

ASSIGNMENT 7

A.Tejasri

Download all python codes from

<https://github.com/tejasri3657/Assignment-7/blob/main/Assignment-7.py>

Latex-tikz codes from

<https://github.com/tejasri3657/Assignment-7/tree/main>

Coordinates of foci are given by,

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \mathbf{p}_1 \quad (2.0.9)$$

where, $\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ since the equation of hyperbola is in standard form. Substituting the values in (2.0.9) we have,

$$\mathbf{F} = \pm \begin{pmatrix} 5 \\ 0 \end{pmatrix}. \quad (2.0.10)$$

Eccentricity of the hyperbola is given by,

$$e = \frac{\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u})(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}}}{\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}} \quad (2.0.11)$$

substituting the values in (2.0.11), we have

$$e = \frac{5}{3}. \quad (2.0.12)$$

Length of the latus rectum is given by,

$$l = \frac{2 \left(\sqrt{\frac{f - \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u}}{\lambda_2}} \right)^2}{\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}} \quad (2.0.13)$$

substituting the values in (2.0.13), we have

$$l = \frac{32}{3} \quad (2.0.14)$$

Plot of the hyperbola:

1 QUESTION No 2.38(A)

Find the coordinates of the foci and the vertices, the eccentricity, the length of the latus rectum of the hyperbola $\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & -\frac{1}{16} \end{pmatrix} \mathbf{x} = 1$.

2 SOLUTION

Given equation of the hyperbola,

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & -\frac{1}{16} \end{pmatrix} \mathbf{x} = 1 \quad (2.0.1)$$

we have,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & -\frac{1}{16} \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 1 \quad (2.0.3)$$

$$\mathbf{c} = -\mathbf{V}^{-1} \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$\lambda_1 = \frac{1}{9}, \lambda_2 = -\frac{1}{16} \quad (2.0.5)$$

Axes of hyperbola is given by

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} = 3 \quad (2.0.6)$$

$$\sqrt{\frac{f - \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u}}{\lambda_2}} = 4 \quad (2.0.7)$$

The vertices are given as

$$\pm \begin{pmatrix} 3 \\ 0 \end{pmatrix} \text{ and } \pm \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (2.0.8)$$

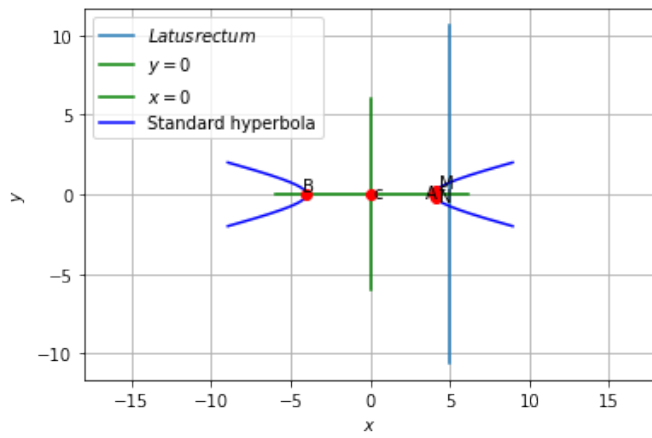


Fig. 2.1: Hyperbola $\frac{x^2}{9} + \frac{y^2}{16} = 1$