohr magneton. Compare the interaction energy of such magnetic moments in a 1.5 T magnetic field with the average translational kinetic energy of molecules at 300 K.

The potential energy from the interaction between the atomic magnetic moment and the external magnetic field is $U = -\boldsymbol{\mu} \cdot \mathbf{B}$. At most, the magnitude of the atomic magnetic moment is around the Bohr magneton μ_B, so the maximum for the potential energy is

$$U_{\max} \approx \mu_B B = (9.27 \times 10^{-24}\ \text{J/T})(1.5\ \text{T}) = 1.4 \times 10^{-23}\ \text{J}.$$

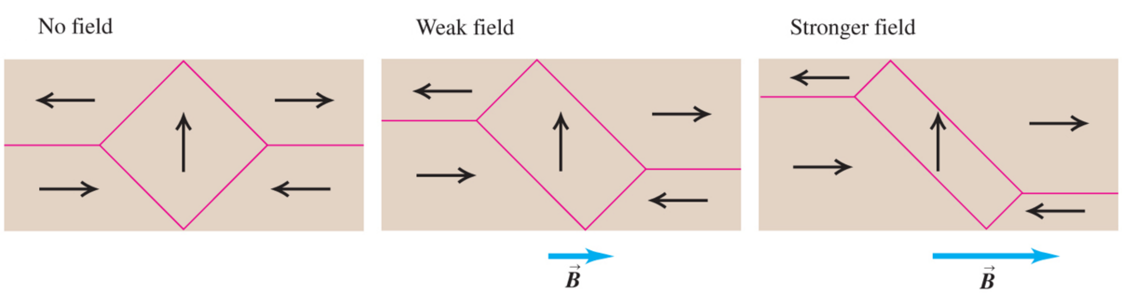
Meanwhile, the average translational energy K of a single NO molecule is

$$K = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(300\ \text{K}) = 6.2 \times 10^{-21}\ \text{J},$$

which more than two orders of magnitude larger than the energy magnetic moment interaction. Thus, at a temperature of 300 K, the magnetic interaction energy is only about 0.2% of the thermal kinetic energy, so we expect only a slight degree of alignment of the magnetic moments with the external field. This is why paramagnetic susceptibilities at ordinary temperatures are usually very small.

Ferromagnetism

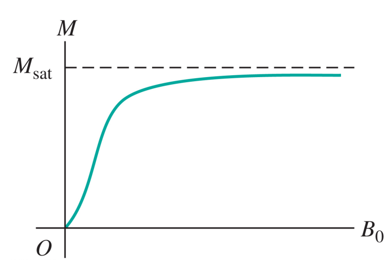
- In **ferromagnetic** materials (such as iron, nickel, and cobalt), there are strong interactions between the atomic magnetic moments that cause them to align parallel with each other even when there is no external magnetic field. Such regions are called **magnetic domains.**

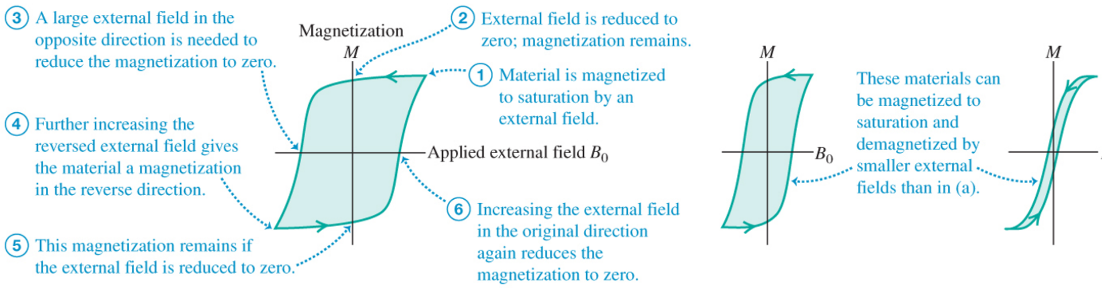


- When there is no external $\mathbf{B}_0$ field present, the domains are randomly oriented.
- Applying an external $\mathbf{B}_0$ field causes the domain boundaries to shift, and the domains that are aligned with $\mathbf{B}_0$ grow larger, while the other domains shrink.
- Such materials have large values of K_{m}, on the order of 10^3 to 10^5.

Hysteresis

- For ferromagnetic materials, a strong enough $\mathbf{B}_0$ field will eventually cause the magnetization to reach its saturation value $\mathbf{M}_{\text{sat}}$.
- The relationship between $\mathbf{M}$ and $\mathbf{B}_0$ is different depending on whether or not $\mathbf{B}_0$ is increasing or decreasing. This behavior is called **hysteresis,** and the effect depends on the material.





Example: A Ferromagnetic Material

A cube-shaped permanent magnet is made of a ferromagnetic material with a magnetization M of about 8×10^5 A/m. The side length is 2 cm. (a) Find the magnetic dipole moment of the magnet. (b) Estimate the magnetic field due to the magnet at a point 10 cm from the magnet along its axis.

(a) The total magnetization is

$$\mu_{\text{total}} = MV = (8 \times 10^5 \text{ A/m})(2 \times 10^{-2} \text{ m})^3 = 6 \text{ A} \cdot \text{m}^2.$$

(b) The magnetic field can be approximated by treating the cube as a current loop. This approximation is justified since the distance $x = 10$ cm at which we're evaluating the magnitude of the magnetic field is significantly larger than the side length $a = 2$ cm of the cube. Furthermore, since $x^2 \gg a^2$, we can approximate the $x^2 + a^2$ term as x^2, and we have

$$B = \frac{\mu_0 \mu_{\text{total}}}{2\pi(x^2 + a^2)^{3/2}} \approx \frac{\mu_0 \mu_{\text{total}}}{2\pi x^3} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(6 \text{ A} \cdot \text{m}^2)}{2\pi (0.1 \text{ m})^3} = 1 \times 10^{-3} \text{ T}.$$