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Practicable solution approaches for differentiated pricing of vehicle sharing systems

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Abstract

Vehicle sharing systems have become increasingly popular. However, one-way vehicle sharing system providers face a major challenge. The uneven distribution of vehicles across locations caused by the uneven nature of the demand patterns poses a problem, since there are accumulations of vehicles where the demand is low. This challenge can be solved with an appropriate pricing approach that creates incentives for user-based relocation by considering supply-side network effects. While the literature mostly focuses on trip-based pricing, we were inspired by the majority of car sharing providers who use origin-based minute pricing that differentiates based on the origins of rentals, such as Share Now. Therefore, we develop two different and *practicable* solution approaches to determine spatially and temporally differentiated origin-based minute prices that take into account supply-side network effects. The first solution approach does not differentiate between rentals and demand and calculates continuous prices for every period and location. The second solution approach determines the vehicle distribution for each period and then calculates the optimal prices for each period backwards. Extensive computational experiments show that our solution approaches anticipate supply-side network effects and thus generate a near-optimal profit in less computational time compared to more complex benchmarks from the literature. In a sensitivity analysis we additionally show that the results are robust against stochasticity of demand and that the solution approaches perform well for different price sets.

Keywords Vehicle sharing systems · Car sharing · Differentiated pricing · Static pricing · Origin-based · Optimization · Backwards solution approach

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1 Introduction

Vehicle sharing systems (VSSs), such as car sharing, bike sharing, or scooter sharing, are specific shared mobility systems (Mourad et al. 2019). They allow users to flexibly and spontaneously rent vehicles for individual trips for a short period of time (Ataç et al. 2021). In contrast to other popular shared mobility systems, these individual trips are conducted by the user. VSSs are one component of a sustainable mobility concept in light of the ongoing debate about the climate crisis (Turan et al. 2023). For example, ideally, car, bike or scooter sharing systems with widely available vehicles do not only partially replace private vehicles but can also help to reduce emissions, especially, if the trips are made with an electric vehicle.

This, in combination with a high flexibility, are reasons why the concept of car sharing has become increasingly popular among providers and customers in recent years. For example, in the EU, the share of sold cars used for new mobility (car sharing, ride hailing, ride sharing, and robocabs) is predicted to rise from 2% in 2015 up to 15% in 2025 (Destatis 2017).

VSS providers operate in a business area with a given fleet-size. The VSS can be operated by different institutions, such as public municipalities or private companies. Customers use a mobile application to get the location and price of available vehicles. The VSS provider cannot select customers for trips (no trip selection), but it does set prices for locations in different periods, which affects demand. The VSS providers are interested in the central objective of maximizing profit (minimizing costs) (Pantuso 2022). In the best case, this objective is achieved with the help of a *practicable* solution approach. *Practicable* here means that the provider is able to successfully execute or implement the solution approach in practice without any further knowledge, e.g. about additional preprocessing steps.

We make a distinction between one-way or free-floating and two-way VSS. In one-way VSS, the customer can pick up the vehicle at one station and drop it off at any other station or at the same station (or in free-floating VSS, the customer can pick up the vehicle and drop it off anywhere in the business area). In two-way VSS, the customer has to drop off the vehicle at the same station where she picked it up. We focus on one-way (and free-floating) VSS as a flexible and convenient alternative to private vehicles, which enables users to pick up and drop off vehicles anywhere within the provider's business area. However, one-way (and also free-floating) VSS providers face a main challenges: The distribution of vehicles is uneven across locations, because origin and destination locations are influenced by the customer's preferences. These preferences and thus the demand vary during the day (uneven nature of the travel pattern). This is the so-called "tide phenomenon", which represents the oscillation of demand intensity throughout the day (spatio-temporal demand asymmetries) (Côme 2014; Jorge and Correia 2013). Besides, the current number of available vehicles in an location affects the number of future rentals in that location. Therefore, it must be taken into account that a rental not only decreases the current amount of vehicles in a origin location and increases the amount of vehicles in the destination location, but also has an impact on future rentals in these two locations (supply-side network effects). Neglecting these supply-side network effects leads to an accumulation or absence of vehicles at popular locations. As a result, the system is no longer able to serve demand and may lose customers (Di Febbraro et al. 2012).

With regards to the imbalance, a possible solution is relocation. More precisely, relocation can be distinguished into operator-based and user-based relocation. Operator-based relocation involves repositioning of vehicles by employees, while user-based relocation is performed by customers and is incentivized by the provider. The operator-based relocation increases the operational costs, as providers need additional staff and extra equipment, e.g. operator-based relocation of bikes is handled with trucks (Dötterl et al. 2017). Although operator-based relocation is one of the main cost drivers (Jorge and Correia 2013) and (time) inefficient (vehicle cannot be used while it is relocated by staff) (Schiffer et al. 2021), almost all VSS providers use it to obtain reduced imbalance.

A more cost-effective method is user-based relocation (Brendel et al. 2016). Ideally, the provider sets prices to encourage customers to drive from a low-demand location to a high-demand location (Angelopoulos et al. 2016). Furthermore, no additional vehicles or additional trips are needed. In short, user-based relocation is more preferable from an environmental and economic perspective (Clemente et al. 2017), as it is a more sustainable and cost-effective alternative to operator-based relocation (Stokkink and Geroliminis 2021).

In addition, providers should also consider customer preferences regarding pricing. Customers have two distinct preferences:

1. Customers prefer an easy and comfortable booking process in VSSs, i.e. booking a vehicle without much effort. They cannot and/or do not want to disclose their destination or the duration of the intended rental. Therefore, VSS providers do not know the destination in advance. If VSS providers ask customers to (truthfully) disclose their intended destination, this would considerably change the customers experience of VSSs and thus would be unacceptable in most practical settings.
2. Customers prefer clear and transparent prices. This means that customers would like to know (minute) prices before the rental starts.

The easiest way to satisfy both customer preferences is to set a unit price (minute price is the same regardless of location and time). However, a one-size-fits-all price fails to meet the VSS providers challenge (uneven vehicle distribution). Another way to price the two customer preferences is to choose discrete prices from a predefined price set that vary (only) based on where and when a rental starts (*origin-based pricing*). This predefined price set has a clear number of price points. The alternative, i.e., displaying prices for all potential trips combinations (trip-based pricing) in advance of a rental, is impracticable in general, given that free-floating VSS or one-way station-based VSS providers often discretize their business area into up to hundred zones (Müller et al. 2023).

Thus, the challenge of imbalance, the objective of maximization, and customer preferences can be reasonably addressed with an appropriate origin-based differentiated pricing approach with discrete prices form a pre-defined price set. It incentivizes customers to improve future vehicle distribution (user-based relocation) by considering a longer time horizon (e.g., one day, one week), which means that supply-side network effects are sufficiently taken into account. This is in line with the business decision of Share Now (origin-based prices from a price set with three different price points).

Against this background, this paper focuses on origin-based pricing, where the free-floating or one-way VSS provider sets prices depending on a rental's time and origin and from a discrete price set. Since origin-based pricing is most commonly used in current practice, the providers of such VSSs do not have to change their booking process by asking customers for the destination or rental duration. This means a more efficient interaction between user and provider and an easier implementation. To be precise, in this work, we consider the problem of differentiated pricing of free-floating or one-way VSS providers with a focus on its *practicability*, by using different heuristic solution approaches.

The contributions of our work are the following:

- First, to the best of our knowledge, we are one of the first to focus on *practicable*, *origin-based* differentiated pricing, which is highly relevant in practice. The proposed pricing mechanism is transparent to the customer. The problem's practical relevance is ensured by, among other things, a cooperation with Share Now, Europe's largest car sharing provider.
- Second, the solution approaches we develop are problem-specific, easy to implement and new. The first one is a simplified model, so that the problem can be solved quickly, even for large instances. The second solution approach is a backwards algorithm for determining the best prices for all location-period-combinations. Thus, the major advantage of both approaches is that they require no pre-processing and yet produce the same results as existing, more complex benchmarks in clearly less time. Thus, these new solution approaches are beneficial for practitioners to get a *practicable* and straightforward solution and for researchers to benchmark other upcoming solution approaches.
- Third, we generate a number of relevant managerial insights based on an extensive computational study and a sensitivity analysis with different problem sizes, considering various relevant parameter settings and demand patterns.

The remainder of the paper is organized as follows. In Sect. 2, we review the relevant literature, focusing on differentiated pricing problems using optimization. In Sect. 3, we describe the problem and present two proposed solution approaches. Section 4 contains the computational study. After the computational study, we perform a sensitivity analysis in Sect. 5. Section 6 concludes the paper and gives an outlook on future research. The appendix provides additional results for the computational experiments and underlying mathematical model.

2 Literature

The literature on VSS optimization is quite extensive, so for general overviews we refer to the following papers:

- bike sharing: DeMaio (2009), Fishman et al. (2013), Ricci (2015)
- car sharing: Jorge and Correia (2013), Ferrero et al. (2015a), Ferrero et al. (2015b), Illgen and Höck (2019), Nansubuga and Kowalkowski (2021), Esfandabadi et al. (2022)
- VSS in general: Laporte et al. (2015, 2018), Ataç et al. (2021), with a focus on sustainability: Turan et al. (2023, Chapter 5.1)

In our literature review, we focus on differentiated pricing in VSSs in the sense that the pricing does not depend on the system's current state (e.g. current vehicle distribution, see Ataç et al. 2021). Furthermore, we only consider papers that apply collective, not individual pricing (targeted to all customers, see Pantuso 2022) using optimization. We exclude papers that apply business rules (e.g. Ruch et al. 2014; Brendel et al. 2016; Wagner et al. 2015; Barth et al. 2004).

In the following, we introduce dimensions for differentiated pricing approaches (Sect. 2.1). Using these dimensions, Sect. 2.2 considers differentiated pricing and Sect. 2.3 presents further literature that developed single-period solution approaches in a rolling-horizon fashion for dynamic pricing.

2.1 Dimensions of differentiated pricing

We propose two dimensions of the customer perspective and four dimensions of the provider perspective to structure the different solution approaches (see also Table 1 in Appendix 1). The following dimensions describe the customer perspective:

1. Spatio-temporal pricing: Origin-based prices depend only on the time and location of a rental's start. Other variants are destination-based prices (prices depend on location and time of destination) or trip-based prices (prices depend on both origin and destination).
2. Number of possible prices: Some pricing approaches set only one price for all locations and periods, whereas others set different prices selected from a discrete set of prices (price list). Still others either have a defined upper and lower bound for prices, or the restriction that prices must be positive, or no restrictions regarding the prices at all.

The following four dimensions characterize the provider perspective:

1. Control of rentals: There are providers that can influence the number of rentals only by the price (price control). However, there are also providers that can additionally reject requests (trip selection).

2. Objective: Different pricing approaches aim either at improving the distribution (balance) of vehicles in the VSS or at increasing profit (or reducing costs).
3. Foresight: Some pricing approaches determine prices based on the current period without considering the supply-side network effects for the next period(s) (myopic). In contrast, other pricing approaches additionally consider how pricing decisions in the current period affect future vehicle supply, and thus future rentals at each location in subsequent periods, by considering supply-side network effects (anticipative).
4. Additional parameters: Some pricing approaches require some pre-processing, for instance, additional estimation of parameters in advance to perform the price determination.

2.2 Literature on differentiated pricing

Most of the published papers considering differentiated pricing in VSS deal with trip-based prices. There are solution approaches that use a fluid approximation to determine prices. In fluid approximations, the model sets the prices so that the rentals of a station match the demand, i.e. there is no distinction between demand and rentals. This means that the price is not constrained (even negative prices are possible). Waserhole and Jost (2012) propose a fluid approximation for the revenue-maximizing trip-based pricing problem, which is the upper bound of the stochastic model if demand and supply are scaled to infinity. Guo and Kang (2022) also propose a fluid model to maximize profit, which considers the pricing and re-balancing problem of electric vehicles.

However, other papers distinguish between demand and rentals. More precisely, rentals depend on supply, demand and prices. Only positive prices are determined here. We distinguish between papers where the provider cannot reject users and controls rentals only by price (i.e. price control) and papers where the provider can reject users (i.e. trip selection).

