

Pricing Approach to Balance Demands for One-way Car-sharing Systems *

Lei Wang, *Member, IEEE* and Wanjiang Ma

Abstract— Carsharing is an alternative transportation mode for urban mobility. One-way carsharing model presents possible imbalance problem in fleet distribution and demands. Dynamic pricing approach can affect users' behavior to change the users' demands and the moving of vehicles in order to keep the system in balance. This paper presents a method to determine the pricing schemes. Firstly the paper reveals the mechanism of reaction of users who had received the variable pricing offers, and establishes a price-demand model. The price-demand model takes into account both the elasticity price-demand effect and the changing of departure and destination station of users. Subsequently, we formulate an optimization model to find out proper pricing schemes which can keep the station vehicle inventory at a proper range. Finally the paper presents a numerical example by adopting the pricing scheme method we put forward.

Keywords: urban mobility; car sharing; dynamic pricing; price-demand model; optimization.

I. INTRODUCTION

The rapid modern urbanization brings growing transportation demands, and challenges the future urban transportation towards humanized mobility. More and more innovative transportation modes have been emerging during the last decade despite the traditional public transportation still in service, especially with the booming development of mobile internet and shared economy. Carsharing is one of the alternative transportation modes which allows users get access to a fleet without owning a car and pay as they drive. This new concept of mobility provides better flexibility, costs fewer than owning a car, reduces car ownership, increases car utilization, and saves parking space [1].

One-way carsharing system is a business model which allows users to pick up vehicles from one station and return to another station. This model enables fleets moving in the carsharing networks, and causes possible imbalances of the fleet allocation against demands. The lacking of available vehicles in high demand area impacts user's experiences and decreases customer satisfaction. Meanwhile, the idle vehicles in low demand area limits the efficiency and profits from the service provider's side, and finally causes the operation into a negative loop.

To balance the fleets and the demands, one approach is to relocate the vehicles. The relocation tasks should be

conducted by the carsharing service providers. However, the capability of relocating labors is quite confined. Normally a team of relocation workers can move only one vehicle during one trip, while in bikesharing systems they can shift hundreds of bicycles using a truck at a time.

The immoderate growth of the carsharing business in recent years also makes the problem striking. For instance, EVCARD – an electric vehicle sharing system operated in China – has achieved a scale with almost 9,000 vehicles and 5,000 stations by the end of 2017. Traditional vehicle relocation approaches hardly worked in such a large scale system. This approach requires a large group of relocation workers and increases the operation costs. What's worse, the position of workers also needs to be relocated. The allocation of working area constrained by workers' living area makes the problem even more complex, especially facing with the randomness of demands.

A promising approach is to influence the balance from the demand side by applying dynamic pricing scheme. Pricing leverage and proper incentives may influence the demand pattern, and lead to a more balanced circumstance. From the perspective of carsharing service provider's side, users could also be regarded as participants of vehicle relocating, besides the relocation workers. In some studies this approach is also revealed as the user-based relocation. This approach breaks through the limit of relocation labors and becomes a possible solution for growing large scale carsharing systems. To the best of our knowledge, the dynamic pricing scheme towards one-way carsharing systems has not been adequately studied yet, basically from two sides. One is that the users' behavior towards the prices has not been practically calibrated, and the other is that the method to quantify exact solution of pricing scheme is still not clear.

This paper presents a dynamic pricing method which exports variable prices which influence users' behavior to maintain the system balance. Firstly the paper reveals the mechanism of reaction of users who had received the variable pricing offers. Based on the users' behavior, we deduces the demand functions according to the changing pricing schemes. Subsequently, we build up an optimization model to find out proper pricing schemes which can keep the station vehicle inventory at a proper range. Finally the paper presents a numerical example by adopting the method we put forward.

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L. Wang is with the College of Transportation Engineering, Tongji University and the Key Laboratory of Road and Traffic Engineering of the

Ministry of Education, Tongji University, 4800 Cao'an Road, Shanghai, P.R. China (e-mail: wangleicui@gmail.com).

W. Ma is with the Key Laboratory of Road and Traffic Engineering of the Ministry of Education, Tongji University, 4800 Cao'an Road, Shanghai, P.R. China (corresponding author to provide phone: +86-21-6958-4674; fax: +86-21-6958-9475; e-mail: mawanjiang@tongji.edu.cn).

