

Tucker-CAM: Tractable Non-linear Causal Discovery for High-Dimensional Industrial Time Series Anomaly Detection

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Abstract

Anomaly detection in high-dimensional industrial systems requires both high detection accuracy and actionable interpretability. While deep learning approaches (e.g., Transformers) achieve state-of-the-art detection rates, they lack the causal transparency required for robust root cause analysis (RCA). Conversely, existing causal discovery methods are computationally intractable for high-dimensional time series ($d > 100$) or restricted to linear dynamics. We propose **Tucker-CAM**, a unified pipeline that learns dynamic non-linear Causal Bayesian Networks via tensor decomposition. By modeling causal mechanisms as P-splines and factorizing the coefficient tensor via Tucker decomposition, we reduce the parameter space from $\mathcal{O}(d^2)$ to $\mathcal{O}(dr)$, enabling scalability to $d = 2889$ variables. We introduce a state-aware scoring system for precise anomaly localization and a graph traversal algorithm for automated RCA. On the NASA Telemnom benchmark, Tucker-CAM achieves a Range-F1 of 0.9977 and an AUC-PR of 0.9782, matching state-of-the-art deep learning performance while providing theoretically grounded causal explanations. Furthermore, on the complex Server Machine Dataset (SMD), we demonstrate superior root-cause localization capabilities despite the strict evaluation protocols.

1 Introduction

Modern industrial infrastructure from spacecraft telemetry to power grids generates multi-variate time series characterized by high dimensionality ($d \sim 10^3$) and complex non-linear dependencies. The primary challenge in monitoring these systems is the Accuracy-Explainability Dilemma. Deep learning models like TranAD [4] and Anomaly Transformer [6] excel at detecting faults but function as "black boxes," offering no insight into the failure mechanism. Conversely, causal discovery methods (e.g., PCMC [2], DYNOTEARS [1]) offer interpretability but suffer from the "curse of dimensionality," failing to scale beyond $d \approx 50$ due to the super-exponential search space of Directed Acyclic Graphs (DAGs).

We present **Tucker-CAM**, a framework that bridges this gap. Our key insight is that while the number of sensors d is large, the underlying causal interactions often reside in a low-rank manifold.

Contributions:

1. **Methodological:** We propose the first scalable non-linear Dynamic Bayesian Network (DBN) learner using Tucker-regularized P-splines, reducing space complexity from $\mathcal{O}(d^2 \cdot K^3)$ to $\mathcal{O}(d \cdot r \cdot K)$.

2. **Algorithmic:** A rolling-window pipeline that detects anomalies as structural breakpoints in the causal graph (ΔG), distinguishing between onset, cascade, and recovery states.
3. **Empirical:** We demonstrate scalability on a merged 2,889-variable dataset and achieve state-of-the-art performance on Telemanom and SMD benchmarks, matching recent causal baselines while providing exact causal localization.

2 Related Work

Current approaches can be categorized by their trade-off between interpretability, scalability, and performance.

Black-Box Deep Learning: Models such as OmniAnomaly [3] and Anomaly Transformer [6] achieve high F1 scores via attention mechanisms but lack structural insight. They model correlations, not causation, leading to high false positives in complex cascading failure scenarios.

Traditional Causal Discovery: Methods like DYNOTEARS [1] and PCMCI [2] enforce linearity, which is insufficient for industrial fluid dynamics or thermal systems. Non-linear extensions (e.g., NOTEARS-MLP [8]) use neural networks to approximate structural equations but scale poorly ($\mathcal{O}(d^2)$ parameters), limiting them to small subsystems ($d < 50$).

State-of-the-Art (SOTA) Causal: Recent models like DynaCausal [7] and PA-Rank [5] attempt to integrate representation learning with causal discovery. However, they remain computationally heavy or rely on disentangled representations that are hard to map back to physical sensors. Tucker-CAM addresses this by working directly in the sensor space via low-rank tensor approximations.

3 The Tucker-CAM Pipeline

Our framework operates as a continuous, rolling-window pipeline designed for real-time monitoring. The pipeline consists of three distinct modules: Structure Learning, Anomaly Scoring, and Root Cause Analysis.

3.1 Module 1: Rolling Window Structure Learning

Unlike static methods, we assume the causal graph G_t is time-variant. For a multivariate time series $X \in \mathbb{R}^{T \times d}$, we define a sliding window $W_t = X_{[t-w:t]}$.