First, we consider papers where the provider can (indirectly) reject users. Xu et al. (2018) formulate a mixed-integer non-linear and non-convex program. On this basis, they develop a computationally tractable mixed-integer convex program which has the same objective in the optimum, and solve the latter arbitrarily close to optimality. Jiao et al. (2020) integrate trip selection and price incentives with user-based relocation in one model. Therefore this model distinguishes between three different types of demand: (1) potential travel demand, (2) adopted demand (demand after customers see the price), and (3) final served demand (after trip selection). They consider a mixed-integer non-linear program to maximize the profit and propose an iterative algorithm between the two decomposition programming sub-problems (a linear master sub-problem and a non-linear sub-problem). Huang et al. (2020) compare operator-based and user-based relocation. They formulate two mixed-integer non-linear programs for the user-based relocation. The first one sets trip-based prices, whereas the second optimizes pick-up and drop-off fees. They solve both programs with a combined rolling-horizon and iterated local search heuristic. Lu et al. (2021) use another model formulation, i.e. a bi-level non-linear program in which the provider determines profit-maximizing prices on the upper level. In this case, the prices are within the previously defined bounds. The lower level's objective minimizes customers' total cost by a binary choice between two modes of transportation (shared vehicles vs. private vehicles). In an interpretation of a discrete choice model, rentals are additionally bounded from above by a logit model. The authors transform the bi-level program to a single-level one using Karush–Kuhn–Tucker conditions, and heuristically solve it with a genetic algorithm.

Second, we consider papers where the providers only control rentals by price. Jorge et al. (2015) formulate a profit-maximizing trip-based pricing problem as mixed-integer non-linear, non-concave program, which is not tractable for real-world-instances. They propose an iterated local search meta-heuristic to solve the program. Ren et al. (2019) extend the previous program to include a vehicle-grid interaction of electric vehicles. They use non-linear solvers.

Huang et al. (2020) combine tactical and operational decisions for a one-way station-based car sharing system on a mixed-integer non-linear program. This program optimizes profit by considering fleet size, pricing (both tactical) and relocation. They linearize this program, decompose it into two interdependent stages, and develop a gradient search method to solve the two stages. Zhang and Kan (2018) formulate a non-linear, non-concave program that maximizes the profit for an entire planning horizon of a station-based, one-way car sharing system by setting trip-based prices. Particle swarm optimization is used to solve the program.

In contrast to the presented literature above, Soppert et al. (2022) consider origin-based, differentiated pricing for a one-way or free-floating VSS. They formulate a profit-maximizing, mixed-integer linear program that distinguishes between rentals and demand. This program determines origin-based prices from a discrete price set. The rentals of a location are calculated as a minimum of demand and supply (number of idle vehicles). The demand can be influenced only by the price (price control). Since the model cannot be solved due to its complexity, Soppert et al. (2022) propose a solution method using value function approximation. To apply this method, the provider has to estimate some parameters in pre-processing.

We also consider a one-way or free-floating VSS provider, who determines origin-based prices from a discrete price set (i.e. price control). We suggest two solution approaches to solve this mixed-integer linear program without the need to estimate any parameters in pre-processing. This makes them more practicable.

2.3 Further literature

In addition, there are other models for VSSs that are developed in such a way that they have to be solved for each single period with the currently available data (without considering supply-side network effects). Although they apply dynamic pricing, they have similarities with our problem.

The following papers do not distinguish between demand and rentals. They assume that prices do not affect the demand, but affect the customers' destinations. Pfrommer et al. (2014) propose a model predictive control approach. The objective of the quadratic program is a weighted sum of the deviation from an optimal vehicle distribution and the cost of incentive payments. Chemla et al. (2013) use a linear program to determine the number of customers who change their travel plans due to the price incentive in order to reach the given target inventory of vehicles for each station. Haider et al. (2018) formulate a bi-level program, where the upper level determines prices and minimizes vehicle imbalance, while the lower level represents the cost-minimizing route choice of customers. The problem is transformed into a single-level program. Wang and Ma (2019) consider the objective of keeping the vehicle inventory within a certain range for a period. For this purpose, they define lower and upper thresholds for each station. The number of rentals from or to a station can be affected by pick-up and drop-off fees. To this end, they formulate a simple quadratic program to calculate such optimal dynamic fees.

Other papers distinguish between demand and rentals. Pantuso (2020, 2022) formulates an extensive mixed-integer two-stage stochastic program, which maximizes the profit by setting trip-based prices and decides about operator-based relocation. Pantuso (2020) also proposes a compact integer programming reformulation and compares the two formulations in terms of ease of solution. Pantuso (2022) proposes an exact solution algorithm for the mixed-integer two-stage stochastic program.

3 Solution approaches for differentiated pricing

In this section, we first define the origin-based differentiated pricing problem in VSSs (Sect. 3.1) to introduce the problem, and then describe two solution approaches for differentiated pricing. One solution approach is to use a simplified model (Sect. 3.2), the other is to calculate prices backwards (Sect. 3.3).

3.1 Problem statement and notation

There is a free-floating VSS provider, which discretizes the business area in Z zones $\mathcal{Z} = 1, ..., Z$) which can be treated as stations. This VSS provider sets differentiated origin-based prices from a discrete price set P^M. This means for all rentals, which have the same origin, the same minute price $p_{i,t}$ is charged. The considered time horizon is subdivided into T periods ($\mathcal{T} = 1, ..., T$). The VSS provider maximizes its profit by setting minute prices $p_{i,t}$ for every location $i \in \mathcal{Z}$ and period $t \in \mathcal{T}$, regardless of the destination. The minute prices are chosen from a given price set $\mathcal{P}^{\mathcal{M}} = p^1, ..., p^M$ with M price points.

We also have the following assumptions concerning *demand*, *rental realization* and *dynamics*. *Demand* depends on the price. A base demand (demand at the median price) $d_{i,j,t}$ for each rental combination $i - j$ of possible origins and destinations at period t is given. We assume that if the price is lower (than the median price), demand increases and if the price is higher (than the median price) demand decreases. Thus, we scale the base demand with sensitivity factors f^m, which depend on the chosen price $p_{i,t} = p^m$ from a given price set $\mathcal{P}^{\mathcal{M}} = p^1, ..., p^M$ with M price points (like in Soppert et al. 2022). A low price corresponds to a high sensitivity factor and vice versa (like in Özkan 2020; Soppert et al. 2022). The advantage of using these price points $p^m \; \forall \, m = 1..M$ from the price sets and the associated sensitivity factors $f^m \; \forall \, m = 1..M$ is that all (even nonlinear) demand functions can be represented by these price sets and sensitivity factors without affecting the (linear) model. The sensitivity factors $f^m \; \forall \, m = 1..M$ can be determined using either data analysis or limit conjoint analysis. Data analysis involves setting a base price for the same type of vehicle on a particular working day (for several weeks), followed by the lowest price for the same working day (for several weeks) and then the highest price for the same working day (for several weeks). Ideally, all weeks should be comparable and not vary greatly due to weather, major events or the time of year. The difference in average demand on that working day provides the sensitivity factors. Another option is to take a representative sample of customers and use them to do a limit conjoint analysis, where one characteristic is price.

For the *rental realization*, we assume that the provider cannot reject any user at a location, if there are vehicles available (no trip selection, but price control). Therefore, the number of trips is the minimum of the supply (number of idle vehicles at the location) and the demand at each location i. Furthermore, we assume that the rentals split proportionally to the demand regarding their destinations. Thus, the provider can only affect the system and thereby the rentals by price.

For the *dynamics*, we assume that rentals start at the beginning of a period t and the vehicles, at latest, always become available again at the beginning of the respective next period $t + 1$. The average rental duration $l_{i,j} \in \mathbb{R}_0^+$ (in minutes) is shorter than the period length, but can vary according to the spatial distance between different locations $i - j$. This assumption is common in the literature (see e.g. Soppert et al. 2022; Huang et al. 2020; Xu et al. 2018). Also, the initial inventory $\hat{a}_{i,0}$ of each location is given at the beginning of the period $t = 0$.

3.2 Simplified model for differentiated pricing

This section describes the first solution approach, which is a simplified model. The idea behind this approach is as follows: We only consider the available vehicles ($a_{i,t}$) and the scaled sensitivity factors ($q_{i,t}$, see the following paragraphs for a more detailed description) as continuous variables in a relaxed model. In this approach, the scaled continuous sensitivity factors $q_{i,t}$ (and thus indirectly the price) determine the level of demand. Compared to a model with discrete prices chosen from a set of prices (p^m from price set $\mathcal{P}^{\mathcal{M}} = p^1, ..., p^M$ with M price points) with corresponding sensitivity factors $f^m \ \forall\, m = 1..M$, it is more straightforward and can be solved much faster since there are *no* integer variables. In a final step, the defined inverse demand function converts continuous sensitivity factors to continuous prices, which are then converted back to discrete prices p^m from the price set $\mathcal{P}^{\mathcal{M}}$ using rounding.

The simplified model does not distinguish between demand and rentals, i.e. demand equals rentals. In addition, demand is continuous and scaled by the sensitivity factor. This has one clear advantage. We do not need additional decision variables for rentals in the model, and all variables are continuous. Therefore, we do not need to define constraints regarding rentals, e.g. ensuring that rentals do not exceed demand or that rentals are the minimum of demand and supply (available vehicles). We only need the upper and lower bounds of the scaled sensitivity factor which scale the demand. This makes the model faster compared to other origin-based pricing models, which considers discrete prices and distinguishes between demand and rentals. However, one drawback is that the continuous prices in the simplified model must be converted to discrete prices.

The model also includes the continuous price indirectly as an inverse demand function $f(q_{i,t})$ for the sensitivity factors. We obtain this inverse demand function $f(q_{i,t})$, which depends on the scaled sensitivity factor $q_{i,t}$, in a three-step process. In the first step, we determine a demand function of the form $f^m(p^m) = a - b \cdot p^m$, where $f^m(p^m)$ is the price-dependent sensitivity factor. This is always 1 for the base demand and e.g. 0.75 for the higher price of $p^2 = 0.36$ €/min and 1.25 for the lower price of $p^1 = 0.24$ €/min. For this example, this means the demand function is: $f^m \approx 2.25 - 4.1666671 \cdot p^m$. In the second step, this demand is scaled so that a low price results in a scaled sensitivity factor of $q_{i,t} = 1$. We do this because we want to limit prices downward to the lowest price. Thus, in the example, the demand function for the scaled sensitivity factors is $q_{i,t} \approx 2 - 4.1666671 \cdot p_{i,t}$. In the third step, we use the inverse demand function to determine the price $f(q_{i,t}) = \frac{a-q}{b}$. For the example, this results in $p = f(q_{i,t}) \approx \frac{2-q}{4.1666671}$. For the simplified model we also need the scaled demand $d^{scaled}_{i,j,t}$. It is defined as the demand $d_{i,j,t}$ at the lowest price p^1. We obtain this scaled demand ($d^{scaled}_{i,j,t}$) by multiplying the base demand $d_{i,j,t}$ at the median price with the sensitivity factor for the lowest price f^1 ($d^{scaled}_{i,j,t} = d_{i,j,t} \cdot f^1$).

We formulate the simplified model based on a deterministic network flow problem in which vehicles move through a spatio-temporal network. The resulting model considers expected values of the demand and available vehicles in the vehicle sharing system.

The model is embedded in an algorithm (see Algorithm 1, line 3) that converts the continuous prices of the simplified model into prices of the discrete price set $\mathcal{P}^{\mathcal{M}}$

Algorithm 1 Applying Simplified Model

1: Results: Best prices for the whole day
2: Step 1: Find a linear price function $f(q_{it})$ that describes the price as a function of price sensitivity
3: Step 2: Solve the following NLP:

$$\max_{\mathbf{q},\mathbf{a}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}} (f(q_{i,t}) - c^{var}) \cdot l_{ij} \cdot d^{scaled}_{i,j,t} \cdot q_{i,t} \tag{1}$$

$$\text{s.t.} \quad a_{i,t+1} = a_{i,t} - \sum_{j\in\mathcal{Z}} d^{scaled}_{i,j,t} \cdot q_{i,t} + \sum_{k\in\mathcal{Z}} d^{scaled}_{k,i,t} \cdot q_{k,t} \quad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{2}$$

$$a_{i,0} = \hat{a}_{i,0} \quad \forall i \in \mathcal{Z} \tag{3}$$

$$a_{i,t} \geq \sum_{j\in\mathcal{Z}} d^{scaled}_{i,j,t} \cdot q_{i,t} \quad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{4}$$

$$q_{i,t} \leq 1 \quad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{5}$$

$$q_{i,t} \in \mathbb{R}^+_0 \quad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{6}$$

$$a_{i,t} \in \mathbb{R}^+_0 \quad \forall i \in \mathcal{Z}, t \in \{1, \ldots, T+1\} \tag{7}$$

4: Step 3: Round to the nearest discrete price of price set $\mathcal{P}^{\mathcal{M}}$

(line 4).

The contribution margin (1) is the product of the rentals ($d^{scaled}_{i,j,t} \cdot q_{i,t}$), the rental time ($l_{i,j}$), and the price minus the variable costs ($f(q_{i,t}) - c^{var}$). Note that maximizing contribution margin here is equivalent to optimizing profit, since decisions about fixed costs cannot be made at this point and are therefore out of scope. Constraints (2) provide flow conservation. They ensure a constant fleet size at all times. The initial vehicle distributions are defined through Constraints (3). Constraints (4) set upper bounds on the number of rentals, so that the rentals are limited by the number of vehicles available in a zone. The next constraints (5) define the upper bounds of the sensitivity factors $q_{i,t}$. The last constraints (6 and 7) define that all variables are real positive numbers. The range of the scaled sensitivity factors are between 0 and 1. Thus, the price cannot be lower than the lowest price (sensitivity factor $q_{i,t} = 1$) and is bounded to b (see demand function).

