

ASSIGNMENT 6

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Download all python codes from

https://github.com/tejasri3657/Assignment-6/blob/main/Assignment_6.py

Latex-tikz codes from

<https://github.com/tejasri3657/Assignment-6/new/main>

Now,

$$\mathbf{F} = \begin{pmatrix} \pm \sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \\ 0 \end{pmatrix} \quad (2.0.6)$$

$$\|\mathbf{F}\|^2 = \frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} \quad (\because Foci = F = c) \quad (2.0.7)$$

$$\Rightarrow 45 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} \quad (2.0.8)$$

And, length of latus rectum = 8

$$\Rightarrow \frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a \quad (2.0.9)$$

$$\Rightarrow \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} = 4 \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \quad (2.0.10)$$

Putting above equation (2.0.10) in (2.0.8), Find

$$\lambda_1, \lambda_2 \quad (2.0.11)$$

$$\lambda_1 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{81} \quad (2.0.12)$$

$$\lambda_2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{36} \quad (2.0.13)$$

Final equation is :

$$\frac{\mathbf{y}^T \mathbf{D} \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.14)$$

$$\Rightarrow \frac{\mathbf{y}^T \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.15)$$

$$\Rightarrow \mathbf{y}^T \begin{pmatrix} \frac{1}{81} & 0 \\ 0 & \frac{1}{36} \end{pmatrix} \mathbf{y} = 1 \quad (2.0.16)$$

Plot of ellipse:

$$\mathbf{F} = \begin{pmatrix} \pm \sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \\ 0 \end{pmatrix} \quad (2.0.4)$$

Let

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}, b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.5)$$

1 QUESTION No 2.74(F)

In each of the following find the equation for the ellipse that satisfies the given condition:

- 1) Latus rectum length 8, foci $\begin{pmatrix} \pm 3\sqrt{5} \\ 0 \end{pmatrix}$

2 SOLUTION

Given that,

Latus rectum length = 8

$$Foci = F = \begin{pmatrix} \pm c \\ 0 \end{pmatrix} = \begin{pmatrix} \pm 3\sqrt{5} \\ 0 \end{pmatrix} \quad (2.0.1)$$

Lemma 2.1. The standard equation of a conic is given by:

$$\frac{\mathbf{y}^T \mathbf{D} \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.2)$$

$$\text{where, } D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (2.0.3)$$

Lemma 2.2. focus of a conic is given by

Let

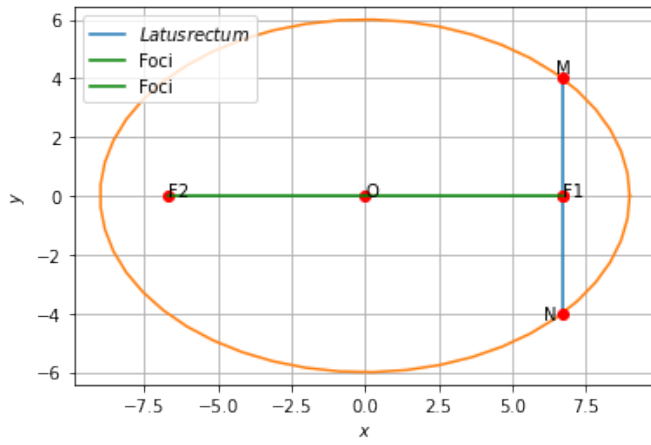


Fig. 2.1: Ellipse $\frac{x^2}{81} + \frac{y^2}{36} = 1$