II. LITERATURE REVIEW

A. Organization-based Vehicle Relocation

Several previous studies had suggested that the on-demand (also known as instant access) one-way carsharing model will lead the future carsharing development trend, because it does not require users to make reservation and allows users to drop off anywhere, which increases flexibility [2-4]. They also pointed out that in such system the fleet is possible to be imbalanced due to the randomness of the demand through time and space. Hence the service providers are responsible for relocating the vehicles to keep the system in sustainable balance. The relocation task conducted by the service providers refers to the organization-based vehicle relocation.

Many studies had been focused on the organization-based relocation. For the carsharing models which require reservation, the vehicle allocation scheme can be solved by deterministic mathematic planning [5-9]. For the on-demand model, demands are not previously known by the service providers, and attracted many researchers to study from different approaches. Kek [10] and Weikl [11] respectively proposed methods from the perspective of inventory control. Our previous study also provided an inventory control strategy to keep the number of vehicles in stations at a proper range, by the constraint of upper and lower thresholds [12]. To apply deterministic mathematical planning method, some involved rolling horizon to dynamically proceed mathematical planning to account for the variable demands [13, 14], and others adopted demand prediction as input [15, 16]. To avoid the influence of the randomness of demands, researchers also adopted stochastic programming [17-19].

Although many works had been conducted toward better organization-based relocation, some researchers also pointed out that the number of relocation workers is the bottleneck of the working coverage and involves more costs. Another problem is the position of the workers themselves will also be imbalanced, which needs to be relocated as well [20, 21]. This reflects that the organization-based relocation is limited and new approaches need to be figured out.

B. User-based Relocation and Pricing

User-based relocation is a promising approach to break through the limitation of relocation labors. The earliest user-based approach is put forward by Barth which is to merge and divide trips to control the moving number of vehicles [22]. Di Febraro provided a strategy to change the users' destination to avoid aggregating of vehicles [13]. Waserhole modeled this problem through price leverage theory [23]. Wagner also developed a user-based relocation platform which pays users as they participate in workers' tasks [24]. Although there have been related works about user-based relocation from different means and aspects, the clear methodology on controlling demand side has not been established yet.

Researchers who focused on user behavior had revealed that the users travel intention may influenced by price and other factors [25, 26]. These studies started from the scope of market penetration of carsharing, and implies that the users travel attitudes can be influenced by many factors include price. And the travel demands depends on the users attitudes. However, these studies were not aiming to calibrate a model

for determine proper prices for balancing the operation. Some paper modeled the pricing problem for carsharing systems from theoretical approach, and in this paper we will present user reflections towards incentives and reveal the price-demand pattern from an observation manner, which helps better understanding of pricing mechanism for carsharing demand control.

III. PRICE-DEMAND MODEL

A. Drawbacks of existing models

In this section, we propose a price-demand model to reflect the dynamics of demands when influenced by changeable prices. A basic theoretical price-demand model was introduced in Waserhole's work [23], which is a S-type function with a high demand limitation with low price while nearly zero demand with high price. Such elasticity of demands can be observed in many market systems. Yet, the elasticity function had not been specified and calibrated in this work. Moreover, the price-demand mechanism should not be simplified as an elasticity model, which is based on an assumption that the users will only take or quit the carsharing mode when the price is changing. Intuitively, the users may not give up their trips or shift to other modes if the price was going up at nearby station, but may change and walk or cycle to another station with normal or lower price offer. This demands change of related stations cannot be revealed by only elasticity demand model.

To model the users' behavior influenced by price, Di Febraro provided a method based on discrete choice model [13], which reflect the reaction of user who will accept or reject the ideal destination suggestion provided by the carsharing service provider, under the factors of walking distance and discount. However, this study limited to destination suggestion. A similar mechanism is possible to apply to the departure stations: users will be or not be attracted to a nearby station with discounts.

In our study, a combined price-demand model is proposed to depict both the elastic demand, the departure station shift and the destination change.

B. Pricing policy in the instance of EVCARD

Pricing policy is quite a complicated problem considering different means of floating price rate, rewards, discounts, and extra charges, etc., which will not be discussed in this paper. We derived our method under the existing policy of the instance of EVCARD.

