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Demand Shift Model of a One-Way Car-Sharing System with Real-Time Pricing

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Abstract— **Car sharing has become popular as a new form of transportation mode in cities. In particular, station-based one-way car sharing gives customers the option to return the vehicle to any available station. It has been gaining popularity owing to its convenience. However, under this type of car-sharing system, user demand is not constant across stations, which leads to the uneven distribution of vehicles among stations. The cost of eliminating this uneven distribution is a major problem in terms of profitability. The goal of this research is to provide a solution to the uneven distribution problem without rebalancing (managed redistribution of vehicles). The main strategy is real-time pricing, with which the user demand moves to cheaper links. This paper proposes a scheme of the real-time pricing, and develops a model describing fluctuations of user demand, not in terms of just increase or decrease, but rather in terms of demand shifts. To demonstrate the effectiveness of the method, the simulations are performed under the same setting as the actual car-sharing service by TOYOTA Corporation, Ha:mo RIDE Toyota operating in Toyota City, Aichi Prefecture.**

I. Introductoin

In recent years, car sharing has become popular as a new type of transportation mode in cities owing to its many advantages over private car ownership, including lower cost, reduction in traffic, and environmental friendliness. Zipcar in the U.S. [1] has surpassed 1 million registered members in 2016, and the German company SHARE NOW [2], which operates services throughout Europe, has more than four million registered members as of 2019. In Japan, the number of car-sharing vehicles and the number of registered members are increasing every year. According to a survey by the Foundation for Promoting Personal Mobility and Ecological Transportation [3], as of March 2020, there are 19,119 car-sharing stations in Japan (up 10.9% from the previous year), 40,290 vehicles (up 15.2%), and 2,046,581 members (up 25.8%), indicating that the market is rapidly expanding. A car-sharing service differs from a car rental service in the following aspects: First, a car-sharing service offers immediate availability 24 hours a day and 7 days a week via online booking, without visiting a storefront. Second, it is intended for short-term use, ranging from a few minutes to an hour. Finally, most vehicles are small single-seater mobility vehicles and fuel-efficient electric vehicles. Compared to a car rental service, a car-sharing service is more convenient for daily use.

In this paper, we focus on one-way type car-sharing services, in which customers can depart from the station of their choice and then return the vehicle to any available station in the area. Because of its convenience in responding flexibly to the needs of users, this service has been deployed on a large scale, mainly in Europe and the U.S., led by Germany's SHARE NOW [2]. Currently, most of the car-sharing market in Japan adopt the round-trip model due to the aforementioned legal restrictions, but a 2014 amendment to the law allowed for the development of one-way services. Therefore, one-way car sharing is currently planned for deployment in Japan [4], and many field trials are underway.

In spite of the convenience of the one-way car-sharing service, it has the serious drawback that vehicles become unevenly distributed among stations depending on the user demands, which causes problems, such as lack of available vehicles at some stations and no available parking spaces in other stations. The uneven vehicle distribution problem is critical to the practical implementation of car-sharing services. The mainstream approach to solving this problem is the redeployment of vehicles from stations with sufficient vehicles to stations with insufficient vehicles, which is called rebalancing. Bicycles and other small-format vehicles can be simultaneously transported in large quantities using a truck. However, in the case of automobiles, rebalancing is generally performed by manually driving the vehicles one by one, which involves serious labor costs. Hence, the development of an inexpensive rebalancing solution is essential for the practical implementation of a one-way car-sharing service.

Several previous studies have addressed the issue on how to reduce the cost of rebalancing. Calafiore et al. used model predictive control to minimize rebalancing based on a control engineering approach [5]. Smith et al. developed a model in which the behavior of workers performing rebalancing is classified into three patterns depending on the distribution of vehicles and customers [6]. Barth et al. conducted a simulation in a resort area in Southern California to determine the number of vehicles that would minimize rebalancing [7]. Mizokami et al. examined the possibility of introducing a system without rebalancing from the perspective of profitability [8]. Other approaches that do not perform rebalancing include that proposed by Nakayama et al. [9], which reduces reservations that would cause uneven distribution, and that of Uesugi et al. [10], which eliminates uneven distribution by asking customers to carpool or split rides. However, these approaches can sacrifice convenience and cause a decline in ridership.

Another way to eliminate uneven distribution without rebalancing is to introduce real-time pricing to encourage and discourage usage. Previous studies have examined the intro-

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duction of real-time pricing in a one-way car-sharing system. Jorge et al. developed a model in which demand is viewed as a function of price with the aim of maximizing profits and then used the model to determine the pricing that would maximize the operating company's profits [11]. This model focuses on cost modeling, including overhead costs, to track changes in profits. Min et al. developed a model for profit maximization that considers the allocation of rebalancing personnel in addition to the use of variable pricing to control vehicles [12]. Other relevant studies include Shuyun et al.'s pricing model that takes into account the scheduling of the charging of electric vehicles in addition to their repositioning [13], Ruiyou et al.'s pricing model that limits demand by considering realistic constraints on the number of vehicles [14], and Kamatani et al.'s pricing model using reinforcement learning [15]. In these existing real-time pricing models, the increase or decrease in demand at a single link between two stations is expressed as the difference between the standard price and the actual price set for using that link. However, to the best of our knowledge, no model that can adequately represent the user shifts to a different link due to real-time pricing has been developed.

