ASSIGNMENT-2

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Download all python codes from

https://github.com/teja3657/Assignment−2/tree/master/CODES

and latex-tikz codes from

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1 Question No 2.9

Given the linear equation $\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

1) intersecting lines
2) coincident lines
3) parallel lines

1)
$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8 \\ \begin{pmatrix} 3 & 2 \end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

2)
$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8 \\ \begin{pmatrix} 4 & 6 \end{pmatrix}\mathbf{x} = 16 \tag{1.0.2}$$

3)
$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8 \\ \begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 4 \tag{1.0.3}$$

2 Solution

1)
$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8 \\ \begin{pmatrix} 3 & 2 \end{pmatrix}\mathbf{x} = 4 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & 3 & 8 \\ 3 & 2 & 4 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 3 & 2 & 4 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 0 & \frac{-5}{2} & -8 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{2R_2}{-5}} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 0 & 1 & \frac{16}{5} \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{3R_2}{2}} \begin{pmatrix} 1 & 0 & \frac{-4}{5} \\ 0 & 1 & \frac{16}{5} \end{pmatrix} \tag{2.0.6}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 2 & 3 & 8 \\ 3 & 2 & 4 \end{pmatrix} \tag{2.0.7}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \tag{2.0.8}$$

is also 2.

$$\begin{aligned} \because Rank\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} &= Rank\begin{pmatrix} 2 & 3 & 8 \\ 3 & 2 & 4 \end{pmatrix} \\ &= dim\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \\ &= 2 \end{aligned} \tag{2.0.9}$$

$\therefore$ Given lines (1.0.1) have unique solution so we can say they intersect.

2)
$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8 \\ \begin{pmatrix} 4 & 6 \end{pmatrix}\mathbf{x} = 16 \tag{2.0.10}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 8 \\ 16 \end{pmatrix} \tag{2.0.11}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}2 & 3 & 8\\4 & 6 & 16\end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix}1 & \frac{3}{2} & 4\\4 & 6 & 16\end{pmatrix} \tag{2.0.12}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 4R_1} \begin{pmatrix}1 & \frac{3}{2} & 4\\0 & 0 & 0\end{pmatrix} \tag{2.0.13}$$

$$\tag{2.0.14}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}2 & 3 & 8\\4 & 6 & 16\end{pmatrix} \tag{2.0.15}$$

results in a matrix with 1 nonzero rows, its rank is 1. Similarly, the rank of the matrix

$$\begin{pmatrix}2 & 3\\4 & 6\end{pmatrix} \tag{2.0.16}$$

is also 1.

$$\because Rank\begin{pmatrix}2 & 3\\4 & 6\end{pmatrix} = Rank\begin{pmatrix}2 & 3 & 8\\4 & 6 & 16\end{pmatrix} = 1$$

$$= dim\begin{pmatrix}2 & 3\\4 & 6\end{pmatrix} = 1 \tag{2.0.17}$$

$\therefore$ Given lines (1.0.2) have infinitely many solutions so we can say they coincide.

3)

$$\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x} = 8$$
$$\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x} = 4 \tag{2.0.18}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}2 & 3\\2 & 3\end{pmatrix}\mathbf{x} = \begin{pmatrix}8\\4\end{pmatrix} \tag{2.0.19}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}2 & 3 & 8\\2 & 3 & 4\end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix}1 & \frac{3}{2} & 4\\2 & 3 & 4\end{pmatrix} \tag{2.0.20}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix}1 & \frac{3}{2} & 4\\0 & 0 & -4\end{pmatrix} \tag{2.0.21}$$

$$\tag{2.0.22}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}2 & 3 & 8\\2 & 3 & 4\end{pmatrix} \tag{2.0.23}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}2 & 3\\2 & 3\end{pmatrix} \tag{2.0.24}$$

is 1.

$$\because Rank\begin{pmatrix}2 & 3\\2 & 3\end{pmatrix} \neq Rank\begin{pmatrix}2 & 3 & 8\\2 & 3 & 4\end{pmatrix} \tag{2.0.25}$$

$\therefore$ Given lines (1.0.3) have no solution so we can say they are parallel. PLOT OF GIVEN LINES -

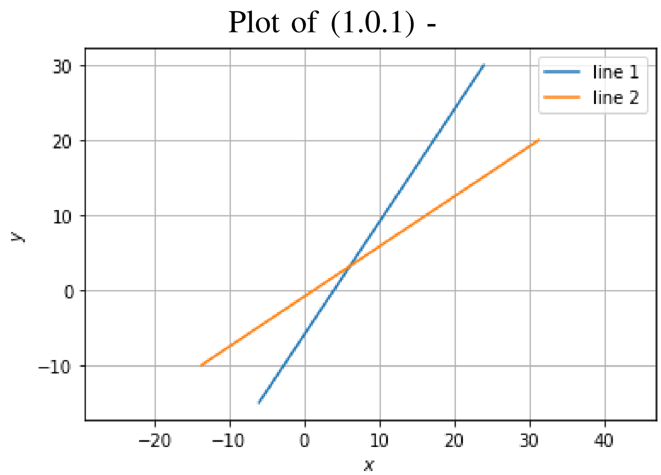

Fig. 2.1: INTERSECTING LINES.

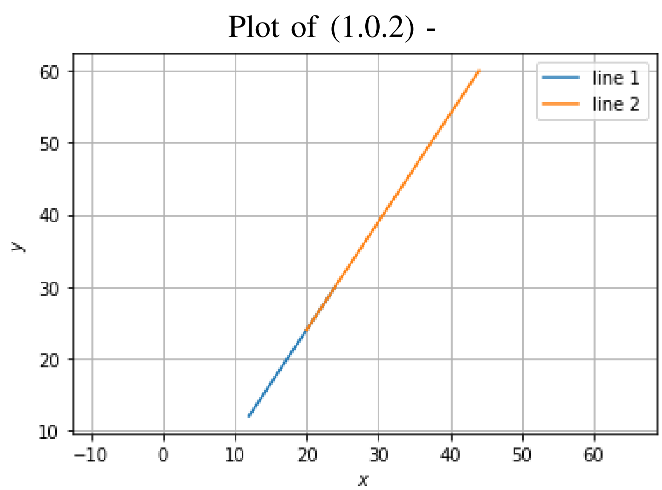

Fig. 2.2: COINCIDENT LINES

Plot of (1.0.3)

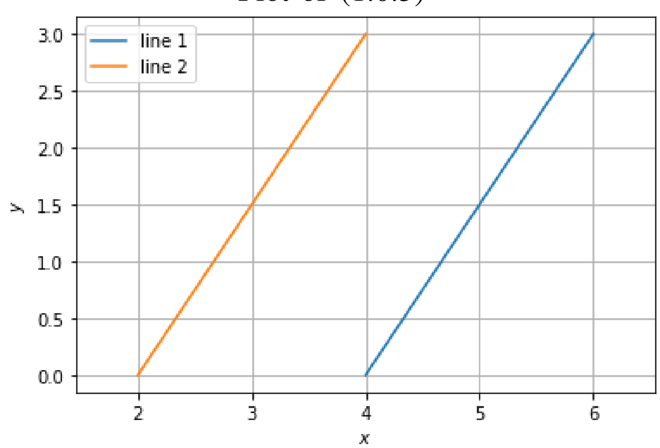

Fig. 2.3: PARALLEL LINES