

ASSIGNMENT-9

A.TEJASRI

1 QUESTION No-2.25(MATRICES)

Using elementary transformations, find the inverse of each of the matrices:

1) $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

2 SOLUTION

1) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \quad (2.0.1)$$

The augmented matrix $[A|I]$ is as given below:-

$$\left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array} \right) \quad (2.0.2)$$

We apply the elementary row operations on $[A|I]$ as follows :-

$$[A|I] = \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array} \right) \quad (2.0.3)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 5 & -2 & 1 \end{array} \right) \quad (2.0.4)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{5}} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{2}{5} & \frac{1}{5} \end{array} \right) \quad (2.0.5)$$

$$\xleftrightarrow{R_2 \leftarrow R_1 + R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & \frac{1}{5} \\ 0 & 1 & -\frac{2}{5} & \frac{1}{5} \end{array} \right) \quad (2.0.6)$$

By performing elementary transformations on augmented matrix $[A|I]$, we obtained the augmented matrix in the form $[I|A]$. Hence we can conclude that the matrix A is invertible and inverse of the matrix is:-

$$\therefore \mathbf{A}^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix} \quad (2.0.7)$$

2) QR decomposition of $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \quad (2.0.8)$$

$$\mathbf{r}_{11} = \sqrt{5} \quad (2.0.9)$$

$$\mathbf{q}_1 = \frac{1}{\|\mathbf{v}_1\|} \mathbf{v}_1 = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix} \quad (2.0.10)$$

$$\mathbf{r}_{12} = \mathbf{v}_1^T \mathbf{v}_2 \quad (2.0.11)$$

$$= \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \sqrt{5} \quad (2.0.12)$$

$$\mathbf{v}_2 = \mathbf{v}_2 - \mathbf{r}_{12} \mathbf{q}_1 \quad (2.0.13)$$

$$= \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \sqrt{5} \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix} \quad (2.0.14)$$

$$= \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad (2.0.15)$$

$$\mathbf{r}_{22} = \|\mathbf{v}_2\| = \sqrt{5} \quad (2.0.16)$$

$$\mathbf{q}_2 = \frac{1}{\|\mathbf{v}_2\|} \mathbf{v}_2 \quad (2.0.17)$$

$$= \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \quad (2.0.18)$$

$$\begin{pmatrix} v_1 & v_2 \end{pmatrix} = \begin{pmatrix} q_1 & q_2 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{pmatrix} \quad (2.0.19)$$

$$\begin{pmatrix} v_1 & v_2 \end{pmatrix} = \mathbf{Q} \mathbf{R} \quad (2.0.20)$$

From above we can see that \mathbf{R} is an upper triangular matrix and

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \quad (2.0.21)$$

Thus obtained QR decomposition is

$$\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & \sqrt{5} \\ 0 & \sqrt{5} \end{pmatrix} \quad (2.0.22)$$