The goal of this research is to solve the problem of uneven distribution of vehicles in a one-way car-sharing system by real-time pricing without rebalancing. To achieve this, a new model that represents the shifts in demand is proposed. Under this model, users voluntarily switch the links they use to neighboring links based on the difference in the usage fee. The model is novel in that it considers fluctuations of user demand, not in terms of just increase or decrease, but rather in terms of demand shifting. Furthermore, the model incorporates realistic constraints on the number of vehicles. A simulation based on the proposed model is run to specifically examine the possibility of resolving the uneven vehicle distribution problem by real-time pricing. To simulate conditions as close to a real environment as possible, the simulations are programmed with the same station layout, information, and usage history as the actual car-sharing service by TOYOTA Corporation, Ha:mo RIDE Toyota [**?**] operating in Toyota City, Aichi Prefecture.

The remainder of this paper proceeds as follows: Section 2 describes the model of a one-way car-sharing system and how the control objective is set. Section 3 describes how shifts in demand are modeled, how usage fees are determined, and how the parameters are set by considering constraints on the number of vehicles. Section 4 presents the results of the simulation. Finally, Section 5 presents the conclusions.

Notation: Let $\mathbb{N}$ denote the set of natural numbers including 0, $\mathbb{R}$ the set of real numbers, and $\mathbb{R}_{+0}$ the set of non-negative real numbers.

II. Statement of the problem

This section describes the transportation model based on [5], as a part of car-sharing system, and provides control objectives employed in this study.

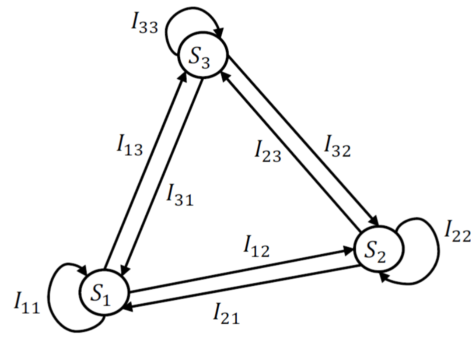

Fig. 1: Sketch of the transportation model

TABLE I: List of variables

Variable	Definition
$z_i(t) \in \{0,\dots,\hat{z}_i\}$	Number of vehicles parked at station S_i at time t
$\hat{z}_i \in \mathbb{N}$	Maximum number of vehicles that can be parked at station S_i
$v_{ij}(t) \in \mathbb{N}$	Number of vehicles traveling on link I_{ij} at time t
$\tilde{q}_{ij}(t,\delta) \in [0,1]$	Probability that a vehicle on link I_{ij} will arrive at destination S_j within δ min from time t
$s_{ij}(t) \in \mathbb{N}$	Demand for link I_{ij} within δ min from time t
$a_{ij}(t) \in \mathbb{N}$	Number of vehicles arriving at station S_j from link I_{ij} within δ min from time t
$d_{ij}(t) \in \mathbb{N}$	Number of vehicles departing station S_i along link I_{ij} within δ min from time t

A. System overview and variables

The transportation system is modeled by a network graph comprising a plurality of stations S_i $(i = 1, 2, \dots, N)$ and links I_{ij} $(i, j = 1, 2, \dots, N)$ connecting them, where N is the total number of stations. A link is not a specific physical path but a representation of travel between stations, regardless of the path. A sketch of this model is shown in Figure 1.

At time t, the number of vehicles parked at station S_i is denoted by $z_i(t) \in \{0, 1, \dots, \hat{z}_i\}$, and the number of vehicles currently traveling over link I_{ij} is denoted by $v_{ij}(t) \in \mathbb{N}$, where $\hat{z}_i$ is the maximum number of vehicles that can be parked at station S_i. Let $\tilde{q}_{ij}(t,\delta) \in [0,1]$ be the arrival rate, that is, the probability that a vehicle on link I_{ij} will arrive at destination S_j within δ min from time t. Let $s_{ij}(t) \in \mathbb{N}$ be the demand for link I_{ij} within δ min from time t. This represents the number of users who wish to travel from S_i to S_j. Within δ min from time t, the number of vehicles arriving from link I_{ij} to station S_j is denoted as $a_{ij}(t) \in \mathbb{N}$, and the number of vehicles departing station S_i along link I_{ij} is denoted as $d_{ij}(t) \in \mathbb{N}$. The variables are listed in Table I.