The prices we get from solving the model are continuous. They must be converted to the discrete prices (line 4) since we only consider discrete prices from a price set $\mathcal{P}^M$. To do this, we first compute the absolute differences of all resulting continuous prices $p^{con}_{i,t}$ for all $i - t$ combinations at all discrete price points p^m. Then, we convert the continuous prices to discrete prices with the smallest absolute difference.

3.3 Backwards algorithm for differentiated pricing

This section describes the second solution approach, called a backwards algorithm (see Algorithm 2 and Fig. 1). The idea of this solution approach is to decompose the problem into smaller problems. These smaller problems can be solved more quickly. To do this, we first compute the vehicle distribution at the beginning of each period (Step 1), taking advantage of the fact that demand is deterministic. Then, starting backwards from period T, we compute prices for each period based on the calculated initial vehicle distributions of this period, taking into account the next periods (Step 2). We start with the last period, because for this period, given that the vehicle

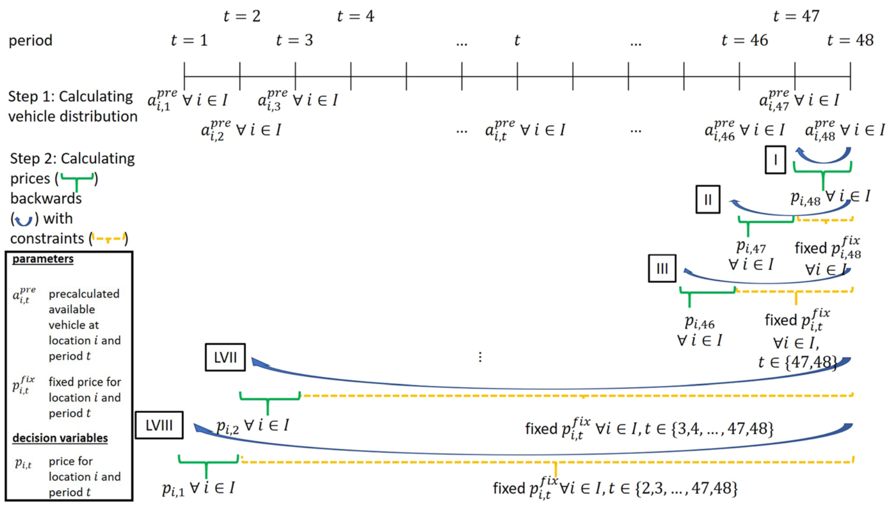


Fig. 1 Illustration backwards algorithm

distribution at the beginning of this period is (approximately) correct, we can calculate the optimal pricing policy. Furthermore, the prices of the first calculation step (i.e. the last period) have no effect on other periods. If we were to calculate prices forward, we could calculate vehicle distribution and prices at the same time, but the calculation of prices for the first period would have a significant impact on the next periods.

The process of the solution approach is shown in Fig. 1. As already mentioned, the first step is to calculate the vehicle distribution across the business area for each period using a simple solution approach. Based on these calculated vehicle distributions, the algorithm calculates the prices backwards by optimizing the price for the current period (green brackets) in different sub-steps (see for example sub-steps I, II, III, XLVII and XLVIII), starting from the last, 48th period (see sub-step I). To obtain a result that considers supply-side network effects, the algorithm takes into account the fixed prices (dashed yellow brackets) calculated in the previous sub-step(s) and the resulting flows of vehicles. For example, in sub-step I, the algorithm optimizes only the prices for the 48th period, and in sub-step XLVIII, the algorithm optimizes the prices for the first period, taking into account the fixed prices of the following periods (period 2 to period 48), considering the resulting network-flows of vehicles until the end of the day.

The advantage of this method lies in its solvability and computational speed, and it takes supply-side network effects into account, at least to some extent. The disadvantage of this solution approach is that the pricing approach can only determine the prices of the period under consideration, but takes into account the fixed prices in the following periods.

Algorithm 2 describe the solution approach in pseudo code. It defines the following two parameters as empty sets (line 2): fixed prices $p^{fix}_{i,t}$ and pre-calculated available vehicles $a^{pre}_{i,t}$ for each zone i at the beginning of each period t. Since there was no calculation of available vehicles and prices yet, the algorithm cannot assign a value to these parameters. Thus, in a first step (line 3), the algorithm solves the problem with a simple and fast solution approach, obtaining the values for the available vehicles $a^{pre}_{i,t}$ for each zone i and period t (e.g. solving the mathematical model of Soppert et al. 2022, with a rolling-horizon approach with horizon length 1, see Appendix 12). This is the basis for the next step.

In Step 2 (line 4), we use pre-calculated vehicle distributions $a^{pre}_{i,t}$ for every period t and zone i from Step 1 as input. The backwards solution approach determines the prices in different sub-steps. Starting from the last period T (line 6), we calculate the prices period by period backwards (line 7) using the mathematical model from Soppert et al. (2022), taking into account the next periods with their fixed prices $p^{fix}_{i,k}\ \forall k \in \{t+1, ..., T\}, i \in \mathcal{Z}$ (line 8). The fixed prices $p^{fix}_{i,k}$ are determined by the previous sub-steps. Thus, only the prices $p_{i,t}$ of the considered period t are the decision variables. However, with the prices of the considered period $p_{i,t}$, the provider also decides on the network flows of the considered period t and thus on the vehicle distributions of the next periods $k \in \{t+1, ..., T\}$, since the vehicle distributions of the next periods depend on the vehicle distribution at the end of the considered period t.

That means, based on the pre-calcuated available vehicles $a^{pre}_{i,T}$ (line 7), the algorithm computes the prices $p_{i,T}$ for each zone i for the last period T (line 8). In this substep, the pricing approach does not need to consider the next periods, since the period T is the last period. After this, the calculated prices $p_{i,T}$ are fixed for the next substep ($p^{fix}_{i,T} = p_{i,T}$, line 9), which is to calculate the prices $p_{i,T-1}$ for each zone i for period $T-1$ based on the pre-calculated available vehicles $a^{pre}_{i,T-1}$ for each zone i for period $T-1$ (line 7). In this substep, the algorithm considers both periods (T and $T-1$), but the prices of period T are fixed. After getting the prices $p_{i,T-1}$ for each zone i for period $T-1$, we fix these prices ($p^{fix}_{i,T-1}$, line 9) as well. This continues until the first period is reached. Here, the algorithm calculates prices based on the pre-calculated available vehicles $a^{pre}_{i,1}$ for each zone i for the first period. For this, the algorithm considers all periods, but only the prices $p_{i,1}$ for each zone i for the first period are decision variables, and the prices of the next periods $t \in \{2, ..., T\}$ are fixed. After this last calculation, the provider has prices for all zones and periods.

Algorithm 2 Backwards Algorithm

1: Results: Best prices for the whole day
2: Step 0: Define fixed prices: $p^{fix}_{i,t} = \emptyset \quad \forall i \in I, t \in \mathcal{T}$ and pre-calculated vehicle-distribution $a^{pre}_{i,t} = \emptyset \quad \forall i \in I, t \in \mathcal{T}$
3: Step 1: Solve the problem using a simple and fast solution approach to get a vehicle distribution $a^{pre}_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$
4: Step 2: Solve the following:
5: **for** $t \in \mathcal{T}$ **do**
6: $t^{backwards} = T - t + 1$
7: Set the initial distribution $a_{i,t^{backwards}} = a^{pre}_{i,t^{backwards}} \quad \forall i \in I$
8:
9: Solve the problem for the period $t^{backwards}$ taking into account the fixed prices $p^{fix}_{i,t} \quad \forall i \in I, t \in t^{backwards}...T$
10: Save the resulting prices $p^{fix}_{i,t^{backwards}} = p_{i,t^{backwards}} \quad \forall i \in I$
11: **end for**

4 Computational study

In extensive computational studies, we investigate the performance of the two solution approaches presented in Sect. 3. We systematically vary the most important influencing factors to compare how the different solution approaches perform against benchmarks. Sect. 4.1 introduces the scenarios and parameters. In Sect. 4.2, we present the solution approaches and the benchmarks we investigate. Section 4.3 presents the computational results.

4.1 Scenarios and parameters

We consider three settings (SMALL, MEDIUM, LARGE) of a free-floating VSS that primarily differ in the size of the operating area and the number of locations Z (SMALL: $Z = 9$, MEDIUM: $Z = 16$ and LARGE: $Z = 25$, all areas are square, e.g.

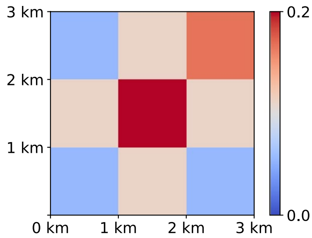


Fig. 2 Initial vehicle distribution in period 0 (SMALL)

see Fig. 2 for $Z = 9$ and $DSR = \frac{2}{3}$). Fig. 2 shows the spatial distribution of vehicles across the business area at the beginning of the day as a percentage of the total fleet for SMALL. The spatial vehicle distribution at the beginning of the day is similar to a realistic vehicle distribution at Share Now and takes into account the spatial differentiation of demand during the day, e.g. the number of available vehicles in SMALL at the beginning is highest in the center, where the cumulative demand is highest. The same idea was used for the initial vehicle distributions in MEDIUM and LARGE. Each of the three settings is examined for three different overall demand levels, which differ in the demand-supply-ratio (DSR). The DSR is the maximum period demand divided by the fleet size and we consider the values $\in \{\frac{1}{3}, \frac{2}{3}, \frac{3}{3}\}$ by scaling demand appropriately. The remaining parameters are constant over all three settings: we discretize the time interval of one day into $T = 48$ periods of 30 min each, in line with practice and literature (see e.g. Kaspi et al. (2016) and Ferrero et al. (2015b)). We select the $M = 3$ price points p^m according to typical prices in practice and literature (see e.g. Lippoldt et al. (2018)). We chose a base price per minute of $p^2 = 0.30$ €/min and price differences of 0.06 €/min to the so-called low and high prices, so that $p^1 = 0.24$ €/min and $p^3 = 0.36$ €/min. Variable costs are $c = 0.075$ €/min. The rental time is set to $l = 15$ min, in line with, for example, Xu et al. (2018), Soppert et al. (2022) and the discussions with our industry partner. The corresponding sensitivity factors $f^{(1)}_{i,j,t} = 0.75, f^{(2)}_{i,j,t} = 1, f^{(3)}_{i,j,t} = 1.25 \quad \forall i, j \in Z, t \in \mathcal{T}$ are chosen according to observations from practice.

4.2 Investigated solution approaches and evaluation metrics

Here, we describe the solution approaches that we investigate.

- MODSIM denotes the solution of a simplified model neglecting the assumption of rental realization (see Sect. 3.2).
- BAW denotes the backwards algorithm that computes the vehicle distribution in a first step using either the rolling-horizon with horizon $H = 1$ (BAW-ROL-1) or MODSIM (BAW-MODSIM) and calculating the optimal prices backwards in a second step.

Besides the solution approaches, we investigate four benchmarks:

- BASE denotes a benchmark using constant uniform pricing. Here we use the base price $p_{i,t} = p^{(2)}\ \forall i \in Z, t \in \mathcal{T}$.
- MOD48h denotes the solution of the model with a given time limit for the solver of 48 h in which all 48 periods are optimized simultaneously.
- ROL-H is a basic rolling-horizon approach and is configured with different horizon lengths H (ROL-1, ROL-4, ROL-8). Note that this benchmark with $H = 1$ represents the myopic solution that only considers one period in each substitute problem.
- ADP-H is the ADP decomposition solution approach presented in Soppert et al. (2022), and is configured with different horizon lengths H (ADP-1, ADP-4, ADP-8). It uses a value function approximation to approximate the future after the horizon H that is being considered.

Each combination of settings and *DSR*s forms an instance in our experiments. We implement the algorithms in Python 3.8 and solve all solution approaches and benchmarks with Gurobi 9.1.2. In all scenarios, we set the optimality gap to zero and the time limit is set to one hour for ADP-H, ROL-H, BAW-ROL-1, BAW-MODSIM and to 48 h for MOD48h and MODSIM. We execute the computations on a workstation with an AMD Ryzen 9 3900 X 12-Core processor with 12 cores and 64 Gigabyte RAM. Please note that we use the MILP of Soppert et al. (2022) (see Appendix 12) to evaluate the computed prices of the solution approaches and the benchmarks.