Under the current operation policy of EVCARD, there have been four pricing tactics in operation regarding to different stations.

- Pick-up rewards. User will receive extra rewards if pick up a car at a station where applies this tactic.
- Pick-up charges. User will receive extra charges if pick up a car at a station where applies this tactic.
- Drop-off rewards. User will receive extra rewards if drop off a car at a station where applies this tactic.
- Drop-off charges. User will receive extra charges if drop off a car at a station where applies this tactic.

C. Influences of Pricing Policy

The four pricing tactics can affect a user's behavior during the user's trip (as shown in Fig. 1). This can results users' behavior and demands in the following three stages:

- Demands inducing and reducing. Before a trip was started, i.e., during the mode deciding stage, the pick-up rewards at a station may generate travel demand or attract traveler from other modes. On the other hand, the pick-up charges at a station may reduce travel demand or push the user to other modes.
- Departure station change. After deciding taking carsharing mode, the user would consider the departure station around the user's walking range. If a station applies pick-up rewards, it will attract users. If a station applies pick-up charges, it will repulse users to other stations nearby.
- Arrival station change. Before arriving at the destination, the user would consider where to drop off the car. If a station applies drop-off rewards, it will attract users to go to. If a station applies drop-off charges, it will push the users to other stations nearby to drop off.

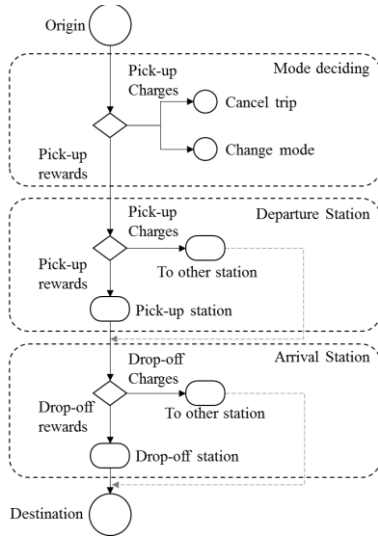


Figure 1. Flow chart of user's trip affected by pricing tactics.

D. Formulae of Price-Demand Model

The demands inducing and reducing can be reflect by a typical S curve as which had been drawn in Waserhole's work [23]. To depict the demands inducing and reducing figure, we implement the S curve by employing the reverse Logistic function.

$$D(D_s^0, P) = \frac{2D_s^0}{1 + \exp(\alpha_s \cdot P)} \quad (1.)$$

Where D denotes the pick-up demand of station s , $\forall s \in S$, and S refers to a set containing all stations. $2D_s^0$ confines the maximum value of variable demands. D_s^0 is the demand that the reward and charge is zero. P denotes the price tactic of station s . If the pick-up rewards tactic applies, P is negative; if the pick-up charges tactic applies, P is positive. There is an parameter α_s to be calibrated, which decides the bending shape of the curve, and depends on station s .

EVCARD had applied rewards and charges tactics to some of experimental stations to examine the effects of pricing influence. The system released rewards of RMB 20, and charges of RMB 5, 10, 20, and 30. Without loss of generality, we divide the demand values of each station by two times of its mean value of demands, and do not consider α_s changing by different station s . Hence we can plot all points from different stations into one figure, as shown in Fig. 2. A general value of α_s was calibrated as 0.166.

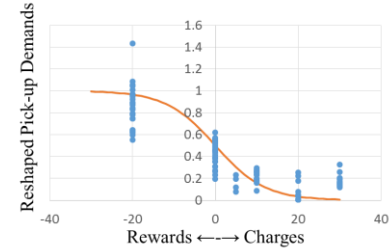


Figure 2. Demands inducing and reducing with prices.

Departure station changes of users result in the change of pick-up demands at every stations. From the perspective of pick-up demand at one specific station, there are two kinds of change of demands: 1) station attracting pick-up demands, and 2) station distracting pick-up demands.

If the pick-up rewards tactic applies to the station, the station attracts pick-up demands from other stations. The probability of changing departure station of a user is

$$p_D^+(P, d) = \frac{1}{1 + \exp(\beta_{D0}^+ + \beta_{D1}^+ P + \beta_{D2}^+ d)}, P < 0 \quad (2.)$$

where P is the price the same as aforementioned. d denotes the distance from any other station to the station.

