ASSIGNMENT 6

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Download all python codes from

https://github.com/tejasri3657/Assignment−6/blob/main/Assignment_6.py

Latex-tikz codes from

https://github.com/tejasri3657/Assignment−6/new/main

1 Question No 2.74(f)

In each of the following find the equation for the ellipse that satisfies the given condition:

1) Latus rectum length 8, foci $\begin{pmatrix} \pm 3\sqrt{5} \\ 0 \end{pmatrix}$

2 Solution

Given that,
Latus rectum length = 8

$$Foci = F = \begin{pmatrix} \pm c \\ 0 \end{pmatrix} = \begin{pmatrix} \pm 3\sqrt{5} \\ 0 \end{pmatrix} \tag{2.0.1}$$

Lemma 2.1. *The standard equation of a conic is given by:*

$$\frac{\mathbf{y}^\top D\mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.2}$$

$$where,\ D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{2.0.3}$$

Lemma 2.2. *focus of a conic is given by*

$$\mathbf{F} = \begin{pmatrix} \pm \sqrt{\frac{(\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \\ 0 \end{pmatrix} \tag{2.0.4}$$

Let

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}, b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} \tag{2.0.5}$$

Now,

$$\mathbf{F} = \begin{pmatrix} \pm \sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \\ 0 \end{pmatrix} \tag{2.0.6}$$

$$\|\mathbf{F}\|^2 = \frac{(\mathbf{u}^T \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} \quad (\because Foci = F = c) \tag{2.0.7}$$

$$\implies 45 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} \tag{2.0.8}$$

And, length of latus rectum = 8

$$\implies \frac{2b^2}{a} = 8 \implies b^2 = 4a \tag{2.0.9}$$

$$\implies \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} = 4\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} \tag{2.0.10}$$

Putting above equation (2.0.10) in (2.0.8), Find

$$\lambda_1, \lambda_2 \tag{2.0.11}$$

$$\lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{81} \tag{2.0.12}$$

$$\lambda_2 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{36} \tag{2.0.13}$$

Final equation is :

$$\frac{\mathbf{y}^\top Dy}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.14}$$

$$\implies \frac{\mathbf{y}^\top \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.15}$$

$$\implies \mathbf{y}^\top \begin{pmatrix} \frac{1}{81} & 0 \\ 0 & \frac{1}{36} \end{pmatrix} = 1 \tag{2.0.16}$$

Plot of ellipse:

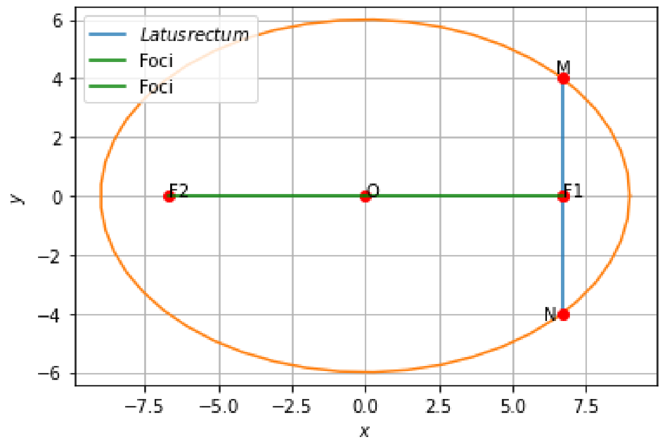

Fig. 2.1: Ellipse $\frac{x^2}{81} + \frac{y^2}{36} = 1$