4.3 Results

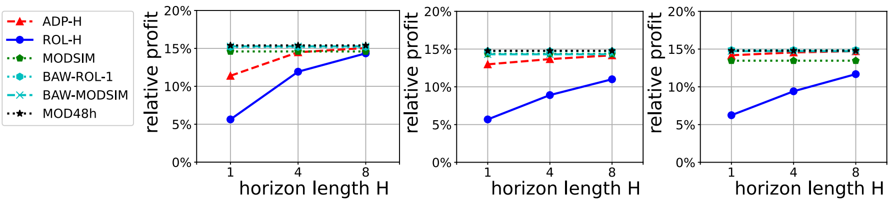
In this section we present the results regarding the analyses of profit (Sect. 4.3.1), pricing (Sect. 4.3.2), rentals (Sect. 4.3.3) and computational time (Sect. 4.3.4).

4.3.1 Profit

We begin with a comparison of the different solution methods and the benchmarks by identifying the improvement over BASE. The potential is graphically shown in Fig. 3. It depicts the profit obtained with the different solution approaches and benchmarks (for the later considered ADP-H and ROL-H in dependence of the horizon lengths H on the horizontal axis) relative to the profit with BASE, which the 0%-line marks. The profits obtained by MODSIM, BAW-ROL-1, BAW-MODSIM and MOD48h are horizontal lines as they do not depend on horizon H.

We observe that MOD48h yields a profit increase of at least 13.5% over BASE. For SMALL with $DSR = 1/3$ MOD48h yields the optimal solution. For LARGE and a higher DSR than 1/3, MOD48h does not find any feasible solution within 48 h. The myopic solution ROL-1 provides at least 5% more profit than BASE.

The difference between ROL-1 and MOD48h (in the instance SMALL, $DSR = 1/3$) shows the whole effect of considering supply-side network effects. The exact supply-side network effect for larger instances cannot be determined because it is not possible to determine the optimal solution within 48 h. In the instance where an optimal solution can still be determined within 48 h (SMALL, $DSR = 1/3$), the



(a) $Z = 9$ (b) $Z = 16$ (c) $Z = 25$

Fig. 3 Relative profit increase (SMALL, MEDIUM, LARGE), $DSR = 1/3$

profits of MODSIM, and BAW-ROL-1, BAW-MODSIM and ADP-8 are very similar to the profit of the optimal solution (MOD48h). Furthermore, it can be shown that the profits of ADP-8, BAW-ROL-1 as well as BAW-MODSIM and MODSIM are almost the same in all other instances (see Fig. 1 in Appendix). BAW-ROL-1 (BAW-MODSIM) generates a profit that is at most 0.55 (0.22) percentage points higher or 0.16 (0.06) percentage points lower than ADP-8. MODSIM generates a profit that is at most 0.19 percentage points higher or 1.26 percentage points lower than ADP-8. This leads to the conclusion that these solution approaches and the benchmarks consider the supply-side network effects best.

In contrast, the profit of ROL-8 is sometimes very similar (e.g. Fig. 3a) to the profit of MOD48h, sometimes clearly lower (e.g. Fig. 3b, c).

Thus, we can draw the following conclusions:

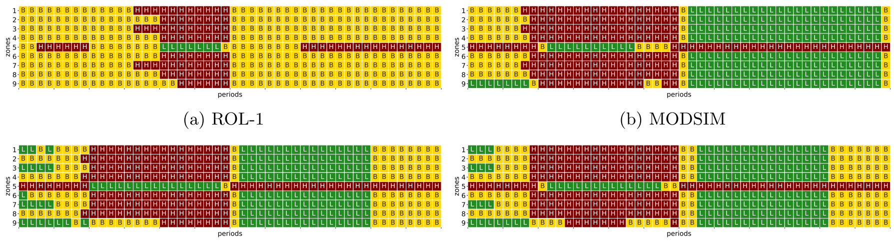
1. Although the proposed solution approaches (MODSIM, BAW-ROL-1, BAW-MODSIM) are rather straightforward, they achieve equivalent results to more complex benchmarks (MOD48h, ADP-8).
2. The comparison of profit of both new solution approaches (MODSIM, BAW) shows that they successfully consider supply-side network effects, as do the benchmarks MOD48h and ADP-8.

4.3.2 Pricing decisions

The anticipative solution approaches MODSIM, BAW-ROL1 and BAW-MODSIM consider supply-side network effects as the benchmarks ADP-8 and MOD48h in contrast to ROL-1 (see Sect. 4.3.1). These supply-side network effects are also reflected in the pricing decisions.

The pricing decisions for selected solution approaches and benchmarks are depicted as price tables in Fig. 4 (see Fig. 13 in Appendix 4 for all solution approaches and benchmarks) for SMALL with $DSR = 1/3$. For the sake of simplicity, we only analyze the pricing decision of the benchmarks ROL-1 (as myopic pricing), ROL-8, MOD-48 h, ADP-1 and ADP-8.

Since MOD48h (which is the optimal solution in this instance) considers the supply-side network effects throughout the day, we compare the price table of the other solution approaches and benchmarks with that of MOD48h. It is obvious that the price table of ROL-1 is very different from the price table of MOD48h, which shows



(a) ROL-1 (b) MODSIM (c) BAW-ROL-1 (d) MOD48h

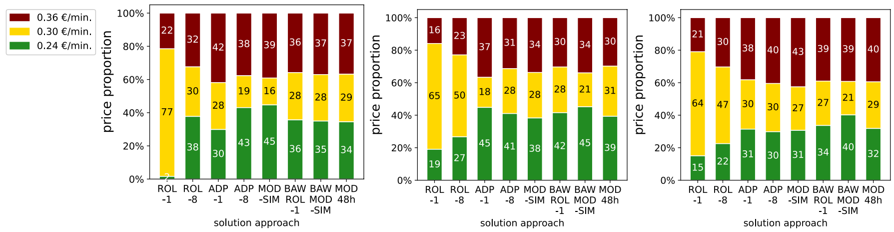
Fig. 4 Pricing different solution approaches and benchmarks (SMALL), $DSR = 1/3$. Green: L = low price, yellow: B = base price, red: H = high price

the strong impact of supply-side network effects. The price tables of ROL-8, ADP-8, MODSIM, BAW-ROL-1 and BAW-MODSIM are very similar to the price pattern of MOD48h. That indicates that these solution approaches and benchmarks consider supply-side network effects.

On an aggregate level, these differences become also visible in comparing the proportion of different prices of the solution approaches and benchmarks. In SMALL with $DSR = 1/3$, for example, ROL-1 results in 2% low, 77% base, and 22% high prices (see Fig. 5). Pricing decisions of MOD48h consists of 34% low, 29% base, and 37% high prices. The proportions of different prices of ADP-8 (43% low, 19% base, 38% high prices), MODSIM (45% low, 16% base, 39% high prices), BAW-ROL-1 (36% low, 28% base, 36% high prices) and BAW-MODSIM (35% low, 25% base, 37% high prices) are also similar to the proportion of different prices of MOD48h, especially for the high prices. This shows that the better supply-side network effects are captured, the more the resulting pricing decisions resemble the optimal pricing.

Thus, we can draw the following conclusions:

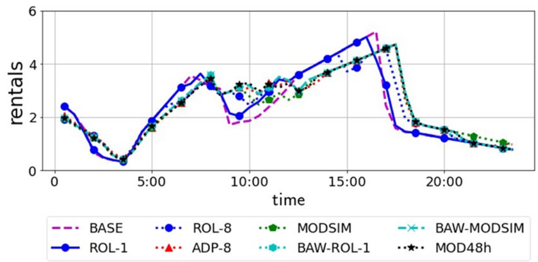
1. Supply-side network effects are visible in the price tables and price proportions of MOD48h.
2. MODSIM, BAW-ROL-1 and BAW-MODSIM create price tables which are very similar to those of MOD48h and ADP-8, which indicates that they consider supply-side network effects effectively.



(a) SMALL, $DSR = 1/3$ (b) MEDIUM, $DSR = 1/3$ (c) LARGE, $DSR = 1/3$

Fig. 5 Price proportions. Green: low price, yellow: base price, red: high price

Fig. 6 Rentals over the day (SMALL), $DSR = 1/3$



4.3.3 Rentals

The consideration of supply-side network effects is evident in profit and pricing. We also consider the course of rentals for SMALL with a $DSR = 1/3$ (see Fig. 6). Since prices affect demand and demand affects rentals, we study the extent to which supply-side network effects are evident for rentals. For the sake of simplicity, we only analyze the pricing decision of the benchmarks BASE, ROL-1 (as myopic pricing), ROL-8, MOD-48 h and ADP-8.

The first thing to notice is that during the periods with low demand, BASE results in fewer rentals than MOD48h and during the peaks it generates more rentals than MOD48h. Furthermore, it is apparent that the rental-curve of ROL-1 fluctuates similar to the rental-curve of BASE, whereas the rental-curve of MOD48h fluctuates less than both. Another remarkable feature is that the rental curves of MOD48h, ADP-8, BAW-ROL-1 and BAW-MODSIM lie almost on top of each other. This means that the rental curves of ADP-8, BAW-ROL-1, and BAW-MODSIM are hidden behind the rental curve of MOD48h in Fig. 6. The rental curve of MODSIM deviates only slightly from these three rental curves.

From this we conclude that all anticipative solution approaches and benchmarks (ROL-8, ADP-8, MODSIM, BAW-ROL-1, BAW-MODSIM) in this setting consider the supply-side network effects similarly and that apart from the similar prices, the rental curves of MOD48h, ADP-8 BAW-ROL-1, BAW-MODSIM and MODSIM are also very similar.

4.3.4 Computational time

An important aspect for the practicability of solution approaches is, among others, the computational time. For this purpose, we compare the computational time for SMALL, MEDIUM and LARGE (see Fig. 7 for $DSR = 1/3$).

First, we consider the different benchmarks. MOD48h (2.5 h up to over 48 h) always takes the longest time for SMALL. For MEDIUM and LARGE, MOD48h takes the full given time of 48 h. The computational times of the rolling-horizon and decomposition solution approaches depend on their horizon. ROL-1 (6 s up to under 1 min) takes the second lowest computational time. ROL-4 (18 s up to 8 min) requires a similar computational time. In contrast, ROL-8 (20 min up to over 12 h) requires a clearly longer computational time than ROL-1 and ROL-4. The

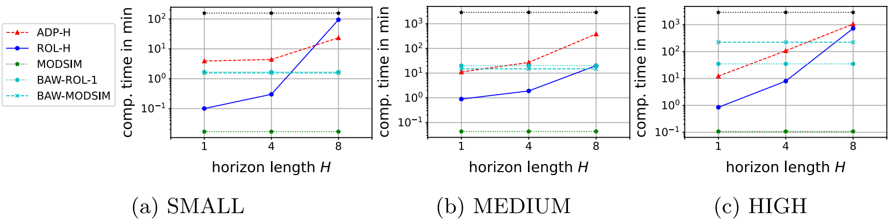


Fig. 7 Computational time (SMALL, MEDIUM, LARGE), $DSR = 1/3$

order of computational times for the different horizons is similar for the decomposition solution approaches. More precisely, ADP-1 (4 min up to over 12 min) and ADP-4 (4 min up to over 1.5 h) still require relatively short computational times, whereas the computational time for ADP-8 (23 min up to over 17.5 h) is clearly longer. Thus, when comparing each different horizon length of the rolling-horizon approaches with the decomposition solution approaches it is obvious that the decomposition solution approaches need longer computational times. This is due to the consideration of future states in the decomposition solution approaches. However, it should also be noted that the benchmarks ADP-1, ADP-4 and ADP-8 require parameter estimation in advance. The additional computational time of parameter estimation for all periods, which lies between less than 1 h for SMALL and roughly 2 h for LARGE, thus, must be considered (Soppert et al. 2022).

Second, we consider the proposed solution approaches. MODSIM (1 s up to 6 s) takes the least computational time for SMALL, MEDIUM and LARGE. BAW-ROL-1 (1.5 min up to 35 min) and BAW-MODSIM (1 min up to over 3.h) need relatively short computational times.

Comparing benchmarks with solution approaches, ADP-8 and MOD48h, which consider the supply-side network effects effectively, require more computational time than MODSIM, BAW-ROL-1 and BAW-MODSIM. In most cases, ADP-4 and ROL-8 require longer computational times than the proposed solution approaches. ROL-4 takes less time than BAW-MODSIM and BAW-ROL-1, but more time than MODSIM.

Fig. 15 in Appendix 5 depicts the results for SMALL, MEDIUM and LARGE with all DSRs. We also investigate the computational time in dependency of the number of zones for $DSR = 1/3$ (see Fig.16in Appendix 6). For this analysis, the time limit for the calculations was removed, but a target optimality gap of 2% was added. All proposed solution approaches provide results for all considered number of zones. This shows that the new solution approaches are also applicable for realistic zone sizes.