If the pick-up charges tactic applies to the station, the station distracts pick-up demands to other stations. The probability of changing departure station of a user is

$$p_D^-(P, d) = \frac{1}{1 + \exp(\beta_{D0}^- + \beta_{D1}^- P + \beta_{D2}^- d)}, P > 0 \quad (3.)$$

Arrival station changes of users result in the change of drop-offs at every stations. Similarly, from the perspective of one specific station, there are two kinds of change of drop-offs: 1) station attracting drop-offs, and 2) station repulsing drop-offs.

If the drop-off rewards tactic applies to the station, the station attracts users to drop off. The probability of changing arrival station of a user is

$$p_R^+(P, d) = \frac{1}{1 + \exp(\beta_{R0}^+ + \beta_{R1}^+ P + \beta_{R2}^+ d)}, P < 0 \quad (4.)$$

If the drop-off rewards tactic applies to the station, the station pushes users to other stations to drop off. The probability of changing arrival station of a user is

$$p_R^-(P, d) = \frac{1}{1 + \exp(\beta_{R0}^- + \beta_{R1}^- P + \beta_{R2}^- d)}, P > 0 \quad (5.)$$

In formula (2) to (5), parameters named β can be separately calibrated. Still we calibrated them by using the experimental data of EVCARD, and the values are listed in Table 1.

TABLE I. VALUES OF PARAMETERS

a. $P_D^+(p, d)$					
	β	Std.	Wals.	df	Sig.
β_{D0}^+	4.675	2.134	4.799	1	0.028
β_{D1}^+	0.101	.021	23.132	1	0.000
β_{D2}^+	0.132	.045	8.604	1	0.003
b. $P_D^-(p, d)$					
	β	Std.	Wals.	df	Sig.
β_{D0}^-	6.988	2.388	8.563	1	0.003
β_{D1}^-	-0.165	.065	6.444	1	0.011
β_{D2}^-	0.127	.022	33.324	1	0.000
c. $P_R^+(p, d)$					
	β	Std.	Wals.	df	Sig.
β_{R0}^+	4.427	3.628	1.489	1	0.222
β_{R1}^+	0.081	.032	6.407	1	0.011
β_{R2}^+	0.179	.078	5.266	1	0.022
d. $P_R^-(p, d)$					
	β	Std.	Wals.	df	Sig.
β_{R0}^-	8.477	4.362	3.777	1	0.052
β_{R1}^-	-0.162	.026	38.822	1	0.000
β_{R2}^-	0.246	.098	6.301	1	0.012

Based on formulae (1) to (5), here we deduce the pick-up demands and drop-off demands of each station.

The pick-up demand of station $\forall s \in S$

$$D_s = \begin{cases} D(D_s^0, P_{Ds}) + \sum_{s' \in (S-s)} D_{s'}^0 \cdot p_D^+(P_{Ds}, d_{ss'}) & P_{Ds} < 0 \\ D(D_s^0, P_{Ds}) - \sum_{s' \in (S-s)} D_{s'}^0 \cdot p_D^-(P_{Ds}, d_{ss'}) & P_{Ds} > 0 \\ D_s^0 & P_{Ds} = 0 \end{cases} \quad (6.)$$

where, $\forall s' \in (S-s)$

$$D_{s'} = \begin{cases} D(D_{s'}^0, P_{Ds'}) - D_{s'}^0 \cdot p_D^+(P_{Ds}, d_{ss'}) & P_{Ds} < 0 \\ D(D_{s'}^0, P_{Ds'}) + D_{s'}^0 \cdot p_D^-(P_{Ds}, d_{ss'}) & P_{Ds} > 0 \\ D_{s'}^0 & P_{Ds} = 0 \end{cases} \quad (7.)$$