B. System model

The number $v_{ij}(t)$ of vehicles on link I_{ij} varies according to

$$v_{ij}(t+\delta) = v_{ij}(t) - a_{ij}(t) + d_{ij}(t). \tag{1}$$

TABLE II: List of variables: expected values

Variable	Definition
$\bar{z}_i(t) \in [0, \hat{z}_i]$	Expected value of the number of vehicles parked at station S_i at time t
$\bar{v}_{ij}(t) \in \mathbb{R}_{+0}$	Expected value of the number of vehicles traveling on link I_{ij} at time t
$q_{ij} \in [0, 1]$	Expected value of the probability that a vehicle on link I_{ij} will arrive at destination S_j within δ min from time t
$\bar{s}_{ij}(t) \in \mathbb{R}_{+0}$	Expected value of demand for link I_{ij} within δ min from time t
$\bar{a}_{ij}(t) \in \mathbb{R}_{+0}$	Expected value of the number of vehicles arriving at station S_j from link I_{ij} within δ min from time t
$\bar{d}_{ij}(t) \in \mathbb{R}_{+0}$	Expected value of the number of vehicles departing station S_i along link I_{ij} within min from time t

The change in the number $z_j(t)$ of vehicles at station S_j can be represented by the following equation:

$$z_j(t+\delta) = z_j(t) + \sum_{i=1}^{N} a_{ij}(t) - \sum_{h=1}^{N} d_{jh}(t). \tag{2}$$

The second term on the right-hand side represents the sum of the number of vehicles arriving at station S_j within time δ, and the third term represents the sum of the number of vehicles departing from station S_j within time δ.

Averaged models are derived from (1) and (2). The expected value of each parameter with respect to time is expressed as a bar, e.g., $\bar{v}_{ij}(t) = \mathrm{E}(v_{ij}(t))$, where $\mathrm{E}(\cdot)$ represents the expectation of a random variable. To simplify the model, it is assumed that the average arrival rate does not depend on the time and number of vehicles. Thus, it is assumed that $\mathrm{E}(\tilde{q}_{ij}(t, \delta)) = q_{ij}$ holds with some $q_{ij} \in [0, 1]$. The variables for the averaged models are listed in Table II. Because $\bar{v}_{ij}(t)$ vehicles arrive at S_j over link I_{ij} within time δ with a probability of q_{ij}, the expected value of the number of arriving vehicles $\bar{a}_{ij(t)}$ can be written as follows:

$$\bar{a}_{ij}(t) = q_{ij}\bar{v}_{ij}(t). \tag{3}$$

Therefore, the averaged models of Equations (1) and (2) can be expressed as follows:

$$\bar{v}_{ij}(t+\delta) = (1 - q_{ij})\bar{v}_{ij}(t) + \bar{d}_{ij}(t), \tag{4}$$

$$\bar{z}_j(t+\delta) = \bar{z}_j(t) + \sum_{i=1}^{N} q_{ij}\bar{v}_{ij}(t) - \sum_{h=1}^{N} \bar{d}_{jh}(t). \tag{5}$$

The number of vehicles in Equations (1) and (2) are integer values, whereas the parameters in Equations (4) and (5) are the expected values and hence the real numbers.

C. Control objective

The control objective is to reduce the uneven distribution of vehicles in the one-way car-sharing system. Therefore, one of the control guidelines is to consider variations in parking rates at all stations. If this variation in parking rates is reduced, then the problem of the uneven distribution of vehicles can be solved. Let $r_i(t) = \bar{z}_i(t)/\hat{z}_i$ be the parking rate at station S_i, $\mu_r(t)$ the average value of the parking rates of all stations, and $\sigma_r(t)$ the standard deviation, defined as follows:

$$\begin{aligned} \mu_r(t) &= \frac{1}{N}\sum_{i=1}^{N} r_i(t) \\ \sigma_r(t) &= \sqrt{\frac{1}{N}\sum_{i=1}^{N}(r_i(t) - \mu_r(t))^2}. \end{aligned} \tag{6}$$

Next, we consider two constraints that must be satisfied by $\bar{d}_{ij}(t)$, the number of vehicles departing from S_i over link I_{ij}. (i) The sum of the vehicles departing from station S_i must be less than or equal to $\bar{z}_i(t)$, the number of vehicles parked at station S_i at time t. This constraint is expressed by the following equation:

$$\sum_{h=1}^{N} \bar{d}_{ih}(t) \le \bar{z}_i(t) \quad \forall i = 1, 2, \ldots, N. \tag{7}$$

(ii) The sum of the expected values $\bar{d}_{ij}(t)$ of the number of vehicles departing from station S_j must be less than or equal to the number of new vehicles that can be accepted at station S_j at time t. The number of vehicles that can be accepted is the number of empty parking spaces $\hat{z}_j - \bar{z}_j(t)$ at S_j minus the sum of the number of vehicles $\bar{v}_{hj}(t)$ on the links moving toward S_j. This constraint can be expressed as

$$\sum_{h=1}^{N} \bar{d}_{hj}(t) \le \hat{z}_j - \bar{z}_j(t) - \sum_{h=1}^{N} \bar{v}_{hj}(t) \quad \forall j = 1, 2, \ldots, N. \tag{8}$$

The above constraints entail that $\bar{d}_{ij}(t)$, the number of departing vehicles, will not necessarily match the demand $\bar{s}_{ij}(t)$. Thus, it is desirable to reduce the difference between them; in other words, the number of canceled cars should be as small as possible. Let $\bar{m}(t)$ be the sum of the number of canceled cars within δ min from time t, defined as

$$\bar{m}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N}(\bar{s}_{ij}(t) - \bar{d}_{ij}(t))^2. \tag{9}$$

For the purpose, the number of departing vehicles $\bar{d}_{ij}(t)$ can be directly determined, and the demand $\bar{s}_{ij}(t)$ can be controlled according to a human decision model through real-time pricing. Now, the control objective considered in this paper is summarized as follows.

