Portfolio Management

Sai Teja, Sai kiran, Aryan, Abhishek

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Indian Institute of Technology Bombay

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Overview

Problem Statement

Calculate optimal instrument ratios under the constraints of maximum profits for a given risk by modeling the historical instrument prices for predicting estimated price for profit calculation and variance for risk

Apart from that, we can also include various constraints which involve investors' requirements like the following

- $B_u := \{u_1, u_2 .... u_n\}$ where u_i is the upper bound on investment set by customer/financial regulators.
- $B_l := \{l_1, l_2 ...... l_n\}$ where l_i represents minimum investment the customer is looking to make.

Description

We describe the mechanism we currently have in mind, this is already used to prepare version 1 of our tool.

Def 1.1 Risk

We characterize the risk of a portfolio allocation as the variance in the return. $R = Var(\Sigma_i(y_i\alpha_i))$, where α_i is the return random variable of an instrument.

Def 1.2 Mean and Variance of historic data

We assume stock prices are controlled by Geometric Brownian Motion(GBM) which is governed by

$$\frac{dS}{S} = \mu dt + \sigma dW$$

Assumptions and Approximations

Assumption 1.1 Independence of instrument returns

We assume that α_i for i in $[1, N]$ are pairwise-independent. This assumption is not a very good assumption in the regular market but provides a decent linear approximation.

Approximation 1.1 Linear definition of risk

To use linear programming, we provide an alternate definition for risk using Jensen's/Cauchy's inequality

$\sqrt{nR_{app}} = \Sigma_i (y1 * std(\alpha_i))$

$R \leq R_{app}$ from Cauchy's inequality

We use the linear definition of risk in combination with mean and variance estimates to generate linear program that can solve the original system

Implied Volatility from Option Prices

Objective

Calculate implied volatility from option prices using a mathematical model.

Definition

Implied volatility represents the market's expectation of future volatility as implied by the current option prices.

Formula

Implied volatility is obtained by solving an option pricing model. A common model is the Black-Scholes formula:

$$C = S_0 e^{-qt} N(d_1) - X e^{-rt} N(d_2)$$

where d_1 and d_2 are calculated based on the stock price (S_0), strike price (X), time to expiration (t), interest rate (r), and volatility (σ).

Implied Volatility from Option Prices

Process

1. Obtain option prices.
2. Formulate the Black-Scholes model.
3. Use an optimization algorithm to solve for implied volatility.

Implied Volatility from Option Prices

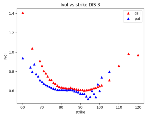
0.3 

Figure: 3 days

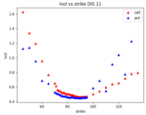
0.3 

Figure: 11 days

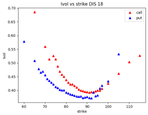
0.3 

Figure: 18 days

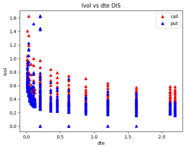
0.3 

Figure: Ivol vs dte

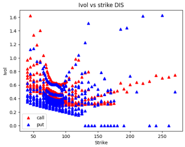
0.3 

Figure: Ivol vs strike

Figure: Volatility vs. Strike

Convex Programming

When correlations are involved in the formulation, linear programming no longer suffices the task, we now formulate our problem as a convex programming instance.

Problem Formulation

$$\text{maximize} p_r^T v$$
$$v^T Q v \leq R$$
$$p^T v \leq B$$
$$v \geq 0$$

where,

- Q:- covariance matrix
- v:- allocation matrix
- p:- price array
- p_r:- predicted return array

CP rounding

Randomized Rounding Strategy

- **Fractional Solution:** After obtaining a fractional solution from convex programming.
- **Rounding Up:** Round up with a probability equal to the fractional part.
- **Rounding Down:** Round down for the remaining probability.

Algorithm

1. For each fractional variable x_i:
2. Generate a random number r from a uniform distribution between 0 and 1.
3. If $r \leq \{x_i\}$, round up; otherwise, round down.