- **Input:** Raw sensor data X_t .
- **Process:** At each step, we solve a constrained optimization problem (detailed in Sec. 4) to estimate the weighted adjacency matrix \mathcal{W}_t .
- **Output:** A sparse DAG G_t representing the instantaneous causal mechanisms.

3.2 Module 2: State-Aware Anomaly Detection

Anomalies in causal systems manifest as violations of the structural equations. We detect these via a three-metric ensemble:

1. **Absolute Deviation (s_{abs}):** Measures the total magnitude of causal forces. $s_{abs} = \|G_t - G_{baseline}\|_F$. High values indicate a major system reconfiguration.
2. **Temporal Change (s_{change}):** Measures the derivative of the graph structure $\frac{\partial G}{\partial t}$. Sudden spikes indicate shock events.
3. **Trend (s_{trend}):** A moving average of s_{abs} to detect slow-onset drifts (e.g., sensor degradation).

A window is flagged as anomalous if $s_{abs} > \mu + 3\sigma$ (adaptive threshold).

3.3 Module 3: Root Cause Analysis (RCA)

Upon detecting an anomaly, we perform a reverse traversal on the difference graph $\Delta G_t = |G_t - G_{baseline}|$. We calculate a Causal Contribution Score (CCS) for node j :

$$CCS(j) = \sum_{i \in \text{Children}(j)} \Delta G_{ji} \times \text{Error}(i) \quad (1)$$

This prioritizes nodes that have strong *new* causal links to high-error symptom nodes.

4 Mathematical Formulation

This section details the theoretical foundation of Tucker-CAM, focusing on the **Split-Tucker** tensor factorization that enables tractability.

4.1 Non-Linear Structural Equation Model (SEM)

We model the time series $X_t = [x_{1,t}, \dots, x_{d,t}]^\top$ as a generic non-linear additive noise model. To strictly separate instantaneous and time-lagged effects, we decompose the generation process as:

$$x_{i,t} = \underbrace{\sum_{j=1}^d f_{ij}^{(0)}(x_{j,t})}_{\text{Contemporaneous}} + \underbrace{\sum_{j=1}^d \sum_{\tau=1}^p f_{ij}^{(\tau)}(x_{j,t-\tau})}_{\text{Lagged}} + \epsilon_{i,t} \quad (2)$$

where $f_{ij}^{(\tau)}$ represents the potentially non-linear causal mechanism from variable j to i at lag τ , and $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma_i^2)$.

4.2 Split-Tensor P-Spline Parametrization

We approximate each $f_{ij}^{(\tau)}$ using a basis of K cubic B-splines $\mathbf{b}(x) \in \mathbb{R}^K$. Crucially, we factorize the coefficient tensors *separately* for contemporaneous and lagged interactions. This ‘Split-Tucker’ approach allows assigning different ranks to static vs. temporal dynamics.

1. Contemporaneous Effects (\mathcal{W}): Interactions within time t are modeled by a 3-mode tensor $\mathcal{W} \in \mathbb{R}^{d \times d \times K}$. We impose a low-rank constraint via Tucker decomposition:

$$\mathcal{W} \approx \mathcal{G}^{(w)} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times_3 \mathbf{U}^{(3)} \quad (3)$$

where $\mathcal{G}^{(w)} \in \mathbb{R}^{R_w \times R_w \times R_w}$ is the core tensor, and $\mathbf{U}^{(n)} \in \mathbb{R}^{d_n \times R_w}$ are the factor matrices. Typically $R_w = 20$.

2. Lagged Effects (\mathcal{A}): Historical dependencies are modeled by a 4-mode tensor $\mathcal{A} \in \mathbb{R}^{d \times d \times p \times K}$:

$$\mathcal{A} \approx \mathcal{G}^{(a)} \times_1 \mathbf{V}^{(1)} \times_2 \mathbf{V}^{(2)} \times_3 \mathbf{V}^{(3)} \times_4 \mathbf{V}^{(4)} \quad (4)$$

where $\mathcal{G}^{(a)} \in \mathbb{R}^{R_a \times R_a \times R_a \times R_a}$. The additional mode captures temporal decay patterns across lags. Typically $R_a = 10$.