Problem 1: In the averaged models given by (4) and (5), we make the standard variation $\sigma_r(t)$ of the parking rates of all stations and the number $\bar{m}(t)$ of canceled cars as small as possible while satisfying the constraints (7) and (8) on the number of departing vehicles $\bar{d}_{ij}(t)$. For this purpose, design a departing strategy to determine $\bar{d}_{ij}(t)$ and a pricing scheme to control $\bar{s}_{ij}(t)$ appropriately.

III. Development of Models

A. Control of departing vehicles

This section considers a departing strategy of vehicles for $\bar{d}_{ij}(t)$ in the system (4) and (5) to reduce the number of canceled cars (9) while satisfying constraints (7) and (8). It is desirable that the number of departing vehicles is equal to the demand, that is,

$$\bar{d}_{ij}(t) = \bar{s}_{ij}(t), \tag{10}$$

as long as constraints (7) and (8) are satisfied. Otherwise, we have to adjust $\bar{d}_{ij}(t)$ to satisfy these constraints in the following way.

First, if (7) is not satisfied by $\bar{d}_{ij}(t) = \bar{s}_{ij}(t)$, it is necessary to make adjustments to ensure that $\sum_{h=1}^{N} \bar{d}_{ih}(t) = \bar{z}_i(t)$. In this case, the number of departing vehicles $\bar{d}_{ij}(t)$ on link I_{ij} is determined by allocating the number of parking vehicles $\bar{z}_i(t)$ according to the ratio of demand at link I_{ij} ($\bar{s}_{ij}(t)$) to the sum of demand at station S_i ($\sum_{h=1}^{N} \bar{s}_{ih}(t)$), that is,

$$\bar{d}_{ij}(t) = \frac{\bar{s}_{ij}(t)}{\sum_{h=1}^{N} \bar{s}_{ih}(t)} \bar{z}_i(t). \tag{11}$$

If (8) is not satisfied by $\bar{d}_{ij}(t) = \bar{s}_{ij}(t)$, it is necessary to make adjustments to ensure that $\sum_{h=1}^{N} \bar{d}_{ij}(t) = \hat{z}_j - \bar{z}_j(t) - \sum_{h=1}^{N} \bar{v}_{hj}(t)$. Employ the same allocation strategy to (11), and the number of departing vehicles is determined as

$$\bar{d}_{ij}(t) = \frac{\bar{s}_{ij}(t)}{\sum_{h=1}^{N} \bar{s}_{hj}(t)} \left(\hat{z}_j - \bar{z}_j(t) - \sum_{h=1}^{N} \bar{v}_{hj}(t) \right). \tag{12}$$

To reduce the number of canceled cars (9) while satisfying constraints (7) and (8), we just choose the minimum value from (10), (11), and (12). Accordingly, the departing strategy of vehicles is proposed as follows:

$$\begin{aligned} \bar{d}_{ij}(t) = \min\{ & \bar{s}_{ij}(t), \frac{\bar{s}_{ij}(t)}{\sum_{h=1}^{N} \bar{s}_{ih}(t)} \bar{z}_i(t), \\ & \frac{\bar{s}_{ij}(t)}{\sum_{h=1}^{N} \bar{s}_{hj}(t)} \left(\hat{z}_j - \bar{z}_j(t) - \sum_{h=1}^{N} \bar{v}_{hj}(t) \right) \}. \end{aligned} \tag{13}$$

This value is inputted into the system (4) and (5).

B. Demand shift by pricing

Next, real-time pricing is introduced as a means of shifting the user demand $\bar{s}_{ij}(t)$. In this section, a demand shift model is developed to describe the effect of real-time pricing.

Let $\bar{o}_{kl}(t) \in \mathbb{R}_{+0}$ be the user demand for link I_{ij} at a fixed price, referred to as the *original demand*. In contrast, $\bar{s}_{ij}(t)$ is the demand changed by pricing, referred to as the *actual demand*. Let $c_{ij}(t) \in \mathbb{R}_{+0}$ be the usage fee of link I_{ij} at time t, which is assumed to change according to the pricing scheme proposed in Section III-C.

Depending on the usage fee $c_{ij}(t)$, part of the original demand $\bar{o}_{ij}(t)$ will shift to another link I_{kl} to determine the actual demand $\bar{s}_{ij}(t)$. This model is constructed as follows.

Let $w_{ijkl}(t) \in \mathbb{R}_{+0}$ be the weight to represent the incentive to switch the link from I_{ij} to I_{kl}. The greater the value of this parameter, the greater the benefit of switching links, and the more demand shift is encouraged. Here, $w_{ijkl}(t)$ is expressed as follows:

$$\begin{aligned} & w_{ijij}(t) = 1 \\ & w_{ijkl}(t) = \begin{cases} \eta_{ik}\eta_{jl}(c_{ij}(t) - c_{kl}(t)) & \text{if} \quad c_{ij}(t) > c_{kl}(t) \\ 0 & \text{if} \quad c_{ij}(t) \leq c_{kl}(t) \end{cases} \\ & \qquad (ij \neq kl), \end{aligned} \tag{14}$$

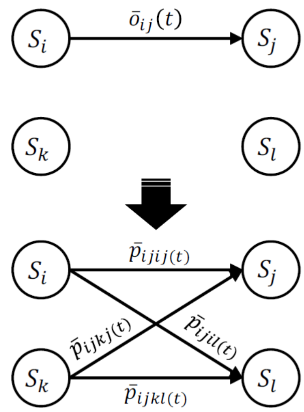

Fig. 2: Allocation of original demand $\bar{o}_{ij}(t)$ to links

where η_{ik} is a coefficient that represents the ease with which a user can change the station from S_i to S_k. In general, this parameter will be higher if S_i and S_k are closer because users will be more likely to switch to a cheaper station if it is closer. If the price of the new link is higher than that of the original link, then there is no shift, and the weight is 0.

