Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
   Z=(X-mu)/sigma
   Z= (50-45)/8=0.625
   Pr(Z<=0.625)
   73.4%
   100-73.4=26.6=0.26

   The probability that the service manager cannot meet his commitment is 0.26.
   Answer is Option B
   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
      We have a normal distribution with   = 38 and   = 6. Let X be the number of employees. So according to question
      a)Probability of employees greater than age of 44= Pr(X>44)
      $Pr(X > 44) = 1 - Pr(X \le 44)$.
      Z = (X -)/ = (X - 38)/6
      Thus the question can be answered by using the normal table to find
      $Pr(X \le 44) = Pr(Z \le (44 - 38)/6) = Pr(Z \le 1)$=84.1345%
      Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%
      So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%
      Therefore the statement that "More employees at the processing center are older than 44 than between 38 and 44" is TRUE.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
      Probabilty of employees less than age of 30 = Pr(X<30).
      Z = (X -)/ = (30 - 38)/6
      Thus the question can be answered by using the normal table to find
      $Pr(X \le 30) = Pr(Z \le (30 - 38)/6) = Pr(Z \le -1.333)$=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912*400=36.48(or 36 employees).
Therefore, statement B of the question is also TRUE.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

As we know that if X ~ N(μ1, σ1^2), and Y ~ N(μ2, σ2^2) are two independent random variables then X + Y ~ N(μ1 + μ2, σ1^2 + σ2^2) , and X − Y ~ N(μ1 − μ2, σ1^2 + σ2^2) .
Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ~ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2).
Therefore in the question
2X1~ N(2 u,4 σ^2) and
X1+X2 ~ N(μ + μ, σ^2 + σ^2) ~ N(2 u, 2σ^2)
2X1-(X1+X2) = N(4μ,6 σ^2)

4. Let X ~ $N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.
The Probability of getting value between a and b should be 0.99.
So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).
The Probability towards left from a = -0.005 (ie. 0.01/2).
The Probability towards right from b = +0.005 (ie. 0.01/2).
So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.
By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.
Z=(X- μ) / σ
For Probability 0.005 the Z Value is -2.57 (from Z Table).
Z * σ + μ = X
Z(-0.005)*20+100 = -(-2.57)*20+100 = 151.4
Z(+0.005)*20+100 = (-2.57)*20+100 = 48.6
So, option D is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

95% of the probability lies between 1.96 standard deviations of the mean.

Thus, range is:

= (12 - 1.96* 5, 12 + 1.96 * 5)
= (\$2.2M, \$22.8M)
= (Rs: 99M, Rs.: 1026M)

B. Specify the 5^{th} percentile of profit (in Rupees) for the company?

Fifth percentile is calculated as:
P (Z <= (p-12)/(5)) = 0.05
From p values of z score table, we get:
(p-12)/ (5) = -1.644
np = 12 - 8.22 = 3.78
Thus at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies.
Or 5th percentile of profit is Rs. 170.1 million.

C. Which of the two divisions has a larger probability of making a loss each year?

Loss is when profit < 0.
Thus: p < 0
The first division of the company thus has larger probability of making a loss in a given year.