ASSIGNMENT-9

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1 QUESTION No-2.25(Matrices)

Using elementary transformations, find the inverse of each of the matrices:

1) $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

2 Solution

1) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \tag{2.0.1}$$

The augmented matrix $[A|I]$ is as given below:-

$$\left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right) \tag{2.0.2}$$

We apply the elementary row operations on $[A|I]$ as follows :-

$$[A|I] = \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right) \tag{2.0.3}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 5 & -2 & 1 \end{array}\right) \tag{2.0.4}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{5}} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{1}{5} \end{array}\right) \tag{2.0.5}$$

$$\xleftrightarrow{R_2 \leftarrow R_1 + R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & \frac{1}{5} \\ 0 & 1 & \frac{-2}{5} & \frac{1}{5} \end{array}\right) \tag{2.0.6}$$

By performing elementary transformations on augmented matrix$[A|I]$, we obtained the augmented matrix in the form $[I|A]$. Hence we can conclude that the matrix A is invertible and inverse of the matrix is:-

$$\therefore \mathbf{A^{-1}} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix} \tag{2.0.7}$$

2) QR decomposition of $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{v_2} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{2.0.8}$$

we can express these as

$$\mathbf{v_1} = \mathbf{r_{11}q_1} \tag{2.0.9}$$

$$\mathbf{v_2} = \mathbf{r_{12}q_1} + \mathbf{r_{22}q_2} \tag{2.0.10}$$

Where

$$\mathbf{r_{11}} = \|\mathbf{v_1}\| \tag{2.0.11}$$

$$\mathbf{q_1} = \frac{1}{\|\mathbf{v_1}\|}\mathbf{v_1} \tag{2.0.12}$$

$$\mathbf{r_{12}} = \mathbf{v_1}^T\mathbf{v_2} \tag{2.0.13}$$

$$\mathbf{v_2} = \mathbf{v_2} - \mathbf{r_{12}q_1} \tag{2.0.14}$$

$$\mathbf{r_{22}} = \|\mathbf{v_2}\| \tag{2.0.15}$$

$$\mathbf{q_2} = \frac{1}{\|v_2\|}\mathbf{v_2} \tag{2.0.16}$$

From(2.0.9)and(2.0.10).

$$\begin{pmatrix} v_1 & v_2 \end{pmatrix} = \begin{pmatrix} q_1 & q_2 \end{pmatrix}\begin{pmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{pmatrix} \tag{2.0.17}$$

$$\begin{pmatrix} v_1 & v_2 \end{pmatrix} = \mathbf{Q}R \tag{2.0.18}$$

From above we can see that R is an upper triangular matrix and

$$\mathbf{Q^TQ} = \mathbf{I} \tag{2.0.19}$$

Now by using equations (2.0.11) to (2.0.16)

$$\mathbf{r_{11}} = \sqrt{5} \tag{2.0.20}$$

$$\mathbf{q_1} = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \tag{2.0.21}$$

$$\mathbf{r_{12}} = \sqrt{5} \tag{2.0.22}$$

$$\mathbf{v_2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \tag{2.0.23}$$

$$\mathbf{r_{22}} = \sqrt{5} \tag{2.0.24}$$

$$\mathbf{q_2} = \begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \tag{2.0.25}$$

Thus obtained QR decomposition is

$$\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}\begin{pmatrix} \sqrt{5} & \sqrt{5} \\ 0 & \sqrt{5} \end{pmatrix} \tag{2.0.26}$$