As shown in Figure 2, the portion of the original demand $\bar{o}_{ij}(t)$ on link I_{ij} that shifts to link I_{kl} is denoted by $\bar{p}_{ijkl}(t)$, determined by $w_{ijkl}(t)$ as follows:

$$\bar{p}_{ijkl}(t) = \frac{w_{ijkl}(t)}{\sum_{m=1}^{N} \sum_{n=1}^{N} w_{ijmn}(t)} \bar{o}_{ij}(t). \tag{15}$$

Then, the actual demand $\bar{s}_{ij}(t)$ is determined as

$$\bar{s}_{ij}(t) = \bar{o}_{ij}(t) - \sum_{k=1}^{N} \sum_{l=1}^{N} \bar{p}_{ijkl}(t) + \sum_{k=1}^{N} \sum_{l=1}^{N} \bar{p}_{klij}(t), \tag{16}$$

where the second term on the right-hand side represents the sum of the demand shift away from link I_{ij}, and the third term on the right-hand side represents the sum of the demand shift toward link I_{ij}. From Equation (15),

$$\begin{aligned} \sum_{k=1}^{N} \sum_{l=1}^{N} \bar{p}_{ijkl}(t) &= \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{w_{ijkl}(t)}{\sum_{m=1}^{N} \sum_{n=1}^{N} w_{ijmn}(t)} \bar{o}_{ij}(t) \\ &= \bar{o}_{ij}(t) \end{aligned}$$

holds, with which Equation (16) becomes a simpler form

$$\bar{s}_{ij}(t) = \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{w_{klij}(t)}{\sum_{m=1}^{N} \sum_{n=1}^{N} w_{klmn}(t)} \bar{o}_{kl}(t). \tag{17}$$

C. Pricing scheme

This section develops a pricing scheme to reduce the standard deviation of the parking rates $\sigma_r(t)$ in (6) according to the demand shift model. The usage fee $c_{ij}(t)$ is determined according to the parking rates $r_i(t), r_j(t)$ of stations i, j as follows:

$$c_{ij}(t) = \hat{c}_{ij} + \beta_{ij}(r_j(t) - r_i(t)), \tag{18}$$

where $\hat{c}_{ij} \in \mathbb{R}_{+0}$ denotes the basic usage fee for link I_{ij}, and $\beta_{ij} > 0$ is a coefficient that sets the weight of the parking rate in real-time pricing.

To verify the effect of the pricing scheme (18), consider a situation that stations S_i and $S_{\hat{i}}$ locate near to each other and so S_j and $S_{\hat{j}}$ do, and that the difference between the parking rates $r_{\hat{j}}(t) - r_{\hat{i}}(t)$ is far larger than $r_j(t) - r_i(t)$. Then, from (18), the price $c_{\hat{i}\hat{j}}(t)$ of link $I_{\hat{i}\hat{j}}$ is far higher than that $c_{ij}(t)$ of I_{ij}, that is, $c_{\hat{i}\hat{j}}(t) \gg c_{ij}(t)$. Next, from (14), the weights satisfy

$$w_{\hat{i}\hat{j}ij}(t) \gg 1, \;\; w_{ij\hat{i}\hat{j}}(t) = 0, \;\; w_{ijij}(t) = w_{\hat{i}\hat{j}\hat{i}\hat{j}}(t) = 1.$$

Then, from (17), the approximate equations

$$\begin{aligned} \bar{s}_{ij}(t) &\simeq \frac{w_{\hat{i}\hat{j}ij}(t)\bar{o}_{\hat{i}\hat{j}}(t)}{w_{\hat{i}\hat{j}\hat{i}\hat{j}}(t) + w_{\hat{i}\hat{j}ij}(t)} + \frac{w_{ijij}(t)\bar{o}_{ij}(t)}{w_{ij\hat{i}\hat{j}}(t) + w_{ijij}(t)} \\ &\simeq \bar{o}_{\hat{i}\hat{j}}(t) + \bar{o}_{ij}(t) \\ \bar{s}_{\hat{i}\hat{j}}(t) &\simeq \frac{w_{\hat{i}\hat{j}\hat{i}\hat{j}}(t)\bar{o}_{\hat{i}\hat{j}}(t)}{w_{\hat{i}\hat{j}\hat{i}\hat{j}}(t) + w_{\hat{i}\hat{j}ij}(t)} + \frac{w_{ij\hat{i}\hat{j}}(t)\bar{o}_{ij}(t)}{w_{ij\hat{i}\hat{j}}(t) + w_{ijij}(t)} \\ &\simeq 0 \end{aligned}$$

hold, which implies that the original demand $\bar{o}_{\hat{i}\hat{j}}(t)$ of link $I_{\hat{i}\hat{j}}$ shifts to I_{ij}. When the number $\bar{d}_{ij}(t)$ of departing vehicles is equal to $\bar{s}_{ij}(t)$ as (10), the models (4), (5) indicate that the difference $r_{\hat{j}}(t) - r_{\hat{i}}(t)$ does not increase while $r_j(t) - r_i(t)$ does. As a result, the parking rate $r_i(t)$ is equalized, and the standard deviation $\sigma_r(t)$ in Equation (6) is reduced to satisfy the control objective.