From the obtained computational times we can conclude the following.

1. ROL-1 and ROL-4 require short computational times but do not consider the supply-side network effects effectively (see Sect. 4.3.1).
2. ADP-8 and MOD48h require long computational times.

3. MODSIM requires a short computational time and and is a preferred option due to the comparable results in Sect. 4.3.1 with the best benchmarks (MOD48h, ADP-8).
4. BAW-ROL-1 and BAW-MODSIM need about the same and clearly less computational time than the benchmarks that give similar results (ADP-8, MOD48h). Thus, they are also preferred options.
5. MODSIM, BAW-ROL-1 and BAW-MODSIM provide results even for realistic instances (e.g. $Z = 81$).

5 Sensitivity analysis

In this section, we perform a sensitivity analysis to show how stable the solutions of the different approaches are. First, we study the solution stability in a stochastic environment (see Sect. 5.1). Second, we analyze the effect of different intervals between prices in price sets and different numbers of prices in price sets by modifying the discrete price set (see Sect. 5.2). Third, we apply the different solution approaches and benchmarks for one week (see Sect. 5.3). Fourth, we study the impact of a start solution on results and computational time (see Sect. 5.4).

5.1 Stochastic demand

We analyze the robustness of the prices generated by the different solution approaches and benchmarks in a stochastic environment. For this purpose, we use the multiplicative stochastic function, which generates a stochastic demand $D_{i,j,t}$ (Talluri and van Ryzin 2004): $D_{i,j,t} = d_{i,j,t} \cdot \xi$ where ξ is a stochastic error term which is assumed to follow a normal distribution $N(1, \sigma)$. We evaluate all scenarios, i.e., SMALL, MEDIUM and LARGE with all DSRs. For each scenario, we consider different degrees of stochasticity, expressed by different standard deviations $\sigma \in 0, 0.1, 0.2, 0.3, 0.4$ of the factor ξ. These values are in the range of demand uncertainties we observed in practice. For each of the resulting combinations of scenario and degree of stochasticity, we draw $S = 1,000$ demand matrices.

Fig. 8 illustrates the results for SMALL, MEDIUM and LARGE with $DSR = 1/3$. On the vertical axis, the mean value of the relative profit increases with respect to BASE. On the horizontal axis, the standard deviation σ is varied. Overall, the solution approaches MODSIM, BAW-ROL-1 and BAW-MODSIM are robust to the stochasticity of demand. Similar to the profits of the benchmarks ROL-1, ROL-8, ADP-1 and ADP-8, the relative profits of MODSIM, BAW-ROL-1 and BAW-MODSIM decrease slightly with increasing stochasticity. The order of the different solution approaches and benchmarks with respect to their performance does not change in most instances. MODSIM, BAW-MODSIM and BAW-ROL-1 deliver profits that are *not* worse than the benchmark ADP-8 for all scenarios and all stochasticities (see Fig. 17 in Appendix 7).

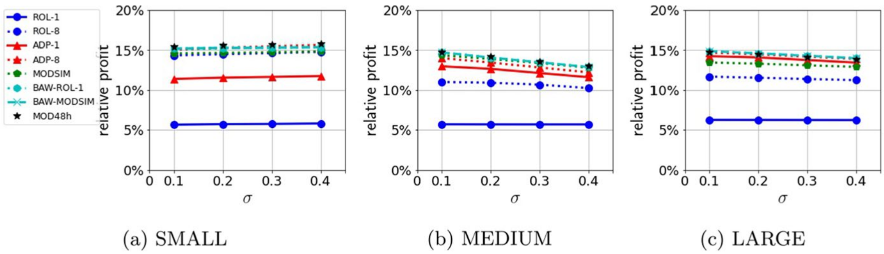


Fig. 8 Stochastic evaluation of solution approaches and benchmarks (SMALL, MEDIUM, LARGE), $DSR = 1/3$

5.2 Different price sets

In this section, we analyze the impact of the price sets on the performance of the different solution approaches and benchmarks. For this purpose, we use a common standard demand pattern (SMALL, $DSR = 1/3$). Furthermore, in each instance we use 0.30 €/min as base price. We conduct two experiments (see Table 2 in Appendix 8). First, we investigate 20 different price sets with three prices each. The intervals between prices are the same within a price set, but differ between price sets. We use intervals of 0.01 €/min, 0.02 €/min up to 0.2 €/min (see Table 2 in Appendix 8). Second, we investigate the impact of the number of prices on the performance. Here, the intervals are the same for each price set, but we increase the number of price points to five, seven and nine. Price sensitivities change in accordance with prices.

Note that we exclude the ADP benchmarks for this analysis due to the high effort of estimating the parameters in pre-processing. Thus, we focus on the solution approaches MODSIM, BAW-ROL-1 and BAW-MODSIM and on the benchmarks ROL-8 and MOD48h only.

Fig. 9 illustrates the results for different intervals between the prices and different number of prices in the price set. On the vertical axis, the mean value of the relative profit increases with respect to BASE. On the horizontal axis, the interval between the prices (see Fig. 9a) or the number of prices (see Fig. 9b) in the price set is varied.

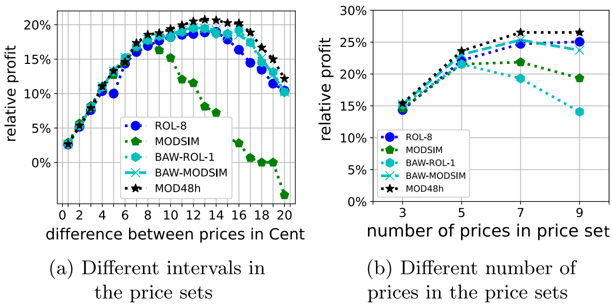


Fig. 9 Sensitivity analysis of different price sets (SMALL), $DSR = 1/3$

In Fig. 9a the relative profit of all solution approaches and benchmarks, except of MODSIM, tends to increase with the difference between the prices in the price set up to the difference of 0.14 €/min, after which it decreases. This shows that the prices should be chosen reasonably. The relative profit of MODSIM increases until the difference of 0.08 €/min. Thereafter, the profit curve drops sharply. We assume that this is due to the difference between the discrete price points $(p^{(1)}, p^{(2)}, p^{(3)})$. MODSIM, which converts the continuous prices of the simplified model into discrete prices, calculates larger price differences between the continuous and the discrete price. Thus, if the price difference between price points is greater than 0.08 €/min, the larger the price difference, the worse the results of MODSIM. In reality, however, differences of 0.05 €/min and 0.06 €/min are observed for Share Now.

In addition, in Fig. 9b the relative profit also increases as the number of price points in the price set increases. The profit increase is degressive. Furthermore, ROL-8, MODSIM, BAW-ROL-1, BAW-MODSIM and MOD48h perform similarily well for three and five price points, whereas ROL-8, BAW-MODSIM and MOD48h perform better than MODSIM and BAW-ROL-1 for seven and nine price points.

With regards to the worse performance of BAW-ROL-1 compared to BAW-MODSIM, the first step of calculating vehicle distributions seems to be decisive. More precisely, BAW-MODSIM, which has a suitable solution approach MODSIM to determine the vehicle distributions, performs better than BAW-ROL-1, which has the benchmark ROL-1 to determine the vehicle distributions.

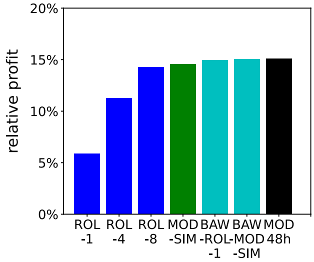
From this we can conclude that BAW-MODSIM is the best practicable solution approach. It provides similar results as MOD48h and ROL-8.

5.3 Pricing for one week without operator-based relocation

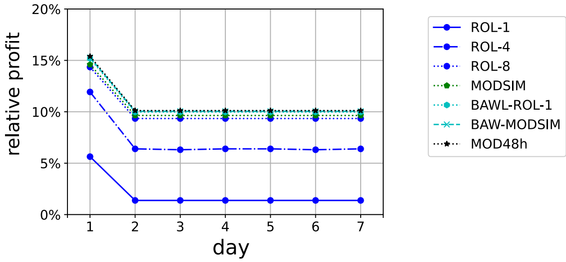
In this section, we examined the determination of prices for an entire week without operator based relocation. For this purpose, we assume that each day has the same demand pattern. For this study, we look at the relative average profit for a day compared to the profit of pricing with BASE, the course of rentals, and the price structure.

5.3.1 Profit

First, we consider the relative average profit per day for one week (see Fig. 10a), which is relative to the average profit per day of BASE. It is obvious that supply-side network effects have to be taken into account. This can be seen in the difference between myopic pricing ROL-1 and anticipative pricing ROL-8, MODSIM, BAW-ROL-1, BAW-MODSIM or MOD48h. Second, we consider the relative profit per day for one week (see Fig. 10b), which is relative to the profit of BASE at the first day. Similar to the observation above, the profit of MODSIM, BAW-ROL-1, BAW-MODSIM, ROL-8 and MOD48h is similar and higher than the profit of ROL-1. It is also obvious that the order of the different solution approaches and benchmarks with regard to profit does not change over the seven days.



(a) Average relative profit of a day for a whole week



(b) Relative profit for each day of the week

Fig. 10 Profit for a week without operator-based relocation (SMALL), $DSR = 1/3$

In addition, we note that the profit is highest on the first day compared to the following days for all solution approaches and benchmarks (see Fig. 10b). However, this shows that operator-based relocation can be worthwhile.

Considering computational times, MOD48h needs 336.44 h, ROL-8 needs 8.46 h, while BAW-ROL-1 needs 7.6 min, BAW-MODSIM 7 min and MODSIM 0.07 min to calculate prices (for a more detailed analysis see Sect. 4.3.4)

5.3.2 Pricing decisions

We examine the pricing decisions (see Figs. 18 in Appendix 9). Except for the first day, we see a repeating price pattern for each day. Moreover, it is obvious that the myopic pricing approach (ROL-1, Fig. 18a in Appendix 9) leads to a clearly different price table than the solution approaches and benchmarks that include supply-side network effects (ROL-8, MODSIM, BAW-ROL-1, BAW-MODSIM, MOD48h). However, there are also differences between them. For example, ROL-8 (see Fig. 18c in Appendix 9) sets high and low prices less frequently than MODSIM, BAW-ROL-1, BAW-MODSIM and MOD48h (see Fig. 18 in Appendix 9). The different price tables can be explained by the different pricing decisions on the first day, which probably lead to different vehicle distributions on the first day and thus different stable price patterns for days 2-7.

5.3.3 Rentals

We examine rentals over the seven days (see Fig. 11). Again, we note that after the initial day, the rental course over the following days is identical. Furthermore, we observe that BASE and the myopic pricing approach (ROL-1) result in numbers of rentals that fluctuate clearly more than for the others. ROL-8, MODSIM, BAW-ROL-1, BAW-MODSIM, MOD48h are similar.

We can therefore conclude the following:

1. Each solution approach and each benchmark creates a (daily) regular rental pattern (see Fig. 11) and pricing (see Fig. 18 in Appendix 9) after the first day. These regular patterns (for day 2 to day 7) can be identified by the equal contribution margins (see Fig. 10b).
2. The consideration of supply-side network effects in solution approaches and benchmarks is useful and leads to clearly higher profits, even in longer periods. The consideration of these supply-side network effects can be observed in the price tables (see Fig. 18 in Appendix 9).
3. The solution approaches MODSIM, BAW-ROL-1 and BAW-MODSIM provide the same results as the benchmarks ROL-8 or MOD48h (see Fig. 10a), but need less computational time.

5.4 Impact of a start solution

In Sect. 4.3, we notice that even for some instances ($Z = 25$, $DSR = 2/3$ and $Z = 25$, $DSR = 3/3$), there are no results for MOD48h since the solver does not find a feasible solution within the given time limit of 48 h. Therefore, we wanted to investigate two aspects in this section: First, whether a start solution improves the solution quality and second, whether and how a start solution affects the computational time. We use the BASE solution as a start solution in all instances.

The comparison leads to the following findings: All applied solution approaches and benchmarks with a start solution result in no noticeable improvement of the results regarding profit (see Table 6 in Appendix 11). Notably, the start solution has a positive effect on the computational time for most instances (a reduction of computational time up to 92 min, see Table 7 in Appendix 11). However, a start solution can also have negative effects on the computational time for some instances. For example, for MOD48h at the smallest instance ($Z = 9$, $DSR = 1/3$), it has a clearly negative time effect (+ 46 h).

Nevertheless, the use of a start solution results in solutions for MOD48h in all instances within the time limit. From this it can be concluded that it makes sense to use BASE as a start solution. This applies in particular to real instances.

