ASSIGNMENT 7

A.Tejasri

Download all python codes from

```
https://github.com/tejasri3657/Assignment-7/blob/
    main/Assignment-7.py
```

Latex-tikz codes from

```
https://github.com/tejasri3657/Assignment-7/tree/
    main
```

1 Question No 2.38 (a)

Find the coordinates of the foci and the vertices, the eccentricity,the length of the latus rectum of the hyperbola $\mathbf{x}^\top \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{-1}{16} \end{pmatrix} \mathbf{x} = 1$.

2 Solution

Given equation of the hyperbola,

$$\mathbf{x}^\top \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{-1}{16} \end{pmatrix} \mathbf{x} = 1 \tag{2.0.1}$$

we have,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{-1}{16} \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f = 1 \tag{2.0.3}$$

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$

$$\lambda_1 = \frac{1}{9}, \lambda_2 = \frac{-1}{16} \tag{2.0.5}$$

Axes of hyperbola is given by

$$\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} = 3 \tag{2.0.6}$$

$$\sqrt{\frac{f - \mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u}}{\lambda_2}} = 4 \tag{2.0.7}$$

The vertices are given as

$$\pm \begin{pmatrix} 3 \\ 0 \end{pmatrix} \text{and} \pm \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{2.0.8}$$

Coordinates of foci are given by,

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \mathbf{p_1} \tag{2.0.9}$$

where, $\mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ since the equation of hyperbola is in standard form. Substituting the values in (2.0.9) we have,

$$\mathbf{F} = \pm \begin{pmatrix} 5 \\ 0 \end{pmatrix}. \tag{2.0.10}$$

Eccentricity of the hyperbola is given by,

$$e = \frac{\sqrt{\frac{(\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u})(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}}}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}} \tag{2.0.11}$$

substituting the values in (2.0.11),we have

$$e = \frac{5}{3}. \tag{2.0.12}$$

Length of the latus rectum is given by,

$$l = \frac{2\left(\sqrt{\frac{f - \mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u}}{\lambda_2}}\right)^2}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}} \tag{2.0.13}$$

substituting the values in (2.0.13),we have

$$l = \frac{32}{3} \tag{2.0.14}$$

Plot of the hyperbola:

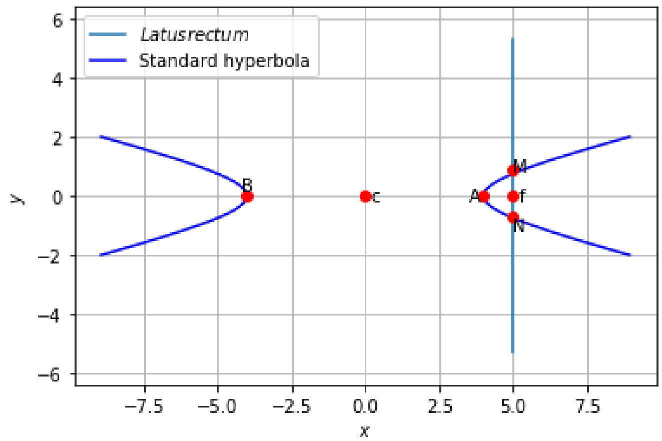

Fig. 2.1: Hyperbola $\frac{x^2}{9} + \frac{y^2}{16} = 1$