IV. Simulation

A. Simulation conditions

To verify the effectiveness of the proposed models, the simulation was programmed with the same station layout, information, and usage history as the actual car-sharing service by TOYOTA Corporation, Ha:mo RIDE Toyota operating in Toyota City, Aichi Prefecture. There are $N = 57$ stations scattered over an area of approximately 12 km × 8 km. Figure 3 shows the layout of the stations with the longitude and latitude coordinates in decimal degrees, where the blue circles in the map represent the stations. Of the 57 stations, 50 stations were located within 1 km of another station. Hence, users are expected to switch links in the proposed model by walking to or from a different station near the desired arrival or departure station.

The system model with (4) and (5) was executed for 48 h (2880 min) worth of data as a discrete-time system with sampling time $\delta = 10$ min. The initial number of vehicles $\bar{z}_i(0)$ at each station and the maximum number of vehicles that can be parked at each station $\hat{z}_i$ were set based on the actual operational data from Ha:mo RIDE. The total number of vehicles is 104, and the sum of the maximum number of vehicles that can be parked is $\sum_{i=1}^{N} \hat{z}_i = 263$. The original demand $\bar{o}_{ij}(t)$ in (17) was determined based on the actual usage history of Ha:mo RIDE.

The other numerical conditions were set to satisfy realistic conditions as follows. There were no moving vehicles at time 0, that is, $v_{ij}(0) = 0$. The arrival rate of vehicles on link I_{ij} to station S_j in (4) and (5) was set to $q_{ij} = 0.9998^{L_{ij}}$, and the coefficient η_{ik} in (14) was set to $\eta_{ik} = 0.1\sqrt{2}\ 0.998^{L_{ik}}$,

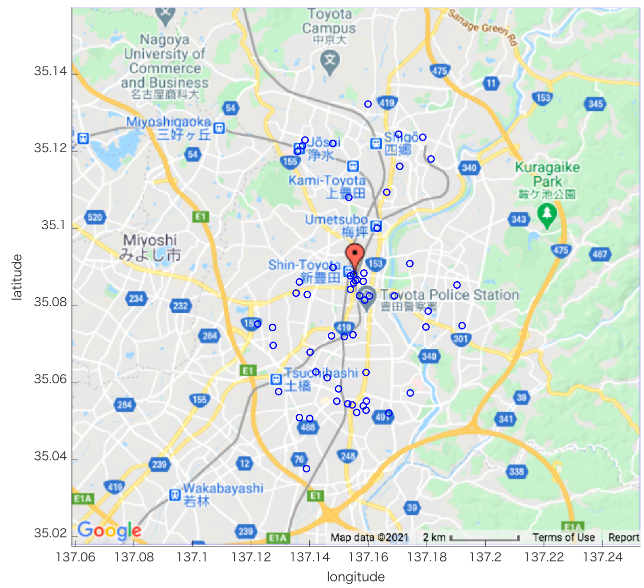

Fig. 3: Layout of $N = 57$ stations in Ha:mo RIDE Toyota

where L_{ij} is the distance between stations i and j. The price range was set to $100 \leq c_{ij}(t) \leq 300$ according to (18), where β_{ij} was set to 100. Under this setting, a difference of 10% in the parking rate causes a price adjustment of 10 yen. Based on real pricing, the basic usage fee was set to $\hat{c}_{ij} = 200$ yen. The price was changed in 10-yen increments, and $c_{ij}(t)$ was rounded to the nearest 10 yen.

B. Simulation results

Simulations were run under two conditions, with real-time pricing (Case 1) and without real-time pricing (Case 2), and the results were compared.

Figures 4 and 5 show how the parking rate $r_i(t)$ changes over time at each station in Cases 1 and 2, respectively. As shown in Figure 4, the unevenness in $r_i(t)$ was particularly reduced during the second day when the usage was high in Case 1. At the end of the simulation, there were no stations where parking rate $r_i(t)$ was 100% and five stations where $r_i(t)$ was 0%. By contrast, Figure 5 shows that there were 12 stations where $r_i(t)$ was 100% and 13 stations where $r_i(t)$ was 0% in Case 2. Figure 6 shows the standard deviation $\sigma_r(t)$ of the parking rates for all stations. The value is lower with real-time pricing (Case 1, solid line) than that without real-time pricing (Case 2, dashed line). From time $t = 23$ [hours], the standard deviation under real-time pricing remained less than or equal to 0.27 (dashed line), demonstrating that real-time pricing reduces the unevenness of the parking rates $r_i(t)$.

