Solutions to Quiz (31st January 2025)

Question 1

Compare $f(n) = n^{1/2}$ and $g(n) = n/\log(n+1)$ using o.

Solution. We note that $g(n)$ is non-zero for $n > 2$, so we can write

$$\frac{f(n)}{g(n)} = \frac{\log(n+1)}{n^{1/2}}$$

Now $\log(n+1) \sim \log(n)$ and $\log(n)$ is $o(n^c)$ for *every* positive real number c. Thus, this limit goes to 0 as n goes to ∞.

Thus, $f(n)$ is $o(g(n))$

Question 2

Which (if any) of the functions $f(n) = 5^{n-5}$ and $g(n) = 3^{n+5}$ is $O(3^n)$?

Solution. We note that $f(n) = 5^{-5} \cdot 5^n$ and 3^n is $o(5^n)$. Thus, $f(n)$ is *not* $O(3^n)$.

We note that $g(n) = 3^5 \cdot 3^n$ so $g(n)$ is $O(3^n)$.

Question 3

Given that $f(n) - n$ is $o(n)$. For a positive real number c give an inequality between c and k that decides whether $f(n)^k$ is $o(n^c)$ or $O(n^c)$ or neither.

Solution. We note that $f(n) \sim n$, which means that $f(n)^k \sim n^k$ for all positive k.

Thus, $f(n)^k$ is $O(n^c)$ for $c \geq k$.

In fact $f(n)^k$ is $o(n^c)$ for $c > k$.

Finally, for $c < k$, we see that n^c is $o(f(n)^k)$. Thus $f(n)^k$ is *not* $O(n^c)$ for $c < k$.