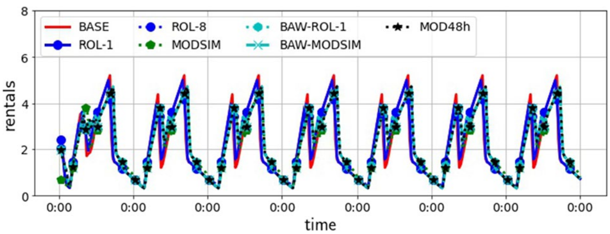


Fig. 11 Rentals over seven days (SMALL), $DSR = 1/3$

6 Conclusion

In this paper we propose two different solution approaches for the problem of origin-based differentiated pricing for VSSs, which maximize the profit by setting spatially and temporarily differentiated origin-based minute prices. Although origin-based pricing is the most *practicable* variant of differentiated pricing, only one paper in the current literature focuses on differentiated origin-based pricing in VSSs.

The first solution approach is a simplified mathematical model (simplified model, MODSIM). The second one is a backwards algorithm (BAW), which computes the vehicle distribution for every period with an appropriate straightforward solution approach in a first step. We apply a myopic solution approach (BAW-ROL-1) and a relaxed solution approach (BAW-MODSIM) for this step. In the second step, we calculate the prices backwards based on the calculated vehicle distributions in the first step.

Extensive computational experiments and the sensitivity analysis with a varying number of zones, demand patterns, overall demand levels, varying price sets and pricing for one week without operator-based relocation show the stability of the results from our solution approaches MODSIM, BAW-ROL-1 and BAW-MODSIM.

In the computational study, the proposed solution approaches perform similarly well as the best performing benchmarks ADP-8, ROL-8 and MOD48h with considerably shorter computational time without having to calculate additional parameters (as with ADP-8). Additionally, they show a considerable improvement with regard to profit over the myopic benchmark (ROL-1, up to 10 percentage points). In the instance (SMALL, $DSR = 1/3$) where the optimal solution can be determined, we can show that the proposed, anticipative solution approaches (MODSIM, BAW-ROL-1, BAW-MODSIM) find a solution close to optimality. Finally, the resulting price tables show high similarity to the optimal price tables, in contrast to the price tables from the myopic pricing approach (ROL-1). This shows that, in contrast to the myopic solution approach, our approaches, just like other benchmarks (ADP-8, ROL-8, MOD48h), take supply-side networks into account. We also show that the proposed solution approaches need clearly less computational time in comparison to the benchmarks ADP-4, ADP-8 and MOD48h.

In the sensitivity analysis, we investigate the stability of the solutions from our solution approaches and benchmarks in regards to four aspects. First, we investigate the stability of the results against a background of an environment with different degrees of stochasticity of the demand. The solution approaches MODSIM, BAW-ROL-1 and BAW-MODSIM are robust against stochasticity of demand. The order of the different solution approaches and benchmarks with respect to profit does not change in most cases. Second, we study the effect of different price sets on the performance of the different solution approaches and benchmarks. We find that MODSIM is only useful for the difference between the discrete prices up to 0.08 €/min, and that MODSIM and BAW-ROL-1 lose performance when the price set has more than five price points. For settings with price sets containing more than five price points, we recommend BAW-MODSIM. Third, we study the determination of prices for an entire week without operator-based relocation. Even in this long considered period the consideration of supply-side network effects is useful and leads to clearly higher profits for the anticipative solution approaches (MODSIM, BAW-ROL-1, BAW-MODSIM) and the benchmarks (ROL-8, MOD48h). Fourth, we study the impact of a start solution. The implementation of a start solution does not noticeably improve the results regarding profit (except for MOD48h for LARGE, where now a feasible solution exists). However, in most cases, it has a positive effect on the computational time.

To conclude, we propose different pricing approaches (MODSIM, BAW-ROL-1, BAW-MODSIM) that can be used for profit maximization in VSSs by considering supply-side network effects with clearly shorter computational times. These pricing approaches do not require a pre-processing for estimating parameters in advance, are straightforward to apply and equal in profit to comparable but more complex benchmarks (e.g. ADP-8).

Based on the presented results and methodology there are some directions for future works. First, a VSS provider competes with public transportation and, in some cities, with other VSS providers. This competition could be explicitly taken into account in the solution approaches. Further research can be done to see how this affects profit, average prices, and rentals. Second, a VSS provider's fleet consists of different types of vehicles, which could also be considered in the solution approaches. Third, the rental duration can be longer than one period or even variable. The effects of changing the rental duration assumption (rental period $\geq$ period length) could be studied for all solution approaches and benchmarks. Fourth, the computational study examines the effects of pricing for car sharing with its typical demand patterns. These solution approaches can also be applied to bike sharing, moped sharing, or scooter sharing. The effects for these different vehicle sharing systems can also be studied. Intuitively, however, no different results are expected.

Literature overview

See Table 1.

Table 1 Literature overview on differentiated pricing and further related literature in VSSs

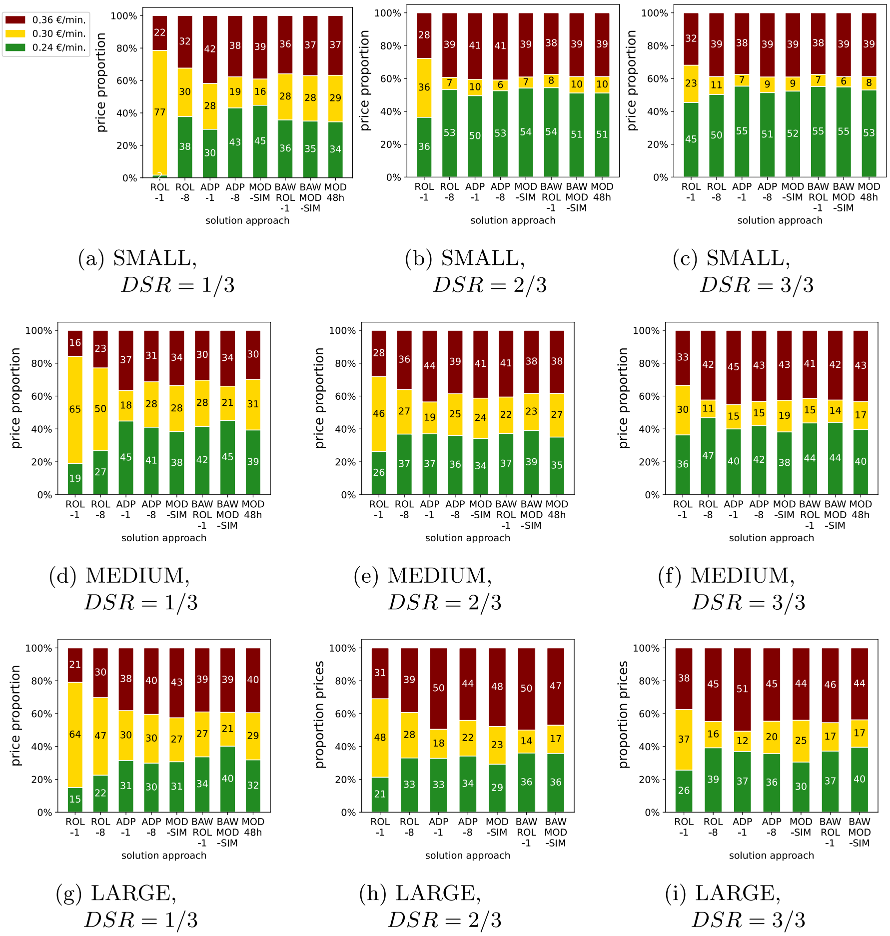
	Spatio-temporal pricing			Number of possible prices					Control of rentals		Objective		Fore-sight		Additional parameters	
	Origin	Destination	Trip	One	Price list	Upper and lower bound	Positive	No restriction	Price control	Price control, trip selection	Profit	Balance	Myopic	Anticipative	Norequirement	Requiresestimation
Differentiated pricing																
Waserhole and Jost (2012)			x					x	x		x			x	x	
Guo and Kang (2022)			x					x	x		x		x		x	
Xu et al. (2018)			x				x			x	x			x	x	
Jiao et al. (2020)			x				x			x	x			x	x	
Huang et al. (2020)			x				x			x	x			x	x	
Lu et al. (2021)			x			x				x	x			x	x	
Zhang and Kan (2018)			x			x			x		x			x	x	
Jorge et al. (2015)			x				x		x		x			x	x	
Ren et al. (2019)			x				x		x		x			x	x	
Huang et al. (2020)			x				x		x		x			x	x	

Table 1 (continued)

	Spatio-temporal pricing			Number of possible prices					Control of rentals		Objective		Fore-sight		Additional parameters	
	Origin	Destination	Trip	One	Price list	Upper and lower bound	Positive	No restriction	Price control	Price control, trip selection	Profit	Balance	Myopic	Anticipative	Norequirement	Requiresestimation
Zhang and Kan (2018)			x			x			x		x			x	x	
Soppert et al. (2022)	x				x				x		x			x		x
Müller (2023)	x				x				x		x			x	x	
Further related literature (VSS only)																
Pfrommer et al. (2014)		x				x			x			x	x		x	
Chemla et al. (2013)		x				x			x			x	x		x	
Haider et al. (2018)			x			x			x			x	x		x	
Wang and Ma (2019)			x					x	x			x	x		x	
Pantuso (2022)			x		x				x		x		x		x	

Price proportion for different scenarios

See Fig. 12.



(a) SMALL, $DSR = 1/3$

(b) SMALL, $DSR = 2/3$

(c) SMALL, $DSR = 3/3$

(d) MEDIUM, $DSR = 1/3$

(e) MEDIUM, $DSR = 2/3$

(f) MEDIUM, $DSR = 3/3$

(g) LARGE, $DSR = 1/3$

(h) LARGE, $DSR = 2/3$

(i) LARGE, $DSR = 3/3$

Fig. 12 Price proportion (SMALL, MEDIUM, LARGE). Green: low price, yellow: base price, red: high price

Pricing for all solution approaches and benchmarks

See Fig. 13.

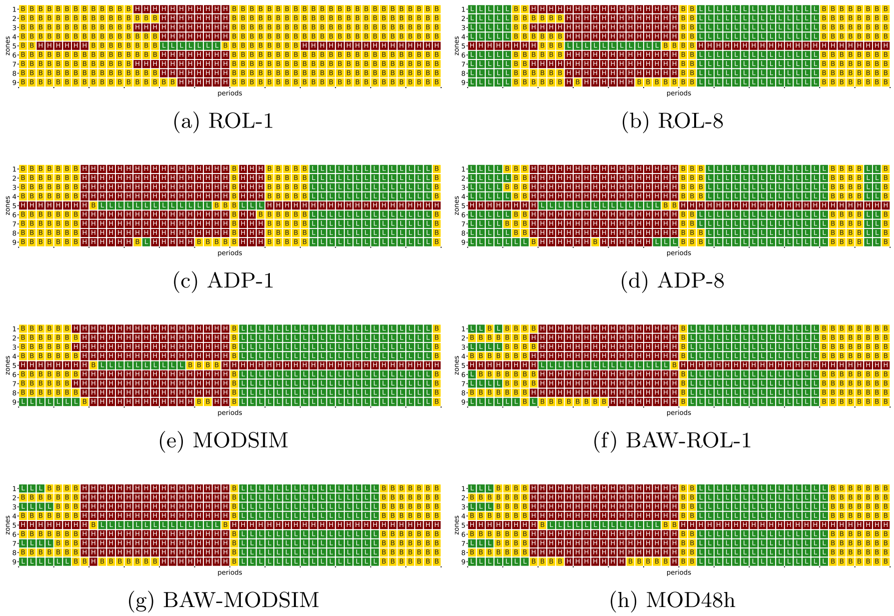


Fig. 13 Pricing with different solution approaches and benchmarks (SMALL), $DSR = 1/3$. Green: L = low price, yellow: B = base price, red: H = high price

Profit for different scenarios

See Fig. 14.