Figure 7 plots the cumulative values of the number of departing vehicles, $\sum_{\tau=0}^{t} \sum_{ij} \bar{d}_{ij}(\tau)$, by the solid line, shifted demand, $\sum_{\tau=0}^{t} \sum_{ijkl} \bar{p}_{ijkl}(\tau)$, by the dashed line, and original demand, $\sum_{\tau=0}^{t} \sum_{ij} \bar{o}_{ij}(\tau)$, by the dotted line in Case 1. The original demand increased to 188.0 as the dotted line, while 87.7 vehicles shifted to different links as the dashed line, and the number of departing vehicles increased to

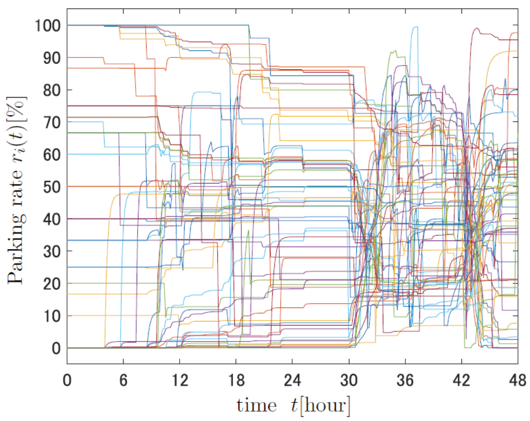

Fig. 4: Parking rate $r_i(t)$ (Case 1)

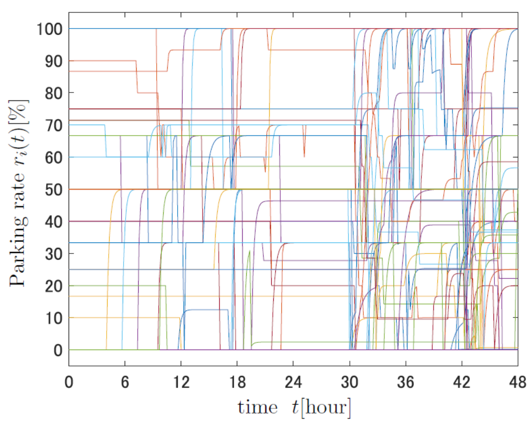

Fig. 5: Parking rate $r_i(t)$ (Case 2)

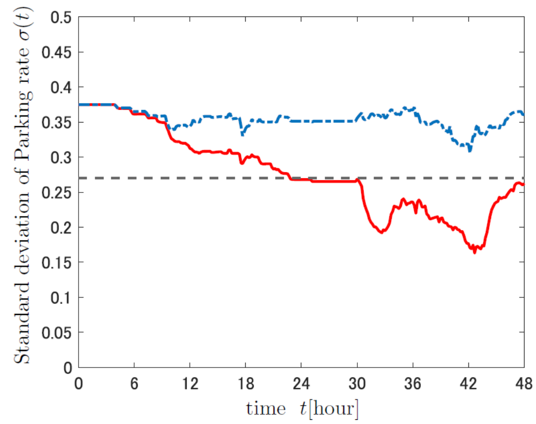

Fig. 6: Standard deviation of the parking rate $\sigma_r(t)$ (Case 1 (solid line), Case 2 (dashed line))

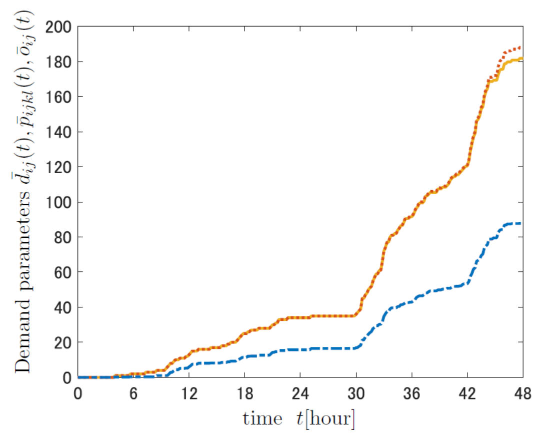

Fig. 7: Cumulative number of departing vehicles $\sum_{\tau=0}^{t}\sum_{ij}\bar{d}_{ij}(\tau)$ (solid line), shifted demand $\sum_{\tau=0}^{t}\sum_{ijkl}\bar{p}_{ijkl}(\tau)$ (dashed line), and original demand $\sum_{\tau=0}^{t}\sum_{ij}\bar{o}_{ij}(\tau)$ (dotted line) in Case 1

181.8 as the solid line. This result means that the shifted demand accounts for 48.2% of the departing vehicles. In contrast, Figure 8 plots those in Case 2, which shows that the cumulative number of departing vehicles, $\sum_{\tau=0}^{t}\sum_{ij}\bar{d}_{ij}(\tau)$, is 154.4 (solid line), less than that in Case 1.

Figure 9 depicts the cumulative number of canceled cars, $\sum_{\tau=0}^{t}\bar{m}(\tau)$, on all links over time in Cases 1 and 2 by solid and dashed lines, respectively. In Case 1, a total of 6.2 cars were canceled, while in Case 2, a total of 33.7 cars were canceled. This result shows that the proposed method reduces the number of canceled cars by 81.5%.