CP rounding

Considerations

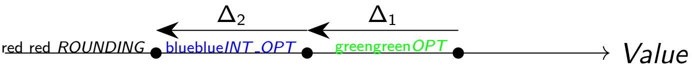
- This approach introduces randomness and provides a feasible solution with some optimally.
- The effectiveness depends on problem characteristics and the quality of the rounding algorithm.

Rounding Algorithm and Heuristics

Let the solution we obtain be v, For any linear function LP

$$E[LP(v^{round})] = LP(v)$$

CP rounding



$$\Delta_1 + \Delta_2 \geq \Delta_2$$

CP rounding results

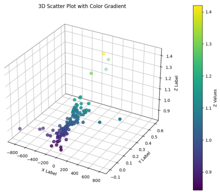


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Figure: error vs deviation

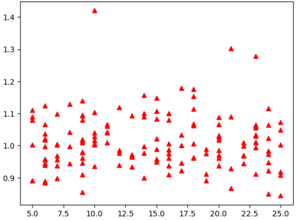

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Figure: error vs num stocks

Theoritical Risk Foundations

Consider the random variable X with mean $\mathrm{E}[X]$ and variance $\mathrm{Var}[X]$.

Probability Inequality

$$\Pr(X - \mathrm{E}[X] \leq -t) \leq e^{-\frac{t^2}{2(\mathrm{Var}[X] - \frac{t}{3})}}$$

This inequality provides an upper bound on the left tail probability.
We claim that our risk formulation provides a meaning which can be inferred from the above bound on inequality. We provide a probability that we lose more than 90% of our projected value(given by our algorithm).

Alternative Risk Defintion

We say that, given Var[X], we can back-calculate the upper bound on the probability that we go to less than 90% of the projected value.

$$Pr = e^{-\frac{(ln(0.9))^2}{2Var[x]}}$$

Momentum Trading

Definition

Momentum trading is a strategy that involves taking advantage of the continuation of existing trends in financial markets.

Key Concepts

- Buy assets that have shown upward trends.
- Relies on the belief that trends tend to persist.

Indicators

Common indicators include moving averages, relative strength, and price rate of change but we use predicted mean in our case.

Comparison with Momentum Portfolio

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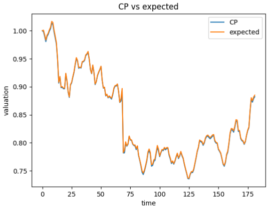

Figure: CP vs expected

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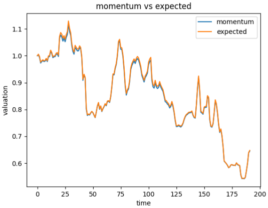


Figure: momentum vs expected

Comparison with Momentum Portfolio

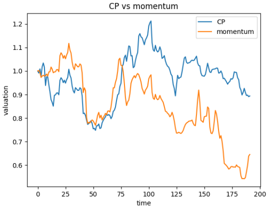
0.3 

Figure: CP vs expected

Figure: Results on CP vs momentum

Here, we answer the question of how Implied Volatility can help us to get better estimates of future trends.

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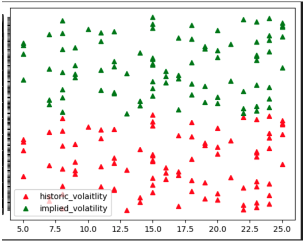


Figure: Return vs n

Figure: Results on Implied Volatility vs Historical Volatility

Here, we answer the question of how Implied Volatility can help us to get better estimates of future trends.

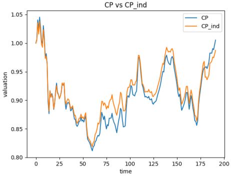
0.3 

Figure: LP vs CP

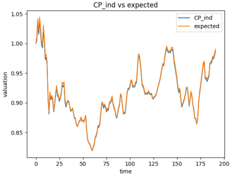
0.3 

Figure: LP vs Expected

Results for different combinations of instruments

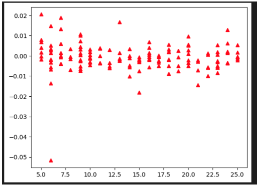
0.3 

Figure: n vs deviation from expected return

Figure: Results on different combinations of instruments for the same day

References

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Empirical study on portfolio size and risk diversification
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Thank You