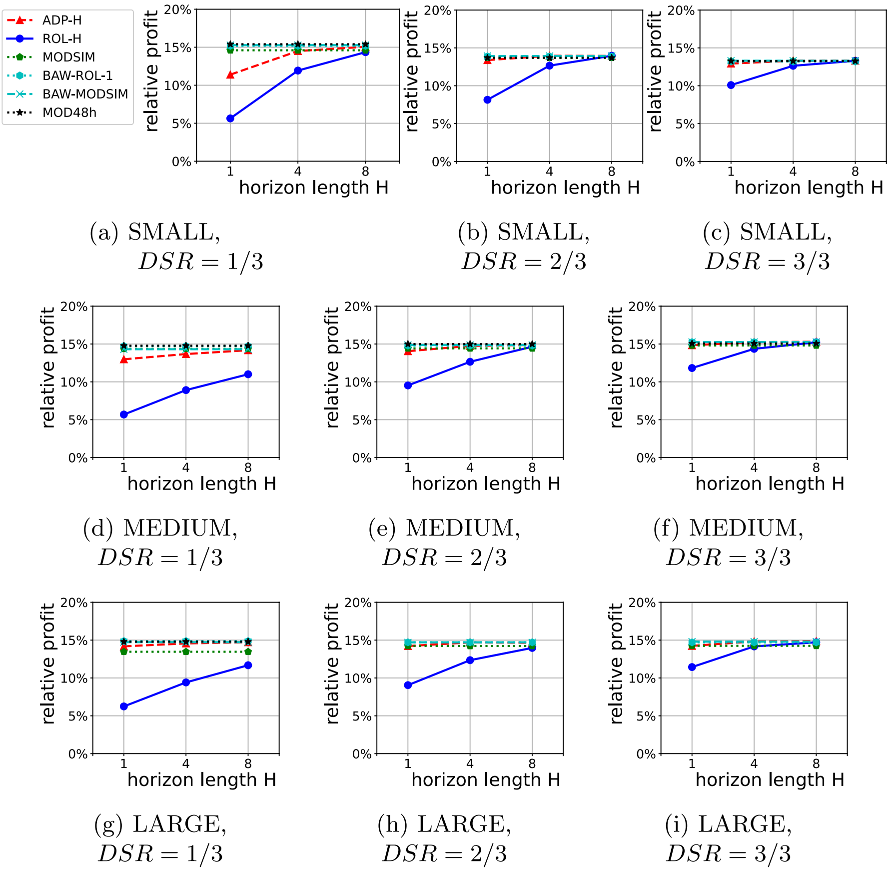
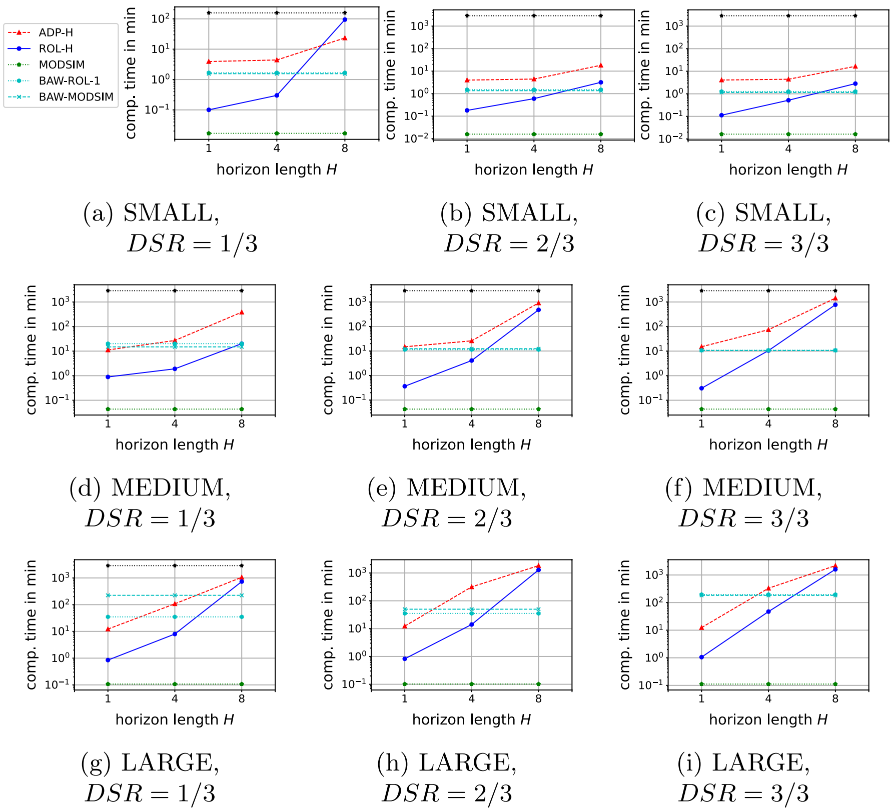


Fig. 14 Relative profit increase (SMALL, MEDIUM, LARGE)

Computational time for different scenarios

See Fig. 15.



(a) SMALL, $DSR = 1/3$

(b) SMALL, $DSR = 2/3$

(c) SMALL, $DSR = 3/3$

(d) MEDIUM, $DSR = 1/3$

(e) MEDIUM, $DSR = 2/3$

(f) MEDIUM, $DSR = 3/3$

(g) LARGE, $DSR = 1/3$

(h) LARGE, $DSR = 2/3$

(i) LARGE, $DSR = 3/3$

Fig. 15 Computational time (SMALL, MEDIUM, LARGE)

Computational time for different amount of zones

See Fig. 16.

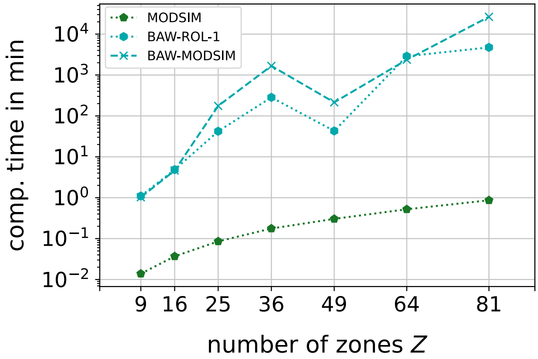
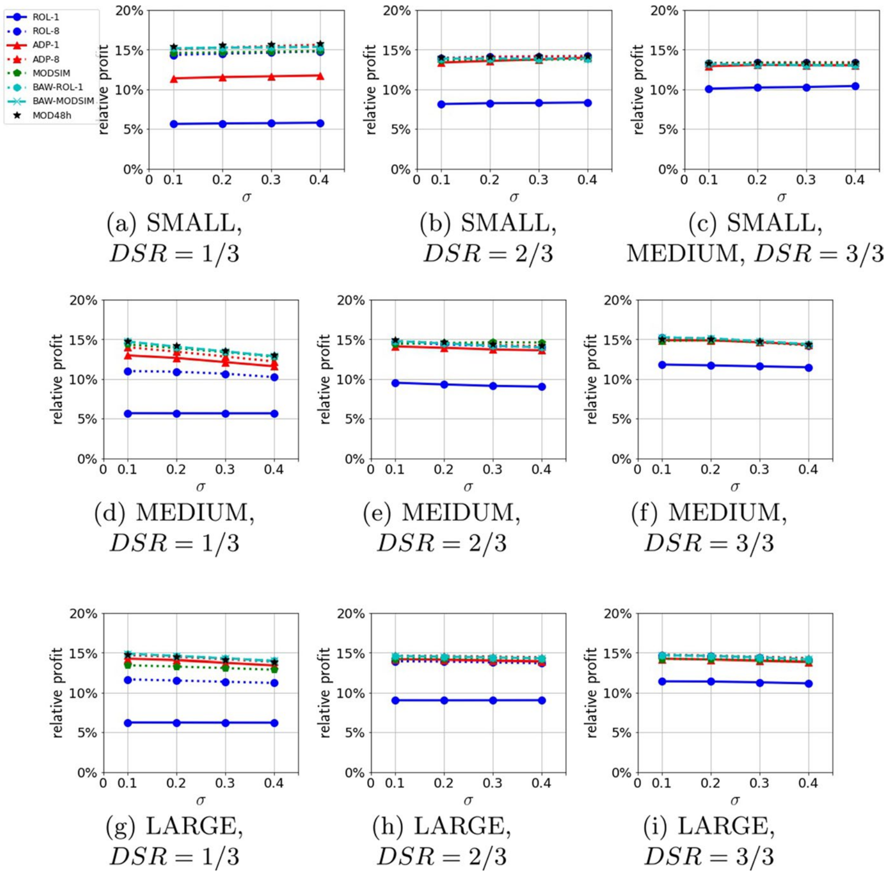


Fig. 16 Computational time for different amount of zones, $DSR = 1/3$

Stochastic demand

As a technical remark, note that in the stochastic demand model, demand realization $D_{ijt} < 0$ could potentially result in particular for high values of σ (see the corresponding discussion in Talluri and van Ryzin (2004, Chapter 7.3.4)). We correct for this by setting negative draws to 0. Note that the small positive bias resulting from this truncation is not relevant to our study, as for each degree of stochasticity, we use the same 1000 scenarios for all approaches we compare (Fig. 17).



(a) SMALL, $DSR = 1/3$

(b) SMALL, $DSR = 2/3$

(c) SMALL, MEDIUM, $DSR = 3/3$

(d) MEDIUM, $DSR = 1/3$

(e) MEIDUM, $DSR = 2/3$

(f) MEDIUM, $DSR = 3/3$

(g) LARGE, $DSR = 1/3$

(h) LARGE, $DSR = 2/3$

(i) LARGE, $DSR = 3/3$

Fig. 17 Pricing with different solution approaches (SMALL), $DSR = 1/3$

Overview over different price lists

See Table 2.

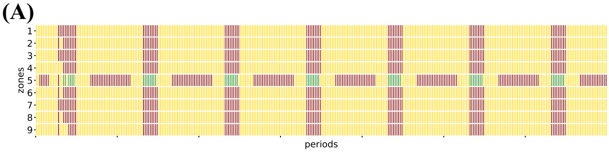
Table 2 Overview over different price lists

Experiment 1: different intervals			Experiment 2: different number of prices		
Price interval	Price set	Sensitivity factors	Number of prices	Price set	Sensitivity factors
0.01 €/min	{0.29, 0.30, 0.31}	{1.04, 1, 0.96}	3	{0.24, 0.30, 0.36}	{1.25, 1, 0.75}
0.02 €/min	{0.28, 0.30, 0.32}	{1.08, 1, 0.92}	5	{0.18, 0.24, 0.30, 0.36, 0.42}	{1.50, 1.25, 1, 0.75, 0.50}
0.03 €/min	{0.27, 0.30, 0.33}	{1.12, 1, 0.87}	7	{0.12, 0.18, 0.24, 0.30, 0.36, 0.42, 0.48}	{1.75, 1.50, 1.25, 1, 0.75, 0.50, 0.25}
0.04 €/min	{0.26, 0.30, 0.34}	{1.17, 1, 0.83}	9	{0.06, 0.12, 0.18, 0.24, 0.30, 0.36, 0.42, 0.48, 0.54}	{2.00, 1.75, 1.50, 1.25, 1, 0.75, 0.50, 0.25, 0}
0.05 €/min	{0.25, 0.30, 0.35}	{1.21, 1, 0.79}			
0.06 €/min	{0.24, 0.30, 0.36}	{1.25, 1, 0.75}			
0.07 €/min	{0.23, 0.30, 0.37}	{1.29, 1, 0.71}			
0.08 €/min	{0.22, 0.30, 0.38}	{1.33, 1, 0.67}			
0.09 €/min	{0.21, 0.30, 0.39}	{1.37, 1, 0.62}			
0.10 €/min	{0.20, 0.30, 0.40}	{1.42, 1, 0.58}			
0.11 €/min	{0.19, 0.30, 0.41}	{1.46, 1, 0.54}			
0.12 €/min	{0.18, 0.30, 0.42}	{1.50, 1, 0.50}			
0.13 €/min	{0.17, 0.30, 0.43}	{1.54, 1, 0.46}			
0.14 €/min	{0.16, 0.30, 0.44}	{1.58, 1, 0.42}			
0.15 €/min	{0.15, 0.30, 0.45}	{1.62, 1, 0.37}			
0.16 €/min	{0.14, 0.30, 0.46}	{1.67, 1, 0.33}			
0.17 €/min	{0.13, 0.30, 0.47}	{1.71, 1, 0.29}			
0.18 €/min	{0.12, 0.30, 0.48}	{1.75, 1, 0.25}			
0.19 €/min	{0.11, 0.30, 0.49}	{1.79, 1, 0.21}			
0.20 €/min	{0.10, 0.30, 0.50}	{1.83, 1, 0.17}			

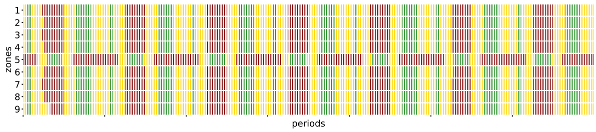
One week pricing

See Fig. 18.