Figure 10 shows the cumulative total income $\sum_{\tau=0}^{t}\sum_{ij}c_{ij}(\tau)\bar{d}_{ij}(\tau)$ from the car-sharing system over time. In the first half of the time, the income in Case 1 (solid line) is lower than that in Case 2 (dashed line), but the positions are gradually reversed starting at approximately $t = 33$ [hours]. The reason for the reversal can be understood by referring to Figure 9 as follows: After $t = 33$ [hours], as the number of canceled cars begins to increase in Case 2 (dashed line), that in Case 1 remains small (solid line). The final total income was approximately 32,937 yen in Case 1 and 30,870 yen in Case 2. Figure 7 shows that the total number of departing vehicles $\bar{d}_{ij}(t)$ in Case 1 is 181.8, which makes the average price per use approximately 181.2 yen. Under variable pricing in Case 1, the average fee is lower than that in Case 2 (200 yen), but the number of canceled cars is also lower, which is why higher income can be generated compared to that in Case 2.

Overall, a comparison of the simulations under Case 1 and Case 2 reveals that, under the proposed method (Case 1), 48.2% of users switched to cheaper links. As a result, the parking rate becomes evener, reducing the number of canceled cars by 81.5% and the number of stations with a parking rate of 100% or 0% at the end. Furthermore, even though the average usage fee fell, the total income rose because of the decrease in the number of canceled cars.

V. Conclusions

This study investigated the application of real-time pricing to solve the problem of uneven distribution of vehicles in a one-way car-sharing system. An algorithm was proposed that sets the usage fees according to the uneven distribution of vehicles, and a model was proposed that represents shifts in demand in terms of links between stations, taking

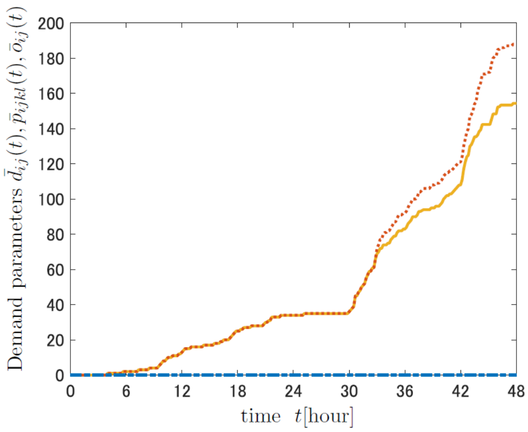

Fig. 8: Cumulative number of departing vehicles $\sum_{\tau=0}^{t}\sum_{ij}\bar{d}_{ij}(\tau)$ (solid line), shifted demand $\sum_{\tau=0}^{t}\sum_{ijkl}\bar{p}_{ijkl}(\tau)$ (dashed line), and original demand $\sum_{\tau=0}^{t}\sum_{ij}\bar{o}_{ij}(\tau)$ (dotted line) in Case 2

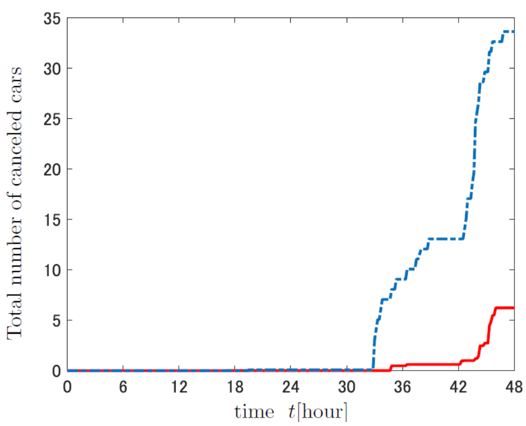

Fig. 9: Cumulative number of canceled cars $\sum_{\tau=0}^{t}\bar{m}(\tau)$ (Case 1 (solid line), Case 2 (dashed line))

into account constraints on the number of vehicles. Then, a simulation was conducted based on the actual data of the car-sharing system, Ha:mo RIDE Toyota by TOYOTA Corporation. The simulation results demonstrated that the proposed method eliminates the uneven distribution of vehicles under realistic scenarios. As future work, there is room for improvement in the estimates used for setting the demand and other parameters, and further study is required to demonstrate the validity of the method in real environments.

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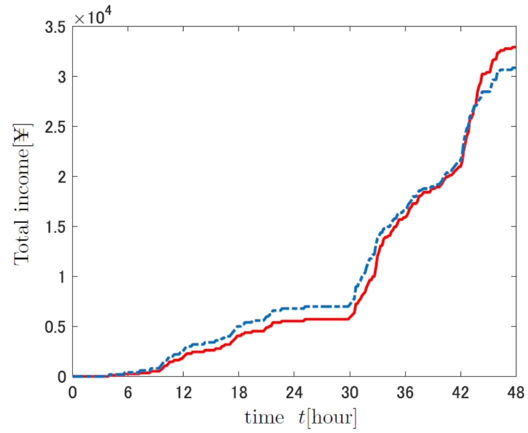

Fig. 10: Total income $\sum_{\tau=0}^{t}\sum_{ij}c_{ij}(\tau)\bar{d}_{ij}(\tau)$ (Case 1 (solid line), Case 2 (dashed line))

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