And the drop-off demand of station $\forall s \in S$

$$R_s = \begin{cases} R_s^0 + \sum_{s' \in (S-s)} R_{s'}^0 \cdot p_R^+(P_{Rs}, d_{ss'}) & P_{Rs} < 0 \\ R_s^0 - \sum_{s' \in (S-s)} R_{s'}^0 \cdot p_R^-(P_{Rs}, d_{ss'}) & P_{Rs} > 0 \\ R_s^0 & P_{Rs} = 0 \end{cases} \quad (8.)$$

where, $\forall s' \in (S-s)$

$$R_{s'} = \begin{cases} R_{s'}^0 - R_{s'}^0 \cdot p_R^+(P_{Rs}, d_{ss'}) & P_{Rs} < 0 \\ R_{s'}^0 + R_{s'}^0 \cdot p_R^-(P_{Rs}, d_{ss'}) & P_{Rs} > 0 \\ R_{s'}^0 & P_{Rs} = 0 \end{cases} \quad (9.)$$

And R_s^0 refers to the original drop-off demand at the station. In formulae (6) to (9), $d_{ss'}$ is the walking distance from station s to station s' , and is regarded as known values. The independent variables are P_{Ds} and P_{Rs} , which determine the demands D_s and $R_s \forall s \in S$.

IV. PRICING SCHEME OPTIMIZATION

This section presents a practicable method to calculate the pricing scheme under the pricing tactics of EVCARD. The aim of the optimization is to keep the vehicle inventory of each station within a reasonable range. The method is based on the price-demand model aforementioned. The optimization model returns a set of pricing tactics of each

station which should apply pricing tactics to keep vehicle inventory and demands in balance.

The objective is to keep the number of vehicles at each station in a reasonable range. In the authors' previous study, a practical method to determine the suitable range of vehicle inventory had been provided [12]. We calculated an upper threshold and a lower threshold for a station to confine a suitable range of vehicle storage. If the number of vehicles exceed the upper threshold, the station is under the risk of overflowing, which needs more pick-up demands and pushing drop-off demands to nearby other stations. On the contrary, if the number of vehicles is below the lower threshold, the station is under the risk of vehicle insufficiency, which needs to reduce pick-up demands and attract drop-off demands.

Carsharing system usually presents complex system dynamics. The properties of one-way availability and electric fleet make the system modeling more complicated. Many studies regarding to fleet relocation problem were based on time step transition or time space networks. In our study, since the work is based on the determined upper and lower thresholds which involves the factors of different time steps, the desired vehicle inventory and pricing scheme can be calculated in one time step independently. Under that condition, we give the objective function as follows, which implies the minimum difference of the desired vehicle inventory after applying pricing tactics and the two thresholds:

$$\min \sum_{s \in S} (x_s - S_{upper,s})^2 + (S_{lower,s} - x_s)^2 \quad (10.)$$

Where x_s is the desired vehicle inventory at station s , and $S_{upper,s}$ and $S_{lower,s}$ denote the upper and lower thresholds of station s respectively. In the following part we will give the function of x_s in detail.

And x_s contains the decision variables P_{Ds} and P_{Rs} , which means the pick-up pricing and drop-off pricing respectively. The pricing is capable to mark rewards or charges. Negative indicates rewards and positive value indicates charges.

To deduce x_s , we start from

$$x_s^1 = x_s^0 + R_s^0 - D_s^0, \forall s \in S \quad (11.)$$

Where x_s^1 is the untreated number of vehicles of station s at the end of the studying time step. x_s^0 is the number of vehicles of station s at the beginning of the time step, and can be detected from the operating system. R_s^0 is the number of vehicles that will return to station s during the time step. D_s^0 is the pick-up demands at station s .

We introduce two parameters to indicate whether the number of vehicles will be out of thresholds:

$$\lambda_s^u = \begin{cases} 1, x_s^1 - S_{upper,s} \geq 0 \\ 0, x_s^1 - S_{upper,s} < 0 \end{cases}, \forall s \in S \quad (12.)$$

$$\lambda_s^l = \begin{cases} 1, S_{lower,s} - x_s^1 \geq 0 \\ 0, S_{lower,s} - x_s^1 < 0 \end{cases}, \forall s \in S \quad (13.)$$

$$P_{Ds} = (\lambda_s^u + \lambda_s^l - \lambda_s^u \lambda_s^l) P_{Ds}, \forall s \in S \quad (14.)$$

$$P_{Rs} = (\lambda_s^u + \lambda_s^l - \lambda_s^u \lambda_s^l) P_{Rs}, \forall s \in S \quad (15.)$$

Formula (12) marks the states of exceeding upper threshold of station s ; formula (13) marks the states of being under lower threshold of station s . Formulae (14) and (15) keep the pricing tactics at zero if vehicle number is within the range of station s .

