

Week 2 Assignment

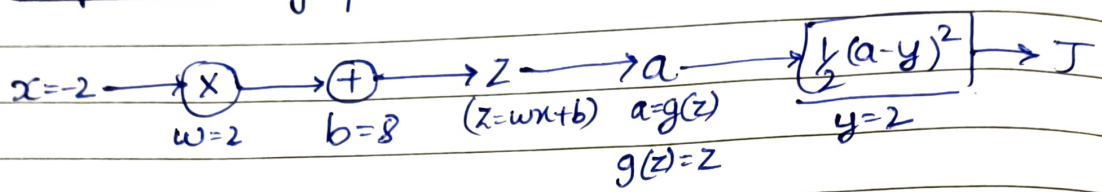
Given that

$w=2, b=8$, Input: $x=-2$, Label: $y=2$,

Activation function: $g(z)=z$

Cost function: $J(w,b) = \frac{1}{2}(a-y)^2$

Computational graph:



Forward Pass: $z = wx + b \Rightarrow z = 2(-2) + 8 = 4$

$a = g(z) = z = 4$

$J = \frac{1}{2}(a-y)^2 = \frac{1}{2}(4-2)^2 = 2$

Backpropagation:

$$\frac{\partial J}{\partial a} = a - y = 4 - 2 = 2$$

$$\because g(z) = z, \quad \frac{\partial a}{\partial z} = 1 \Rightarrow \frac{\partial J}{\partial z} = \frac{\partial J}{\partial a} \cdot \frac{\partial a}{\partial z} = 2$$

$$\frac{\partial J}{\partial w} = x = -2$$

$$\frac{\partial J}{\partial w} = \frac{\partial J}{\partial z} \cdot \frac{\partial z}{\partial w} = 2 \cdot (-2) = -4$$

$$\frac{\partial z}{\partial b} = 1, \quad \frac{\partial J}{\partial b} = 2$$

If w is changed by a small amount ϵ , $\Delta J \approx \frac{\partial J}{\partial w} \epsilon$

$$\Rightarrow \boxed{\Delta J = -4\epsilon}$$

This shows that $\uparrow w \downarrow \text{cost}$.

\therefore Backpropagation indicates that \uparrow the weight & \downarrow the bias will reduce the loss.