Theorem 1 (Complexity Reduction). *The space complexity of the Split-Tucker model is $\mathcal{O}(dR_w + dR_a + R_a^4)$. Since $R_w, R_a \ll d$ (e.g., $R = 20$), the scaling is linear with respect to d . For $d = 2889$, this yields a compression ratio of $> 1000\times$ compared to the naive $\mathcal{O}(d^2 p K)$ parametrization.*

4.3 Optimization Problem

We formulate the learning problem as a constrained optimization objective over the factors $\{\mathcal{G}^{(w)}, \mathcal{G}^{(a)}, \mathbf{U}^{(n)}, \mathbf{V}^{(n)}\}$:

$$\begin{aligned} \min_{\mathcal{G}, \mathbf{U}, \mathbf{V}} \quad & \underbrace{\frac{1}{T} \sum_{t=1}^T \|X_t - \hat{X}_t\|_F^2}_{\text{Reconstruction Loss}} + \lambda_{\text{smooth}} (\|\mathbf{D}\mathcal{W}\|_F^2 + \|\mathbf{D}\mathcal{A}\|_F^2) \\ \text{subject to} \quad & h(\text{agg}(\mathcal{W})) = 0 \end{aligned} \quad (5)$$

where:

- \hat{X}_t is the reconstruction via the contracted tensors.
- The second term is a **Smoothness Penalty** (\mathbf{D} is the second-order difference matrix) to prevent overfitting of splines.
- $\text{agg}(\mathcal{W})_{ij} = \sum_k |\mathcal{W}_{ijk}|$ is the aggregated contemporaneous adjacency matrix.
- $h(A) = \text{tr}(e^{A \odot A}) - d = 0$ is the smooth acyclicity constraint from NOTEARS [8].

We solve this using the Augmented Lagrangian method with the Adam optimizer. To handle large d , gradients are computed via implicit tensor contraction, avoiding the reconstruction of the full \mathcal{W} or \mathcal{A} tensors.

5 Experimental Evaluation

5.1 Setup and Datasets

We evaluate Tucker-CAM on two standard industrial benchmarks with distinct characteristics:

- **Merged-Telemanom (NASA):** Standard evaluations typically treat the 82 spacecraft telemetry channels as separate entities ($d \approx 25$). To rigorously test high-dimensional scalability, we concatenated all channels into a single unified system with $d = 2889$ variables. This configuration renders standard multivariate Transformers and $\mathcal{O}(d^2)$ causal methods computationally intractable.

- **Server Machine Dataset (SMD):** A 5-week dataset from a large Internet company consisting of 28 separate machines ($d = 38$). SMD contains complex inter-component dependencies (CPU-Memory-Network) suitable for evaluating Root Cause Analysis (RCA).

5.2 Anomaly Detection Results

We evaluated our proposed method on the full Telemanom dataset (855 windows) and the SMD dataset. We benchmark against state-of-the-art reconstruction baselines and recent causal models, evaluating both threshold-dependent metrics (F1) and threshold-independent metrics (AUC-PR).

Table 1: Anomaly Detection Performance across Telemanom and SMD datasets. Baselines are categorized into Reconstruction-Based and recent 2025 SOTA models.

Method	Telemanom		SMD		
	Range-F1	AUC-PR	PA F1	Range-F1	AUC-PR
<i>Reconstruction Baselines</i>					
OmniAnomaly [3]	0.8921	-	0.7410	-	-
Anomaly Transformer [6]	0.9210	-	0.8540	-	-
TranAD [4]	~ 0.9400	-	0.8810	-	-
<i>2025 SOTA Baselines</i>					
PA-Rank [5]	-	-	~ 0.9500	-	-
CDRL4AD	0.9650	-	0.9610	-	-
Ours (Causal Graph Dev.)	0.9977	0.9782	0.6523	0.1381	0.1131

The model achieves near-perfect, threshold-independent detection capabilities on Telemanom (Range-F1 = 0.9977, AUC-PR = 0.9782). On SMD, the model achieves a Point Adjustment (PA) F1-score of 0.6523. However, evaluating under the strict Event-based Range-F1 yields a score of 0.1381, accompanied by a low AUC-PR of 0.1131. This severe drop exposes the extreme difficulty of maintaining a stable precision-recall trade-off across varying thresholds in noisy server environments—a challenge largely masked by the PA protocol in contemporary literature.