Consequently, the function of x_s should be:

$$x_s = x_s^0 + D_s(D_s^0, P_{D_s}) - R_s(R_s^0, P_{R_s}), \forall s \in S \quad (16.)$$

Where D_s and R_s refer to formulae (6) and (8).

It should be noted that x_s^0 and x_s should be integer, since the number of vehicles should only be decimals. This also makes D_s and R_s to be integers.

This quick model is an abnormal problem due to the presence of exponential, quadratic, piecewise functions and even integers. Typical optimization solver would not suitable for this problem. Here we employed heuristic approach of genetic algorithm to fast search the feasible solutions. This paper does not place emphasis on linearization as well as solution algorithm. In our practice, the matlab R2015b toolbox of optimization was adopted.

V. NUMERICAL EXAMPLE

In this section we build up a small scale numerical example to illustrate the pricing method under the framework of EVCARD practice. Fig. 3 shows the network and parameters of this numerical example which consists of five stations. Stations are presented as circles filled with blue color and marked in white numbers. The station set $S = \{1, 2, 3, 4, 5\}$.

All the parameters in this example describe the properties of the system in a unit time, or a time step. The number x in the figure is the current number of vehicles in each station. The number D is the number of pick-up demands during this time step. The number R is the number of drop-off demands during this time step. Arcs between each circles represent the walking distance between two stations. Not all the circles are directly connected for drawing the graph, and the distance between stations without direct connection is set as the distance of the shortest path. The red arrow and green arrow represents the upper threshold and lower threshold of each station respectively.

In this numerical example, parameters in the price-demand function applies the calibrated functions the same as in section III.D.

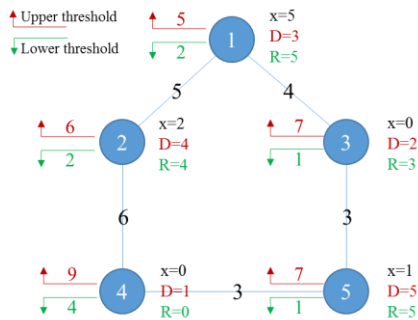


Figure 3. Parameters of the numerical example.

If no pricing strategy applied, the number of vehicles at each station should be $x_s^1 = 7, 2, 1, -1, 1$. It is obvious that the number of vehicles in station 1 will exceed the upper threshold. The number of vehicles in station 4 will be -1, which indicates that the demand cannot be satisfied in station 4.

As we have mentioned in Fig. 2, the prices are configured as multiple of 5, we search for the solutions in multiples of 5. The result of this example is $P_{D1} = -40$, $P_{R1} = 40$, and $P_{D4} = 20$, $P_{R4} = -50$. This result shows that the station 1 should apply the pick-up rewards of RMB 40, and drop-off charges of RMB 40; the station 4 should apply the pick-up charges of RMB 20, and drop-off rewards of RMB 50.

After applying pricing tactics, the number at every stations will be $x_s = 2, 2, 1, 3, 0$. At station 4 and station 5, the vehicle inventory is one less than the lower threshold respectively. Other stations in the system satisfy the inventory control range between upper and lower thresholds. However, the system satisfies two more pick-up demands than the situation without pricing tactics. Pick-up demand at station 4 will not be satisfied if no vehicle moves to it. By applying the pricing scheme, demand at station 4 can be satisfied and the lower threshold of car inventory can be met. A detailed number of demands and number of vehicles are shown in Table 2.

TABLE II. CHANGE OF VARIABLES

	s1	s2	s3	s4	s5
x_s^0	5	2	0	0	1
D_s^0	3	4	2	1	5
R_s^0	5	4	3	0	5
x_s^1	7	2	1	-1	1
D_s	7	3	2	1	4
R_s	4	3	3	4	3
x_s	2	2	1	3	0

Additionally, the revenues with pricing schemes and without pricing schemes can be calculated. To calculate the trips which make revenues, we consider all trips involved in the system including pick-ups and drop-offs. Table III shows the satisfied demands, drop-offs, and revenues considering pricing tactics. We set RMB 30 as the average revenue per trip.

