Chris Piech
CS109

Section #1: Analytic Probability

1. Assume that birthdays happen on any of the 365 days of the year with equal likelihood (we'll ignore leap years).

    a. What is the probability that of the n people in your section, at least two people share the same birthday?

    b. What is the probability that at least one person has a birthday that falls on one of the 8 days we'll have section?

2. Shazam is an application which can predict what song is playing. Based on the frequency of requests it's been getting these days, Shazam has the following prior beliefs as to what song is playing:

    - 80% chance of event X_1, the song is Hold Up by Beyonce
    - 20% chance of event X_2, the song is Can't Get Used to Losing You by Andy Williams

    When a request is made, Shazam receives an audio sample (H) that it uses to update its belief. From the audio sample, Shazam estimates that:

    - $P(H|X_1) = 0.50$
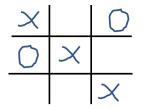   - $P(H|X_2) = 0.90$

    What is the updated probability that the song is Beyonce given the audio sample heard?

3. The probability that a Netflix user likes a movie M_i is p_i.

    Assume that liking movie M_a and M_b are independent events for all a and b. Express all your answers in terms of p's.

    a. What is the probability of a user liking M_1, M_2 and M_3?

    b. What is the probability of a user liking M_1, M_2 or M_3?

4. Extra practice: consider a TicTacToe board where each location can either have an empty space, an X, or an O. Each location is distinct (thus, even if two boards can be rotated to look the same, we consider them different). Here is an example board:

    a. How many unique ways are there of placing Xs and Os on a TicTacToe board such that each square is either empty, has an X, or has an O? You do not have to follow the rules of the game and you do not have to fill each square.

    b. The "Turn Rule" states that players take turns and X always starts. After 5 moves, how many unique TicTacToe boards are there that satisfy the Turn Rule (X has played three times and O has played twice)?

    c. If both players play randomly, what is the probability that X will win after 5 moves (X has played three times and O has played twice)? X wins if their three pieces make a vertical, diagonal or horizontal line. There are 8 such lines.

Problems 2, 3 and 4 were on previous CS109 midterms. Problem 4 was from last year.