CONTROL SYSTEMS

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February 2020

Problem Statement

Consider the following state-space representation of a linear time-invariant system $\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x(t)$;

$y(t) = c^T x(t)$;

$c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$;

The value of $y(t)$ for $t = log_e 2$ is

Given $x^{\cdot}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x(t)$

also we know that $x^{\cdot}(t) = Ax(t)$

on comparing we get $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$;

now $y(t) = c^T x(t)$; $x(t) = e^{At}x(0)$;

where $e^{At} = L^{-1}[(SI - A)^{-1}]$;

$$SI - A = \begin{bmatrix} s-1 & 0 \\ 0 & s-2 \end{bmatrix}$$

$$(SI - A)^{-1} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-2} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} e^t \\ e^{2t} \end{bmatrix}$$

$$y(t) = \begin{pmatrix}1\\1\end{pmatrix}\begin{bmatrix}e^{t}\\e^{2t}\end{bmatrix} = \left[e^{t} + e^{2t}\right]$$

now for $t = log_e 2$

$$y(t) = \left[e^{log_e 2} + e^{2 log_e 2}\right]$$

$$= [2 + 4] = 6$$

$$y(t) = 6$$