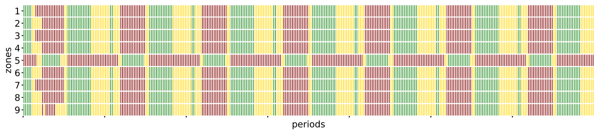
(A)



(a) ROL-1

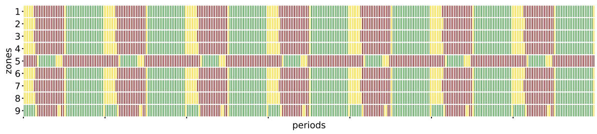


(b) ROL-4

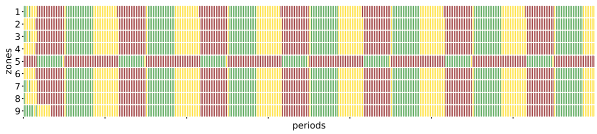


(c) ROL-8

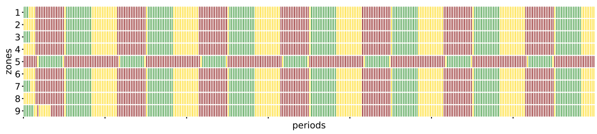
(B)



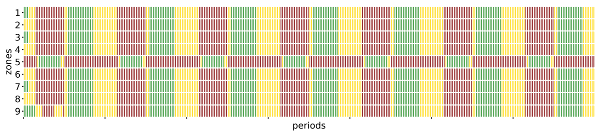
(a) MODSIM



(b) BAW-ROL-1



(c) BAW-MODSIM



(d) MOD48h

Fig. 18 Pricing for one week with different solution approaches (SMALL) $DSR = 1/3$. Green: low price, yellow: base price, red: high price

Stochastic evaluation

See Tables 3, 4 and 5.

Table 3 Mean profit increase (SMALL, MEDIUM, LARGE), $DSR = 1/3$

	Mean profit increase with respect to BASE in %														
	$Z = 9$					$Z = 16$					$Z = 25$				
	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
BASE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROL-1	6.24	5.71	5.73	5.79	5.78	9.05	5.67	5.67	5.67	5.61	11.44	6.24	6.23	6.22	6.22
ROL-4	9.41	12.06	12.16	12.29	12.36	12.35	8.84	8.78	8.45	8.15	14.17	9.24	9.07	8.90	8.74
ROL-8	11.68	14.50	14.64	14.79	14.83	13.97	10.91	10.67	10.24	9.83	14.71	11.54	11.38	11.24	11.09
ADP-1	14.25	11.55	11.65	11.75	11.81	14.20	12.66	12.13	11.60	11.00	14.25	14.10	13.77	13.44	13.12
ADP-4	14.57	14.59	14.79	15.03	15.20	14.67	13.20	12.57	11.95	11.35	14.82	14.36	14.05	13.72	13.40
ADP-8	14.72	15.27	15.46	15.62	15.72	14.65	13.45	12.83	12.22	11.64	14.78	14.48	14.20	13.91	13.60
MODSIM	13.46	14.67	14.74	14.85	14.97	14.23	13.90	13.39	12.80	12.26	14.25	13.30	13.11	12.90	12.69
BAW-ROL-1	14.87	15.26	15.31	15.36	15.37	14.59	14.08	13.48	12.88	12.32	14.70	14.61	14.30	13.99	13.66
BAW-MODSIM	14.73	15.36	15.41	15.47	15.49	14.71	13.62	12.97	12.34	11.73	14.80	14.41	14.06	13.72	13.36

Table 4 Mean profit increase (SMALL, MEDIUM, LARGE), $DSR = 2/3$

	Mean profit increase with respect to BASE in %														
	$Z = 9$					$Z = 16$					$Z = 25$				
	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
BASE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROL-1	8.14	8.26	8.29	8.35	8.28	9.53	9.31	9.13	9.04	8.99	9.05	9.04	9.05	9.05	9.07
ROL-4	12.65	12.84	12.91	12.99	12.98	12.65	12.43	12.20	12.05	11.99	12.35	12.30	12.26	12.22	12.17
ROL-8	13.96	14.09	14.16	14.22	14.14	14.64	14.37	14.15	14.02	13.97	13.97	13.93	13.82	13.73	13.63
ADP-1	13.69	13.59	13.77	14.00	14.10	14.42	13.94	13.73	13.62	13.55	14.23	14.13	14.03	13.93	13.84
ADP-4	13.87	14.00	13.98	13.99	13.90	14.79	14.44	14.15	13.96	13.85	14.59	14.57	14.46	14.35	14.24
ADP-8	13.93	14.10	14.12	14.16	14.07	14.87	14.53	14.31	14.16	14.06	14.71	14.60	14.52	14.43	14.34
MODSIM	13.38	13.73	13.74	13.80	13.77	14.12	14.55	14.63	14.61	14.60	14.20	14.29	14.30	14.25	14.18
BAW-ROL-1	13.96	13.88	13.87	13.89	13.82	14.80	14.50	14.20	13.99	13.84	14.67	14.51	14.41	14.31	14.22
BAW-MODSIM	13.94	13.94	13.94	13.97	13.90	14.79	14.62	14.33	14.12	13.99	14.65	14.64	14.53	14.43	14.33

Table 5 Mean profit increase (SMALL, MEDIUM, LARGE), $DSR = 3/3$

	Mean profit increase with respect to BASE in %														
	$Z = 9$					$Z = 16$					$Z = 25$				
	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
BASE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROL-1	10.09	10.23	10.30	10.42	10.45	11.83	11.72	11.60	11.48	11.51	11.44	11.41	11.31	11.18	11.08
ROL-4	12.64	12.78	12.74	12.73	12.64	14.36	14.31	14.08	13.64	13.30	14.17	14.09	13.93	13.73	13.55
ROL-8	13.29	13.37	13.39	13.38	13.30	15.23	15.03	14.69	14.25	13.95	14.71	14.56	14.37	14.13	13.90
ADP-1	13.26	13.08	13.04	13.03	12.95	14.81	14.87	14.64	14.34	13.98	14.25	14.16	14.02	13.87	13.75
ADP-4	13.20	13.24	13.17	13.13	13.06	15.25	15.04	14.73	14.35	14.02	14.70	14.67	14.52	14.35	14.21
ADP-8	13.31	13.38	13.40	13.39	13.30	15.26	14.95	14.63	14.26	13.81	14.80	14.66	14.50	14.34	14.18
MODSIM	12.93	13.33	13.30	13.29	13.21	14.90	14.90	14.69	14.28	13.84	14.25	14.18	14.09	13.99	13.89
BAW-ROL-1	13.31	13.14	13.06	13.01	12.94	15.25	15.12	14.78	14.40	14.04	14.82	14.60	14.42	14.18	13.95
BAW-MODSIM	13.31	13.24	13.17	13.13	13.06	15.14	15.05	14.68	14.20	13.79	14.78	14.70	14.55	14.39	14.21

Impact of a start solution

See Tables 6 and 7.

Table 6 Profit with and without start solution

	Difference in profit increase with respect to BASE in %								
	$Z = 9$			$Z = 16$			$Z = 25$		
	$DSR = 1/3$	$DSR = 2/3$	$DSR = 3/3$	$DSR = 1/3$	$DSR = 2/3$	$DSR = 3/3$	$DSR = 1/3$	$DSR = 2/3$	$DSR = 3/3$
ROL-4	0.02	0.00	0.00	0.00	0.37	0.09	− 0.04	− 0.15	− 0.04
ROL-8	0.02	0.00	0.00	− 0.06	− 0.02	− 0.09	− 0.33	− 0.01	0.04
MOD48h	0.00	0.00	0.00	0.07	0.02	0.25	0.30		
ADP-1	0.00	0.00	− 0.06	0.00	− 0.04	0.00	0.00	0.00	0.02
ADP-4	0.00	0.00	0.00	0.00	0.04	0.14	0.00	− 0.01	− 0.01
ADP-8	0.00	0.00	0.00	− 0.07	0.05	0.01	− 0.04	0.00	− 0.04
BAW-ROL-1	− 0.01	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00
BAW-MODSIM	0.00	0.00	0.00	0.00	− 0.04	0.00	0.00	0.00	− 0.05

Table 7 Computational time with and without start solution

	Difference in computational time in min								
	$Z = 9$			$Z = 16$			$Z = 25$		
	$DSR = 1/3$	$DSR = 2/3$	$DSR = 3/3$	$DSR = 1/3$	$DSR = 2/3$	$DSR = 3/3$	$DSR = 1/3$	$DSR = 2/3$	$DSR = 3/3$
BASE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROL-4	0.04	− 0.18	− 0.12	− 0.98	− 1.44	− 4.50	2.91	8.10	11.36
ROL-8	− 23.55	− 1.25	− 0.75	− 5.94	− 91.07	− 87.91	− 84.10	− 91.59	265.08
MOD48h	2723.46	1.00	0.42	− 0.16	0.08	0.00	0.06		
ADP-1	0.14	0.07	− 9.94	− 3.89	− 7.61	22.76	3.05	0.00	0.01
ADP-4	0.04	− 0.05	− 9.64	− 3.71	− 7.72	− 36.43	− 30.74	0.00	− 0.01
ADP-8	− 6.45	− 2.90	0.46	47.06	− 89.43	− 1394.14	73.93	40.21	7.44
BAW-ROL-1	0.46	0.83	0.13	− 15.44	− 3.46	− 2.24	16.17	21.76	65.43
BAW-MODSIM	0.21	− 0.07	0.10	− 9.42	− 4.92	− 2.51	− 30.54	63.86	74.60

Mathematical model

Soppert et al. (2022) define the mathematical model of optimization of profit with differentiated prices for a VSS as follows:

$$\max_{\mathbf{y},\mathbf{q},\mathbf{r},\mathbf{a},\mathbf{s}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{i,j,t}\cdot l_{,ij}\cdot(p^m-c) \tag{8}$$

$$\text{s.t.}\quad a_{i,t}=\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{i,j,t}+s_{i,t}\forall i\in\mathcal{Z},t\in\mathcal{T} \tag{9}$$

$$\sum_{i\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{i,j,t}+s_{j,t}=a_{j,t+1}\forall j\in\mathcal{Z},t\in\mathcal{T} \tag{10}$$

$$a_{i,0}=\hat{a}_{i,0}\forall i\in\mathcal{Z} \tag{11}$$

$$\sum_{m\in\mathcal{M}} y^m_{i,t}=1\forall i\in\mathcal{Z},t\in\mathcal{T} \tag{12}$$

$$r^m_{i,j,t}\le d^m_{i,j,t}\cdot y^m_{i,t}\forall i,j\in\mathcal{Z},t\in\mathcal{T},m\in\mathcal{M} \tag{13}$$

$$r^m_{i,j,t}\le d^m_{i,j,t}/\sum_{k\in\mathcal{Z}} d^m_{i,k,t}\cdot a_{i,t}\forall i,j\in\mathcal{Z},t\in\mathcal{T},m\in\mathcal{M} \tag{14}$$

$$\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} d^m_{i,j,t}\cdot y^m_{it}-a_{i,t}\le\bar{M}\cdot q_{i,t}\forall i\in\mathcal{Z},t\in\mathcal{T} \tag{15}$$

$$\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} -d^m_{i,j,t}\cdot y^m_{i,t}+a_{i,t}\le\bar{M}\cdot(1-q_{i,t})\forall i\in\mathcal{Z},t\in\mathcal{T} \tag{16}$$

$$\sum_{m\in\mathcal{M}} d^m_{i,j,t}\cdot y^m_{i,t}\le\sum_{m\in\mathcal{M}} r^m_{i,j,t}+\bar{M}\cdot q_{i,t}\forall i,j\in\mathcal{Z},t\in\mathcal{T} \tag{17}$$

$$s_{i,t}\le\bar{M}\cdot(1-q_{,it})\forall i\in\mathcal{Z},t\in\mathcal{T} \tag{18}$$

$$y_{i,t}^{m} \in \{0,1\} \forall i \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{19}$$

$$q_{i,t} \in \{0,1\} \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{20}$$

$$r_{i,j,t}^{m} \in \mathbb{R}_0^+ \forall i,j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{21}$$

$$s_{i,t} \in \mathbb{R}_0^+ \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{22}$$

$$a_{i,t} \in \mathbb{R}_0^+ \forall i \in \mathcal{Z}, t \in \{0,1,\dots,T\} \tag{23}$$

See Table 8.

Table 8 List of notation

Symbol	Description
Sets	
$\mathcal{M}$	Set of discrete price points
$\mathcal{T}$	Set of periods
$\mathcal{Z}$	Set of locations
Parameters	
$\hat{a}_{i,0}$	Initial number of available vehicles at location i
c	Variable costs per minute of a rental
$d_{i,j,t}^{m}$	Demand for trips from location i to location j at period t at price m
$l_{i,j}$	Rental duration from location i to location j
$\bar{M}$	Big-M
p^m	Price $m \in \mathcal{M}$
Variables	
$a_{i,t}$	Available vehicles at location i at period t
$q_{i,t}$	Auxiliary binary variable
$r_{i,j,t}^{m}$	Rentals from location i to location j at period t at price m
$s_{i,t}$	Vehicle, that remains at location i in period t
$y_{i,t}$	Binary variable, pricing decision, takes the value 1, if and only if price p^m is set in location i at period t

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Declarations

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