(b) Decay Model (exponential case only)

Exponential Decay :

If a function x (t) decreses continuously at a rate k > 0, its differential equation is given by

$dx/dy = -kx, \quad k > 0$

then x (t) has the form

$x(t) = x_o e^{-kt}$

where x_o is the initial amout x (0). In this case, the quantity x (t) is said to exhibit exponential decay, and k is the decay rate.

Example - 4

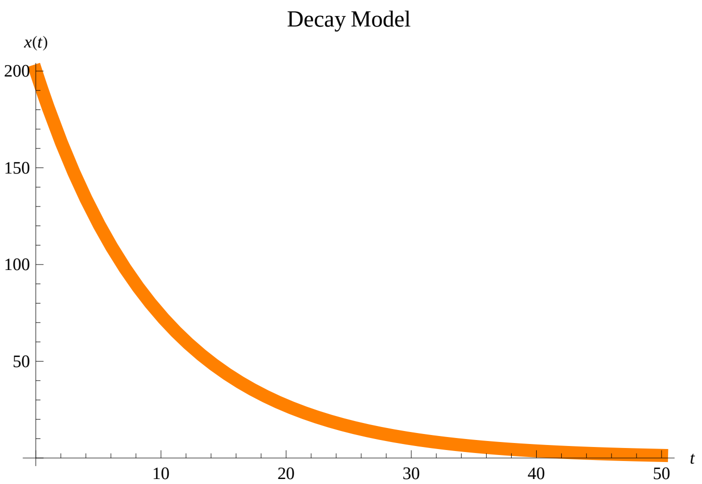
Suppose that a certain radioactive element has an annual decay rate of 10 %. Starting with a 200 gram sample of the element, how many grams will be left in 3 years ?

$x_o = 200$, k = 10 % = .1, x[t] in 3 years = ?

In[2]:=
```
Sol = DSolve[x'[t] == -k * x[t], x[t], t]
```
Out[2]= $\{\{x[t] \to e^{-k\,t}\,C[1]\}\}$

In[3]:=
```
Sol1 = Evaluate[x[t] /. Sol[[1]] /. {k → .1, C[1] → 200}]
```
Out[3]= $200\,e^{-0.1\,t}$

In[9]:=
```
Plot[Sol1, {t, 0, 50}, PlotStyle → {Orange, Thickness[0.02]},
 PlotLabel → "Decay Model", AxesLabel → {t, x[t]}]
```
Out[9]=



In[8]:=
```
x[3] = Evaluate[Sol1 /. {t → 3}]
```
Out[8]= 148.164

Conclusion : Hence, grams left after 3 years will be 148.164.

In[10]:=
```
ClearAll
```
Out[10]= ClearAll

Example - 5

Using the same element as Example - 4, if a particular sample of the element decays to 50 grams after 5 years, how big was the original sample?

k = 10 % = 0.1, t = 5, x (t) = 50, x[0] = ?

