ASSIGNMENT 4

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1 QUESTION No 2.39

Find the equation of the plane through the intersection of the planes $\begin{pmatrix} 3 & -1 & 2 \end{pmatrix}\mathbf{x} = 4$ and $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\mathbf{x} = -2$ and the point $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$.

2 SOLUTION

Given,

$$\begin{pmatrix} 3 & -1 & 2 \end{pmatrix}\mathbf{x} = 4 \tag{2.0.1}$$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\mathbf{x} = -2 \tag{2.0.2}$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \tag{2.0.3}$$

Equation can be written as,

$$\mathbf{n_1^T x} = c_1 \tag{2.0.4}$$

$$\mathbf{n_2^T x} = c_2 \tag{2.0.5}$$

Where,

$$\mathbf{n_1} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.6}$$

$$c_1 = 4, c_2 = -2 \tag{2.0.7}$$

Required equation of the plane containing (2.0.4) and (2.0.5) is,

$$\mathbf{n_1^T x} + \lambda \mathbf{n_2^T x} = c_1 + \lambda c_2 \tag{2.0.8}$$

$$\implies (\mathbf{n_1^T} + \lambda \mathbf{n_2^T})\mathbf{x} = c_1 + \lambda c_2 \tag{2.0.9}$$

By substituting the intersection of the plane of the point (2.0.3).so,

$$\mathbf{A}(\mathbf{n_1^T} + \lambda \mathbf{n_2^T})\mathbf{x} = c_1 + \lambda c_2 \tag{2.0.10}$$

$$\lambda(\mathbf{A n_2^T} - c_2) = c_1 - (\mathbf{A n_1^T}) \tag{2.0.11}$$

$$\lambda = \frac{(c_1 - \mathbf{A n_1^T})}{(\mathbf{A n_2^T} - c_2)} \tag{2.0.12}$$

$$\implies \lambda = \frac{-2}{7} \tag{2.0.13}$$

$\therefore$ By substituting $\lambda, n_1, n_2, c_1, c_2$ values in (2.0.9) we get required plane equation as,
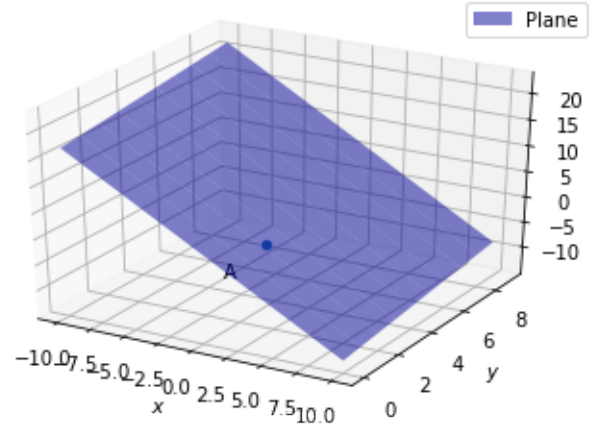
$$\begin{pmatrix} 19 & -9 & 12 \end{pmatrix}\mathbf{x} = 32 \tag{2.0.14}$$

Plot of the plane :

Fig. 2.1: Plot of the plane