ASSIGNMENT-2

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1 QUESTION No 2.9

Given the linear equation $\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

1) Intersecting lines   3) Parallel lines
2) Coincident lines

1)

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8$$
$$\begin{pmatrix} 3 & 2 \end{pmatrix}\mathbf{x} = 4 \qquad (1.0.1)$$

2)

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8$$
$$\begin{pmatrix} 4 & 6 \end{pmatrix}\mathbf{x} = 16 \qquad (1.0.2)$$

3)

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8$$
$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 4 \qquad (1.0.3)$$

2 SOLUTION

1)

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8$$
$$\begin{pmatrix} 3 & 2 \end{pmatrix}\mathbf{x} = 4 \qquad (2.0.1)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \qquad (2.0.2)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & 3 & 8 \\ 3 & 2 & 4 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 3 & 2 & 4 \end{pmatrix} \qquad (2.0.3)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 0 & \frac{-5}{2} & -8 \end{pmatrix} \qquad (2.0.4)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{2R_2}{-5}} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 0 & 1 & \frac{16}{5} \end{pmatrix} \qquad (2.0.5)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{3R_2}{2}} \begin{pmatrix} 1 & 0 & \frac{-4}{5} \\ 0 & 1 & \frac{16}{5} \end{pmatrix} \qquad (2.0.6)$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 2 & 3 & 8 \\ 3 & 2 & 4 \end{pmatrix} \qquad (2.0.7)$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \qquad (2.0.8)$$

is 2.

$$\because Rank\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} = Rank\begin{pmatrix} 2 & 3 & 8 \\ 3 & 2 & 4 \end{pmatrix}$$
$$= dim\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$$
$$= 2 \qquad (2.0.9)$$

$\therefore$ Given lines (1.0.1) have unique solution so we can say they intersect.

2)

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 8$$
$$\begin{pmatrix} 4 & 6 \end{pmatrix}\mathbf{x} = 16 \qquad (2.0.10)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 8 \\ 16 \end{pmatrix} \qquad (2.0.11)$$

The augmented matrix for the above equation

is row reduced as follows

$$\begin{pmatrix} 2 & 3 & 8 \\ 4 & 6 & 16 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 4 & 6 & 16 \end{pmatrix} \quad (2.0.12)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 4R_1} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.13)$$

$$(2.0.14)$$

∵ row reduction of the 2×3 matrix

$$\begin{pmatrix} 2 & 3 & 8 \\ 4 & 6 & 16 \end{pmatrix} \quad (2.0.15)$$

results in a matrix with 1 nonzero rows, its rank is 1. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} \quad (2.0.16)$$

is also 1.

$$\because Rank \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} = Rank \begin{pmatrix} 2 & 3 & 8 \\ 4 & 6 & 16 \end{pmatrix} = 1$$

$$= dim \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} = 1 \quad (2.0.17)$$

∴ Given lines (1.0.2) have infinitely many solutions so we can say they coincide.

3)

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} = 8$$
$$\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} = 4 \quad (2.0.18)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \quad (2.0.19)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & 3 & 8 \\ 2 & 3 & 4 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 2 & 3 & 4 \end{pmatrix} \quad (2.0.20)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & \frac{3}{2} & 4 \\ 0 & 0 & -4 \end{pmatrix} \quad (2.0.21)$$

$$(2.0.22)$$

∵ row reduction of the 2×3 matrix

$$\begin{pmatrix} 2 & 3 & 8 \\ 2 & 3 & 4 \end{pmatrix} \quad (2.0.23)$$

results in a matrix with 2 nonzero rows, its rank

is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix} \quad (2.0.24)$$

is 1.

$$\because Rank \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix} \neq Rank \begin{pmatrix} 2 & 3 & 8 \\ 2 & 3 & 4 \end{pmatrix} \quad (2.0.25)$$

∴ Given lines (1.0.3) have no solution so we can say they are parallel. PLOT OF GIVEN LINES -
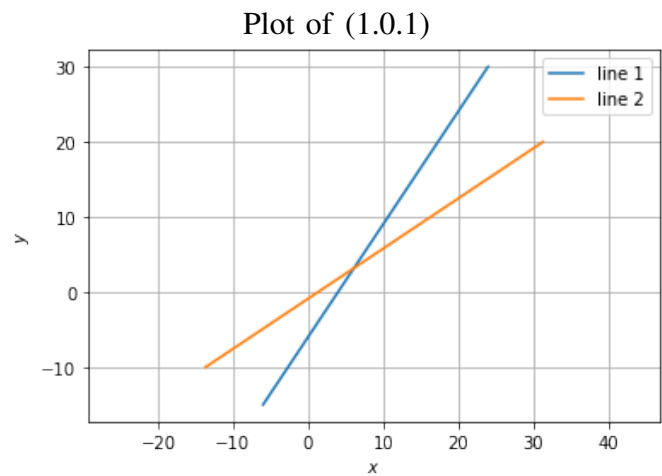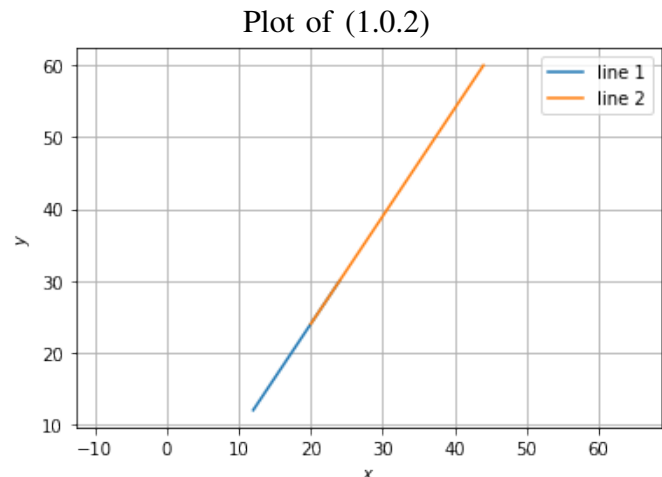

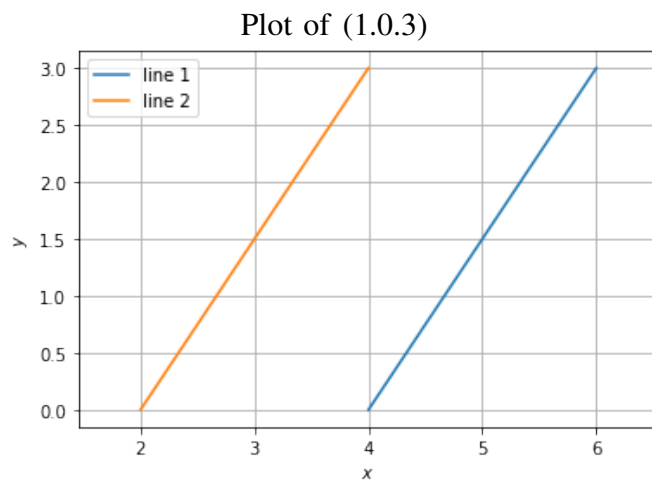Fig. 2.1: INTERSECTING LINES.

Fig. 2.2: COINCIDENT LINES

Fig. 2.3: PARALLEL LINES