In[15]:=
```
Clear[t, k, x]
```

In[16]:=
```
Sol = DSolve[x'[t] == -k * x[t], x[t], t]
```

Out[16]= $\{\{x[t] \to e^{-k\,t}\, C[1]\}\}$

In[17]:=
```
Sol1 = Evaluate[x[t] /. Sol[[1]] /. {k → .1, C[1] → xo, t → 5}]
```

Out[17]= $0.606531\, x_o$

In[18]:=
```
Sol2 = Solve[Sol1 == 50, xo]
```

Out[18]= $\{\{x_o \to 82.4361\}\}$

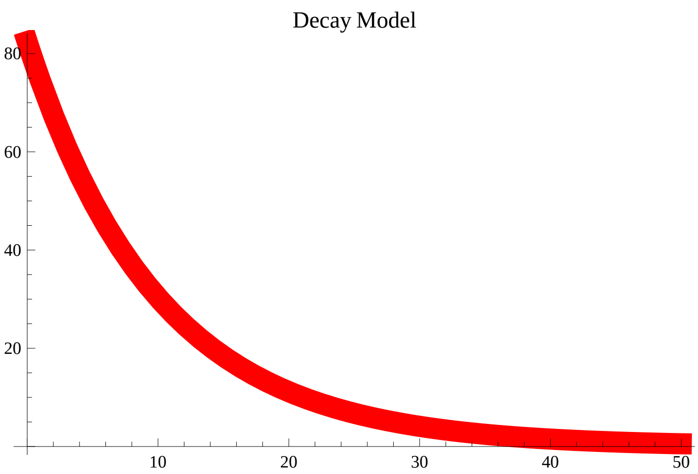
In[19]:=
```
Sol3 = xo /. Sol2[[1]]
```

Out[19]= 82.4361

In[20]:=
```
Sol4 = x[t] /. Sol[[1]] /. {k → 0.1, C[1] → Sol3}
```

Out[20]= $82.4361\, e^{-0.1\,t}$

In[21]:=
```
Plot[Sol4, {t, 0, 50}, PlotStyle → {Red, Thickness[0.031]}, PlotLabel → "Decay Model"]
```

Out[21]=


Conclusion : Hence, original sample was 82.4361 grams.

Example - 6
Suppose that a certain radioactive isotope has an annual decay rate of 5 %. How many years will it take for a 100 gram sample to decay to 40 grams?

Here, k = 5 % = 0.05, x (0) = 100, x (t) = 40, t = ?

In[22]:=
```
Clear[t, k, x]
```

In[23]:=
```
Sol = DSolve[x'[t] == -k * x[t], x[t], t]
```

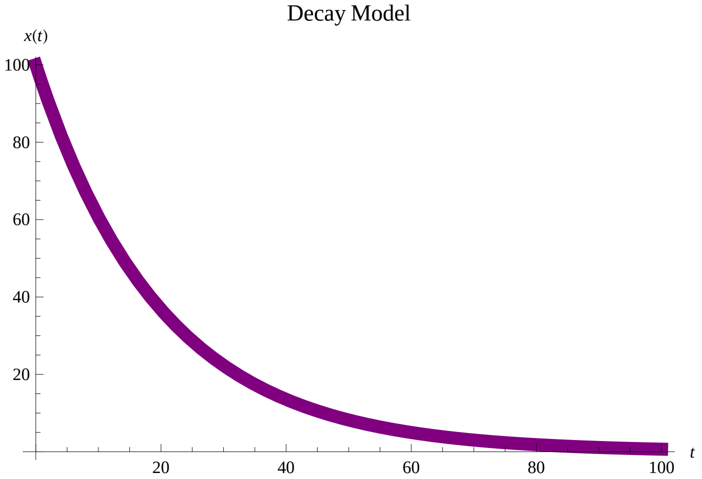
Out[23]= $\{\{x[t] \to e^{-k\,t}\, C[1]\}\}$

In[24]:=
```
Sol1 = Evaluate[x[t] /. Sol[[1]] /. {k → .05, C[1] → 100}]
```

Out[24]= $100\, e^{-0.05\,t}$

In[27]:=
```
Plot[Sol1, {t, 0, 100}, PlotStyle → {Purple, Thickness[0.02]},
 PlotLabel → "Decay Model", AxesLabel → {t, x[t]}]
```

Out[27]=



In[26]:=
```
Solve[Sol1 == 40, t]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

Out[26]= {{t → 18.3258}}

Conclusion : Hence it will take 18.3258 years for 100 gram sample to decay to 40 grams.

Example - 7
Using the same element as in Example - 6, what is the half life of the element?
Here, k = 5 % = 0.05, x (0) = 100, x (t) = 40, t = ?

In[28]:=
```
Sol = DSolve[x'[t] == -k * x[t], x[t], t]
```

Out[28]= $\{\{x[t] \to e^{-k t} C[1]\}\}$

In[29]:=
```
Sol1 = Evaluate[x[t] /. Sol[[1]] /. {k → 0.05, C[1] → 100}]
```

Out[29]= $100\, e^{-0.05 t}$

In[30]:=
```
Solve[Sol1 == 50, t]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

Out[30]= {{t → 13.8629}}

Conclusion : Hence, half - life of radioactive isotope is 13.8629 years.