TABLE III. CHANGE OF VARIABLES

		s1	s2	s3	s4	s5	sum
without pricing	satisfied demands	3	4	2	0	5	14
	drop-offs	5	4	3	0	5	17
	relocate-out	2					2
	relocate-in				4		4
	pick-up revenue	90	120	60	0	150	630
	drop-off revenue	50	120	90	-200	150	
with pricing	satisfied demands	7	3	2	1	4	17
	drop-offs	4	3	3	4	3	17
	ds	7	3	2	1	4	17
	rs	4	3	3	4	3	17
	pick-up revenue	-70	90	60	50	120	720
	drop-off revenue	280	90	90	-80	90	

To keep the vehicle inventory within the threshold, vehicle relocation is needed. Table III gives the number of relocations at station 1 and 4. Relocating one time will cost RMB 50, and will be involved in revenue calculation. The result shows that the total revenue of pricing approach (720) is better than the relocation approach (630).

VI. CONCLUSIONS

This paper presented a pricing approach to deal with the fleet unbalance problem. The pricing approach is based on the price-demand model which depicts the relationship between price and demand. Being aware of the fact that users may change their departure station and destination station as well as they may take or abandon carsharing mode under different prices, we reconstructed a combined price-demand model beyond traditional elastic pricing model and discrete choice model. Based on the relationship of price and demand, we established an optimization model and solved the problem on an example. In the example, the pricing scheme can keep the vehicle inventory in a proper range, and lead to better revenue.

However, further works still remain. The randomness of users' behavior and price-demand relationship needs further analysis. The optimization model here is an unusual model with complicated forms, which is possible to be reformed in more concise and efficient way. Future modeling may also find other objectives such as maximizing the revenue which will help carsharing providers improve their profits.