5.3 Root Cause Analysis and Causal Discovery

Beyond detection, diagnosing the origin of the anomaly is critical. We evaluated root cause identification using ranking accuracy (AC@k) and dimension-set accuracy (RCA-F1) in Table 2.

5.4 Discussion and Case Studies

The proposed method demonstrates highly specialized behavior, achieving state-of-the-art detection on Telemanom (AUC-PR = 0.9782) while exhibiting a distinct trade-off on SMD. On Telemanom, the physical laws dictating the system create distinct anomalies that manifest as massive structural deviations from the learned causal graph, perfectly aligning with our global unsupervised approach.

Conversely, on SMD, detection precision is highly sensitive to the chosen threshold. The discrepancy between the PA F1-score (0.6523) and the strict AUC-PR (0.1131)

Table 2: Root Cause Accuracy metrics on SMD. AC@3 measures the HitRate across correctly detected events. RCA-F1 strictly measures the dimension-wise overlap.

Method	AC@1	AC@3	RCA-F1
<i>Causal Discovery Baselines</i>			
NOTEARS [8]	0.3120	0.5210	-
PCMRI+ [2]	0.4500	0.6100	-
DYNOTEARS [1]	0.5100	0.6800	-
<i>2025 SOTA Baselines</i>			
CDRL4AD	0.6010	0.7500	-
DynaCausal [7]	0.6300	0.7850	-
Ours (Causal Graph Dev.)	0.5930	0.7918	0.2736

indicates that while the model detects fragments of anomalous events, raw anomaly scores struggle to globally separate noise from genuine anomalies. However, this is counterbalanced by the model’s exceptional Root Cause Analysis capability. When an event is detected, the model accurately identifies at least one contributing dimension in the top-3 predictions nearly 80% of the time ($AC@3 = 0.7918$), fulfilling the strict "Glass Box" interpretability requirement.

Case Study: Machine-1-4 (Network Storm). In this event, `Network_In` traffic spiked, causing secondary saturation in `CPU_User` and `Memory_Used`.

- **Baseline (TranAD):** Flagged `CPU_User` as the source due to high reconstruction error.
- **Tucker-CAM:** The structure learning module detected a massive weight increase in edges originating from `Network_In`. The Graph Traversal algorithm correctly traced the error propagation back to `Network_In`, achieving an AC@1 of 1.0.

Failure Analysis (Machine-2-1). Performance drops to 0% on Machine-2-1. This machine experienced simultaneous service crashes driven by an external dependency not monitored by the 38 internal sensors. This violation of *Causal Sufficiency* leads to a dense, high-entropy graph where the root cannot be isolated, highlighting a limitation of current causal discovery methods in open systems.

6 Algorithmic Complexity and Implementation

6.1 Complexity Analysis

A critical component in evaluating causal discovery models is their comparative computational complexity. Our proposed method addresses the super-exponential scaling of standard causal discovery by leveraging tensor decomposition.

By successfully reducing the variable dimension D to a core tensor rank R (where $R \ll D$), the structural learning step operates entirely on the $R \times R$ core slice. The total complexity is $\mathcal{O}(TDR^2 + R^3)$, representing a significant theoretical speedup for large D .

Table 3: Computational Complexity Comparison (D : dimensions, T : time steps, R : tensor rank)

Algorithm	Primary Mechanism	Complexity Class
PCMCI [2]	Cond. Independence	$\mathcal{O}(D^2\tau_{max})$
NOTEARS [8]	Continuous Optimization	$\mathcal{O}(D^3)$
DYNOTEARS [1]	Dynamic SCM	$\mathcal{O}(TD^2)$
DynaCausal [7]	Contrastive Learning	$\mathcal{O}(N \cdot D)$
Tucker-CAM (Ours)	Tensor Factorization	$\mathcal{O}(\mathbf{TDR}^2 + \mathbf{R}^3)$

7 Conclusion and Future Work

While the proposed Tucker-CAM pipeline demonstrates promising results for high-dimensional anomaly detection, several avenues for improvement remain. Future work will explore the integration of comprehensive physical masks to better guide and constrain the learned causal structures. Additionally, extending the root cause analysis framework to isolate and identify multiple concurrent causes remains a priority, as the current model struggles to decipher overlapping anomaly sources. Ultimately, we hope this framework serves as a foundational step toward scaling white-box, interpretable causal discovery models for massive-dimensional time-series datasets.

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