REFERENCES

- [1] M. Barth and S. Shaheen, "Shared-Use Vehicle Systems: Framework for Classifying Carsharing, Station Cars, and Combined Approaches," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1791, pp. 105-112, 2002.
- [2] G. H. D. A. Correia, D. R. Jorge, and D. M. Antunes, "The Added Value of Accounting For Users' Flexibility and Information on the Potential of a Station-Based One-Way Car-Sharing System: An Application in Lisbon, Portugal," *Journal of Intelligent Transportation Systems*, vol. 18, pp. 299-308, 2014.
- [3] G. Alfian, J. Rhee, Y.-S. Kang, and B. Yoon, "Performance Comparison of Reservation Based and Instant Access One-Way Car Sharing Service through Discrete Event Simulation," *Sustainability*, vol. 7, pp. 12465-12489, 2015.
- [4] S. Shaheen, N. Chan, A. Bansal, and A. Cohen, "Shared Mobility: A Sustainability & Technologies Workshop: Definitions, Industry Developments, and Early Understanding," University of California, Berkeley, Transportation Sustainability Research Center; California Department of Transportation 2015.
- [5] J. Lee and G.-L. Park, "Design of a Team-Based Relocation Scheme in Electric Vehicle Sharing Systems," in *Computational Science and Its Applications – ICCSA 2013: 13th International Conference, Ho Chi Minh City, Vietnam, June 24-27, 2013, Proceedings, Part III*, B. Murgante, S. Misra, M. Carlini, C. M. Torre, H.-Q. Nguyen, D. Taniar, et al., Eds., ed Berlin, Heidelberg: Springer Berlin Heidelberg, 2013, pp. 368-377.
- [6] D. Jorge, G. H. A. Correia, and C. Barnhart, "Comparing Optimal Relocation Operations With Simulated Relocation Policies in One-Way Carsharing Systems," *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, pp. 1667-1675, 2014.
- [7] M. Nourinejad and M. J. Roorda, "A dynamic carsharing decision support system," *Transportation Research Part E: Logistics and Transportation Review*, vol. 66, pp. 36-50, 2014.
- [8] B. Boyaci, K. G. Zografos, and N. Geroliminis, "An optimization framework for the development of efficient one-way car-sharing systems," *European Journal of Operational Research*, vol. 240, pp. 718-733, 2015.
- [9] A. Carlier, A. Munier-Kordon, and W. Klaudel, "Mathematical Model for the Study of Relocation Strategies in One-way Carsharing Systems," *Transportation Research Procedia*, vol. 10, pp. 374-383, 2015.
- [10] A. G. H. Kek, R. L. Cheu, Q. Meng, and C. H. Fung, "A decision support system for vehicle relocation operations in carsharing systems," *Transportation Research Part E: Logistics and Transportation Review*, vol. 45, pp. 149-158, 2009.
- [11] S. Weikl and K. Bogenberger, "Relocation Strategies and Algorithms for Free-Floating Car Sharing Systems," *IEEE Intelligent Transportation Systems Magazine*, vol. 5, pp. 100-111, 2013.
- [12] G. Cao, L. Wang, Y. Jin, J. Yu, W. Ma, Q. Liu, et al., "Determination of the Vehicle Relocation Triggering Threshold in Electric Car-Sharing System," 2016.
- [13] A. Di Febraro, N. Sacco, and M. Saeednia, "One-way carsharing: solving the relocation problem," *Transportation Research Record: Journal of the Transportation Research Board*, pp. 113-120, 2012.
- [14] G. Santos and G. Correia, "A MIP Model to Optimize Real Time Maintenance and Relocation Operations in One-way Carsharing Systems," *Transportation Research Procedia*, vol. 10, pp. 384-392, 2015.
- [15] M. Bruglieri, A. Colomi, and A. Luè, "The relocation problem for the one-way electric vehicle sharing," *Networks*, vol. 64, pp. 292-305, 2014.
- [16] M. Repoux, B. Boyaci, and N. Geroliminis, "Simulation and optimization of one-way car-sharing systems with variant relocation policies," in *Transportation Research Board*, 2015, p. 19p.
- [17] P. Briest and C. Raupach, "The car sharing problem," presented at the Proceedings of the twenty-third annual ACM symposium on Parallelism in algorithms and architectures, San Jose, California, USA, 2011.
- [18] R. Nair and E. Miller-Hooks, "Fleet Management for Vehicle Sharing Operations," *Transportation Science*, vol. 45, pp. 524-540, 2011.
- [19] W. Fan, "Management of Dynamic Vehicle Allocation for Carsharing Systems: Stochastic Programming Approach," *Transportation Research Record: Journal of the Transportation Research Board*, pp. 51-58, 2013.
- [20] H.-J. Kim, G.-L. Park, and J. Lee, "Incorporative Relocation Team Planning and Staff Member Allocation in Electric Vehicle Sharing Systems," *International Journal of Multimedia and Ubiquitous Engineering (ISSN: 1975-0080)*, vol. 9, pp. 249-256, 2014.
- [21] M. Nourinejad, S. Zhu, S. Bahrami, and M. J. Roorda, "Vehicle relocation and staff rebalancing in one-way carsharing systems," *Transportation Research Part E: Logistics and Transportation Review*, vol. 81, pp. 98-113, 2015.
- [22] M. Barth, M. Todd, and L. Xue, "User-based vehicle relocation techniques for multiple-station shared-use vehicle systems," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1887, pp. 137-144, 2004.
- [23] A. Waserhole and V. Jost, "Pricing in vehicle sharing systems: optimization in queuing networks with product forms," *EURO Journal on Transportation and Logistics*, vol. 5, pp. 293-320, 2014.
- [24] S. Wagner, C. Willing, T. Brandt, and D. Neumann, "Data Analytics for Location-Based Services: Enabling User-Based Relocation of Carsharing Vehicles," in *International Conference on Information Systems*, 2015.
- [25] S. Herrmann, F. Schulte, and S. Voß, "Increasing Acceptance of Free-Floating Car Sharing Systems Using Smart Relocation Strategies: A Survey Based Study of car2go Hamburg," in *Computational Logistics: 5th International Conference, ICCL 2014, Valparaiso, Chile, September 24-26, 2014. Proceedings*, R. G. González-Ramírez, F. Schulte, S. Voß, and J. A. Ceroni Díaz, Eds., ed Cham: Springer International Publishing, 2014, pp. 151-162.
- [26] S. M. Zoepf and D. R. Keith, "User decision-making and technology choices in the U.S. carsharing market," *Transport Policy*, vol. 51, pp. 150-157, 2016.